On the Impossibility of Algebraic Vector Commitments in Pairing-Free Groups

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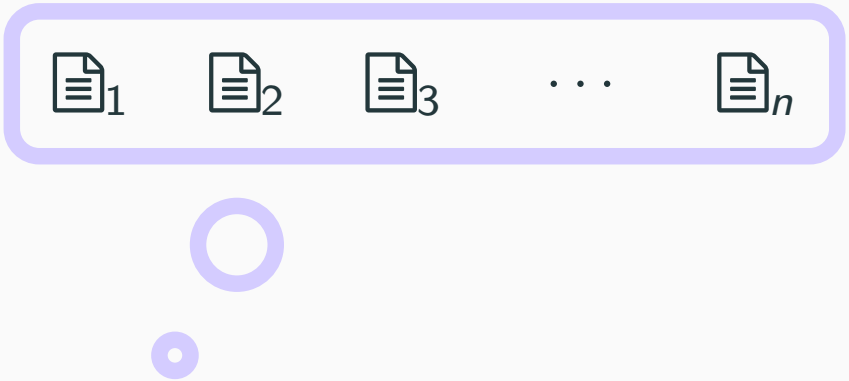
Università di Catania

Vector Commitments

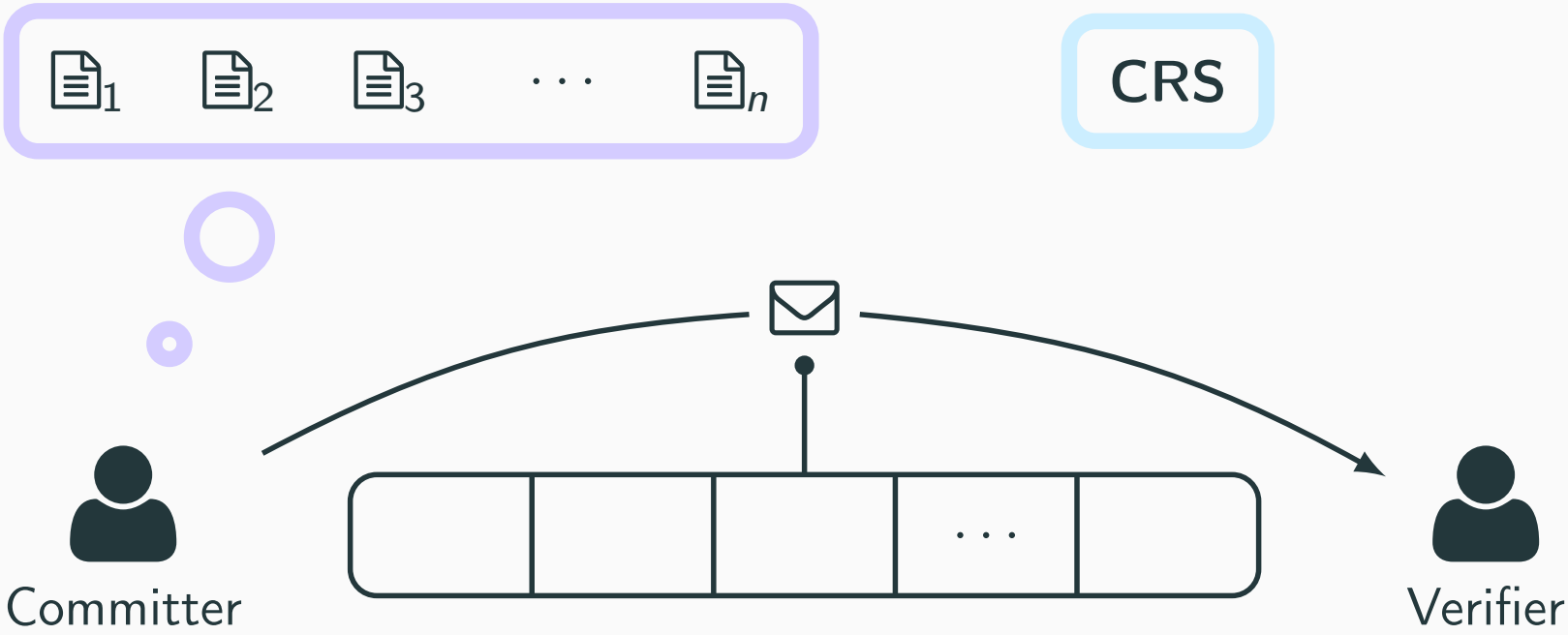


CRS

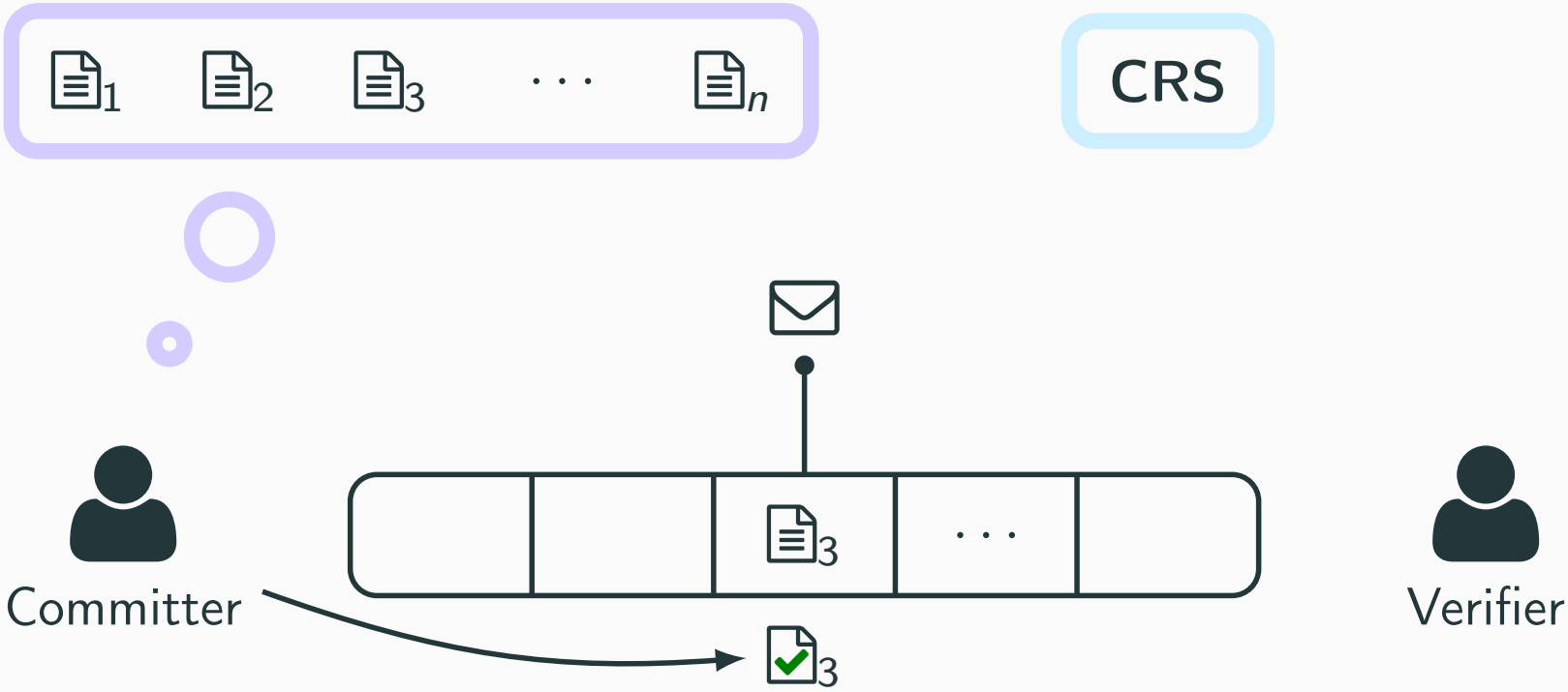
Committer

Verifier

Vector Commitments



Vector Commitments



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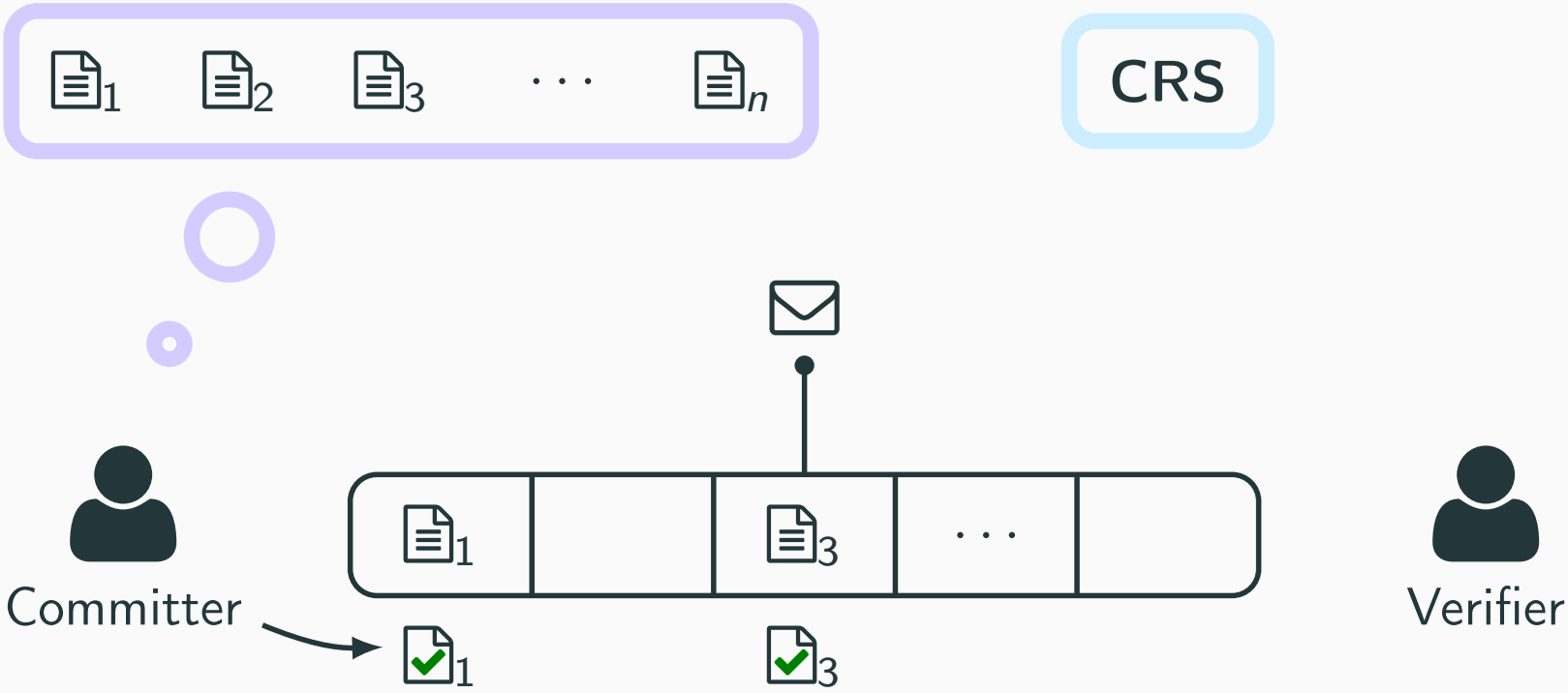

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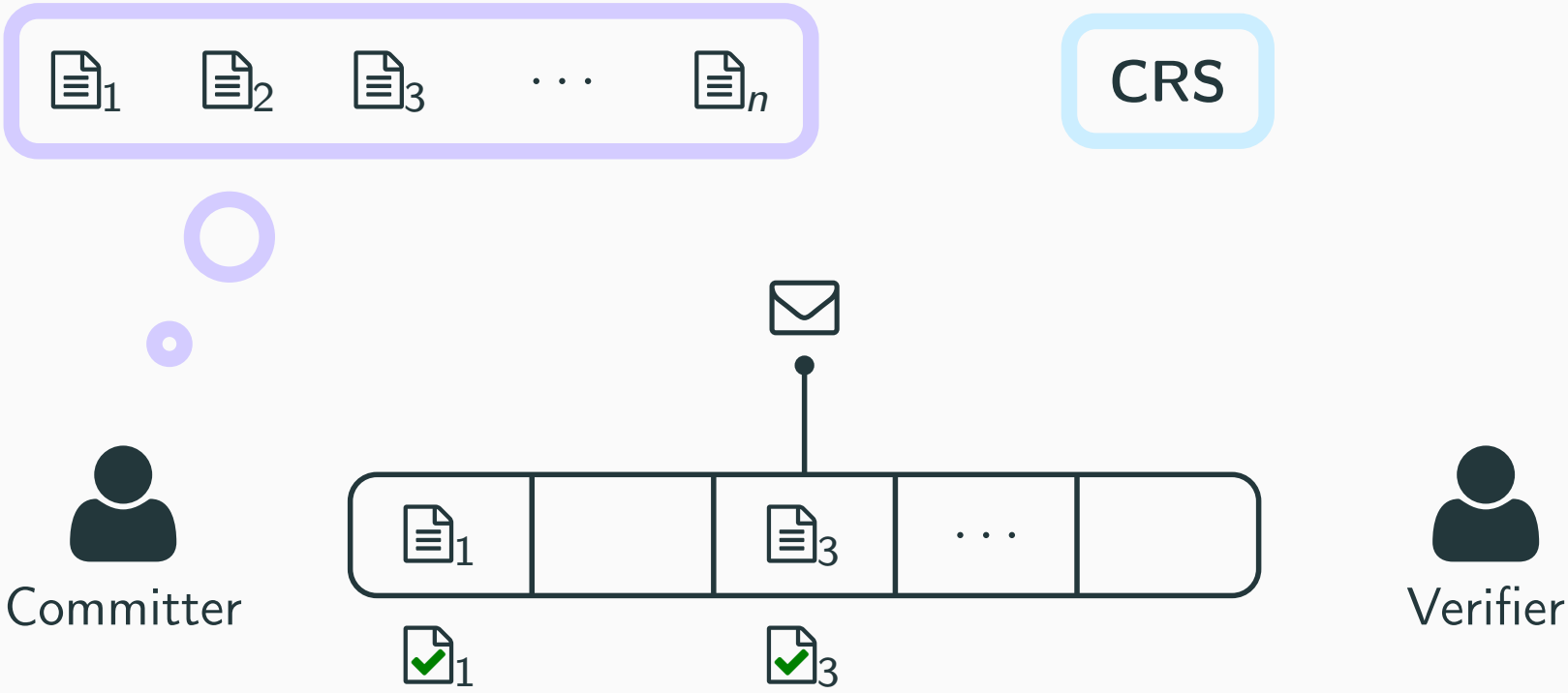
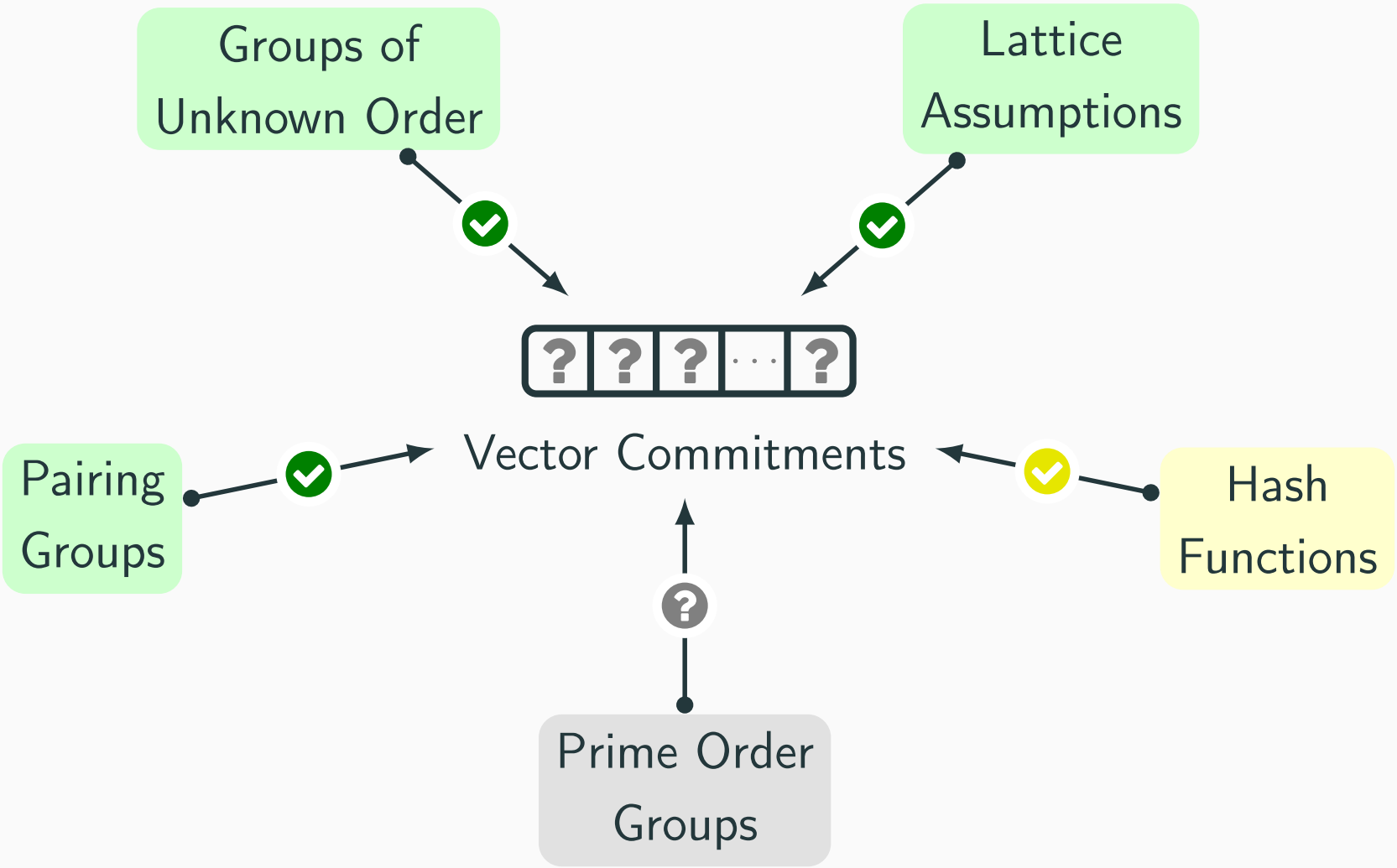


Succinctness: $\square$ and $\square$ have length $O(\log n)$.

Position Binding: hard to open $\square$ to two different m_i, m_i^* at position i.

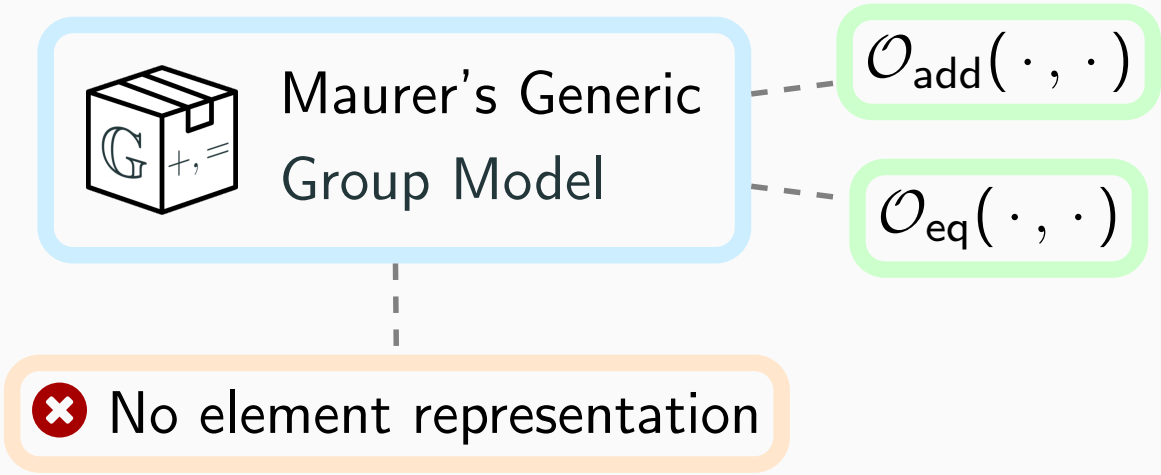
State of the Art



Algebraic Vector Commitments

A Vector Commitment is **Algebraic** if

- Uses a **black box** pairing-free prime order group $\mathbb{G}$.
- Its security reduces **only** to hard problems in $\mathbb{G}$.



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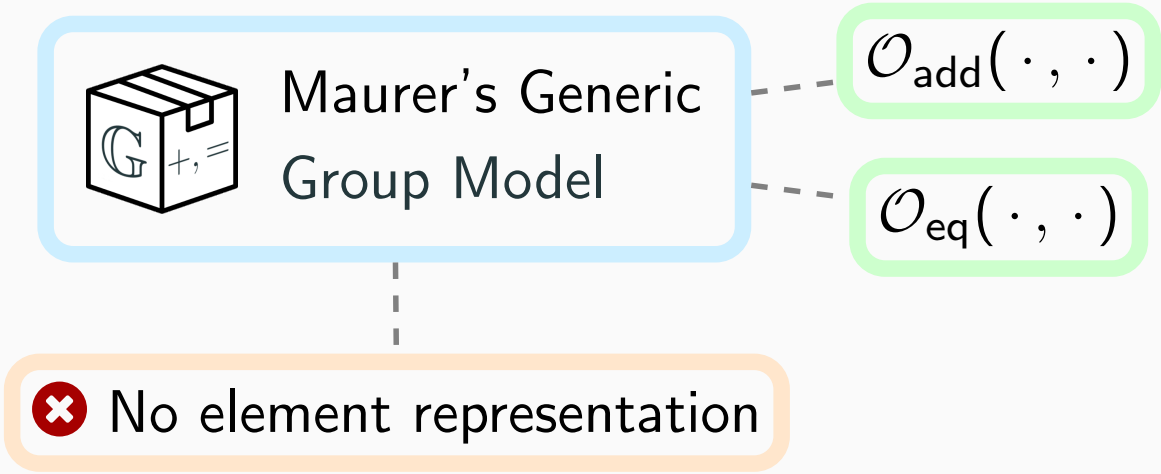

Question: Do Algebraic Vector Commitments exists?

Our Result - Impossibility of Algebraic VC

Strictly Linear Verification VC

$$\boxed{} = (\mathbf{Y}, \mathbf{z}) \in \mathbb{G}^k \times \mathbb{F}_q^h$$

Verification[1]: $A(\mathbf{z})\mathbf{X} == B\mathbf{Y}$
with $A(\mathbf{z})$ affine function of $\mathbf{z}$

Unconditionally Impossible:

$$\ell(\boxed{}) \cdot \ell(\boxed{}) = \Omega(n)$$

[1] $\mathbf{X} \in \mathbb{G}^\nu$ denotes the CRS and commitment's group elements

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Unconditionally Impossible:
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Generic Verification VC

No restriction

Black-Box Separation:
$\ell(\boxtimes) \cdot \ell(\boxed{\checkmark}) = \Omega(n)$

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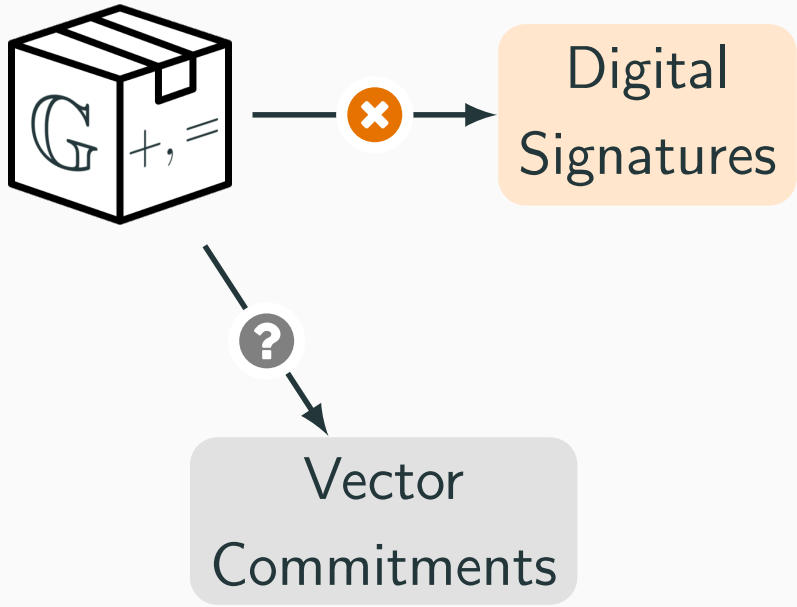
Black-Box Separation Roadmap

Roadmap

1. **Starting point:** [DHH+21].



Black-Box Separation Roadmap

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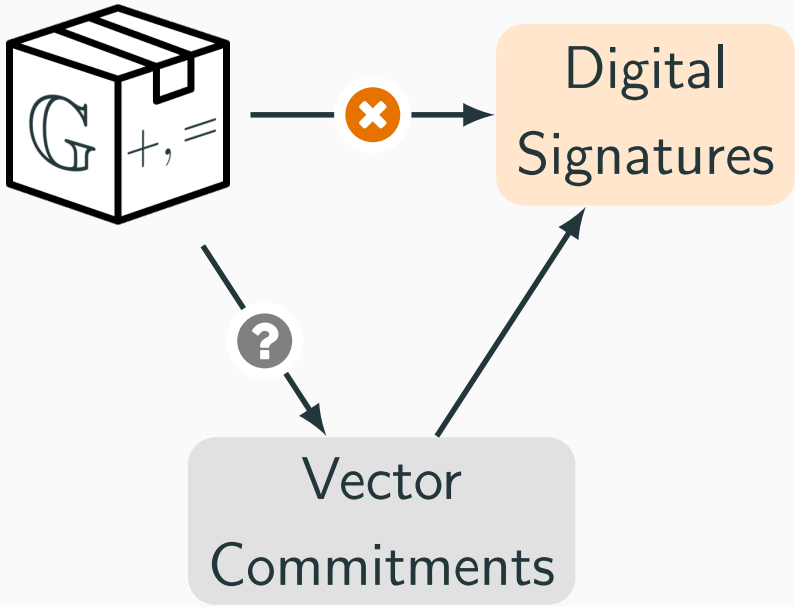
1. **Starting point:** [DHH+21].
2. Build DS from VC.



Black-Box Separation Roadmap

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1. **Starting point:** [DHH+21].
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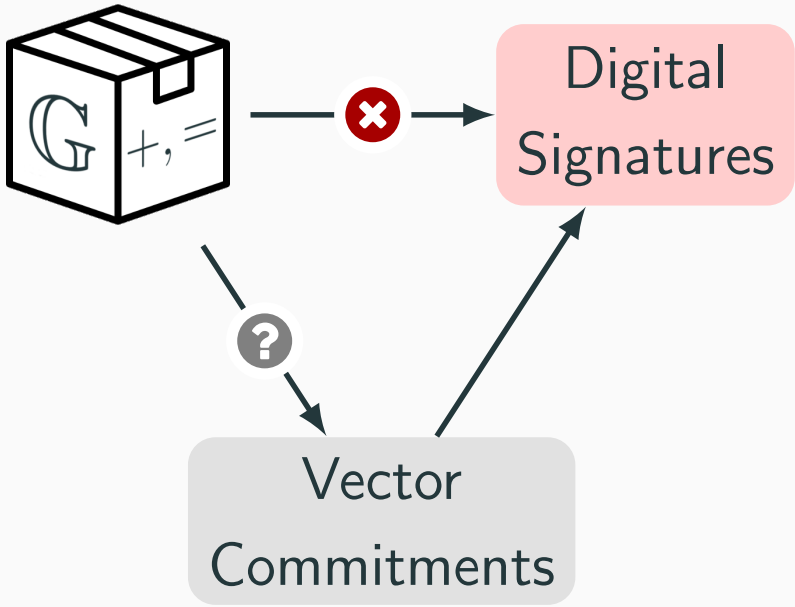
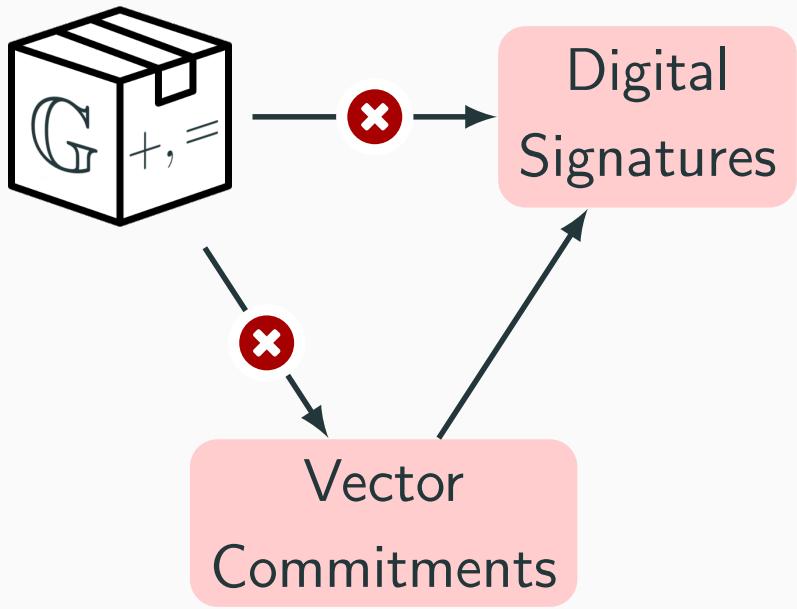
3. Describe an attack for Algebraic Signatures.



Black-Box Separation Roadmap

Roadmap

1. **Starting point:** [DHH+21].
2. Build DS from VC.
3. Describe an attack for Algebraic Signatures.
4. Conclude Algebraic VC are impossible.



Signatures from Vector Commitments

Message Space

$1 \quad 2 \quad 3 \quad \cdots \quad n$

CRS

Signer

Verifier

Signatures from Vector Commitments

Signer

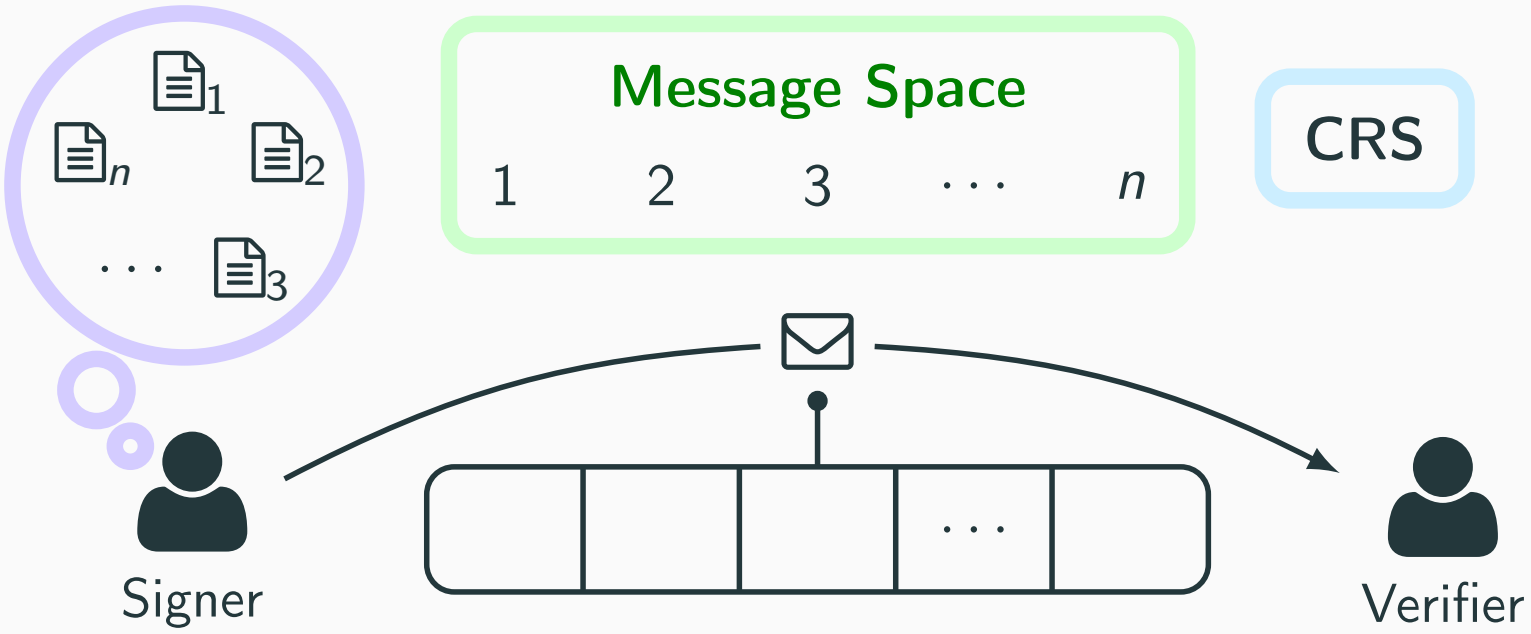
Message Space

1 2 3 ··· n

CRS

Verifier

Signatures from Vector Commitments



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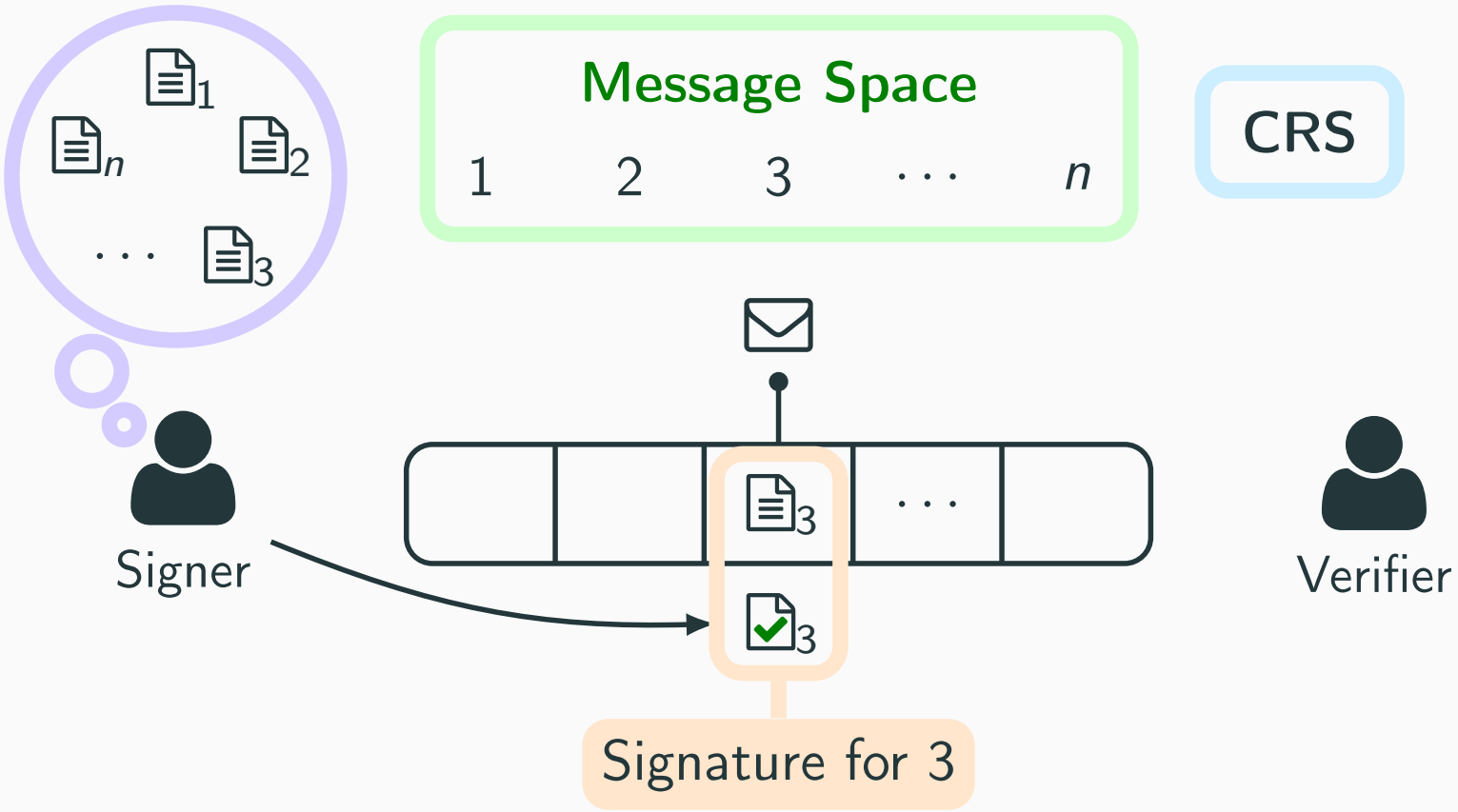

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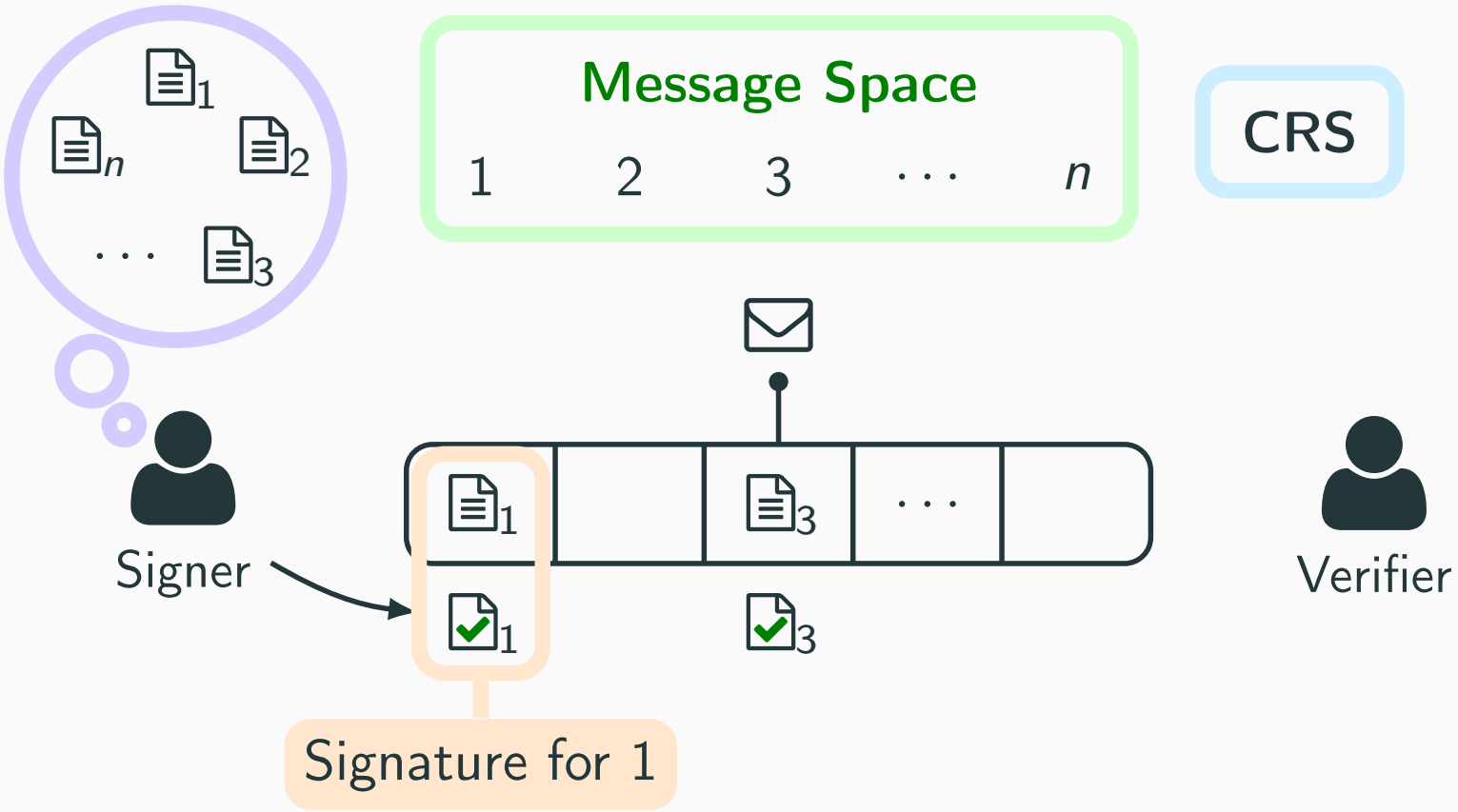

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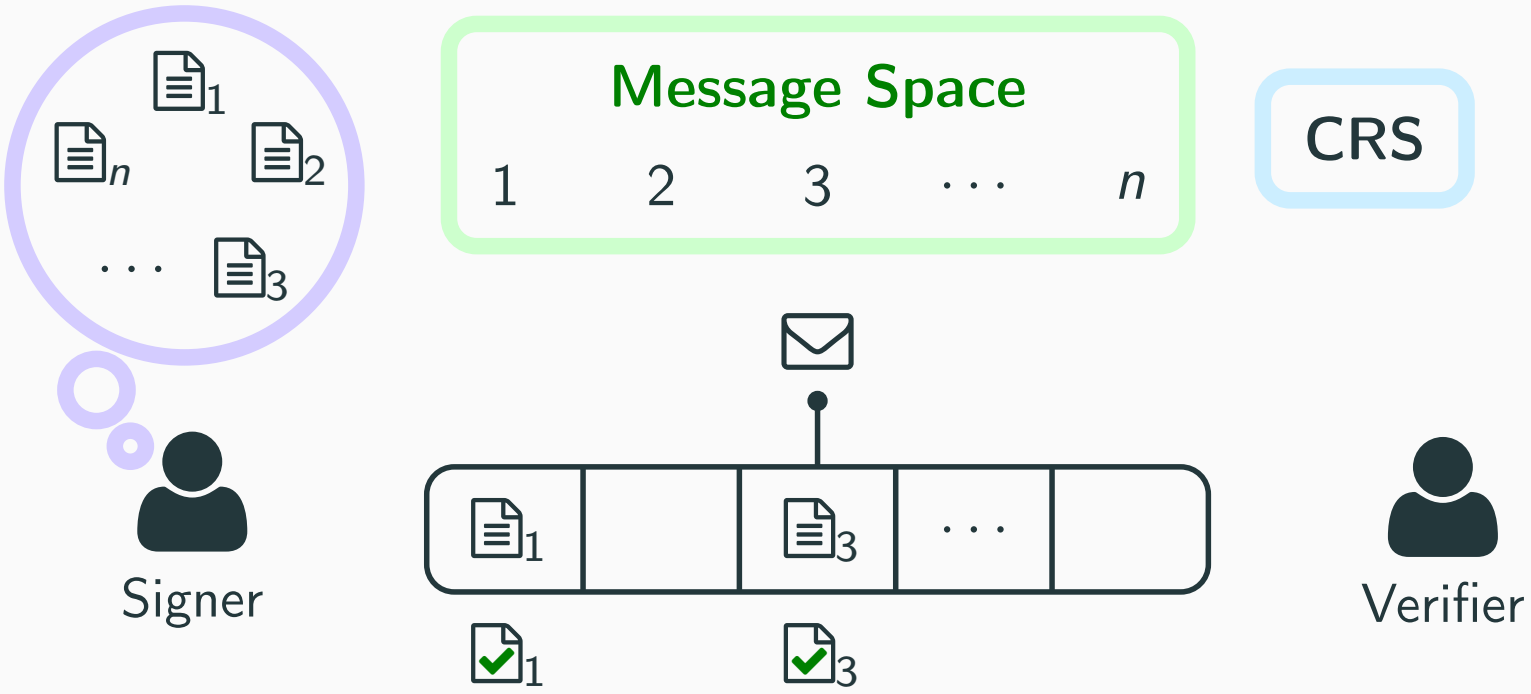
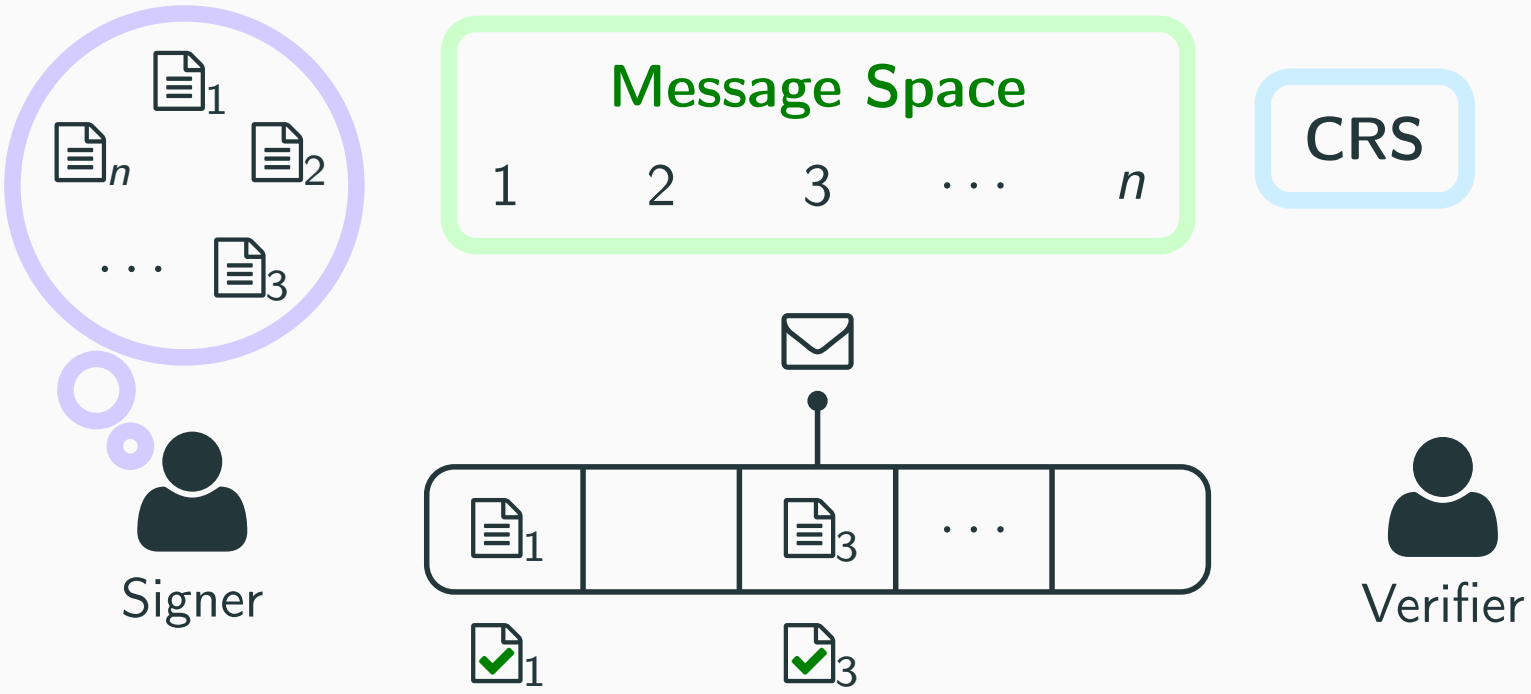


Question: Is this **Unforgeable**?

Signatures from Vector Commitments



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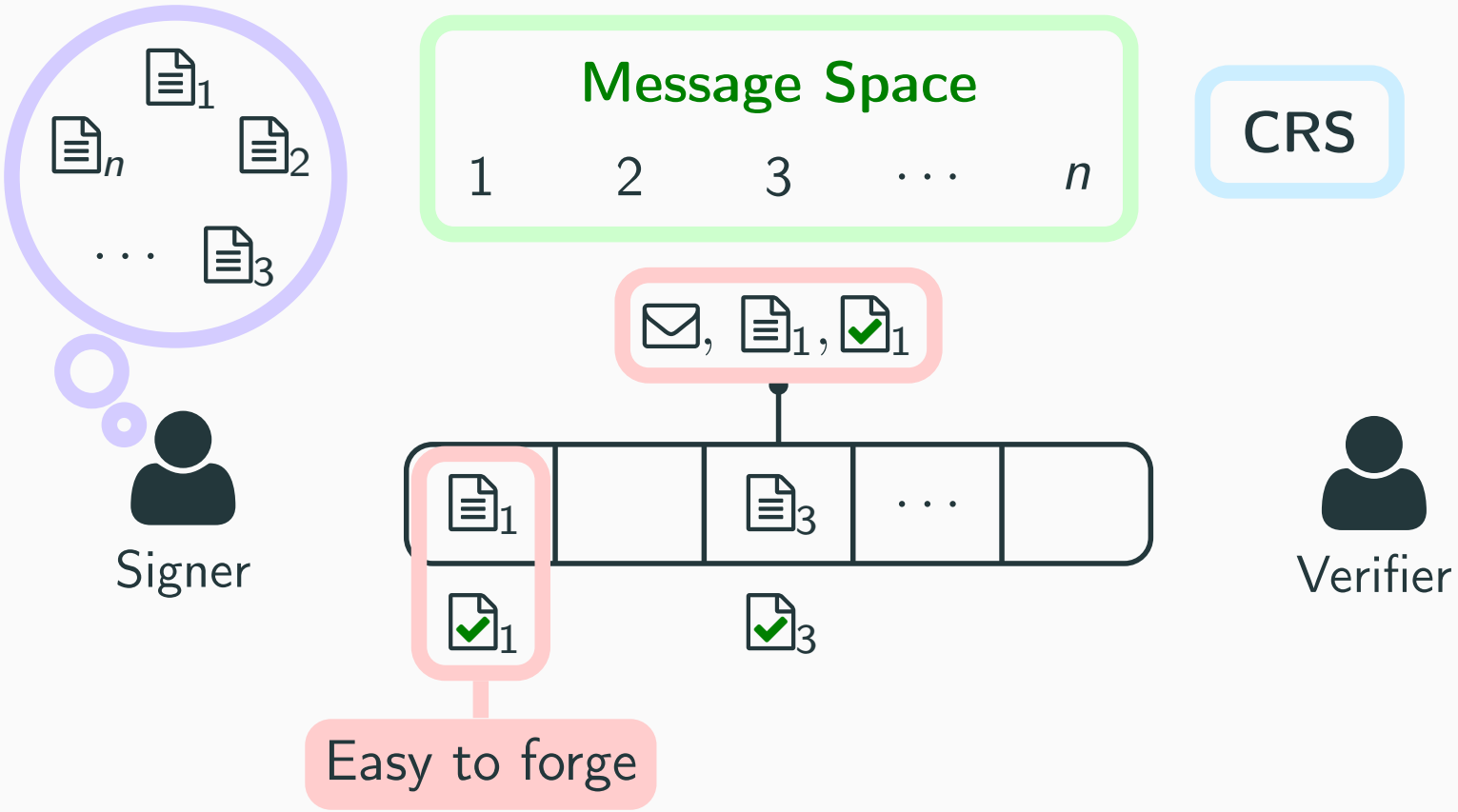

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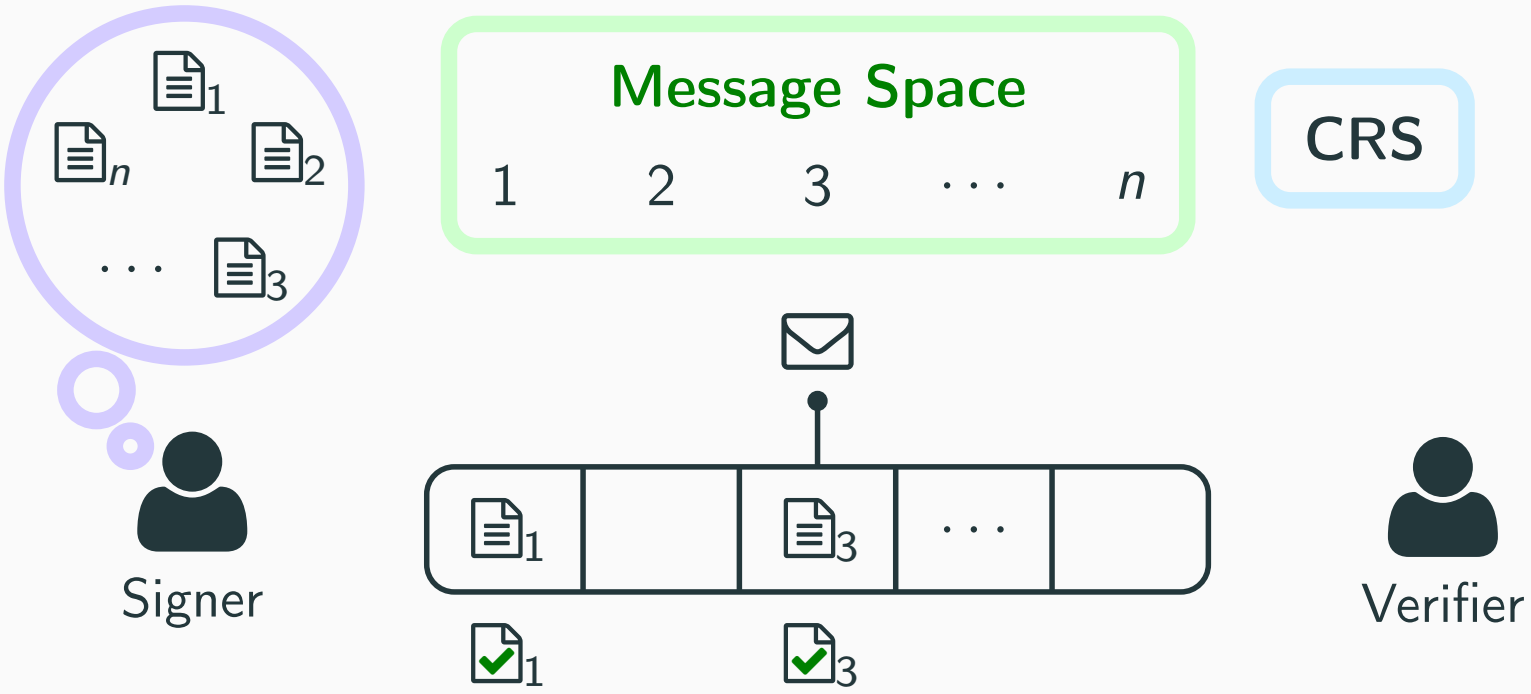
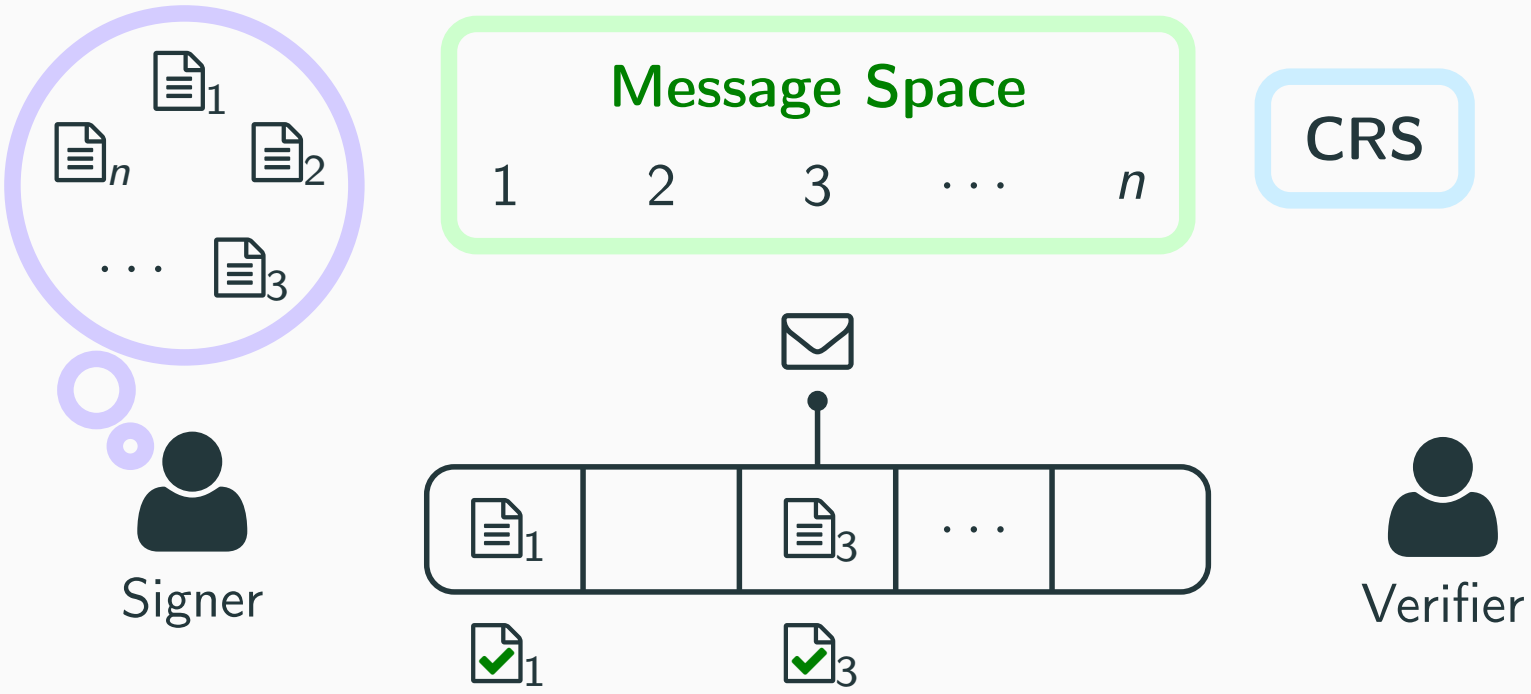


Question: Is this **Unforgeable**?

Question: Is this ϑ-**Unforgeable**?

Not Necessarily!

Signatures from Vector Commitments



Question: Is this **Unforgeable**? ✗ Not Necessarily!

Question: Is this ϑ-**Unforgeable**? ✓ Yes!

Impossibility of Algebraic Signatures

Result: There is no ϑ-unforgeable **Algebraic Signature** such that

- The verification key (excluding the CRS) has n group elements.
- The message space has size $\geq n + \vartheta$.

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[DHH+21]

Verification Equation
$A\mathbf{X} = B\mathbf{Y}$

Does not exclude
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Corollary: Signatures with large message space are impossible in Maurer's GGM.

Putting all Together

Signatures from VC

- ϑ-unforgeable for $\vartheta \approx \ell(✉) + |Q| \cdot \ell(📄)$.
- vk has at most $\ell(✉)$ group elements (not in the CRS).

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Impossibility for Signatures

There exists no ϑ-unforgeable algebraic signature s.t.

- vk has m group elements (not in the CRS).
- The **message space** has size $n \geq \vartheta + m$

Putting all Together

Signatures from VC

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Impossibility for Vector Commitments

$\ell(✉) \cdot \ell(✔) = \Omega(n)$

Conclusions and Open Problems

In Maurer's Generic Group Model **Vector Commitments** cannot be both **position binding** and **succinct**.

Consequences: **Succinct** Polynomial and Functional Commitments are also impossible in Maurer's GGM.

Open problems: Extension to **Shoup's GGM**.

Thanks for your attention!