Beyond Uber:
Instantiating Generic Groups via PGGs

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Background and Motivation

Two step process for building cryptographic schemes

- Design + proof in idealized model
- Heuristic instantiation

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This paradigm is not sound [CGH98; MRH04; Den02]

Find standard-model assumptions which

- Can be satisfied by building blocks
- Yield interesting applications

[CGH98]: Canetti, Goldreich, Halevi. The Random Oracle Methodology, Revisited. *STOC 1998*.

[MRH04]: Maurer, Renner, Holenstein. Indifferentiability, Impossibility Results on Reductions, and Applications to the Random Oracle Methodology. *TCC 2004*.

[Den02]: Dent. Adapting the Weaknesses of the Random Oracle Model to the Generic Group Model. *ASIACRYPT 2002*.

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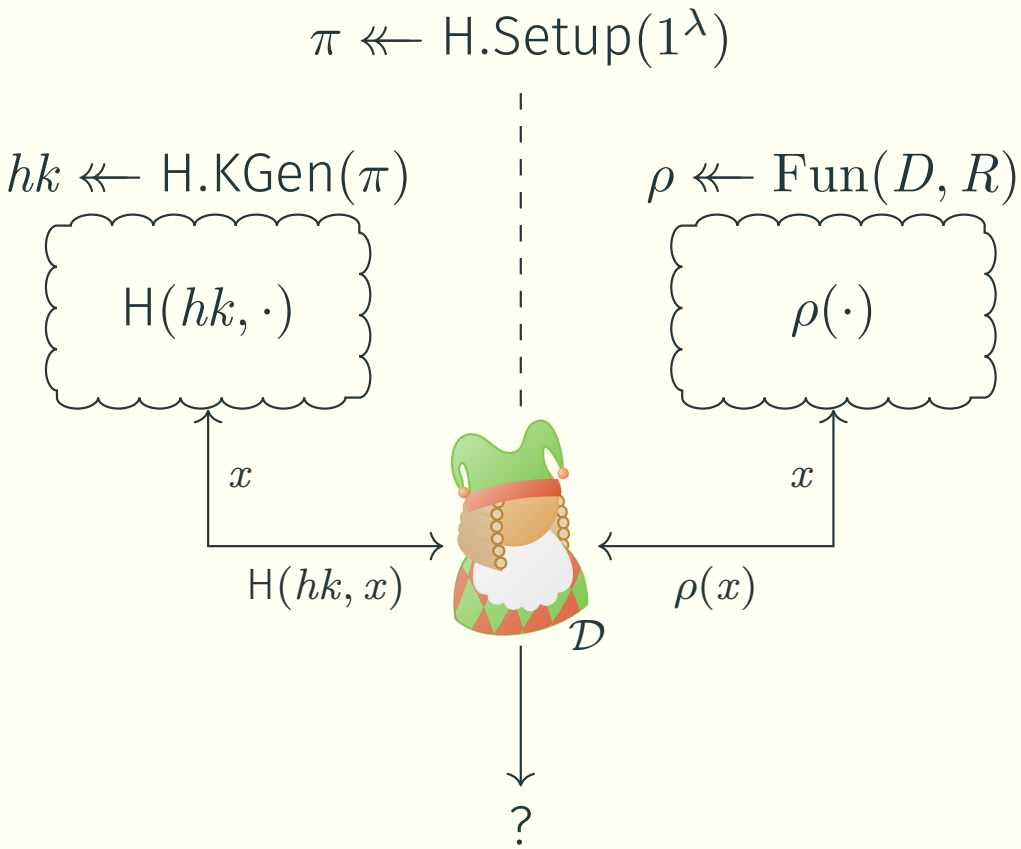
UCE Security for Hash Functions [BHK13]



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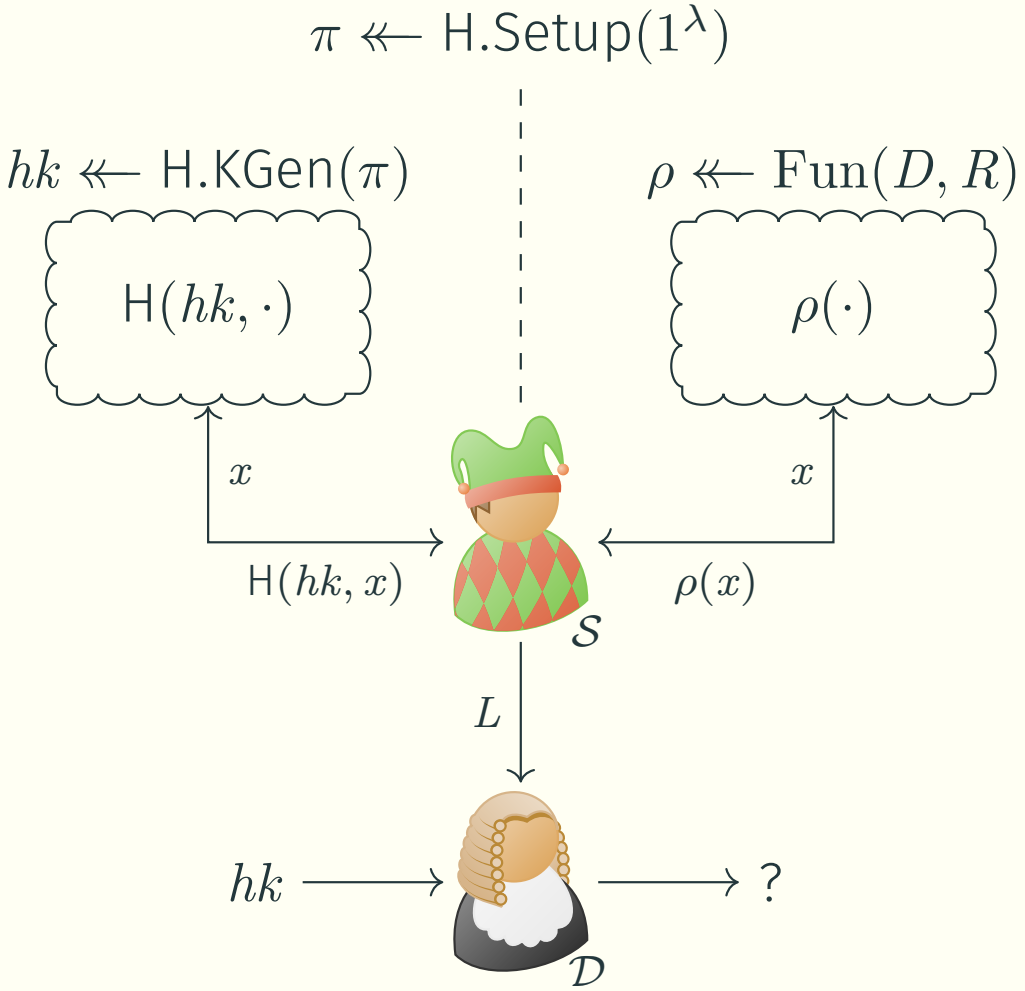

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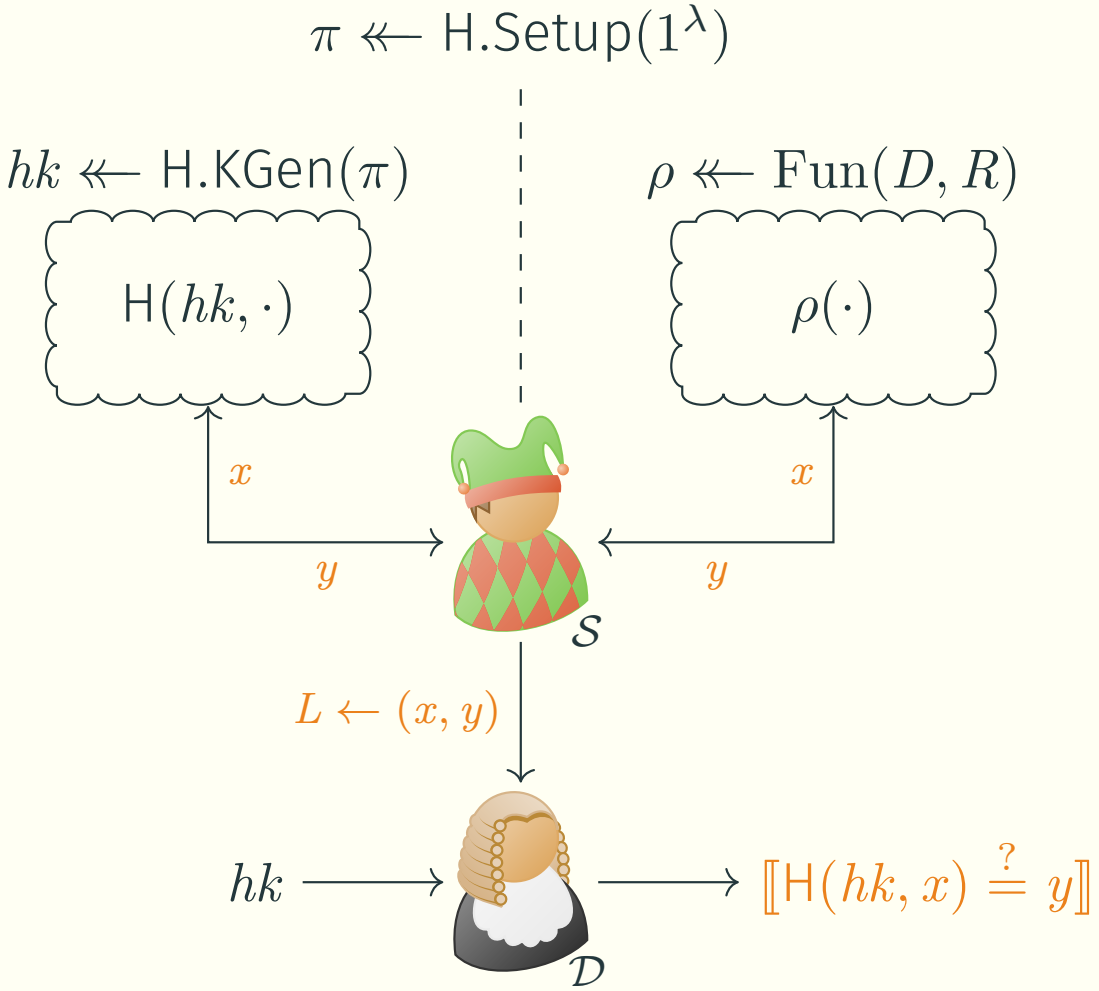

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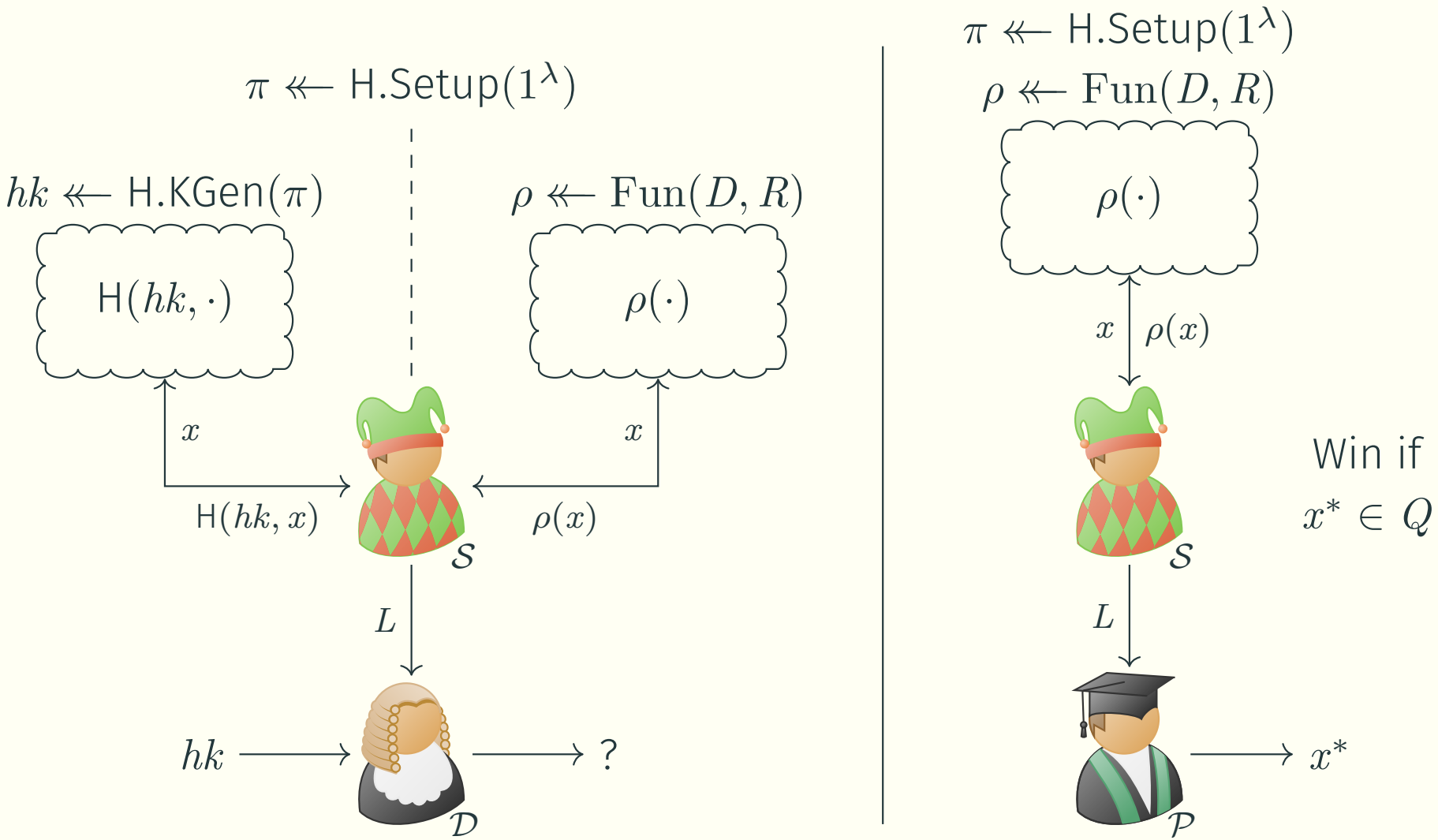

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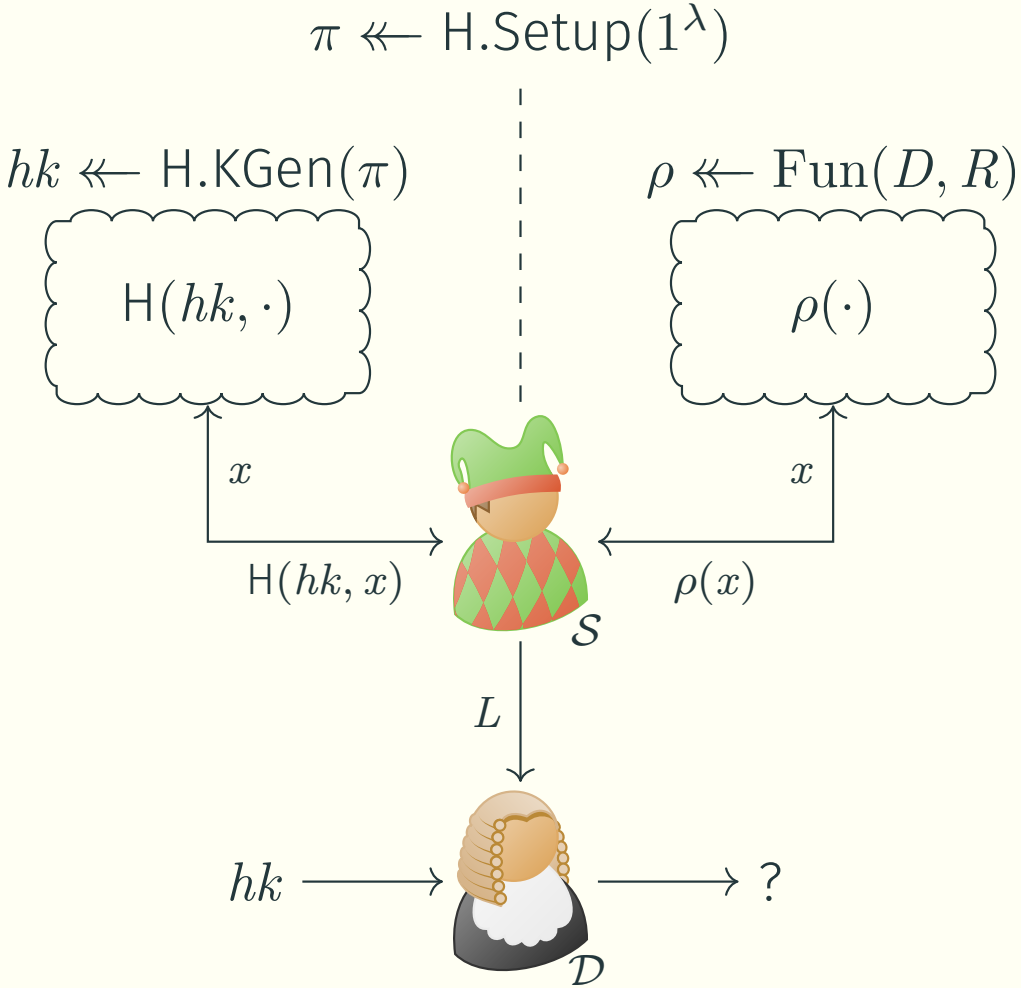
From Hash Functions to Groups



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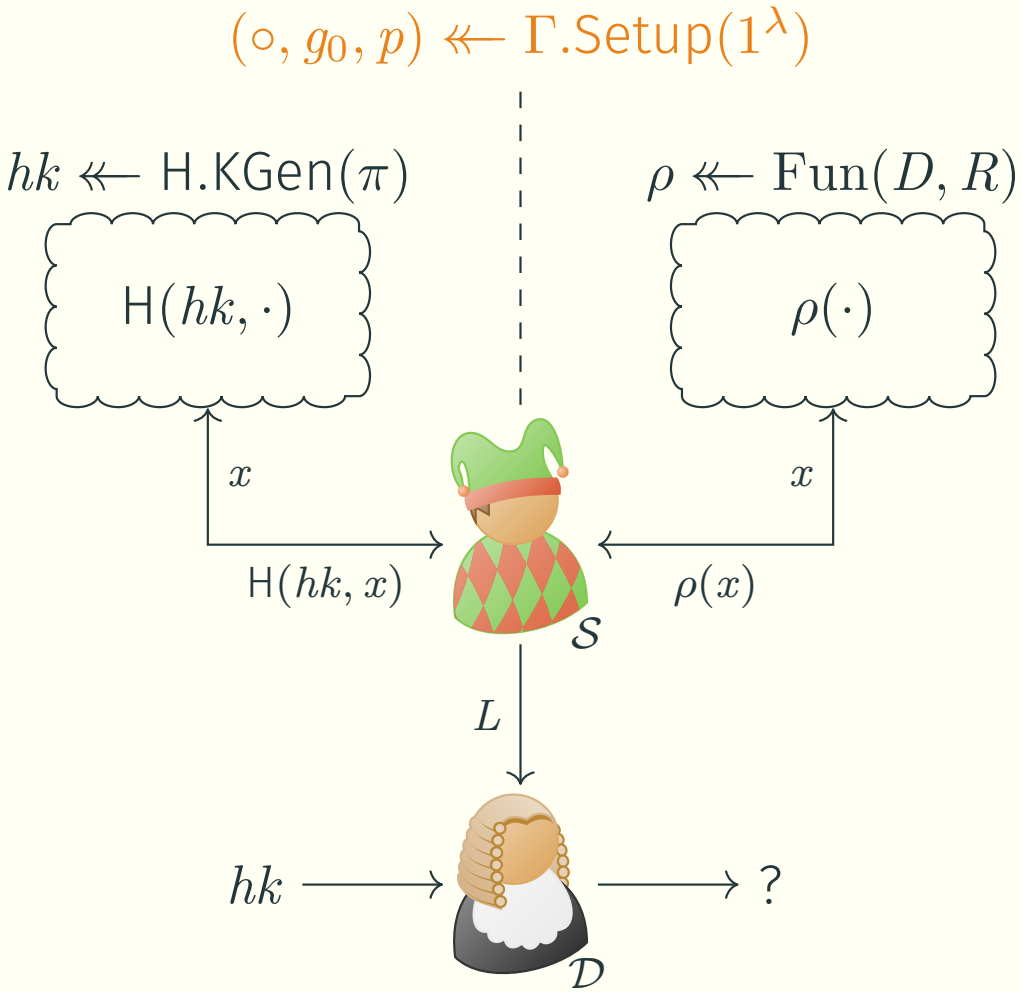

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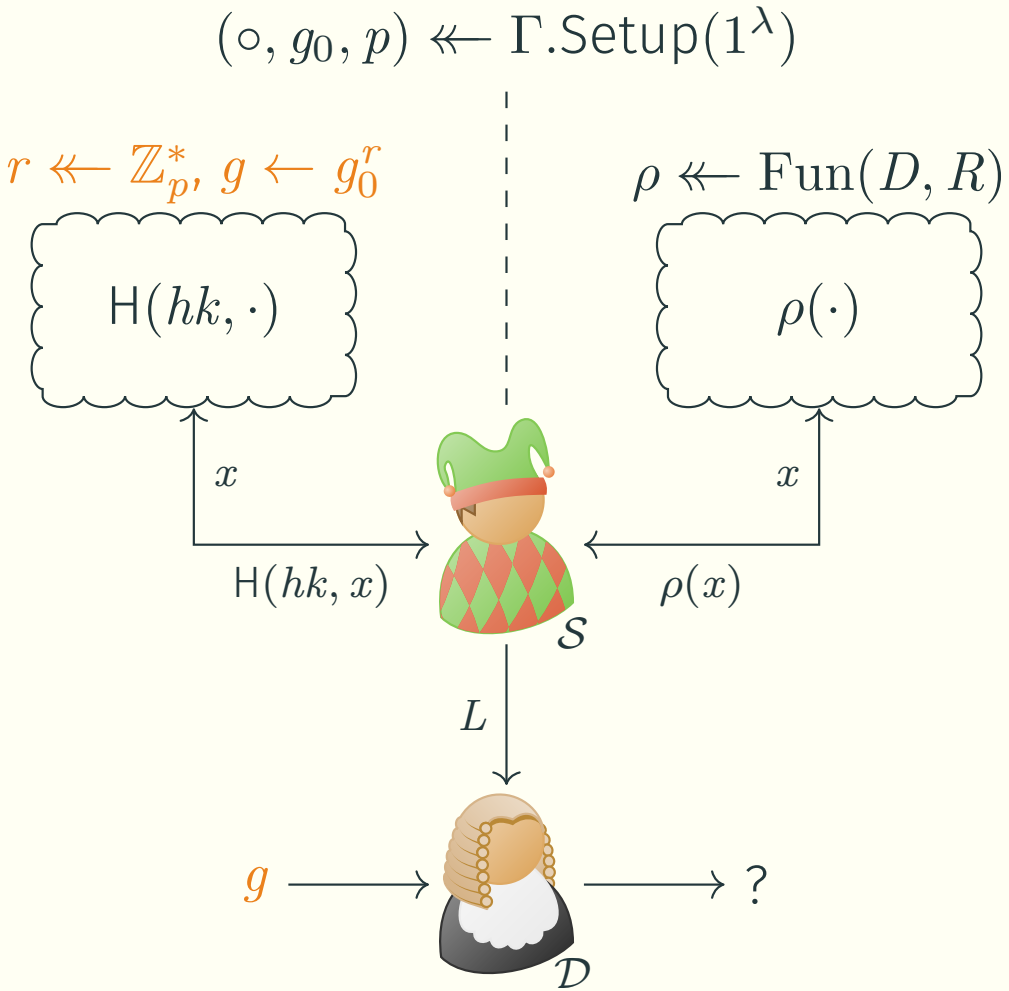

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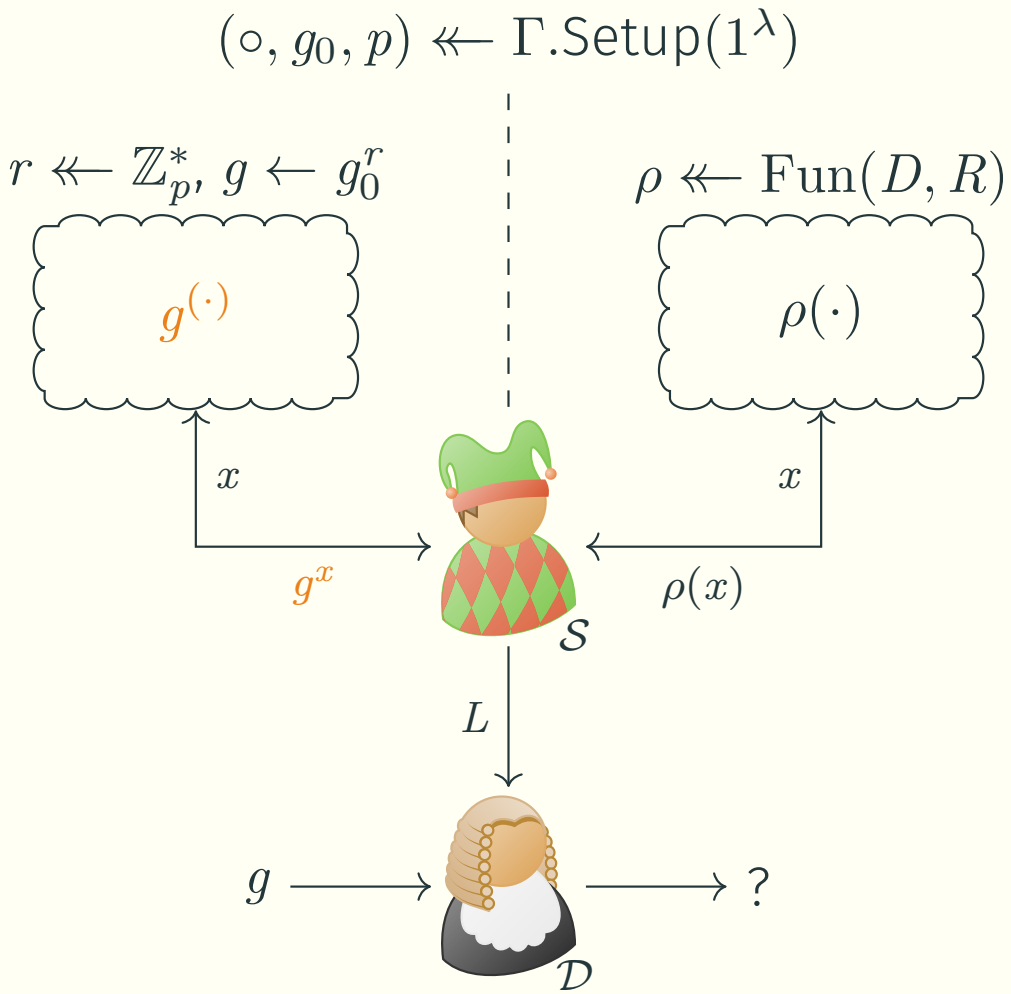

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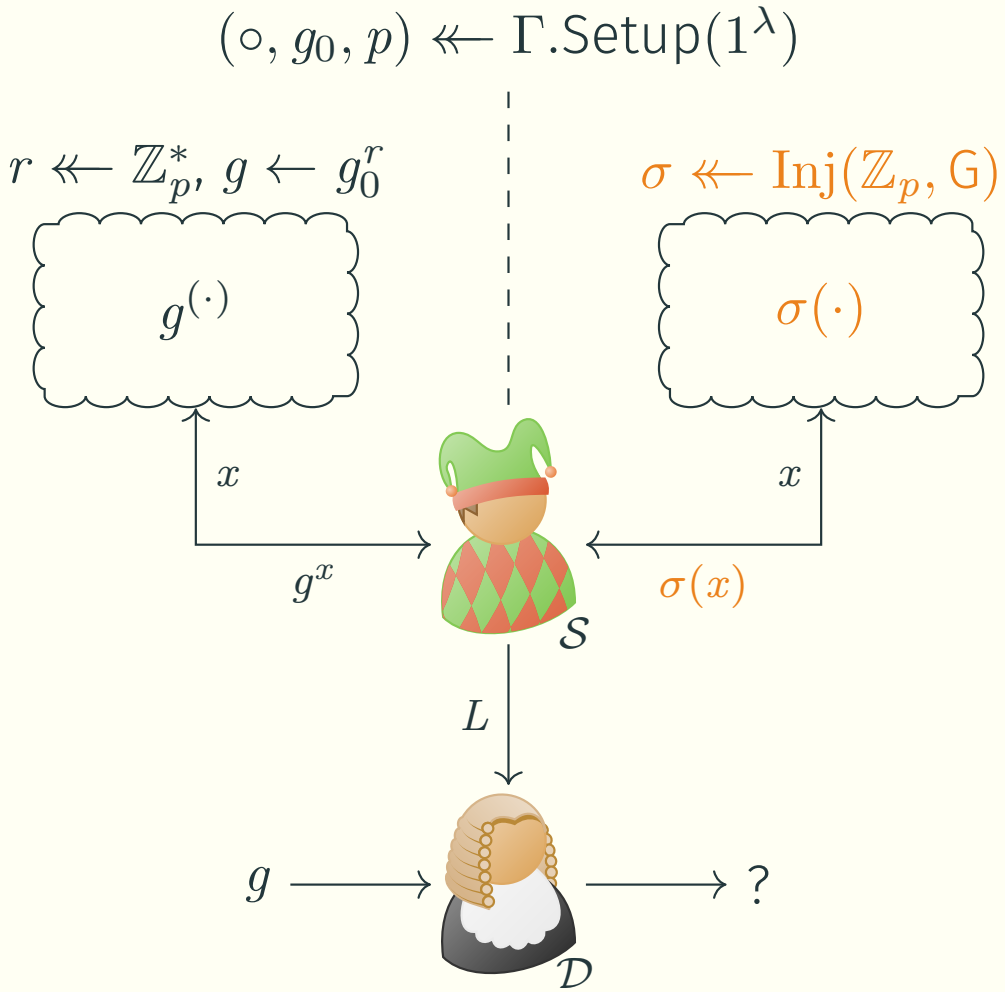

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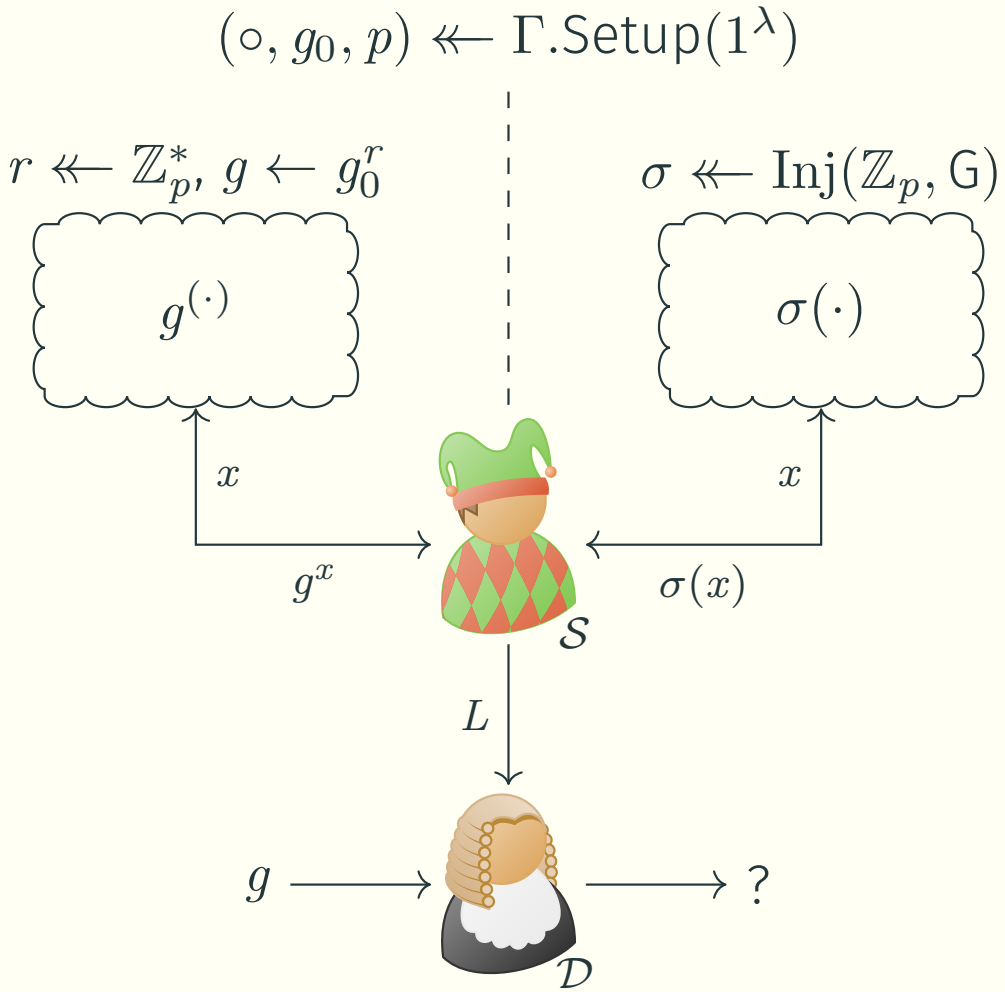
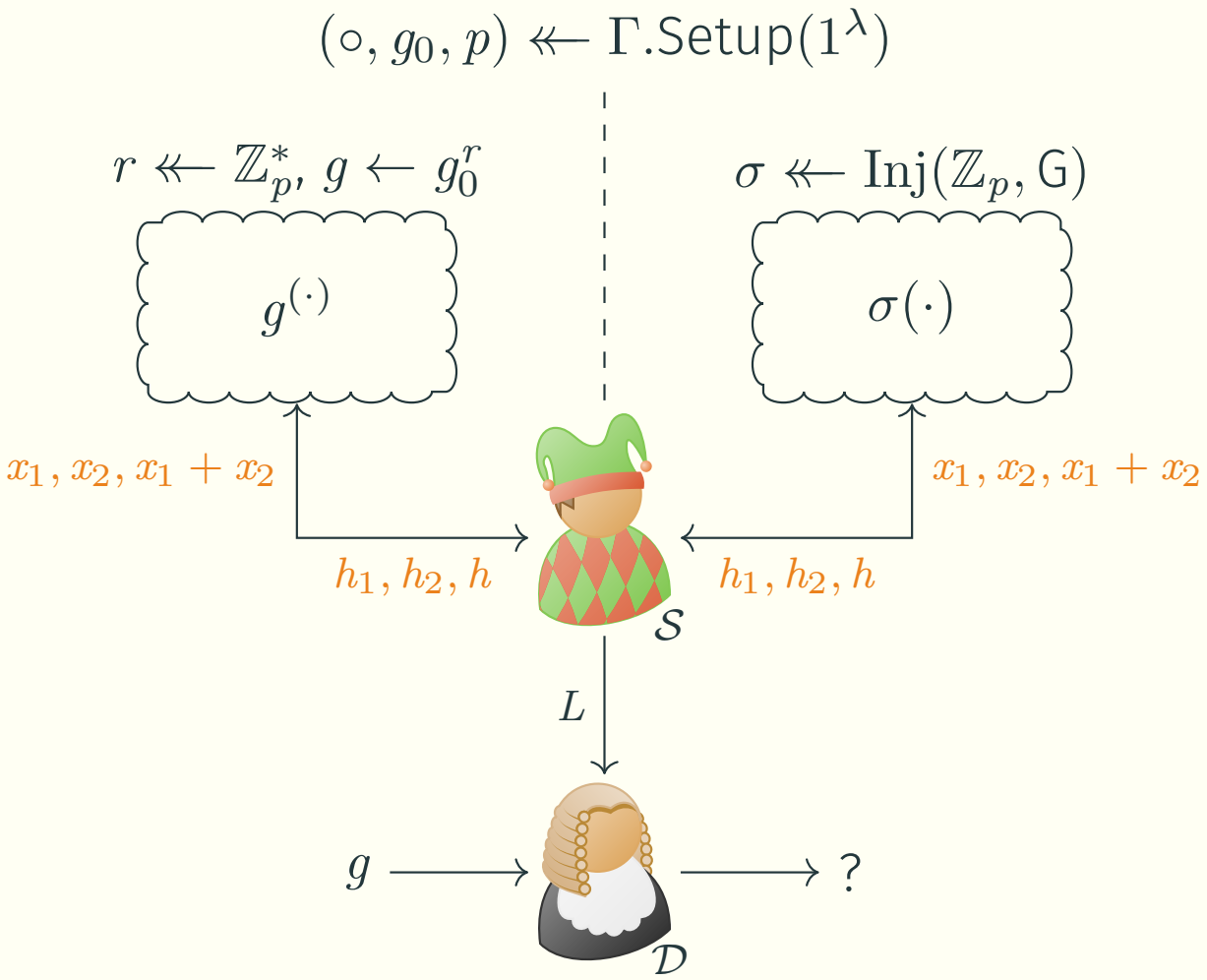


New Attacks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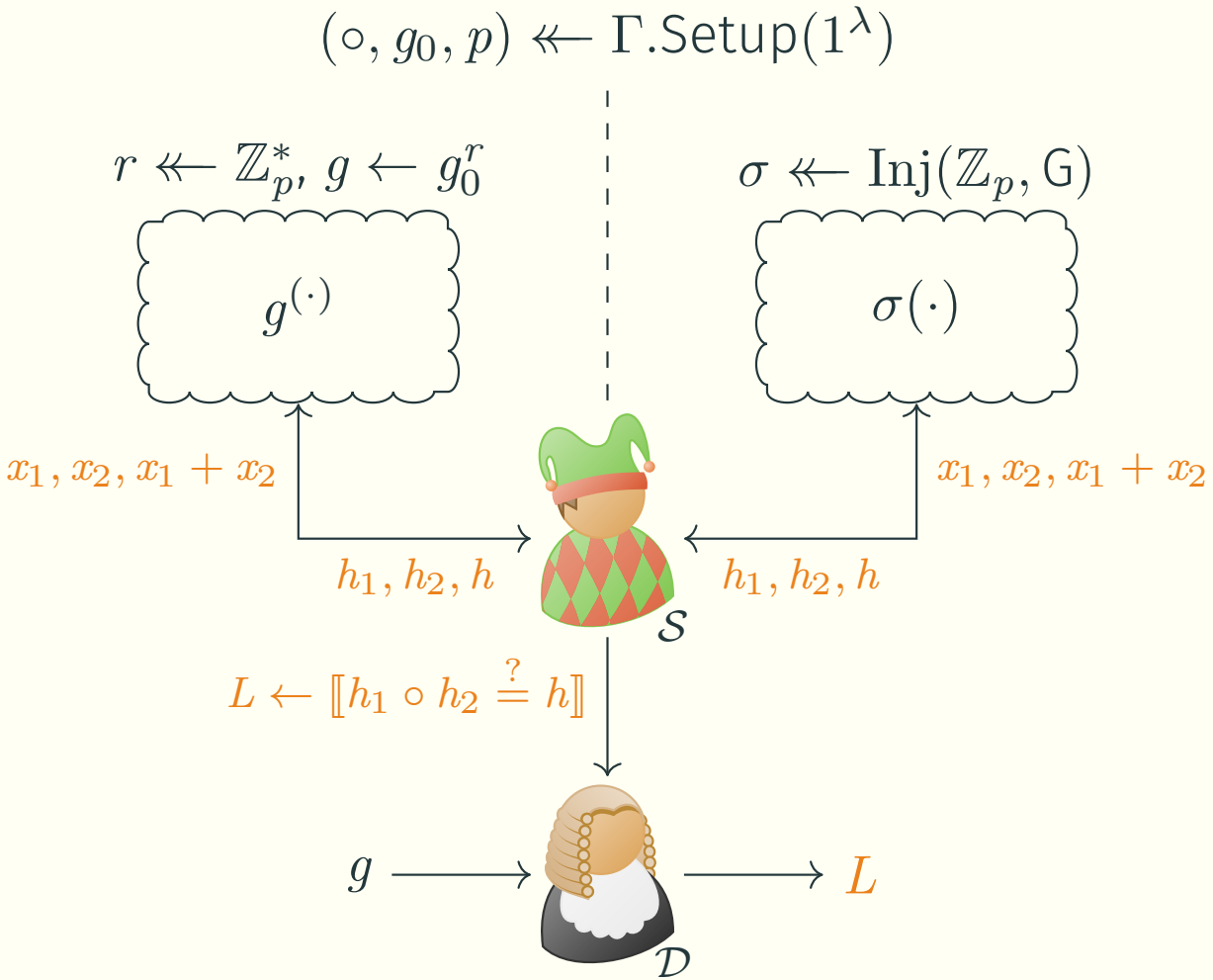
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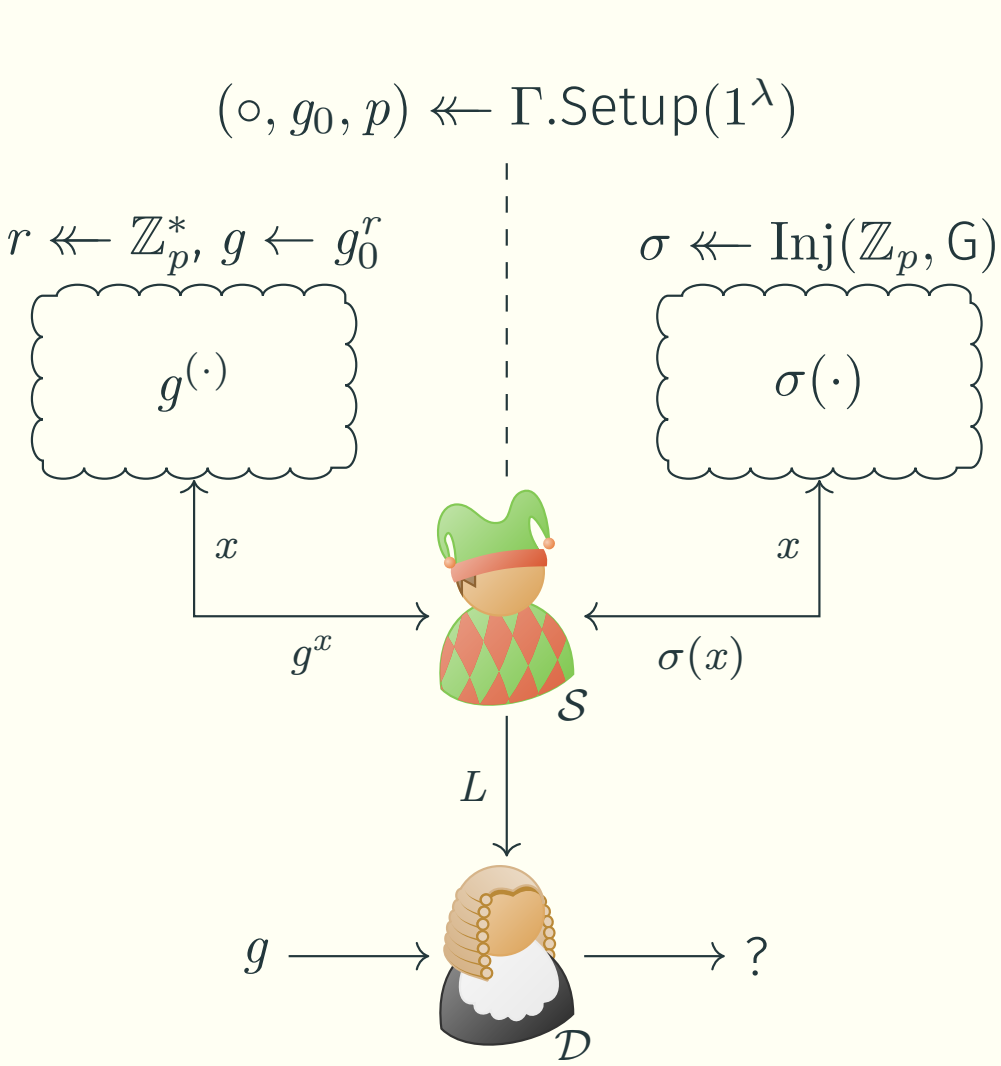
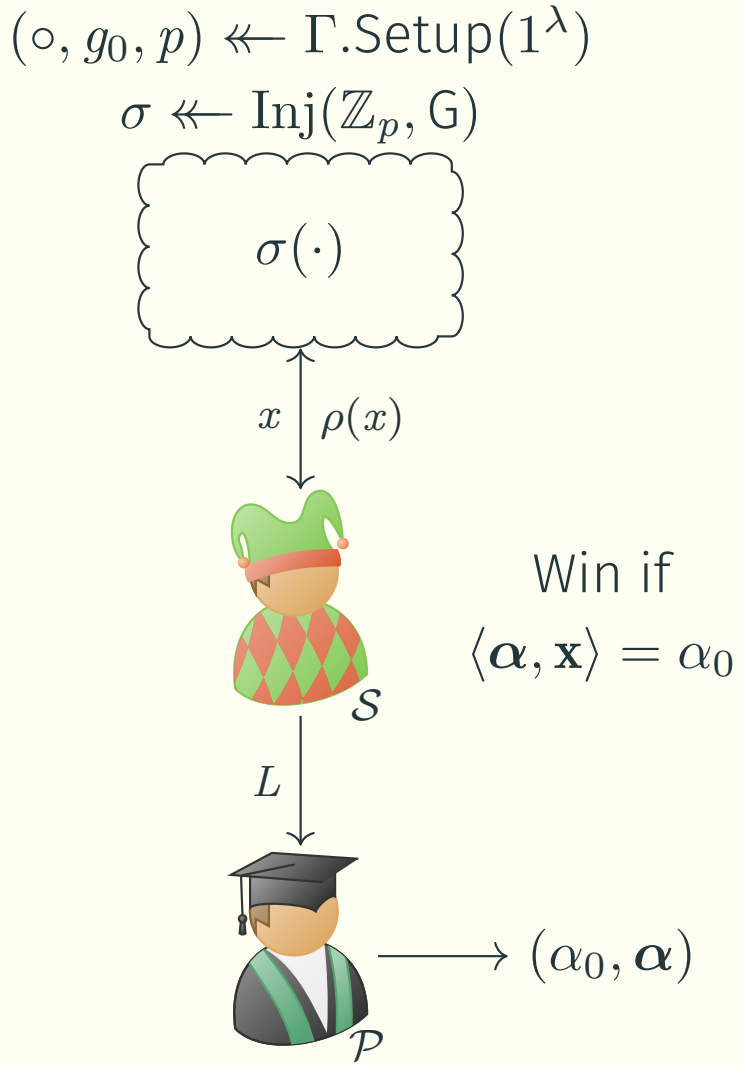
Algebraic Unpredictability





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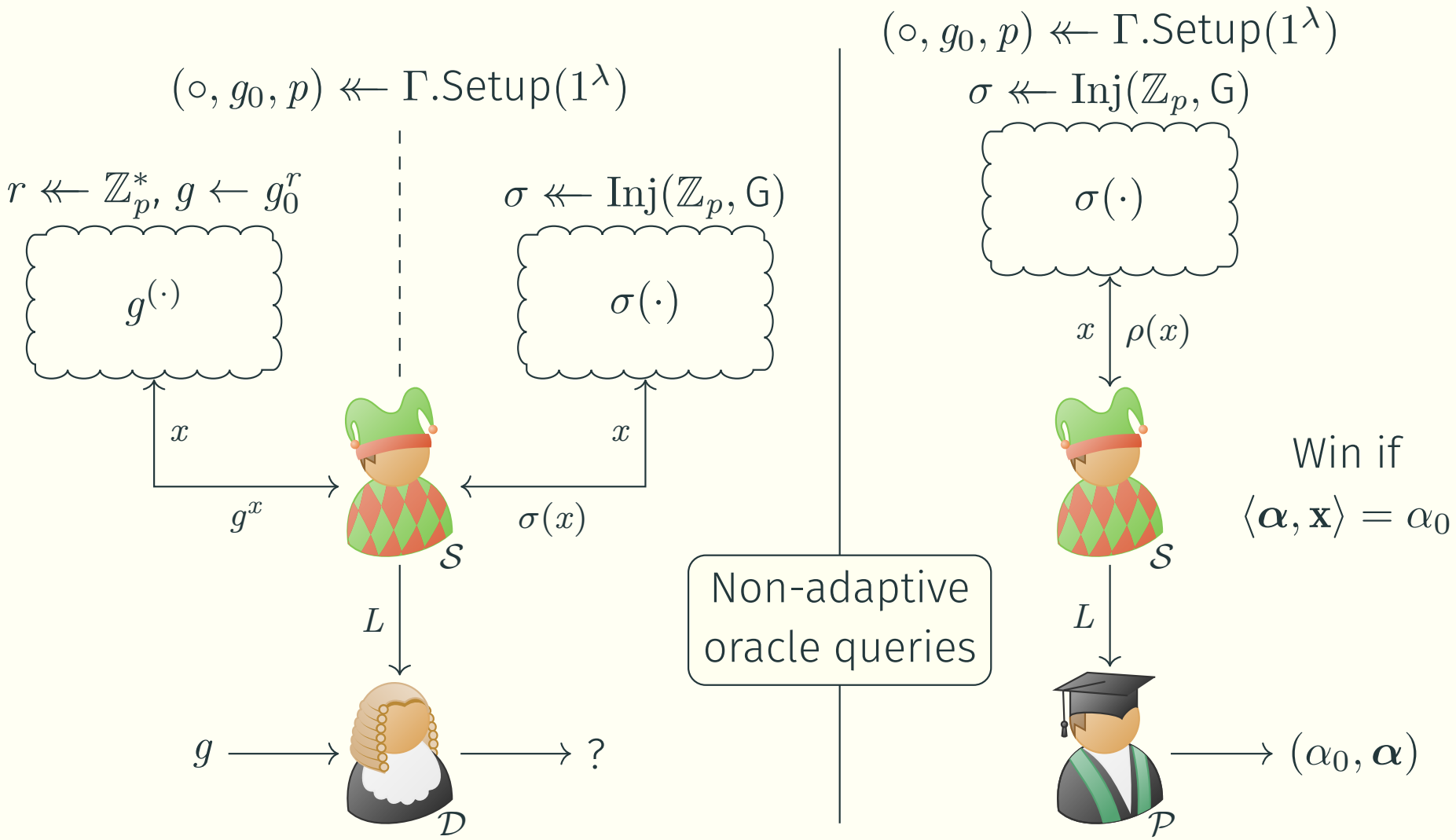
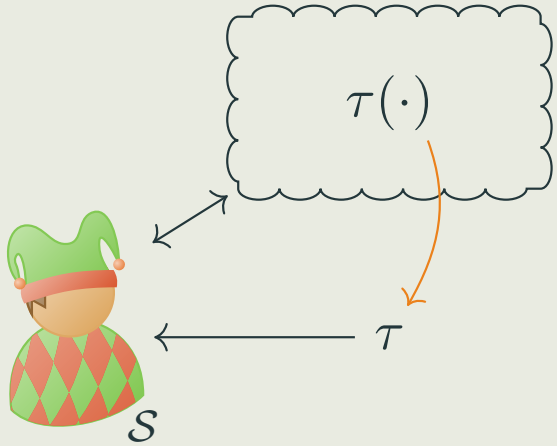


Soundness of the Definition: GGM Feasibility

Modeling of sources

- Sources are unbounded
- Receive τ in full, not only oracle access
- Capture more applications and pre-processing attacks
- Use bit-fixing [CDG18]



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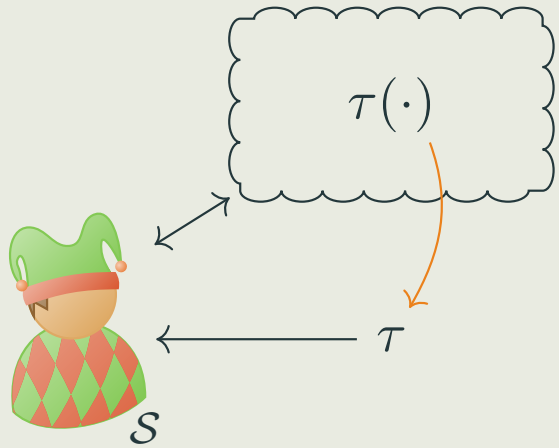

Different proof strategy

- Schwartz–Zippel lemma $\rightsquigarrow$ Algebraic unpredictability
- Reinterpret Schwartz–Zippel as an assumption on sources

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Overview of Applications

Uber assumption

- Also generalizations thereof, e.g., [Can97]

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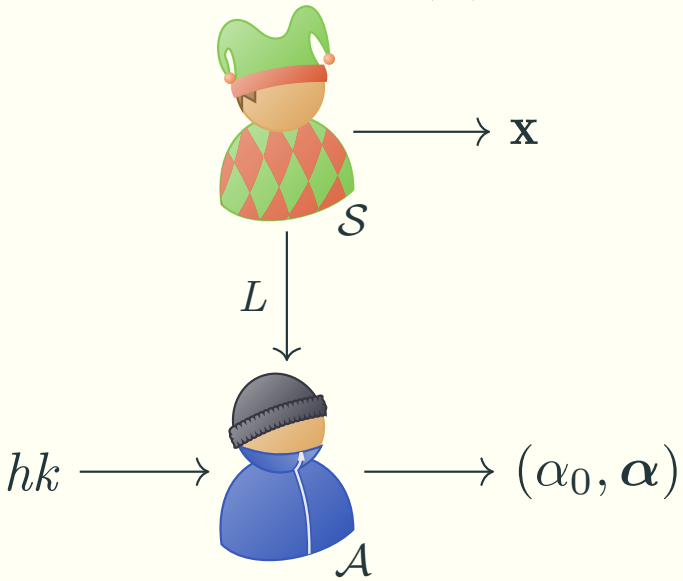

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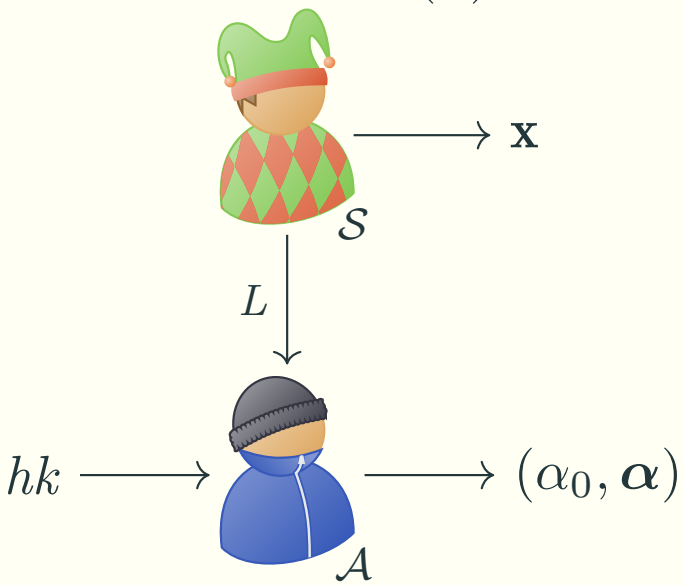

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Results on LDDs

- Provide instantiation for low-degree sources
- Show that a random function is an LDD in certain parameter ranges

Conclusion and Open Problems

Paper summary

- Provide a new definitional framework for assumptions on groups
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