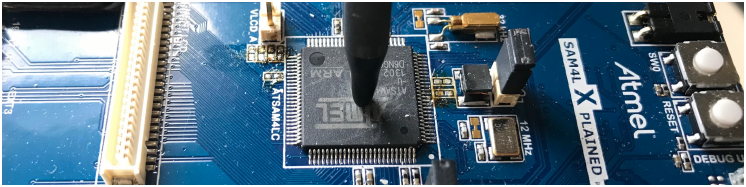


Bitslicing Arithmetic/Boolean Masking Conversions for Fun and Profit

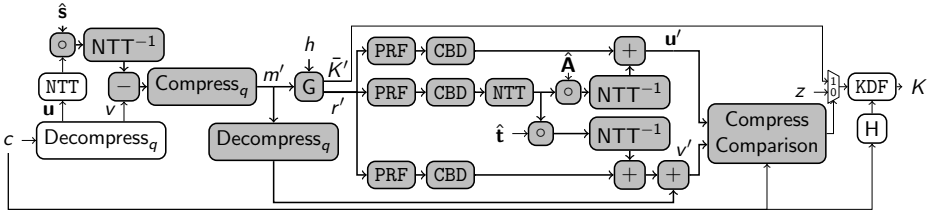
with Application to Lattice-Based KEMs

Olivier Bronchain Gaëtan Cassiers

CHES 2022, Leuven, Belgium

Contents

Masked Kyber computation (Inspired by Bos et al., TCHES 2021.)



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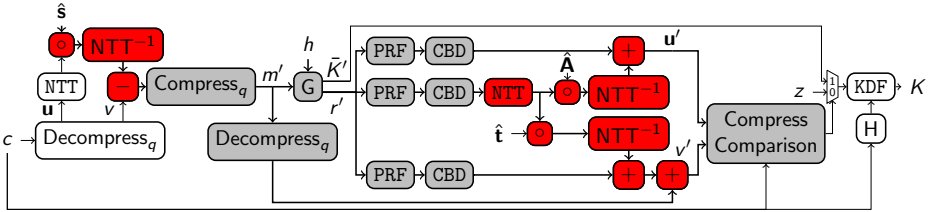

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- Arith. masking.
- Linear overheads.

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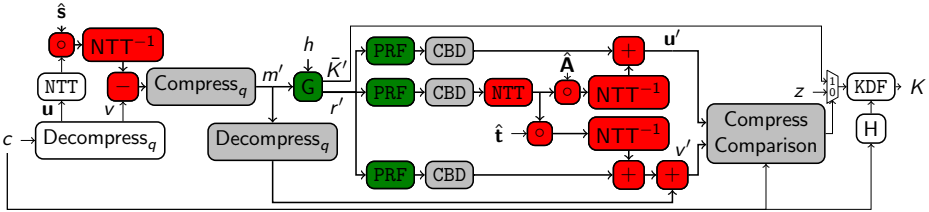

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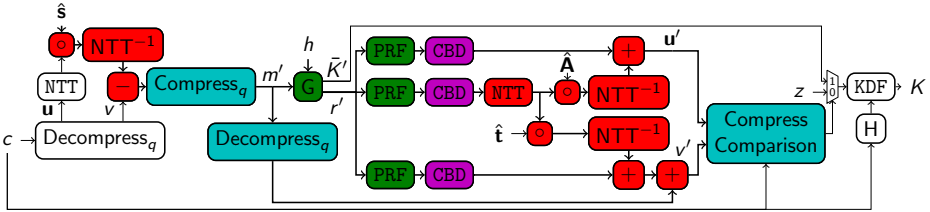

Poly. arithmetic (■):

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Poly. sampl. (■) & compress. (■):

- Boolean & arith. masking.
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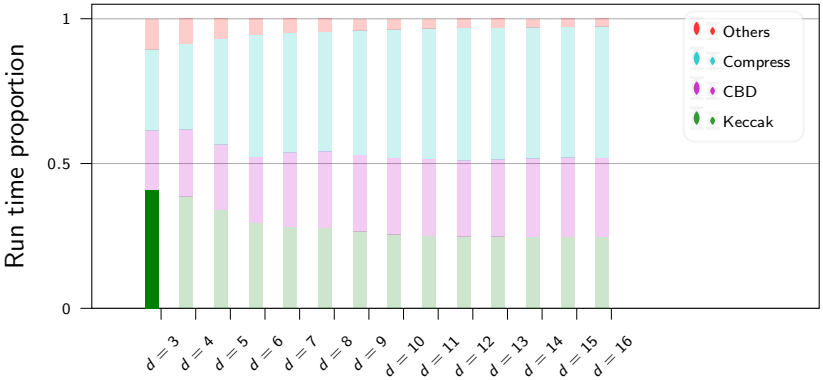
Masked Kyber performance



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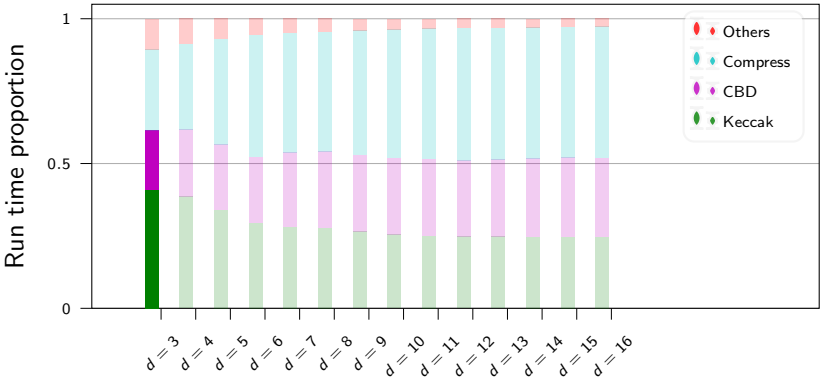

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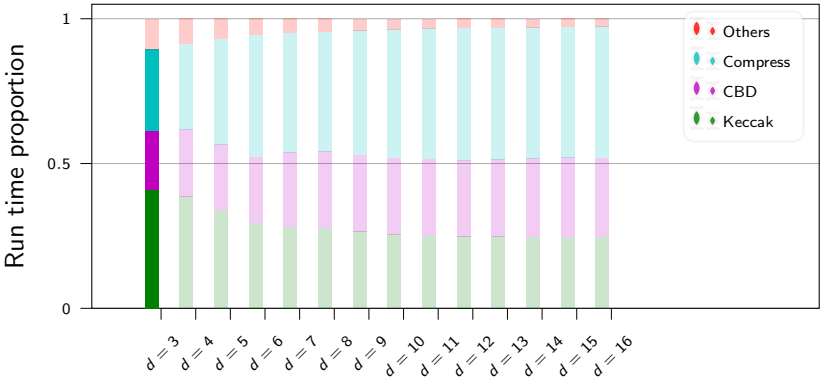

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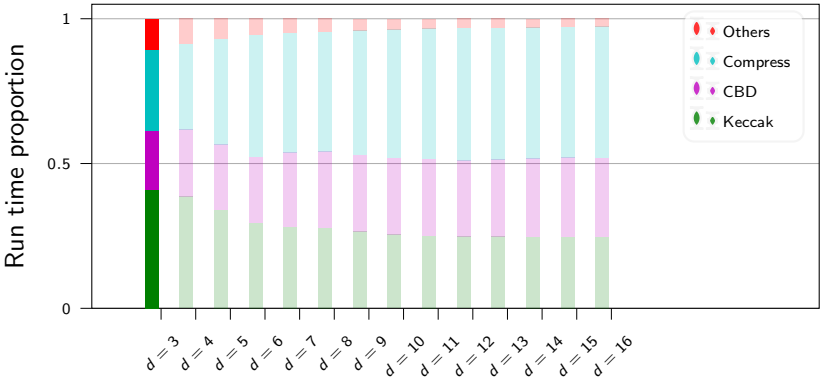

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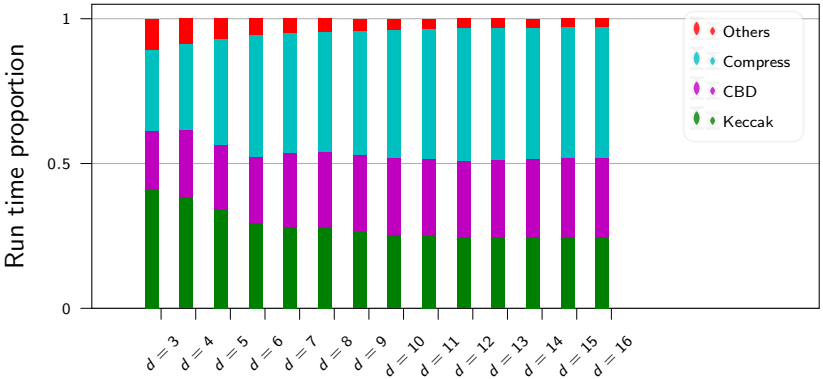

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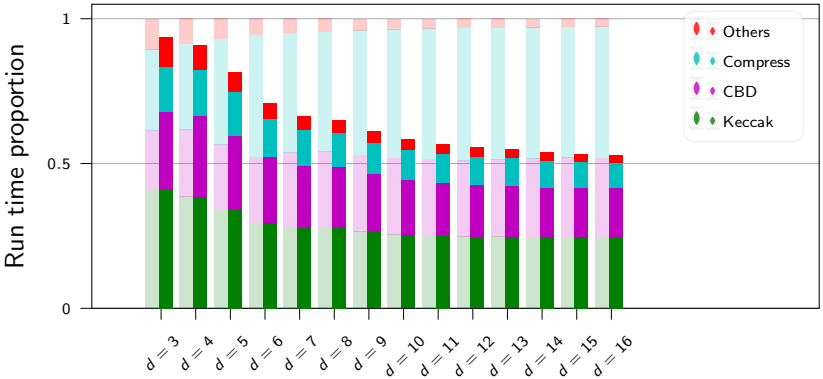
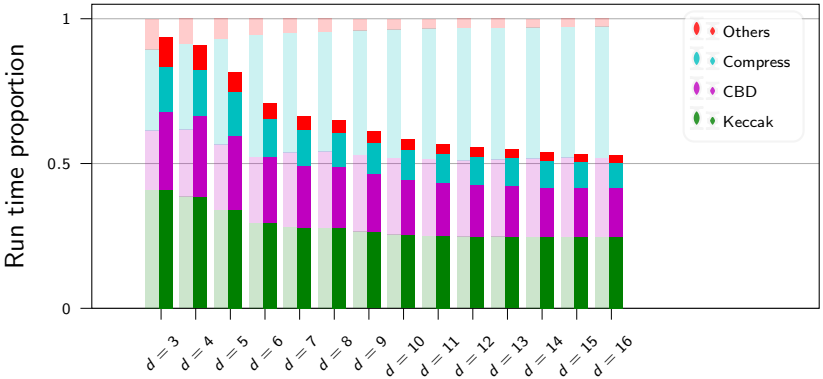


How have we improved the performances of masked Kyber ?

- Optimize CBD and Compressions.

Masked Kyber performance



How have we improved the performances of masked Kyber ?

- Optimize CBD and Compressions.
- Optimize masking conversion algorithms.

Contents

Canonical vs bitslice representations

State $x \in \mathbb{Z}_{3329}[X]/(X^{256}+1)$ represented with 256 13-bit integers.

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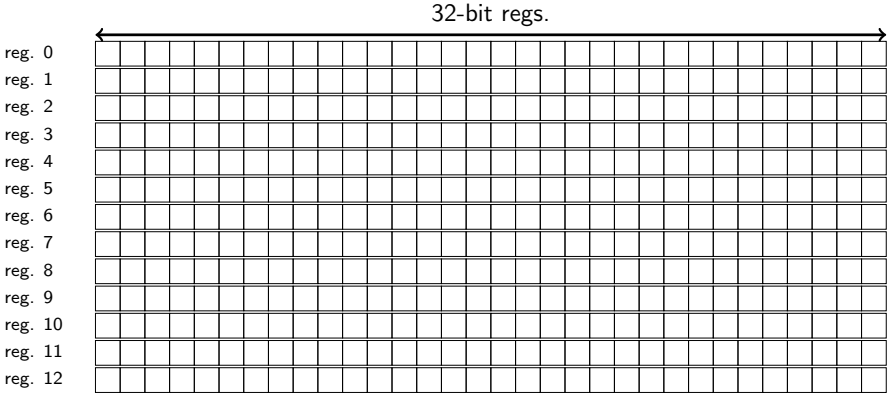

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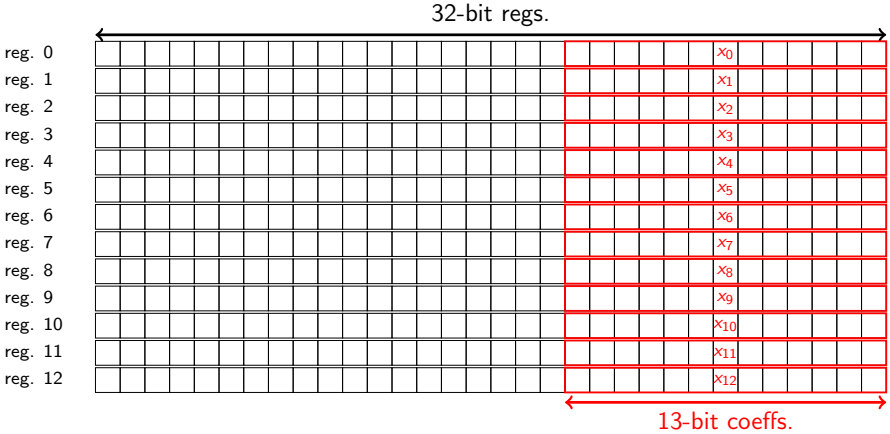

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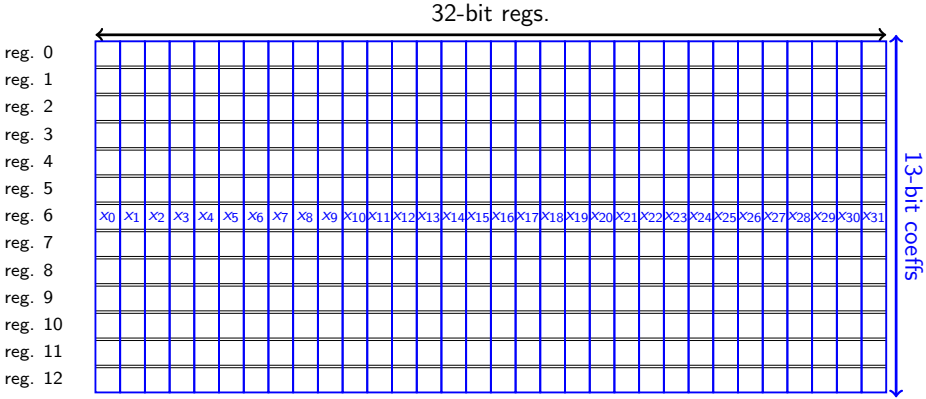
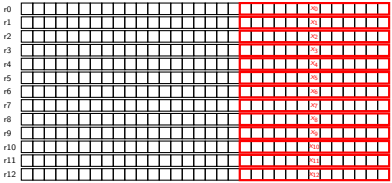


Canonical vs bitslice representations: performance

Canonical



+ Arithmetic operations ($+$, $\times$).
– Single-bit processing.
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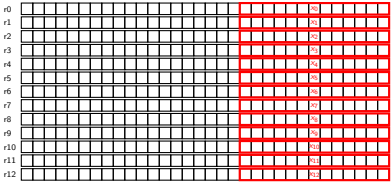

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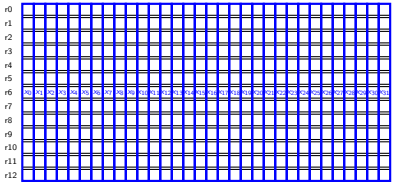

+ Bitwise operations: throughput.
+ Security: registers fully used.
− Representation change cost.

Canonical vs bitslice representations: performance

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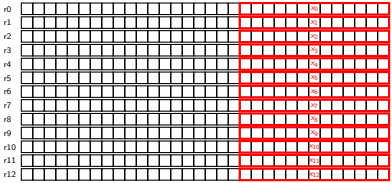
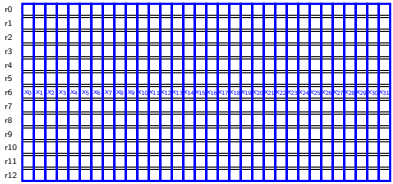


Bitslice



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Representation change: $\mathcal{O}(w \log w)$ instructions for $w \lfloor w / \log p \rfloor$ coefficients.

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$\texttt{SecAdd}^d_{k+1}$: arithmetic addition of Boolean shares

What is SoTA ?

- Masked Kogge-Stone adders.
- $\log k$ times $\texttt{SecAnd}^d_k$.

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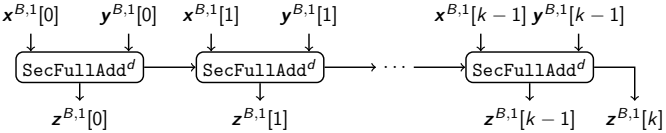
Taking advantage of bitslice:

- Masked ripple-carry adder:
  - Carry bit: $ab + ac + bc = (a + b)(a + c) + a$
  - Output bit: $a + b + c$
  - One single $\texttt{SecAnd}^d_1$ per $\texttt{SecFullAdd}^d$.
- k times $\texttt{SecAnd}^d_1$.



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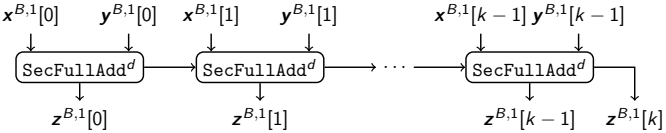

$\rightarrow$ Security: trivial composition with PINI.

$\rightarrow$ **Reduced randomness and operations.**

$\texttt{SecAddModp}_p^d$: addition with arbitrary modulus.

SoTA:

New:

$\texttt{SecAddModp}_p^d$: addition with arbitrary modulus.

SoTA:

1. Add x and y.

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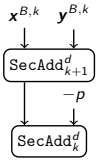

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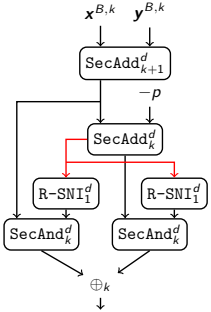
$\texttt{SecAddModp}_p^d$: addition with arbitrary modulus.

SoTA:

1. Add x and y.
2. Subtract p.
3. Select output with underflow bit.



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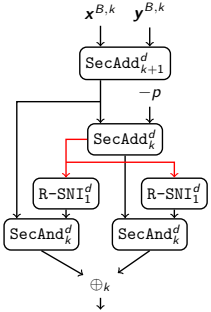

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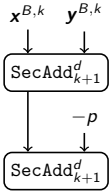

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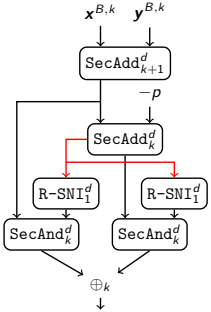

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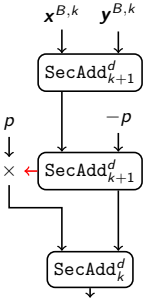

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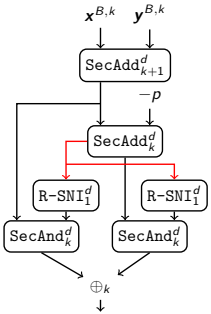

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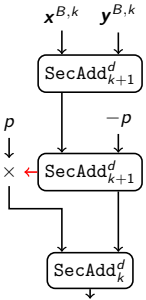
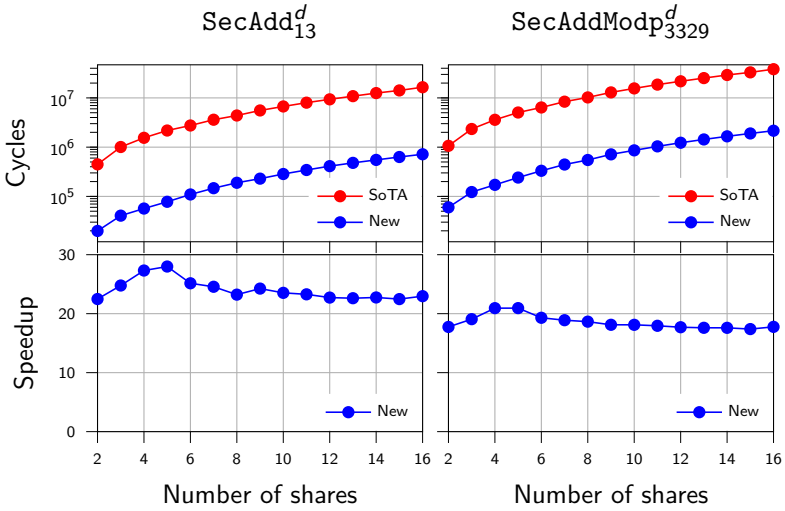


Taking advantage of bitslice:

- $\texttt{SecAdd}_k^d \approx \texttt{SecAnd}_k^d$.
- Replace 2 $\texttt{SecAnd}_k^d$ with 1 $\texttt{SecAdd}_k^d$.
- Secure composition: PINI.

SecAdd and SecAddModp performances



$\texttt{SecA2B}_k^d$: arithmetic to Boolean masking conversions

How to convert $\boldsymbol{c}^{A_{2^k}}$ to $\boldsymbol{c}^{B,k}$ with:

$$c = \sum_i \boldsymbol{c}_i^{A_{2^k}} \mod 2^k = \bigoplus_i \boldsymbol{c}_i^{B,k}$$

How to perform the conversion ?

- $(\boldsymbol{c}_i^{A_{2^k}}, 0)$ is a Boolean sharing of $\boldsymbol{c}_i^{A_{2^k}}$.
- Add all the $\boldsymbol{c}_i^{A_{2^k}}$'s with $\texttt{SecAdd}_k$'s.

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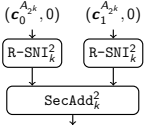

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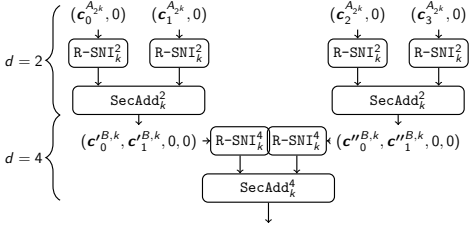

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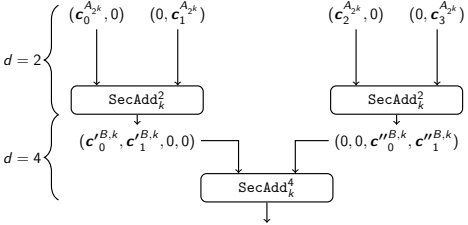
New PINI construction:



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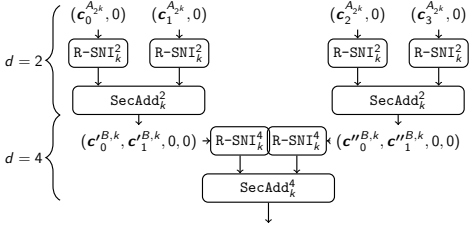
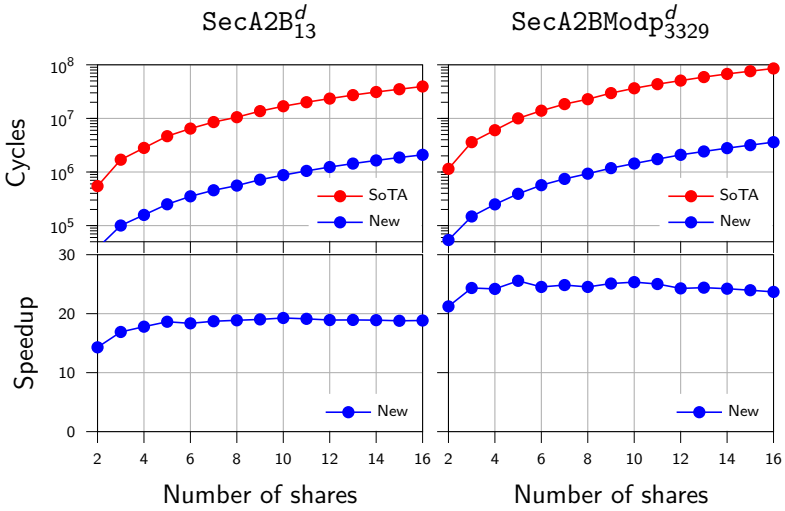


Differences:

- PINI removes refreshes.
- Input shares in different positions.
- Proofs based on *"Gadget Embeddings"*.

SecA2B and SecA2BModp performances



`SecB2A` and `SecB2AModp`: Boolean to arithmetic masking.

Convert a Boolean k'-bit sharing to arithmetic masking with 2^k modulus ?

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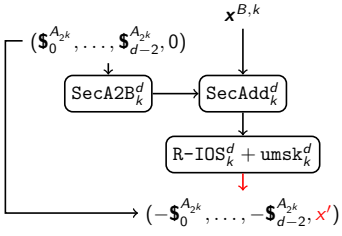
Generic $\texttt{SecB2A}_k^d$:

- Converts any x
- Leverage bitsliced gadgets.



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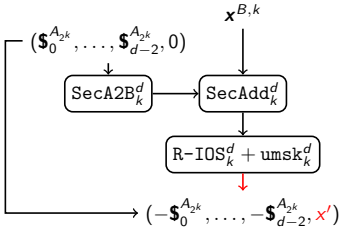
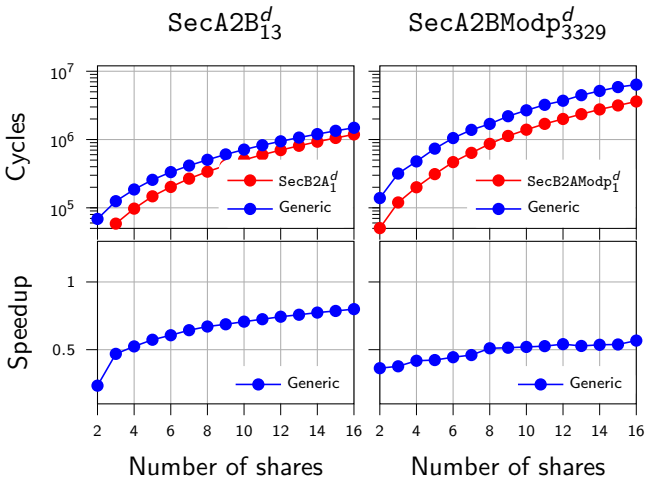


Based on $\texttt{SecB2A}_1^d$:

- Converts $x < 2^{k'}$
- How ?
  - Convert all k' bits with $\texttt{SecB2A}_1^d$.
  - Recombine with arithmetic operations.
- $\texttt{SecB2A}_1^d$ does not benefit from bitslice.

SecB2A and SecB2AModp performances



Observation:

- Better option depends on k'.

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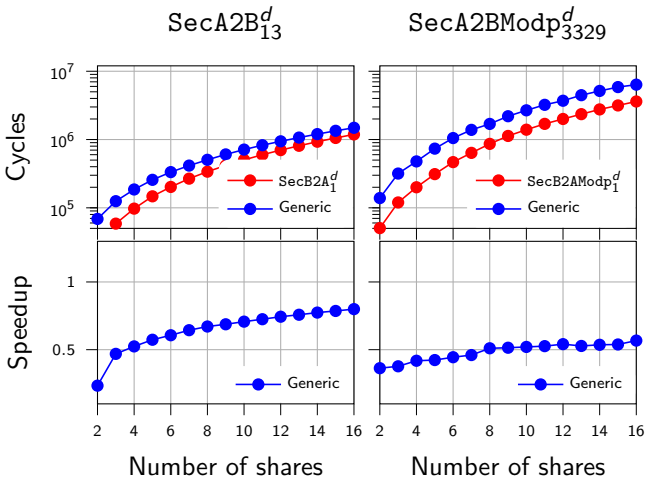

Observation:

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For Kyber:

- SecB2AModp used for binomial sampling.
- $k' = 3$ hence generic is better.

Binomial sampling

Compute $HW(a) - HW(b)$ *(a and b are* $k' = 2$*-bit values for Kyber768).*

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SOTA: *Chains of half-adders*

- Kyber: 8 AND gates per coefficient

New: *Chains of full-adders*

$$\begin{aligned} HW(a) - HW(b) &= HW(a) + (HW(\neg b) - k') \\ &= HW(a \| \neg b) - k' \end{aligned}$$

a_0

a_1

$\neg b_0$

$\neg b_1$

(subtract k' after `SecB2AModp`)

- Kyber: 3 AND gates per coefficient

Binomial sampling

Compute $HW(a) - HW(b)$ (a and b are $k' = 2$-bit values for Kyber768).

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New: *Chains of full-adders*

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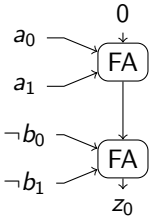

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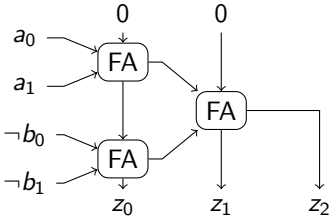
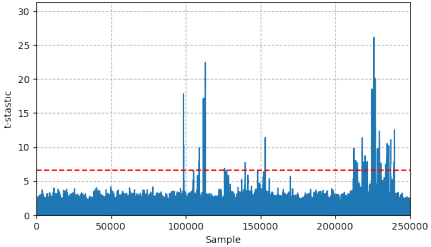


Contents

Implementation on Cortex-M4

(example with $d = 2$)



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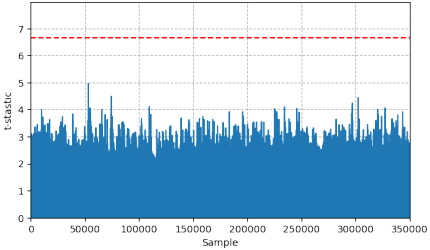

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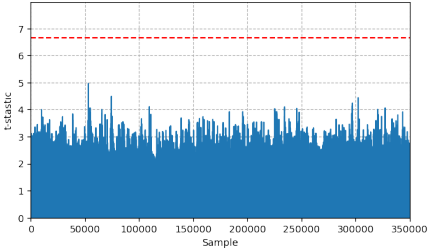

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see: https://github.com/uclcrypto/pqm4_masked

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