Asymptotically Free Broadcast in Constant Expected Time via Packed VSS

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Broadcast for perfect MPC is essentially free*!

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Settings

- Perfect:
  - Computationally unbounded adversary
  - Zero-probability of error
- Optimal resilience: $t < n/3$

BenOr, Goldwasser, Wigderson 88:

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x_1	x_2	x_{n-1}	x_n
VSS	VSS	VSS	VSS

VSS $= O(n^2)$ **p2p** + $O(n^2)$ **broadcast**

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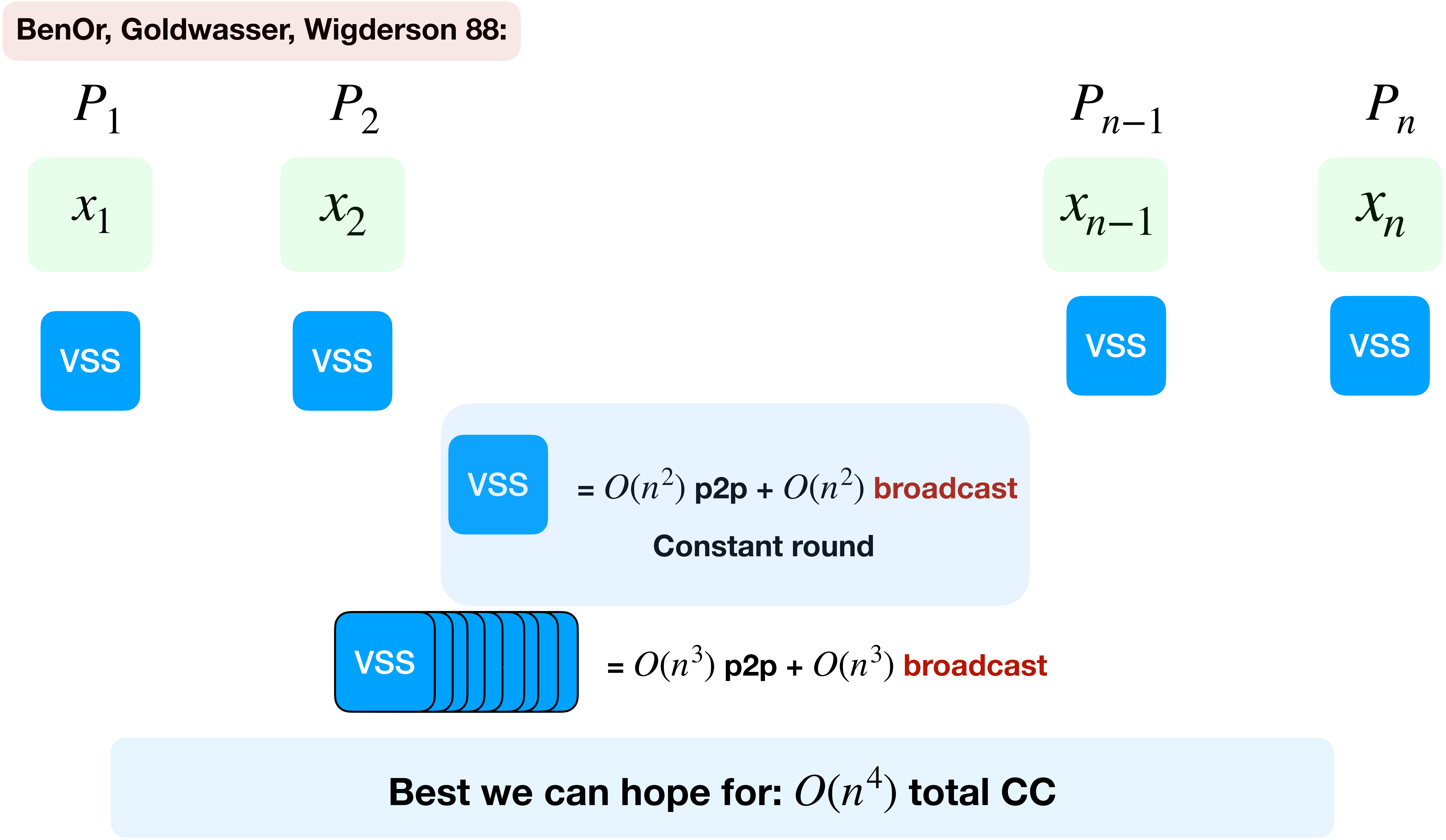

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Best we can hope for: $O(n^4)$ total CC

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$$n \times \mathrm{BC}(n^2)$$

State of the Art: $n \times \mathrm{BC}(n^2)$

for simplicity, counting "words" and not "bits" -> i.e., ignoring $\log n$ factor

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Communication Complexity p2p	$n \times \text{BC}(n^2)$	$O(n^4)$	$O(n^6)$
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A circuit with depth 10 and
$n = 300$ participants

Instead of $\approx$ 10 rounds
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x90,000 improvement

Perfect MPC in the Broadcast-Hybrid Model

For constant depth circuit		Efficient but slow	Fast but inefficient
		[HMP00, BTH08, GLS19]	[BGW88, CCD88, GRR98, CDM00, **A**LR11, A**A**Y21]
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	Broadcast		
Round Complexity			

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STOC Test of Time Award!

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Round Complexity			

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Due to "player elimination"

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Very Slow MPC
Slow and inefficient MPC
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Round Complexity	$O(n^2)$	$O(n)$ **Expected**	$O(1)$ **Expected**

MPC:	Efficient but slow	Efficient but slow	Fast but inefficient
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Broadcast:	Efficient but slow	Our Broadcast	Our Broadcast
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Round Complexity	$O(n^2)$	$O(n)$ Expected	$O(1)$ Expected

MPC:	Efficient but slow	Efficient but slow	Fast but inefficient	Coming up!
	+	+	+	+
Broadcast:	Efficient but slow	Our Broadcast	Our Broadcast	Our Broadcast
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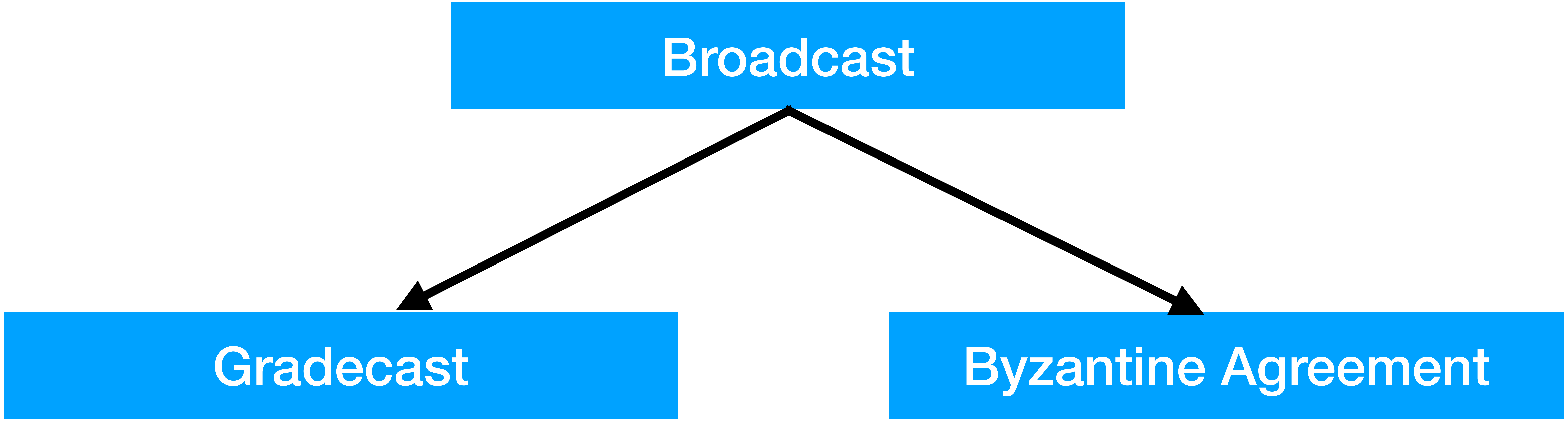
Communication Complexity	$O(\|C\|n + n^3)$	$O(\|C\|n + n^5)$ $O(\|C\|n + n^7)$	$O(\|C\|n^4)$ $O(\|C\|n^6)$	$O(\|C\|n + n^4)$
Round Complexity	$O(n^2)$	$O(n)$ **Expected**	$O(1)$ **Expected**	$O(1)$ **Expected**
				In submission

Structure of Katz and Koo



$$O(n^2L)$$

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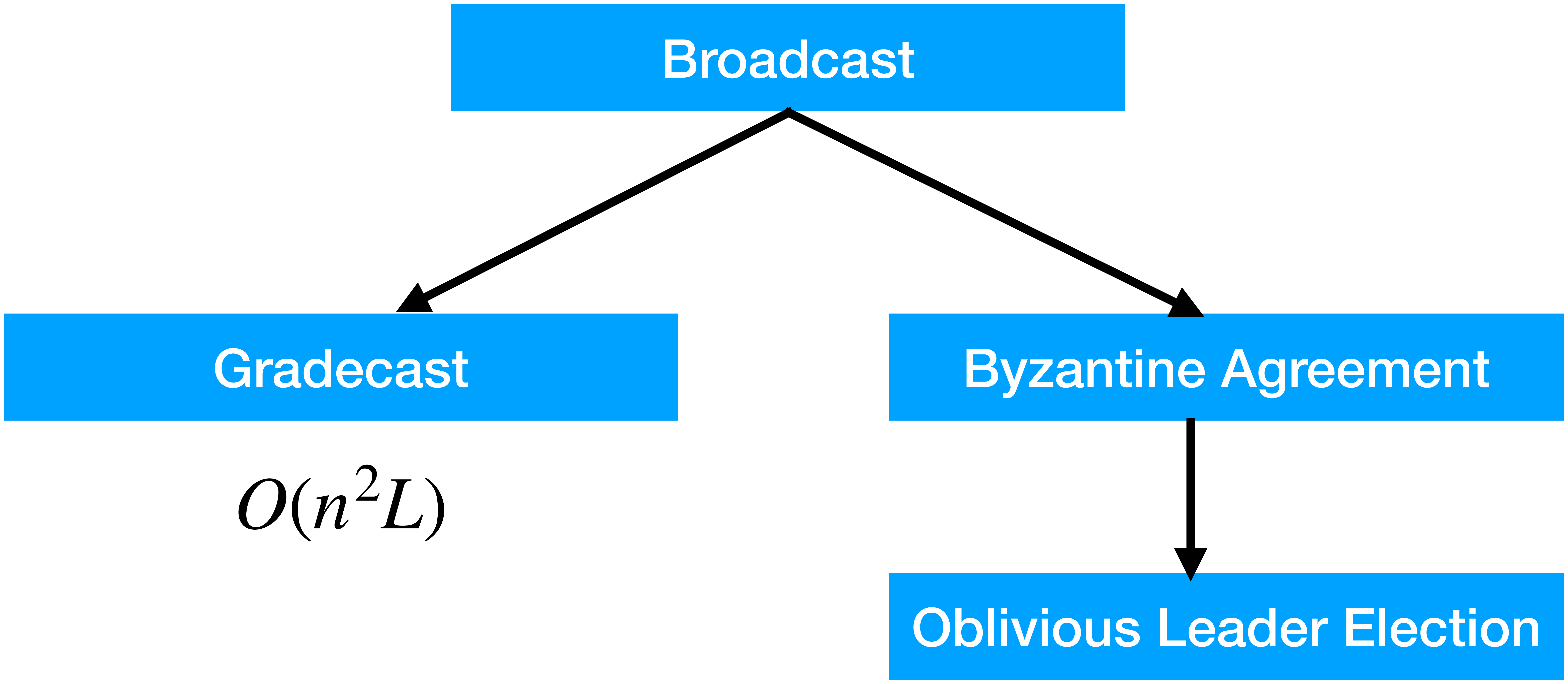

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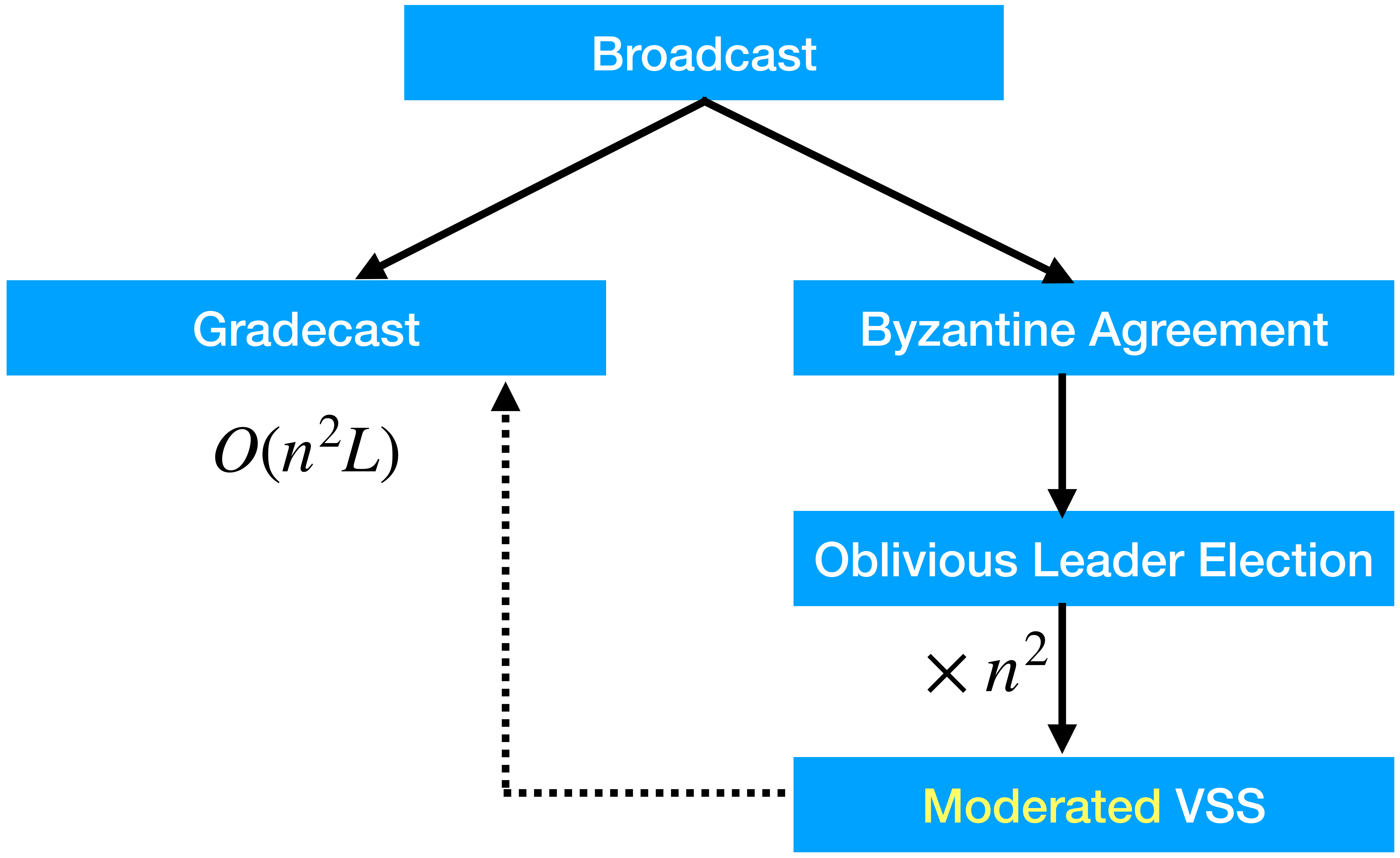

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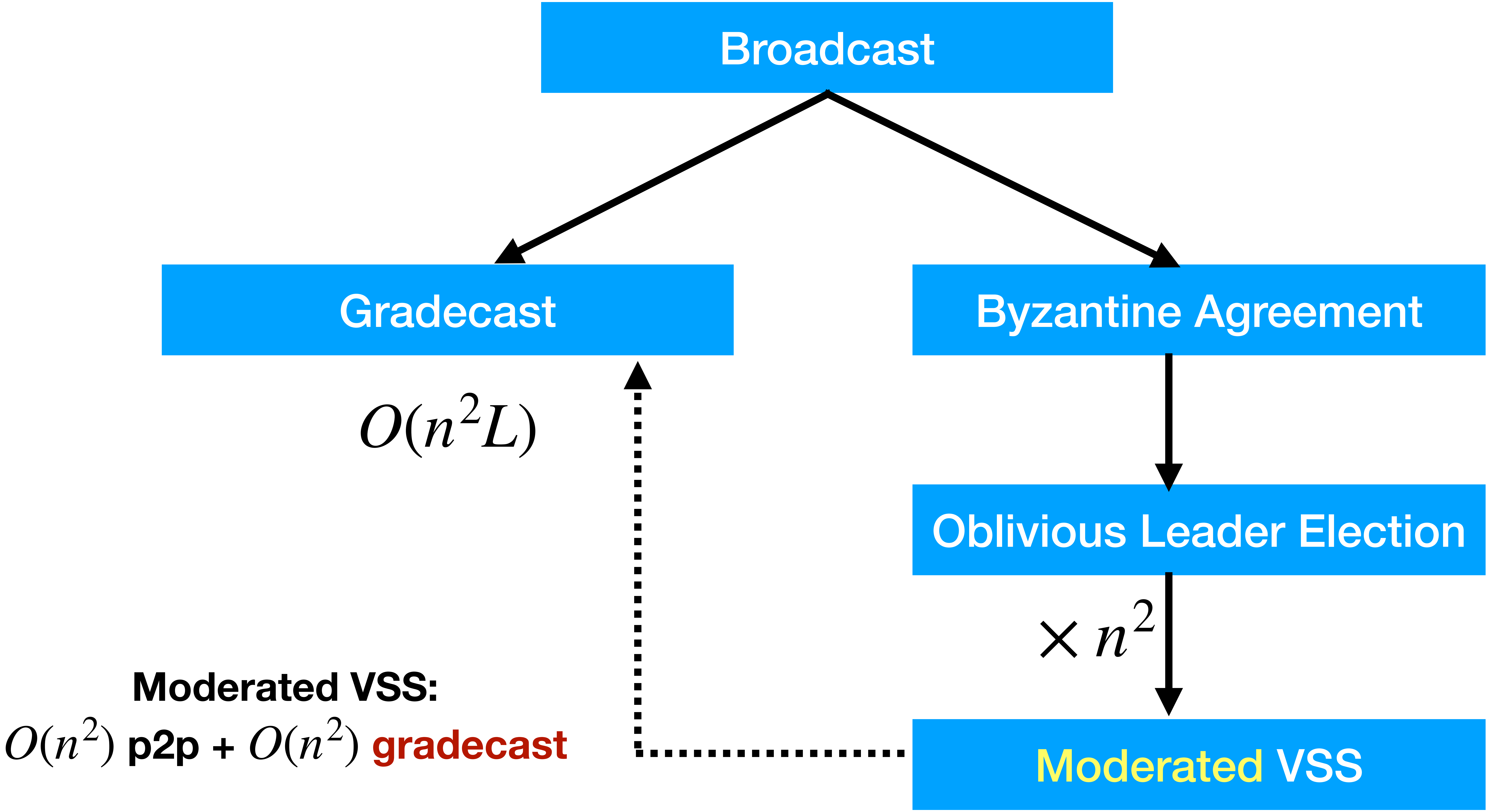

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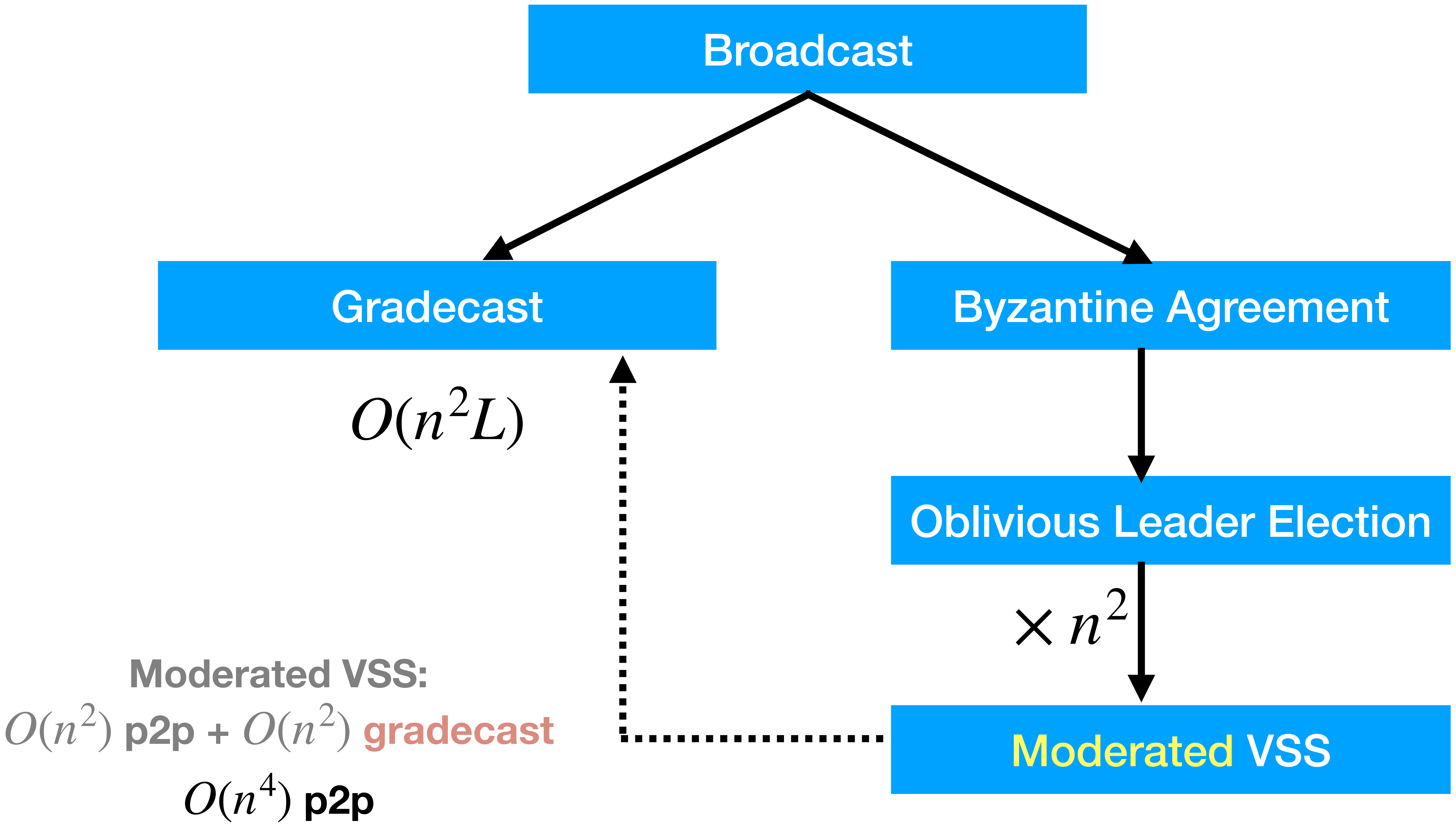

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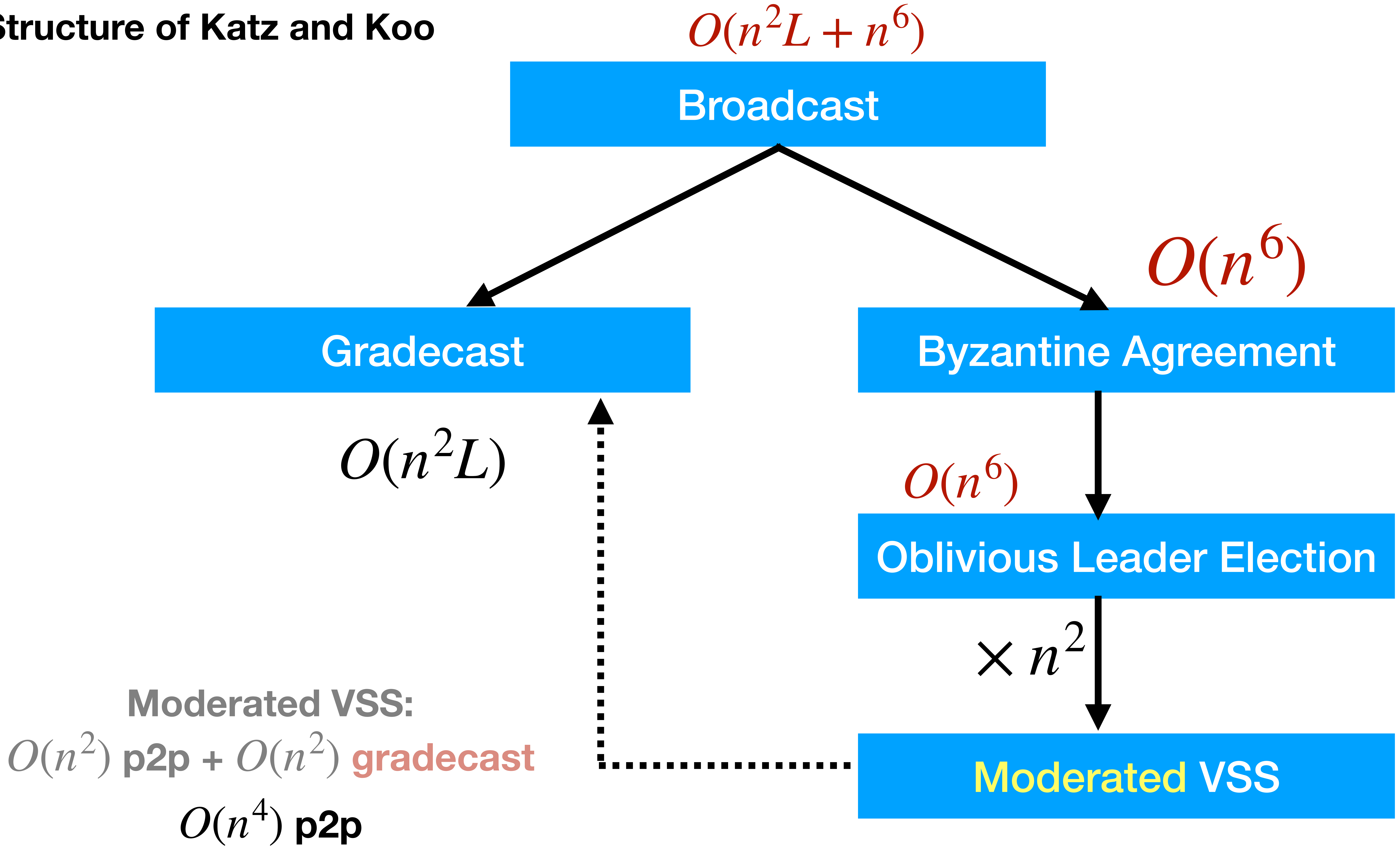
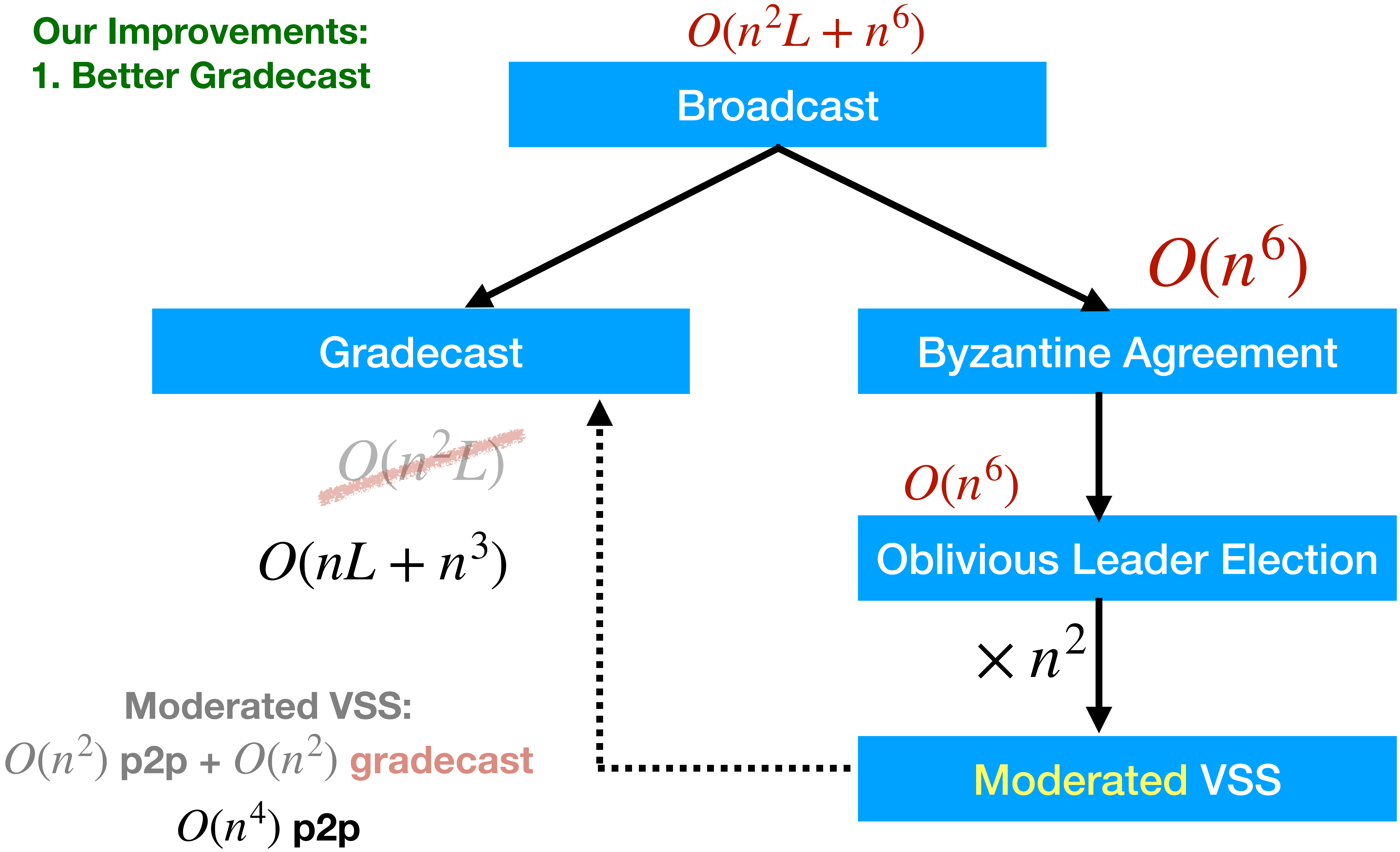
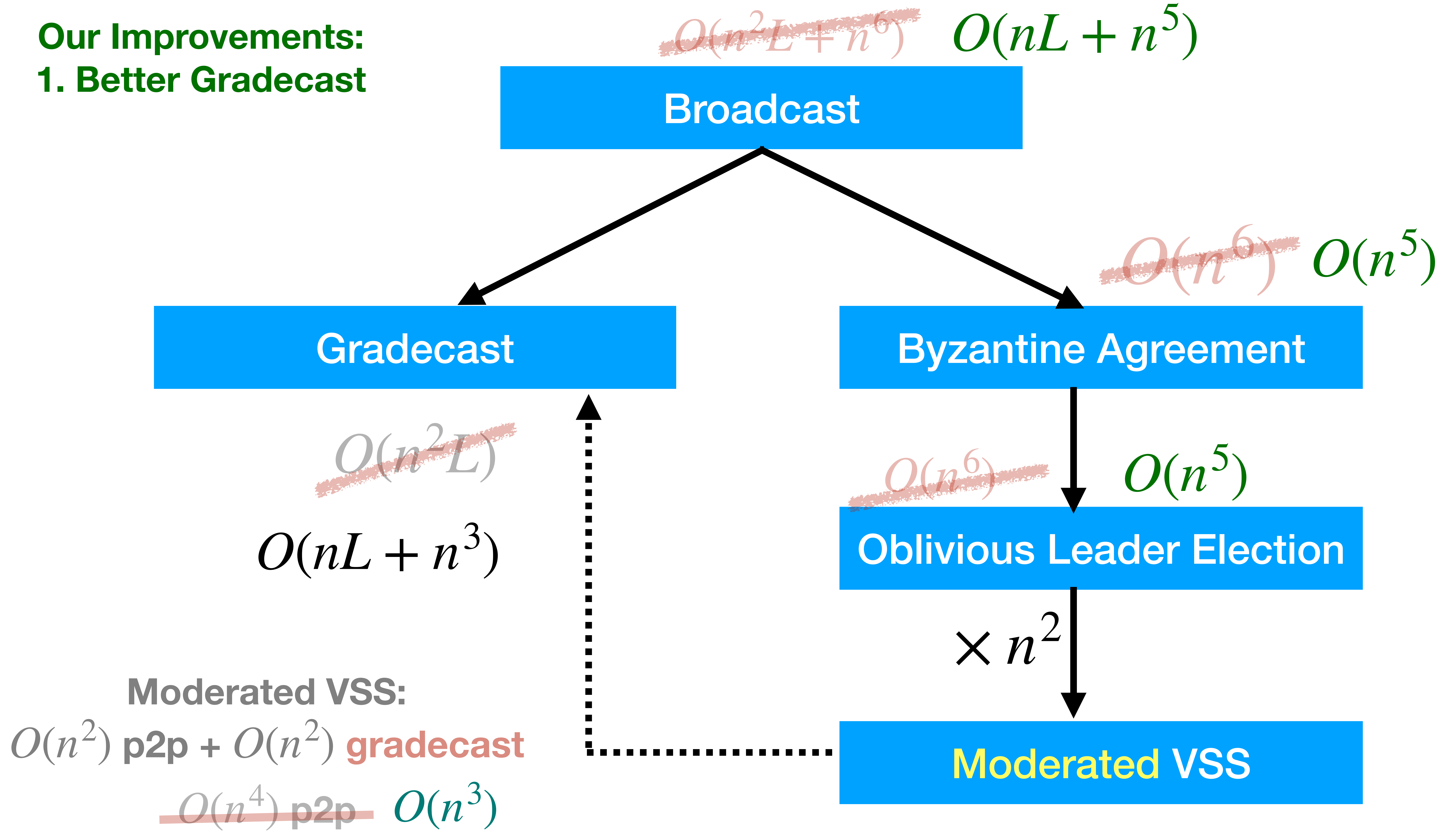


Our Improvements:
1. Better Gradecast





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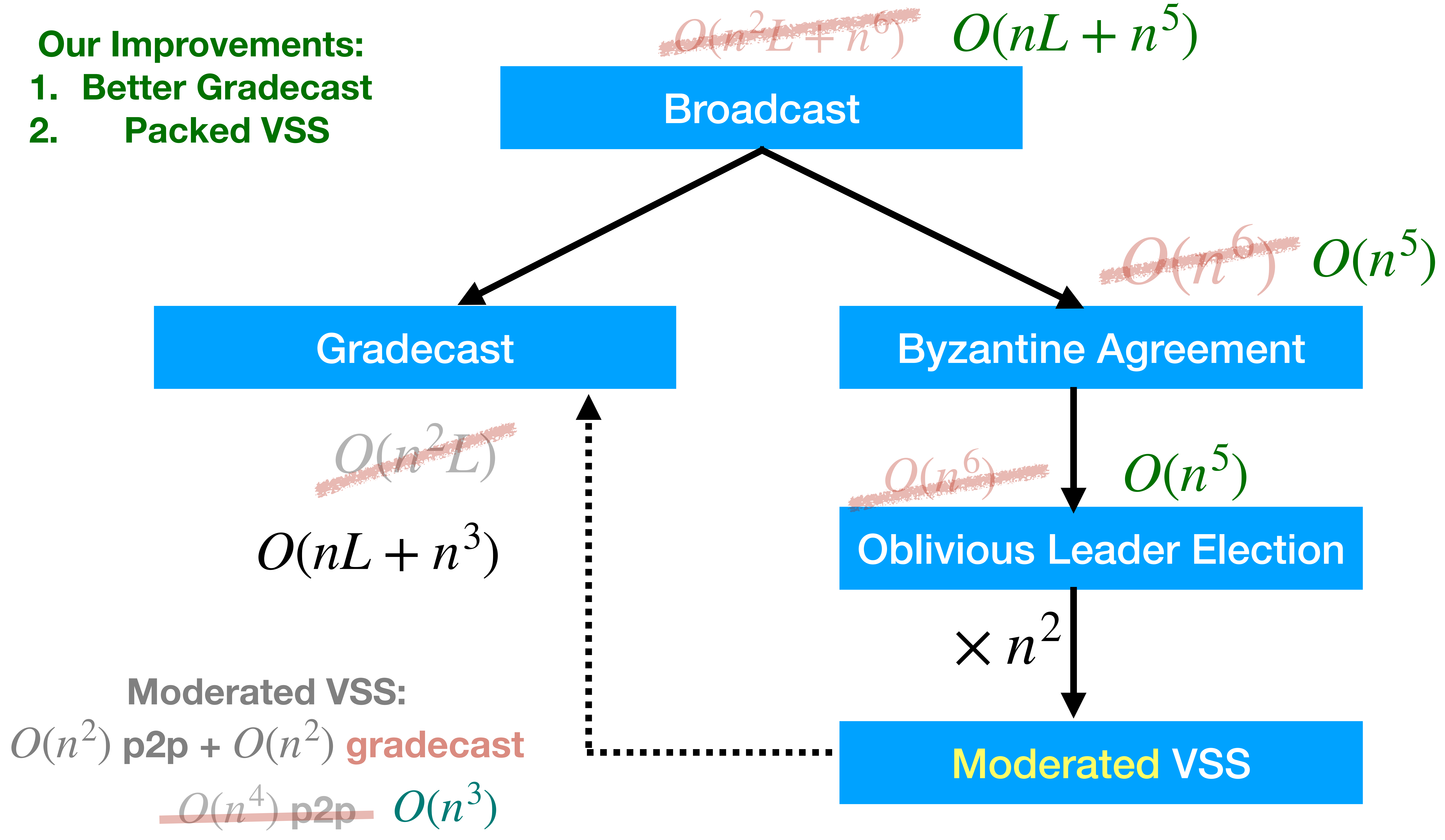
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$O(n)$ improvement over BGW!

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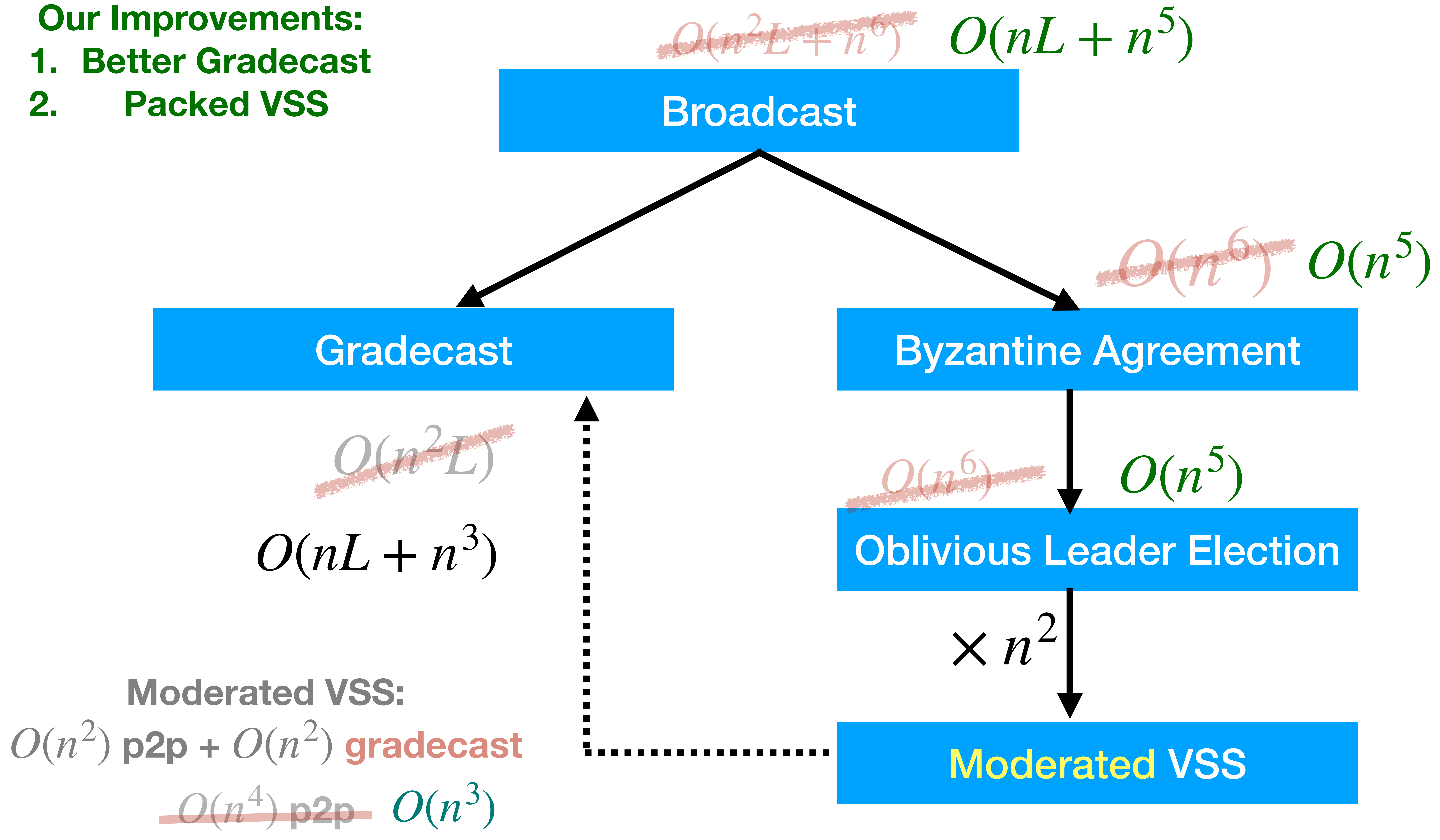
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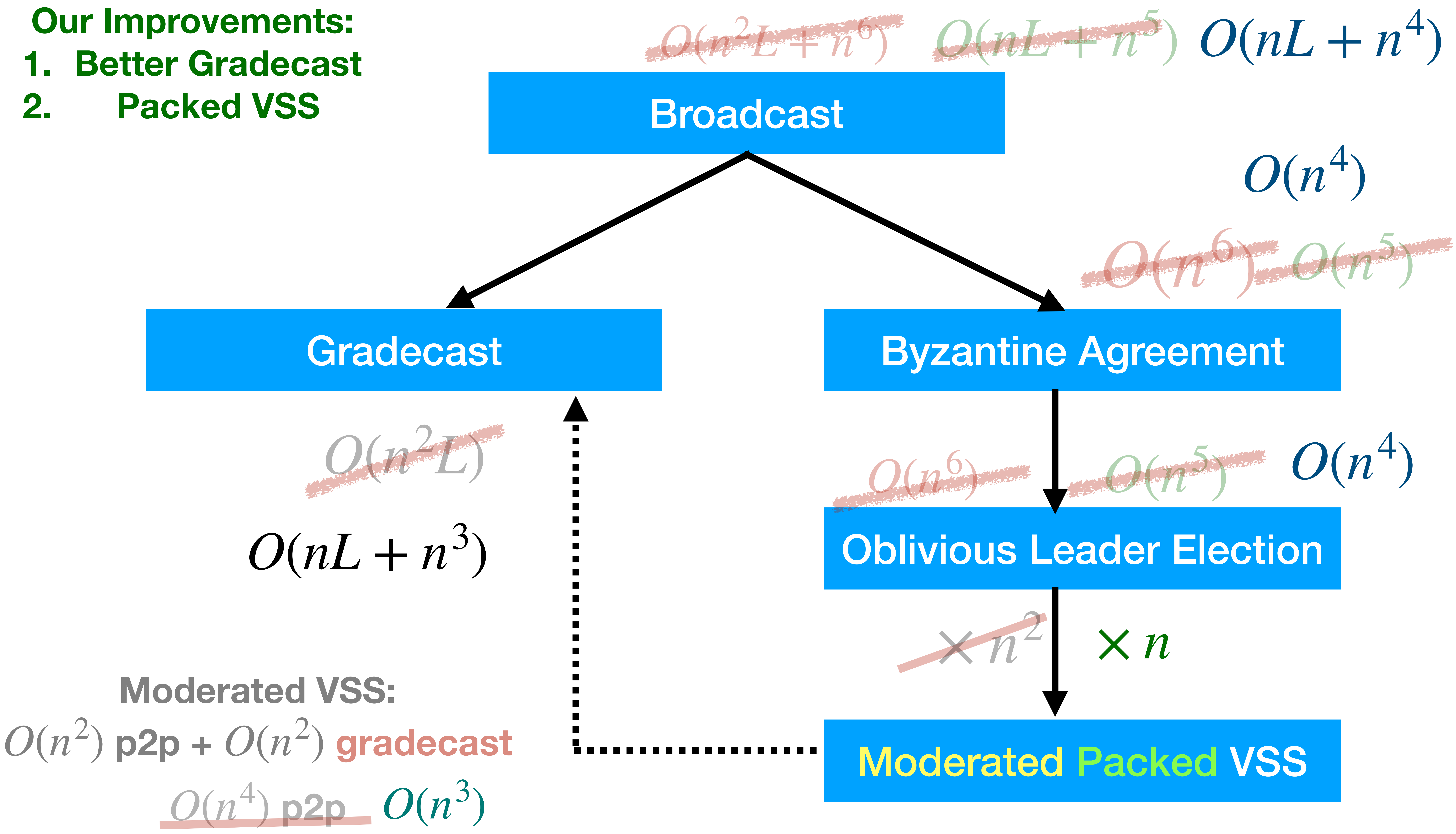

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Thank You!