On the Worst-Case Inefficiency of CGKA

Alexander Bienstock, Yevgeniy Dodis, Sanjam Garg, Garrison Grogan, Mohammad Hajiabadi, Paul Rösler

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- Continuous Group Key Agreement (CGKA) at the core of SGM
  - Creates shared secret for each group operation (used to encrypt msgs, etc)

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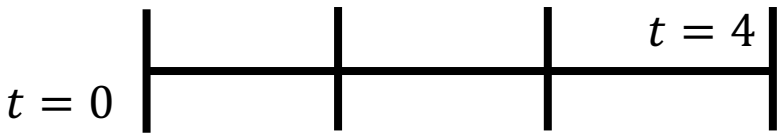

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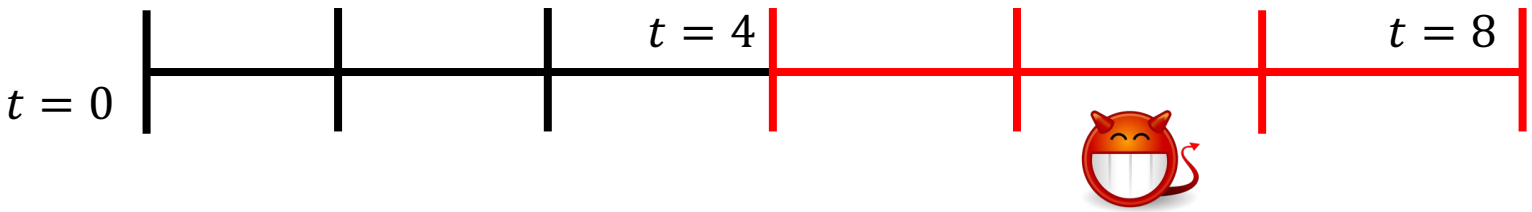

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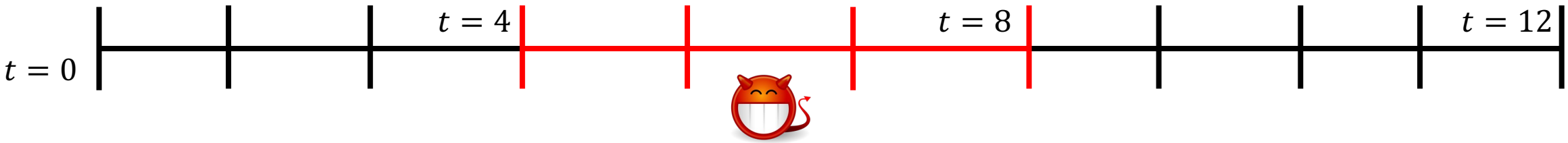

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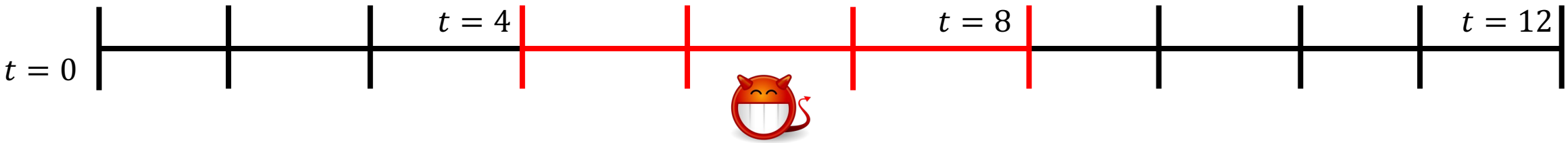

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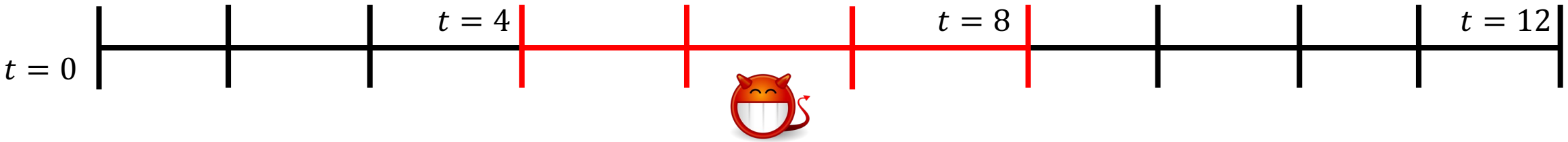

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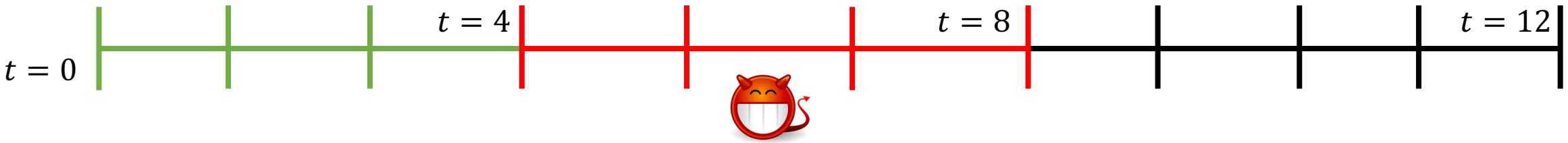

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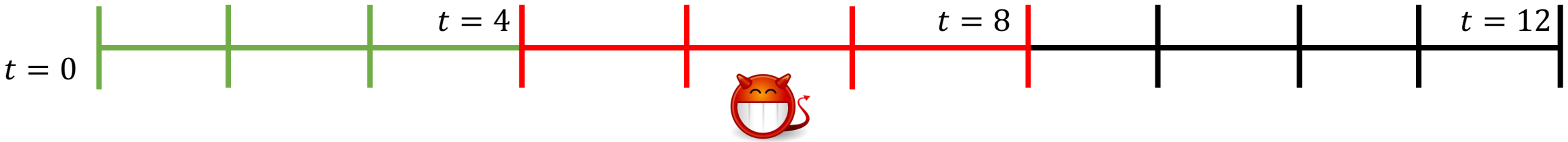

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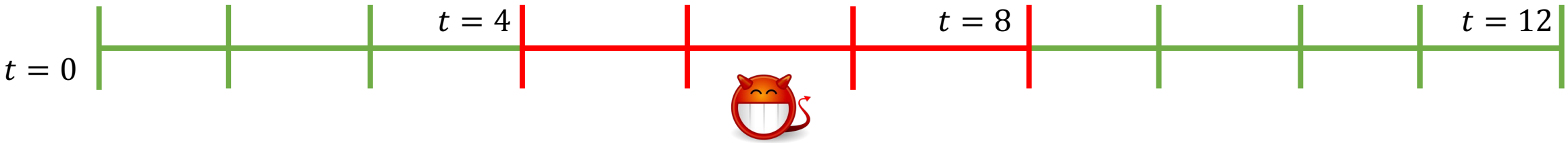

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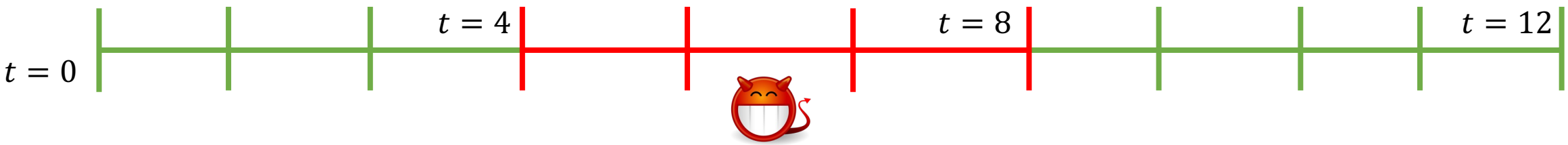

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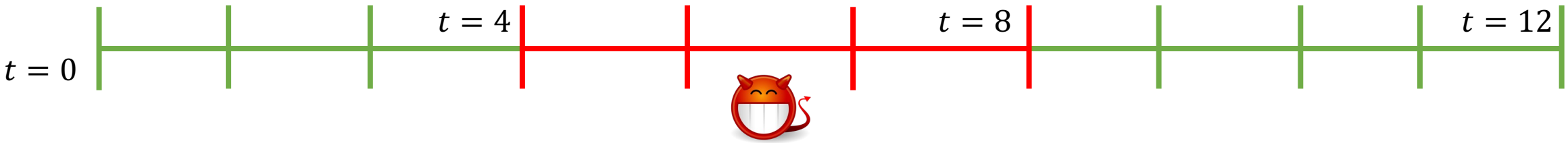

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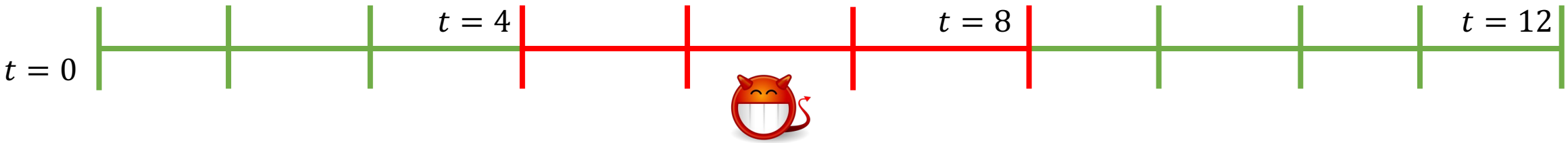

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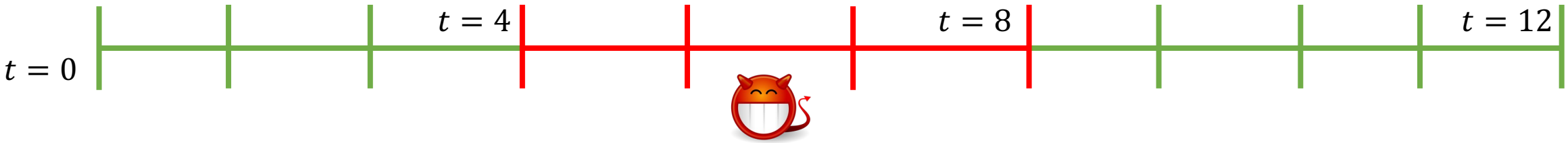

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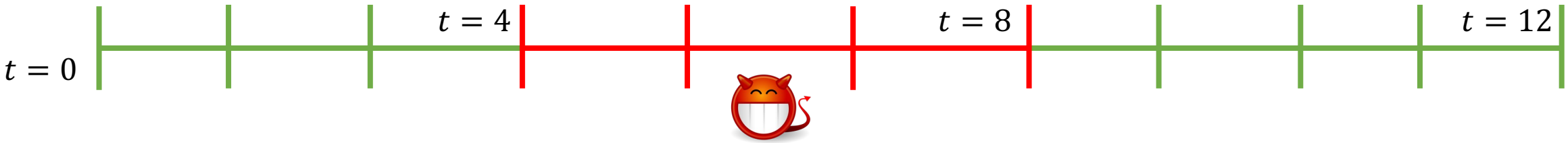

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- Some protocols indeed claim "fair-weather" $O(\log n)$
- But all *known* schemes have $\Omega(n)$ worst-case efficiency
  - Even for communication!
  - Even amortized over long periods of time!
- We show: All CGKA schemes black-box from PKE must suffer from this worst-case inefficiency

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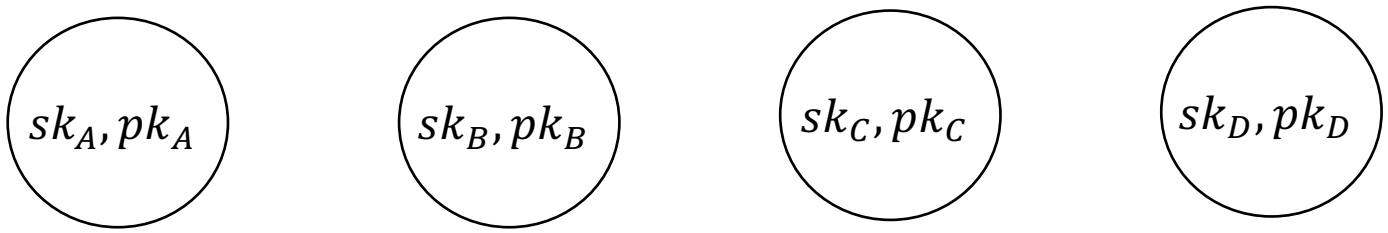

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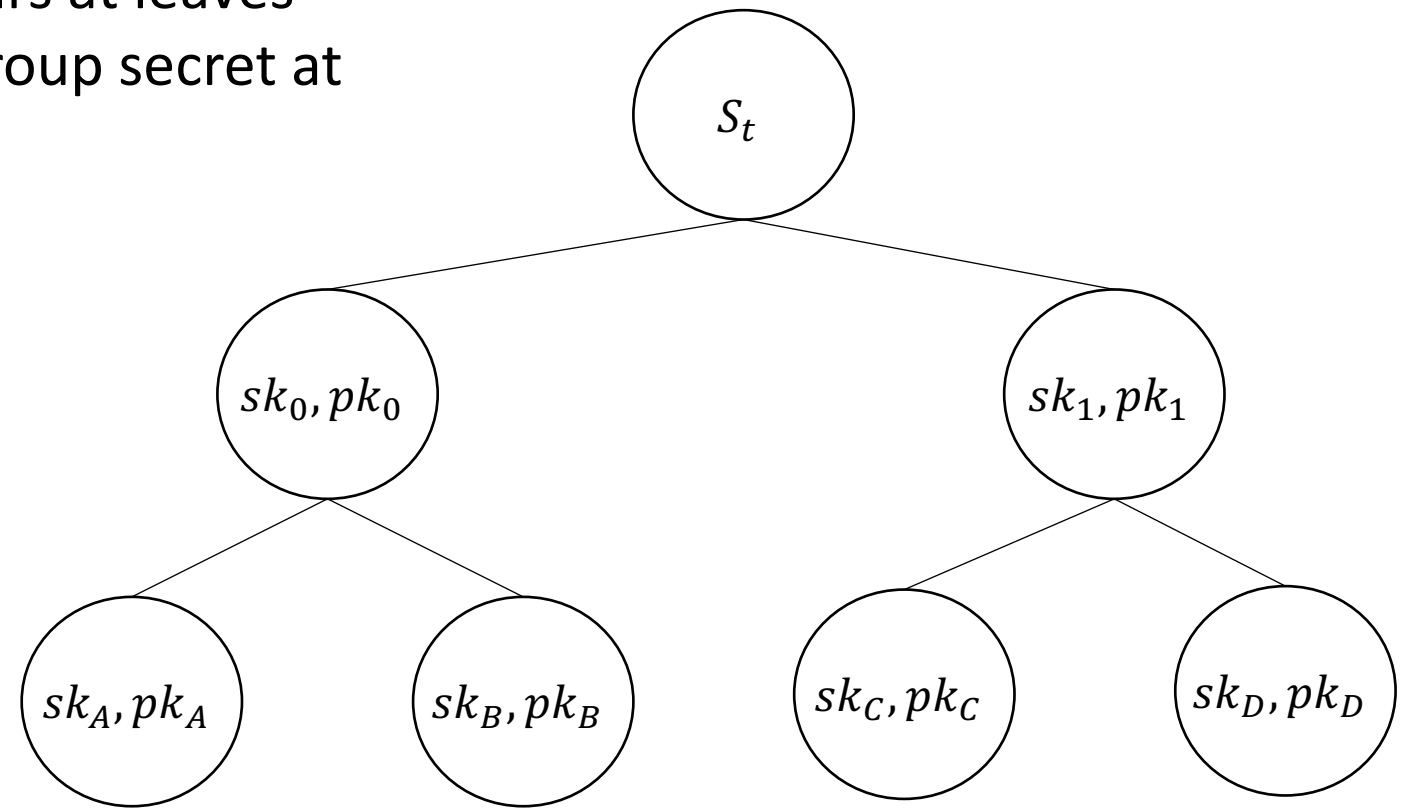

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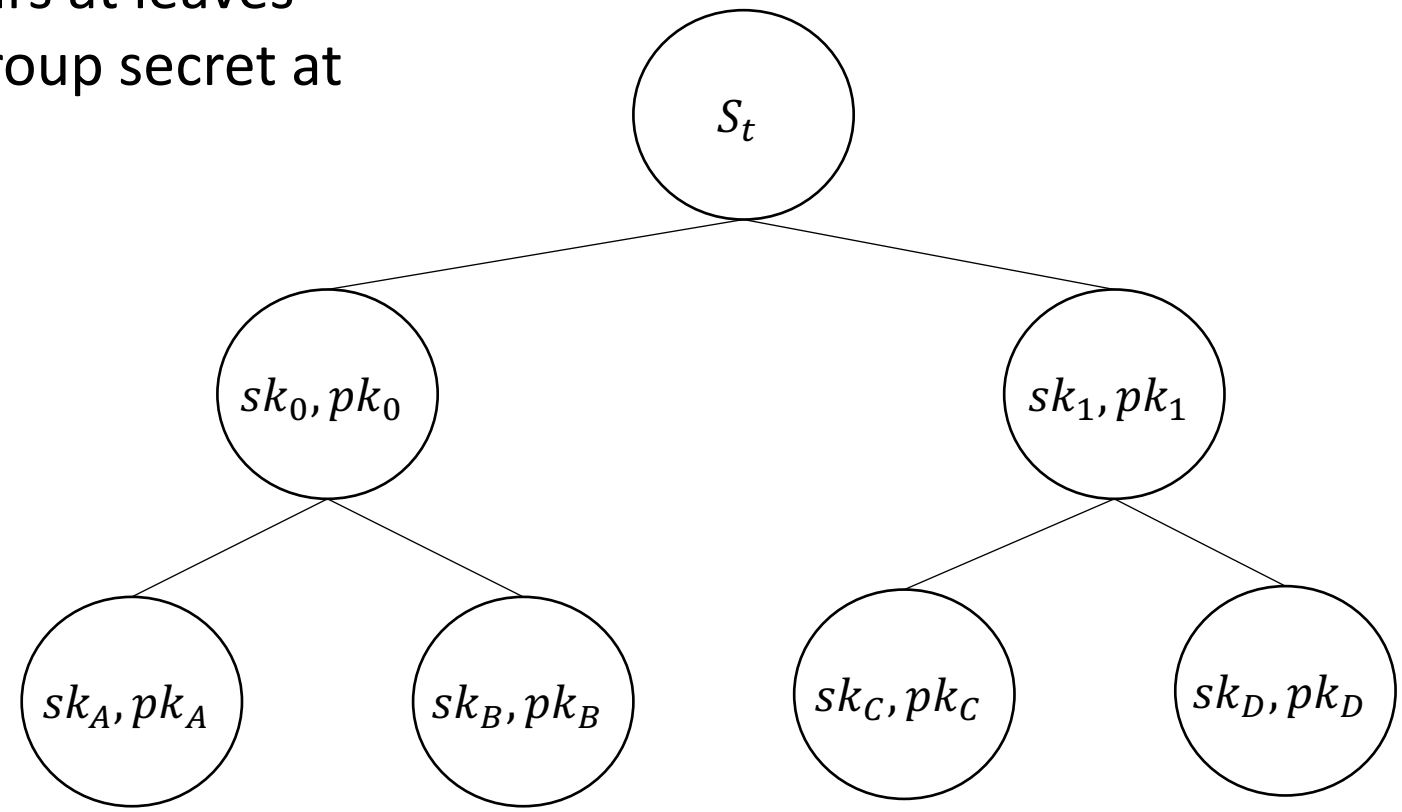

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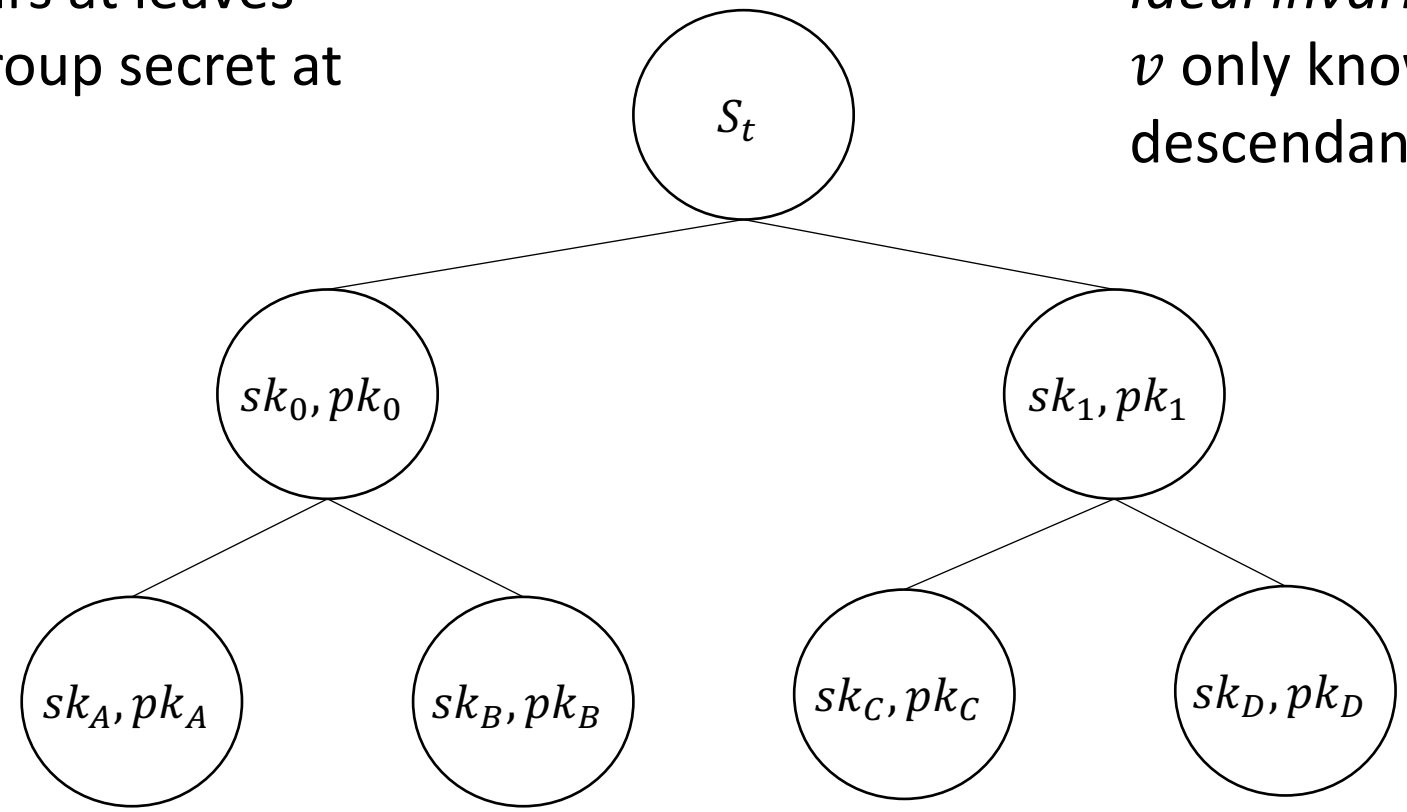

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- *Ideal invariant*: sk_v at node v only known to descendants

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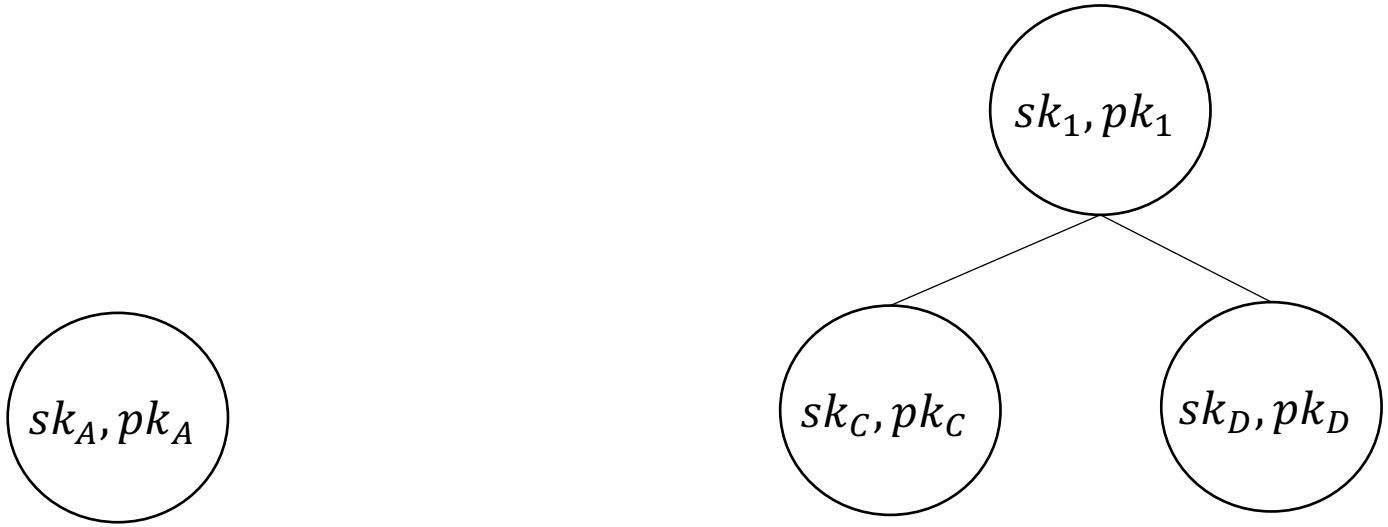

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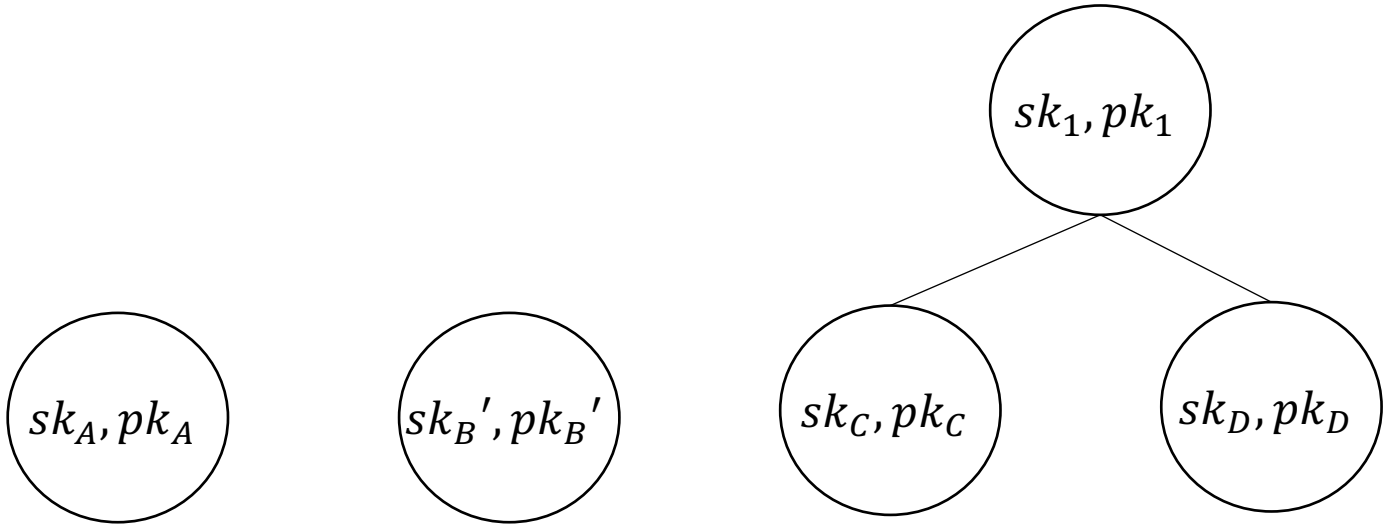

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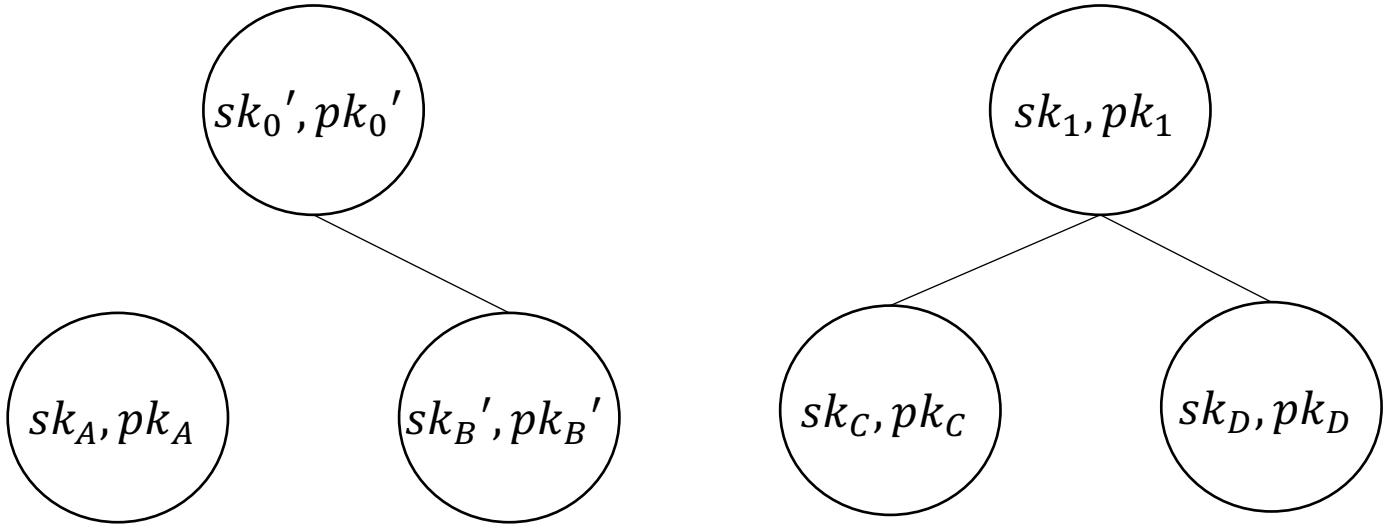

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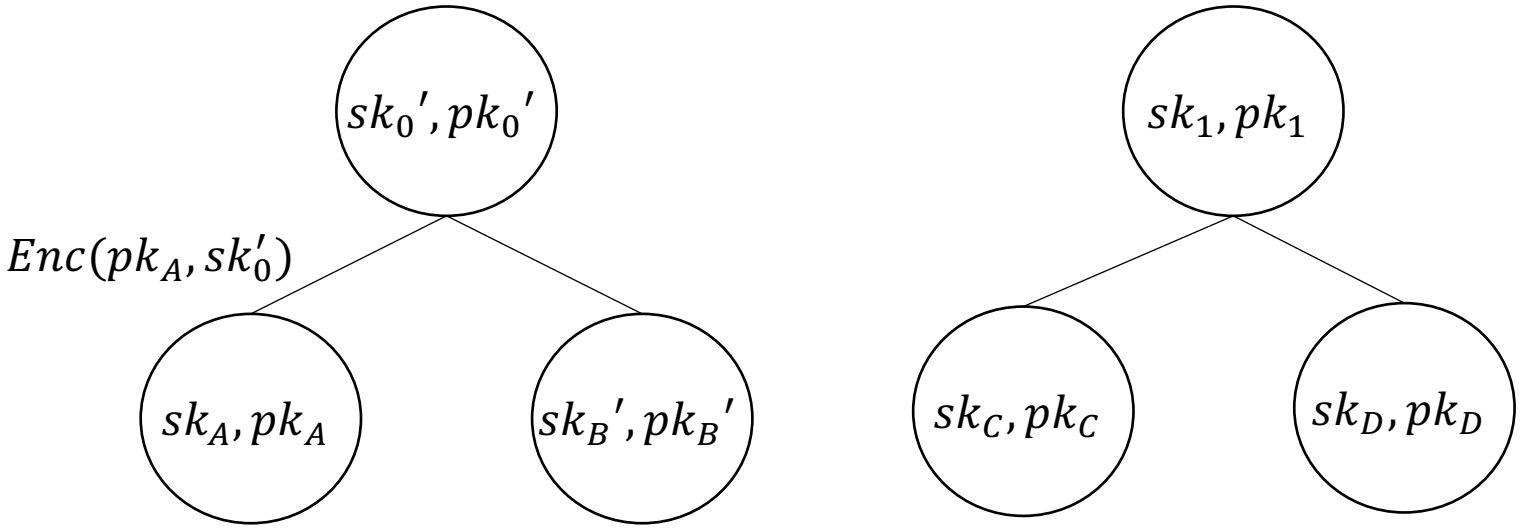

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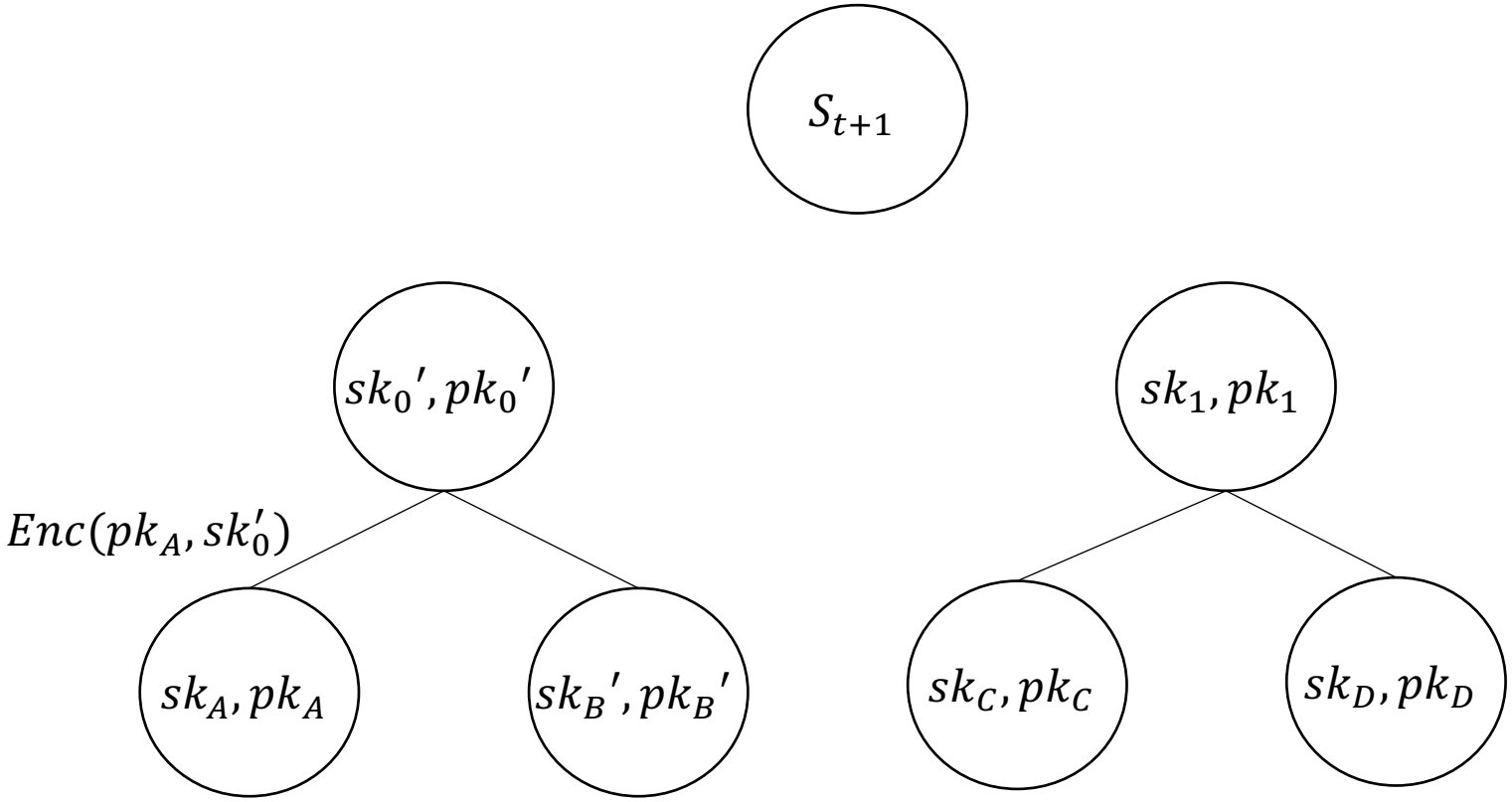

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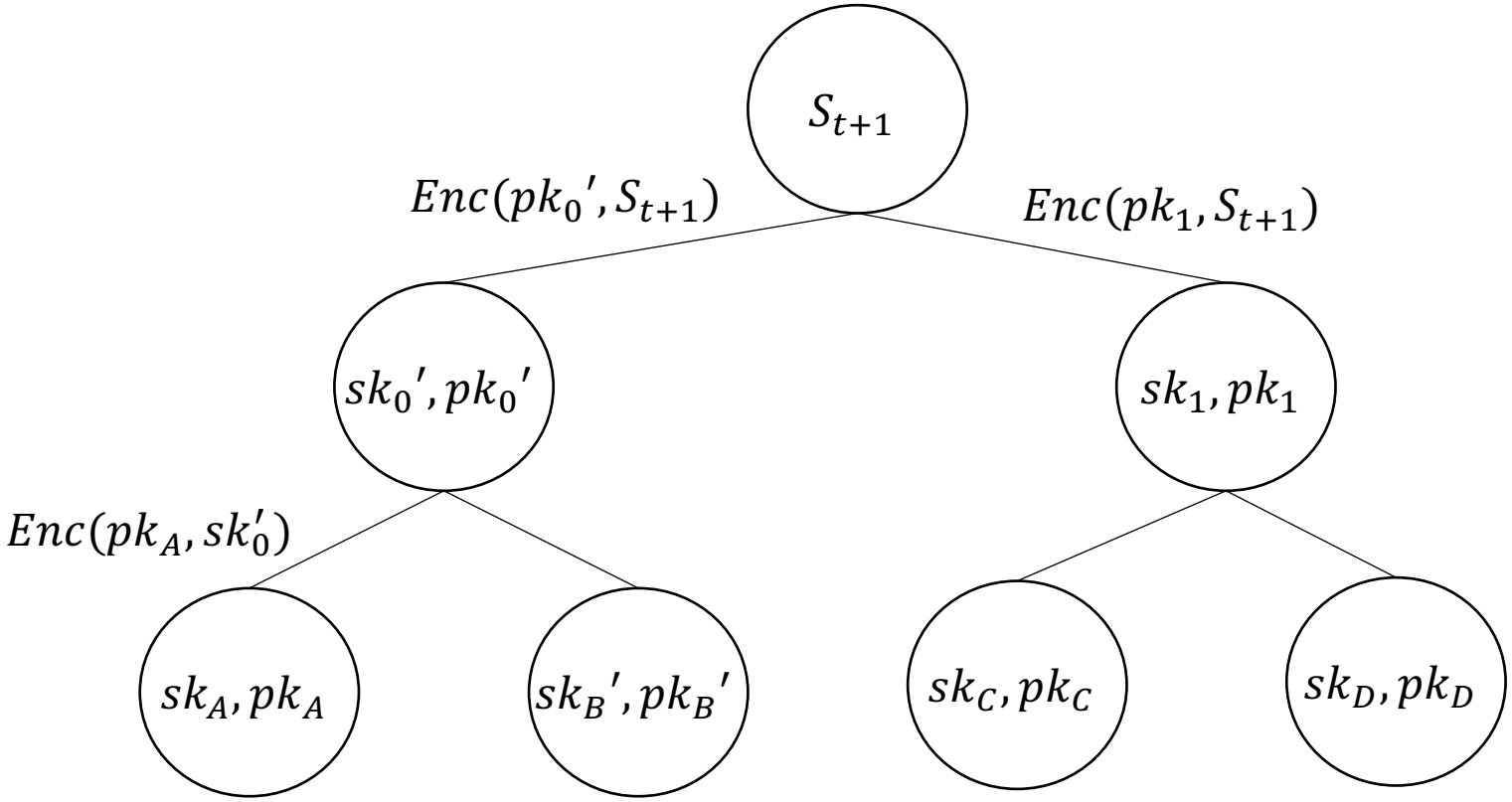

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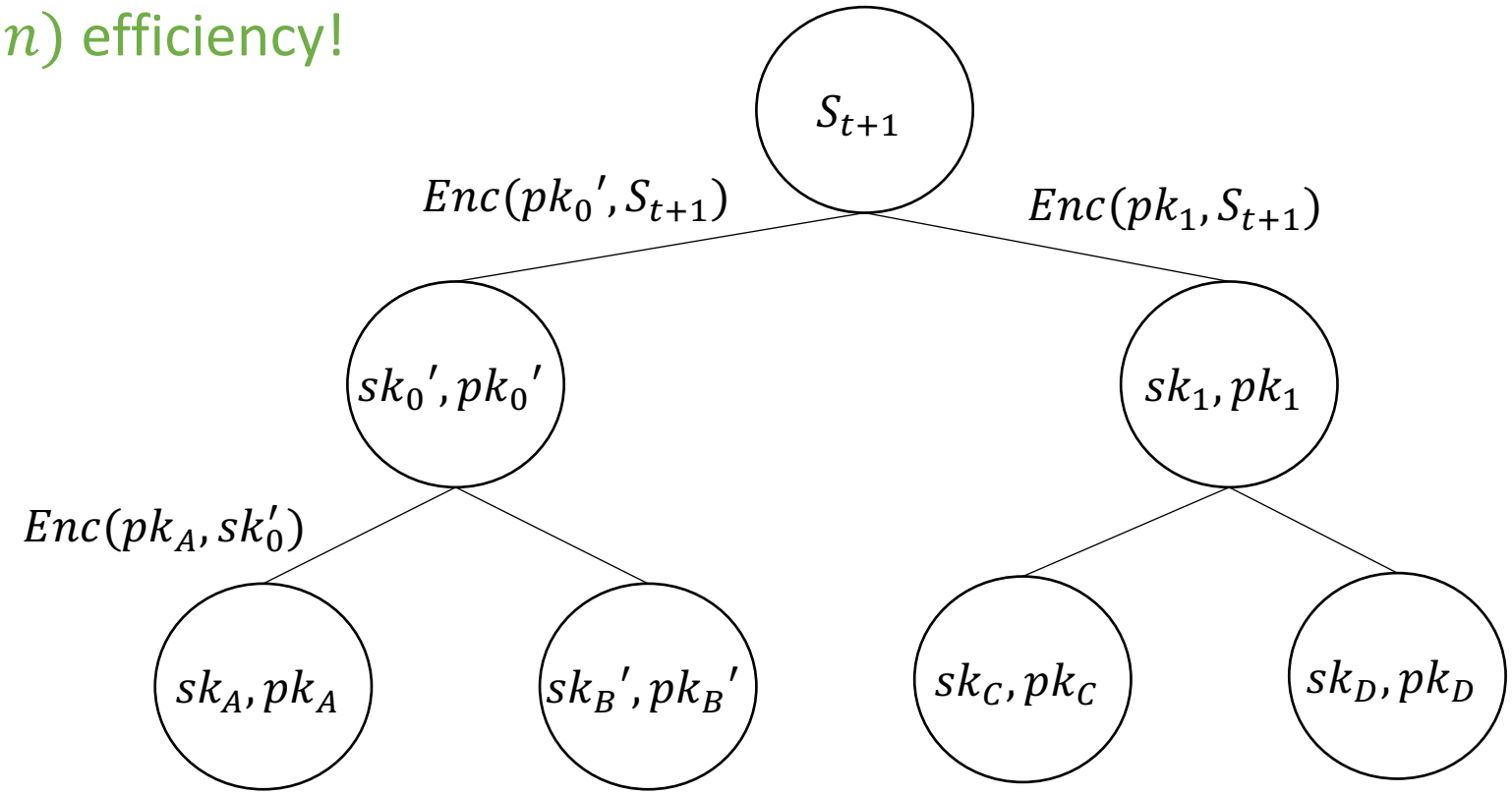

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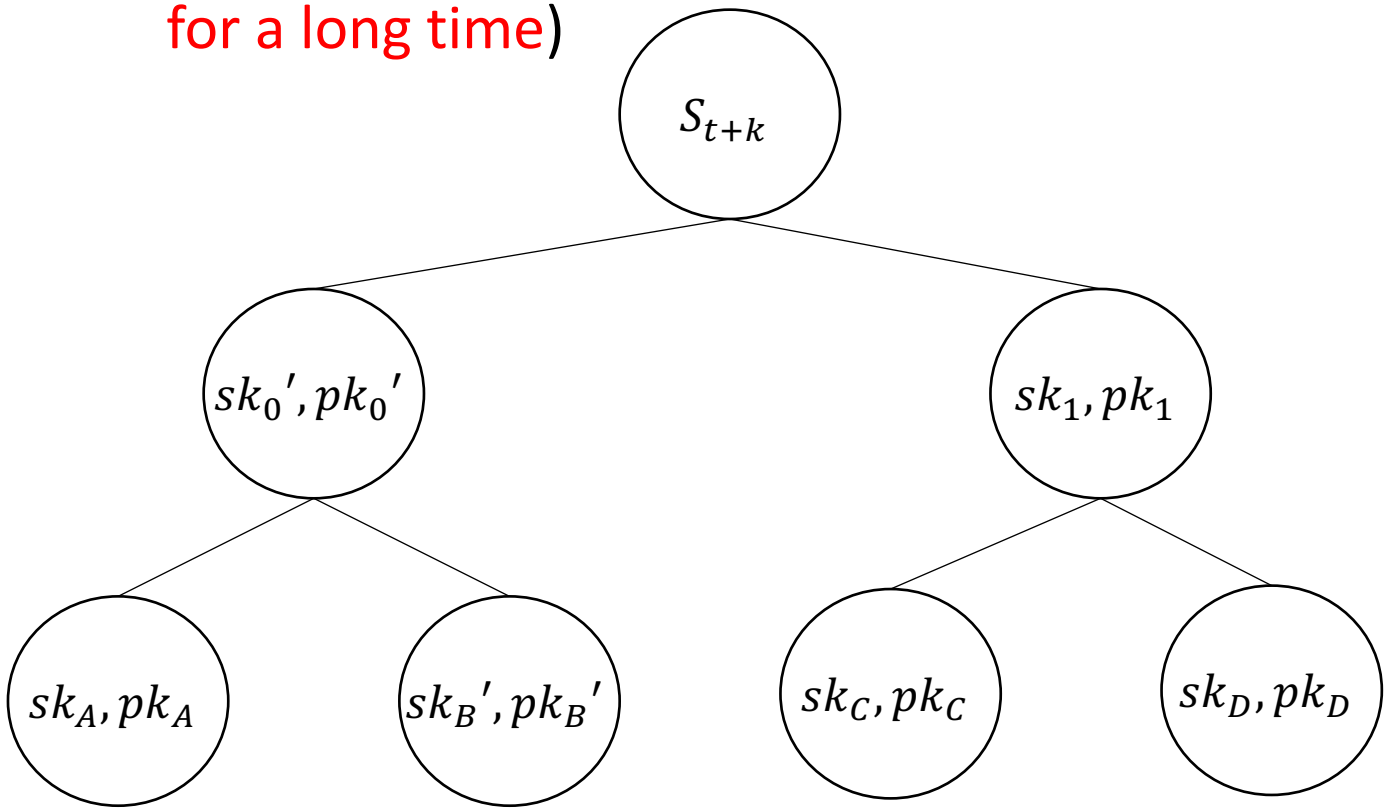

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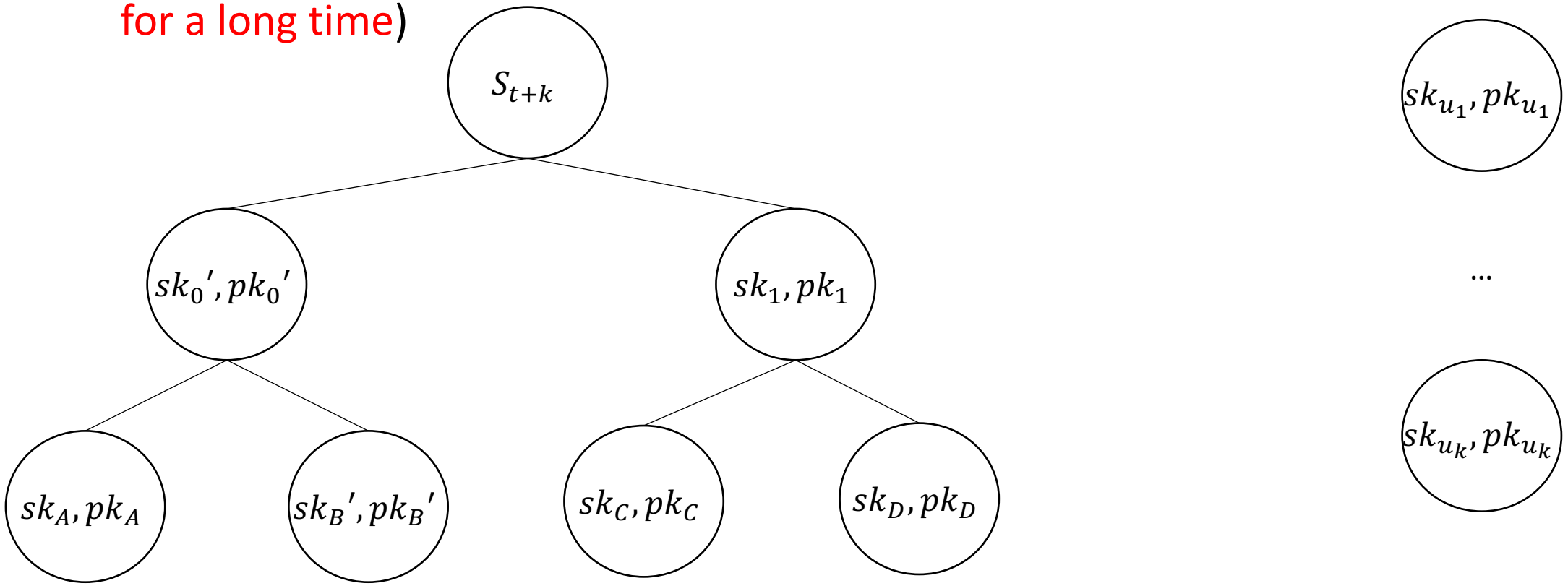

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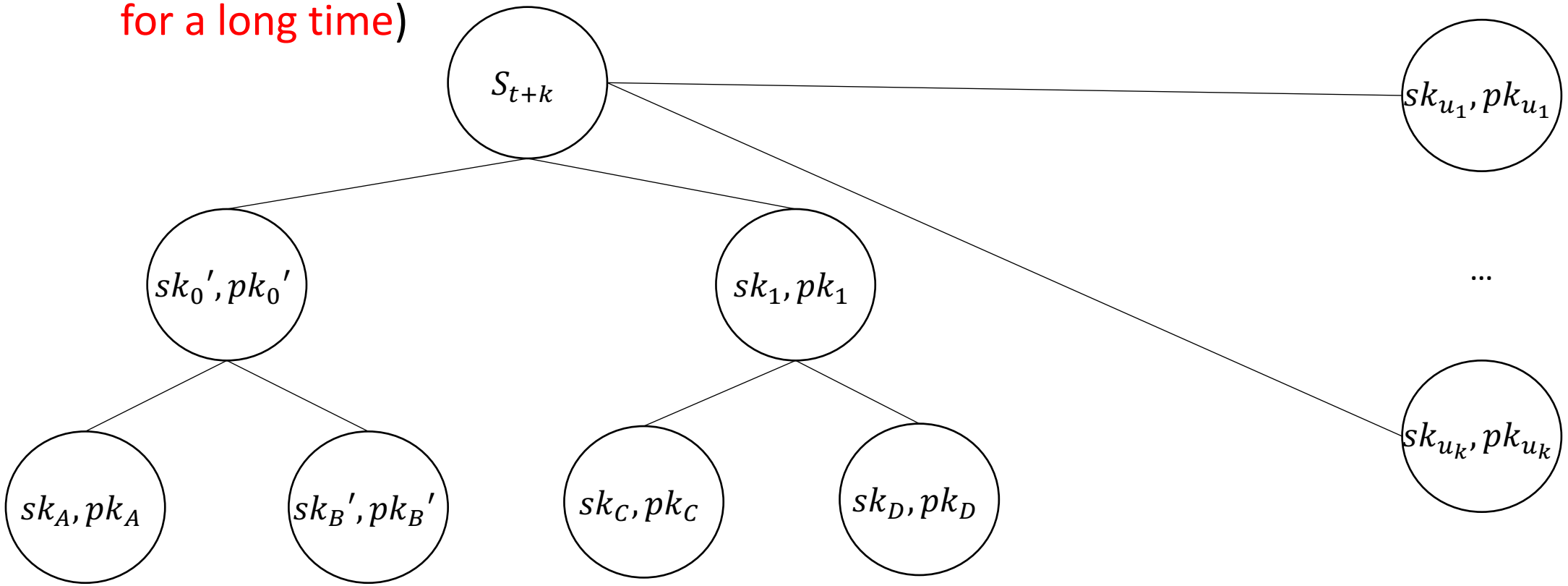

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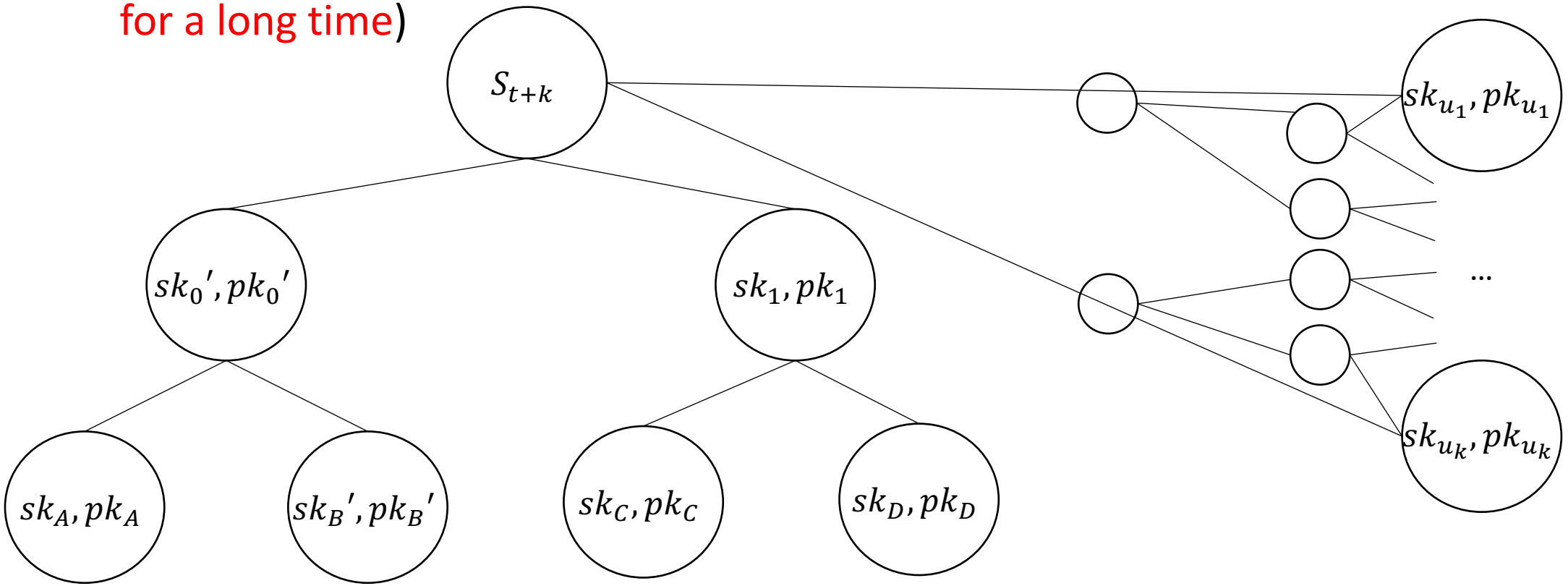

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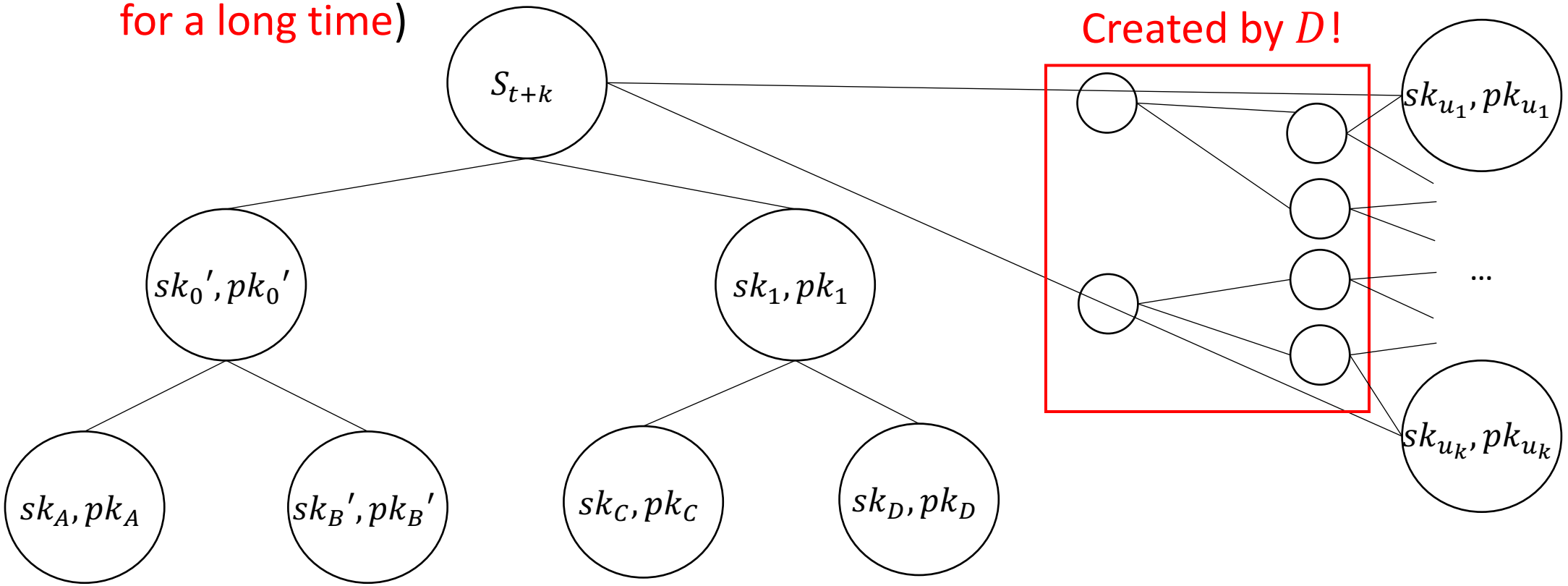

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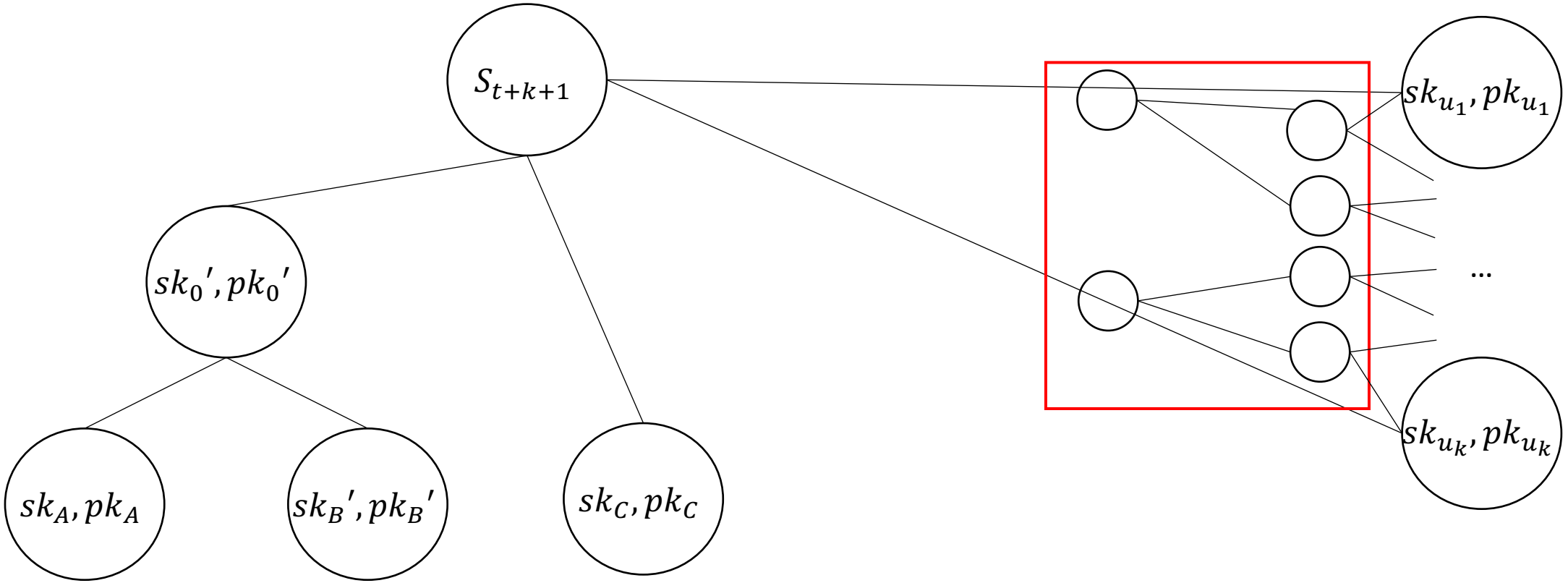

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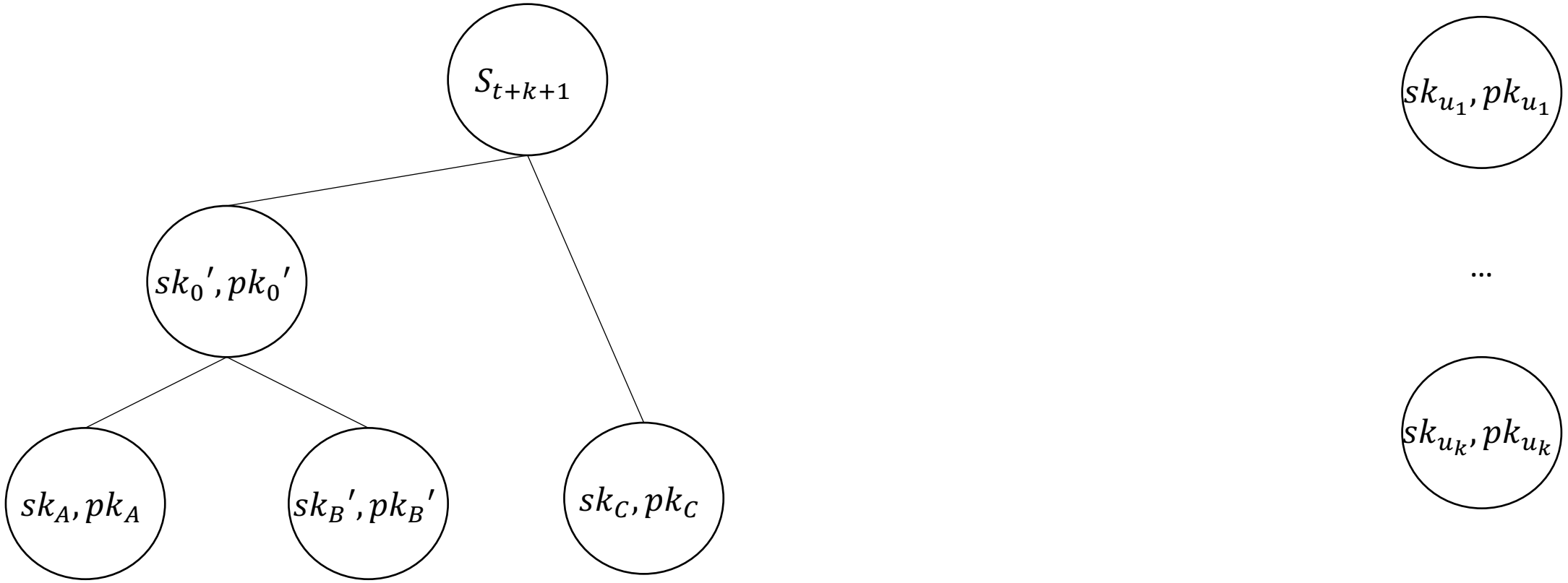

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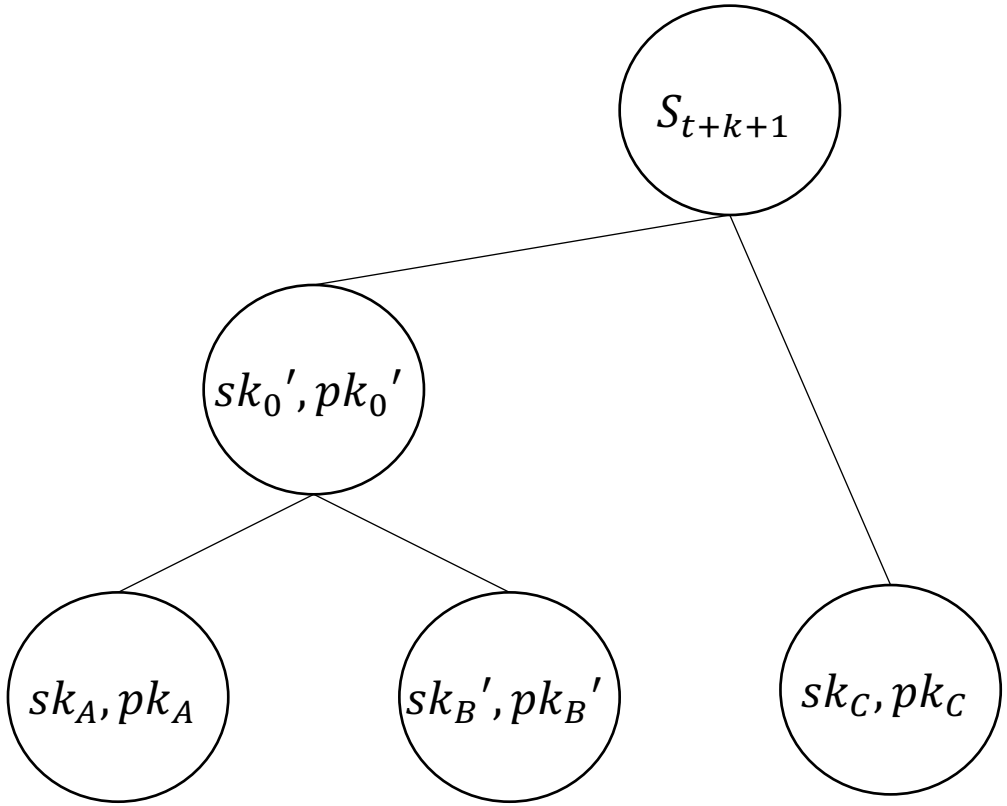

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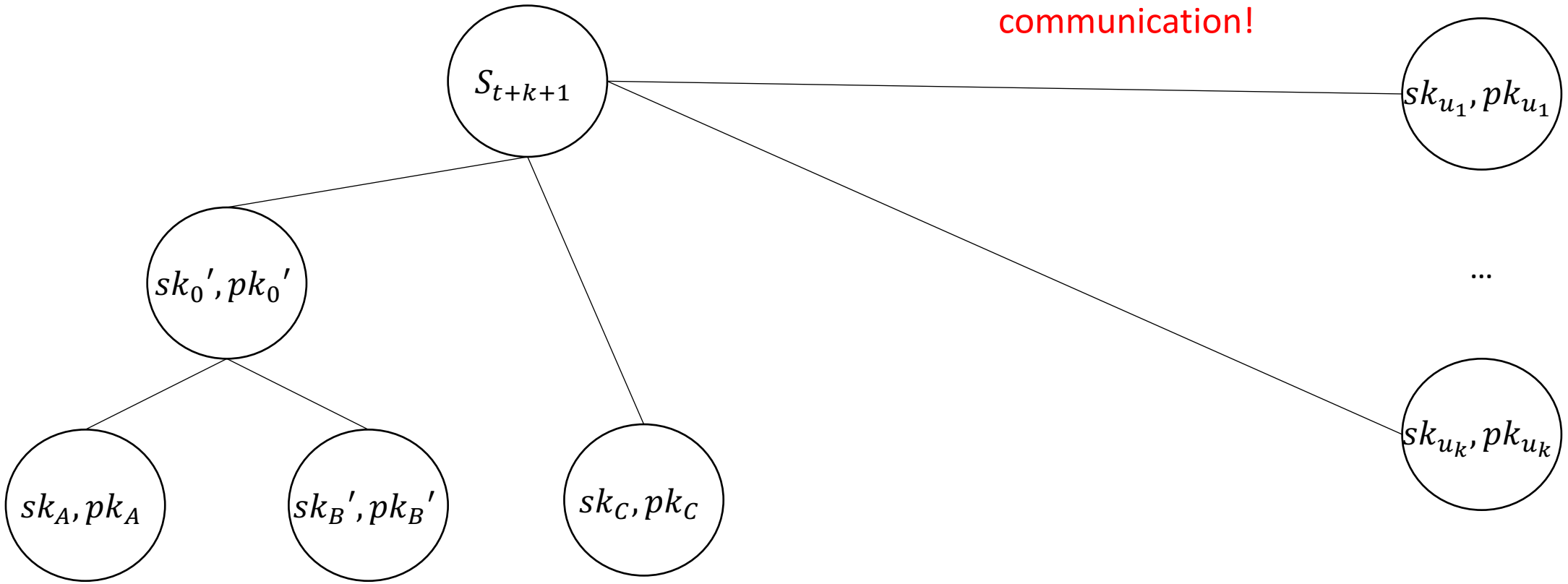

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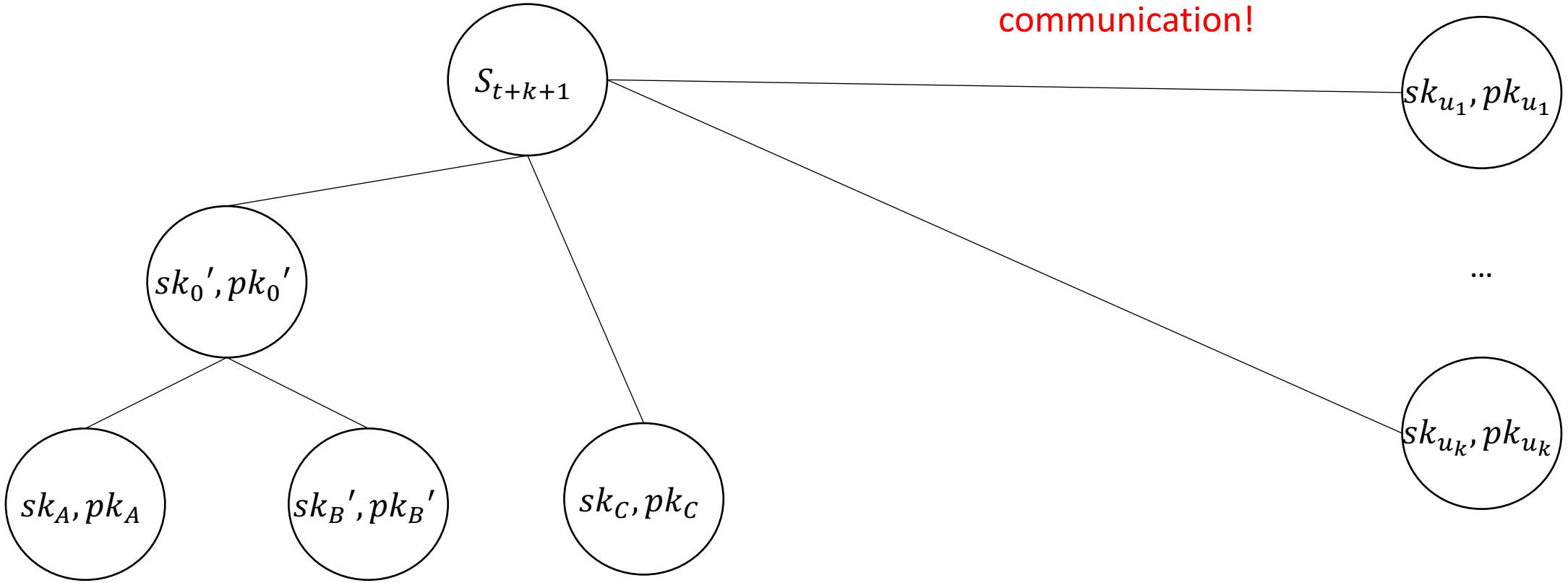

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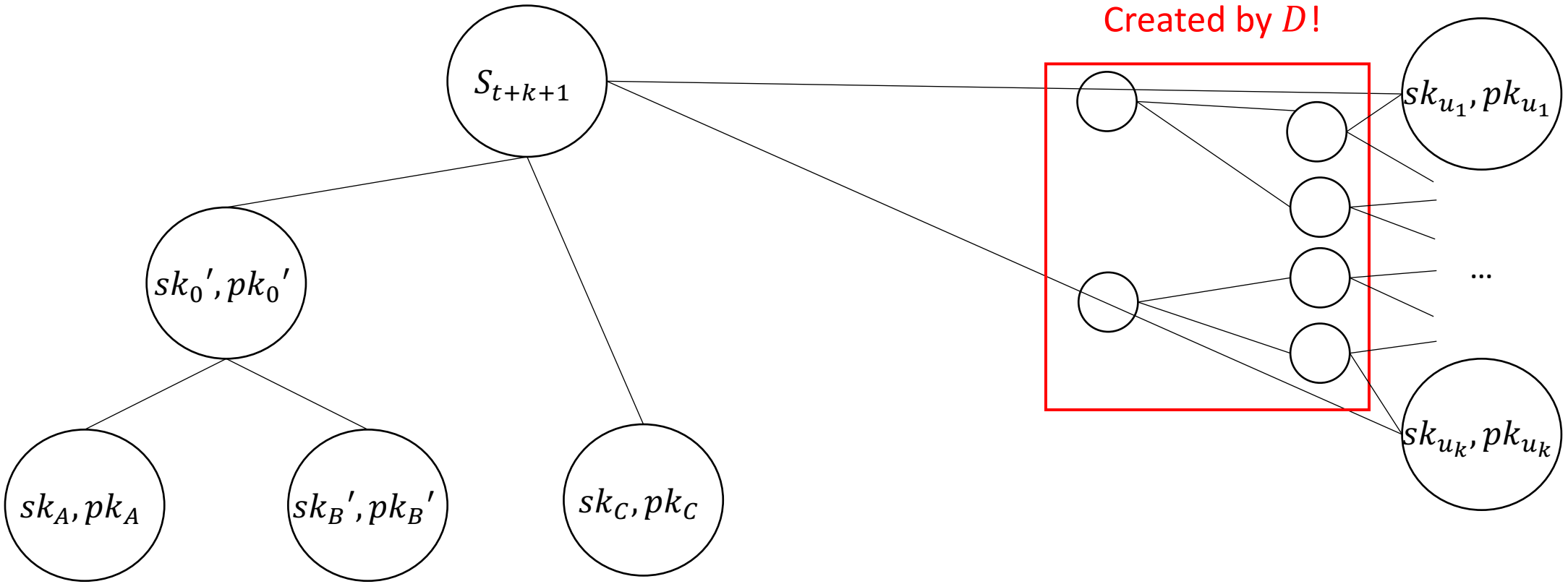

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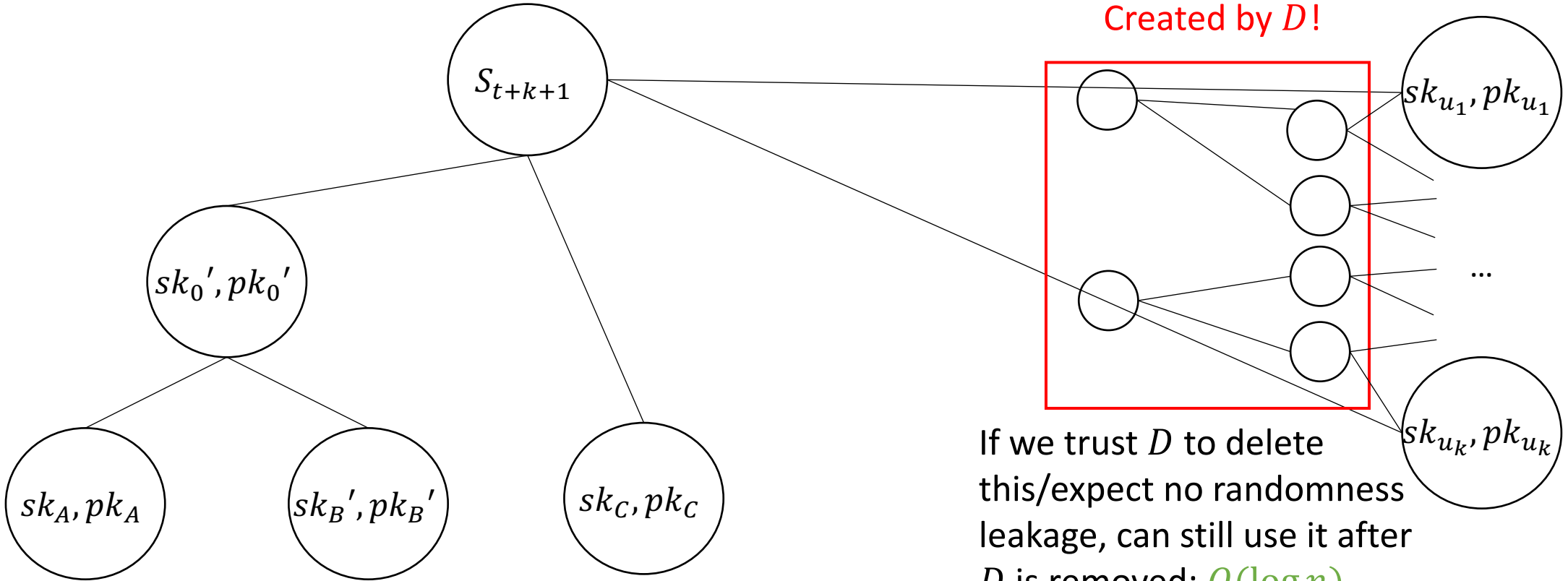

If we trust D to delete this/expect no randomness leakage, can still use it after D is removed: $O(\log n)$ efficiency

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   - So k users' states not independent and can sample correlation successfully
3. Then show that CGKA => CKE, tightly in terms of communication
   - Via example sequence from previous slides -- for *any* protocol the only state added users $u_1, \ldots, u_k$ share with other group members was created by D
   - So after D removed, must communicate fresh key to k independent pk's
   - Also a bit more general (not just D, but multiple users)

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  - Prove the lower bound w.r.t. stronger primitives/assumptions than PKE
  - Prove better “average-case” communication upper bounds
  - What sequences are average? (huge space between trivially good and very bad)