

# GE vs GM: Efficient side-channel security evaluations on full cryptographic keys

## CHES 2022

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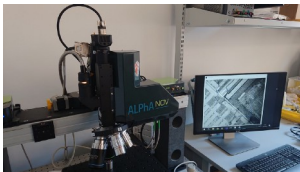
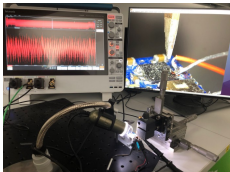


# Thanks Christ, the UPB team and Virgil Gligor from CMU



(The UPB campus – left: our Church; right: the rector offices)

# Side-channel attack security evaluations



Images from <https://medium.com/@charles.guillemet/ledger-donjon-3e04e0ce49a9>

## SCA evaluations necessary:

- During product manufacturing to assess security of products
- For governments, to establish some required standards
- For security industry (e.g. automotive, banking) to ensure that third-party products (e.g. smartcards) have a sufficient level of security
- To obtain a uniform level of security certification (e.g. Common Criteria EAL4+)

# SCA security evaluation tools for short data (e.g. key byte)

- Commonly used security level estimation metrics:  
Success Rate (SR), Guessing Entropy (GE) aka Rank
- Less common (yet...): Massey's Guessing Entropy (GM)
- A mess of guessing entropy measures and notations
  - 1994: James Massey proposes  $E[G]$
  - 1997: Christian Cachin terms it 'Guessing Entropy'  $E[G(X)]$  and present conditional version  $E[G(X|y)]$
  - 2007: Köpf and Basin use the conditional guessing entropy in the context of side-channel attacks
  - 2009: FX Standaert et al. present (empirical) Guessing Entropy in framework for SCA evaluations
- Bigger problem: GE and GM both run in  $O(N \log N)$ 
  - Do not directly scale for large keys (impractical for  $N > 2^{16}$ )
  - We need special methods for full-key security evaluations

# SCA security evaluation tools for full keys (e.g. 128-bit AES key, 4096-bit RSA key)

Two main approaches for full-key security evaluations:

- Key enumeration for large keys ([Charvillon et al. 2012, Poussier et al. 2016])
- **Security level estimation for large keys:**
  - Empirical Guessing Entropy (Rank) estimation ([Charvillon et al. 2013, Glowacz et al. 2015, Zhang et al. 2020])
  - Massey's Guessing Entropy (GM) bounds ([Choudary and Popescu 2017])

# SCA security evaluation tools for full keys (e.g. 128-bit AES key, 4096-bit RSA key)

Our main goal – comparing full-key SCA evaluation tools:

- FSE'15 rank estimation [Glowacz et al. 2015]
  - One of the fastest GE estimation methods to date
  - Works well up to 256 key bytes, with good precision
- GM bounds [Choudary and Popescu 2017]
  - Mathematical, rigorous bounds for GM
  - Fastest and most scalable full-key evaluation method to date
  - Works with 1024-byte keys and beyond
- GEEA rank estimation [Zhang et al. 2020]
  - One of the newest methods for GE estimation on large keys
  - Lower STD than FSE'15

# GM vs GE computation

$$\text{(Massey's)GM} = \frac{1}{N} \sum_{q=1}^N \sum_{i=1}^{|\mathcal{S}|} i \cdot P(k_i | X = \mathbf{X}_q)$$

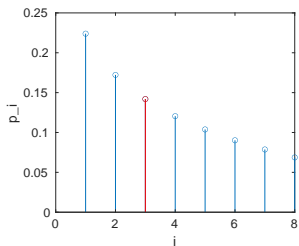
$$\text{(Empirical)GE} = \frac{1}{N} \sum_{q=1}^N \{\text{rank of } k^* \text{ in experiment } q\}$$

$$(P(k_1 | \mathbf{X}_q) \geq \dots \geq P(k_i | \mathbf{X}_q) = P(k^* | \mathbf{X}_q) \geq \dots \geq P(k_{|\mathcal{S}|} | \mathbf{X}_q))$$

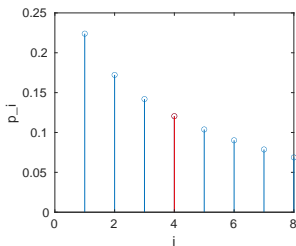
Observations:

- Same complexity (need to sort all the list of probabilities)
- Both dependent on acquired datasets ( $\mathbf{X}_q$ )
- Different use of probabilities
- GE requires knowledge of correct key, GM does not

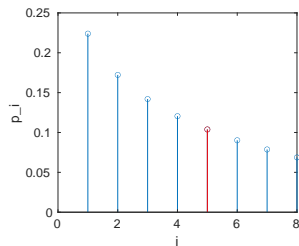
# GM vs GE simple example



GM 3.63



3.63



3.63

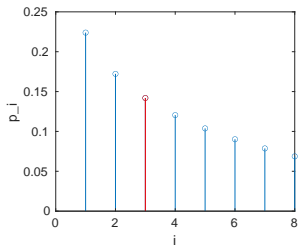
GE 3

4

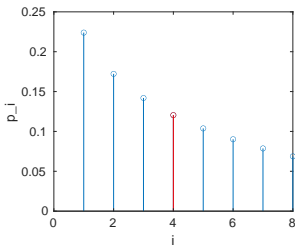
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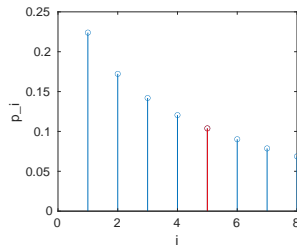
# GM vs GE simple example



GM 3.63



3.63



3.63

GE 3

4

5

→ GE provides actual (empirical) estimation of rank

→ GM is generally a lower bound for GE [KB'07]

# Experimental datasets

- We used three different datasets:
  - *Simulated* dataset (Hamming weight of AES S-box output mixed with Gaussian noise):  $\mathbf{x}_i = \text{HW}(\text{S-box}(k \oplus p_i)) + r_i$
  - *XMEGA* dataset (AVR XMEGA AES engine)

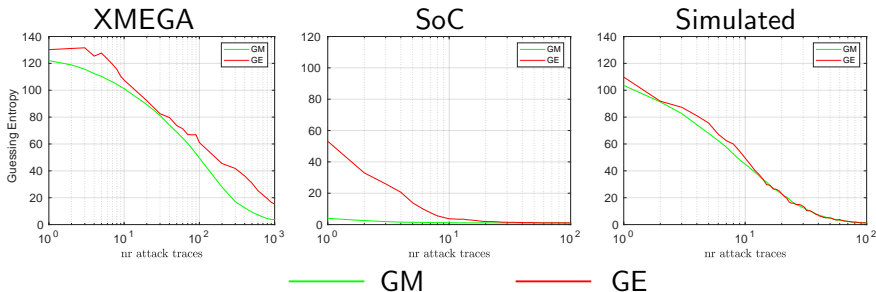


- *SoC* dataset (ChipWhisperer-Lite with STM32F303 32-bit ARM)



- We used Template Attacks to obtain lists of probabilities for each AES key byte  $(p_1, p_2, \dots, p_{256})$

# On the utility of GM



Observation 1: GM is generally a lower bound for GE

→ Can be used to confirm security is above a certain threshold

Observation 2: we may combine both measures to determine the quality of a leakage model

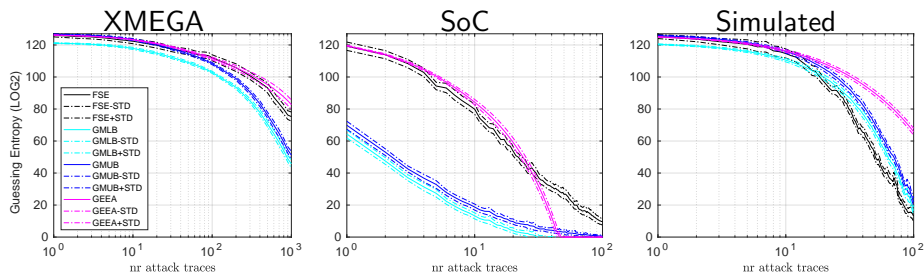
GM close to GE → good model (e.g. in *Simulated* dataset)

GM departs from GE → bad model (e.g. in *SoC* dataset)

# Analysis of full-key evaluation tools

- We focus on the three representative methods
  - FSE'15 (Glowacz et al. 2015)
  - GM Bounds (Choudary and Popescu 2017)
  - GEEA (Zhang et al. 2020).

## Precision analysis on 128-bit data (16-byte results)

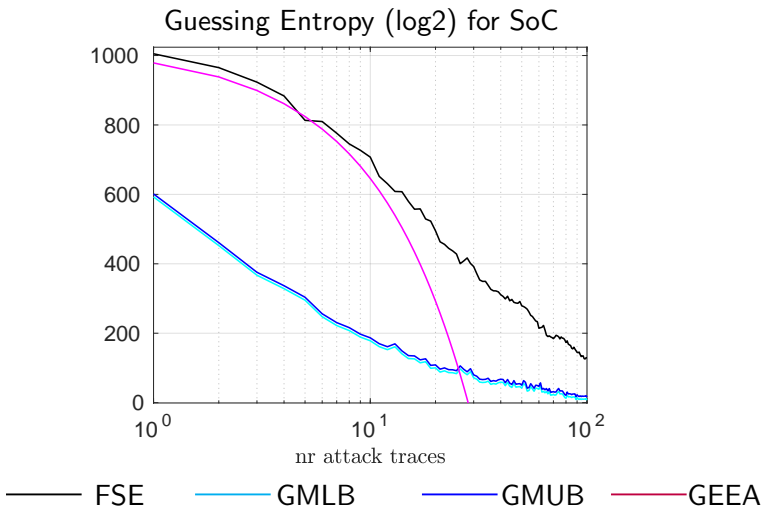


— FSE      — GMLB      — GMUB      — GEEA

Median STD

FSE'15	1.84	2.67	2.89
GM Bounds	0.74	1.34	2.22
GEEA	0.56	0.81	1.77

## Scalability and usability analysis on larger data (128 bytes)



# Scalability and computation analysis on large data (16/128/1024-byte results)

Computation time (s) for XMEGA/SoC/simulated

	16 bytes	128 bytes	1024 bytes
FSE'15	29/60/172	1027/5336/4689	Not practical
GM Bounds	1/1/1	2/6/6	40
GEEA ( $M = 10^4, 10^6$ )	17/18/26	432/415/473	Not practical

# Overall analysis and usability guidelines

- FSE'15:
  - Good approximation of GE
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  - Slow computation for large keys



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- GM Bounds:
  - Guaranteed, tight bounds for GM
  - (Typically) Lower bound for GE/FSE
  - Can be used with very large keys
- GEEA:
  - High accuracy (low STD)
  - Deviates from GE/FSE within similar computation time
  - Needs more analysis to provide some guarantees

# Overall analysis and usability guidelines

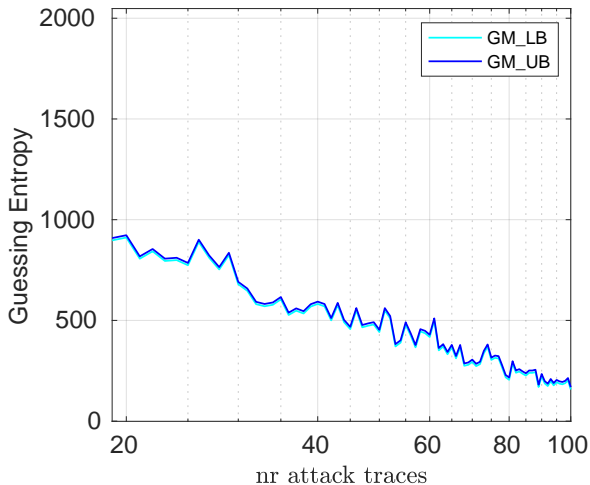
- Conclusions:
  - Use GM Bounds for a very fast security evaluation (lower bound) – works with very large keys  
<https://gitlab.cs.pub.ro/marios.choudary/gmbounds>
  - Use FSE'15 or other GE estimation algorithm for accurate estimate of key rank
  - (Optionally) Use a key enumeration algorithm to output list of keys in decreasing probability

Greetings from the UPB (GM Bounds) Team

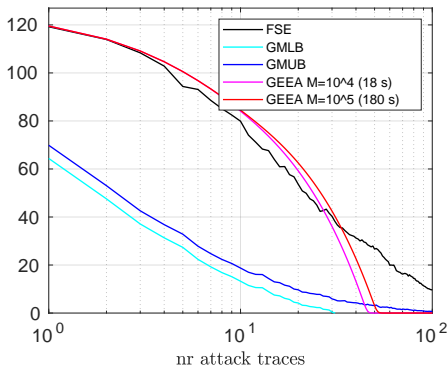


# Appendix

# GM Bounds (log2) on 1024-byte key (SoC data)



# GEEA with varying amount of data (SoC, 16 bytes)



- GEEA computation on large keys uses random selection of subkey computations (comparison vectors)
- Needs very large  $M$  (large computation) to approach GE/FSE
- May not be able to follow GE within given computing power