A Toolbox for Barriers on Interactive Oracle Proofs

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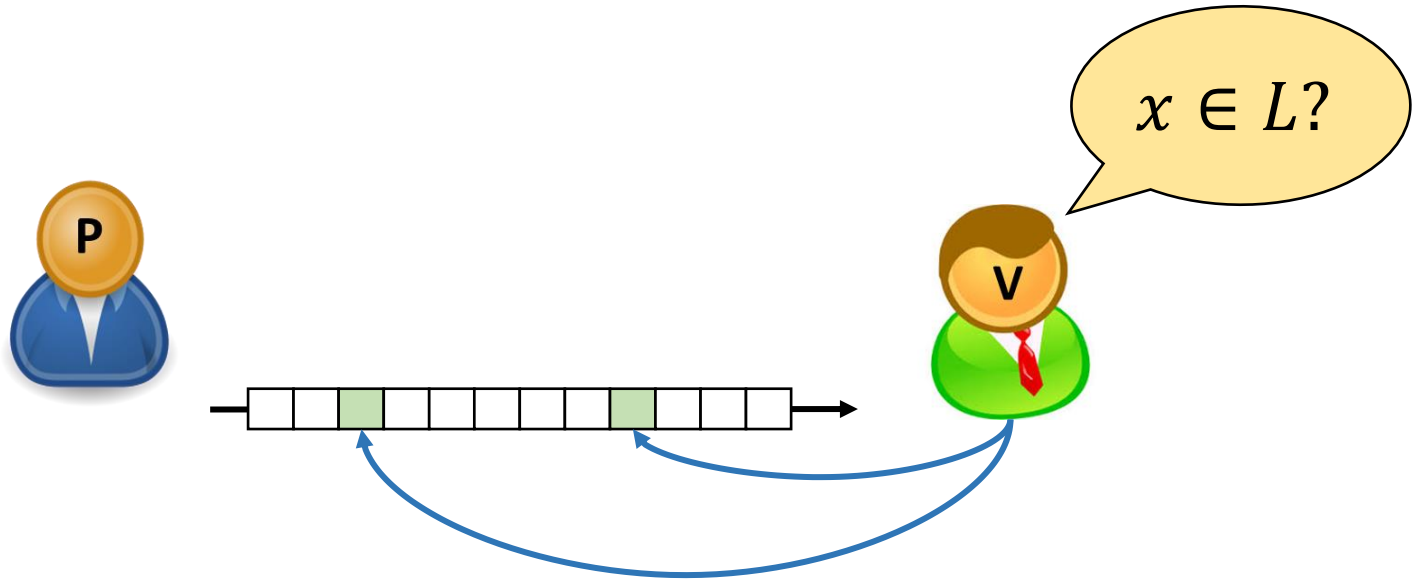
Probabilistically Checkable Proofs



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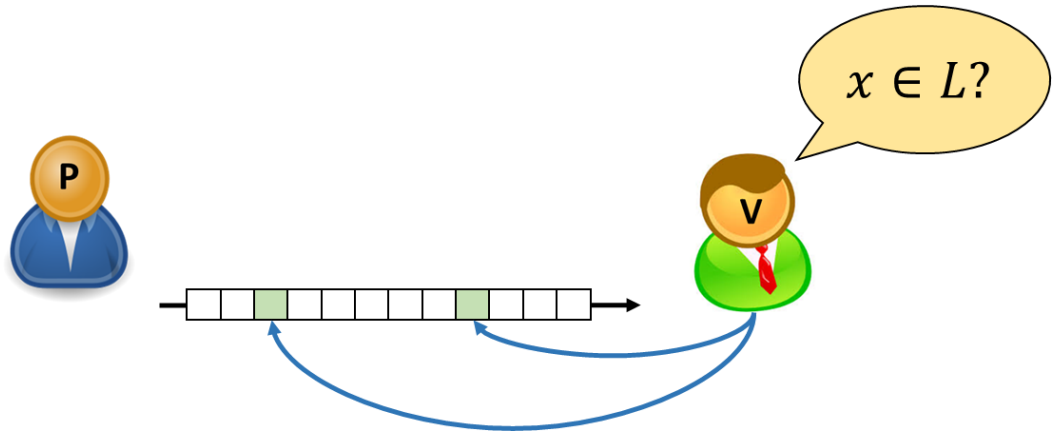

Main parameters of interest:

- Length: number of symbols in proof
- Alphabet size: size of the proof alphabet
- Queries: number of symbols of the proof that are read by Verifier
- Soundness error: probability that the Verifier accepts a false statement

Additional notable parameters

- Completeness error: probability that the Verifier rejects a true statement
- Randomness: number of random coins used by Verifier

(Some of) The known PCP landscape

	Length	Alphabet size	Queries	Soundness error	Possible?
PCP theorem [AS98,ALMSS98]	$poly(n)$	$O(1)$	$O(1)$	$O(1)$	☑

*Some bounds follow from standard hardness assumptions and/or hold for PCPs with non-adaptive verifiers

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Binary-alphabet constant-query PCPs (Folklore)	$2^{o(n)}$	2	2	< 1	☒
	$2^{o(n)}$	2	3	$< \frac{5}{8}$	☒

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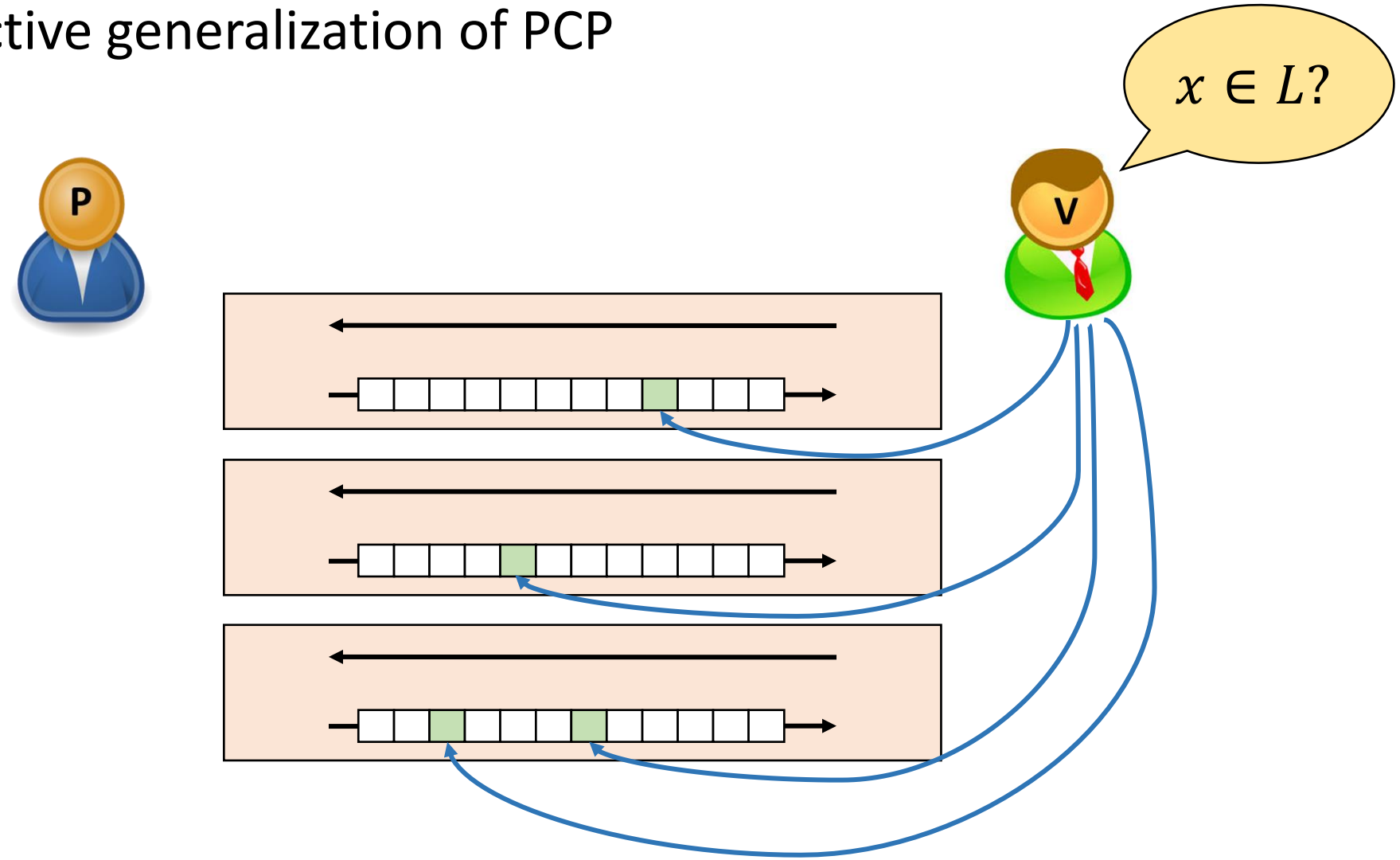
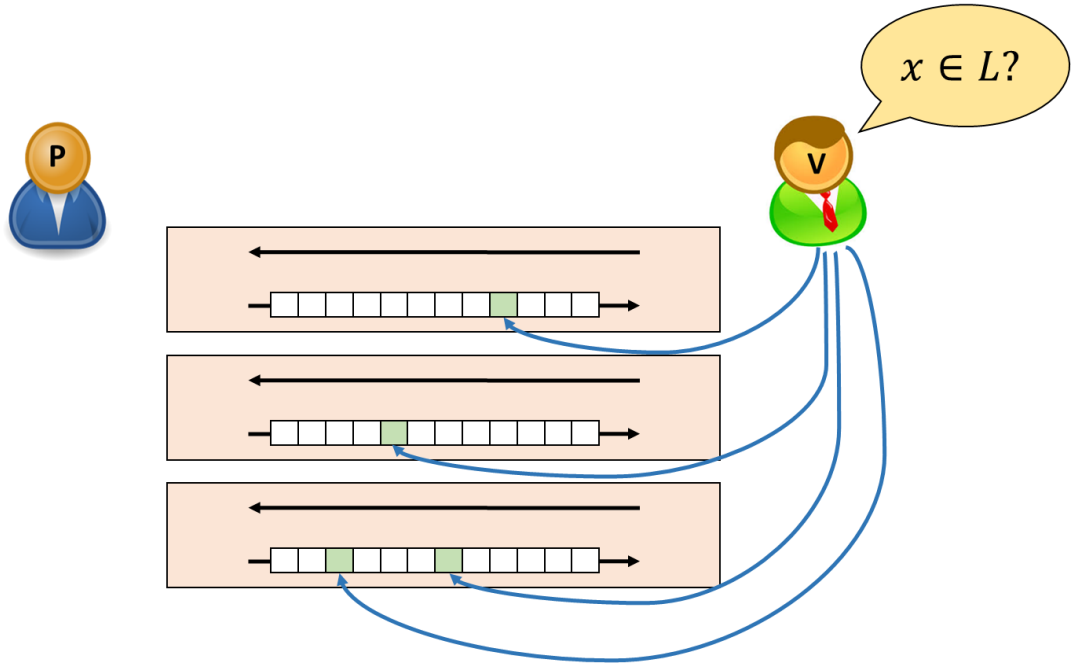
Interactive Oracle Proofs [BCS16,RRR16]

Interactive generalization of PCP



Interactive Oracle Proofs [BCS16,RRR16]



Main parameters of interest:

- Rounds: number of rounds of interaction
- Length: number of symbols in proof
- Alphabet size: size of the proof alphabet
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Additional notable parameters

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Why IOPs?

IOPs have proved useful to achieve many results:

- Succinct arguments [BCS16,…]
- Succinct zero-knowledge proofs [KR09,NR22]
- Delegation of computation [RRR16]
- Hardness of approximation [**A**CY21,**A**CY22]

Are IOPs more powerful than PCPs?

	Length	Alphabet size	Queries	Soundness error	Possible for PCPs?	Possible for IOPs?
Linear-length	$O(n)$	$O(1)$	$O(1)$	$O(1)$	?	?
Sliding-scale Conjecture	$poly(n)$	$poly(n)$	$O(1)$	$\frac{1}{n}$	?	?
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Our results

- We develop a toolbox for deriving barriers for IOPs (and PCPs)

Main results

1. Sliding-scale IOPs are equivalent to sliding-scale PCPs
2. Limitations of binary-alphabet constant-query IOPs
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Result 1: Sliding-scale IOPs are equivalent to sliding-scale PCPs

We show: a transformation from a sliding-scale IOP with $polylog(n)$ rounds to a sliding-scale PCP.

*Given non-uniform advice or under a hardness assumption

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Negative view: a barrier to constructing sliding-scale IOPs. Constructing sliding-scale IOPs is as hard as constructing sliding-scale PCPs.

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Negative view: a barrier to constructing sliding-scale IOPs. Constructing sliding-scale IOPs is as hard as constructing sliding-scale PCPs.

Positive view: efforts towards constructing sliding-scale PCPs can rely on interaction.

Can IOPs help with PCP limitations?

	Length	Alphabet size	Queries	Soundness error	Possible for PCPs?	Possible for IOPs*?
Linear-length	$O(n)$	$O(1)$	$O(1)$	$O(1)$	?	☑ [BCGRS17]
Sliding-scale Conjecture	$poly(n)$	$poly(n)$	$O(1)$	$\frac{1}{n}$	?	?
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Result 2: Limitations of binary-alphabet constant-query IOPs

Folkore: No binary PCPs for NP with

- 2 queries.
- 3 queries and soundness error $< \frac{5}{8}$.

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We show: No $polylog(n)$-round binary IOPs for NP with

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Takeaway: in this regime, interaction doesn't buy us anything.

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Result 3: Limitations of short IOPs

We show: no short IOPs in the [DHK15] parameter regime

	Proof System	Rounds	Length	Alphabet size	Queries	Soundness error	Possible?
[DHK15]	PCP	0	$poly(n)$	$n^{o(1)}$	$polyloglog(n)$	$\frac{1}{n}$	☑
[This Work]	IOP	$polylog(n)$	$\tilde{O}(n)$	$n^{polylog(n)}$	$polyloglog(n)$	$\frac{1}{n}$	☒

*full theorem considers more general trade-offs between parameters

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Additional barriers: related to interactive proofs

Our toolbox

Round and length reduction:

- Round reduction
- Length reduction
- Unrolling to PCP

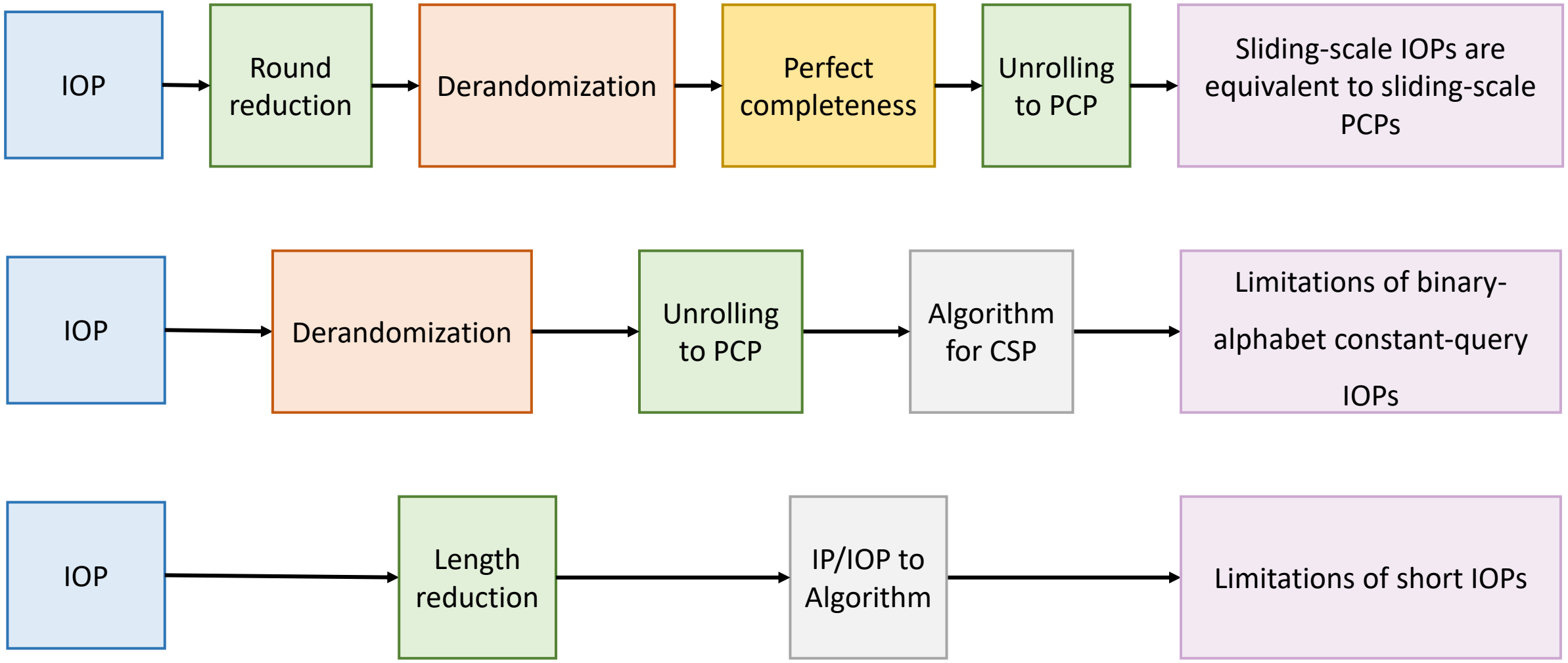
Improving completeness:

- Completeness amplification
- Perfect completeness

Derandomizing IOPs:

- Using non-uniform advice
- Using PRGs

Using the toolbox to get our results



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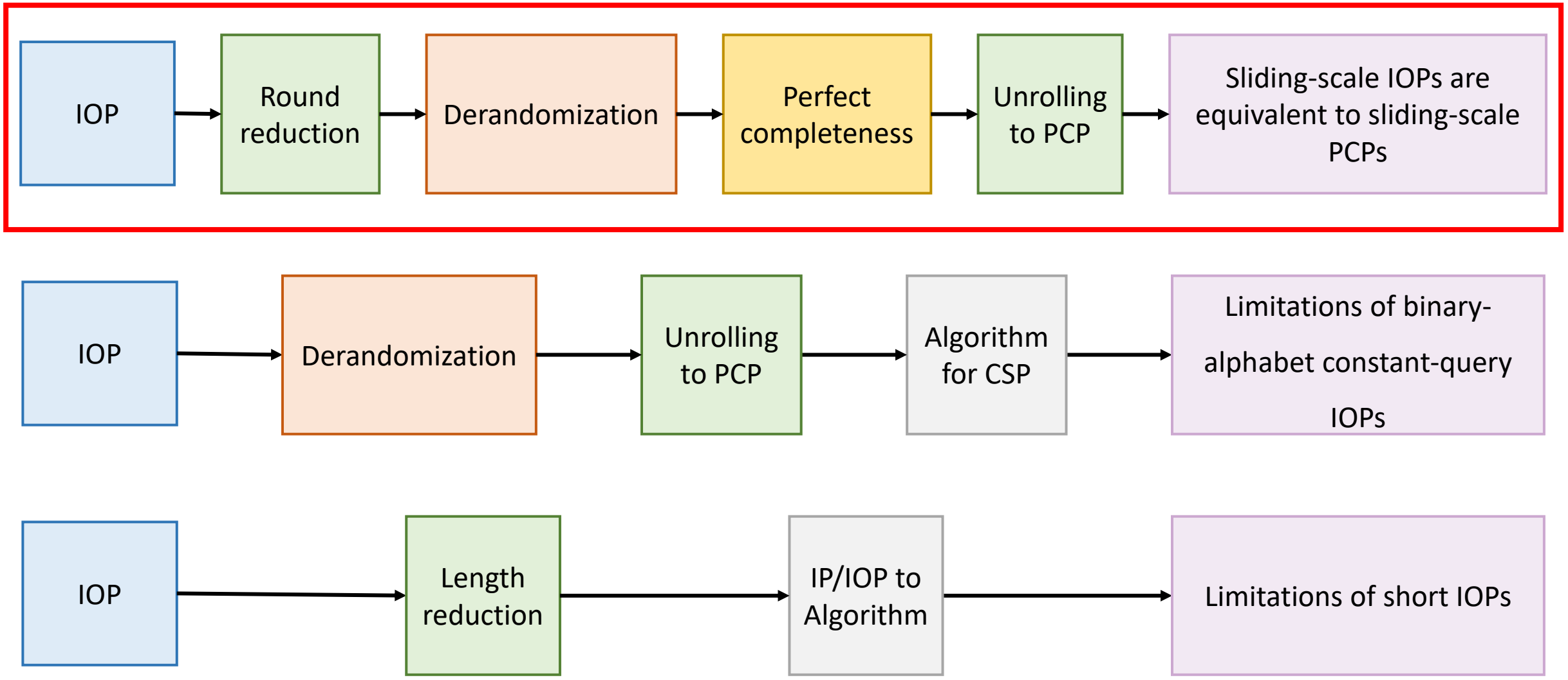

Sliding-scale IOP to sliding-scale PCP

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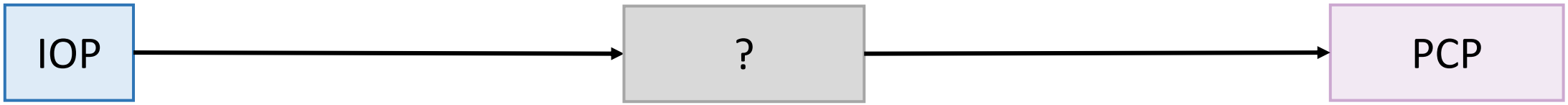
Theorem:*

$$\mathrm{IOP}\left[\begin{array}{c}\text{Rounds} = \text{polylog}(n)\\ \text{Soundness error} = 1/\text{poly(n)}\\ \text{Queries} = O(1)\\ \text{Alphabet size} = poly(n)\\ \text{Length} = poly(n)\end{array}\right] \subseteq \mathrm{PCP}\left[\begin{array}{c}\text{Soundness error} = 1/\text{poly(n)}\\ \text{Queries} = O(1)\\ \text{Alphabet size} = poly(n)\\ \text{Length} = poly(n)\end{array}\right]$$

*Under complexity assumptions or using non-uniform advice

Sliding-scale IOP to sliding-scale PCP

Proof sketch:



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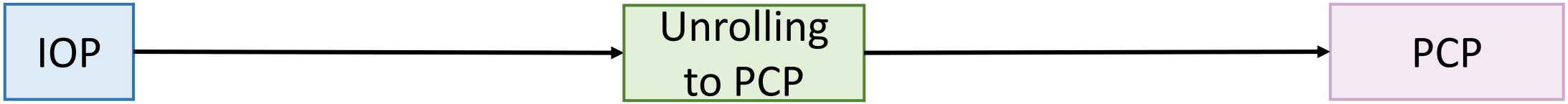

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IOP	$poly(n)$	$polylog(n)$	$poly(n)$
Unroll to PCP	$2^{poly(n)}$	0	$poly(n)$

For all steps: Query complexity = $O(1)$ Soundness error = $1/poly(n)$

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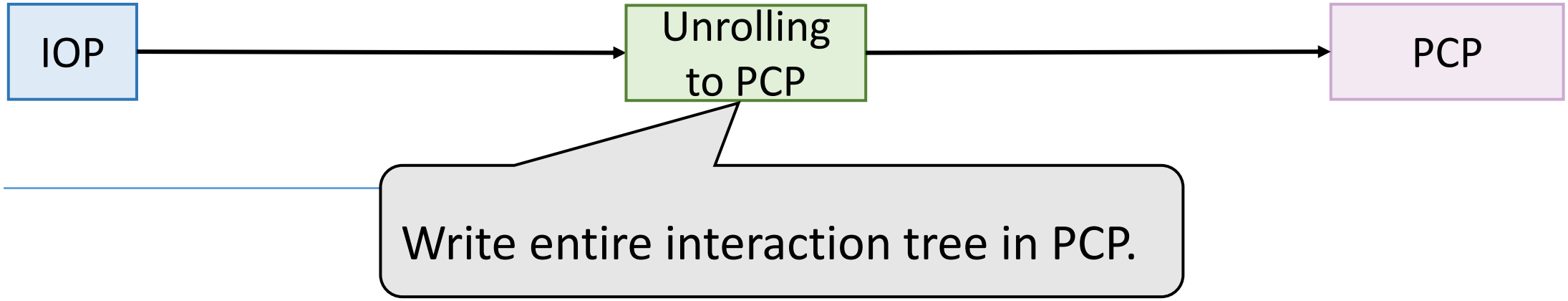

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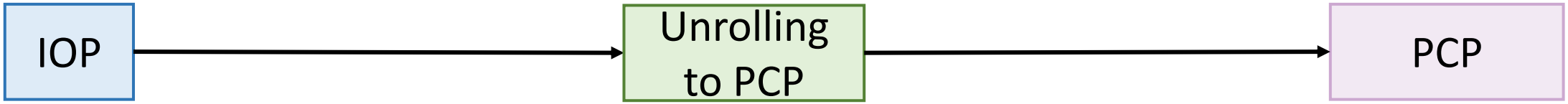

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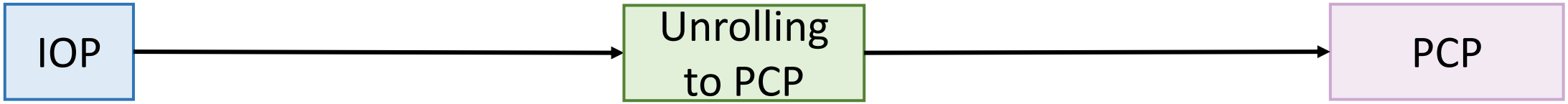

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Length is exponential in #rounds and randomness complexity

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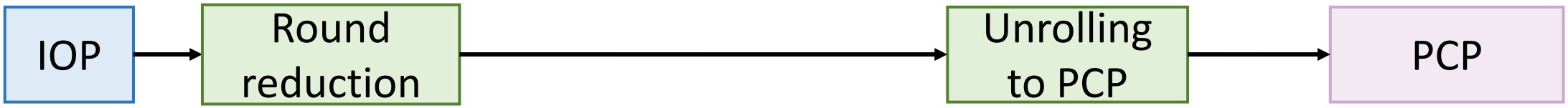

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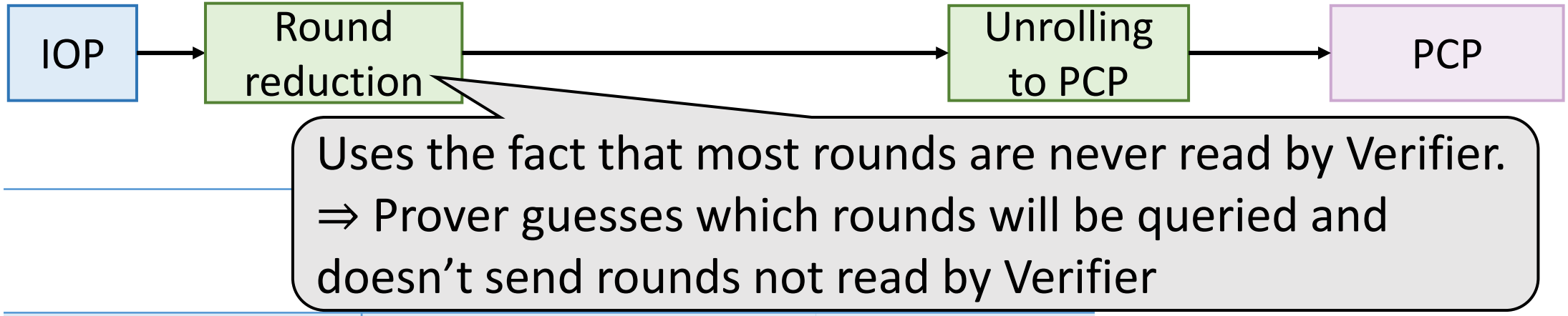

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Sliding-scale IOP to sliding-scale PCP

Proof sketch:

Similar to derandomization of IPs.
Verifier samples from a subset of the random coins

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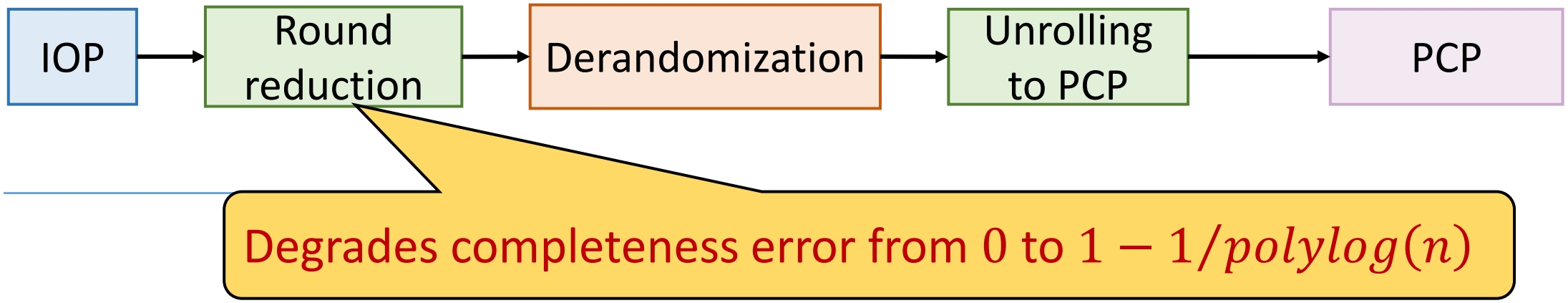

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Sliding-scale IOP to sliding-scale PCP

Proof sketch:

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Round reduction	$poly(n)$	$O(1)$	$poly(n)$	$1 - 1/polylog(n)$
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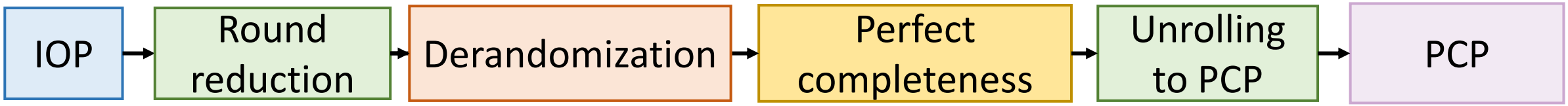

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Derandomization	$poly(n)$	$O(1)$	$O(logn)$	$1 - 1/polylog(n)$
Perfect completeness	$poly(n)$	$O(1)$	$O(logn)$	0
Unroll to PCP	$poly(n)$	0	$O(logn)$	0

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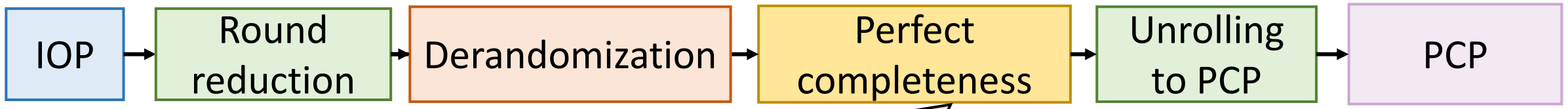

Based on [FGMSZ89] for perfect completeness of IPs, taking care not to harm query complexity

	Leng			
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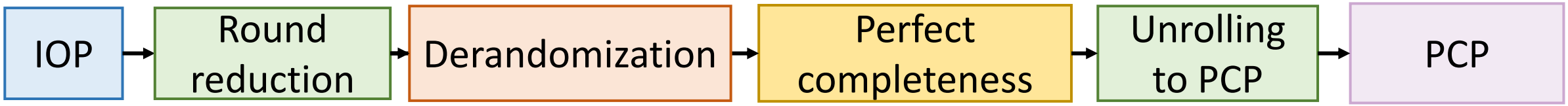

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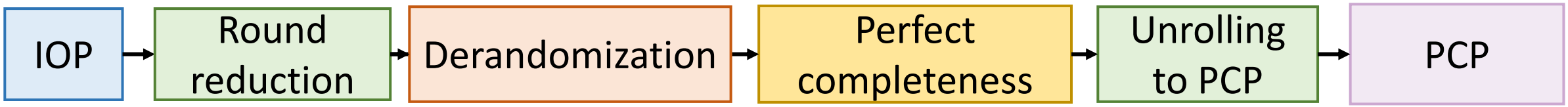
Derandomization	$poly(n)$	$O(1)$	$O(logn)$	$1 - 1/polylog(n)$
Perfect completeness	$poly(n)$	$O(1)$	$O(logn)$	0
Unroll to PCP	$poly(n)$	0	$O(logn)$	0

For all steps: Query complexity = $O(1)$ Soundness error = $1/poly(n)$

Sliding-scale IOP to sliding-scale PCP

Proof sketch:



Order of steps matters!

		Rounds	Randomness	Completeness error
IOP	$poly(n)$	$polylog(n)$	$poly(n)$	0
Round reduction	$poly(n)$	$O(1)$	$poly(n)$	$1 - 1/polylog(n)$
Derandomization	$poly(n)$	$O(1)$	$O(logn)$	$1 - 1/polylog(n)$
Perfect completeness	$poly(n)$	$O(1)$	$O(logn)$	0
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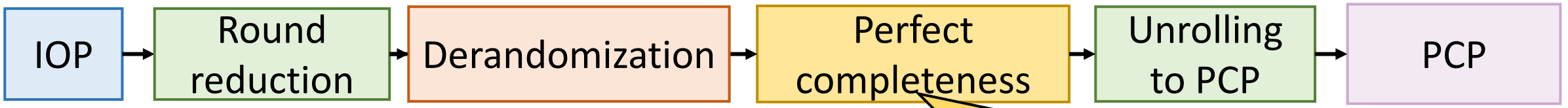

Order of steps matters!

Increases alphabet size as a factor of randomness complexity

		Rounds		error
IOP	$poly(n)$	$polylog(n)$	$poly(n)$	0
Round reduction	$poly(n)$	$O(1)$	$poly(n)$	$1 - 1/polylog(n)$
Derandomization	$poly(n)$	$O(1)$	$O(logn)$	$1 - 1/polylog(n)$
Perfect completeness	$poly(n)$	$O(1)$	$O(logn)$	0
Unroll to PCP	$poly(n)$	0	$O(logn)$	0

For all steps: Query complexity = $O(1)$ Soundness error = $1/poly(n)$

Summary of results

- We develop a toolbox for deriving barriers for IOPs (and PCPs)

Main results

1. Sliding-scale IOPs are equivalent to sliding-scale PCPs
2. Limitations of binary-alphabet constant-query IOPs
3. Limitations of short IOPs

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General question: when are IOPs more powerful than PCPs?

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THANKS!