Side-Channel Masking with Common Shares

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Masking

- Randomize the secret (Enc)
  - Secret variable $x \xrightarrow{\text{rand}}$ shares $\hat{\mathbf{x}}[1], \ldots, \hat{\mathbf{x}}[d]$
    - Boolean masking: $x = \hat{\mathbf{x}}[0] \oplus \ldots \oplus \hat{\mathbf{x}}[d]$
  - Any d shares *are independent of* x
- Private computations (various gadgets, especially multiplication gadgets).
  - Any d intermediates *are independent of* the input secrets: d-privacy, d-probing security

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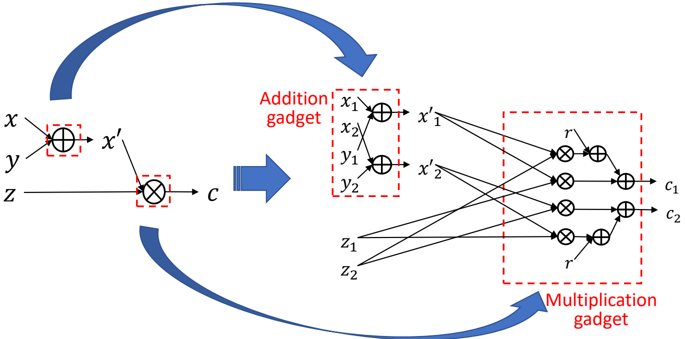

Example: the ISW Multiplicaition with 3 Shares

- Proposed by Yuval **I**shai, Amit **S**ahai and David **W**agner at *CRYPTO '03*.
- Input: $\hat{\mathbf{x}}[1], \hat{\mathbf{x}}[2], \hat{\mathbf{x}}[3]$ and $\hat{\mathbf{y}}[1], \hat{\mathbf{y}}[2], \hat{\mathbf{y}}[3]$, Output: $\hat{\mathbf{z}}[1], \hat{\mathbf{z}}[2], \hat{\mathbf{z}}[3]$

- It requires $\frac{d(d+1)}{2}$ random variables and runs in complexity $\mathcal{O}(d^2)$.

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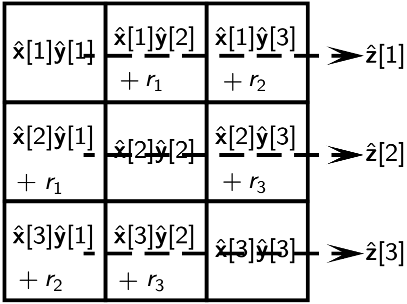

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A Summary of Contributions

Goal: reducing the overheads

- Theoretical contributions
  - Masked multiplication with common shares
  - Precomputation-based design paradigm.
  - New security notion: from parallel to general compositions.
- Application to the masked AES

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Cost amortization

- A part of shares of different variables is the same.
- Randomness and intermediate variables can be reused among different operations.

Two Types of Sharings

- Boolean sharing:
  - Secret variable $x \xrightarrow{\text{rand}}$ shares $\hat{\mathbf{x}}[0], \ldots, \hat{\mathbf{x}}[d]$ such that $x = \hat{\mathbf{x}}[0] \oplus \ldots \oplus \hat{\mathbf{x}}[d]$
  - Common shares are insecure.
    - Sharing of a: $\hat{a}, \hat{\mathbf{s}}[1], \ldots, \hat{\mathbf{s}}[d]$
    - Sharing of b: $\hat{b}, \hat{\mathbf{s}}[1], \ldots, \hat{\mathbf{s}}[d]$
    - $\hat{a} \oplus \hat{b} = a \oplus b$
- Inner product sharing:
  - Secret variable $x \xrightarrow{\text{rand}}$ shares $\hat{\mathbf{x}}[0], \ldots, \hat{\mathbf{x}}[d]$ such that $x = \hat{\mathbf{x}}[0] \oplus \alpha[1]\hat{\mathbf{x}}[1] \oplus \ldots, \alpha[d]\hat{\mathbf{x}}[d]$
  - Common shares can be secure!
    - Sharing of a: $\hat{a}, \hat{\mathbf{s}}[1], \ldots, \hat{\mathbf{s}}[d]$ such that $a = \hat{a} \oplus \alpha_a[1]\hat{\mathbf{s}}[1] \oplus \ldots \oplus \alpha_a[d]\hat{\mathbf{s}}[d]$
    - Sharing of b: $\hat{b}, \hat{\mathbf{s}}[1], \ldots, \hat{\mathbf{s}}[d]$ such that $b = \hat{b} \oplus \alpha_b[1]\hat{\mathbf{s}}[1] \oplus \ldots \oplus \alpha_b[d]\hat{\mathbf{s}}[d]$
    - Still d-probing secure if $(1, \alpha_a)$ and $(1, \alpha_b)$ are linearly independent.

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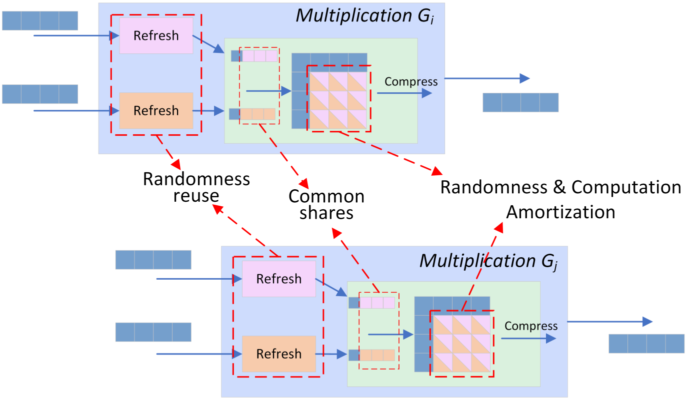
    - Still d-probing secure if $(1, \boldsymbol{\alpha}_a)$ and $(1, \boldsymbol{\alpha}_b)$ are linearly independent.

Masked Multiplications with Common Shares

- Input of Refresh: Boolean sharings.
- Output of Refresh: inner product sharings, allowing:
  - common shares;
  - randomness & computation amortization.
- Output of Multiplicaiton:
  - Boolean shares.



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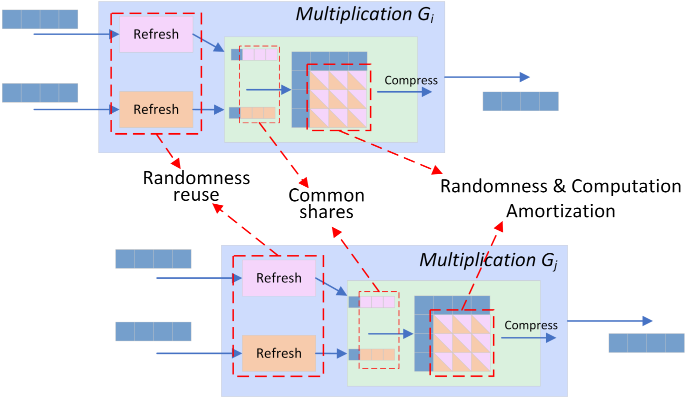
Precomputation-based Design Paradigm



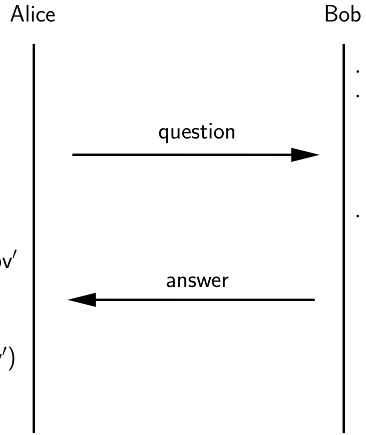
A promising feature: most of the intermediate variables can be precomputed

Precomputation-based Design Paradigm

A masked implementation of a crytographic function f can be divided into:

- Precomputation phase:
  - Precompute a set of vairbles $\mathcal{V}$.
  - Runs in $O(\ell d^2)$ and requires $O(\ell d^2)$ random bits
  - Not one-time effort: it runs for each calls of f.
    - But (important!): the precomputation does not requires the input of f.
- Online-computation phase:
  - Very efficient: runs in $O(\ell d)$ without any random bits.

Precomputation-based Design Paradigm



Bob:

- Generate the randomness
- Perform the precomption
  - Output precomputed values: pv

Alice:

- Generate the randomness
- Perform the precomption
  - Output precomputed values: pv′

Bob:

- Call online masked encryption:
  - answer′ ← E(sharedkey, question, pv)

Alice:

- Call online masked encryption:
  - answer′ ← E(sharedkey, question, pv′)
- Check if answer = answer′

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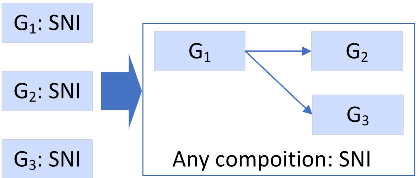
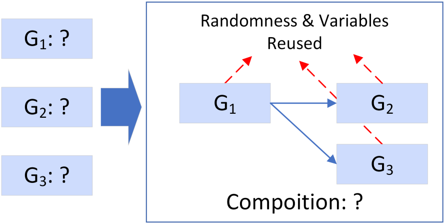
Problem Statement

- Composable security notions: one can concentrate on analyzing every single gadget, and leave the rest to the probe propagation.
  - Non-Inference/Strong Non-Inference (NI/SNI)
  - Probe-Isolating Non-Inference (PINI)
- The randomness used in different gadget should be independent.
  - It is not complied to our case (and other masking schemes with randomness reuse).

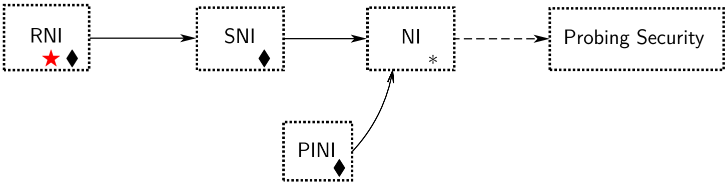




New Security Notion

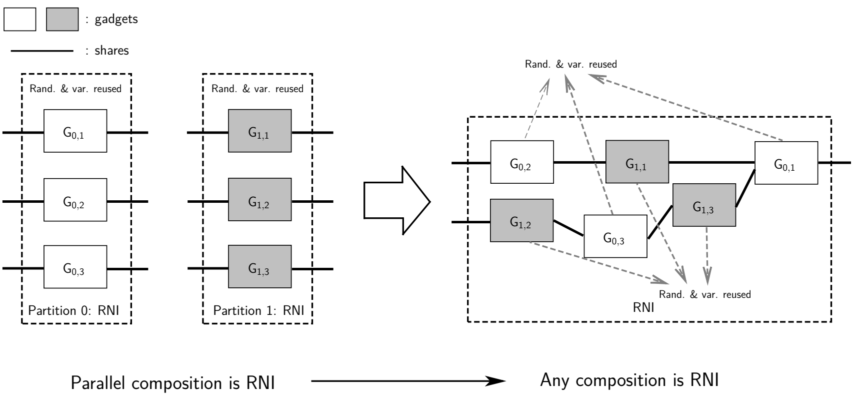
- Randomness Reusable Non-Inference: RNI

⟶: implication
- - -⟶: implication if any d input shares are independent of the secret
★: Supporting compositions even if randomness/variables are reused
♦: Supporting trivial composition if random bits are independent
$*$: Supporting compositions (with SNI refreshing) if random bits are independent



Relations of different security notions

Composability of RNI



An arbitrary composition (on the right) that can be described a bipartite graph is RNI, as long as the parallel compositions (on the left) of gadgets in each partition is RNI.

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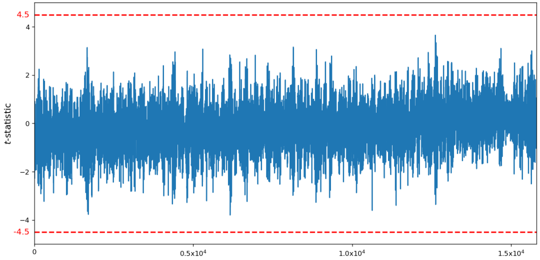
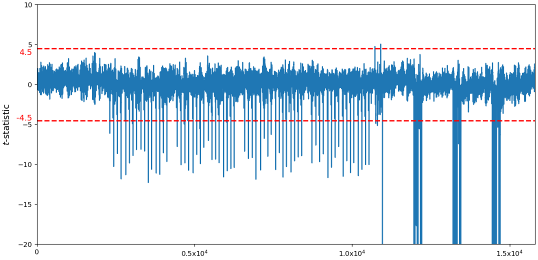
Summary of Masked AES Implementations

	d	KCycles for Precomp.	RAM size	KCycles/penalty factor for online
Unprotected	-	-	-	9.33 / 1
BS method	2	2 880 (random gen.)	1.92 KB	62 / 6.65
LUT method	2	15 360 (random gen.)	10.24 KB	435 / 46.62
Our work	2	144(random gen.) + 705	5.63 KB	**60** / 6.43
BS method	8	34 560 (random gen.)	23.04 KB	330 / 35.36
LUT method	8	245 760 (random gen.)	164 KB	unreported
Our work	8	2 304(random gen.) + 3 66	11 KB	**137** / 14.68

T-test results, security order $d = 1$



(a) New masking



(b) Boolean masking

- In the implementation, we do not attempt to eliminate all the transitional leakage that may damage the independent leakage assumption.

Discussion on the results

- This (good) result for the case of $d = 1$ is a bit surprising, since:
  - there is transitional leakage, but it is still secure
- The new scheme is more robust to some lapses (e.g., transitional leakage) in implementation.
  - We contribute this advantage to the relatively more complex algebraic structure than the Boolean masking.

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Conclusion

- Theoretical contributions:
  - Cost amortized multiplication gadget with common shares
    - The computational decreases: $\tilde{\mathcal{O}}(\ell d^2)$
    - The randomness decreases: $\tilde{\mathcal{O}}(d^2)$
  - Precomputation-based design paradigm for masking
    - Pre-computation phase: $\tilde{\mathcal{O}}(\ell d^2)$ (computational), $\tilde{\mathcal{O}}(d^2)$ (randomness).
    - Online phase: $\tilde{\mathcal{O}}(\ell d)$ (computational), without any randomness.
  - New security notion for proofs: from parallel to general compositions
    - Intrinsically supports randomness/variables reusing.
- Application to AES.
  - A speed-up for the online phase.
  - More robust to some lapses (e.g., transitional leakage) in implementation.

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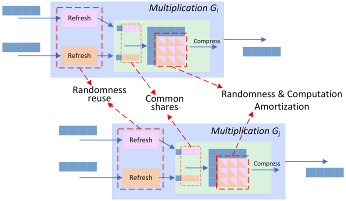
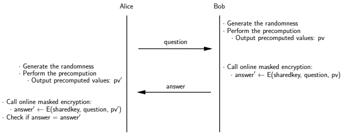
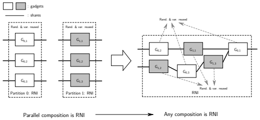


New construction



New paradigm



New proof method

Thank You!