Higher-Order Masked Ciphertext Comparison for Lattice-Based Cryptography

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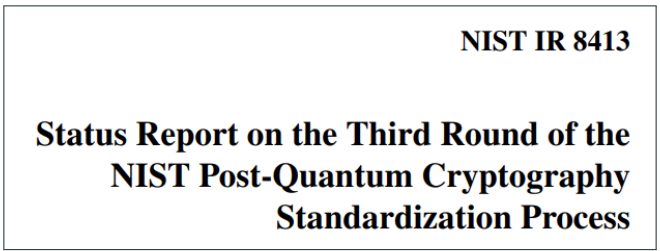
Lattice-based Cryptography

NIST selected Kyber as PQC Key Encapsulation Mechanism (KEM) standard.


NIST IR 8413

Status Report on the Third Round of the NIST Post-Quantum Cryptography Standardization Process

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Higher-order Masking

Implementations require protections again Side-Channel Attacks (SCA)

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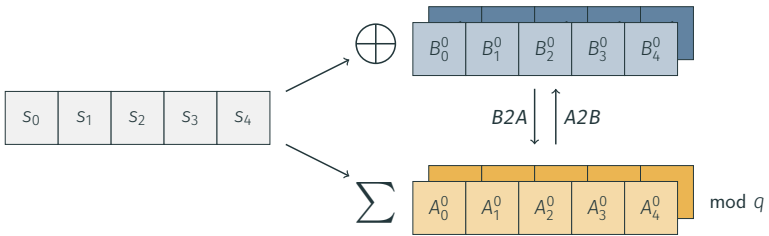

B2A, A2B are key building blocks in masking Lattice-based schemes

Ciphertext Comparison

Kyber uses the FO Transform from IND-CPA PKE to IND-CCA KEM

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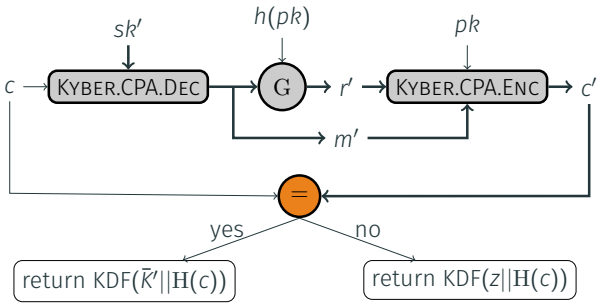

Side-channel Attacks on the FO Transform

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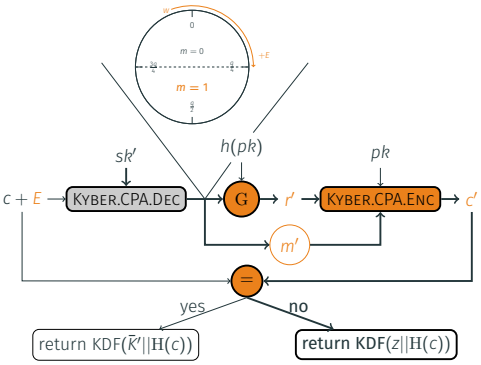

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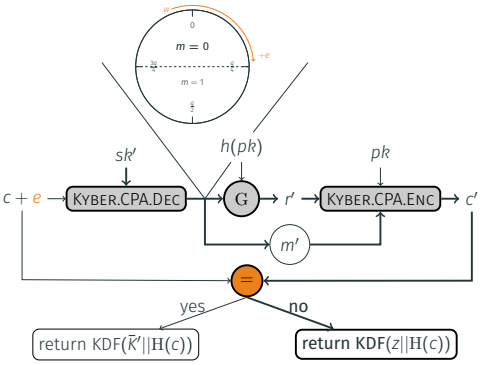

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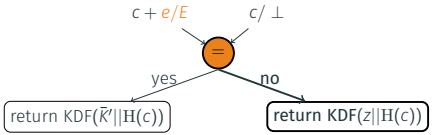

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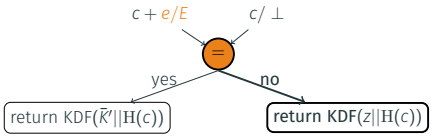

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  1. First-order Hash-based Comparison
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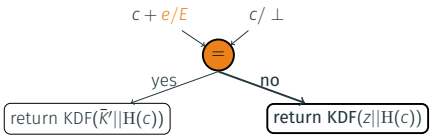

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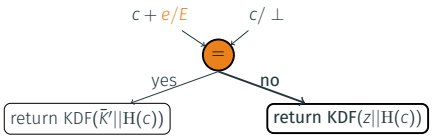

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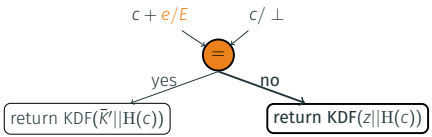

- [BDH+21] Two masked comparisons broken, with fixes proposed:
  1. First-order Hash-based Comparison → **1. we show SCA collision attack**
  2. Higher-order Reduce-Comparisons → **2. we develop into a new method**
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Hash-based Masked Comparison

- Masked Ciphertext Comparison

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- Extra hashing step prevents sensitive leakage

$$H(c - \mathbf{c}'^{(0)}) \stackrel{?}{=} H(\mathbf{c}'^{(1)})$$

SCA on Hash-based Masked Comparison

- Trace contains 2 hash invocations:
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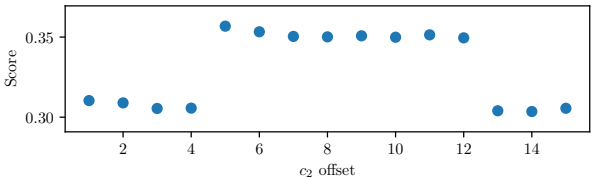

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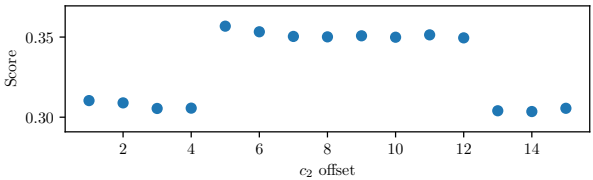
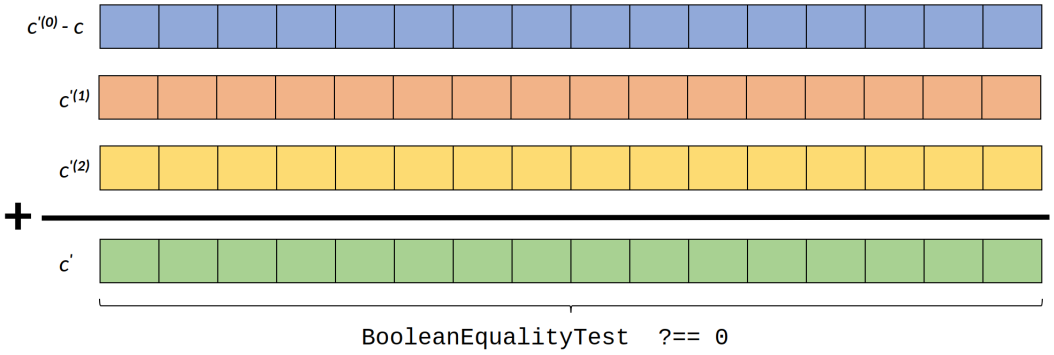


- Single trace for plaintext-checking, 6,000 traces to recover a Kyber512 key
  - **Higher-order Masked Comparison is very necessary**

Reduce-Comparisons



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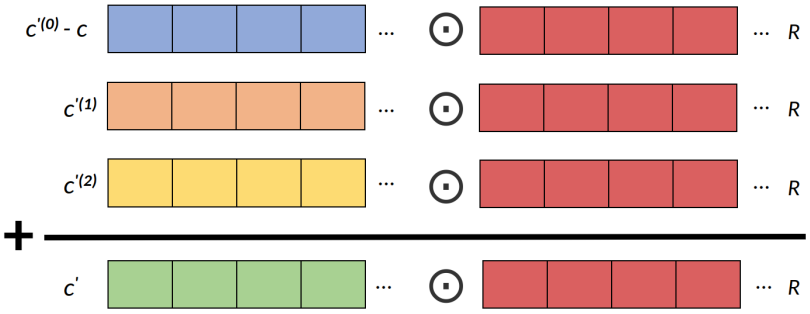

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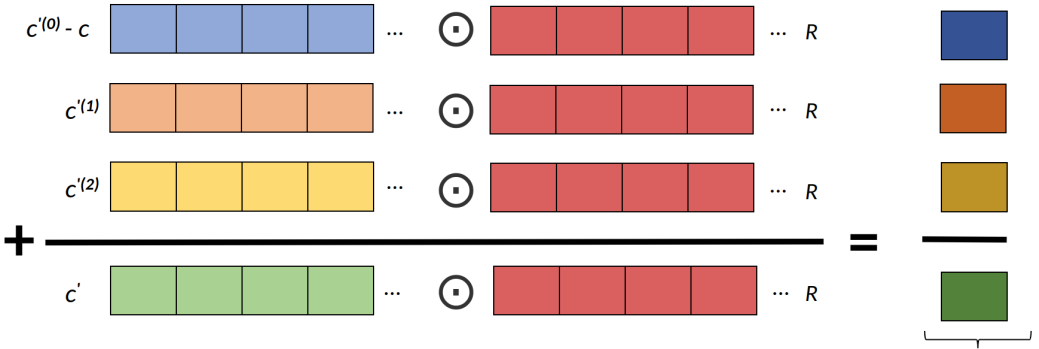

`BooleanEqualityTest ?== 0`

Reduce-Comparisons

Collision probability P_{col} that $e_i \cdot R_i = 0 \bmod q$

1. Prime modulus q: $P_{col} = 1/q$
2. Power-of-two-modulus $q = 2^k$: $P_{col} = 1/2$

Repeat the check L times for probability P_{col}^L

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Saber, but also Kyber is in Category 2 due to ciphertext compression

- $\texttt{Compress}_q(x) = \lfloor (2^k/q) \cdot x \rceil \bmod 2^k$

Random-sum Method

Collision probability $P_{col} = 1/2$ that $e_i \cdot R_i = 0 \mod 2^k$

- $e_i \leftarrow \mathbb{Z}_{2^k}$, $R_i \xleftarrow{\$} \mathbb{Z}_{2^k}$

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More involved implementation :

1. Higher-order MaskedCompress [FVBBR$^+$21]
2. A combination of B2A, A2B to enlarge the masking modulus

Random-sum Method

Input : $\mathbf{c}'$, c. **Output** : $Compress(\mathrm{c}') \stackrel{?}{=} c$

Step 0 : Pre-processing

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  - Higher-order correctness proof for f_{bits} in the paper

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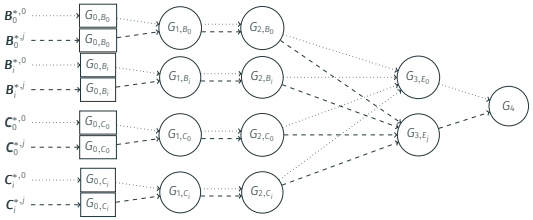
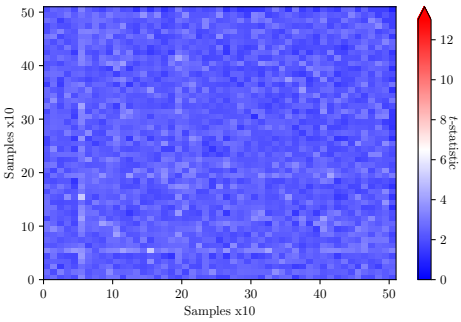
Step 4 : BooleanEqualityTest

Security Proof and Leakage Evaluation

t-SNI security proof



Bivariate 2^{nd}-order *t*-test evaluation



Performance Evaulation

Performance breakdown on ARM-Cortex M4 with collision probability $2^{-s} = 2^{-54}$

Scheme	Comparison	CPU [k]cycles		#random bytes	
		2^{nd}	3^{rd}	2^{nd}	3^{rd}
Saber	**MaskedComp**	**3,363**	**6,543**	**90,264**	**205,104**
Kyber768	**MaskedComp**	**5,253**	**9,712**	**111,256**	**251,184**
	|-Step 0	|-355	|-467	|-0	|-0
	|-Step 1	|-3,485	|-5,742	|-44,544	|-93,184
	|-Step 2	|-1,157	|-3,179	|-57,344	|-147,456
	|-Step 3	|-182	|-197	|-8,192	|-8,192
	|-Step 4	|-73	|-128	|-1,176	|-2,352
	[BGR+21]	22,017 (**4.2x**)	72,568 (**7.5x**)	902,126†	2,434,170†

Original Reduce-Comparisons : $e_i \cdot R_i = 0 \mod 2^k$

Our Reduce-Comparisons : $e_i \cdot R_i = 0 \mod 2^{k+s}$

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Revisiting Higher-Order Masked Ciphertext Comparison for Lattice-Based Cryptography

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Galois-field Reduce-Comparisons : $e_i \odot R_i = 0$

- After A2B, natively in "GF" format

Revisiting Higher-Order Masked Ciphertext Comparison for Lattice-Based Cryptography

Scheme	Comparison	CPU cycles 2^{nd}	CPU cycles 3^{rd}	#random bytes 2^{nd}	#random bytes 3^{rd}
Kyber768	This work (2.1x)	5,2M	9,7M	111K	251K
	GF (1.7x)	4,2M	6,7M	47K	95K
	Simple (1.2x)	3,1M	5,2M	48K	100K
	[CGMZ21] (1.3x)	3.3M	4.5M	80K	94K
	Streamlined Hybrid (1.0x)	2,5M	3,6M	44K	62K

`github.com/KULeuven-COSIC/Masked-Comparison`

`github.com/KULeuven-COSIC/Revisiting-Masked-Comparison`

References

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