Adaptive Multiparty NIKE

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TCC 2022

Non-interactive Key Exchange (NIKE)

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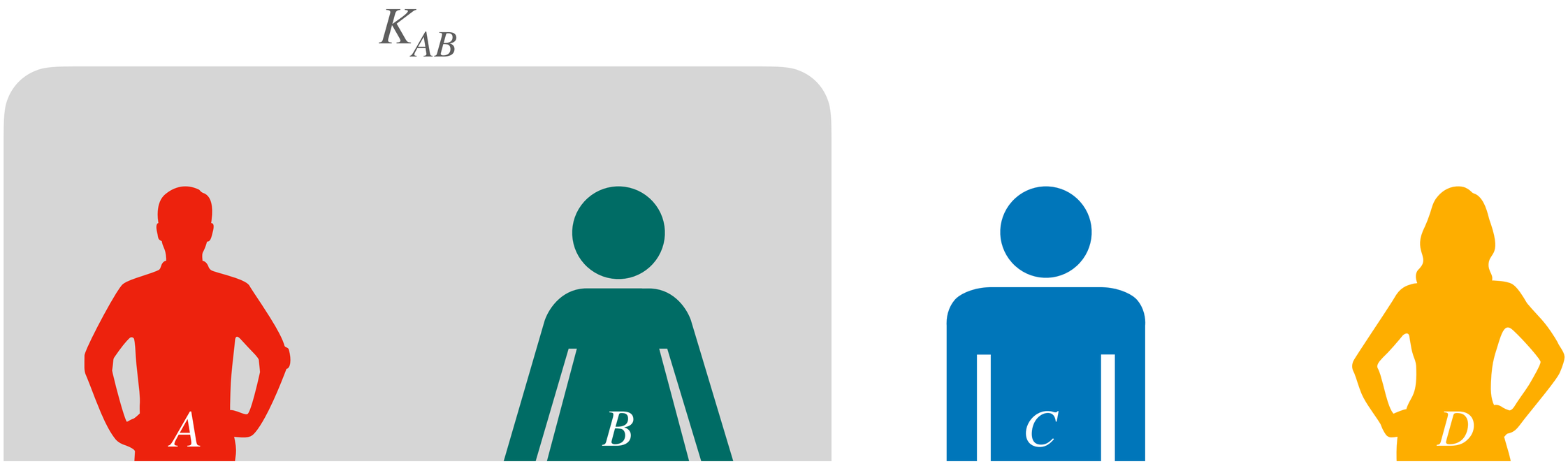
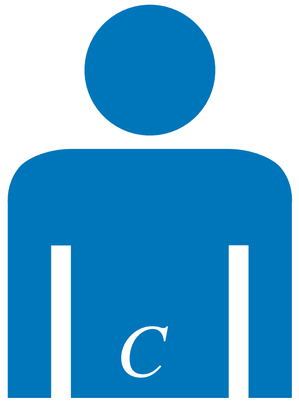


Non-interactive Key Exchange (NIKE)

Public Bulletin Board

pk_A pk_B pk_C pk_D



sk_A sk_B sk_C sk_D

Non-interactive Key Exchange (NIKE)

$\mathsf{KeyGen}(\mathsf{sk}_A, \mathsf{pk}_B) \rightarrow K_{AB}$

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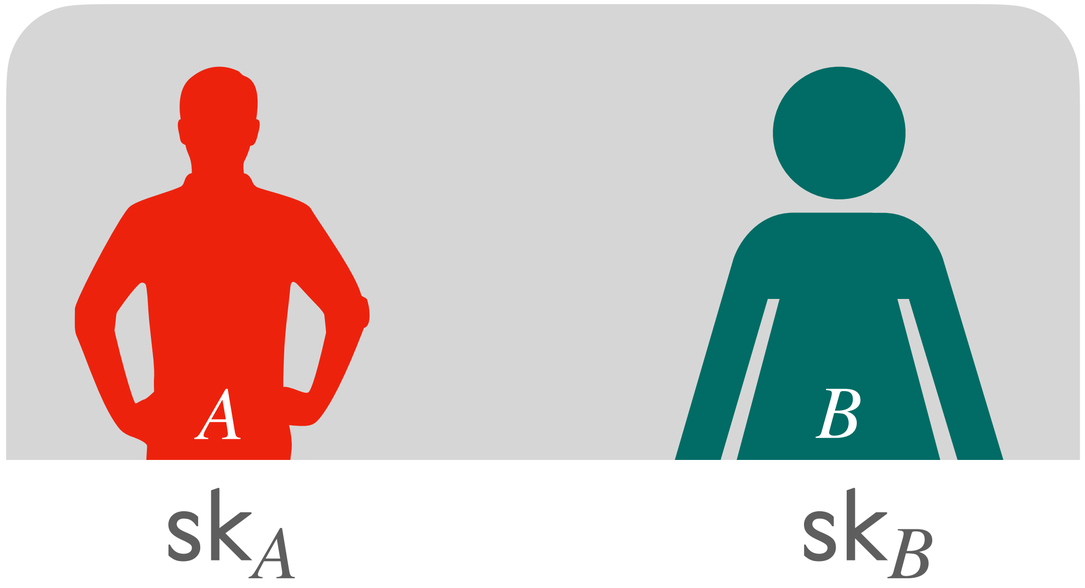

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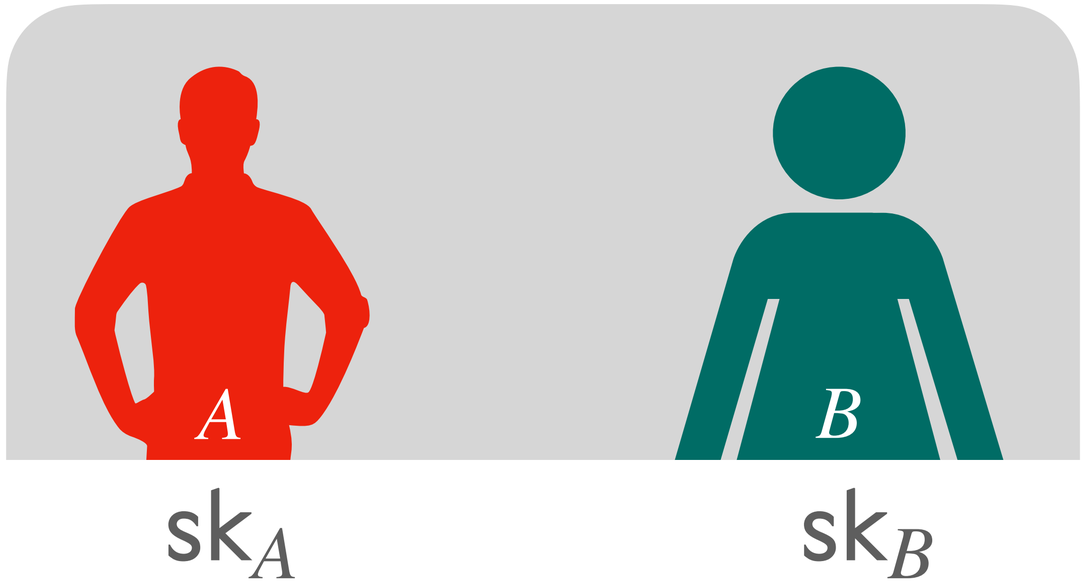
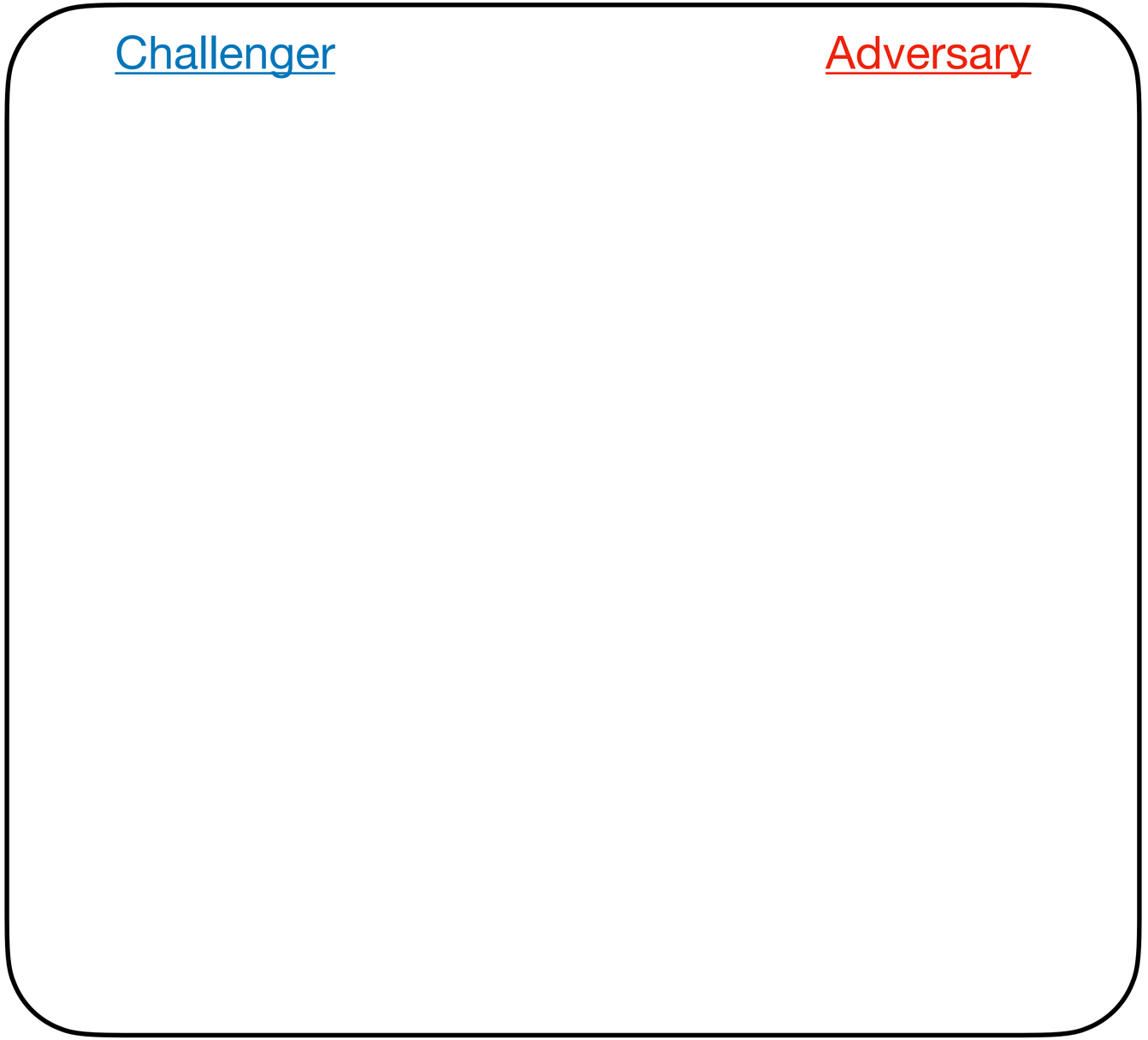


Security:

Adversary cannot guess the secret key for set S

even after seeing many secret keys for users outside S

Adaptive Security for ‘c’ corruptions



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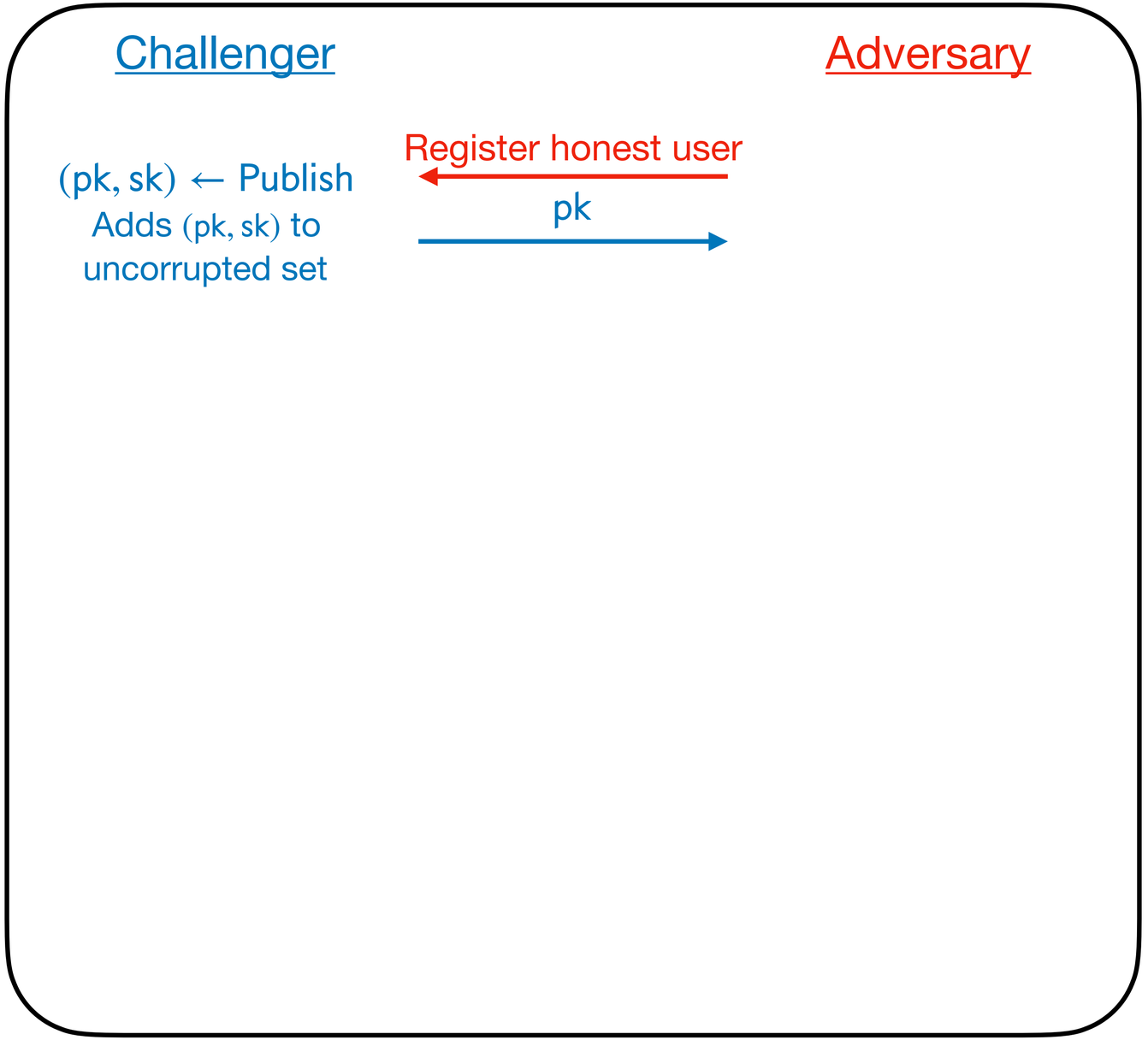

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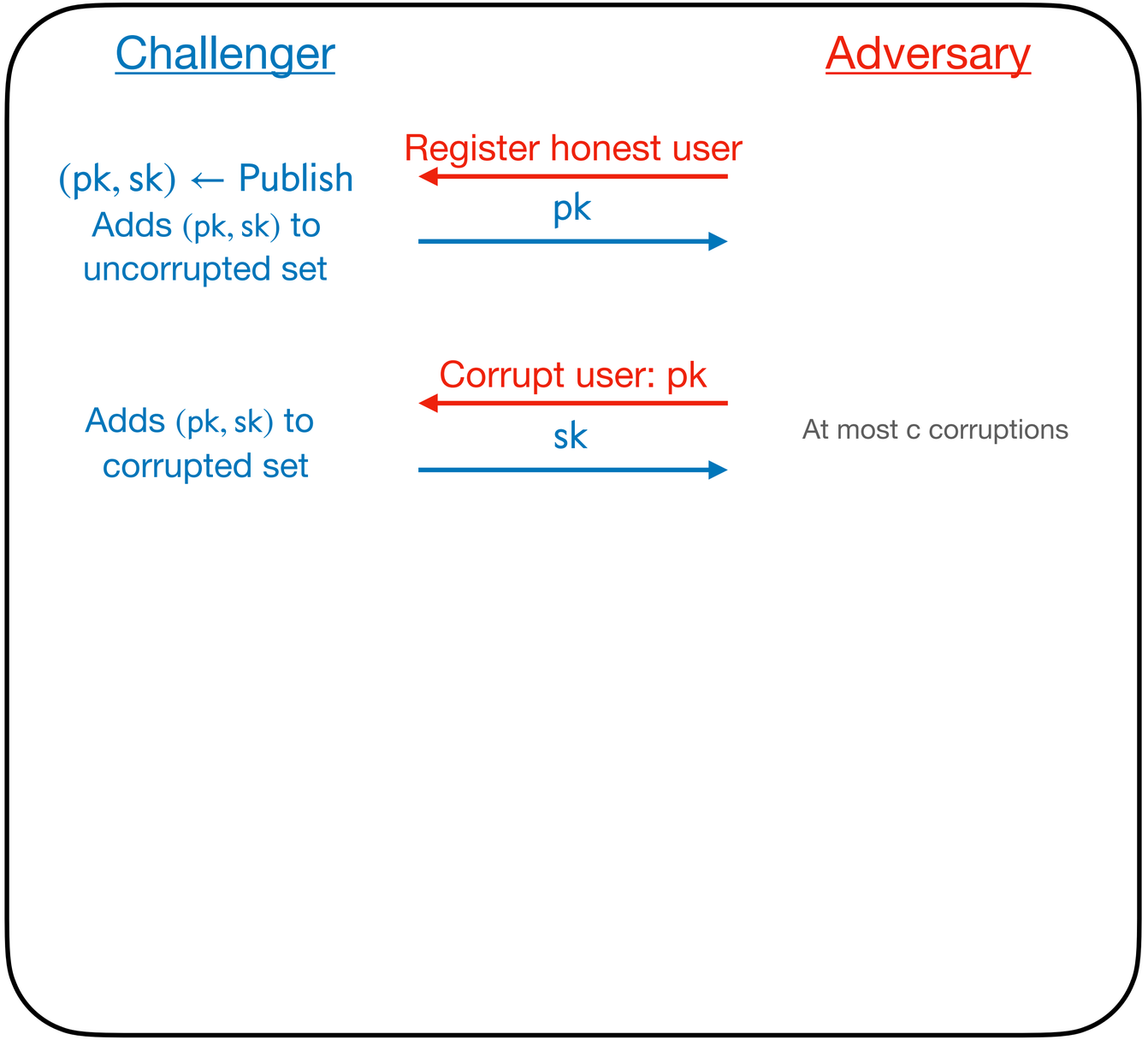

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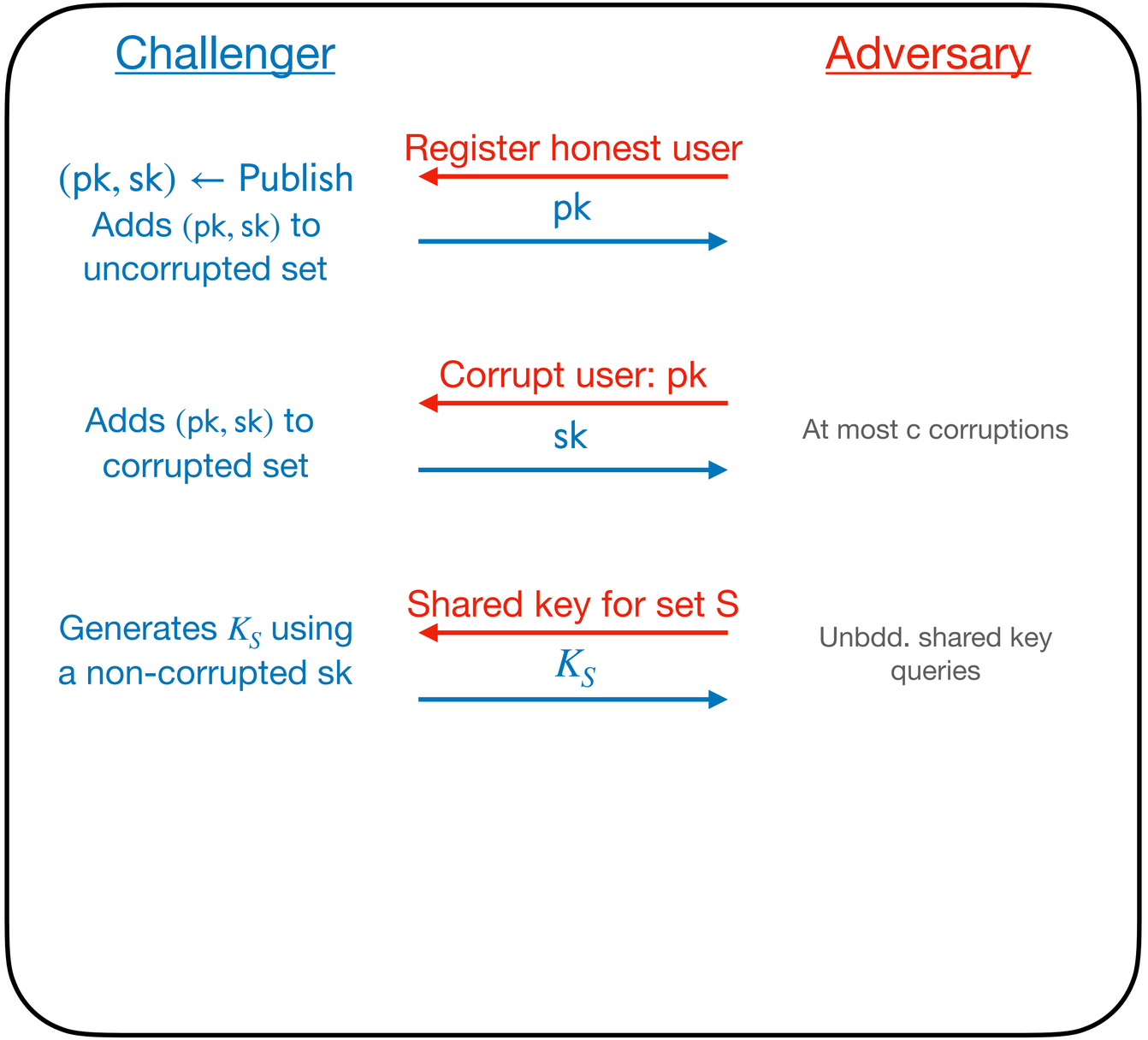

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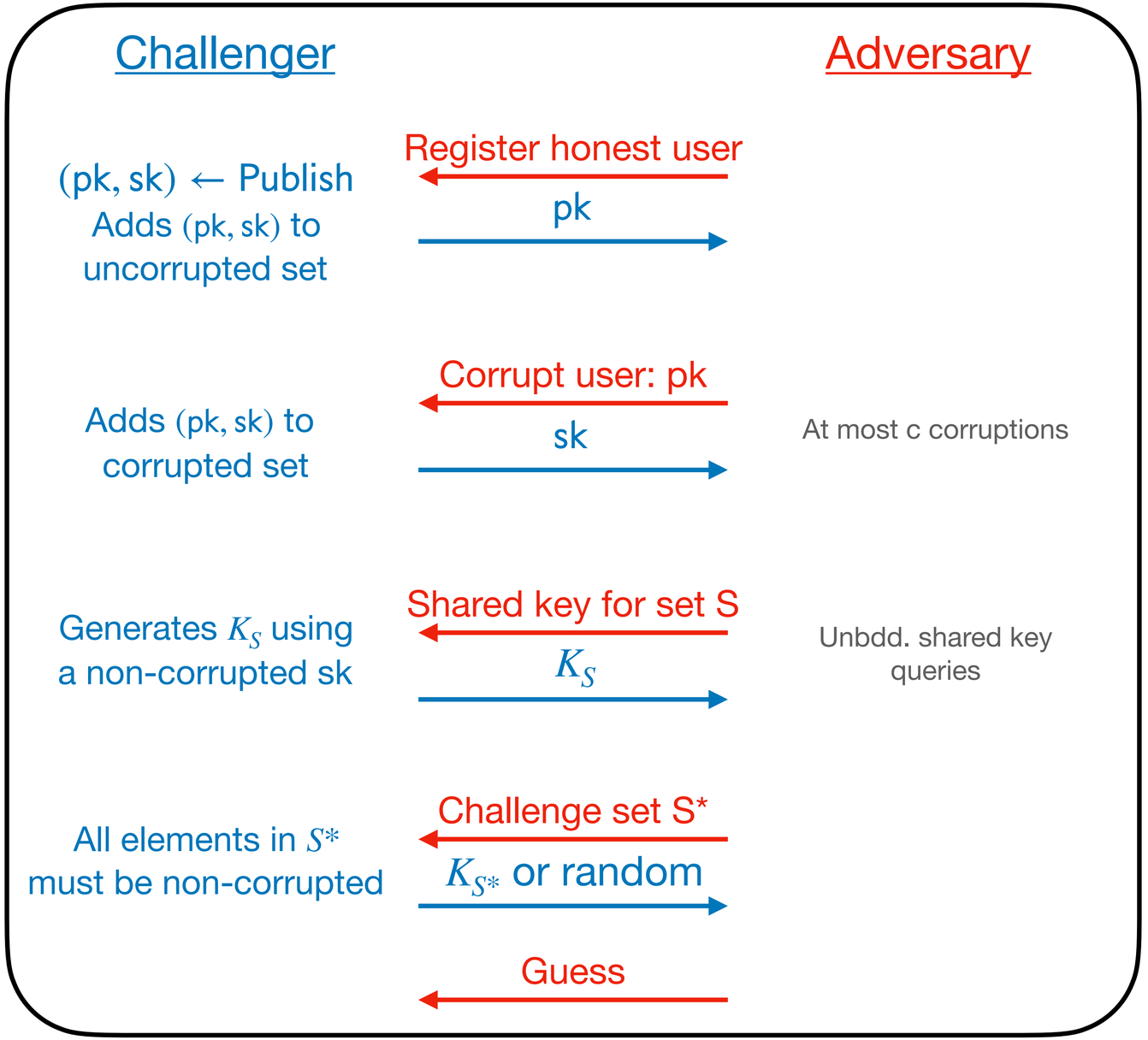

Why Adaptive Security

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A natural and important goal in crypto

- If shared key for one group is leaked, it should not compromise security of other groups
- Adversary may corrupt some users, and then decide which group's security is compromised

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Multiparty key exchange:

- cannot guess the challenge group (N^ℓ factor loss, ℓ can be polynomial)

Prior Works

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2-party :

- Diffie-Hellman (1976)

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3-party :

- Joux (2000) using pairings

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n-party :

- Garg-Gentry-Halevi (2013) using multilinear maps
  - Constructions for super-constant n are broken
- Boneh-Zhandry (2014) using iO
  - Achieves selective security
- Boneh-Zhandry (2014), Rao (2014)
  - Achieve adaptive security using interactive assumptions

Rao 2014 Impossibility

Barriers for achieving adaptive security from non-interactive assumptions

Public keys fix the shared key for every set S

`Admissible reductions' not possible

- Impossibility holds even if the number of corruptions is bounded

All prior constructions are admissible

Our Results and Contributions

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Construction of constrained PRFs for special class, using iO + DDH

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Direct construction of multiparty NIKE with unbounded honest users, bounded corruptions, using iO + standard assumptions on groups

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This talk

A close connection between adaptive NIKE and adaptive constrained PRFs :

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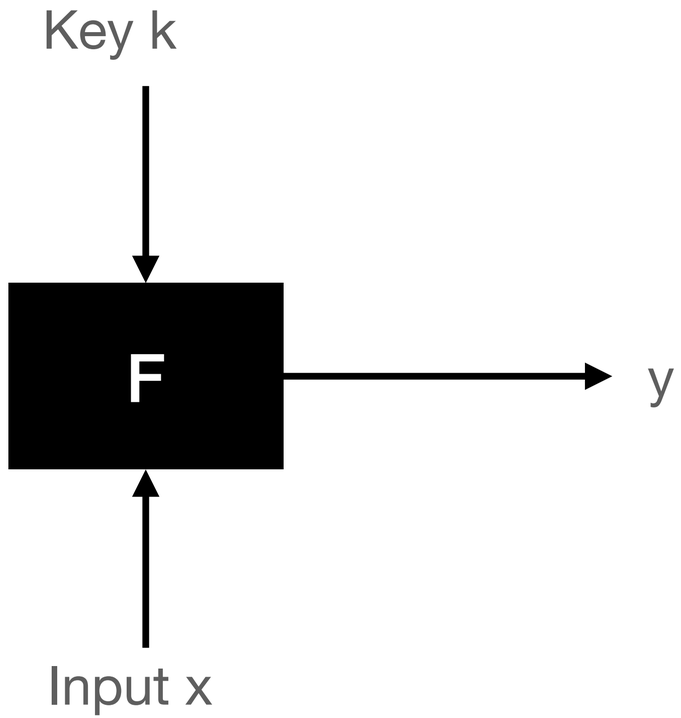
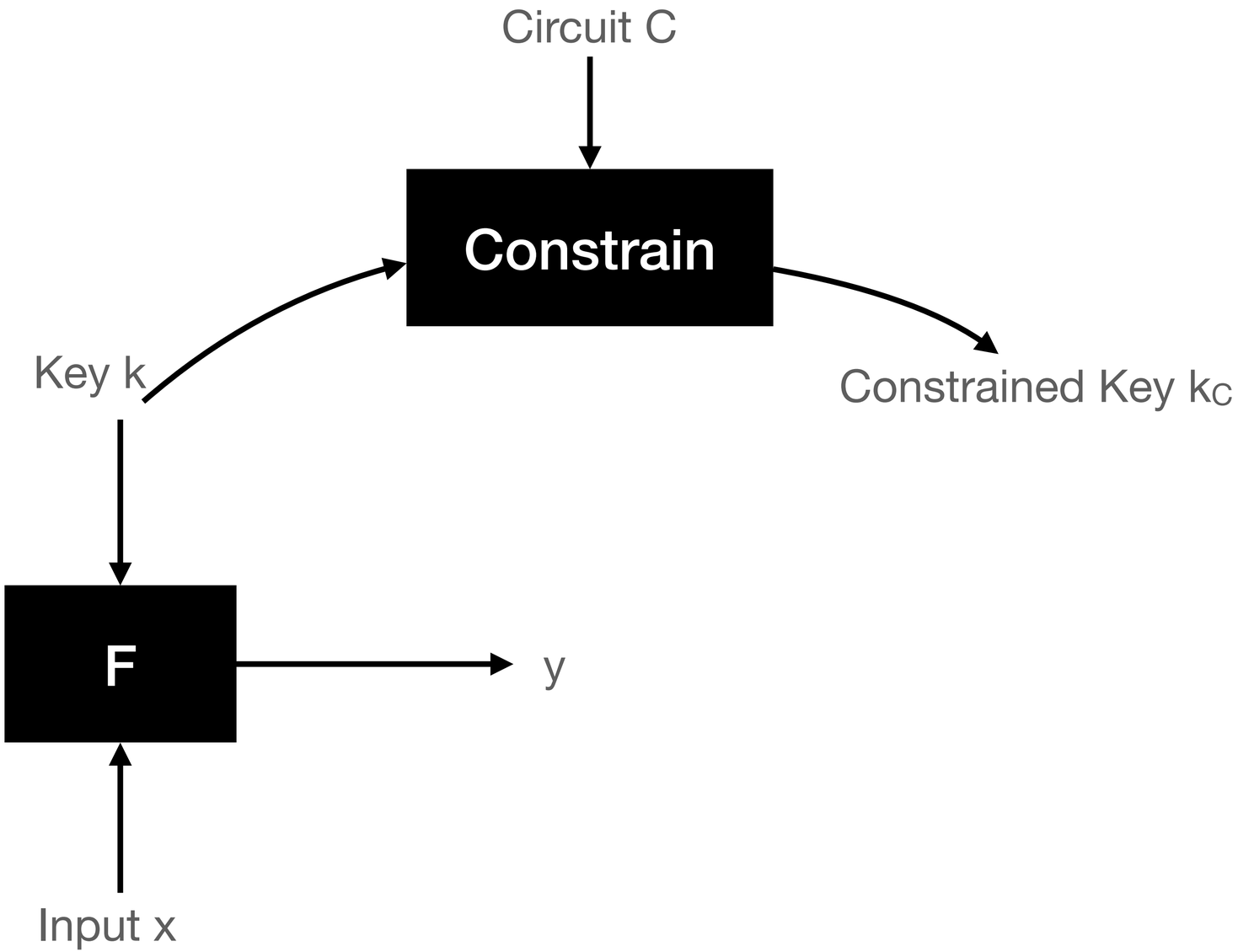
Direct construction of multiparty NIKE with unbounded honest users, bounded corruptions, using iO + standard assumptions on groups

Constrained PRFs



Constrained PRFs



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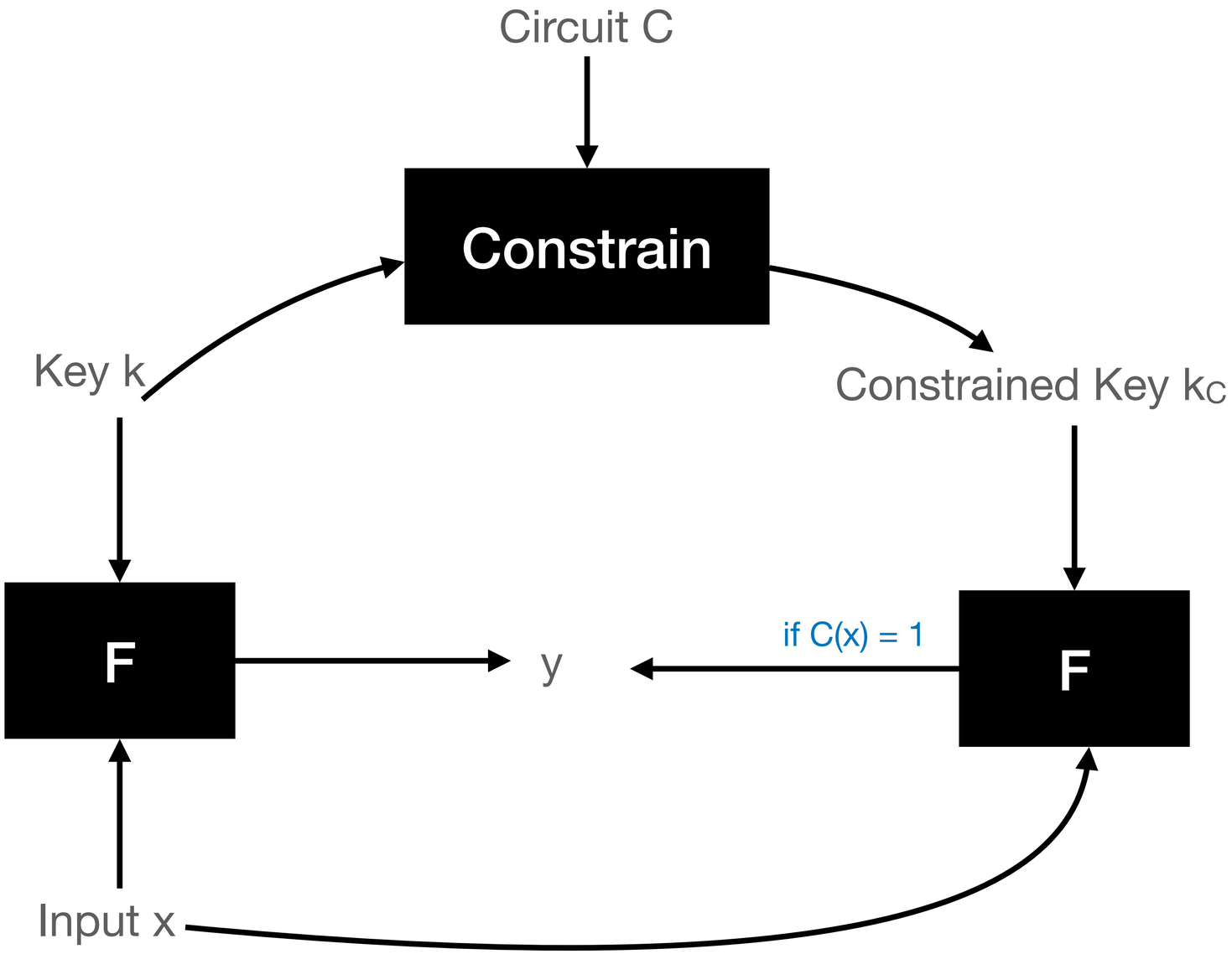

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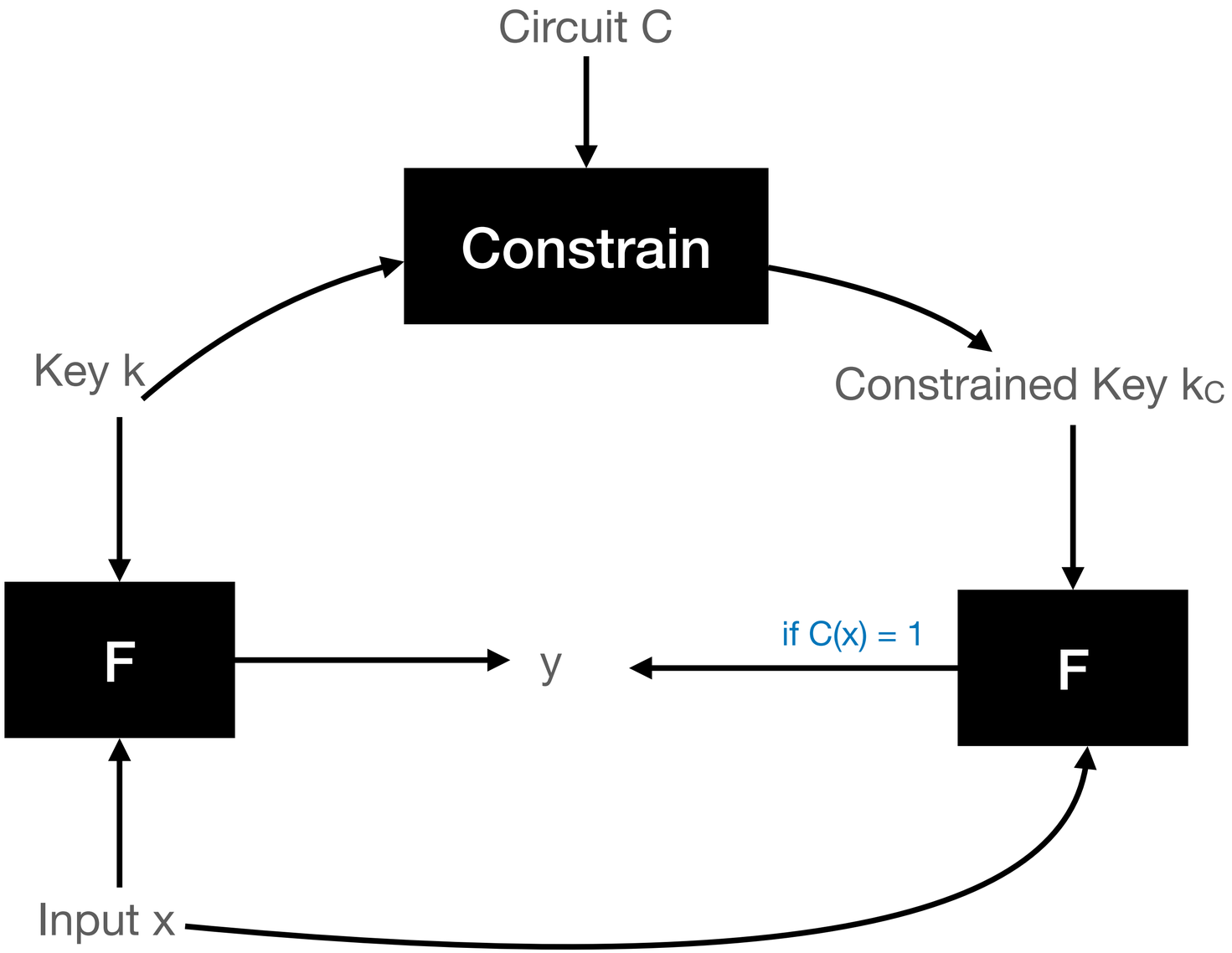


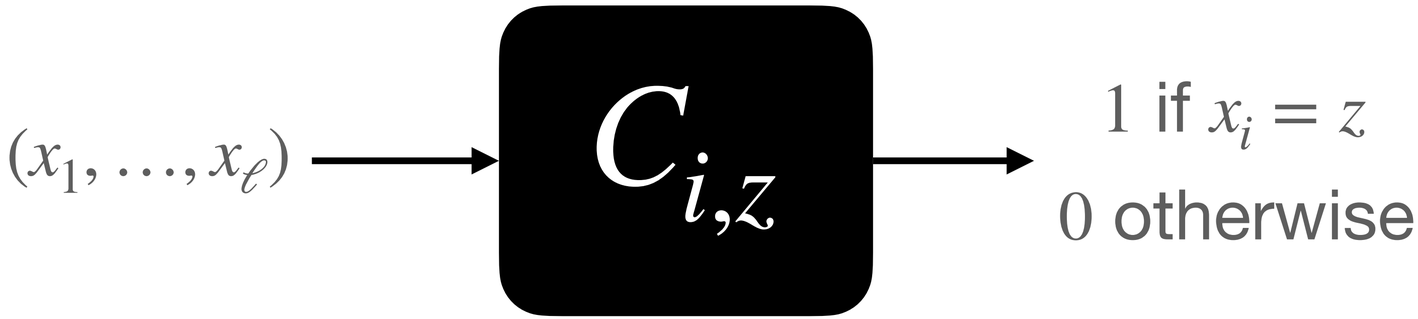
Security

If C(x) = 0, then F(k, x) should look random even if adv. has k_C

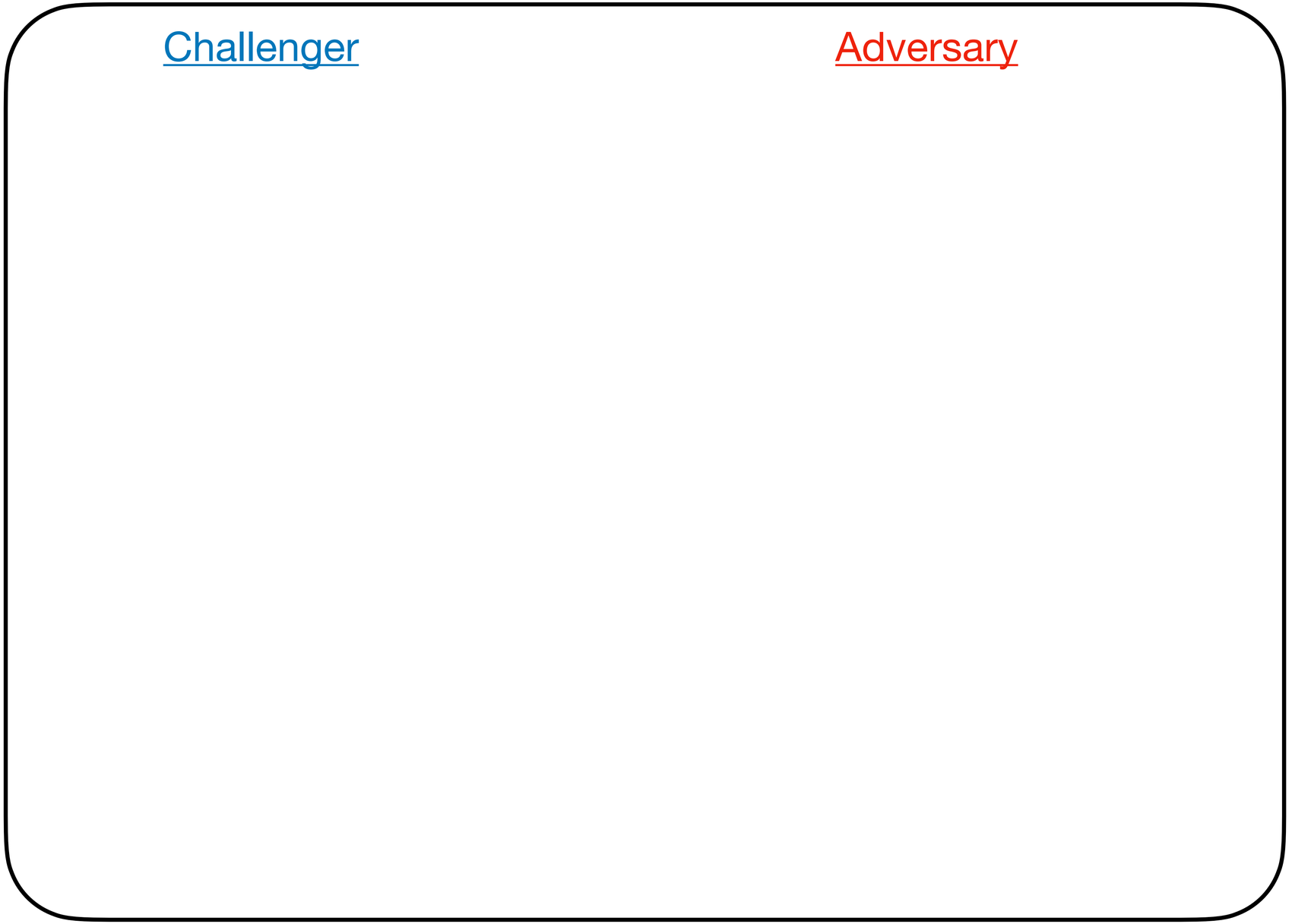
Our Family of Constraints: 1-Symbol Fixing (1-SF)

Defined by alphabet set Σ and length ℓ
$|\Sigma|, \ell$: poly(sec. par.)

Constraints parameterized by $i \in [\ell]$ and $z \in \Sigma$



Adaptive Security for 1-SF-PRF



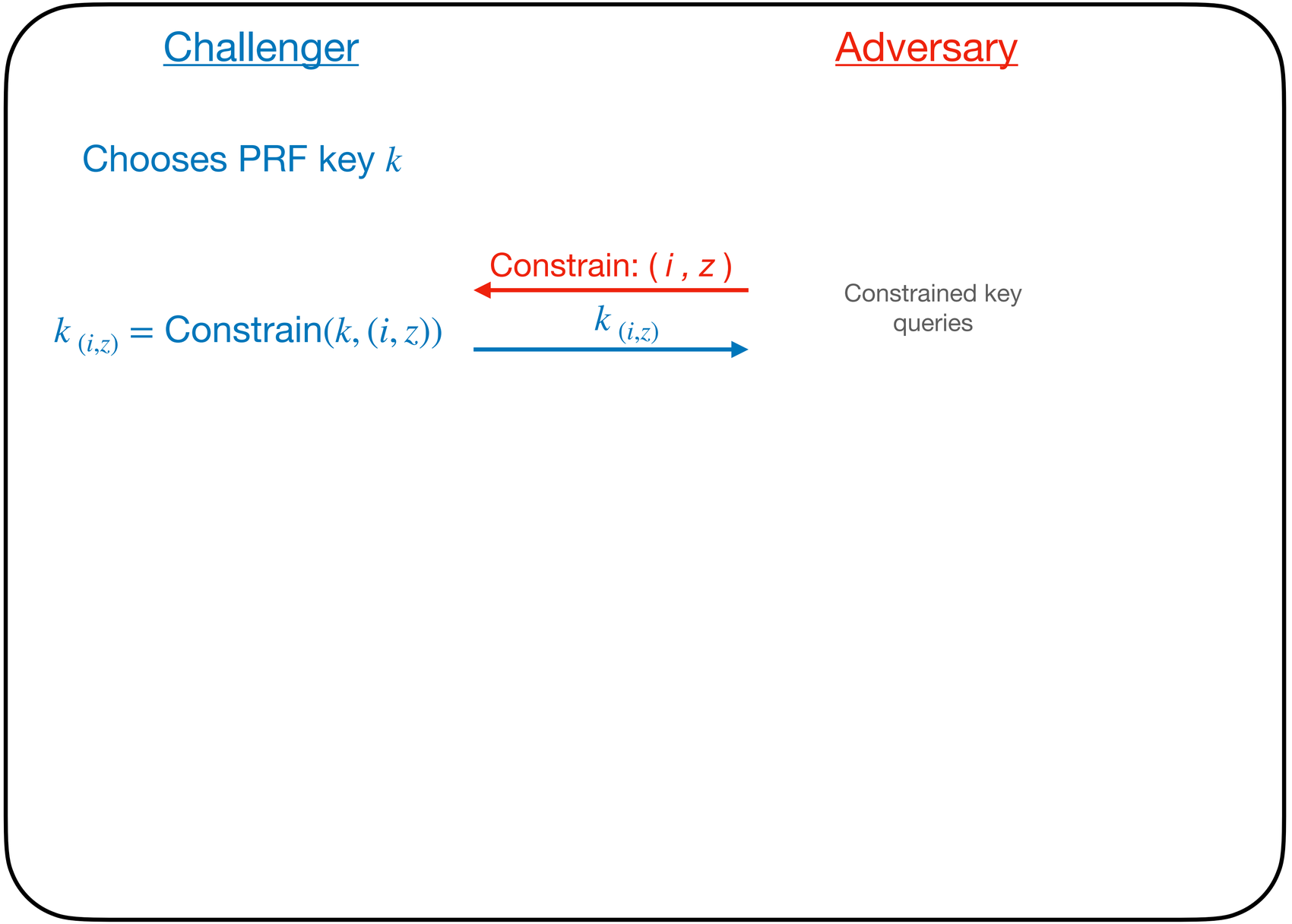
Adaptive Security for 1-SF-PRF

Challenger

Adversary

Chooses PRF key k

Adaptive Security for 1-SF-PRF



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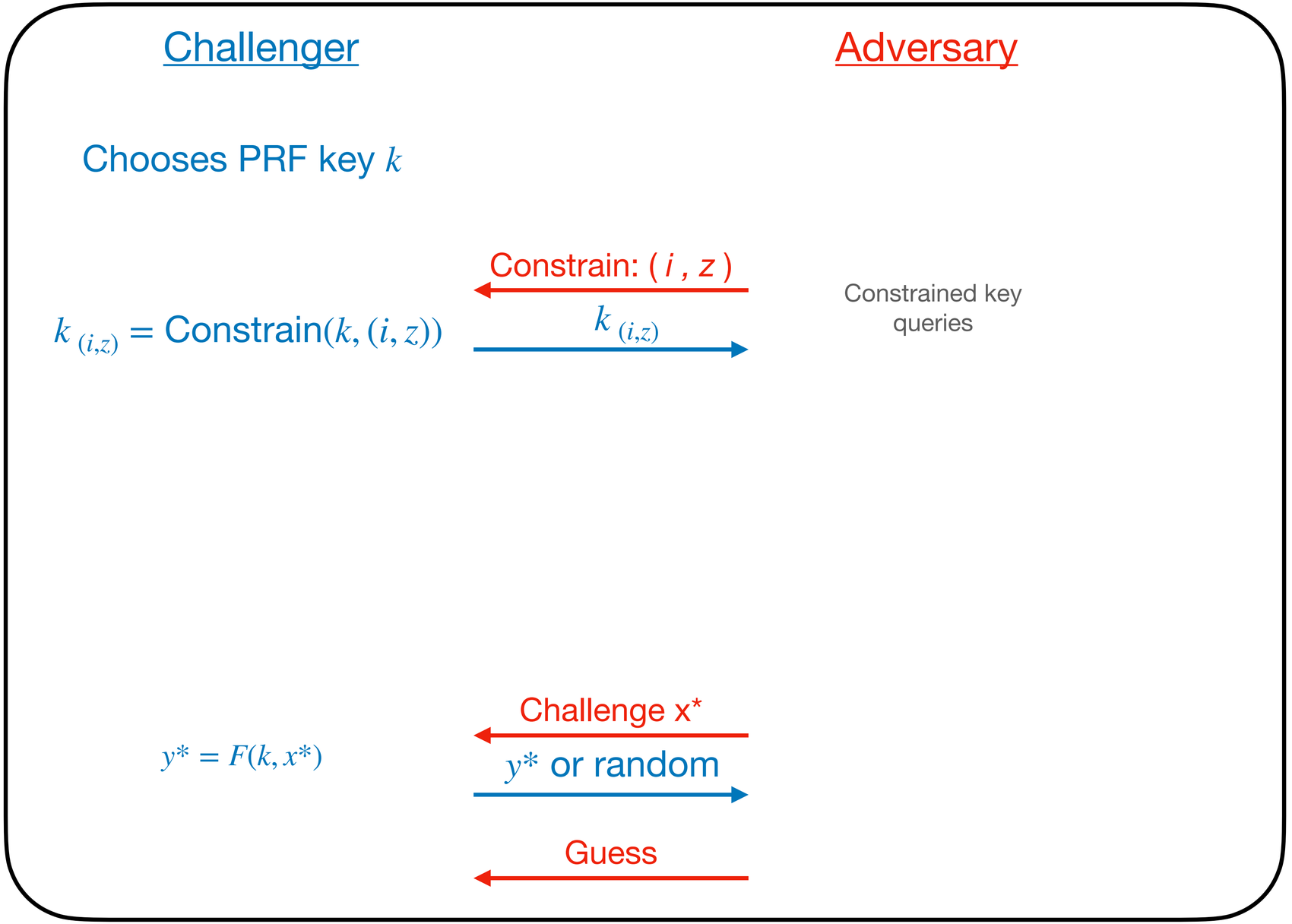

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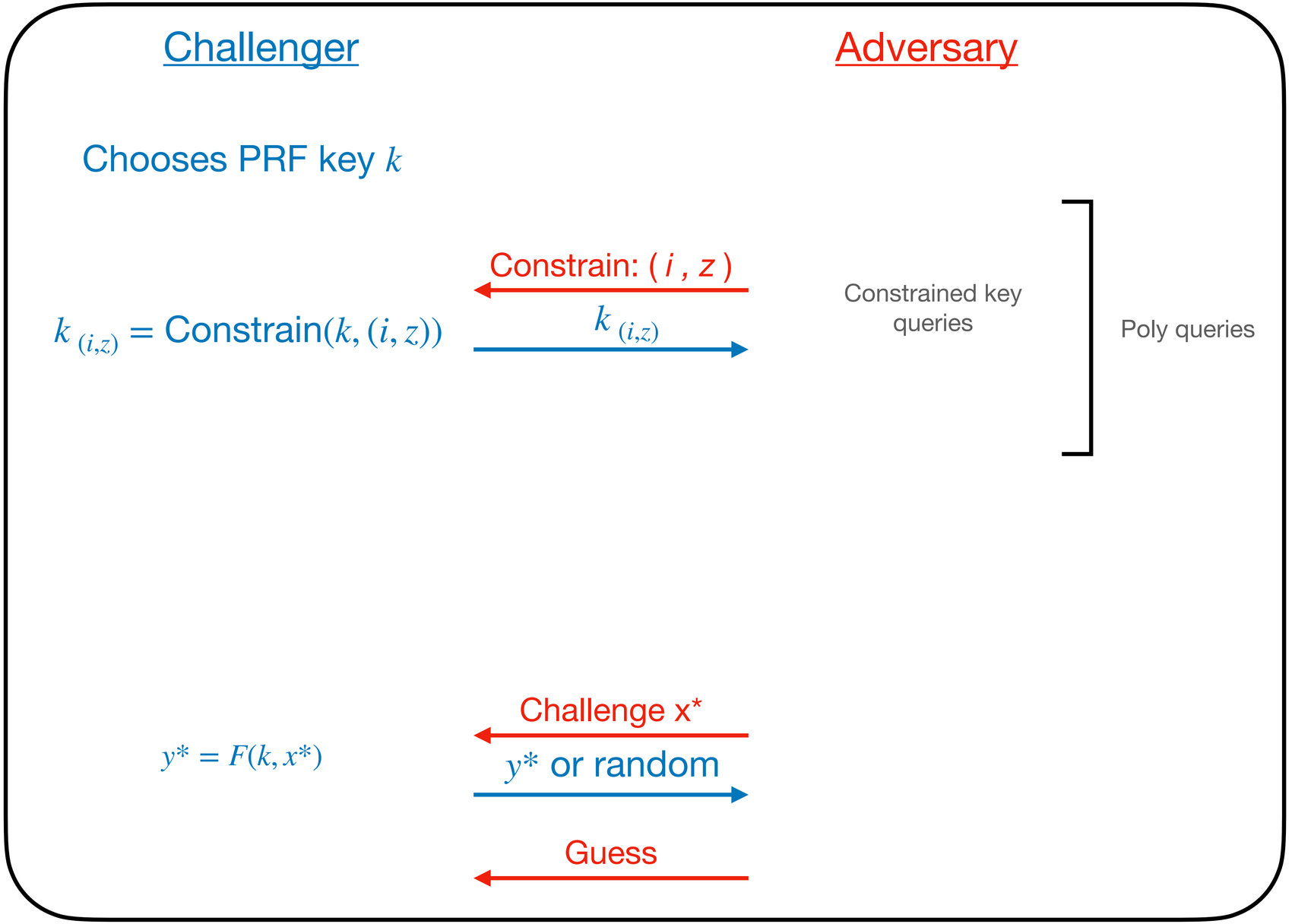


Constructing Adaptively Secure 1-SF-PRF

Our approach: iO + algebraic PRFs

Naor-Reingold PRF

Our Construction

PRF key k : exponents $\{e_{i,w}$ for each $i \in [\ell], w \in \Sigma\}$, group element $h \in \mathbb{G}$

$$F(k, (x_1, \ldots, x_\ell)) = h^{\prod_i e_{i,x_i}}$$

Our Construction

Constrained key for $(\ i^*\ ,\ z\ )$: $iO\big(\ \text{Prog-}(i^*, z)\ \big)$

$(x_1, x_2, \ldots, x_\ell)$ →

Prog-$(\ i^*, z\ )$

Constants: exponents $e_{i,w}$ for all $i \neq i^*, w \in \Sigma$

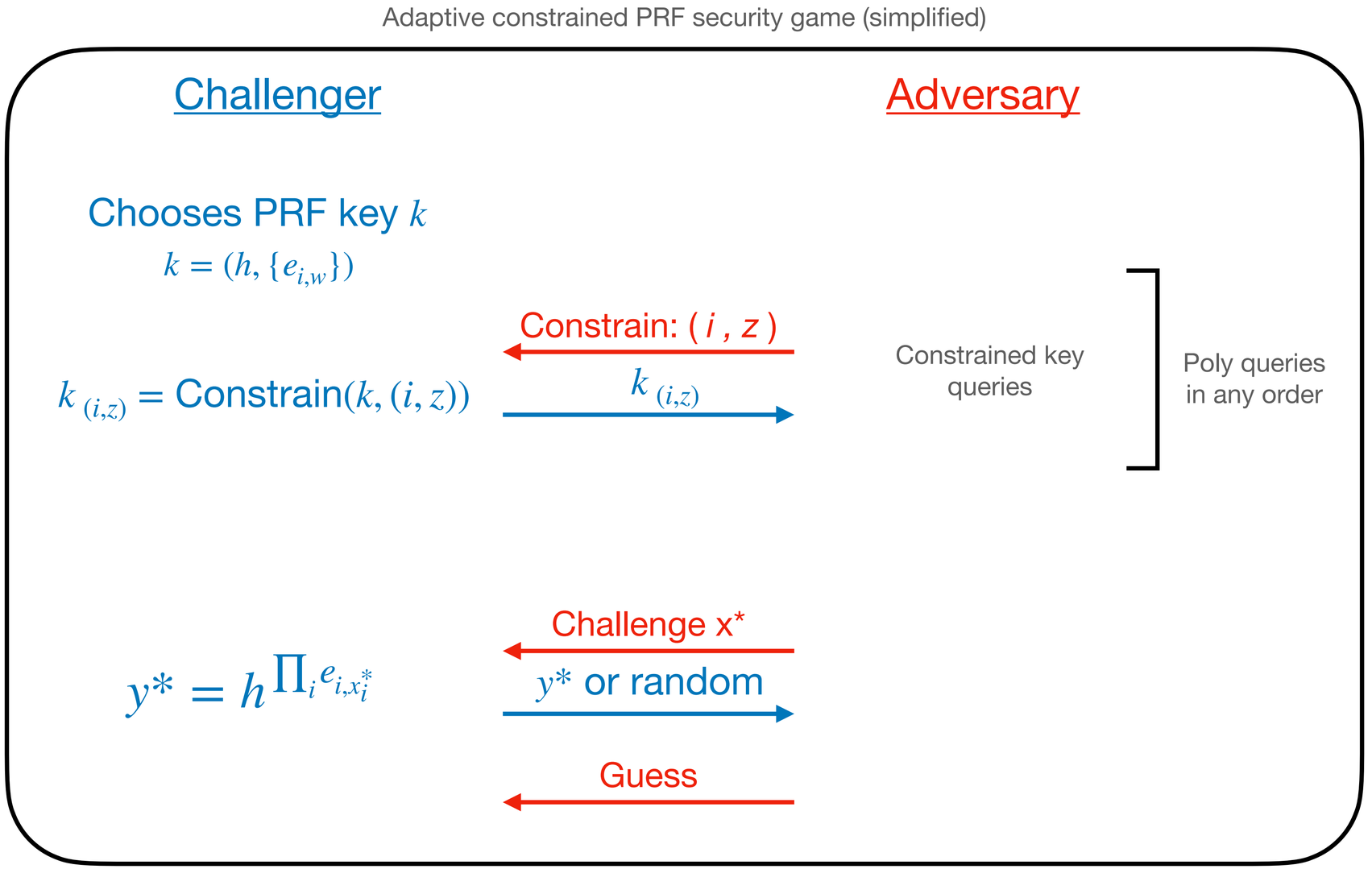
exponent $e_{\ i^*,z}$

group element $h \in \mathbb{G}$

If $x_{i^*} \neq z$, output $\perp$.

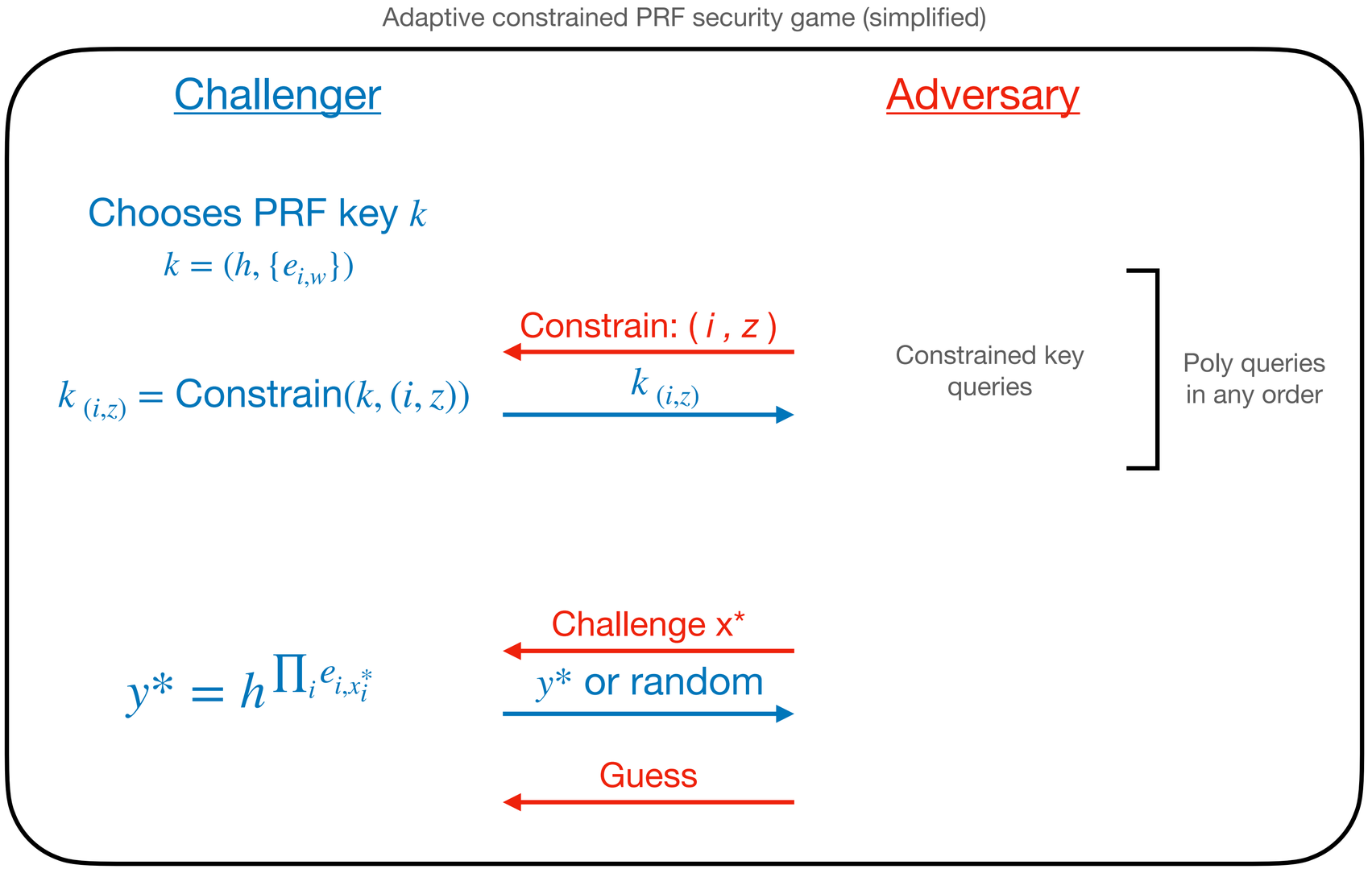
Else compute $v = \prod e_{\ i,x_i}$, output h^v .

Proof Idea:


Adaptive constrained PRF security game (simplified)
Challenger
Adversary
Chooses PRF key k
$k = (h, \{e_{i,w}\})$
$k_{(i,z)} = \text{Constrain}(k, (i, z))$
Constrain: (i , z)
$k_{(i,z)}$
Constrained key queries
Poly queries in any order
$y^* = h^{\prod_i e_{i,x_i^*}}$
Challenge x*
y^* or random
Guess

Proof Idea:

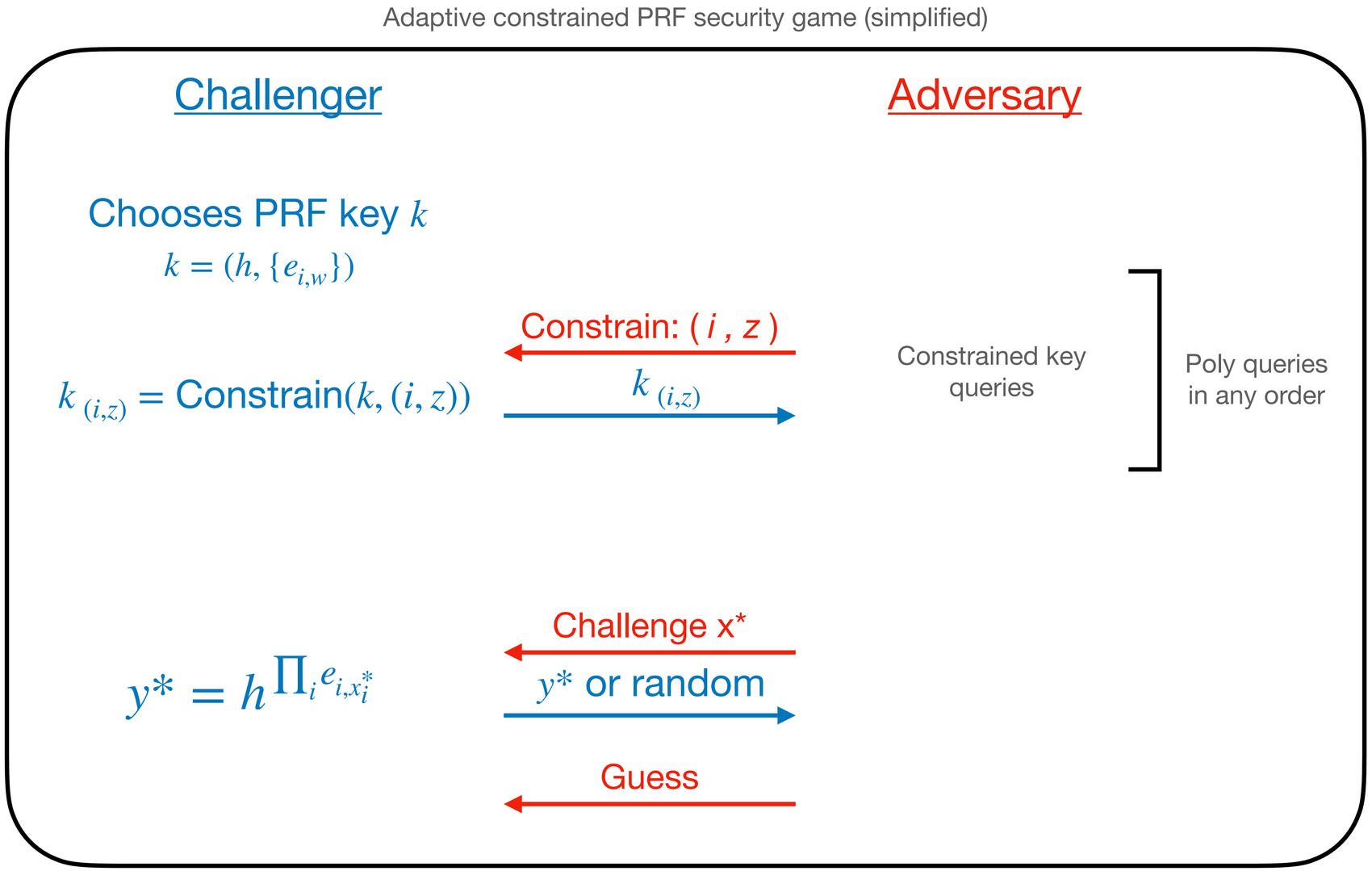
Group element h is contained in the constrained keys.



Proof Idea:

Group element h is contained in the constrained keys.

Can we remove h from the constrained keys?



Idea 1: A different h_i for each constrained key

i^{th} Constrained key for $(\ i^*\ ,\ z\ )$: $iO\big(\ \text{Prog}_i - (i^*, z)\ \big)$

$$\text{Prog}_i - (\ i^*, z\ )$$

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i^{th} Constrained key for $(\ i^*\ ,\ z\ )$: $iO\big(\ \text{Prog}_i - (i^*, z)\ \big)$

The constrained keys need to be consistent !

$$\text{Prog}_i - (\ i^*, z\ )$$

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Idea 1: A different h_i for each constrained key

$x = (z, z, \ldots)$

1^{st} key for constraint $(1\ ,\ z)$

2^{nd} key for constraint $(2\ ,\ z)$

$\text{Prog}_1\text{-}(1\ ,\ z)$

Constants: exponents $e_{i,w}$ for all $i \neq 1, w \in \Sigma$

exponent $e_{1,z}$ $\quad$ $h_1 \in \mathbb{G}$

If $x_2 \neq z$, output $\perp$.

Else compute $v = \prod e_{i,x_i}$, output h_1^v.

$\text{Prog}_1\text{-}(2\ ,\ z)$

Constants: exponents $e_{i,w}$ for all $i \neq 2, w \in \Sigma$

exponent $e_{2,z}$ $\quad$ $h_2 \in \mathbb{G}$

If $x_2 \neq z$, output $\perp$.

Else compute $v = \prod e_{i,x_i}$, output h_2^v.

h_1^v $\quad$ Inconsistent outputs ! $\quad$ h_2^v

Idea 2: In i^{th} key, hardwire all previous h_j and their constraints

Modify program logic to maintain consistency across keys

2^{nd} key : constraint (i^*, z)

Prog_2-(i^*, z)

Constants: exponents $e_{i,w}$ for all $i \neq i^*, w \in \Sigma$

exponent $e_{i^*,z}$ $\quad$ $h_2 \in \mathbb{G}$

$h_1 \in \mathbb{G}$ $\quad$ constraint (i_1, z_1) for 1st query

If $x_2 \neq z$, output $\perp$.

If $x_{i_1} = z_1$, compute $v = \prod e_{i,x_i}$, output h_1^v .

Else compute $v = \prod e_{i,x_i}$, output h_2^v .

second constrained key is consistent with the first key

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Using iO + DDH, we can switch the constrained keys one by one to not use h (but the challenge PRF evaluation still uses h)

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Bypasses Rao’s impossibility result

Open Questions:

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Remove the bound restriction on group size?

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Remove bound on corruptions?

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Thank you!