ECDSA White-Box Implementations
Attacks and Designs from WhibOx 2021 Contest

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Black-Box, Grey-Box, White-Box

- Plaintext → Ciphertext
- Cryptanalysis
- Side-channels/Faults
- Read/modify binary
Designers
- Post C codes computing ECDSA
- Challenges gain strawberries (depending on performances and time until break)

Attackers
- Try to extract the secret key
- Receive bananas (number of strawberries of the challenge)
Our Contributions

zerokey
- Posted the 2 winning challenges
- Described the implementations

TheRealIdefix
- Broke the most challenges
- Described attacks, showing which ones succeeded for each candidate
ECDSA

- Let $G$ be a point of order $n$ on an elliptic curve $E$
- Let $d$ be a 256-bit key
- Let $m$ be a message and $e = H(m)$ its hash value

Algorithm 1: ECDSA signature

1. $k \leftarrow \mathbb{Z}[1, n - 1]$
2. $R \leftarrow kG$
3. $r \leftarrow x_R \mod n$
4. $s \leftarrow k^{-1}(e + rd) \mod n$
5. if $r == 0$ or $s == 0$ then
   6. Go to step 1
7. end
8. Return $(r, s)$
ECDSA Sensitive Values

- Let $G$ be a point of order $n$ on an elliptic curve $E$
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5. **if** $r == 0$ *or* $s == 0$ **then**
   - Go to step 1
6. **end**
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Let $G$ be a point of order $n$ on an elliptic curve $E$
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Deterministic ECDSA

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**Algorithm 1: ECDSA signature**

1. $k \leftarrow \{1, n-1\}$  \hspace{1cm} WB model $\Rightarrow$ No reliable source of randomness!
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**Algorithm 1: ECDSA signature**

1. $k \leftarrow f(e)$  
   WB model $\Rightarrow$ No reliable source of randomness!
2. $R \leftarrow kG$
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Section 1

Breaking the Candidates
Hooking Shared Libraries

Idea

Find some secret values that could be manipulated in the clear

- Use of the GMP library suggested by the contest rules
- Hook the calls to GMP functions (LD_PRELOAD)
- Update a log of the given parameters
- Search for $d$, $k$ or related values in the log
First possibility

Find collisions: signatures generated with the same nonce

- Find \((r_1, s_1)\) and \((r_2, s_2)\) such that \(r_1 = r_2\) (so \(k_1 = k_2\))
- Solve the following system in \(k_1, d\):

\[
\begin{align*}
    s_1 &= k_1^{-1}(e_1 + r_1 d) \\
    s_2 &= k_1^{-1}(e_2 + r_1 d)
\end{align*}
\]
Biased Nonce

First possibility

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\]

Second possibility

Exploit biases in the nonce generation

- Use lattice-based attacks
- Allows to recover \(d\) from a few bits of \(k\) for several signatures.
Grey-box Attacks in the White-Box Model

- Side-channel attacks
  - Difficult to apply (huge size of the traces)

- Fault injections
  - Modify the binary, use debugging tools
  - Many fault attacks on deterministic ECDSA, for example:

\[
\begin{align*}
\text{Valid signature} \\
\hspace{1cm} r &= xR \mod n \\
\hspace{1cm} s &= k - 1 \left( e + rd \right) \mod n \\
\end{align*}
\]

\[
\begin{align*}
\text{Faulty signature} \\
\hspace{1cm} r' &= xR \mod n \\
\hspace{1cm} s' &= k - 1 \left( e + r'd \right) \mod n \\
\end{align*}
\]

\[
d = \left( s \left( r - r' \right) \left( s - s' \right) - 1 - r \right) \mod n
\]
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  Valid signature
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  s = k^{-1}(e + rd) \mod n
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  Faulty signature
  \[
  r' = x_{R'} \mod n \\
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  \]
Grey-box Attacks in the White-Box Model

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- Fault injections
  - Modify the binary, use debugging tools
  - Many fault attacks on deterministic ECDSA, for example:

Valid signature
\[ r = x_R \mod n \]
\[ s = k^{-1}(e + rd) \mod n \]
\[ d = (s(r - r')(s - s')^{-1} - r)^{-1}e \mod n \]

Faulty signature
\[ r' = x_{R'} \mod n \]
\[ s' = k^{-1}(e + r'd) \mod n \]
Percentage of Challenges Broken by Each Attack

- **Bad nonce**: 74%
- **Faults**: 78%
- **Hooking**: 33%

The diagram shows that faults are the most successful attack, followed by bad nonce and then hooking.
Section 2

Design of the Winning Challenges
How to Win the Strawberries

- The implicit framework for white-box implementation
  - A novel encoding method
How to Win the Strawberries

- The implicit framework for white-box implementation
  - A novel encoding method

- Techniques from multivariate public key crypto
  - Additional countermeasures
How to Win the Strawberries

- The implicit framework for white-box implementation
  - A novel encoding method

- Techniques from multivariate public key crypto
  - Additional countermeasures

- Obfuscator
  - Use Tigress to obfuscate the source codes
Implicit Framework

Implicit Function and Implicit Evaluation

\[ F(x) = y \iff T(x, y) = 0 \]

Evaluate \( F(a) \) by substituting \( x = a \) and solving \( T(a, y) = 0 \).
Implicit Framework

Implicit Function and Implicit Evaluation

\[ F(x) = y \iff T(x, y) = 0 \]

Evaluate \( F(a) \) by substituting \( x = a \) and solving \( T(a, y) = 0 \)

Quasilinear Implicit Function (QIF)

\[ \forall x, \text{ function } y \mapsto T(x, y) \text{ is affine} \]

This enables fast solving of \( y \)
Implicit Implementation

\[ F = F^{(t)} \circ F^{(t-1)} \circ \ldots \circ F^{(1)} \]

- **Encoded implementation**

\[ \overline{F} = \overline{F^{(t)}} \circ \ldots \circ \overline{F^{(1)}} = (B^{(t)} \circ F^{(t)} \circ A^{(t)}) \circ \ldots \circ (B^{(1)} \circ F^{(1)} \circ A^{(1)}) \]

- \( T \) is a QIF of \( F^{(i)} \) \( \implies \) \( T' = M \circ T \circ (A, B^{-1}) \) is a QIF of \( F^{(i)'} = B \circ F^{(i)} \circ A \), for any linear permutation \( M \)
Implicit Implementation

\[ F = F(t) \circ F(t-1) \circ \ldots \circ F(1) \]

- **Encoded implementation**

\[ \overline{F} = \overline{F(t)} \circ \ldots \circ \overline{F(1)} = (B(t) \circ F(t) \circ A(t)) \circ \ldots \circ (B(1) \circ F(1) \circ A(1)) \]

- **\( T \) is a QIF of \( F^{(i)} \) \( \implies \) \( T' = M \circ T \circ (A, B^{-1}) \) is a QIF of \( F^{(i)'} = B \circ F^{(i)} \circ A \), for any linear permutation \( M \)**

Implicit implementation = Encoded implementation + QIF
Algorithm 2: White-box ECDSA for winning challenges

1. $e \leftarrow e \mod p$
2. $(v_1, v_2, v_3) \leftarrow E^{(1)}(e)$  // implicit evaluation of $kG$
3. for $o \in \mathcal{L}$ do
4. \hskip1em $(u_1, u_2, u_3) \leftarrow (v_1, v_2, v_3) + p \cdot o$
5. \hskip1em $(r, s) \leftarrow E^{(2)}(u_1, u_2, u_3)$  // implicit evaluation of $(r, s)$
6. \hskip1em if $\text{VerifySignature}(r, s, e) = \text{valid}$ then
7. \hskip2em return $(r, s)$
8. \hskip1em end
9. end

- A round-based implementation of scalar multiplication
- Precompute $\mathcal{L}$ to deal with overflows when converting $\mod p$ to $\mod n$
Obfuscation by multiplying random polynomials

- Implicit evaluation can be preserved w.h.p.

\[ R(a)T(a, y) = 0 \implies T(a, y) = 0 \]

- Set an initial value to prevent failures

Masking of the nonce

Avoid bias of the most significant part of the nonce \(k\)
Implicit implementation + Multiplying random polynomials

<table>
<thead>
<tr>
<th></th>
<th>$T^{(1)}$</th>
<th>${T^{(2)}, \ldots, T^{(t-1)}}$</th>
<th>$T^{(t)}$</th>
<th>$T^{(t+1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>input variables</td>
<td>2+4</td>
<td>5+4</td>
<td>5+3</td>
<td>3+2</td>
</tr>
<tr>
<td>number of components</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>degree</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>number of coefficients</td>
<td>$27 \times 4$</td>
<td>$130 \times 4$</td>
<td>$255 \times 3$</td>
<td>$18 \times 2$</td>
</tr>
</tbody>
</table>

- Compiled binary: 4.42 MB
- Average running time: 0.04s
- Average RAM consumed: 6.14 MB
- Code obfuscation did not impact the running time but increased the binary size by 8% and the average RAM by 3%
**Implicit implementation + Multiplying random polynomials + Masking**

<table>
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<tr>
<th></th>
<th>$T^{(1)}$</th>
<th>${T^{(2)}, \ldots, T^{(t-1)}}$</th>
<th>$T^{(t)}$</th>
<th>$T^{(t+1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>input variables</td>
<td>2+6</td>
<td>7+6</td>
<td>7+5</td>
<td>5+2</td>
</tr>
<tr>
<td>number of components</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>degree</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>number of coefficients</td>
<td>37×6</td>
<td>322×6</td>
<td>854×5</td>
<td>504×2</td>
</tr>
</tbody>
</table>

- Compiled binary: 15.44 MB
- Average running time: 0.15s
- Average RAM consumed: 17.27 MB
- Impact of code obfuscation < 1%
Security Analysis

Automated attacks do not work
- ✗ Hooking shared libraries
- ✗ Biased nonces

✓ Fault attacks [BDG+22]
  - remove only one line of code for each challenge
  - induce uncontrolled fault in $r$
  - defeat the verification steps
We present automated attacks breaking a large number of challenges

Fault attacks are the most efficient and effective

We apply implicit implementation framework to ECDSA

The best implementations were broken by fault attacks

Securing white-box ECDSA is still an open problem
Thank you for your attention!

Any questions?