ECDSA White-Box Implementations

Attacks and Designs from WhibOx 2021 Contest

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COSIC

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IBM

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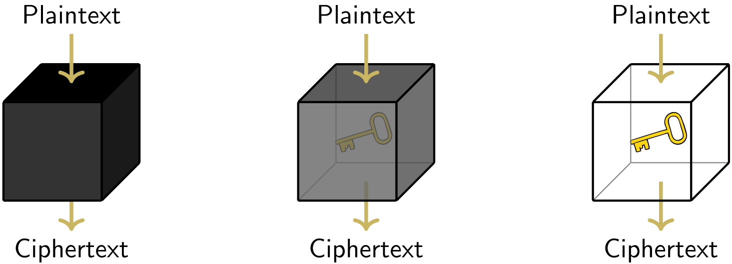
Black-Box, Grey-Box, White-Box



Cryptanalysis	Side-channels/Faults	Read/modify binary

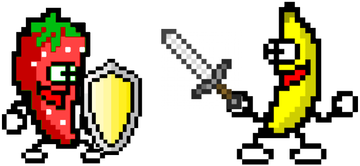
CHES 2021 Challenge - the WhibOx Contest

Designers

- Post C codes computing ECDSA
- Challenges gain strawberries (depending on performances and time until break)

Attackers

- Try to extract the secret key
- Receive bananas (number of strawberries of the challenge)



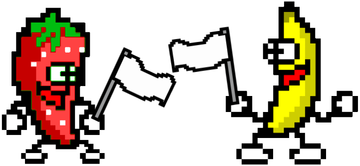
Our Contributions

zerokey

- Posted the 2 winning challenges
- Described the implementations

TheRealIdefix

- Broke the most challenges
- Described attacks, showing which ones succeeded for each candidate



ECDSA

- Let G be a point of order n on an elliptic curve E
- Let d be a 256-bit key
- Let m be a message and $e = H(m)$ its hash value

Algorithm 1: ECDSA signature

1 $k \xleftarrow{\$} [\![1, n-1]\!]$
2 $R \leftarrow kG$
3 $r \leftarrow x_R \bmod n$
4 $s \leftarrow k^{-1}(e + rd) \bmod n$
5 **if** $r == 0$ *or* $s == 0$ **then**
6   Go to step 1
7 **end**
8 Return (r,s)

ECDSA Sensitive Values

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```
1 k ← f(e)            WB model ⇒ No reliable source of randomness!
2 R ← kG
3 r ← x_R mod n
4 s ← k^{-1}(e + rd) mod n
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```
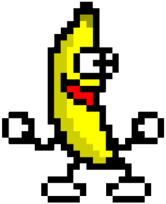

Section 1

Breaking the Candidates



Hooking Shared Libraries

Idea

Find some secret values that could be manipulated in the clear

- Use of the GMP library suggested by the contest rules
- Hook the calls to GMP functions (LD_PRELOAD)
- Update a log of the given parameters
- Search for d, k or related values in the log

Biased Nonce

First possibility

Find collisions: signatures generated with the same nonce

- Find (r_1, s_1) and (r_2, s_2) such that $r_1 = r_2$ (so $k_1 = k_2$)
- Solve the following system in k_1, d:

$$\begin{aligned} s_1 &= k_1^{-1}(e_1 + r_1 d) \\ s_2 &= k_1^{-1}(e_2 + r_1 d) \end{aligned}$$

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Second possibility

Exploit biases in the nonce generation

- Use lattice-based attacks
- Allows to recover d from a few bits of k for several signatures.

Grey-box Attacks in the White-Box Model

- Side-channel attacks
  - ≻ Difficult to apply (huge size of the traces)
- Fault injections
  - ≻ Modify the binary, use debugging tools
  - ≻ Many fault attacks on deterministic ECDSA, for example:

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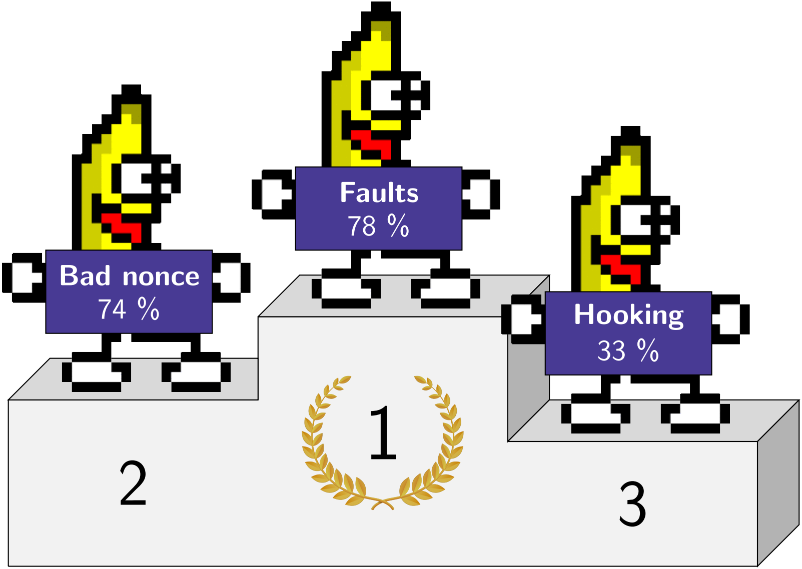
$$d = (s(r - r')(s - s')^{-1} - r)^{-1} e \bmod n$$

Percentage of Challenges Broken by Each Attack



Section 2

Design of the Winning Challenges

How to Win the Strawberries

- The implicit framework for white-box implementation
  - A novel encoding method

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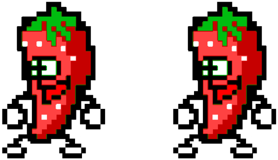
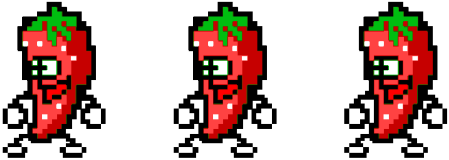
- Techniques from multivariate public key crypto
  - Additional countermeasures



How to Win the Strawberries

- The implicit framework for white-box implementation
  - A novel encoding method
- Techniques from multivariate public key crypto
  - Additional countermeasures
- C obfuscator
  - Use Tigress to obfuscate the source codes



Implicit Framework

Implicit Function and Implicit Evaluation

$$F(\boldsymbol{x}) = \boldsymbol{y} \iff T(\boldsymbol{x}, \boldsymbol{y}) = 0$$

Evaluate $F(\boldsymbol{a})$ by substituting $\boldsymbol{x} = \boldsymbol{a}$ and solving $T(\boldsymbol{a}, \boldsymbol{y}) = 0$

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Quasilinear Implicit Function (QIF)

$$\forall \boldsymbol{x},\ \text{function } \boldsymbol{y} \mapsto T(\boldsymbol{x}, \boldsymbol{y}) \text{ is affine}$$

This enables fast solving of $\boldsymbol{y}$

Implicit Implementation

$$F = F^{(t)} \circ F^{(t-1)} \circ \cdots \circ F^{(1)}$$

- Encoded implementation

$$\overline{F} = \overline{F^{(t)}} \circ \cdots \circ \overline{F^{(1)}} = (B^{(t)} \circ F^{(t)} \circ A^{(t)}) \circ \cdots \circ (B^{(1)} \circ F^{(1)} \circ A^{(1)})$$

- T is a QIF of $F^{(i)} \implies T' = M \circ T \circ (A, B^{-1})$ is a QIF of $F^{(i)'} = B \circ F^{(i)} \circ A$, for any linear permutation M

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Implicit implementation = Encoded implementation + QIF

White-box implementation of ECDSA

Algorithm 2: White-box ECDSA for winning challenges

$e \leftarrow e \bmod p$
$(v_1, v_2, v_3) \leftarrow \overline{E^{(1)}}(e)$ `// implicit evaluation of` kG
for $\mathbf{o}$ *in* $\mathcal{L}$ **do**
  $(u_1, u_2, u_3) \leftarrow (v_1, v_2, v_3) + p \cdot \mathbf{o}$
  $(r, s) \leftarrow \overline{E^{(2)}}(u_1, u_2, u_3)$ `// implicit evaluation of` (r, s)
  if $\mathtt{VerifySignature}(r, s, e) = \text{valid}$ **then**
    return (r, s)
  end
end

- A round-based implementation of scalar multiplication
- Precompute $\mathcal{L}$ to deal with overflows when converting $\bmod p$ to $\bmod n$

Additional Countermeasures

Tricks from the multivariate public-key crypto

Obfuscation by multiplying random polynomials

- Implicit evaluation can be preserved w.h.p.

$$R(\boldsymbol{a})T(\boldsymbol{a},\boldsymbol{y}) = 0 \implies T(\boldsymbol{a},\boldsymbol{y}) = 0$$

- Set an initial value to prevent failures

Masking of the nonce

Avoid bias of the most significant part of the nonce k

keen_ptolemy (Challenge 227)

Implicit implementation + Multiplying random polynomials

	$\overline{T^{(1)}}$	$\{\overline{T^{(2)}}, \ldots, \overline{T^{(t-1)}}\}$	$\overline{T^{(t)}}$	$\overline{T^{(t+1)}}$
input variables	2+4	5+4	5+3	3+2
number of components	4	4	3	2
degree	3	3	4	2
number of coefficients	27×4	130×4	255×3	18×2

- Compiled binary: 4.42 MB
- Average running time: 0.04s
- Average RAM consumed: 6.14 MB

- Code obfuscation did not impact the running time but increased the binary size by 8% and the average RAM by 3%

clever_kare (Challenge 226)

Implicit implementation + Multiplying random polynomials + Masking

	$\overline{T^{(1)}}$	$\{\overline{T^{(2)}}, \ldots, \overline{T^{(t-1)}}\}$	$\overline{T^{(t)}}$	$\overline{T^{(t+1)}}$
input variables	2+6	7+6	7+5	5+2
number of components	6	6	5	2
degree	3	3	4	5
number of coefficients	37×6	322×6	854×5	504×2

- Compiled binary: 15.44 MB
- Average running time: 0.15s
- Average RAM consumed: 17.27 MB
- Impact of code obfuscation $< 1\%$

Security Analysis

Automated attacks do not work

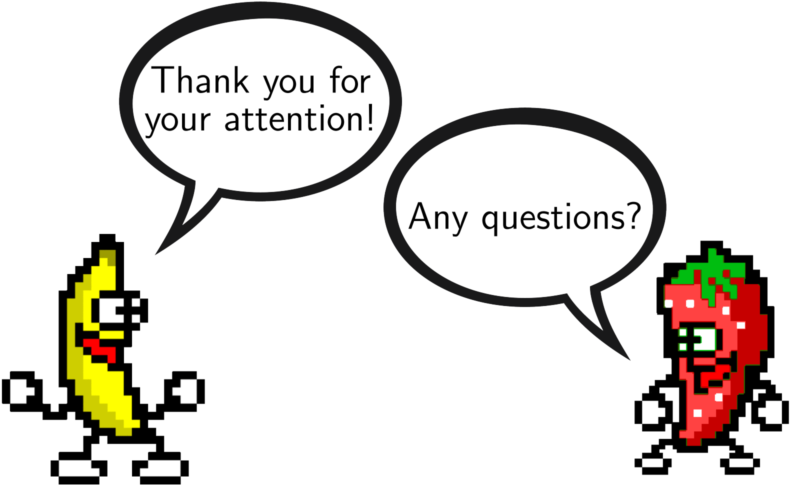
- ✗ Hooking shared libraries
- ✗ Biased nonces

✓ Fault attacks [BDG+22]

- remove only one line of code for each challenge
- induce uncontrolled fault in r
- defeat the verification steps

Summary

- We present automated attacks breaking a large number of challenges
- Fault attacks are the most efficient and effective
- We apply implicit implementation framework to ECDSA
- The best implementations were broken by fault attacks
- Securing white-box ECDSA is still an open problem


Thank you for your attention!
Any questions?