Sublinear Secure Computation from New Assumptions

Elette Boyle[1,2] Geoffroy Couteau[3] Pierre Meyer[1,3]

[1]Reichman University
[2]NTT Research
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TCC 2022

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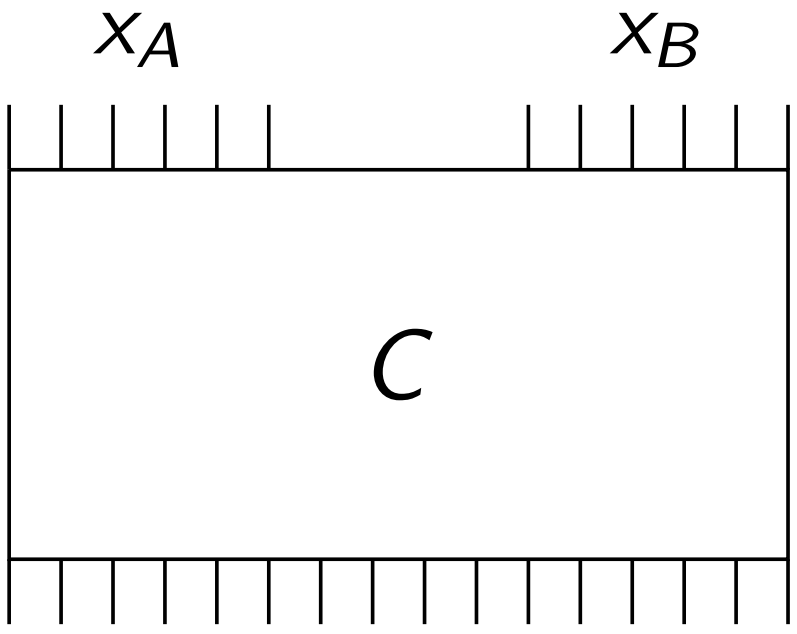
Standard, Well Founded

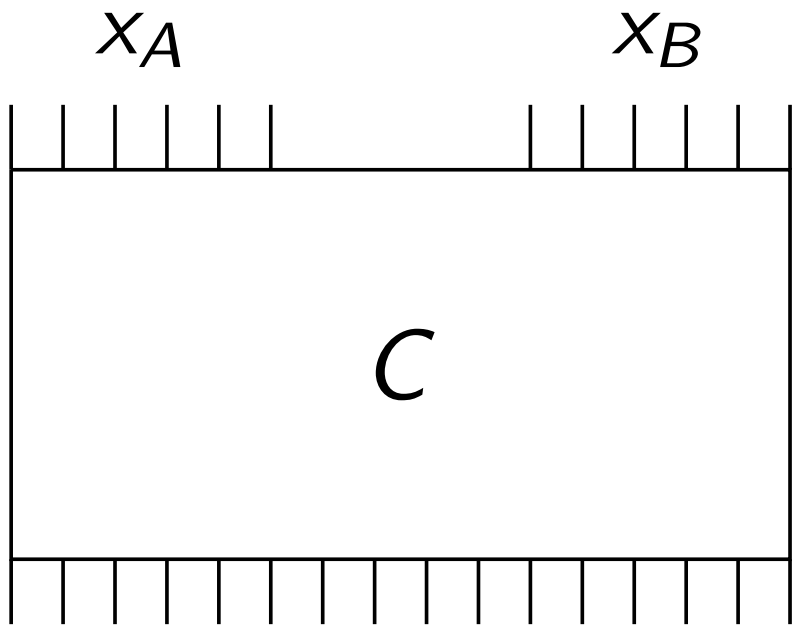
TCC 2022

NEW = Not Previously Known to Imply Sublinear Secure Computation

Sublinear Secure General Computation



Sublinear Secure General Computation

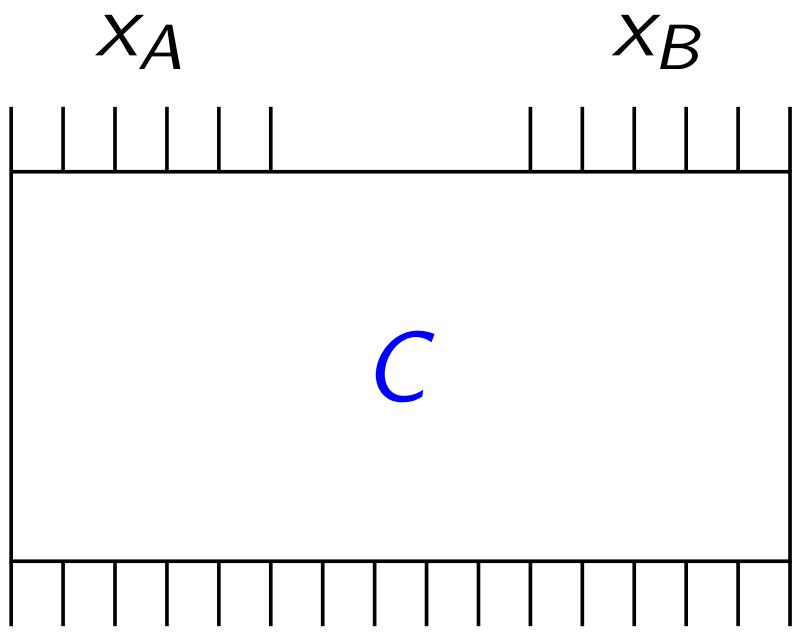


Communication

$$\mathcal{O}(|x_A| + |x_B| + |y|) + o(|C|) + \mathsf{poly}(\lambda)$$

Sublinear in the Circuit-Size

Sublinear Secure General Computation



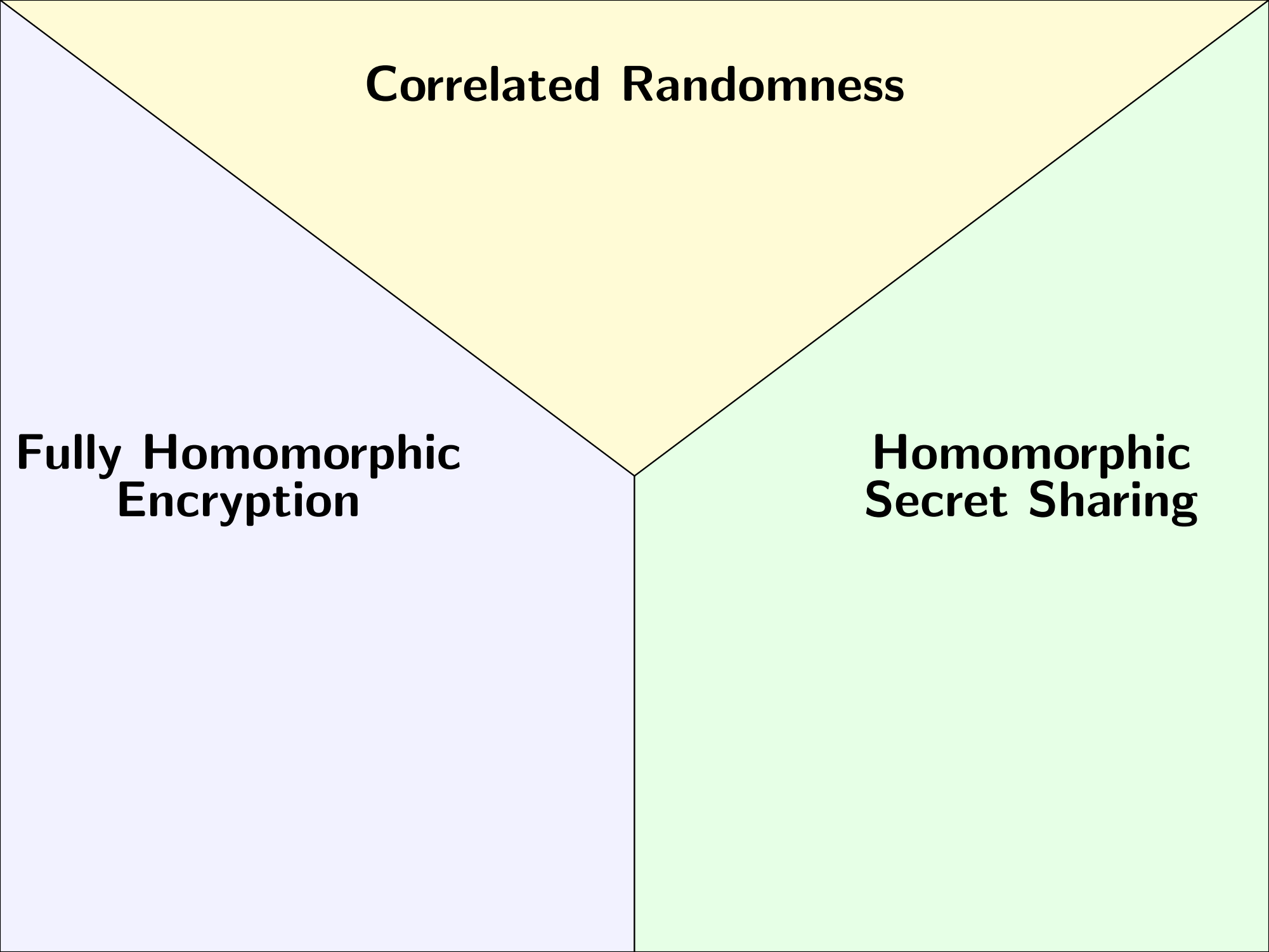
Supported Class?

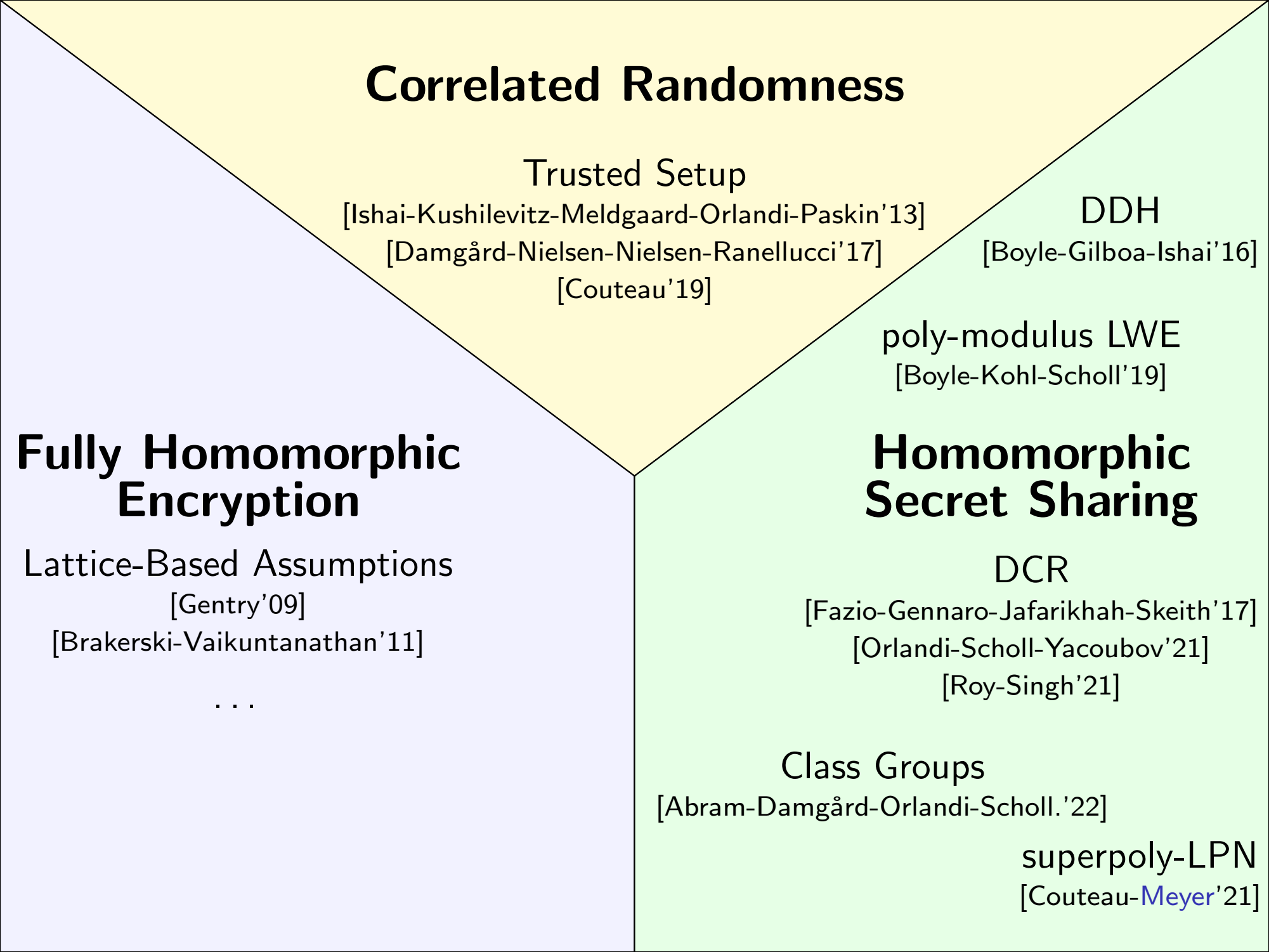
P/Poly
log-depth (NC^1)
log log-depth
Layered Circuits
...

Communication

$$\mathcal{O}(|x_A| + |x_B| + |y|) + o(|C|) + \text{poly}(\lambda)$$

Sublinear in the Circuit-Size

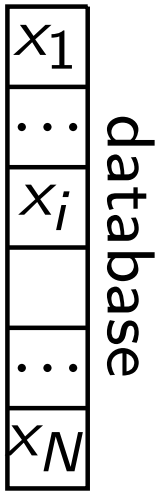

Correlated Randomness
Fully Homomorphic Encryption
Homomorphic Secret Sharing


Correlated Randomness
Trusted Setup
[Ishai-Kushilevitz-Meldgaard-Orlandi-Paskin'13]
[Damgård-Nielsen-Nielsen-Ranellucci'17]
[Couteau'19]
DDH
[Boyle-Gilboa-Ishai'16]
poly-modulus LWE
[Boyle-Kohl-Scholl'19]
Fully Homomorphic Encryption
Lattice-Based Assumptions
[Gentry'09]
[Brakerski-Vaikuntanathan'11]
...
Homomorphic Secret Sharing
DCR
[Fazio-Gennaro-Jafarikhah-Skeith'17]
[Orlandi-Scholl-Yacoubov'21]
[Roy-Singh'21]
Class Groups
[Abram-Damgård-Orlandi-Scholl.'22]
superpoly-LPN
[Couteau-Meyer'21]

(Single-Server) Private Information Retrieval

[Chor-Goldreich-Kushilevitz-Sudan'95] and [Kushilevitz-Ostrovsky'97]

index: i



Client

Server

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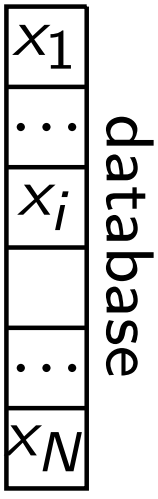

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- **Specialised Computation (Information Retrieval):**
  The client gets x_i (without revealing i).

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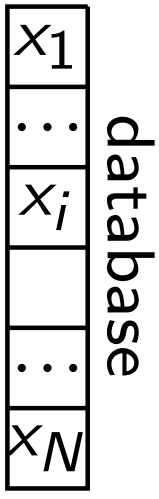

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- **Symmetric PIR (SPIR):** The client *only* gets x_i, *i.e.*

$$C(i, (x_1, \ldots, x_N)) := (x_i, \perp)$$

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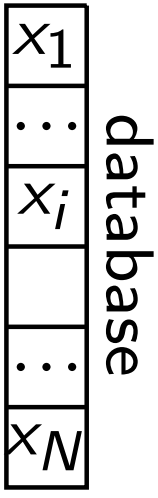

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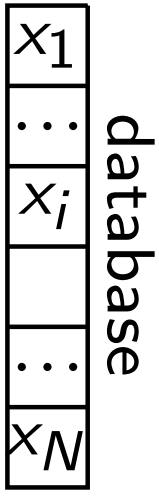

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$$\text{DCR} \vee \text{LWE} \vee \Phi\text{-Hiding} \vee \text{DDH} \vee \text{QR} \implies \text{(Two-Round) Polylog PIR}$$

[Cachin-Micali-Stadler'99], [Lipmaa'05], [Chang'04]
[Ostrovsky-Skeith'07], [Döttling-Garg-Ishai-Malavolta-Mour-Ostrovsky'19]

Our Results

1. Sublinear 2PC for Layered Circuits

- Assumption: Quadratic Residuosity assumption (QR) + Learning Parity with Noise (LPN).

2. Polylogarithmic PIR from CDH

- Assumption: Computational Diffie-Hellman.

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- Key Feature: Uses **polylog communication** and **log rounds**.

Previously: $\Omega(N)$ from CDH
Now: $\mathrm{polylog}(N)$

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From DDH, LWE...: 2-round

Our Results

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- Key Feature: New approach to break the circuit-size barrier.
- Circuit Class: Layered Circuits $\mathcal{O}(|\text{in}| + |\text{out}| + \frac{|C|}{\log\log|C|})$ or LogLog-Depth Circuits $\mathcal{O}(|\text{in}| + |\text{out}| + \sqrt{|C|})$.

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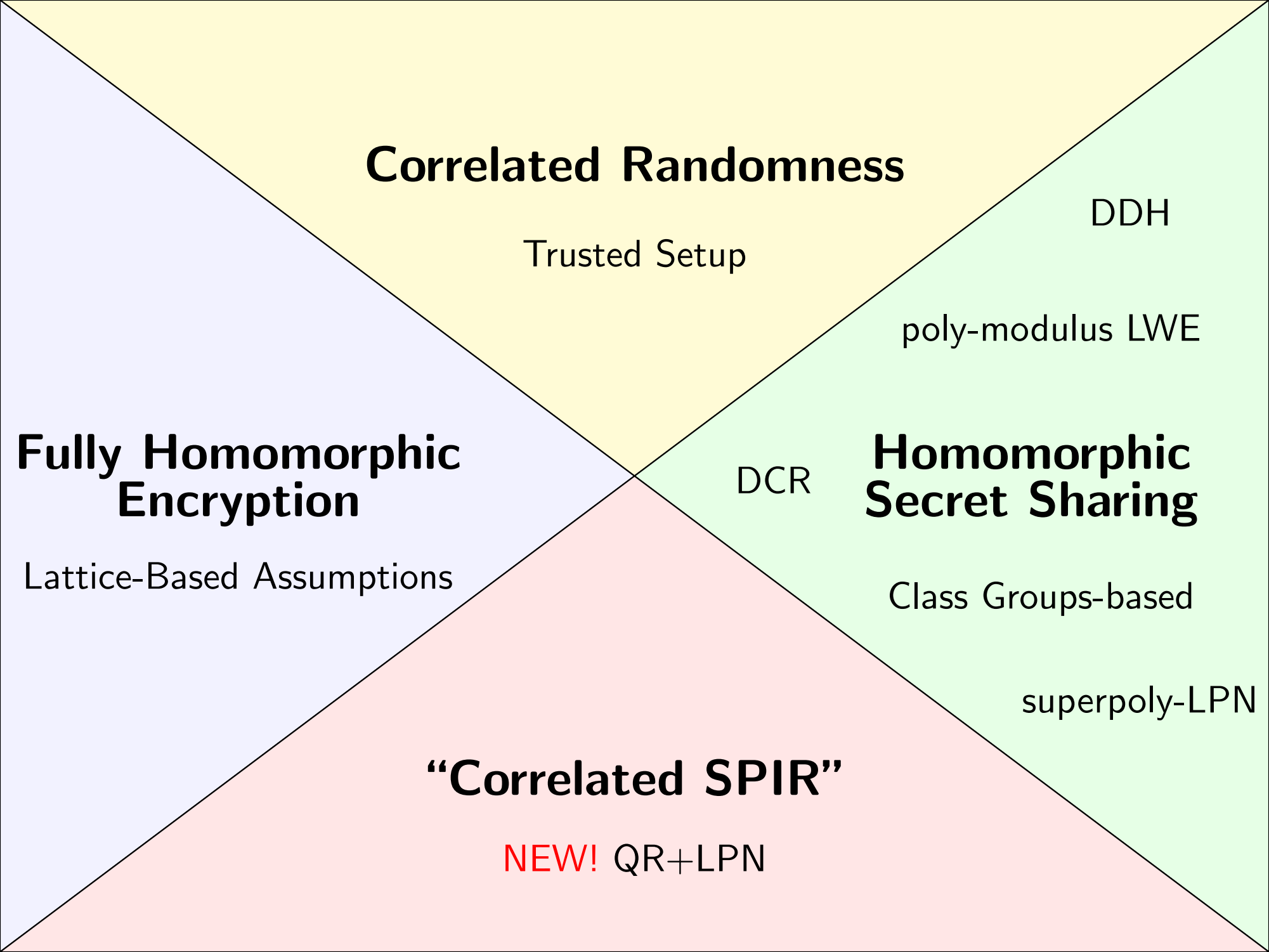
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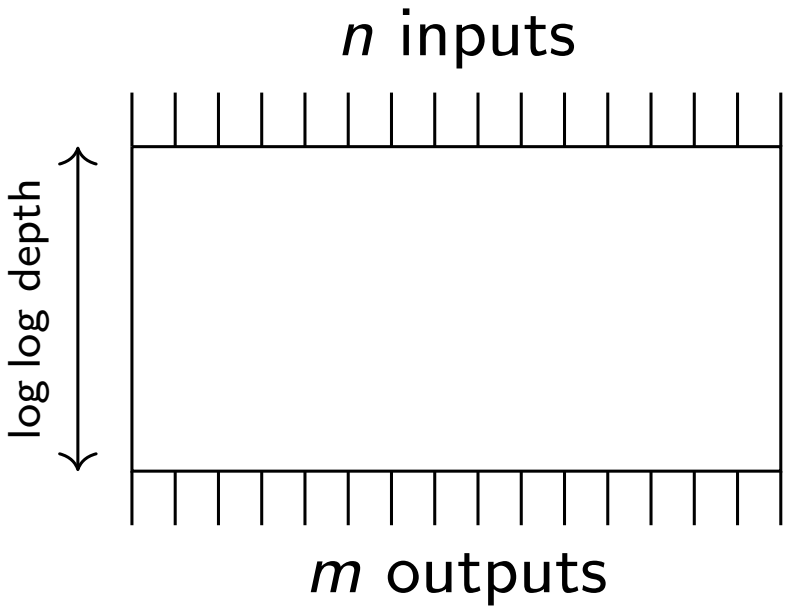
"Correlated SPIR"
NEW! QR+LPN

Reduction to a form of PIR: Exploiting Log-Locality



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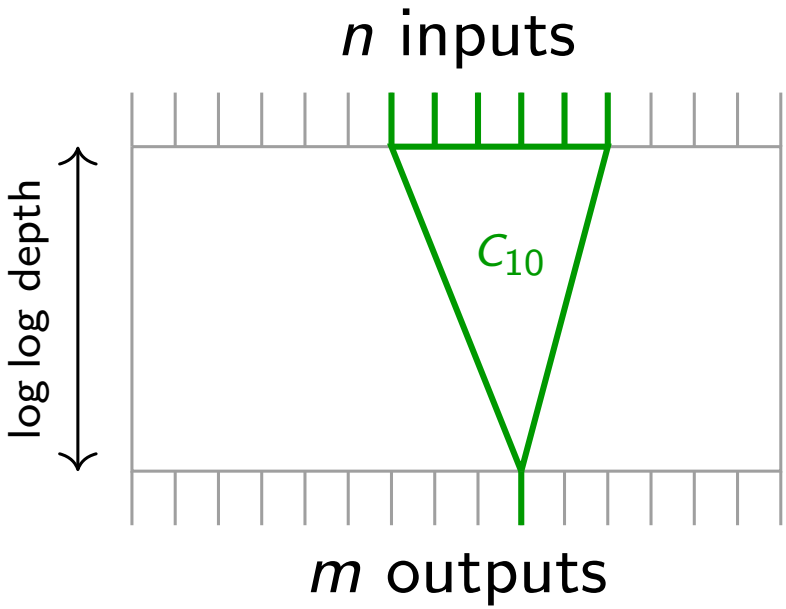

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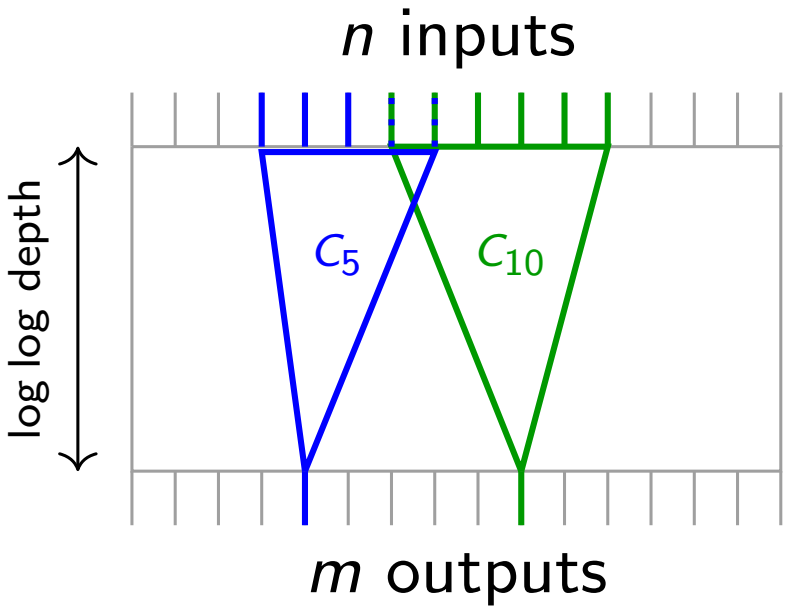

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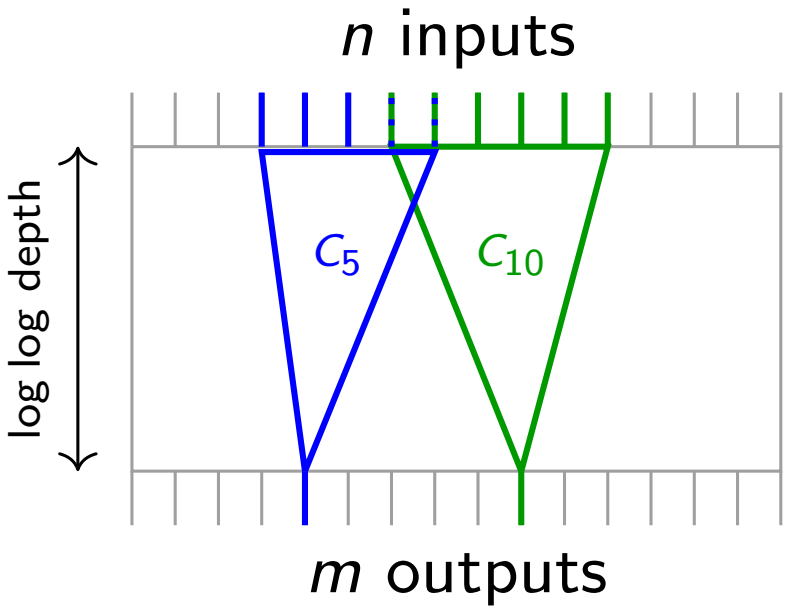

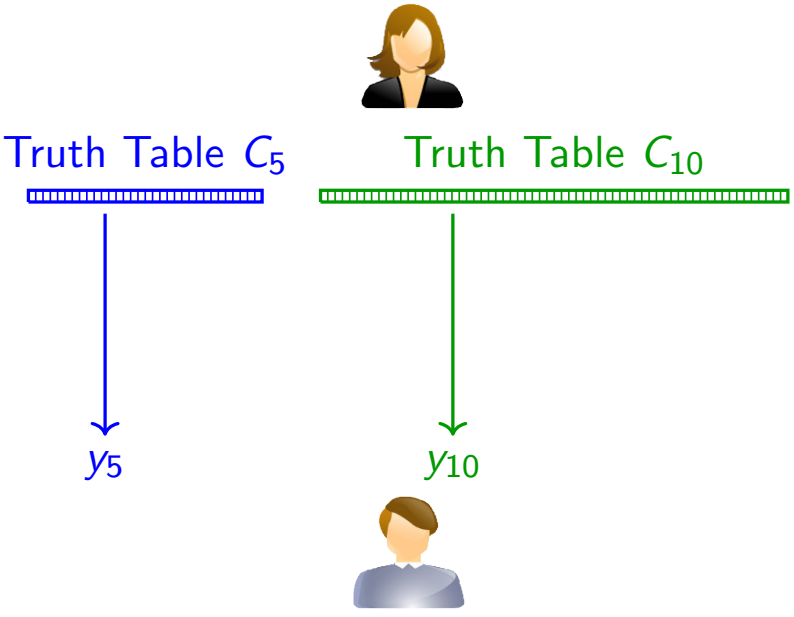

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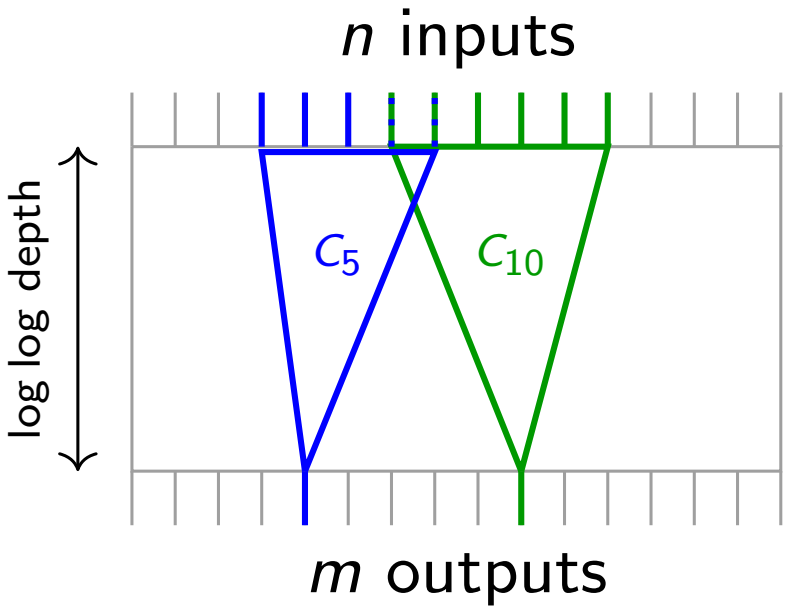
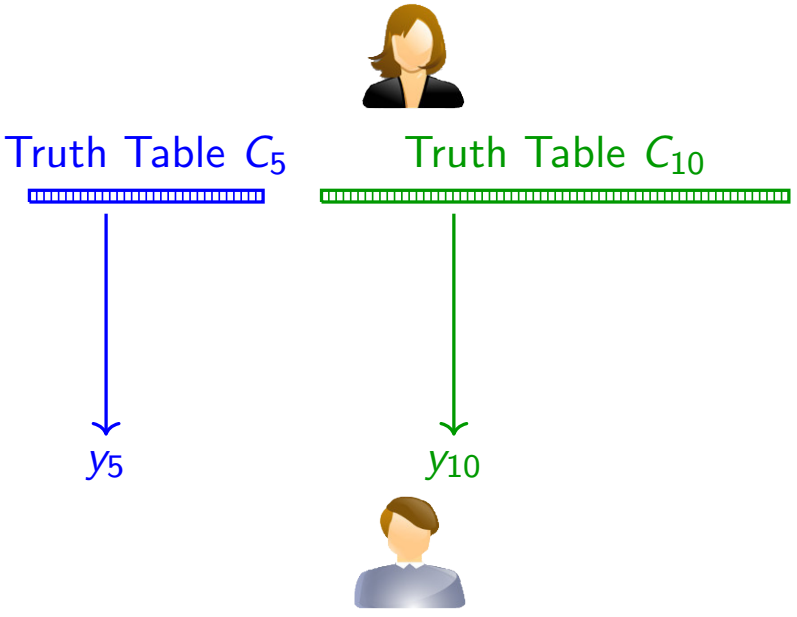
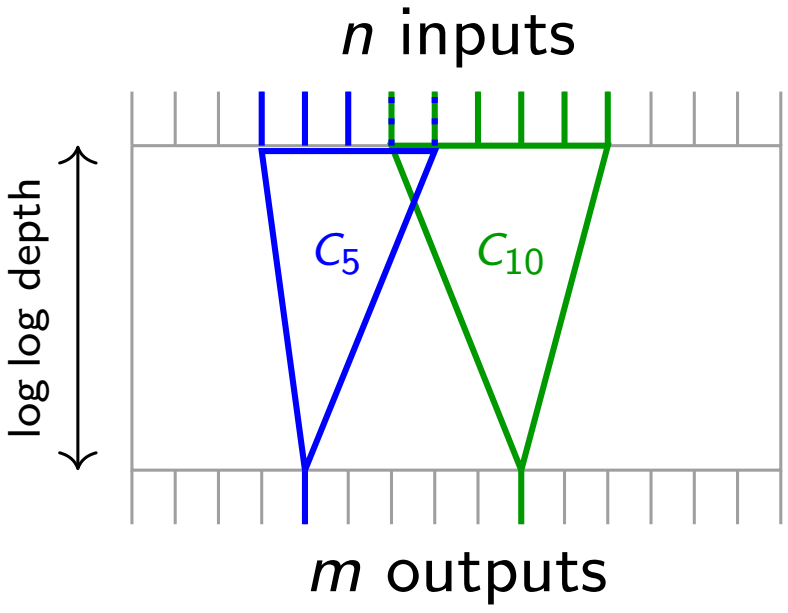
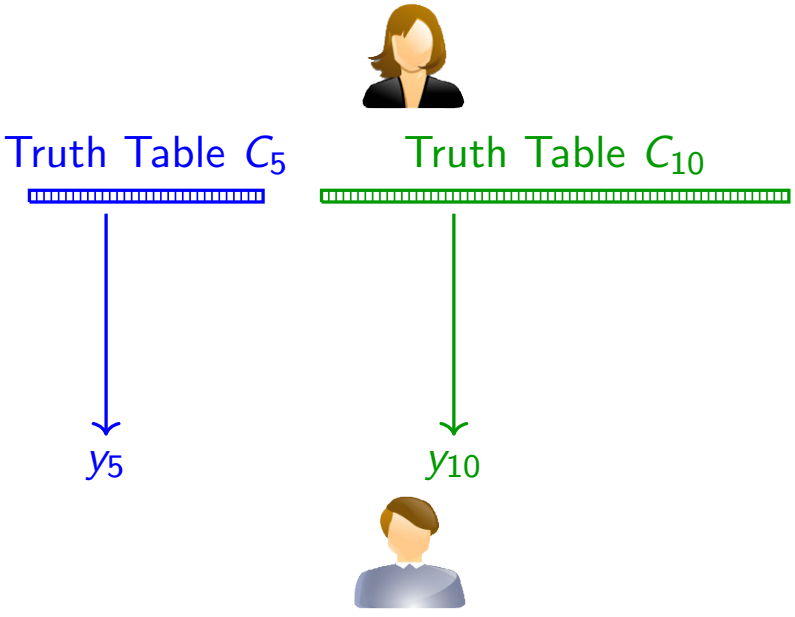




Parallel instances of specialised computation

Reduction to a form of PIR: Exploiting Log-Locality





Parallel instances of specialised computation

- Independent instances?

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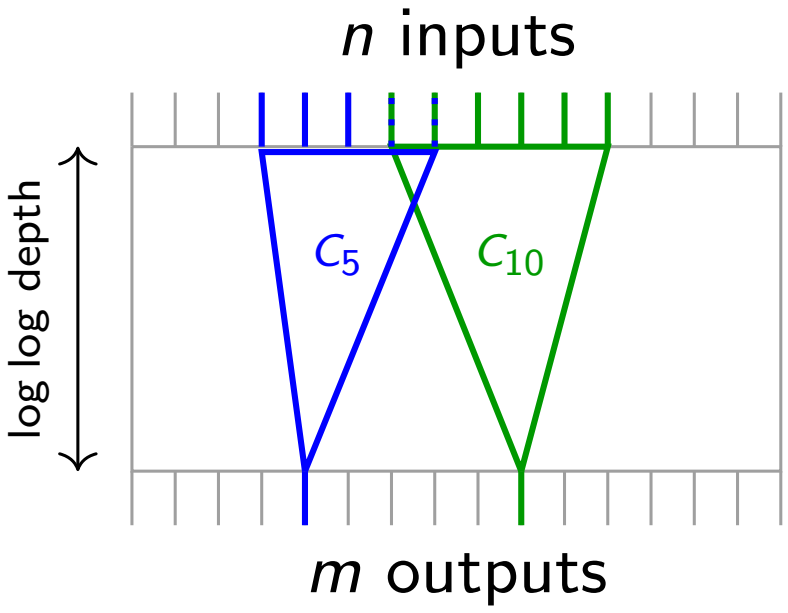

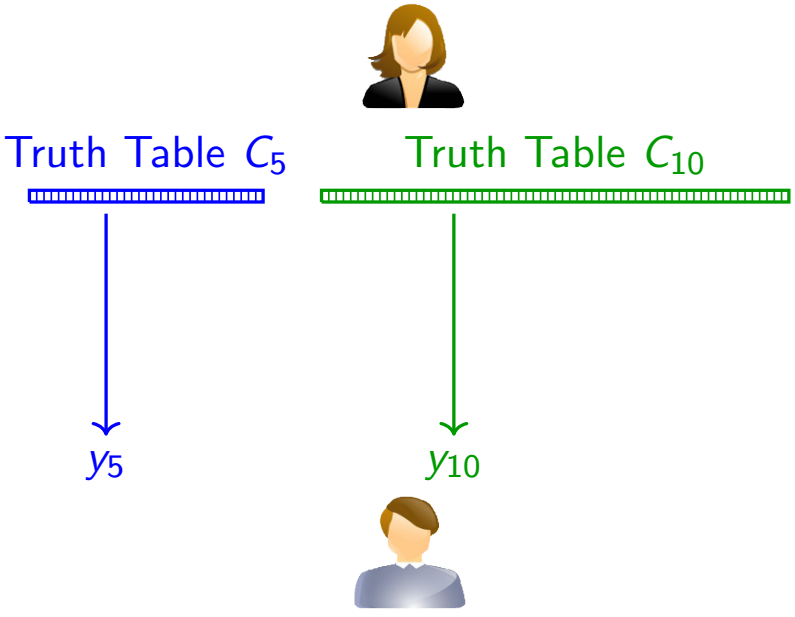

Parallel instances of specialised computation

- Independent instances? Too much communication!

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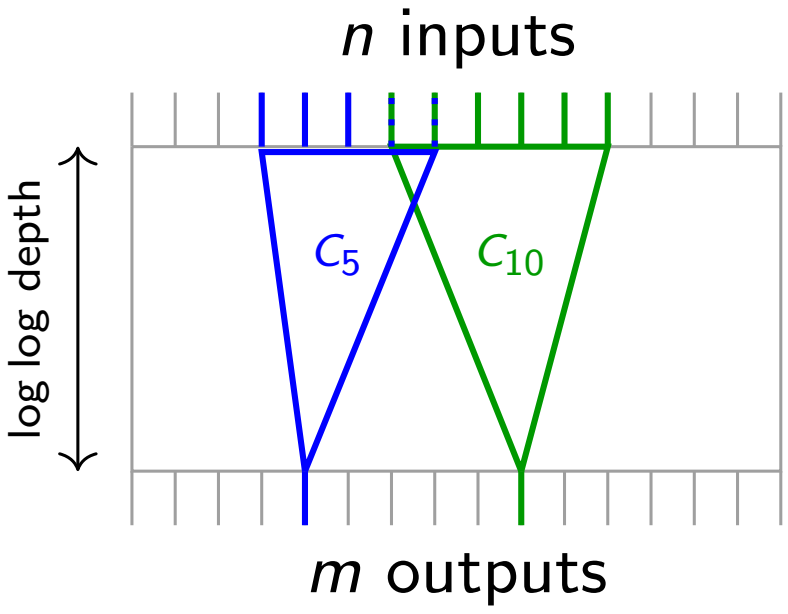

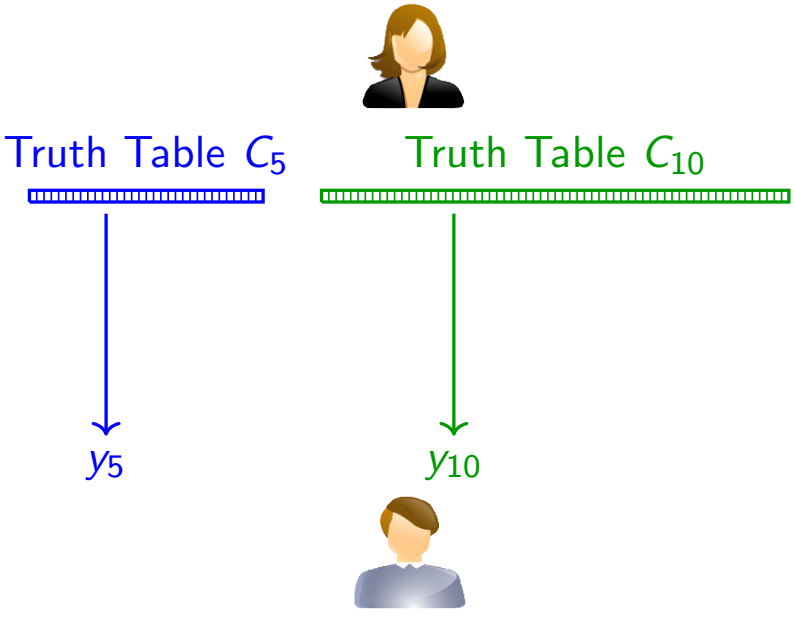

Parallel instances of specialised computation

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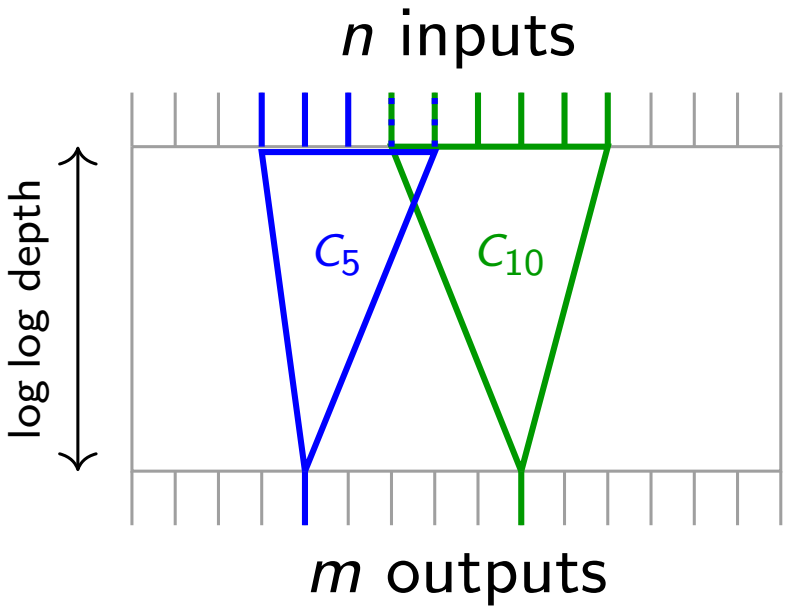
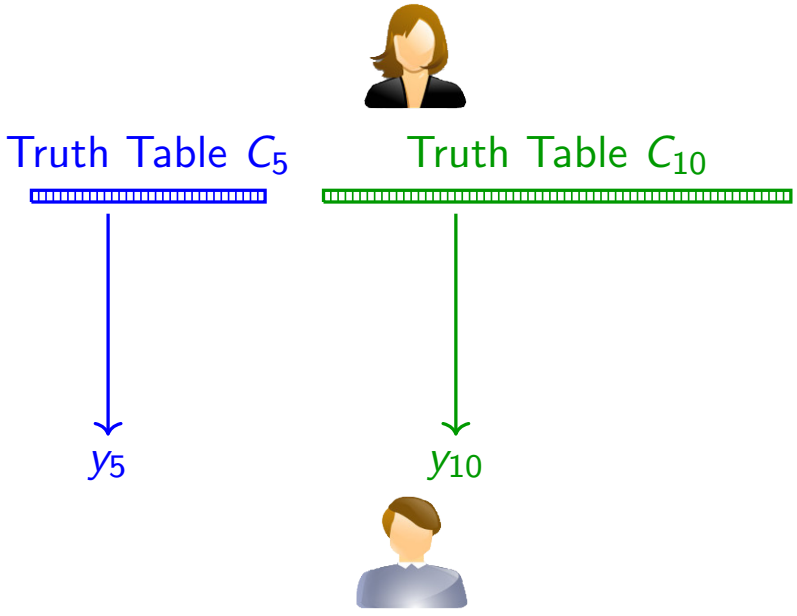
- Exploiting redundancies ("Total Entropy": x_A and x_B)

Reduction to a form of PIR: Exploiting Log-Locality





Parallel instances of specialised computation

- Independent instances? Too much communication!
- Exploiting redundancies ("Total Entropy": x_A and x_B)
  - **With a "Correlated Randomness" approach.** [Couteau'19],[Couteau-Meyer'21].
  - **NEW! Correlated SPIR.** [Boyle-Couteau-Meyer'22]: Up next!

What is Correlated SPIR?

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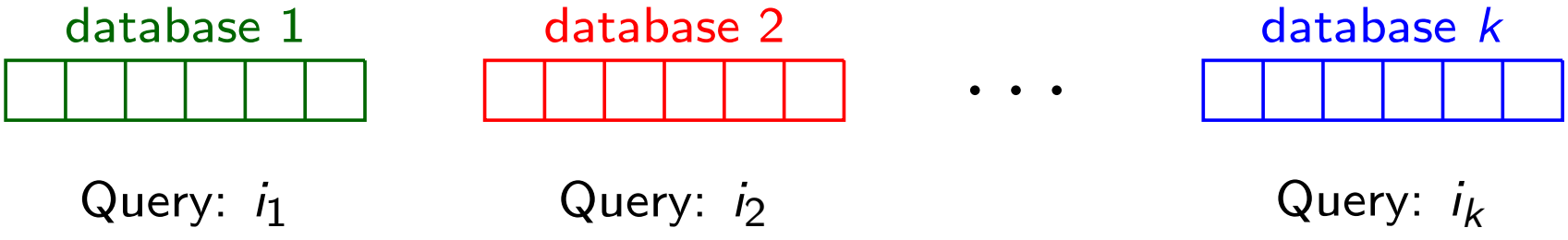
Uncorrelated SPIR



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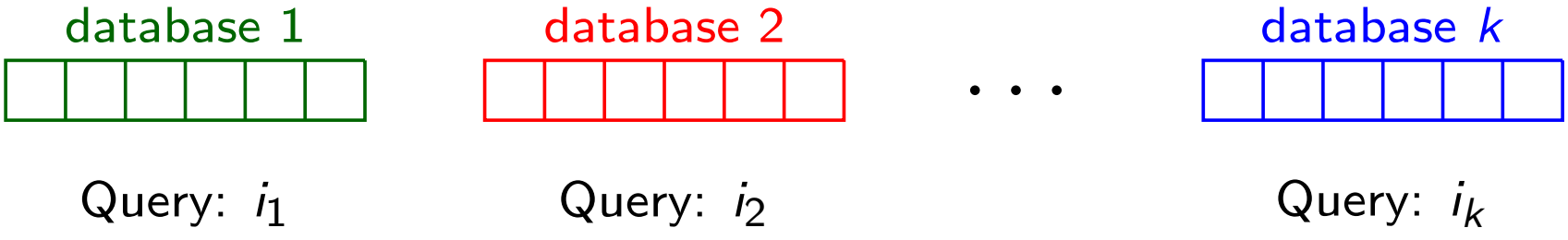
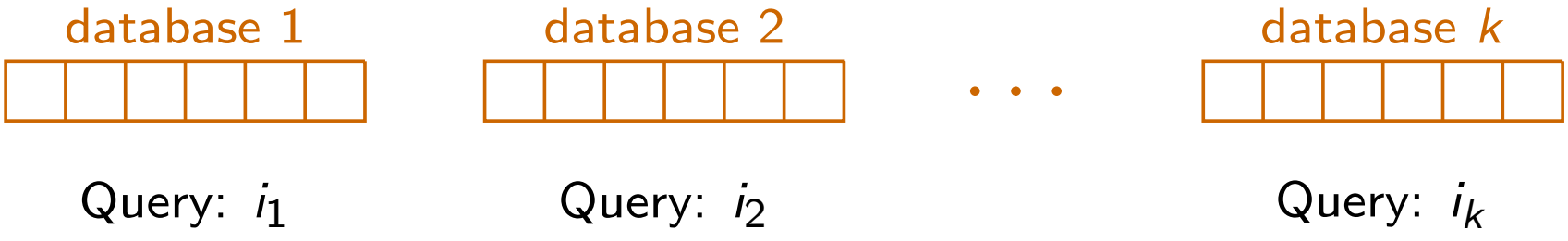


Batch SPIR



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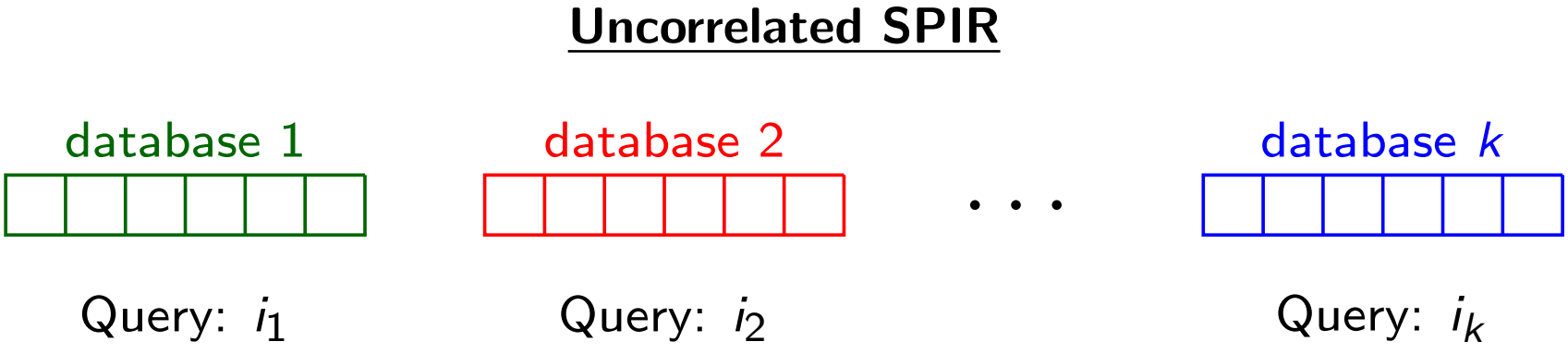

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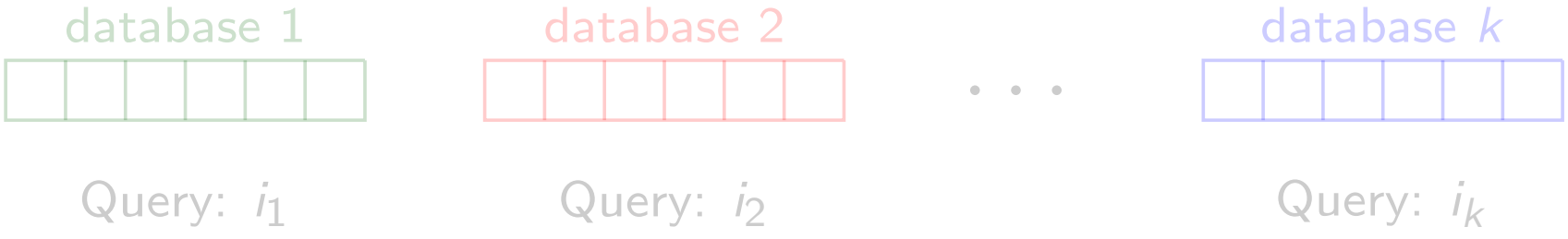
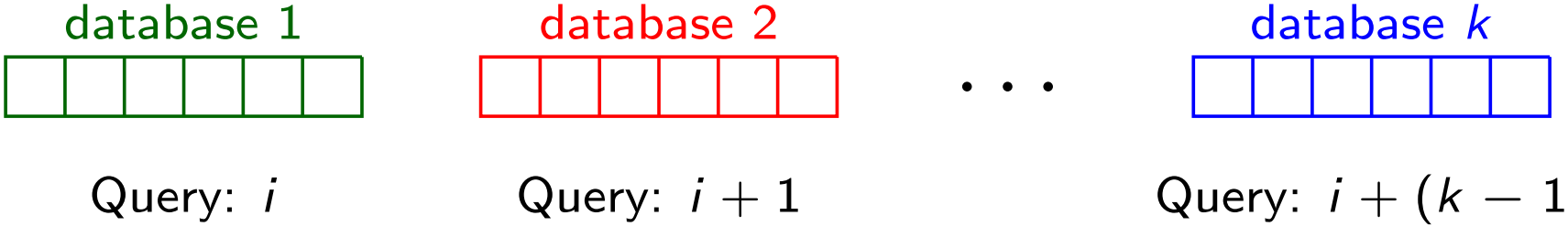
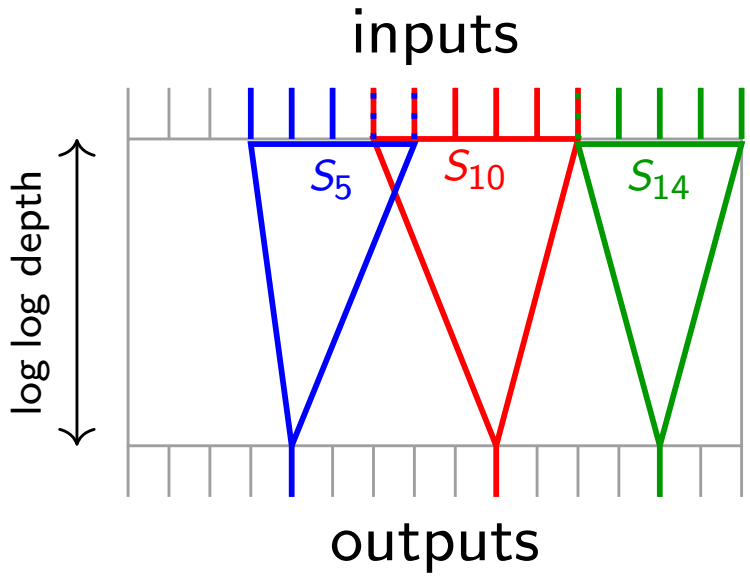


Batch SPIR

Correlated SPIR with Sequential Queries



Correlated SPIR with "Mix and Match" Queries



The j^{th} output depends on subset S_j of Bob's input vector x_B.

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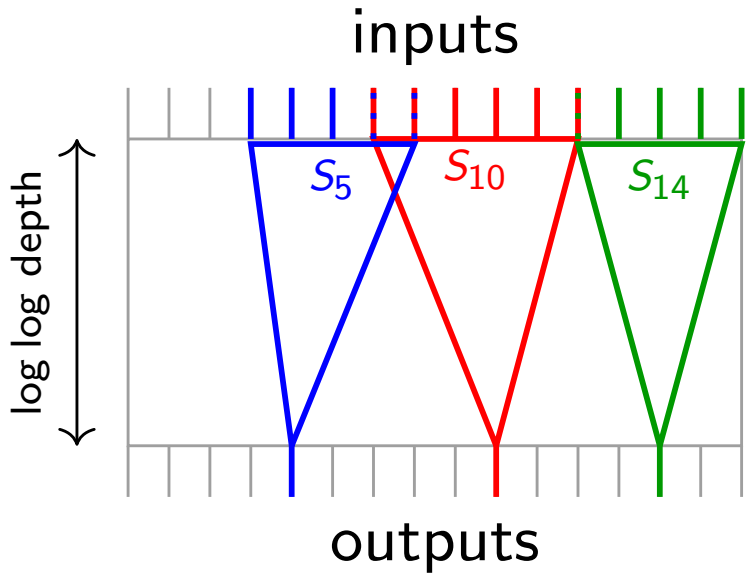


The j^{th} output depends on subset S_j of Bob's input vector x_B.

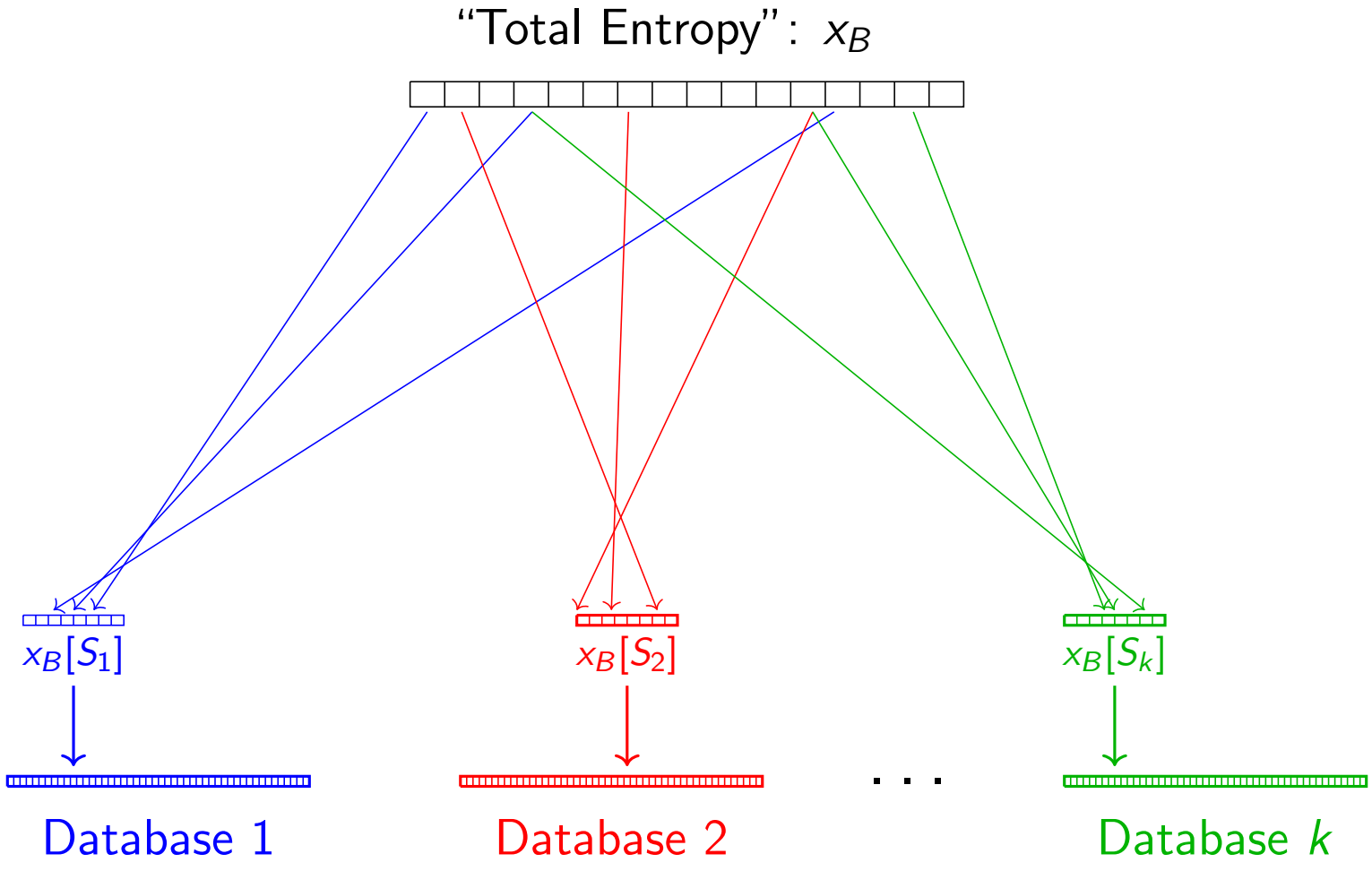
---

database 1 | database 10 | ... | database 14

Query: $x_B[S_5]$ | Query: $x_B[S_{10}]$ | Query: $x_B[S_{14}]$

"Total Entropy": x_B, of n_B bits

Why "Mixing and Matching"?



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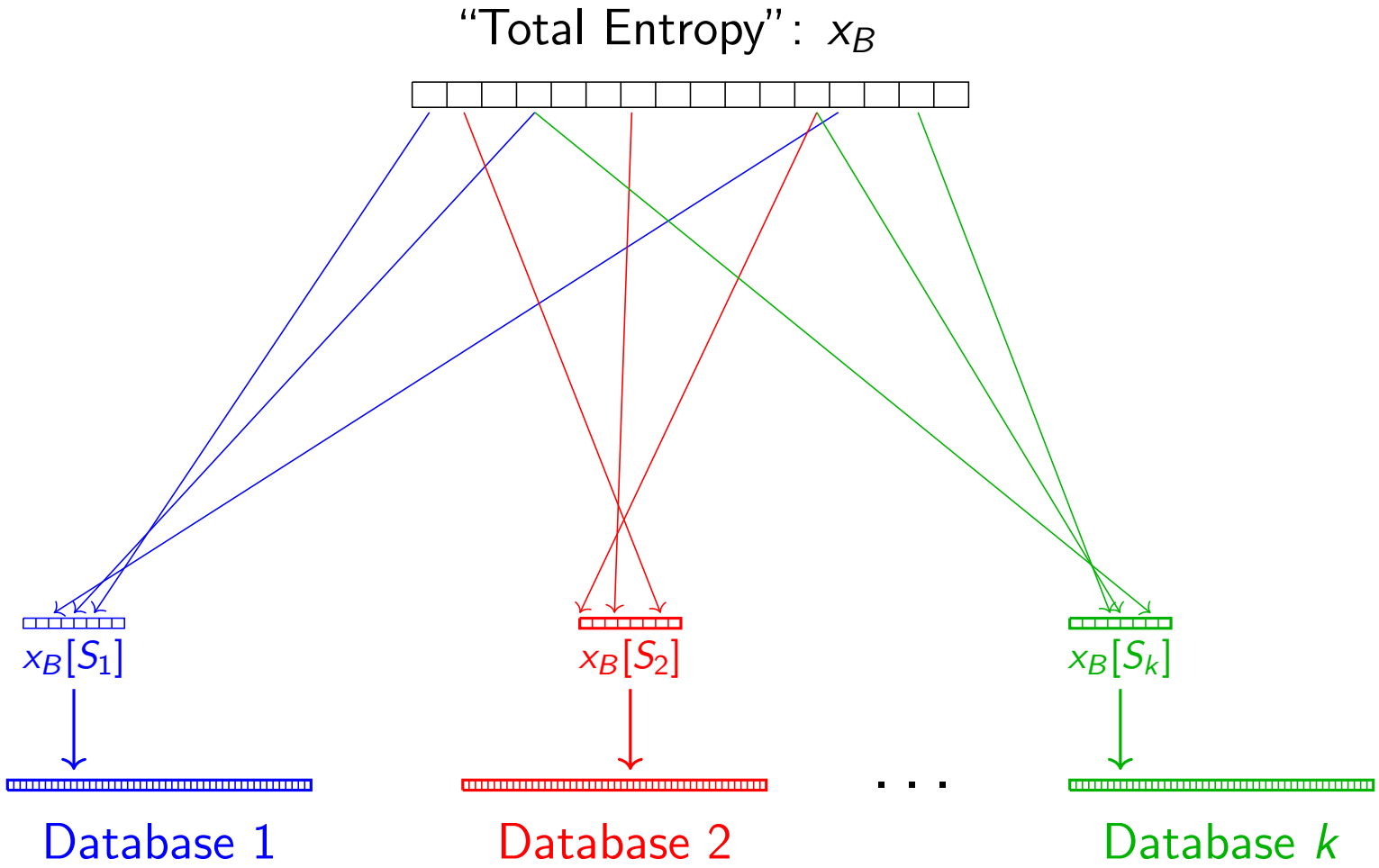
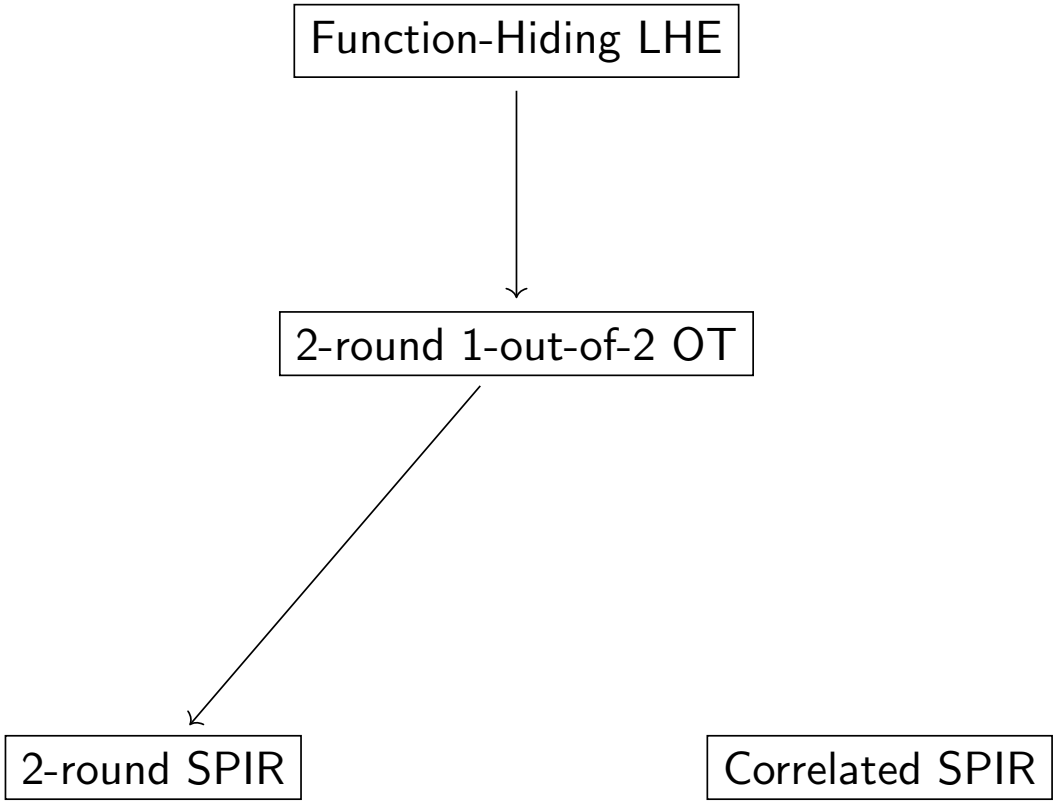


For Sublinear 2PC: Upload $\mathcal{O}(|x_B|)$ and Download $|\text{out}| \cdot (1 + o(1))$

no hidden factor poly(λ)!

How to build Correlated SPIR? — From Rate-1 Batch-LHE

Fact: Function-Hiding Linearly-Homomorphic Encryption implies SPIR



How to build Correlated SPIR? — From Rate-1 Batch-LHE

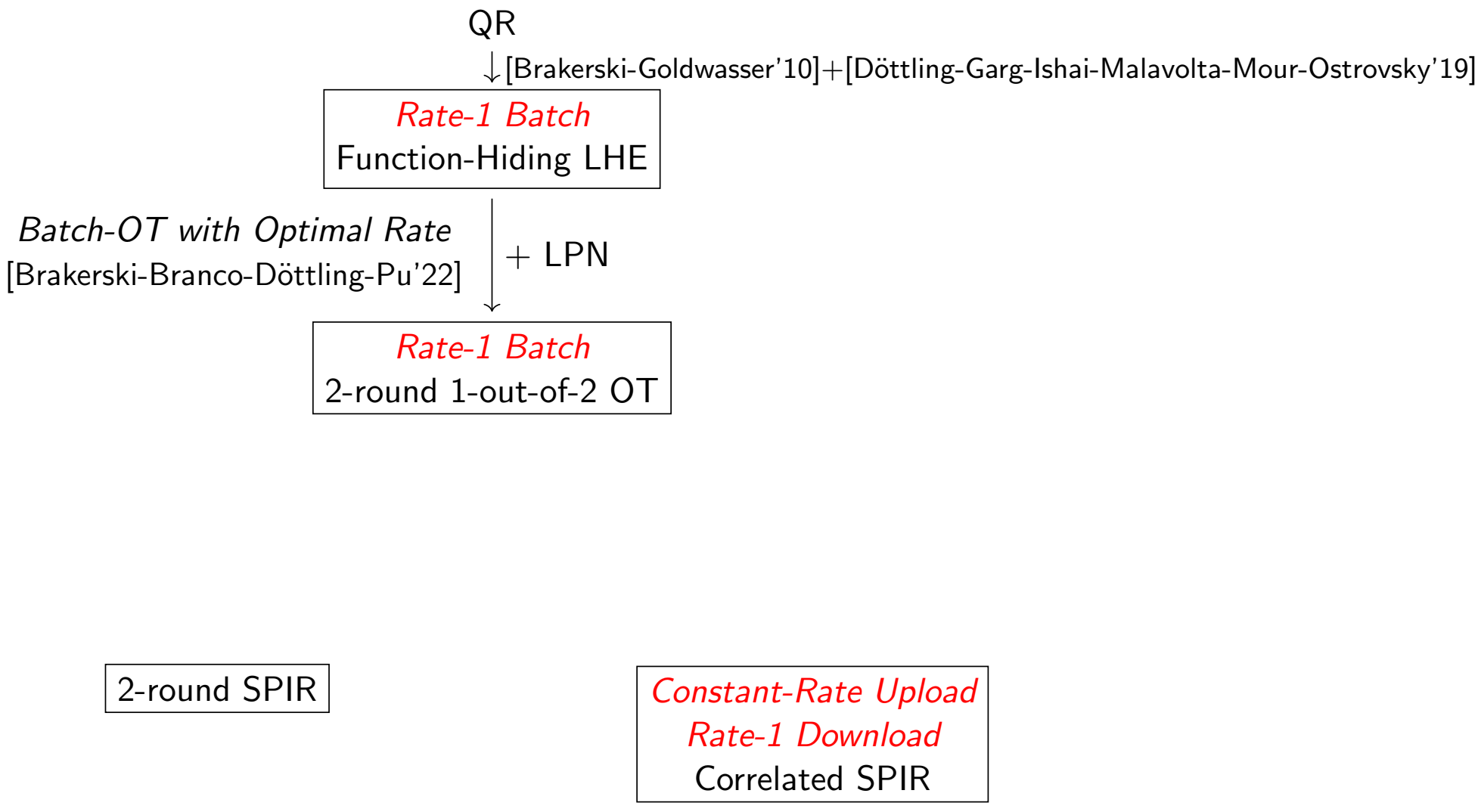
Rate-1 Batch
Function-Hiding LHE

Rate-1 Batch
2-round 1-out-of-2 OT

2-round SPIR

Constant-Rate Upload
Rate-1 Download
Correlated SPIR

How to build Correlated SPIR? — From Rate-1 Batch-LHE



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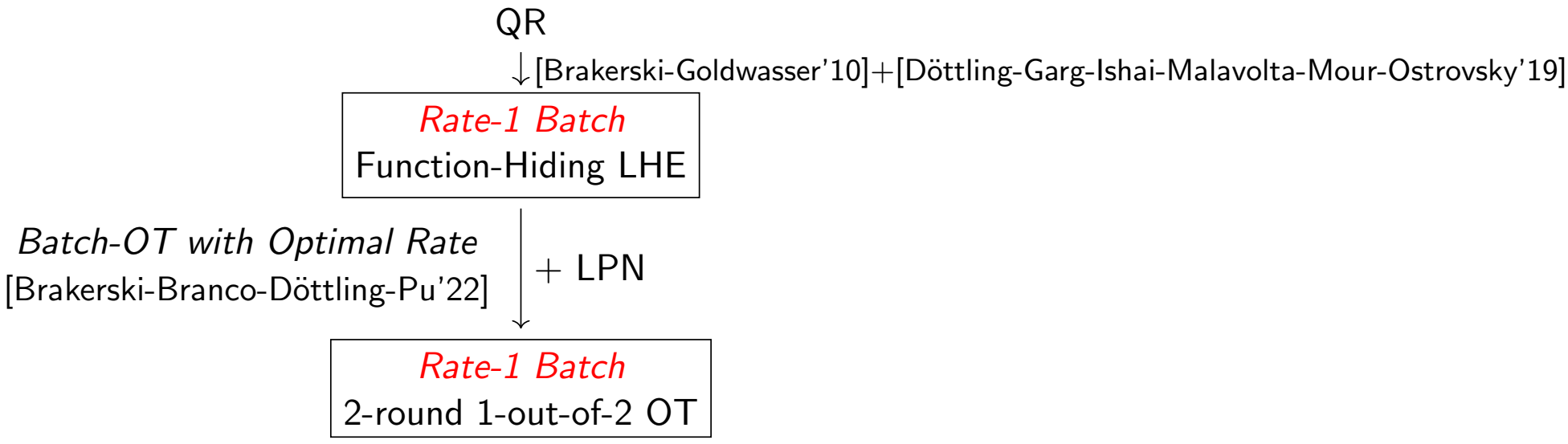


Downside of Batching:

Cannot "Mix and Match"
the atomic 1-out-of-2 OTs

Constant-Rate Upload
Rate-1 Download
Correlated SPIR

How to build Correlated SPIR? — From Rate-1 Batch-LHE

QR

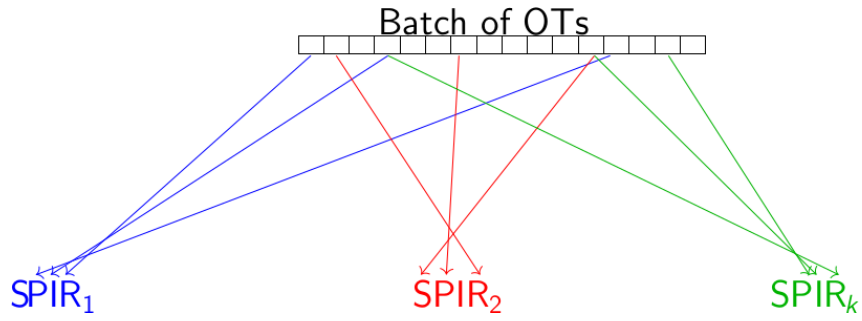
↓

Rate-1 Batch
Function-Hiding LHE

Batch-OT with Optimal Rate
[Brakerski-Branco-Döttling-Pu'22]

↓ + LPN

Rate-1 Batch
2-round 1-out-of-2 OT



Correlated SPIR

Downside of Batching:

Cannot "Mix and Match"
the atomic 1-out-of-2 OTs

Constant-Rate Upload
Rate-1 Download
Correlated SPIR

How to build Correlated SPIR? — From Rate-1 Batch-LHE

QR

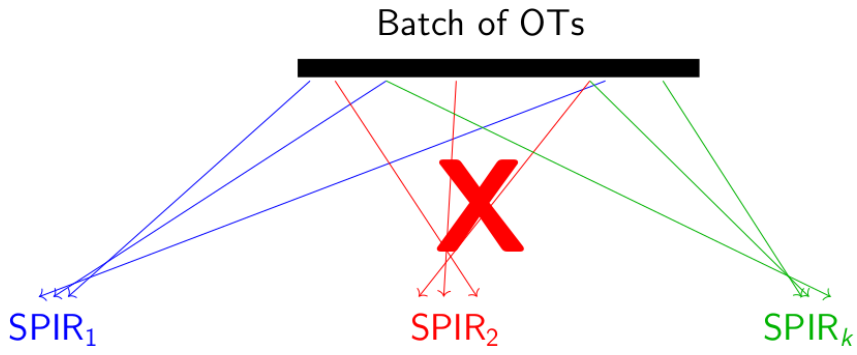
↓

Rate-1 Batch
Function-Hiding LHE

Batch-OT with Optimal Rate
[Brakerski-Branco-Döttling-Pu'22]

↓ + LPN

Rate-1 Batch
2-round 1-out-of-2 OT



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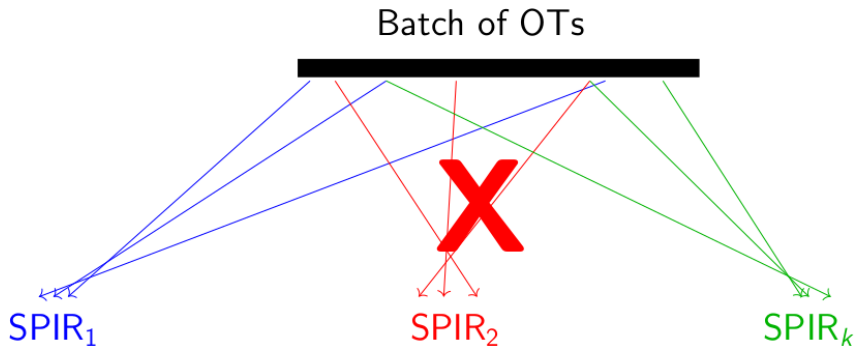
Rate-1 Batch
2-round 1-out-of-2 OT

Sender-Message Decomposability

Constant-Rate Upload
Rate-1 Download
Correlated SPIR



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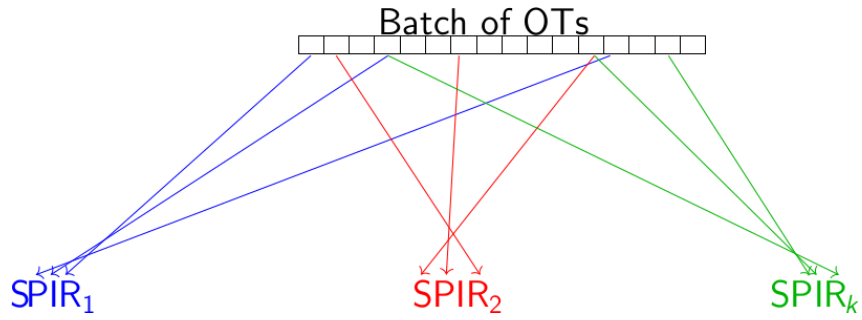
Batch-OT with Optimal Rate
[Brakerski-Branco-Döttling-Pu'22]

↓ + LPN

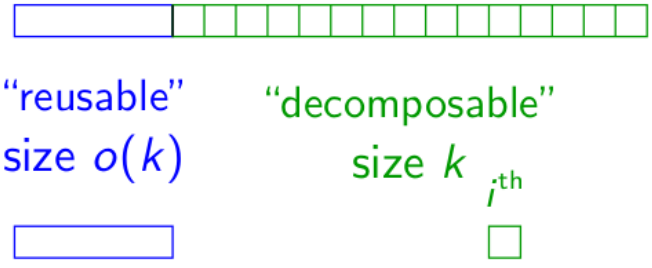
Rate-1 Batch
2-round 1-out-of-2 OT

Sender-Message Decomposability

Constant-Rate Upload
Rate-1 Download
Correlated SPIR



Sender Message for Batch of k OTs



Can be decoded to i^{th} value in batch

Downside of Batching:

Cannot "Mix and Match"
the atomic 1-out-of-2 OTs

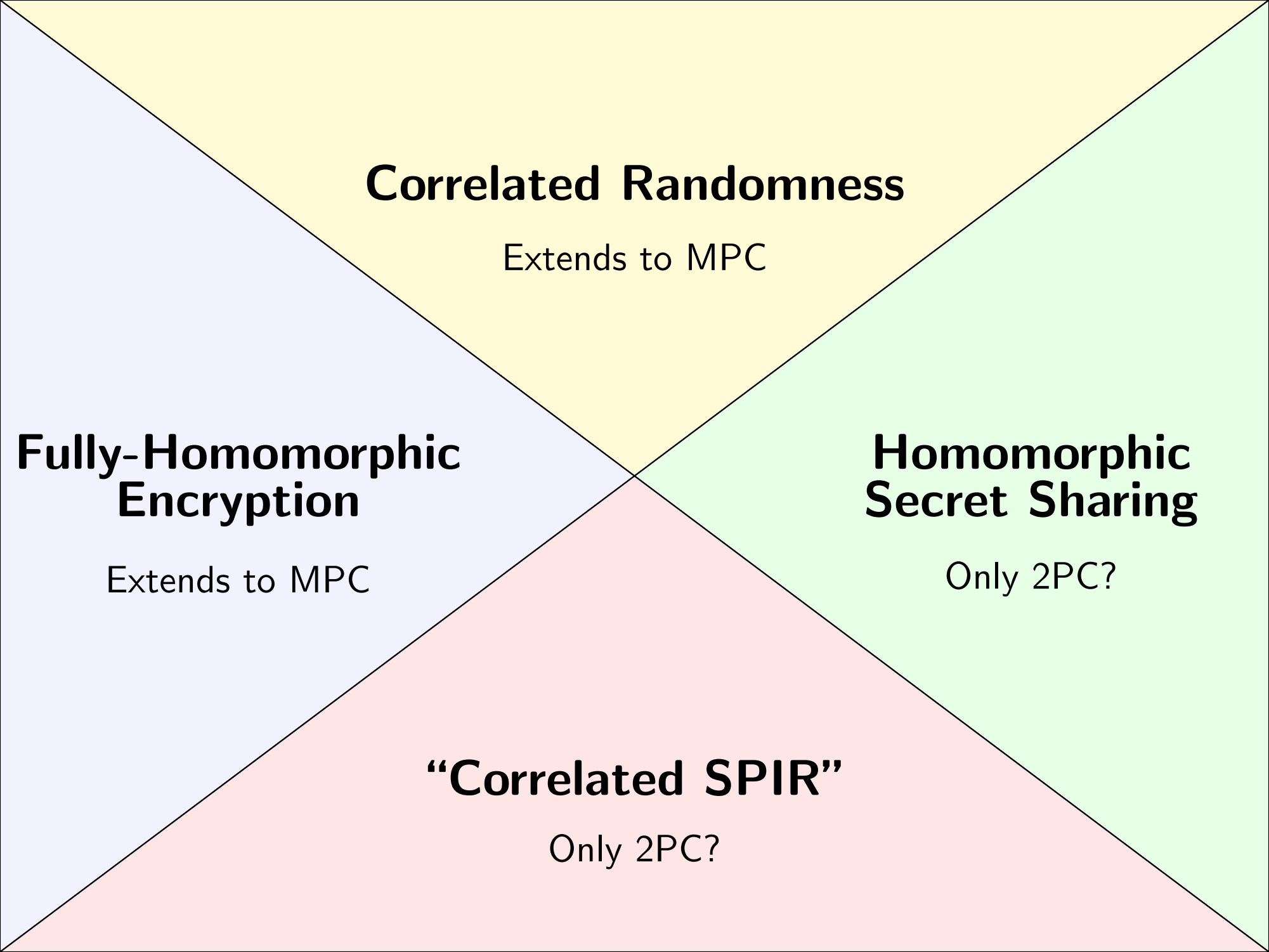
Our Results

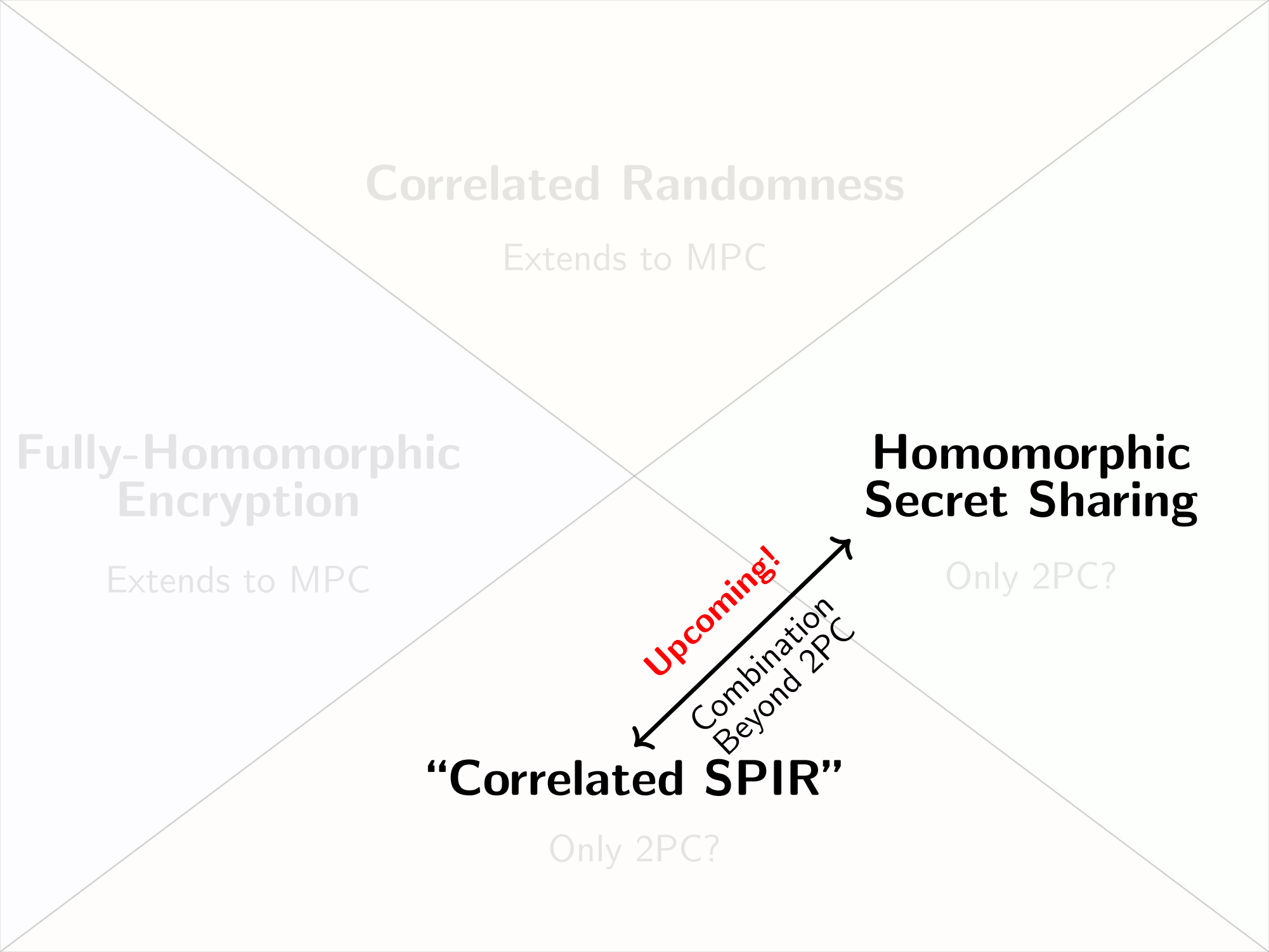
1. Sublinear 2PC for Layered Circuits

- Assumption: QR + LPN.
- Key Feature: New approach to break the circuit-size barrier.
- Circuit Class: Layered Circuits $\mathcal{O}(n + m + \frac{|C|}{\log\log|C|})$ or LogLog-Depth Circuits $\mathcal{O}(n + m + \sqrt{|C|})$.

2. Polylogarithmic PIR from CDH

- Assumption: Computational Diffie-Hellman.
- Key Feature: Uses polylog communication and log rounds.


Correlated Randomness
Extends to MPC
Fully-Homomorphic Encryption
Extends to MPC
Homomorphic Secret Sharing
Only 2PC?
“Correlated SPIR”
Only 2PC?


Correlated Randomness
Extends to MPC
Fully-Homomorphic Encryption
Extends to MPC
Homomorphic Secret Sharing
Only 2PC?
Upcoming!
Combination Beyond 2PC
"Correlated SPIR"
Only 2PC?

Thank you!

Bonus Time!

Polylogarithmic PIR from CDH