## The Hidden Parallelepiped is Back Again: Power Analysis Attacks on Falcon

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September 20, 2022

## THALES



# $\Psi_{\text {Falcon }}$ 

Fast-Fourier Lattice-based
Compact Signatures over NTRU

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## Fast-Fourier Lattice-based Compact Signatures over NTRU

Two Power Analysis attacks on Falcon:

- Efficient DPA attack on the preimage computation
- STA on the trapdoor sampler leading to HPP attack


## Lattice-based cryptography

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Note: It is hard to derivate the good basis from the bad basis.

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## Hidden Parallelepiped attack on the trapdoor sampler

1. Side-channel analysis on the BaseSampler to recover samples
2. Utilisation of the samples to disclose a deformed parallelepiped
3. Application of HPP solver on filtered signatures
4. Private key recovering (possibly with lattice magic)

## 1. Side-channel analysis on the BaseSampler




## BaseSampler():

1. $u \leftarrow$ UniformBits(72)
2. $z^{+} \leftarrow 0$
3. for $i=0 \ldots 16$ do
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We are able to retrieve the value of $z^{+}$through STA.

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Useful observation: Because of the algorithm used in Falcon to compute the GSO (ffLDL algorithm), we have the following:
$\tilde{\mathbf{b}}_{\mathbf{0}}, \ldots, \tilde{\mathbf{b}}_{\mathbf{3}} \approx \mathbf{b}_{0}, \ldots, \mathbf{b}_{3}$ and $\tilde{\mathbf{b}}_{\mathbf{n}}, \ldots, \tilde{\mathbf{b}}_{\mathbf{n}+\mathbf{3}} \approx \mathbf{b}_{n}, \ldots, \mathbf{b}_{n+3}$

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Number of measured signatures

## Conclusion

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Preimage computation: Improvement of State-of-the-Art attack. Trapdoor sampler: Novel attack combining SCA and HPP.

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## Questions ?

## References I

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\begin{array}{ll}
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& \text { and New Records in Lattice Reduction". In: } \\
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& \text { Vincent Rijmen. Vol. 11477. LNCS. Springer, } \\
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{[\text { Dac+20] }} & \text { Dana Dachman-Soled et al. "LWE with Side } \\
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## Partial countermeasure for BaseSampler

Main idea: invert the sign of the operands to replace the (hardware) underflow by a (logical) overflow.

Replace the last substraction by the following:

1. $b \leftarrow 0 \mathrm{xffffff}$
2. $b:=b-\bar{u}+\overline{\operatorname{RCDT}[i]}+c$
3. return $b \gg 24$

State of the register before the last operation:


State of the register after the last operation (original implementation):


State of the register after the last operation (with countermeasure):


