A New Formulation of the Linear Equivalence Problem and Shorter LESS Signatures

Asiacrypt 2023

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In This Talk

Roadmap

- Background
- Signatures from Code Equivalence
- A New Formulation
- Conclusions

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$[n, k]$ Linear Code over $\mathbb{F}_q$

A subspace of dimension k of $\mathbb{F}_q^n$. Value n is called length.

Hamming Metric

$wt(x) = |\{i : x_i \neq 0, 1 \leq i \leq n\}|$, $d(x, y) = wt(x - y)$.
Minimum distance (of $\mathfrak{C}$): $\min\{d(x, y) : x, y \in \mathfrak{C}\}$.

Generator Matrix

$G \in \mathbb{F}_q^{k \times n}$ defines the code as : $x{\in}\mathfrak{C} \iff x = uG$ for $u \in \mathbb{F}_q^k$.
Not unique: $SG, S \in \mathrm{GL}_k(q)$; Systematic form: $(I_k \mid M)$.

Parity-check Matrix

$H \in \mathbb{F}_q^{(n-k) \times n}$ defines the code as: $x{\in}\mathfrak{C} \iff Hx^T = 0$ (syndrome).
Not unique: $SH, S \in \mathrm{GL}_{n-k}(q)$; Systematic form: $(M^T \mid I_{n-k})$.

Information Set: set of columns carrying information symbols (G_J is invertible).

w-error correcting: $\exists$ algorithm that corrects up to w errors.

Decoding Problems

In general, it is hard to decode random codes.

General Decoding Problem (GDP)

Given: $G \in \mathbb{F}_q^{k \times n}$, $y \in \mathbb{F}_q^n$ and $w \in \mathbb{N}$.
Goal: find a word $e \in \mathbb{F}_q^n$ with $wt(e) \leq w$ such that $y - e = x \in \mathfrak{C}_G$.

Easy to see this is equivalent to the following.

Syndrome Decoding Problem (SDP)

Given: $H \in \mathbb{F}_q^{(n-k) \times n}$, $y \in \mathbb{F}_q^{(n-k)}$ and $w \in \mathbb{N}$.
Goal: find a word $e \in \mathbb{F}_q^n$ with $wt(e) \leq w$ such that $He^T = y$.

NP-Complete (Berlekamp, McEliece and Van Tilborg, 1978; Barg, 1994).

Unique solution when w is below a certain threshold (GV Bound).

Very well-studied, solid security understanding Information-Set Decoding (ISD) solvers.

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History suggest that we have to do things a little differently.

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Let $\mathcal{X}$ be a set and $(\mathcal{G}, \cdot)$ be a group. A group action is a mapping

$$\begin{array}{rccc} \star: & \mathcal{G} \times \mathcal{X} & \to & \mathcal{X} \\ & (g, x) & \mapsto & g \star x \end{array}$$

such that, for all $x \in \mathcal{X}$ and $g_1, g_2 \in \mathcal{G}$, $g_2 \star (g_1 \star x) = (g_2 \cdot g_1) \star x$.

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Most famous example: exactly DLP!

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Zero-Knowledge from Group Actions

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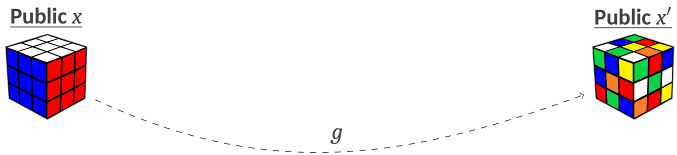

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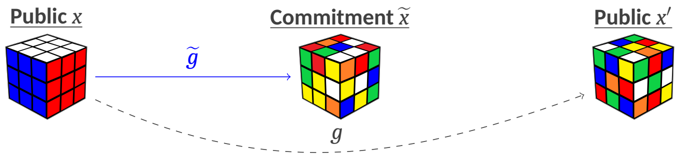

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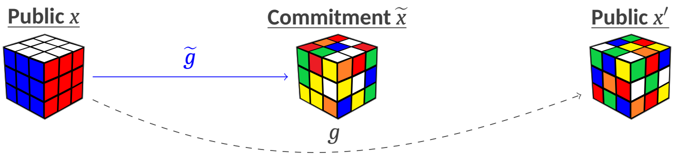

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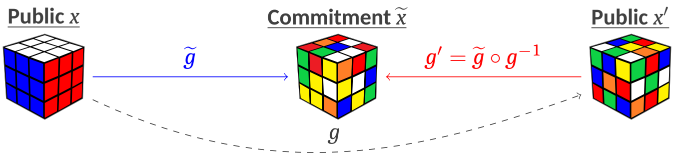

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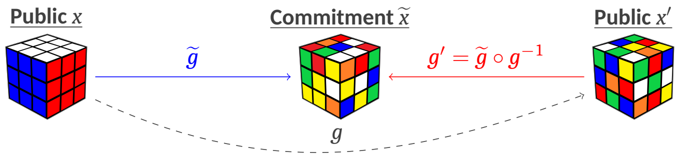

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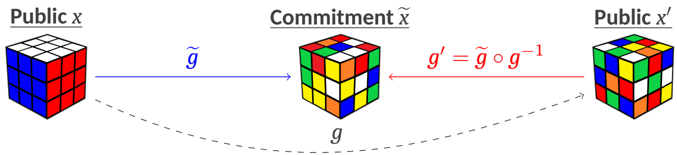

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What about group actions from coding theory?

Maps which preserve the distances (weights).

Isometries in the Hamming Metric

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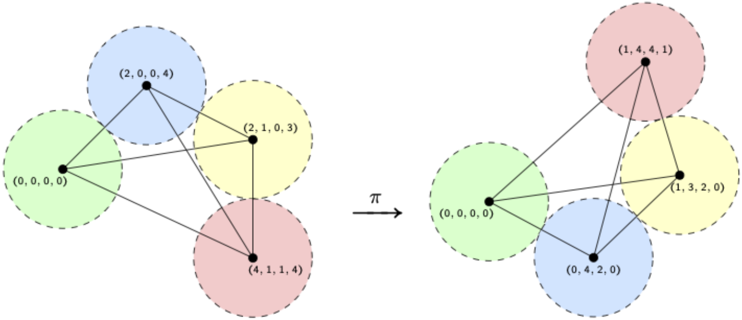

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$$\mathfrak{C} \overset{\mathsf{PE}}{\sim} \mathfrak{C}' \iff \exists (S, P) \in \mathsf{GL}_k(q) \times \mathsf{S}_n \ \text{ s.t. } \ G' = SGP,$$
$$\mathfrak{C} \overset{\mathsf{LE}}{\sim} \mathfrak{C}' \iff \exists (S, Q) \in \mathsf{GL}_k(q) \times \mathsf{M}_n(q) \ \text{ s.t. } \ G' = SGQ,$$

where P is a permutation matrix, and Q a monomial matrix.

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Linear Equivalence Problem (LEP)

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Note that the permutation case (PEP) is just a special case, and for practical applications, we are not interested in the semilinear version of the problem.

The LESS ZKID

A ZK protocol based exclusively on the hardness of the code equivalence problem.

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Prover		Verifier
Choose random monomial matrix $\tilde{Q} \in \mathrm{M}_n(q)$.		
Compute $\tilde{G} = SF(G\tilde{Q})$.		
Set $cmt = \mathbf{H}(\tilde{G})$	$\xrightarrow{cmt}$	
	$\xleftarrow{b}$	Select random $ch \in \{0, 1\}$.
If $ch = 0$ set $rsp = \tilde{Q}$	$\xrightarrow{rsp}$	Verify $\mathbf{H}(SF(G \cdot rsp)) = cmt$.
If $ch = 1$ set $rsp = Q^{-1}\tilde{Q}$		Verify $\mathbf{H}(SF(G' \cdot rsp)) = cmt$.

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Such modifications do not affect security, only requiring small tweaks in proofs or switching to equivalent security assumptions.

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Code parameters chosen using according to this, following conservative criterion. Namely, pick n, k, q so that, for any d and any w:

$$\sqrt{N_d(w)} \cdot C_{\mathsf{ISD}}^{(d)}(n, k, q, w) > 2^{\lambda}.$$

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Can we compress signatures?

Roadmap

- Background
- Signatures from Code Equivalence
- A New Formulation
- Conclusions

Parsing the Information

3 A New Formulation

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Such information is represented by three permutation matrices:

- $n \times n$ permutation matrix $P_{\mathrm{is}} \in \mathsf{S}_{n,k}$
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In particular, for any P:

$$P = P_{\text{is}} \cdot \begin{pmatrix} P_{\text{rows}}^{-1} & 0 \\ 0 & P_{\text{cols}} \end{pmatrix}$$

Splitting the Action

Let $J :=$ set of coordinates that are moved in first k positions; then

$$G \cdot P_{\text{is}} = \Big(\underbrace{G_J}_{k \text{ columns}} , \underbrace{G_{\{1,\cdots,n\} \setminus J}}_{n-k \text{ columns}} \Big).$$

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Then, for any $S \in \mathrm{GL}_k(q)$:

$$\begin{aligned} \mathsf{SF}(SGP) &= \mathsf{SF}\big((S \cdot G_J \cdot P_{\text{rows}}^{-1}, S \cdot G_{\{1,\cdots,n\}\setminus J} \cdot P_{\text{cols}})\big) \\ &= \big(I_k, (S \cdot G_J \cdot P_{\text{rows}}^{-1})^{-1} \cdot S \cdot G_{\{1,\cdots,n\}\setminus J} \cdot P_{\text{cols}}\big) \\ &= \big(I_k, P_{\text{rows}} \cdot G_J^{-1} \cdot G_{\{1,\cdots,n\}\setminus J} \cdot P_{\text{cols}}\big). \end{aligned}$$

Let $\tilde{G} = \mathsf{SF}(G \cdot \tilde{P})$ sent during commitment and $\tilde{P}$ decomposed as before; then

$$\begin{aligned}\tilde{G} &= \left(I_k\ ,\ \tilde{P}_{\text{rows}} \cdot G_J^{-1} \cdot G_{\{1,\cdots,n\}\setminus J} \cdot \tilde{P}_{\text{cols}}\right)\\ &= \left(I_k\ ,\ A\right).\end{aligned}$$

Splitting the Action (cont.)

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Thus, we obtain an invariant up to a column permutation.

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Information Set Linear Equivalence Problem (IS-LEP)

Given $\mathfrak{C}, \mathfrak{C}' \subseteq \mathbb{F}_q^n$, find monomials μ, ζ and an information set J' such that for every $c \in \widetilde{\mathfrak{C}} = \mu(\mathfrak{C})$ there exists $c' \in \mathfrak{C}'$ with $\widetilde{c}_{J'} = c'_{J'}$ and $\widetilde{c}_{\{1,\cdots,n\}\setminus J'} = \zeta\left(c'_{\{1,\cdots,n\}\setminus J'}\right)$. Equivalently, given generators $\widetilde{G}, G' \in \mathbb{F}_q^{k\times n}$, it must be that

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We prove that this is equivalent to LEP (reduction in both ways).

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Conclusions, Current and Future Work

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Future work includes more performance improvements (e.g. Gaussian elimination, pk size), implementation (e.g. AVX2, hardware) and other applications.

Thank you for listening!
Any questions?

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https://www.less-project.com

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