

Updatable Public Key Encryption

From Lattices

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Asiacrypt 2023 - 广州

Updatable Public Key Encryption

Plan

Definitions & Secure Messaging

Construction

A new assumption

Introduction

Public Key Encryption

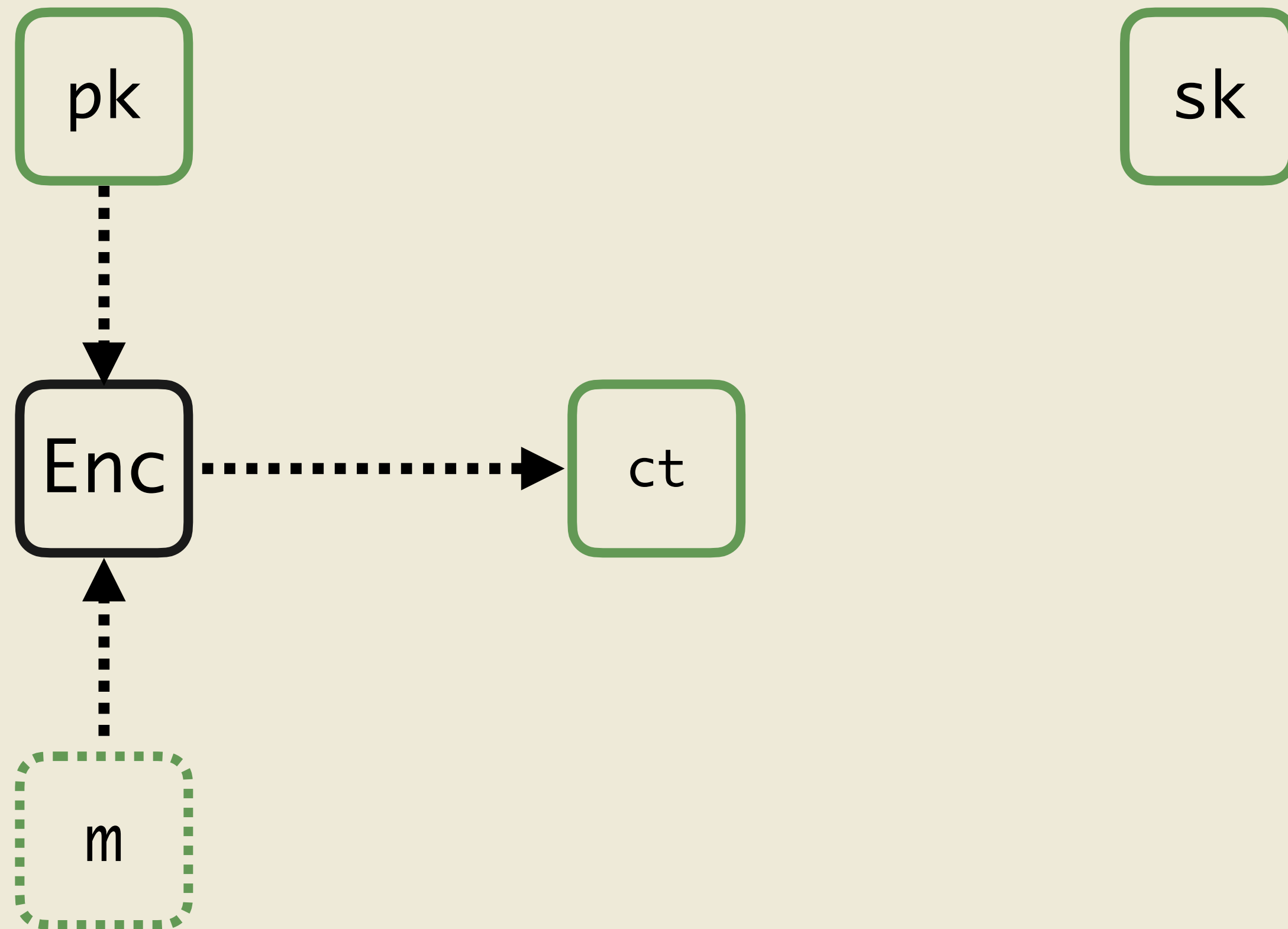
Definition

pk

sk

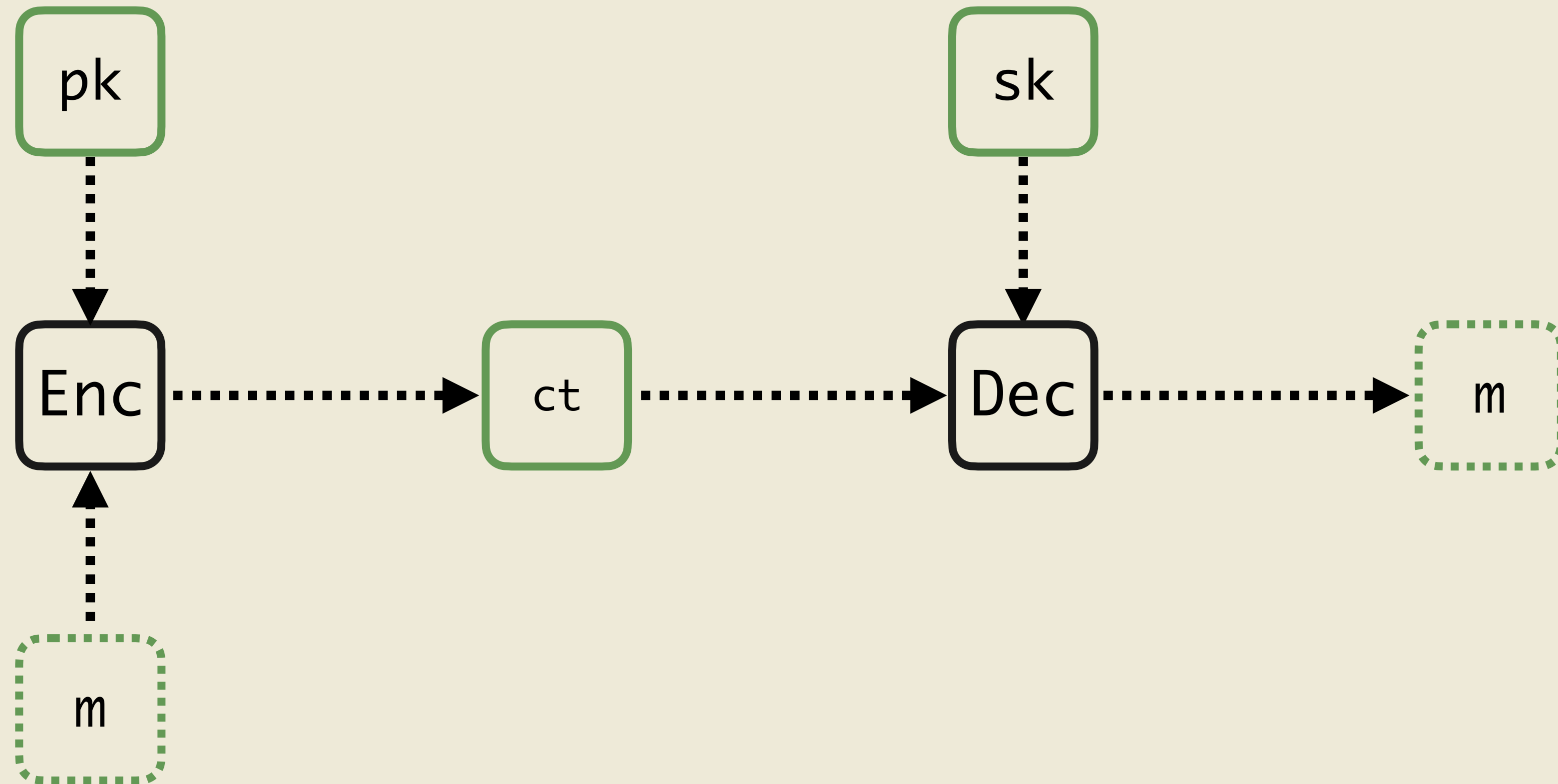
Public Key Encryption

Definition



Public Key Encryption

Definition



Secure Messaging

A quick introduction

 **Signal**

 **WhatsApp**

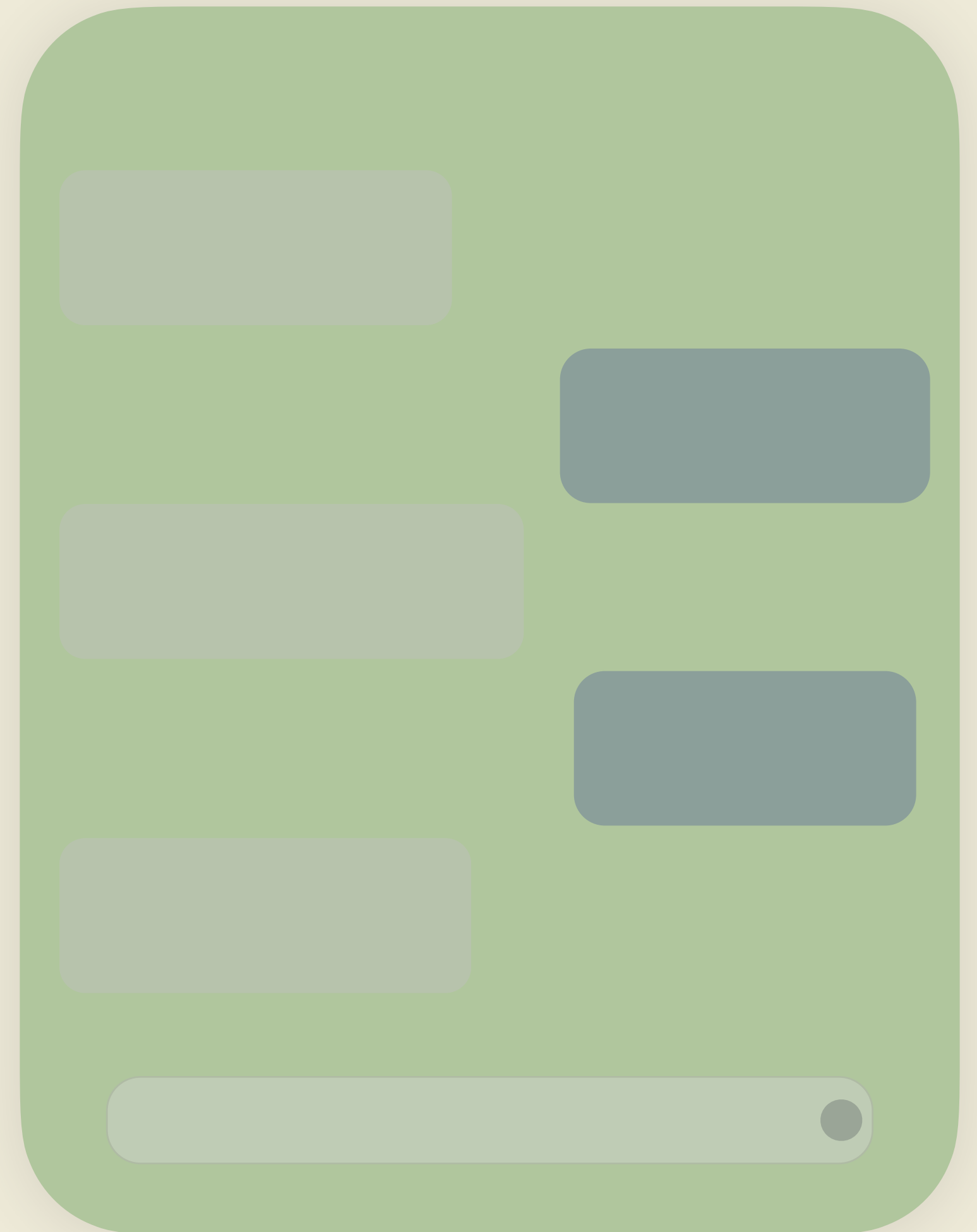
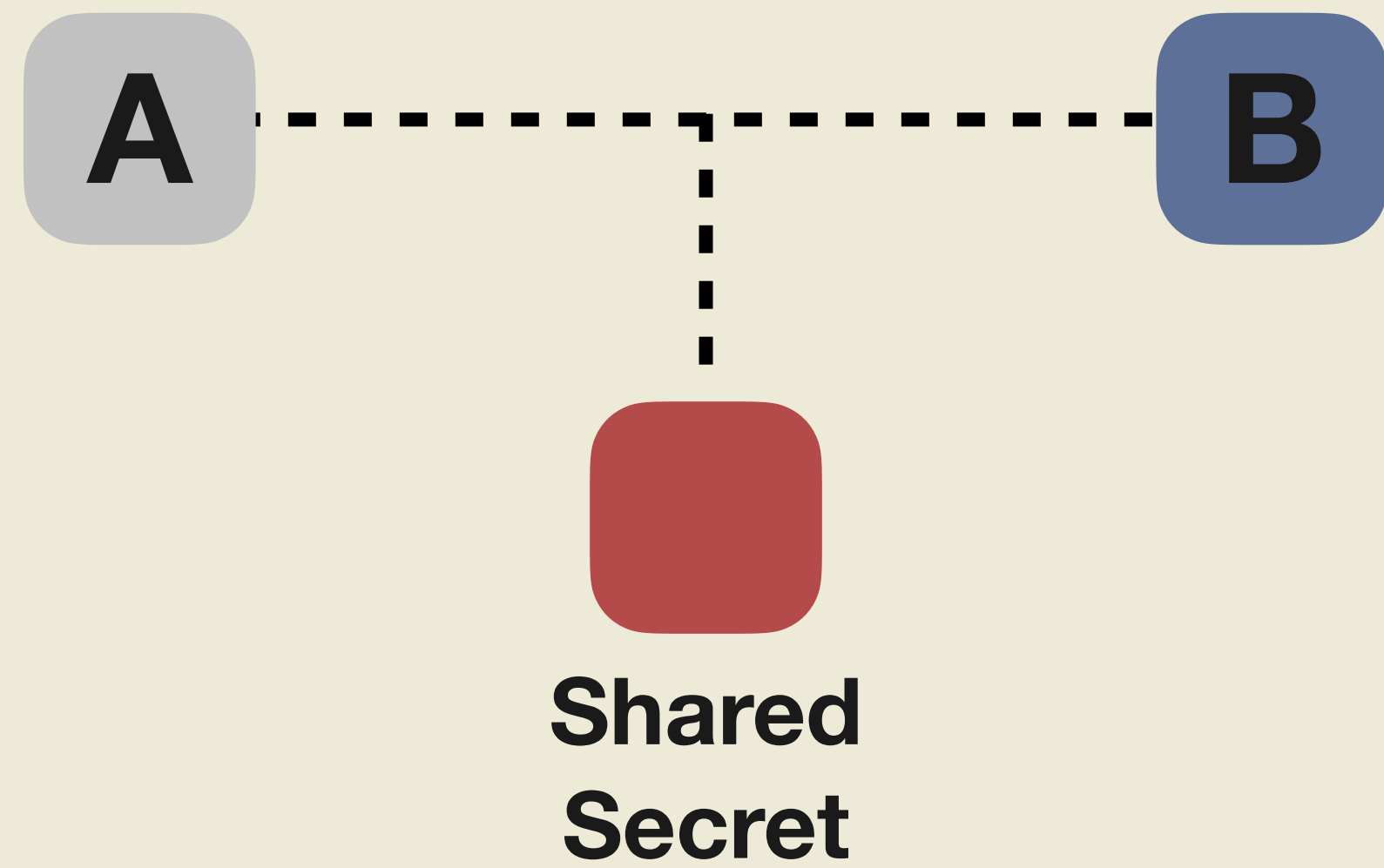
 **Et al.**



Secure Messaging

A quick introduction

Key Exchange Phase



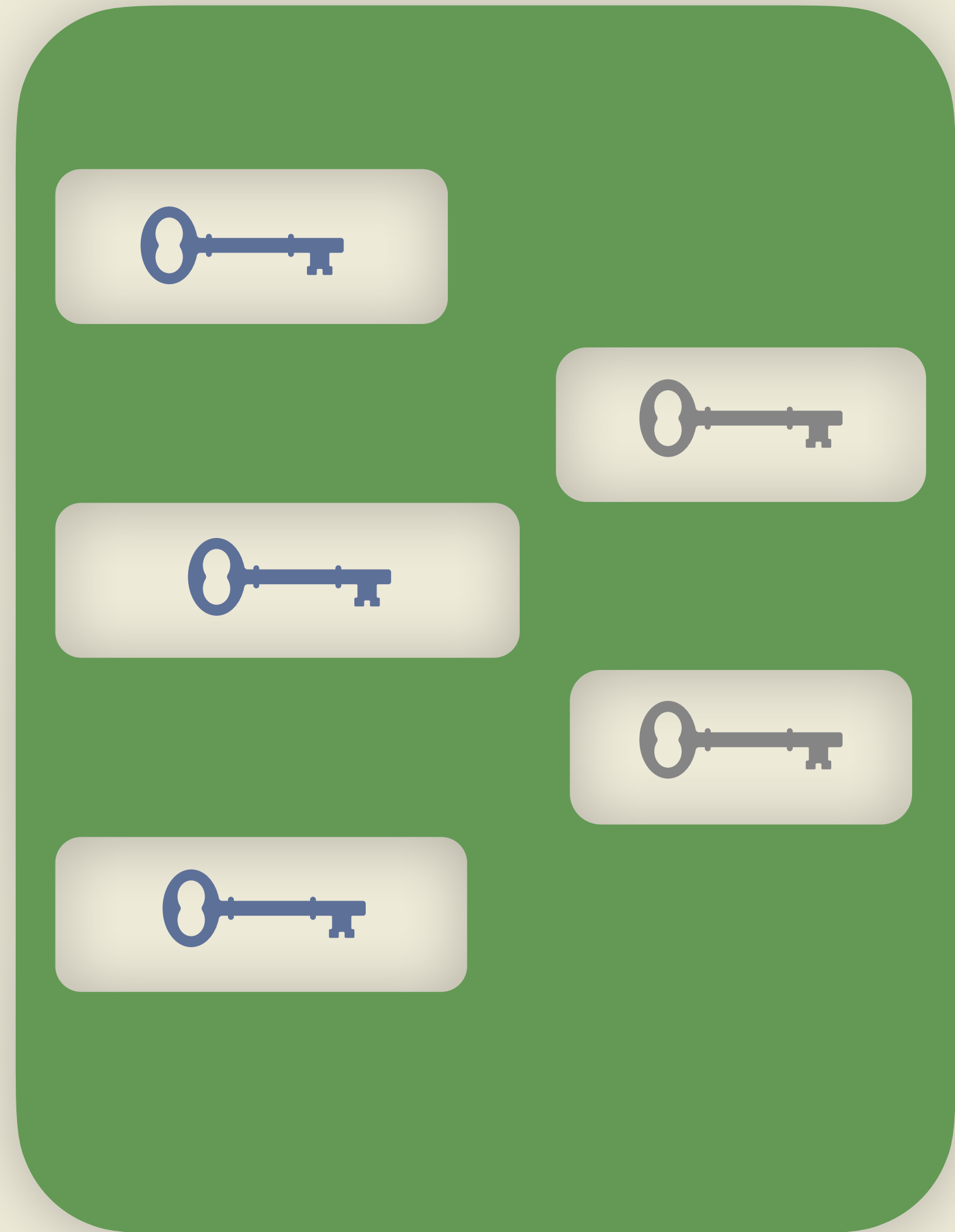
Secure Messaging

A quick introduction



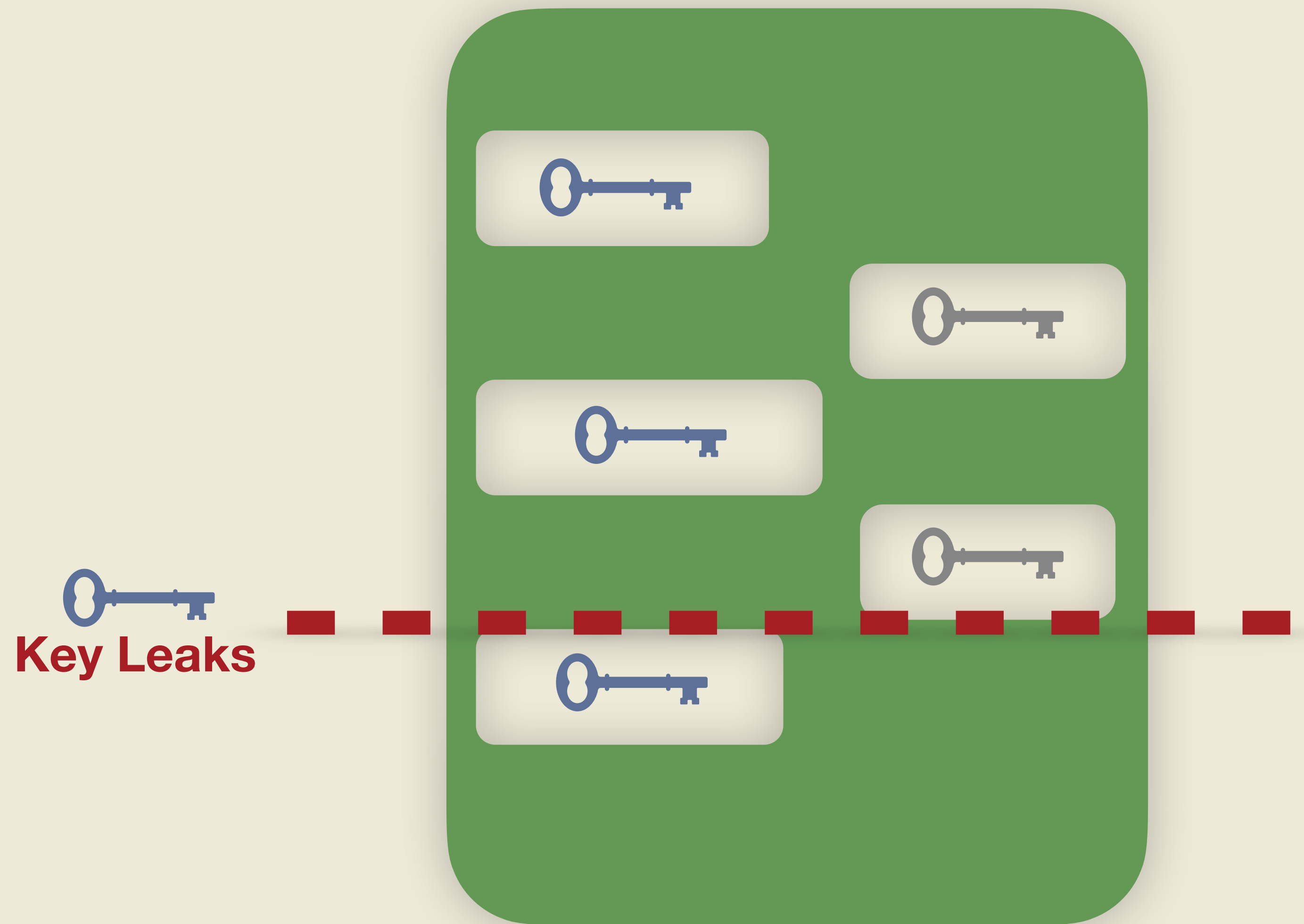
Secure Messaging

A quick introduction



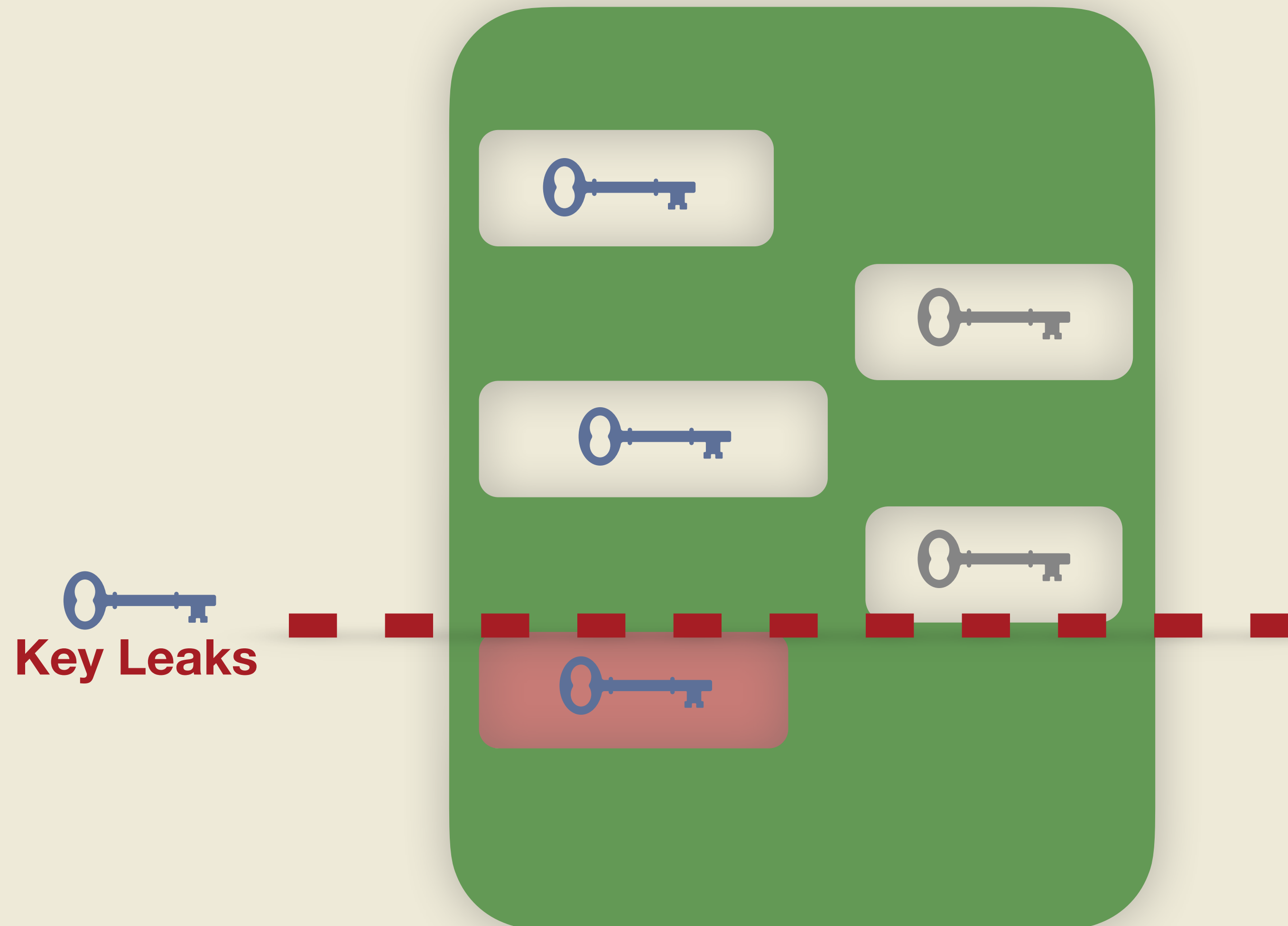
Secure Messaging

A quick introduction



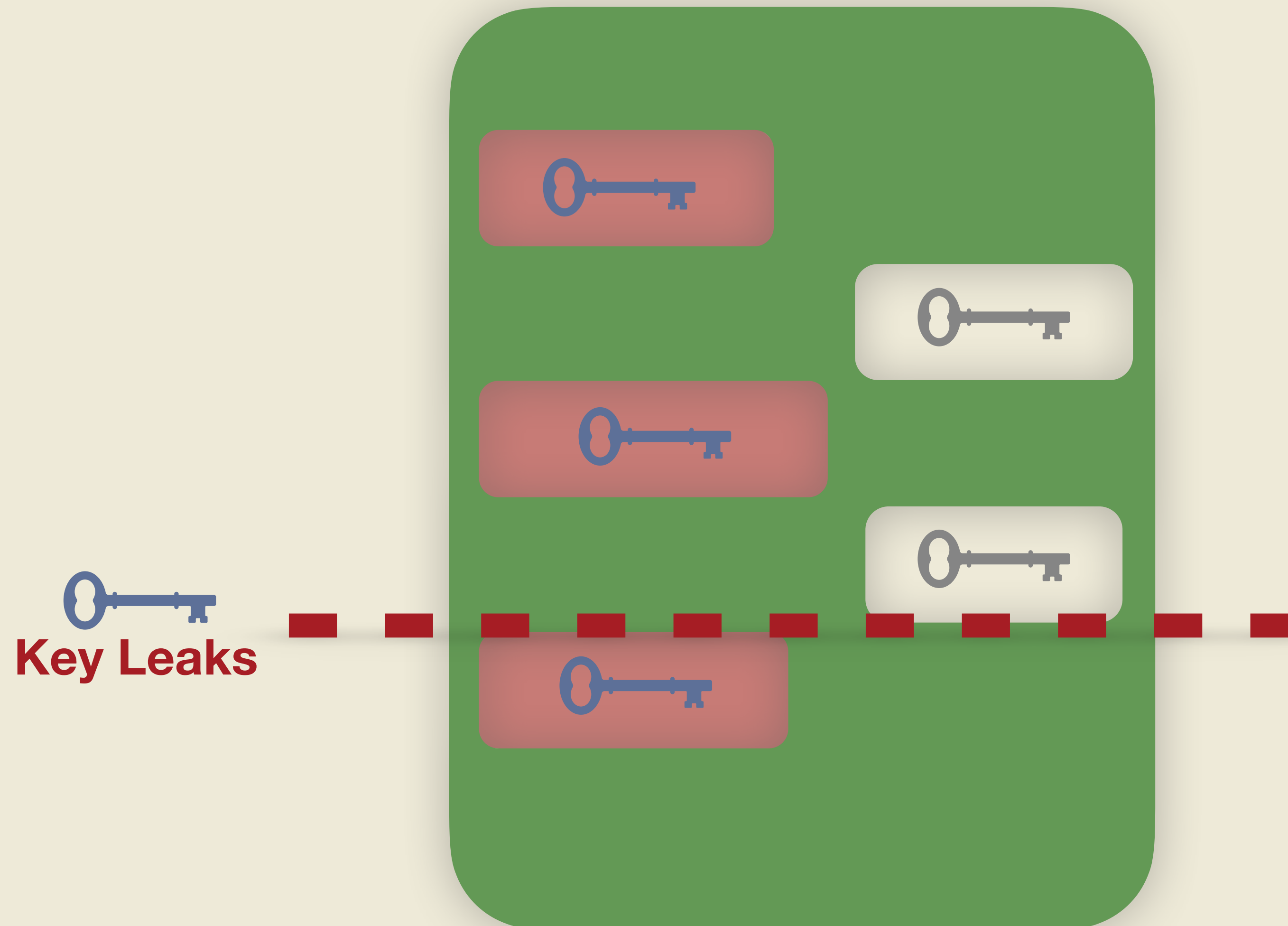
Secure Messaging

A quick introduction



Secure Messaging

A quick introduction



Secure Messaging

A quick introduction



Forward Secrecy

Secure Messaging

A quick introduction



Forward Secrecy

**Post-Compromise
Security**

Secure Messaging

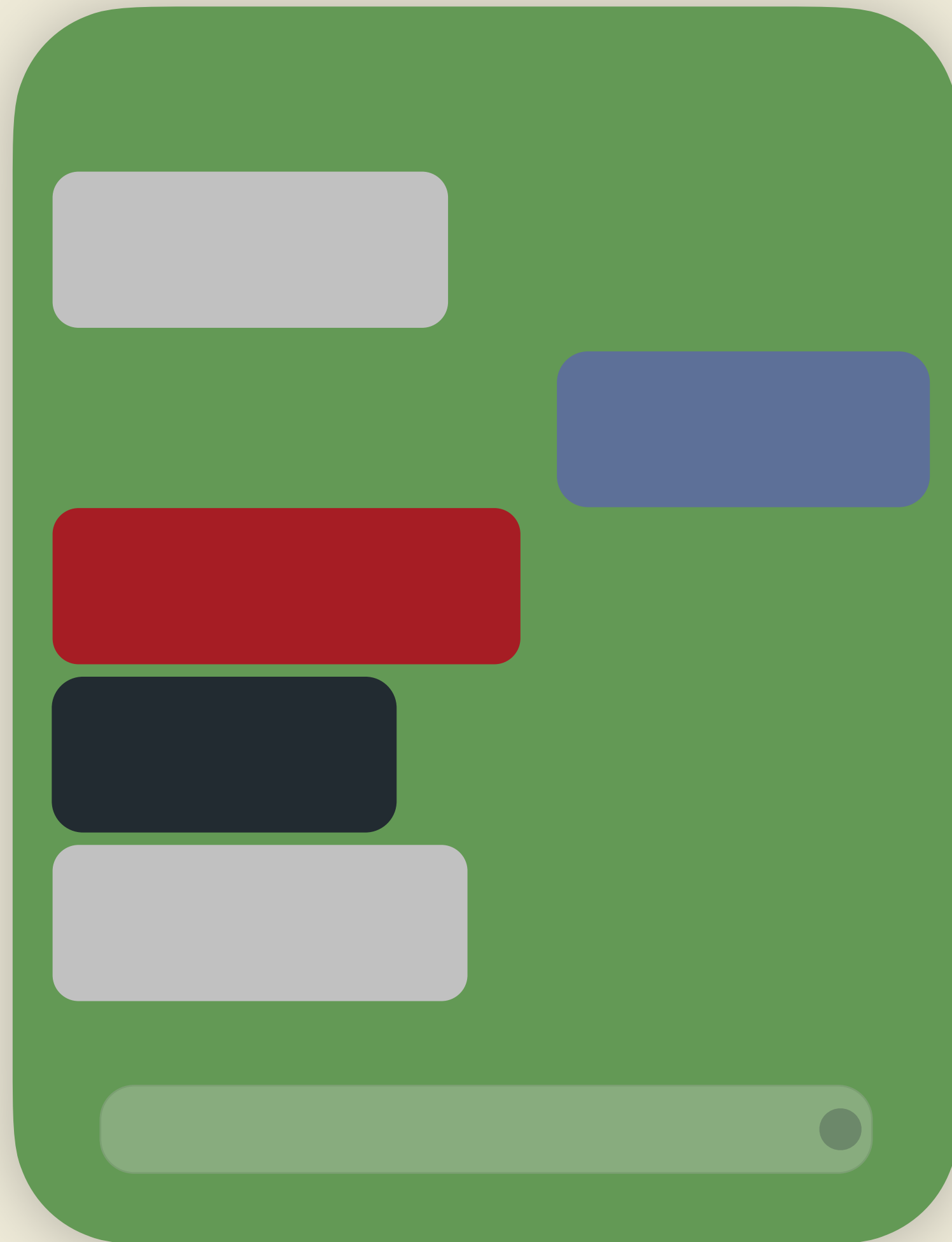
A quick introduction



Forward Secrecy

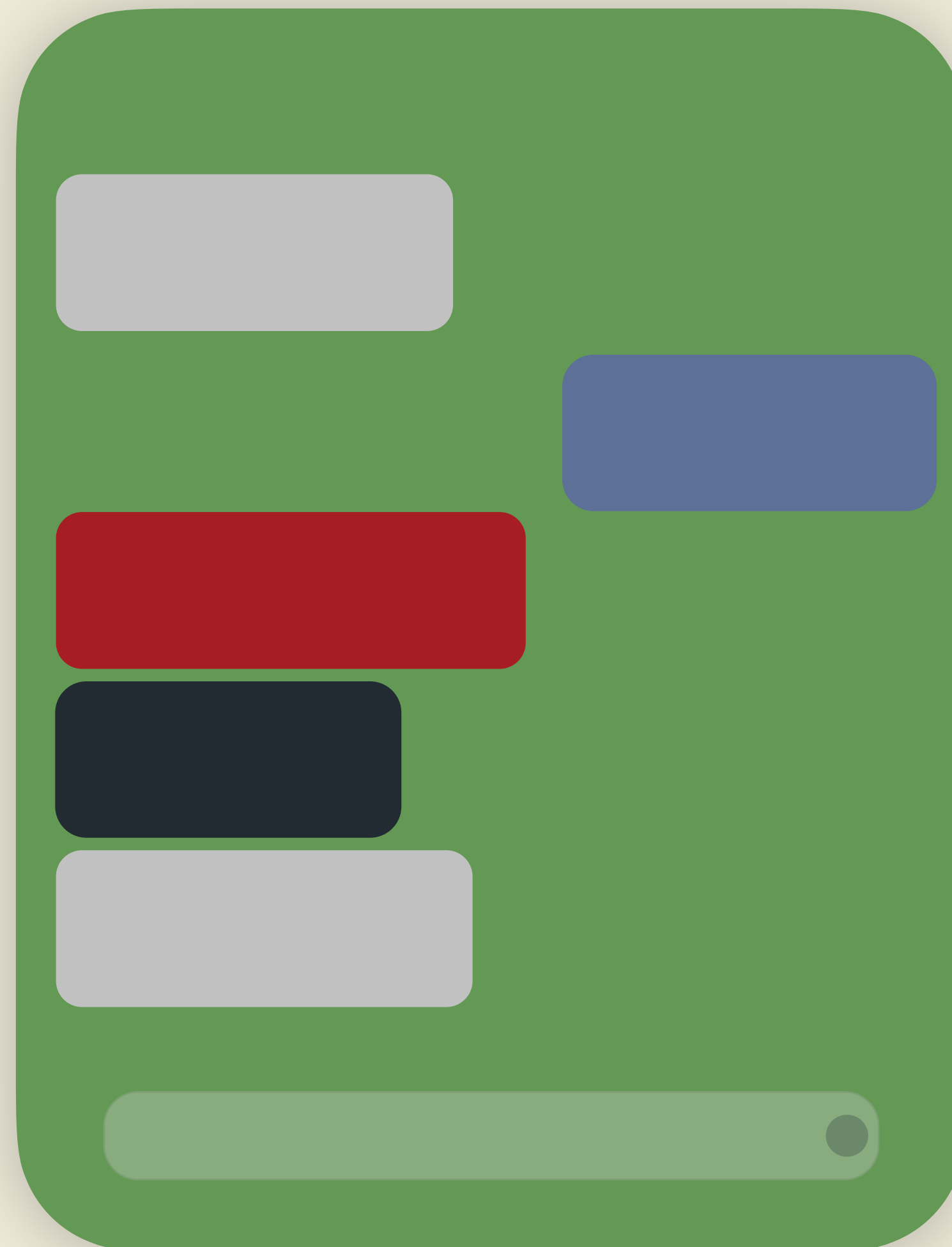
Group Messaging

Here comes trouble



Group Messaging

Here comes trouble

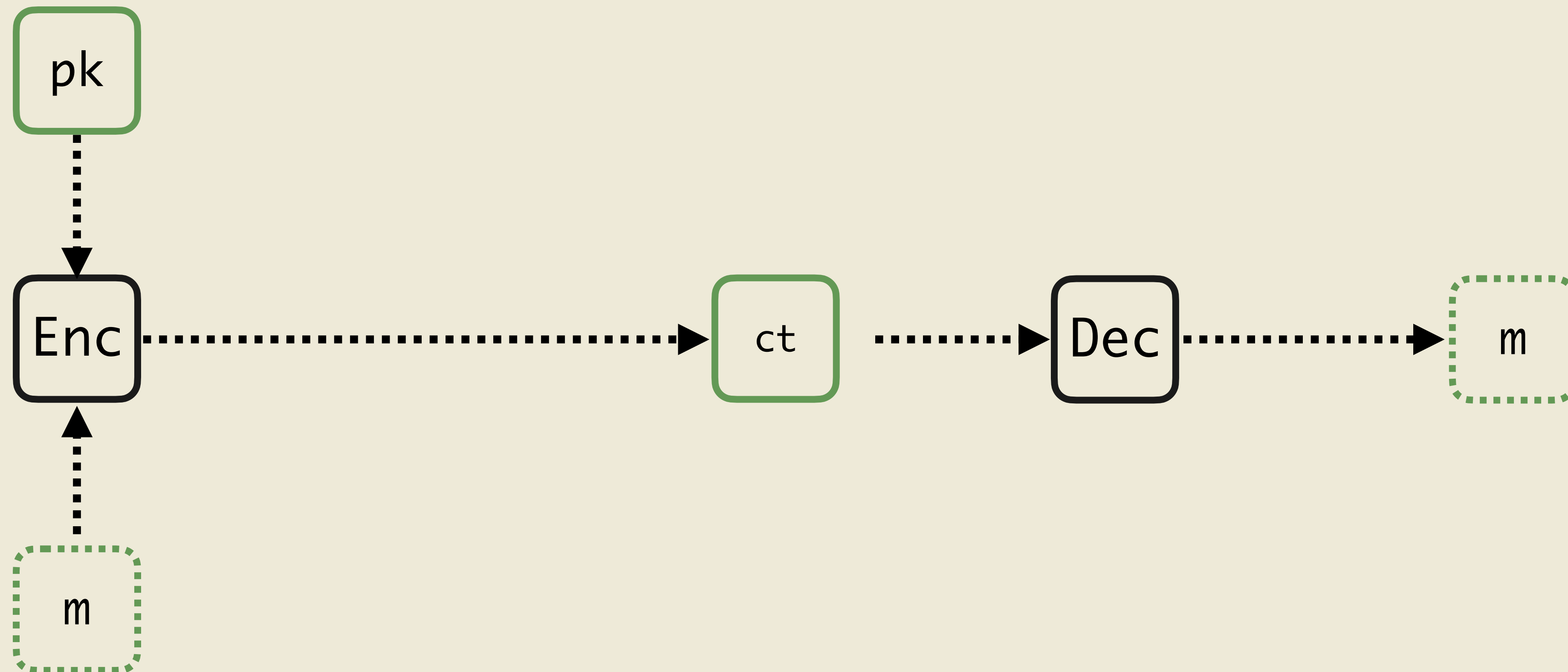


**Need to update
other user's
key to ensure
forward
secrecy**

Definition

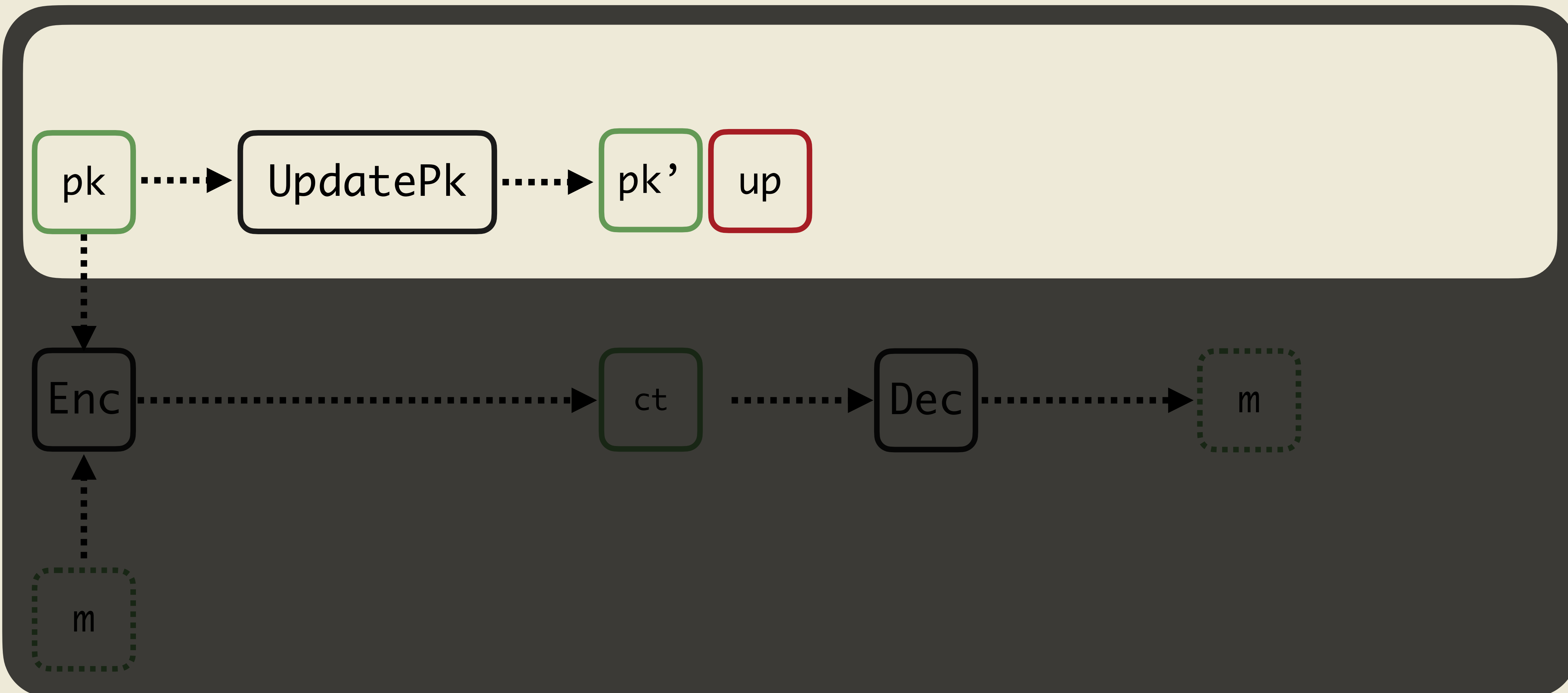
Updatable Public Key Encryption

Definition



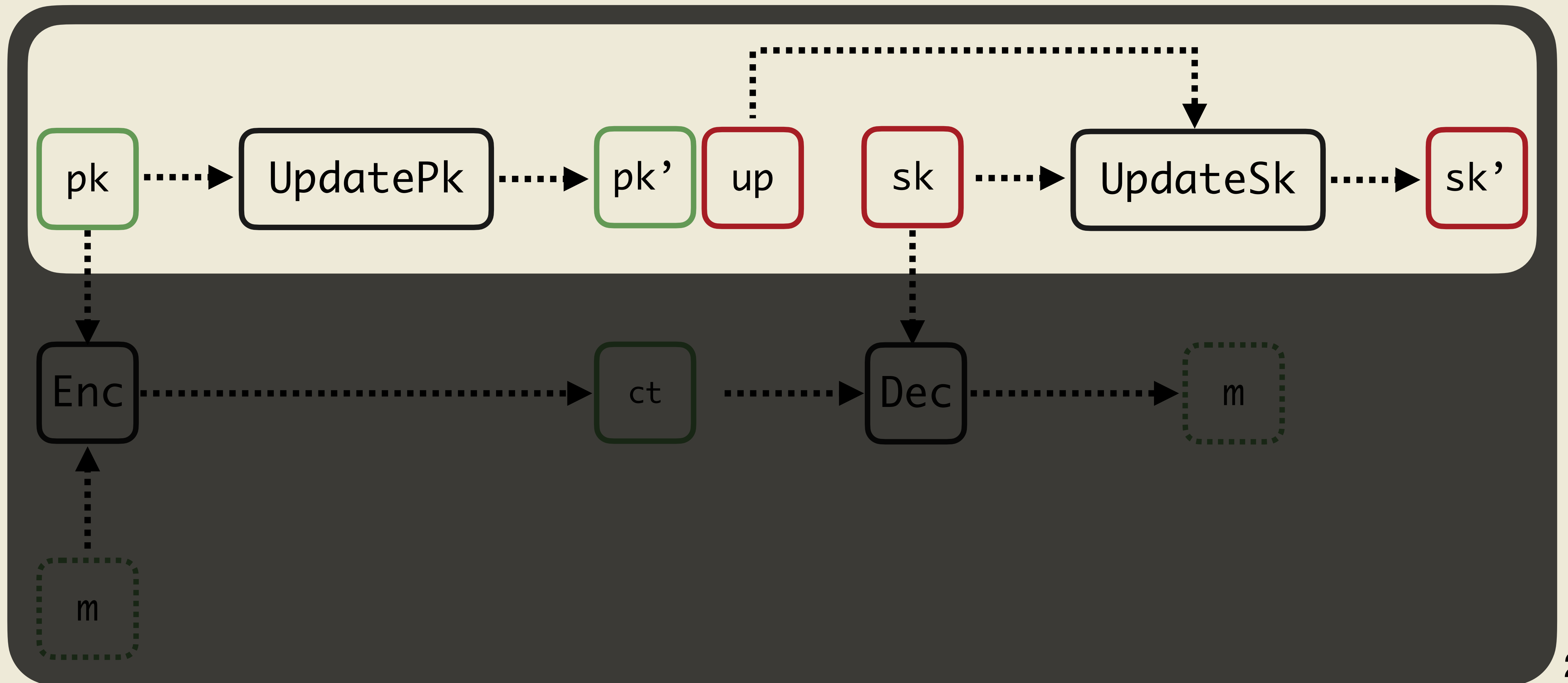
Updatable Public Key Encryption

Definition



Updatable Public Key Encryption

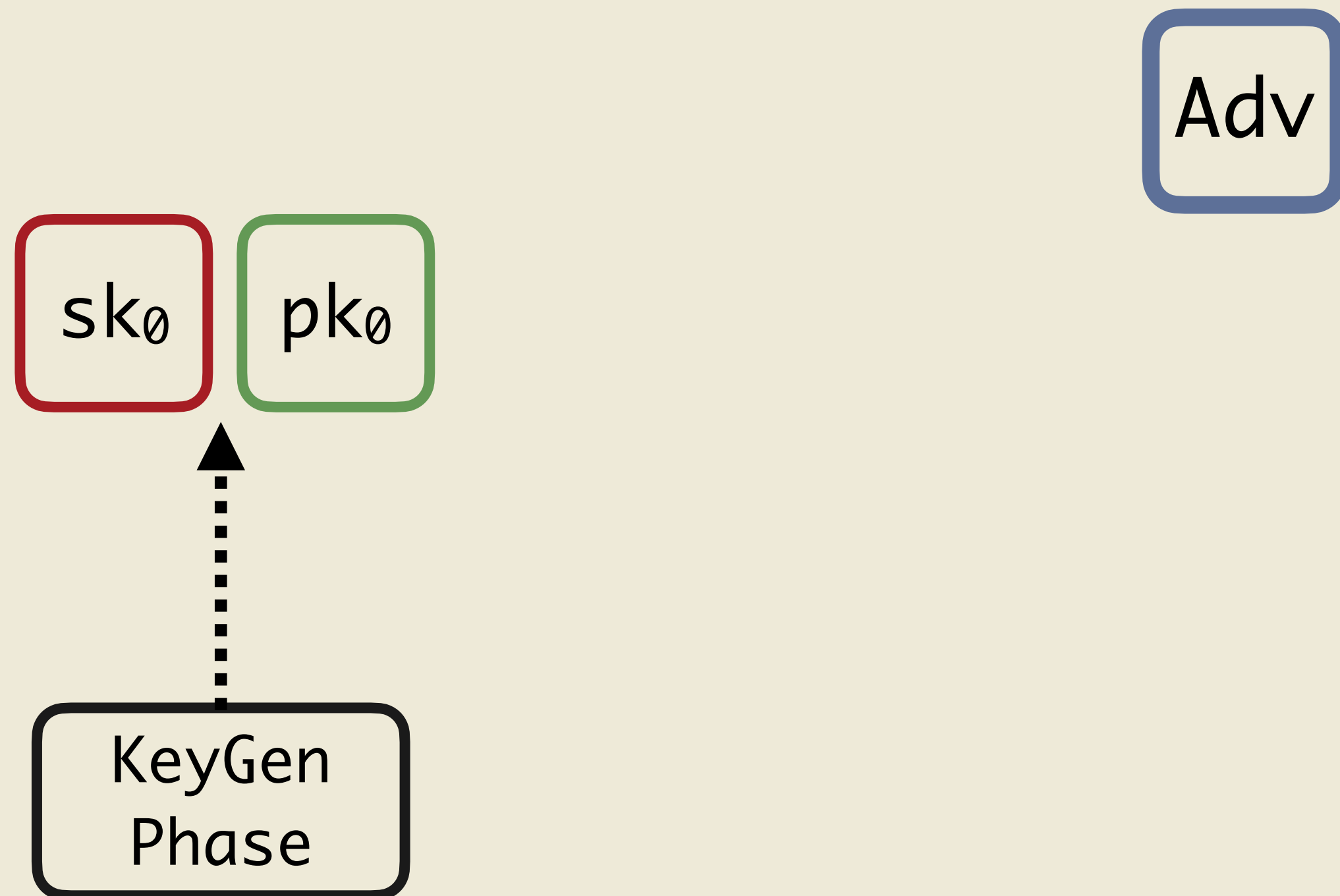
Definition



Security

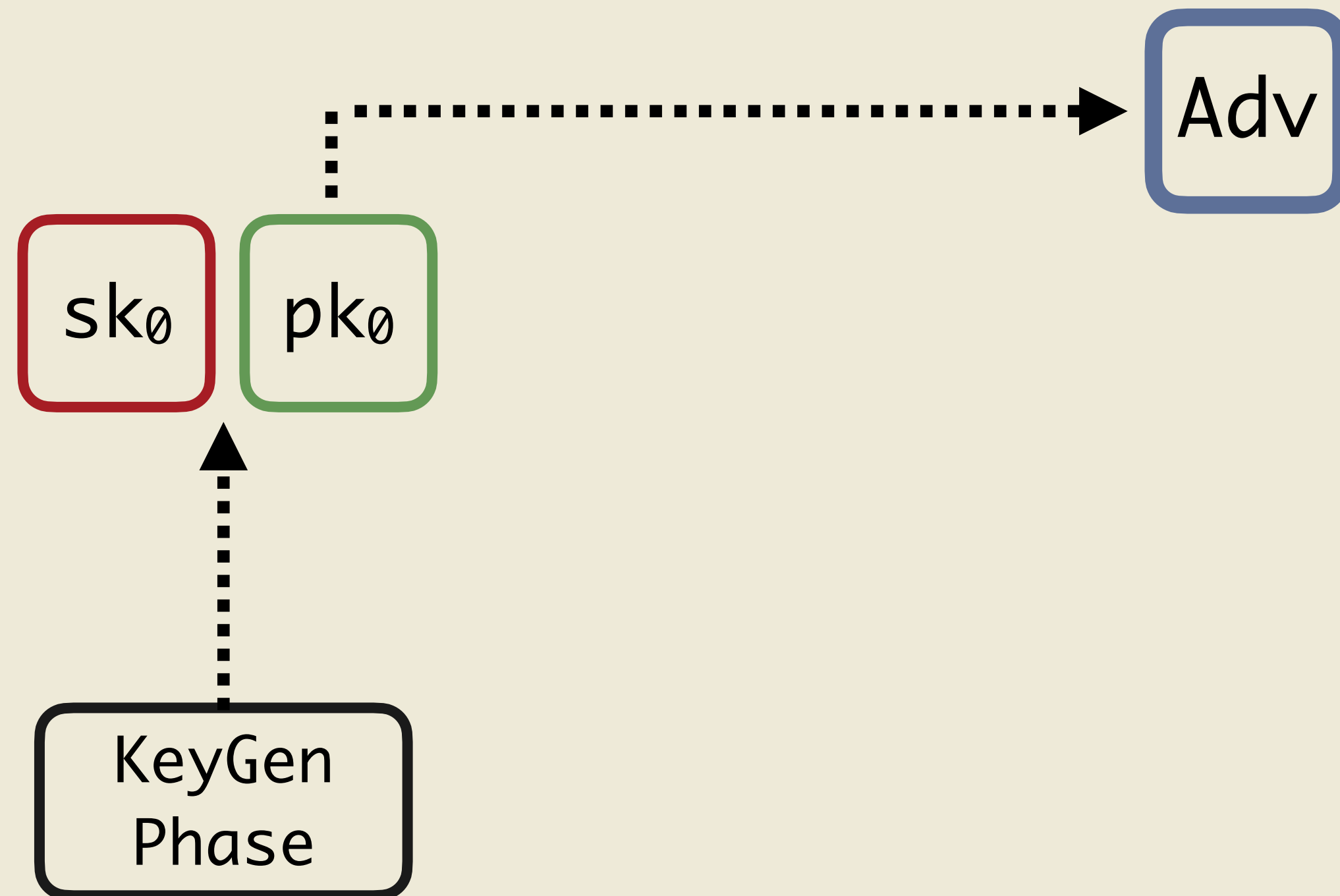
Updatable Public Key Encryption

Definition - Security



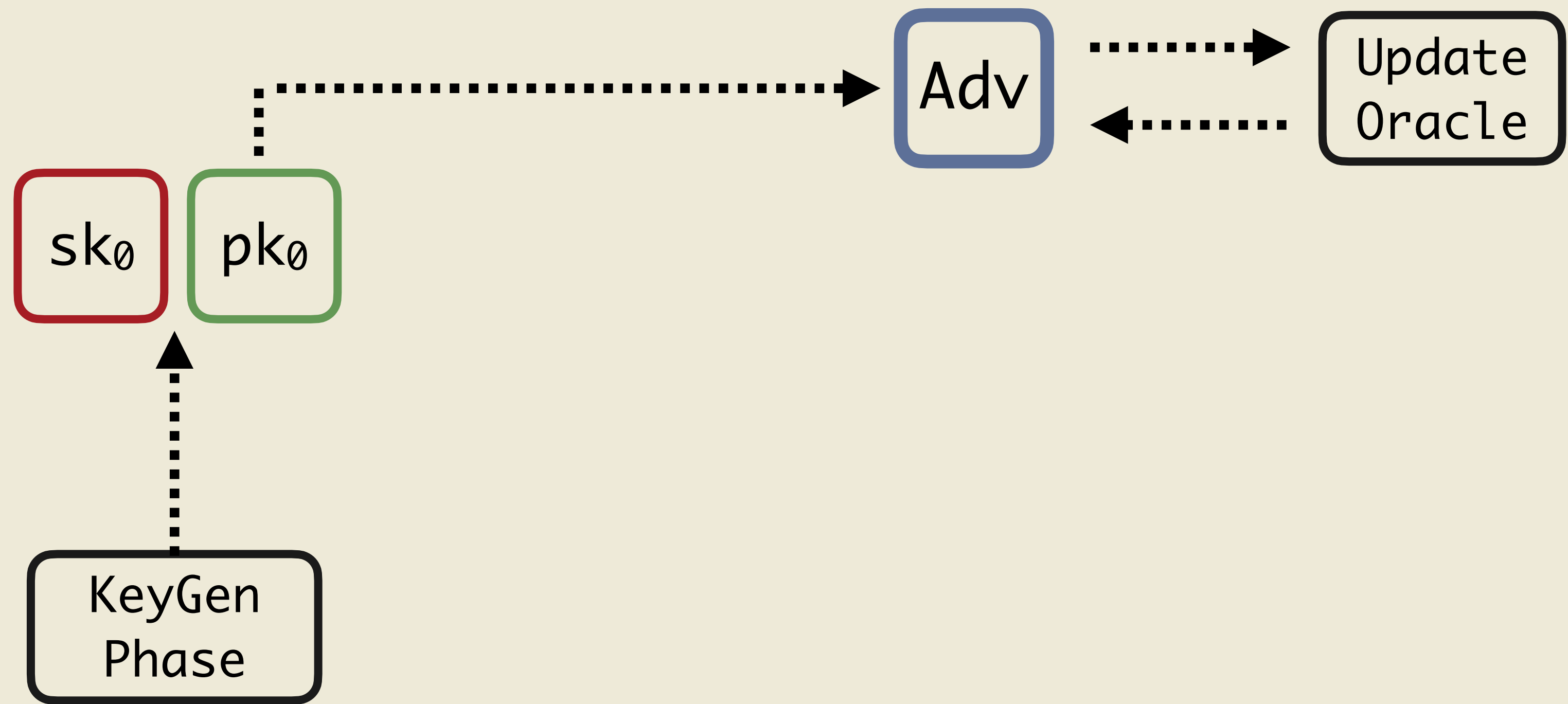
Updatable Public Key Encryption

Definition - Security



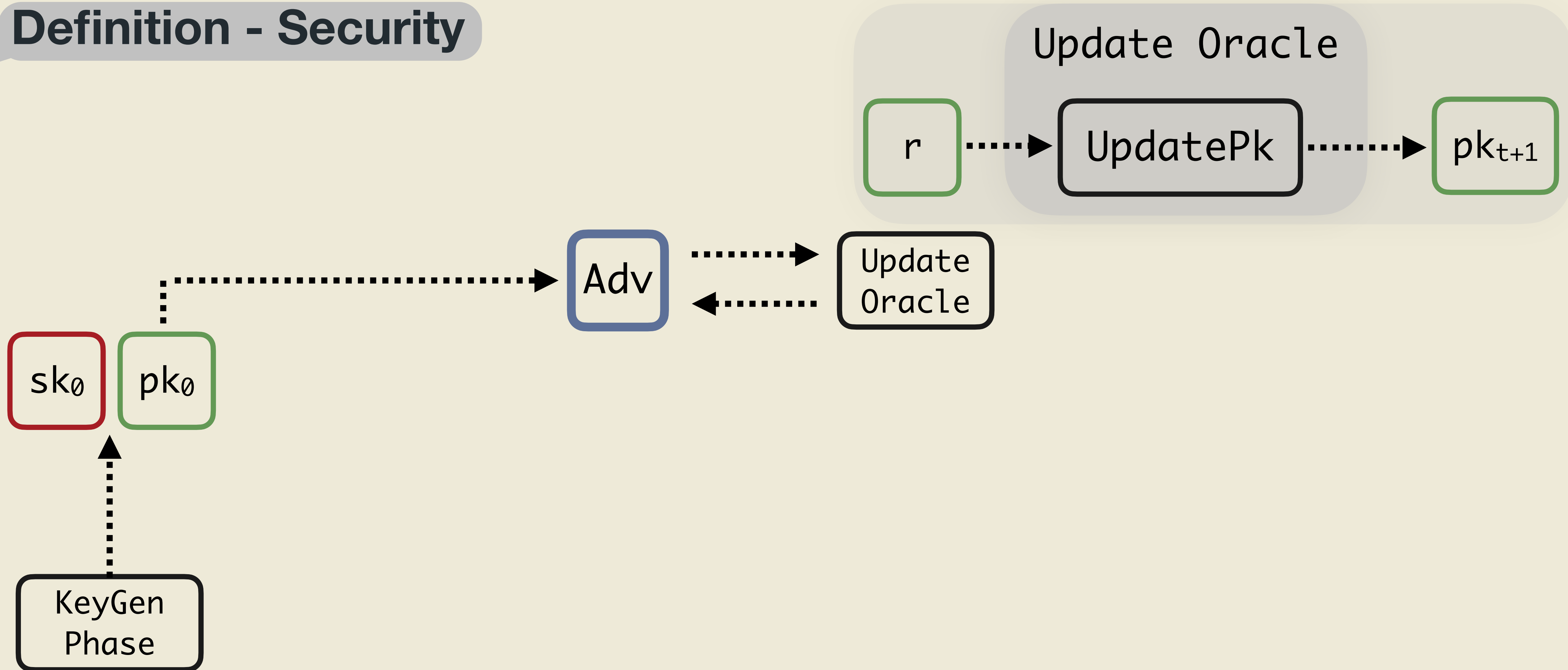
Updatable Public Key Encryption

Definition - Security



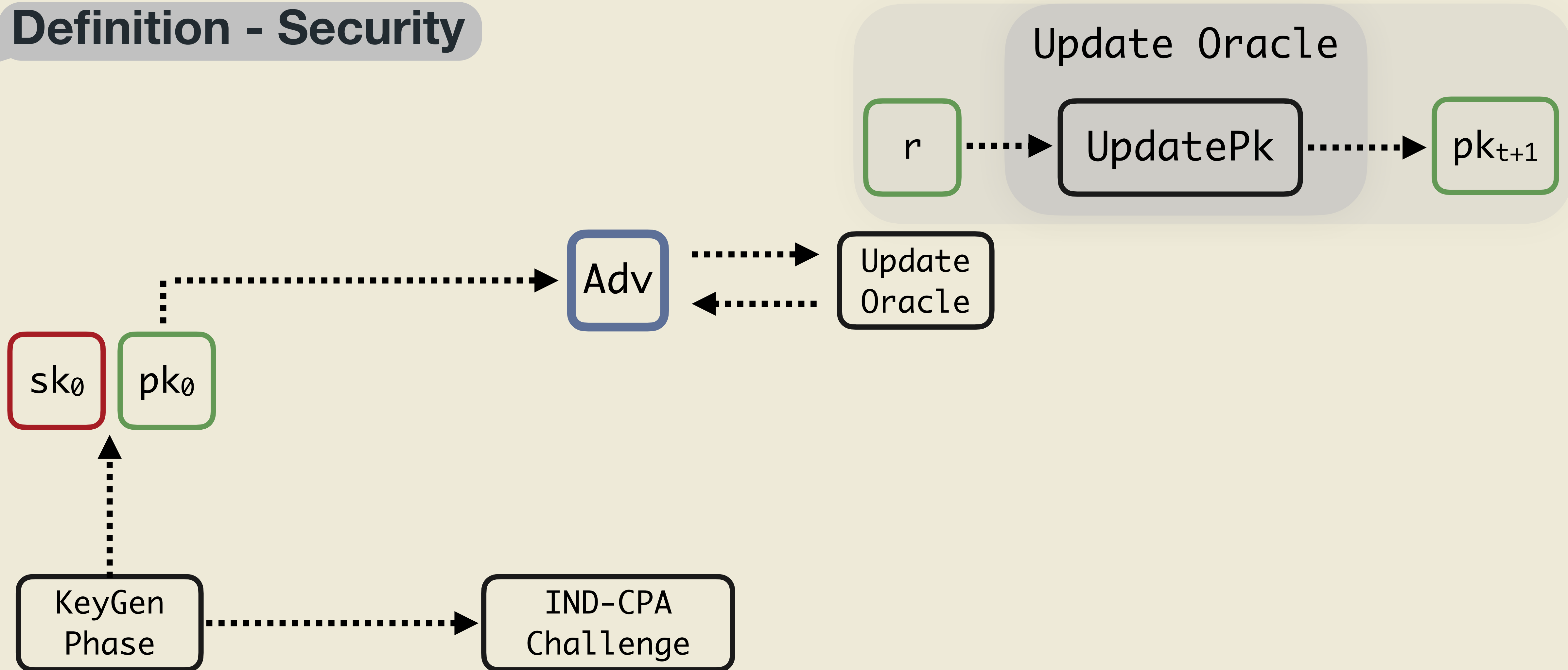
Updatable Public Key Encryption

Definition - Security



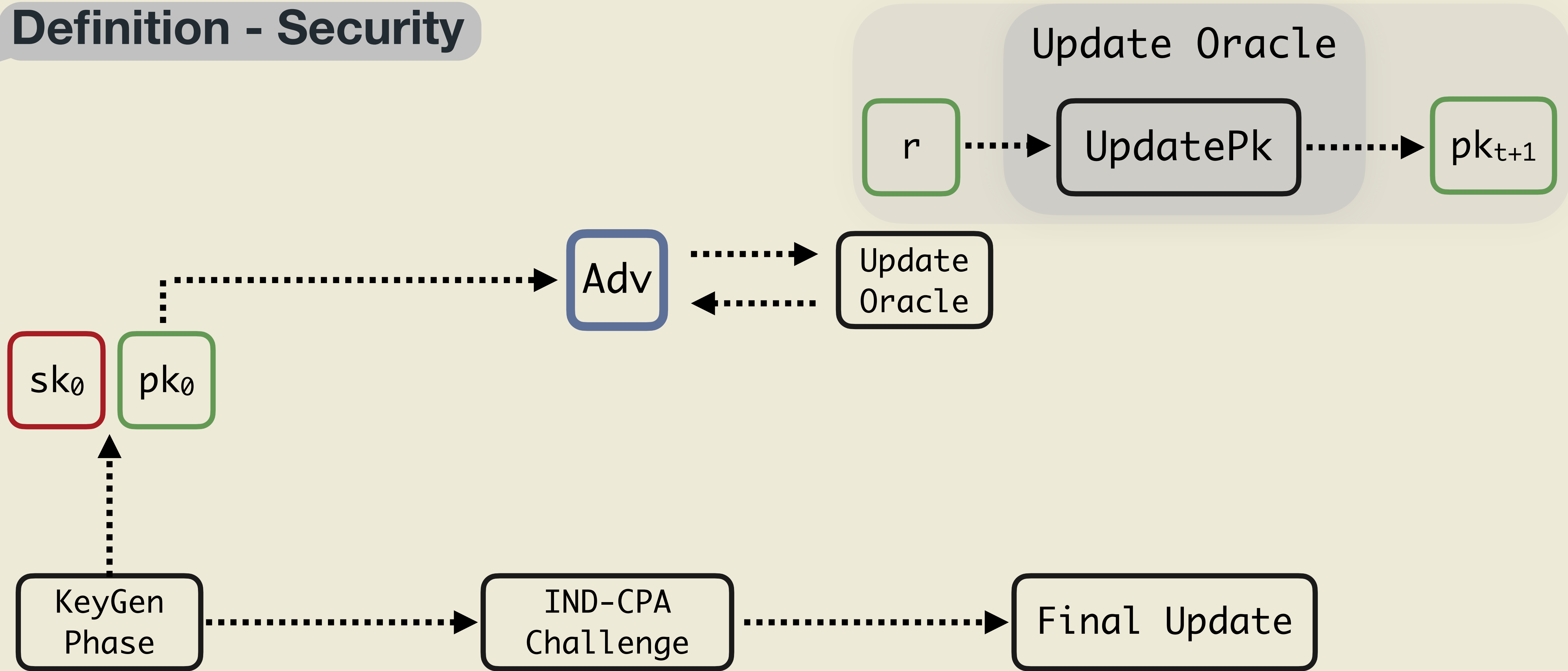
Updatable Public Key Encryption

Definition - Security



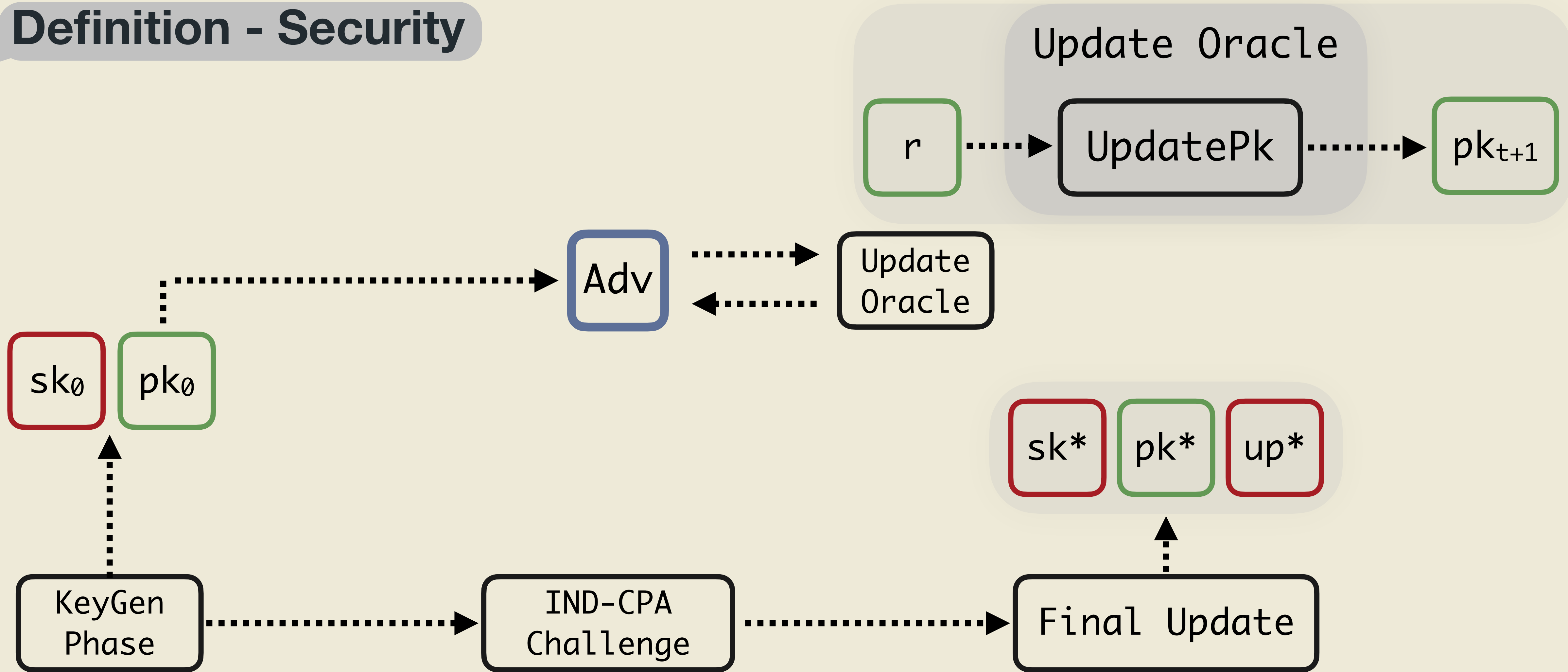
Updatable Public Key Encryption

Definition - Security



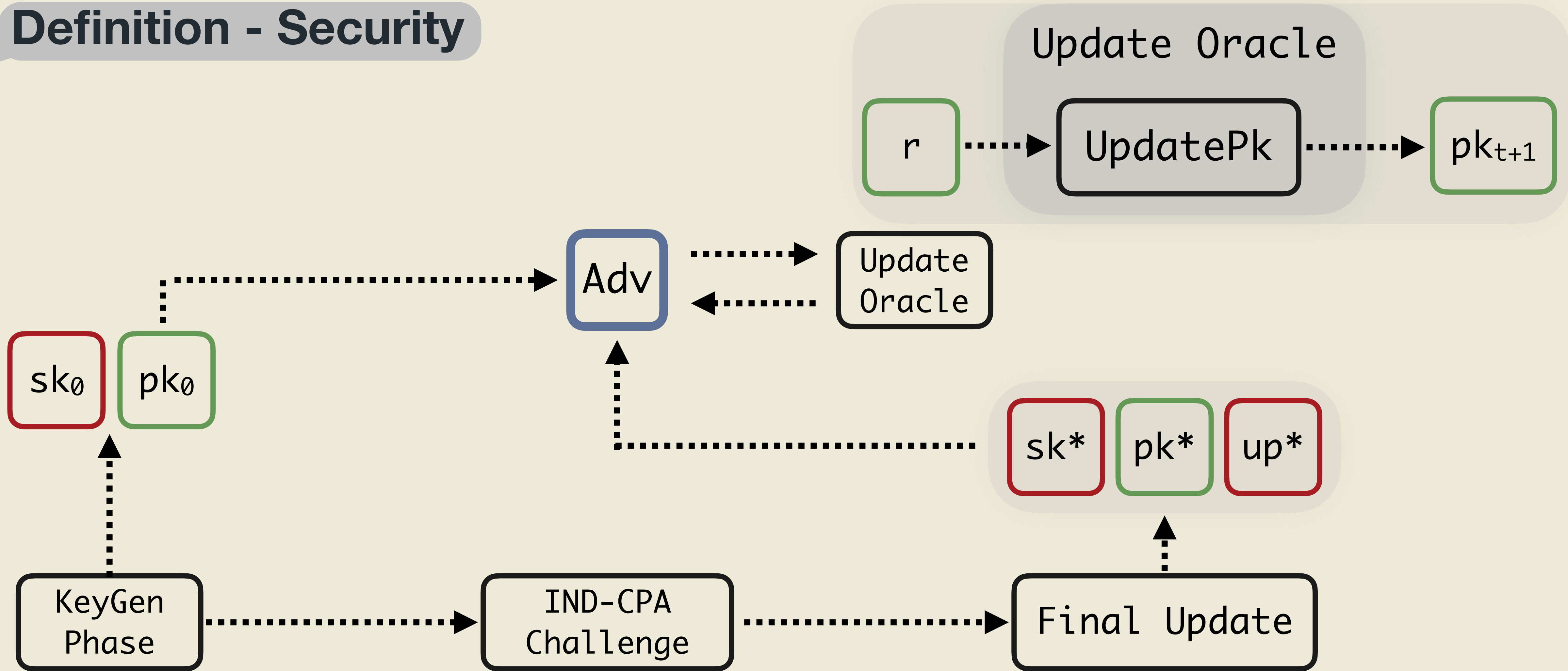
Updatable Public Key Encryption

Definition - Security



Updatable Public Key Encryption

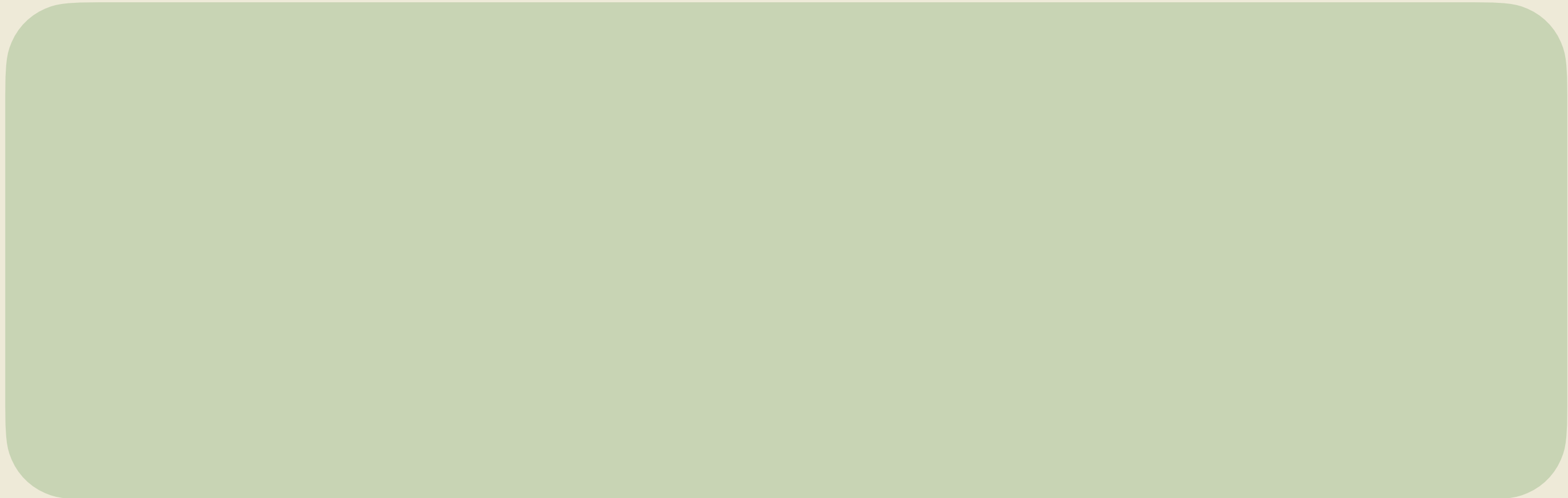
Definition - Security



Construction

UPKE

Construction time



LWE Assumption

UPKE

Construction time



LWE Assumption

UPKE

Construction time

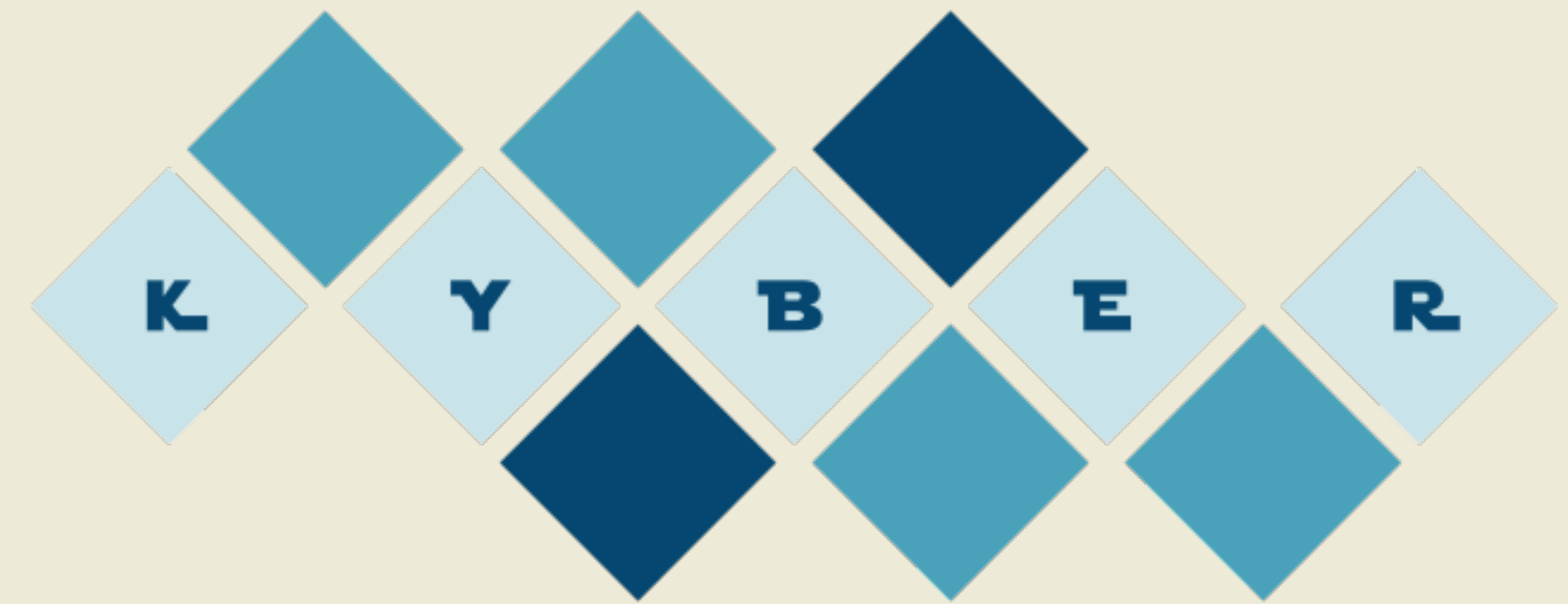


LWE Assumption



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Construction time

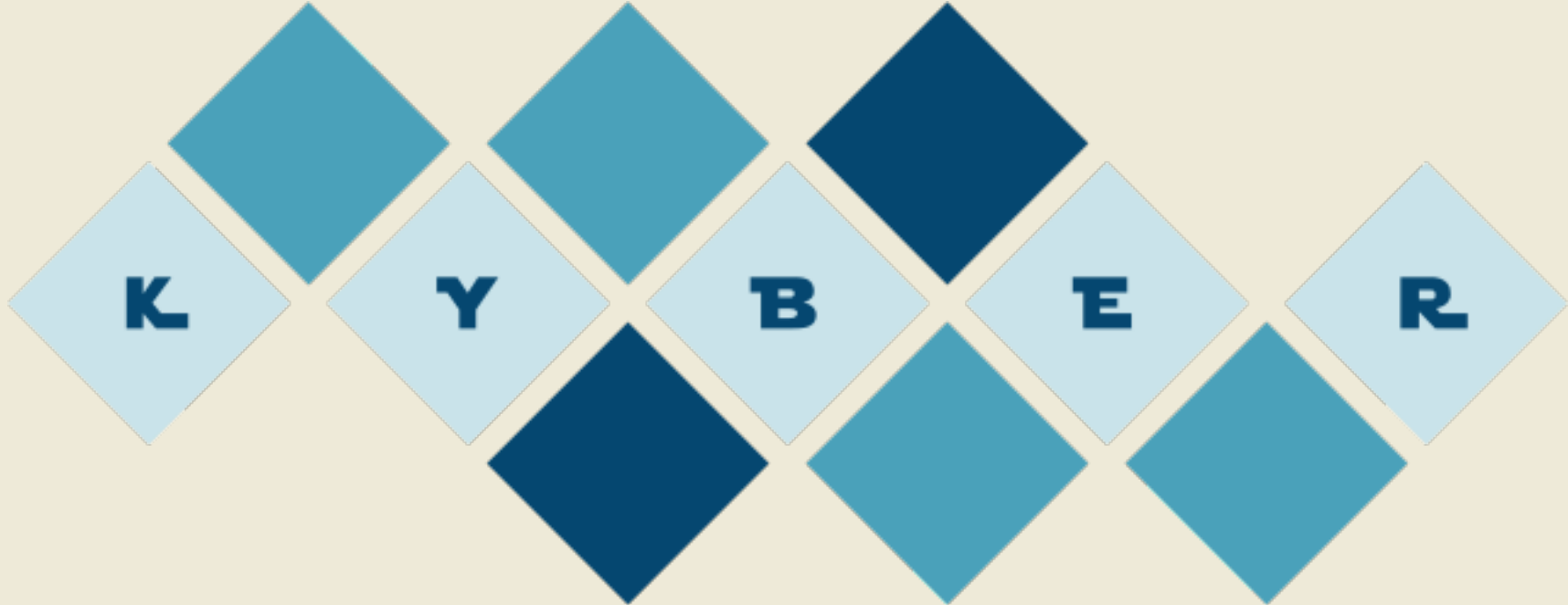


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Construction time

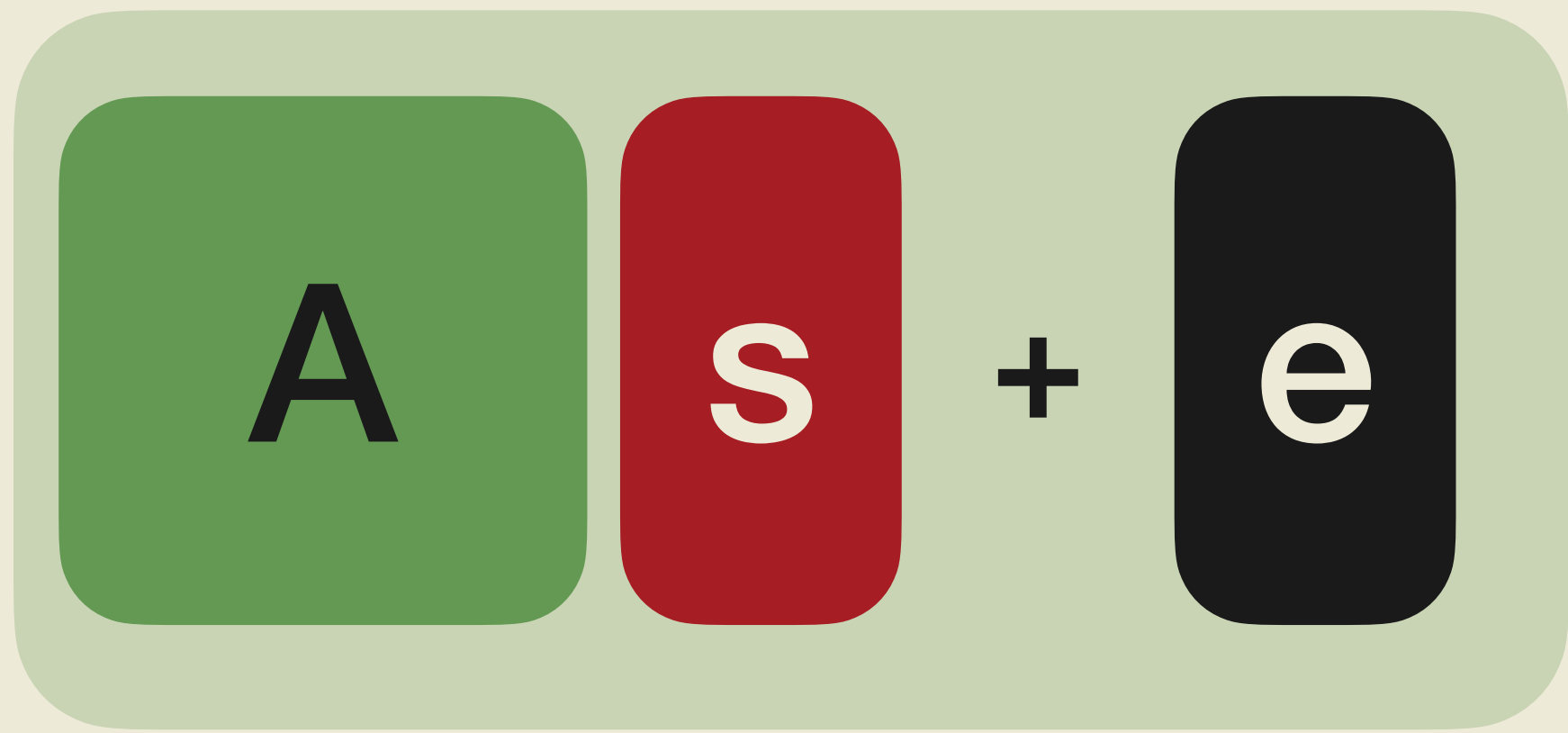


sk



UPKE

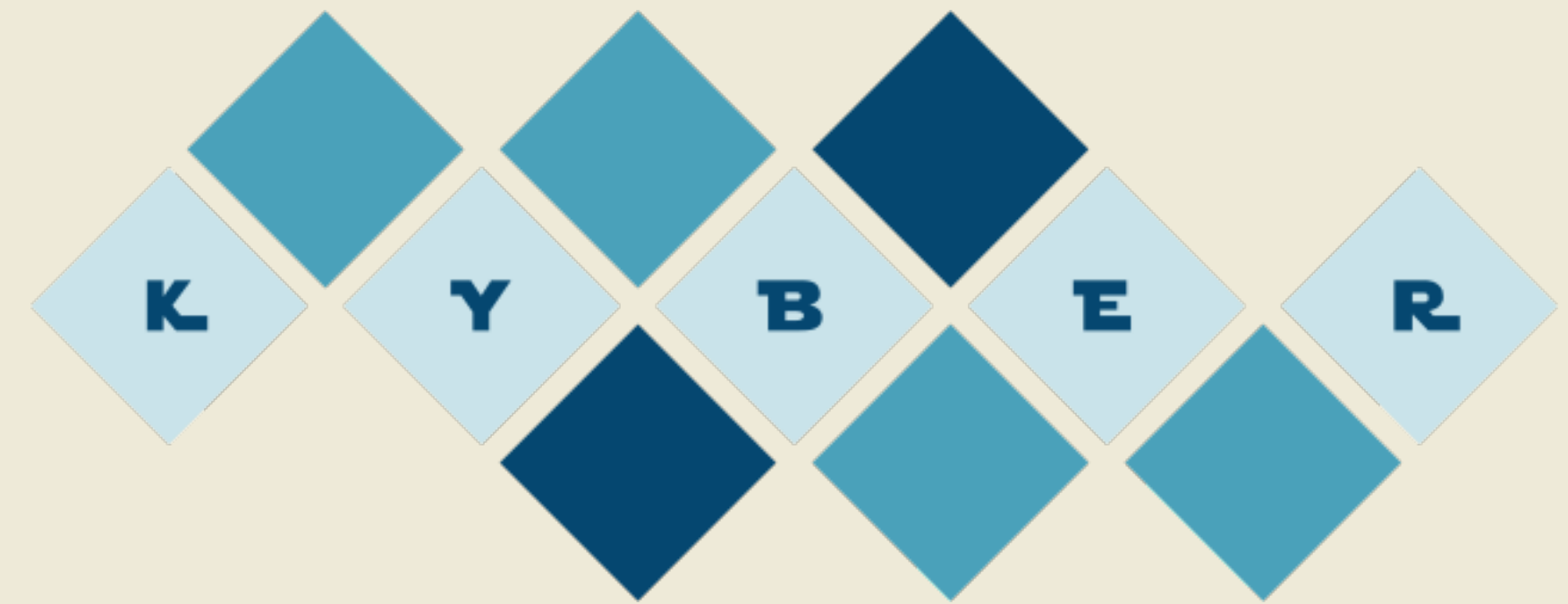
Construction time



pk

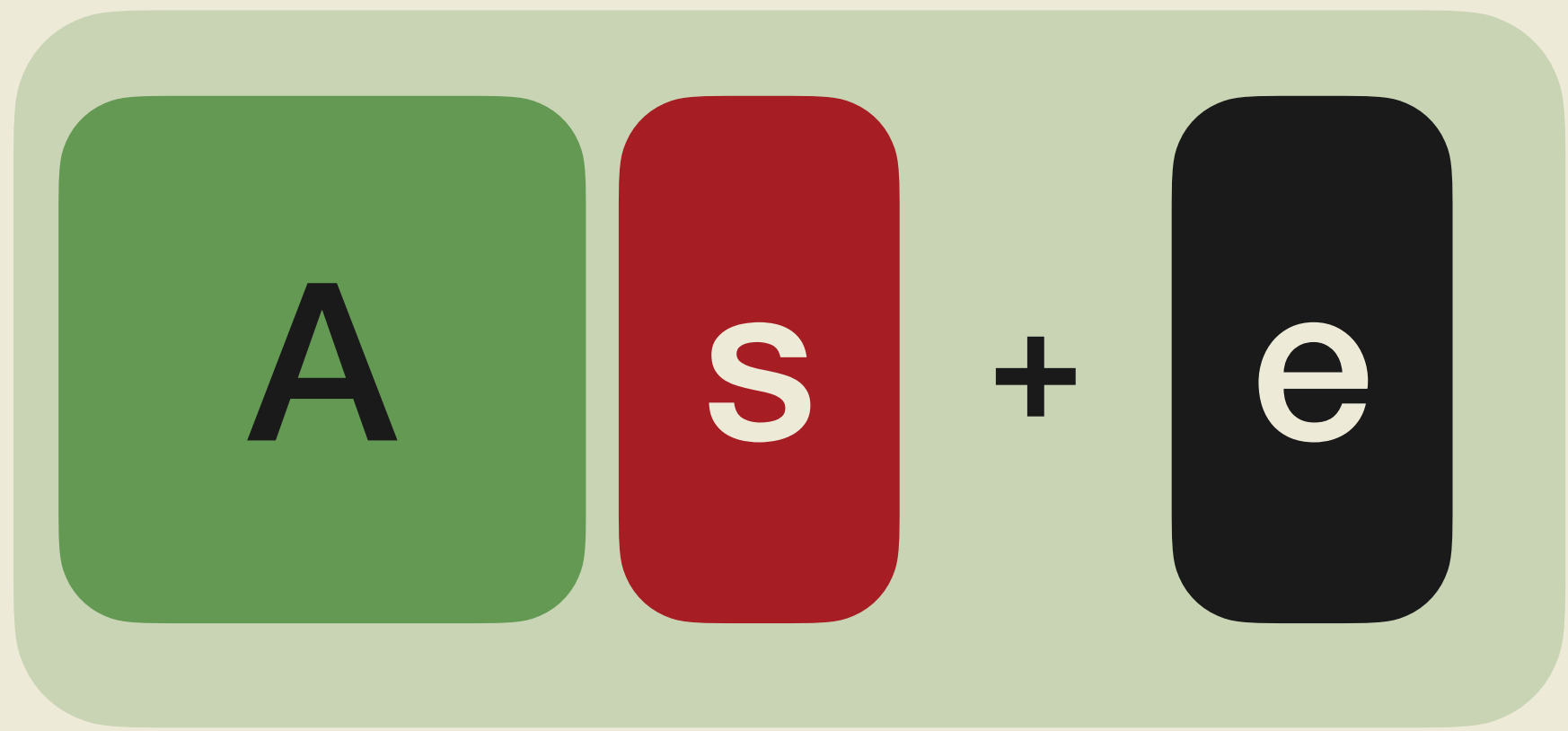


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UPKE

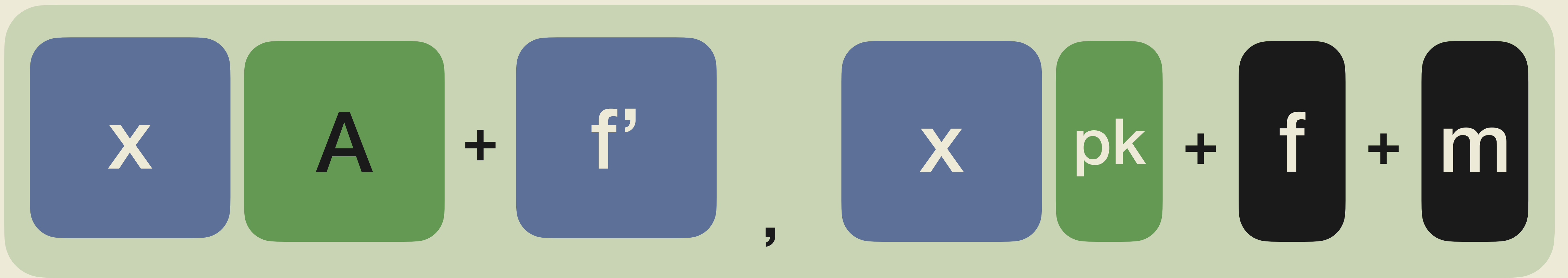
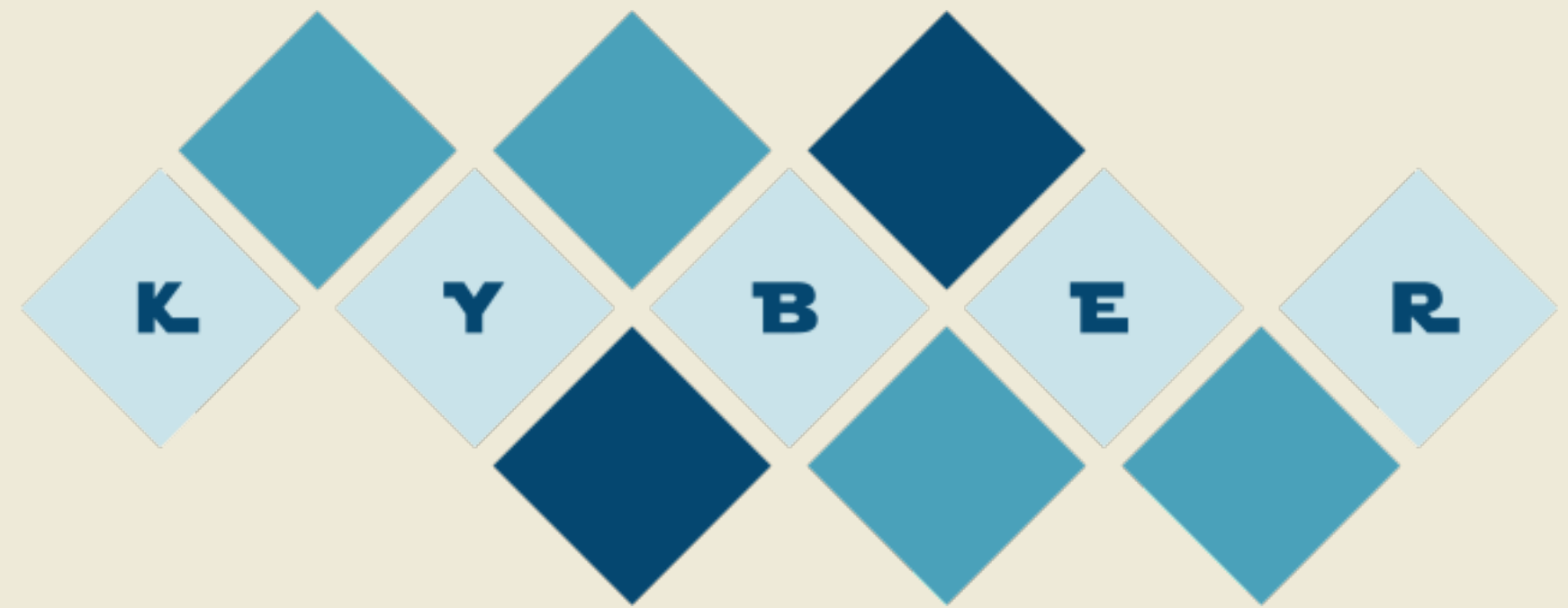
Construction time



pk



sk



Enc(pk, m)

UPKE

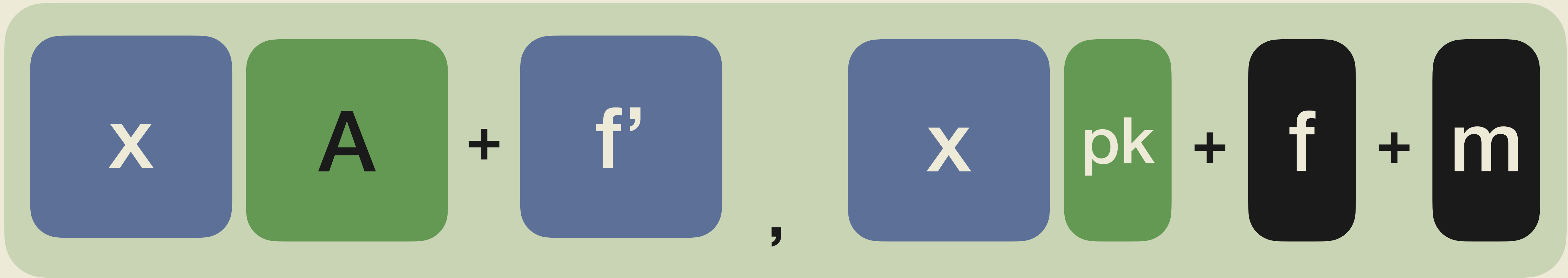
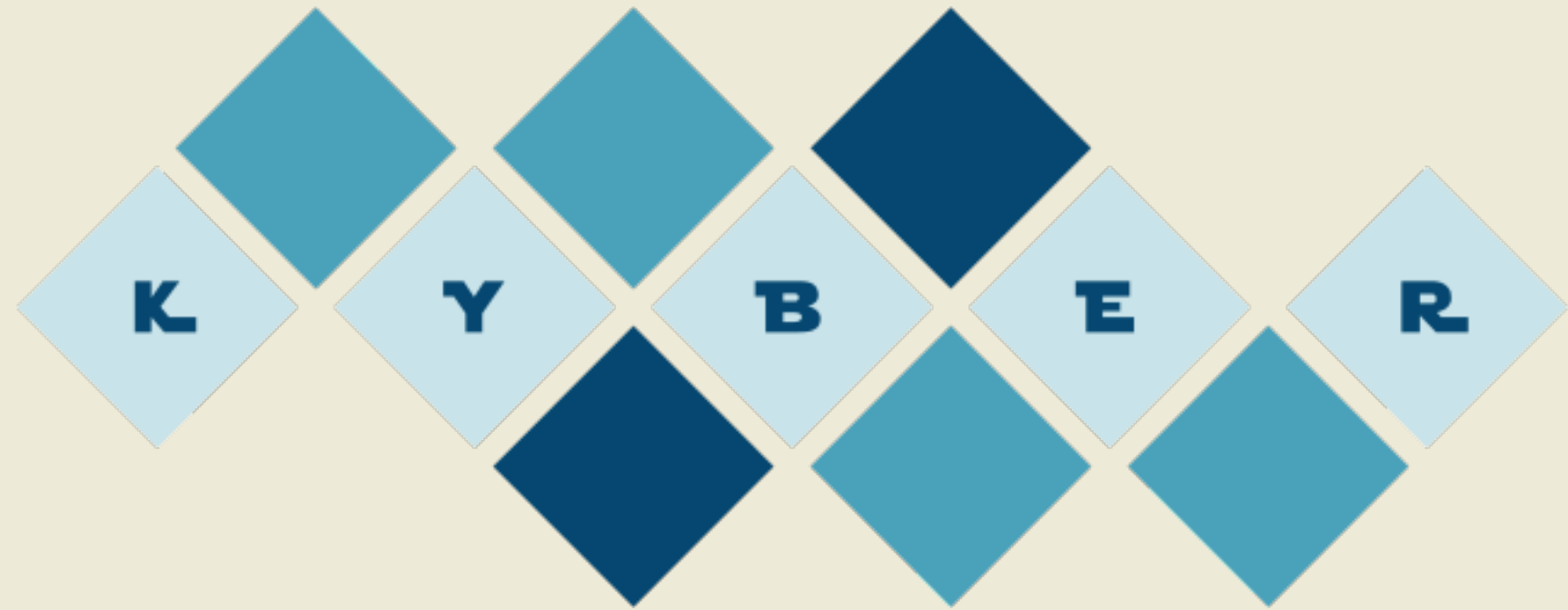
Construction time



pk



sk



Enc(pk, m)

UPKE

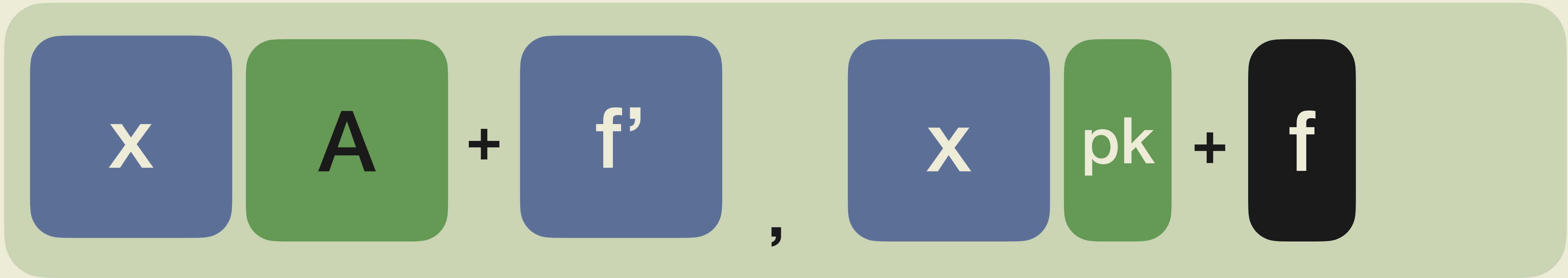
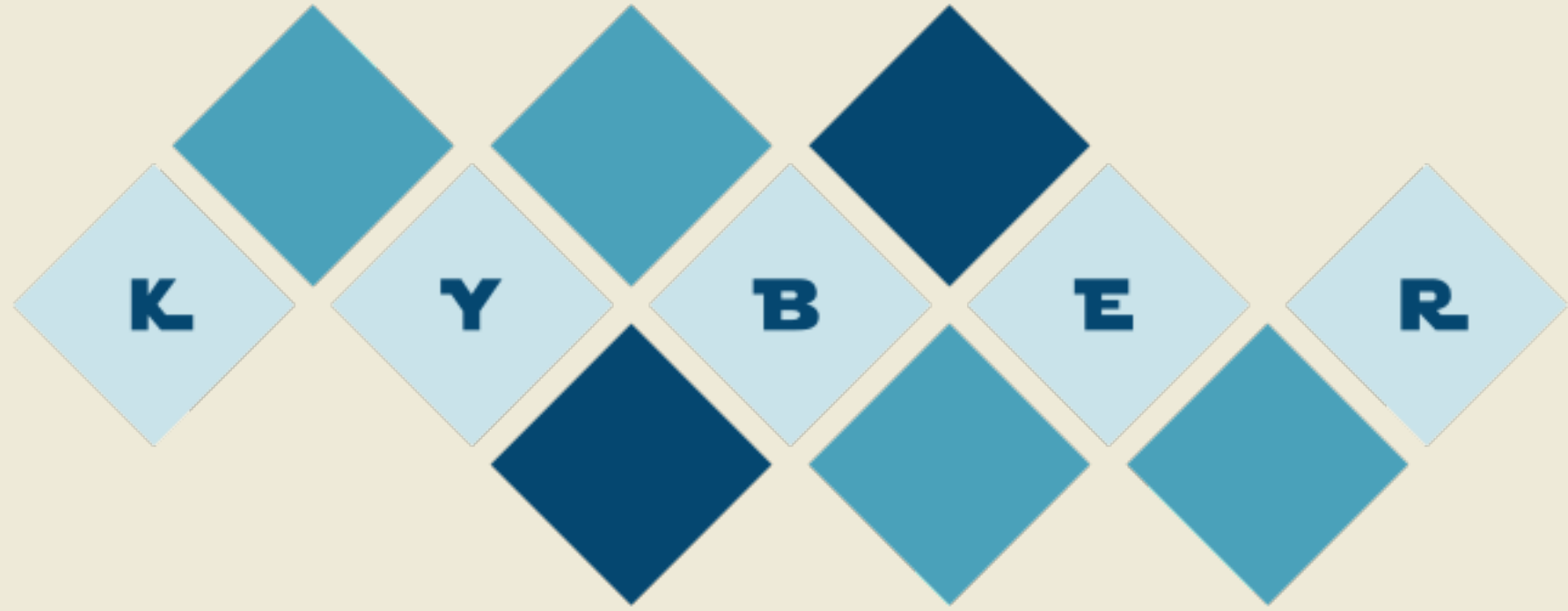
Construction time



pk



sk



Enc(pk, m)

UPKE

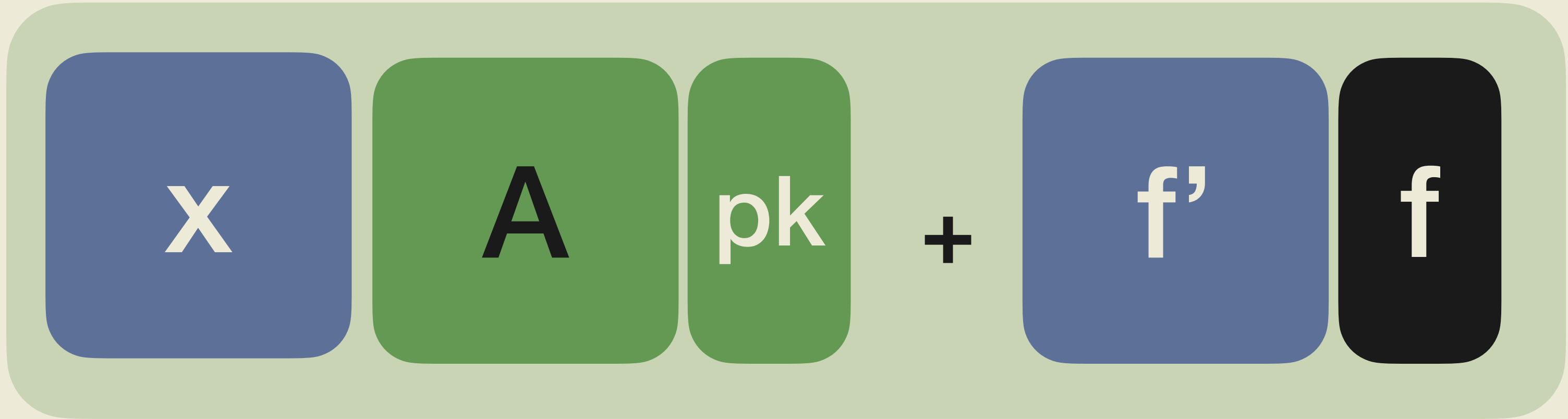
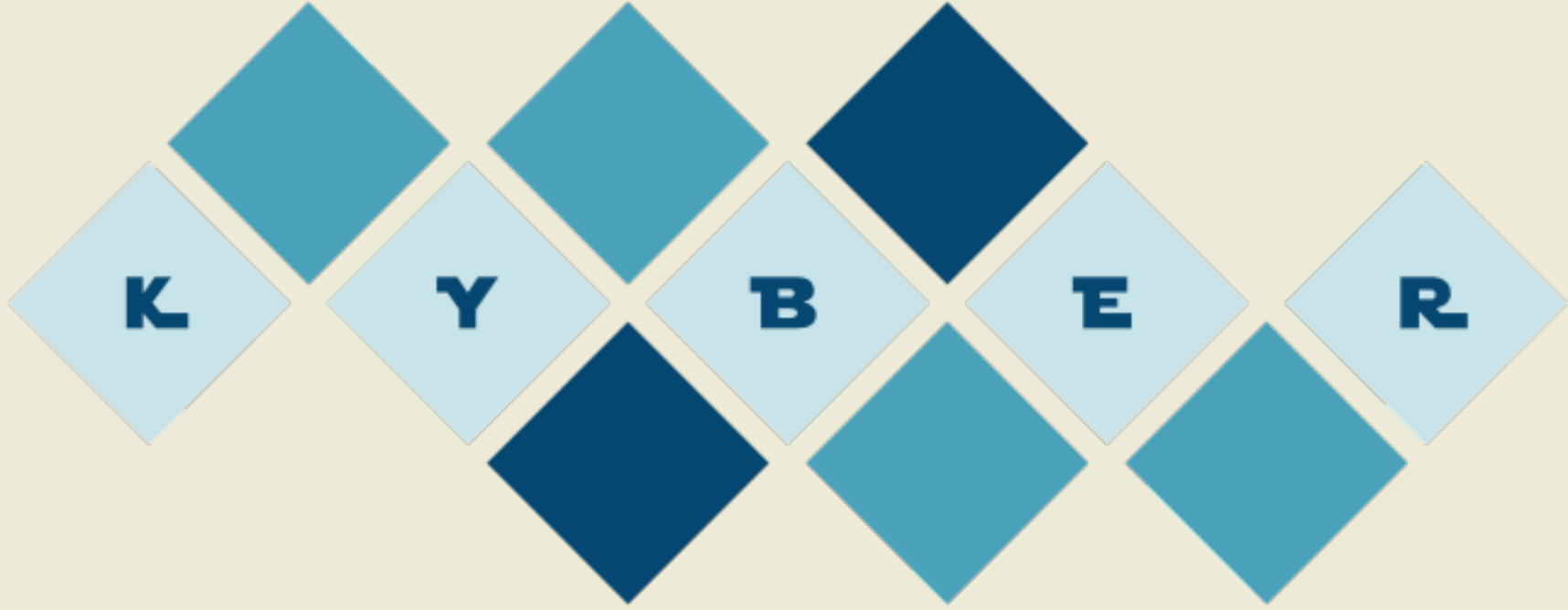
Construction time



pk



sk



Enc(pk, m)

UPKE

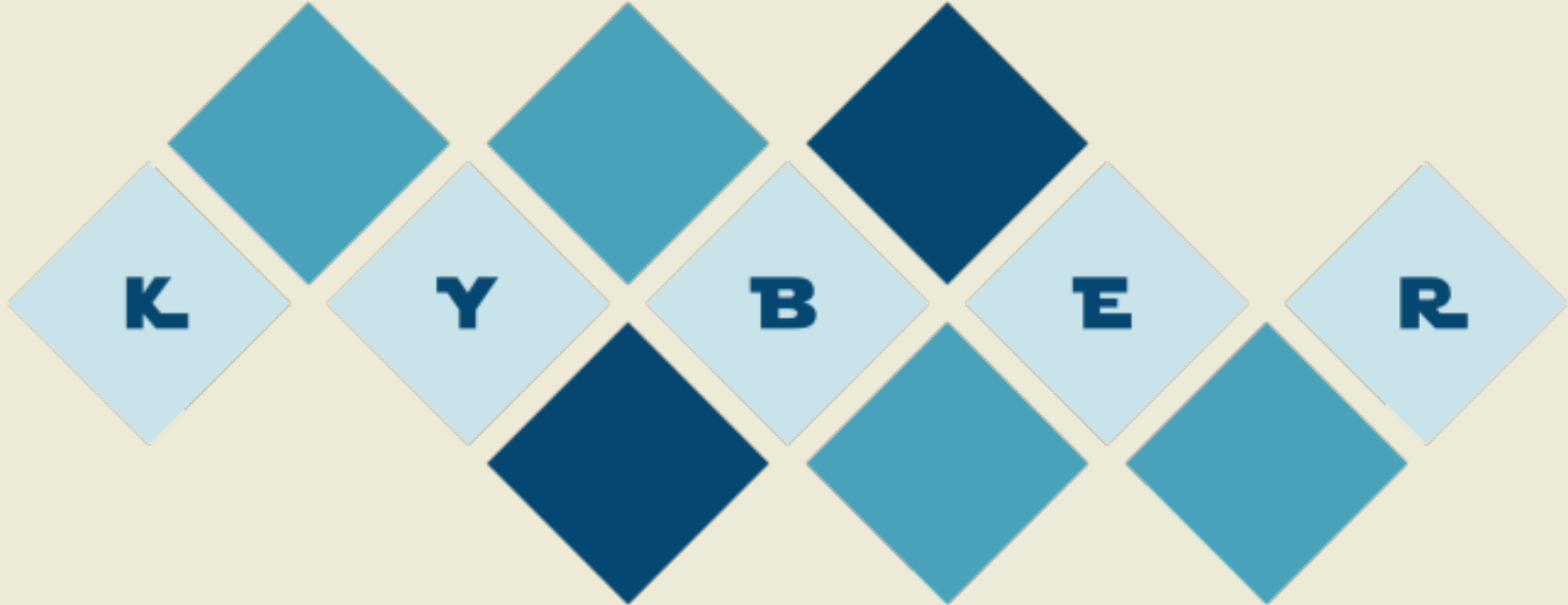
Construction time



pk



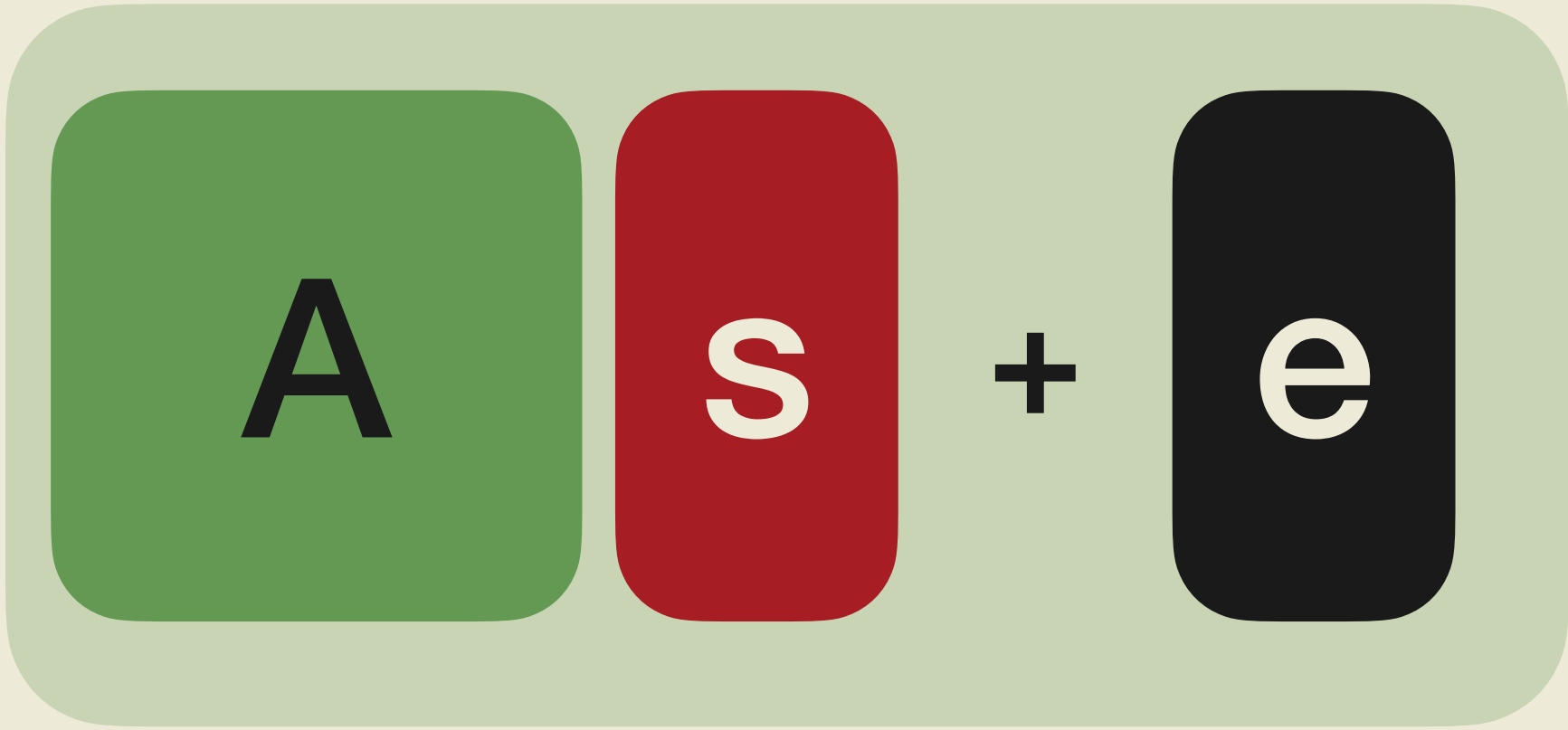
sk



Enc(pk, m)

UPKE

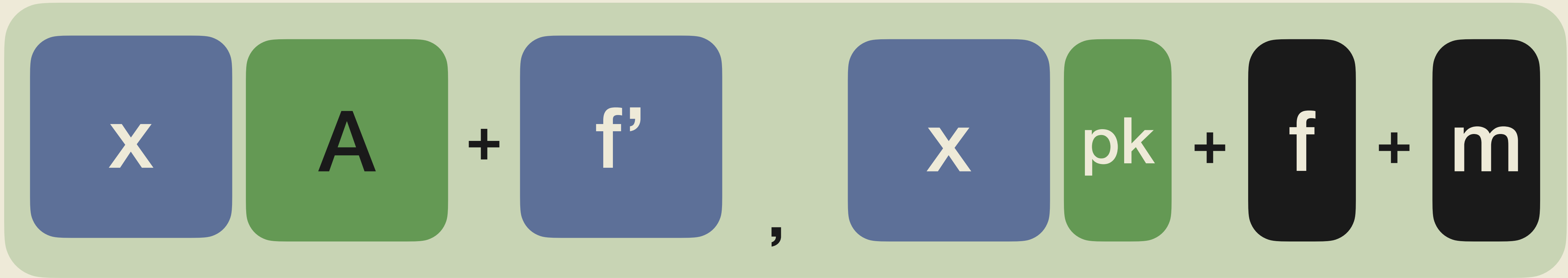
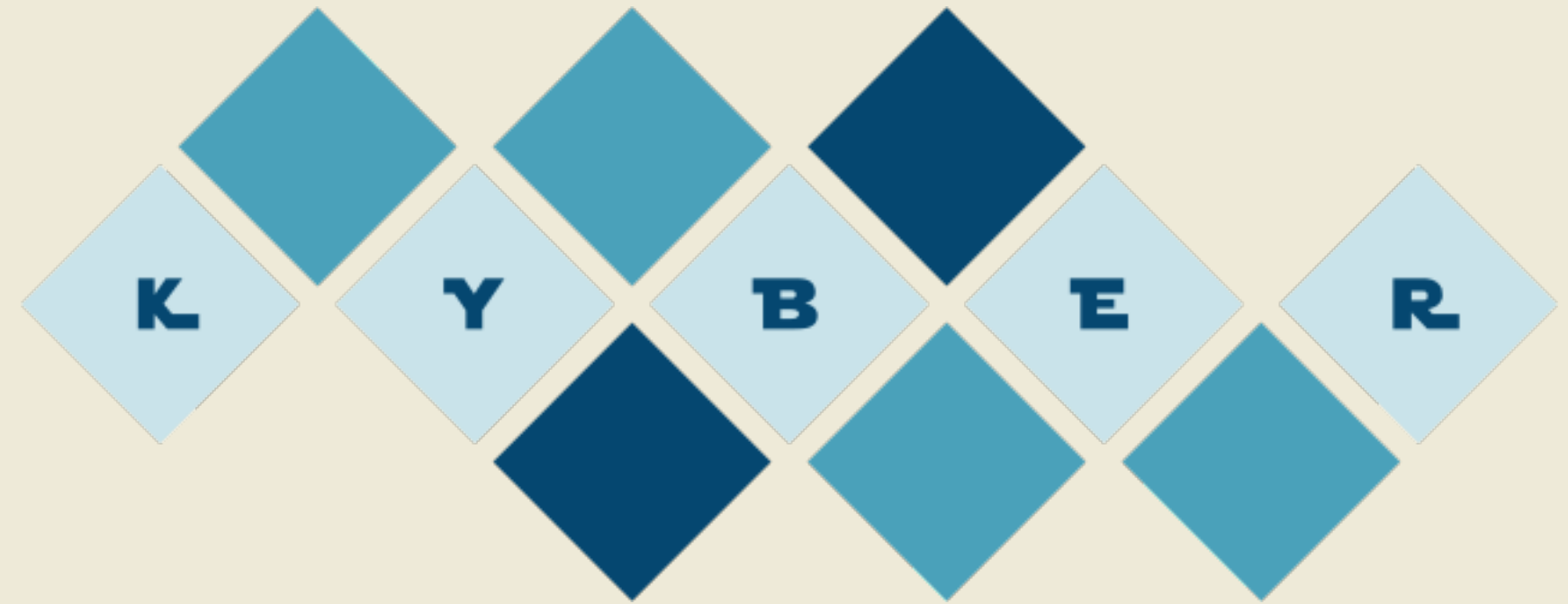
Construction time



pk



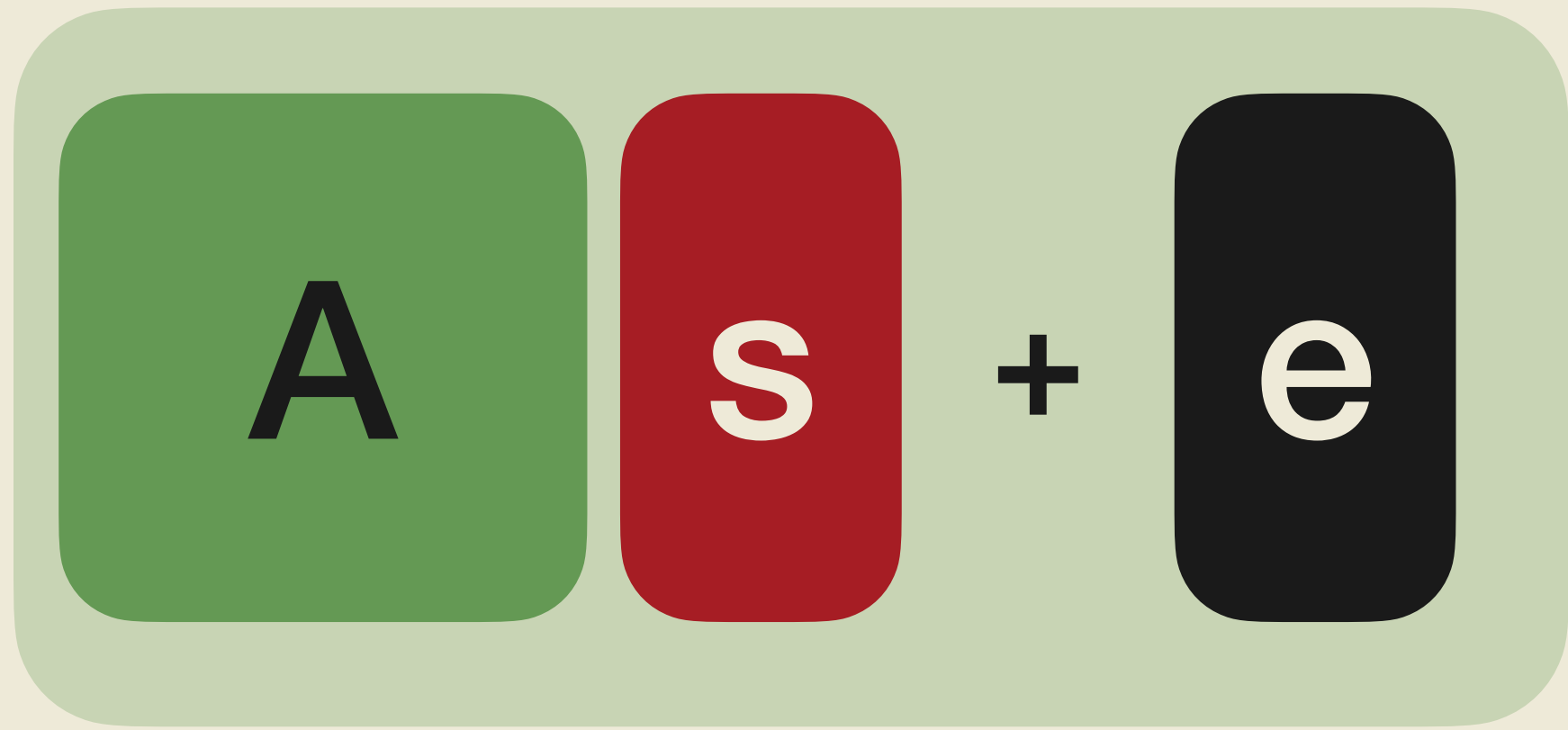
sk



$Enc(pk, m)$

UPKE

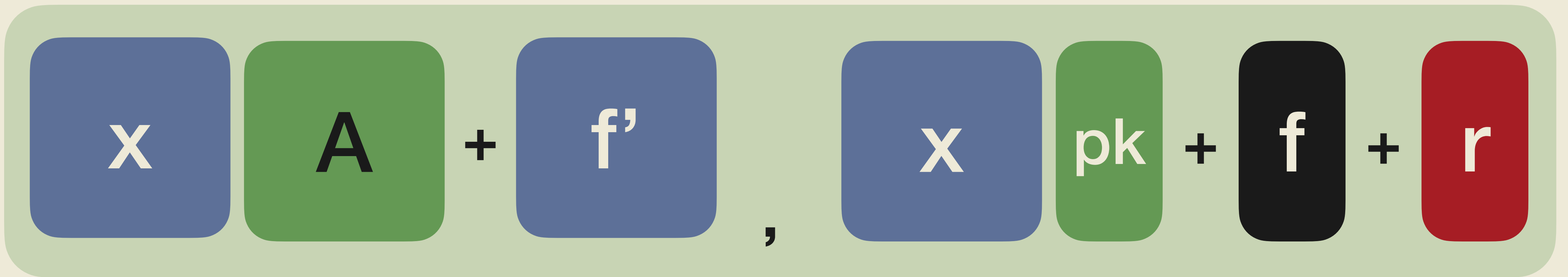
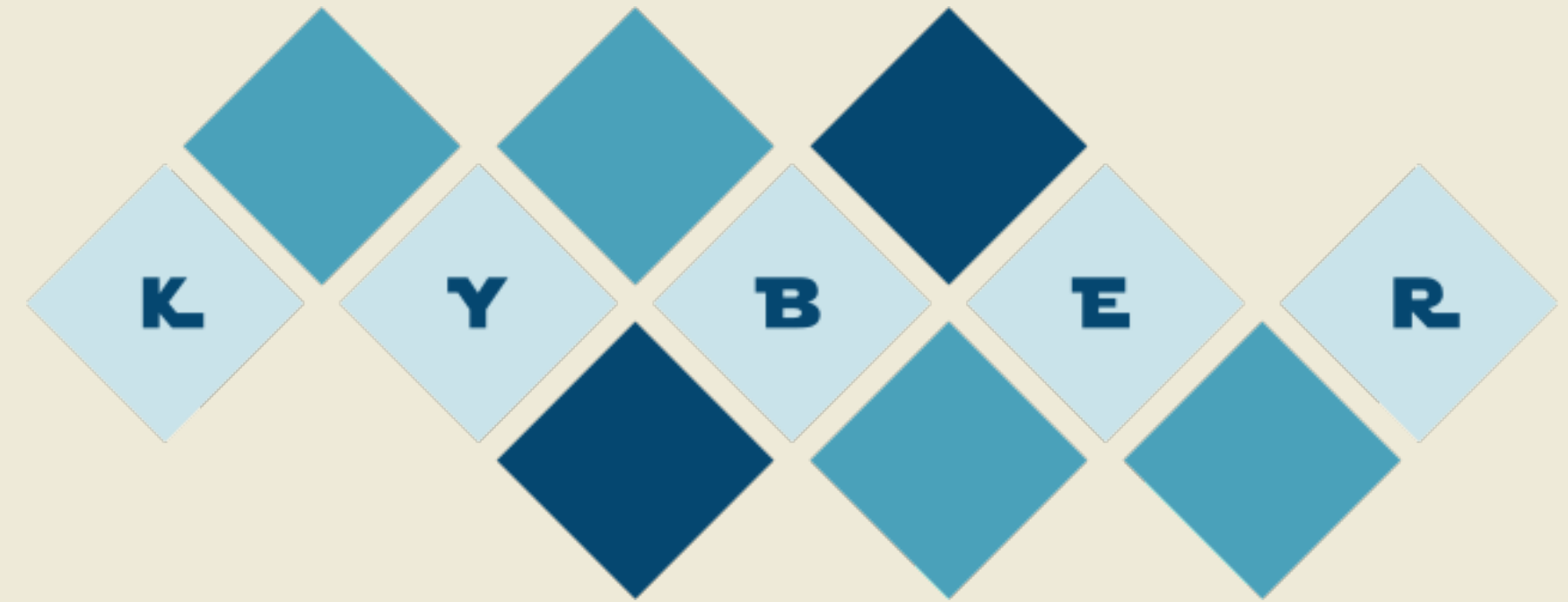
Construction time



pk



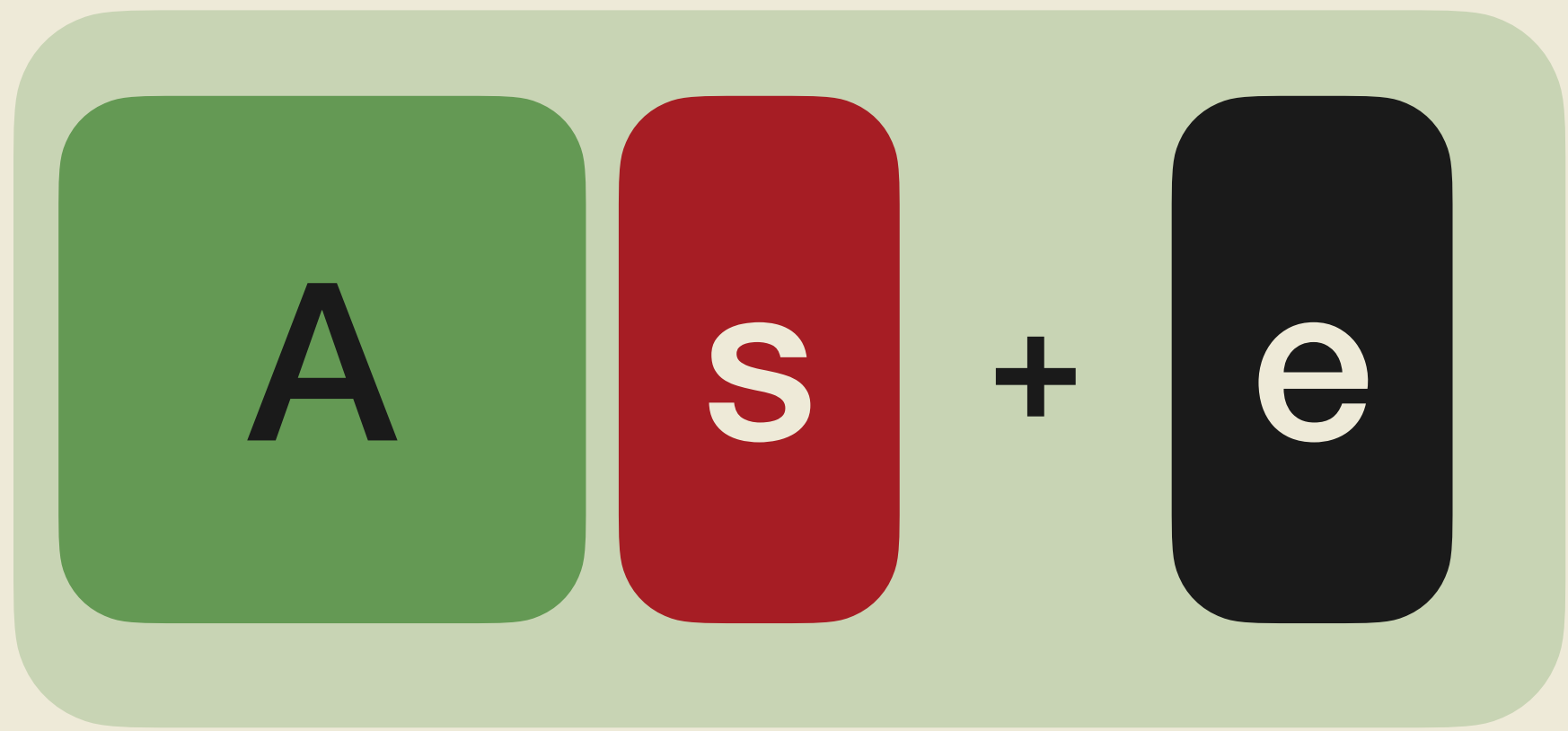
sk



$Enc(pk, r)$

UPKE

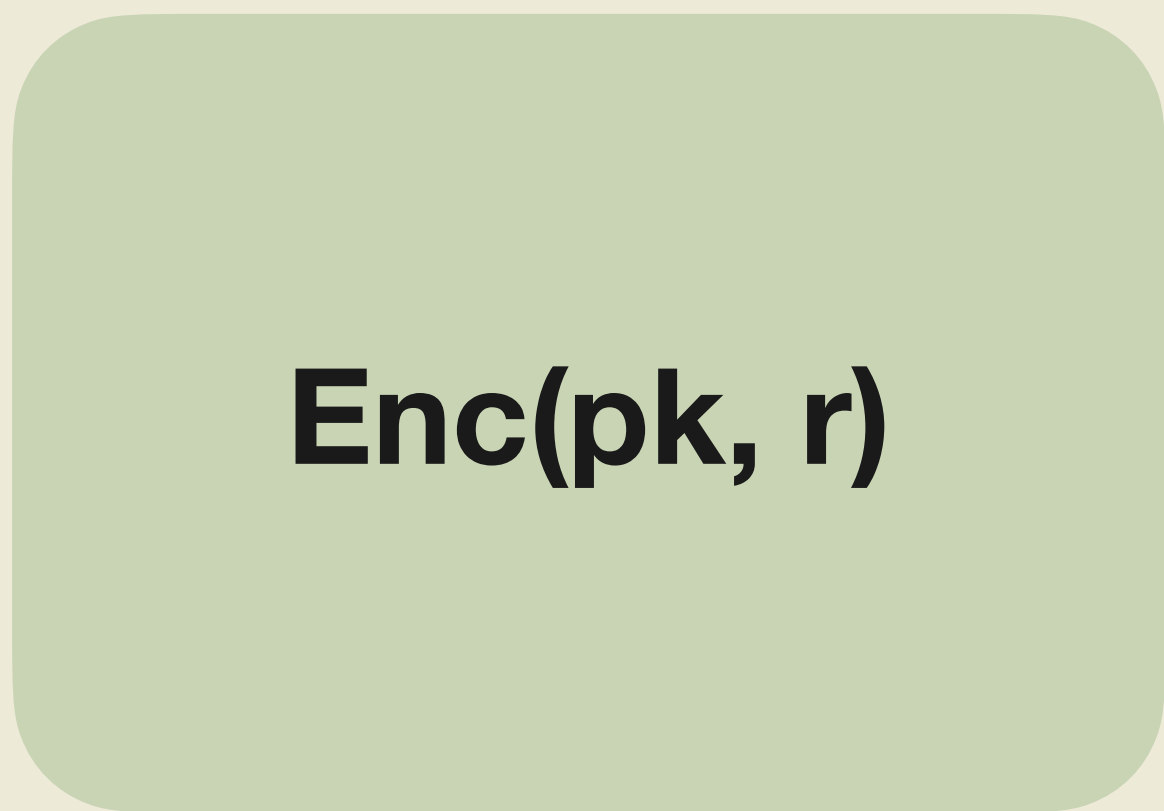
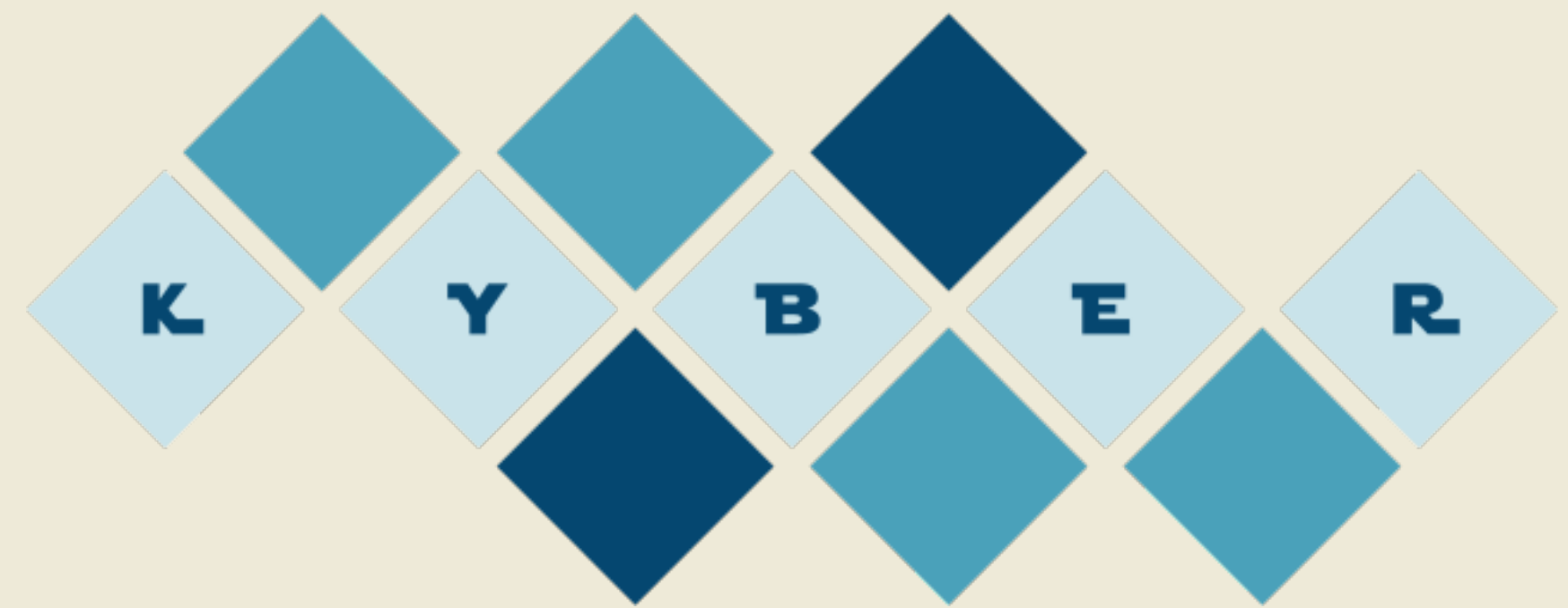
Construction time



pk

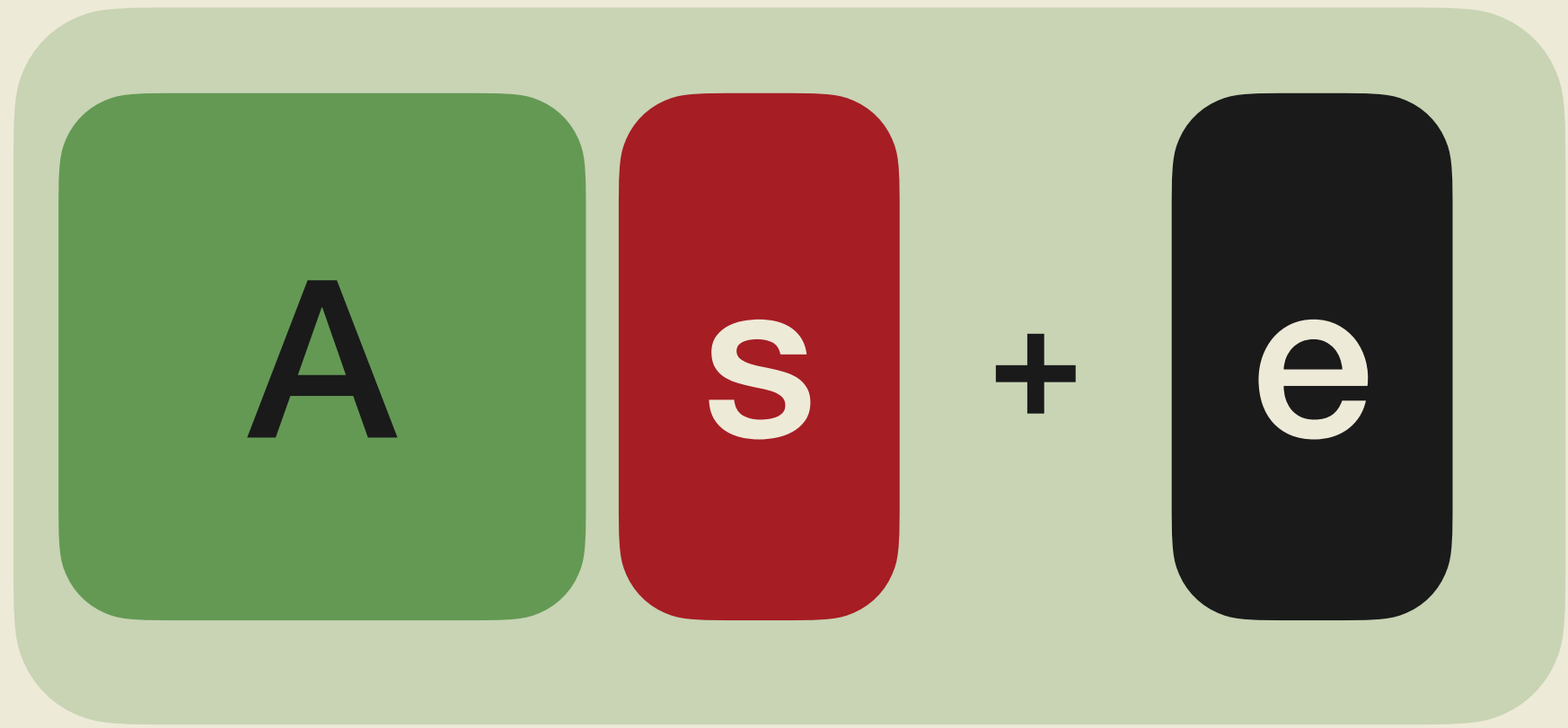


sk



UPKE

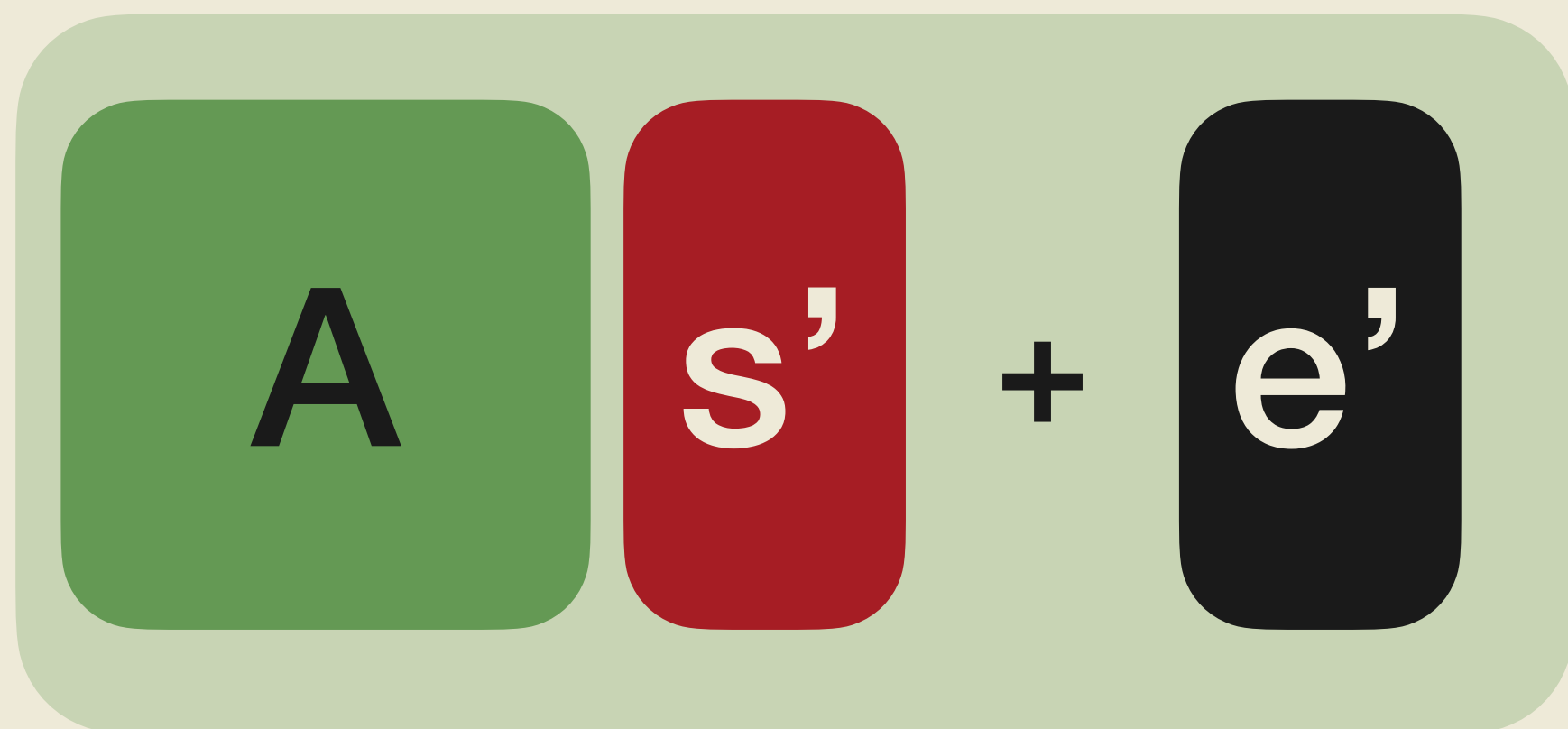
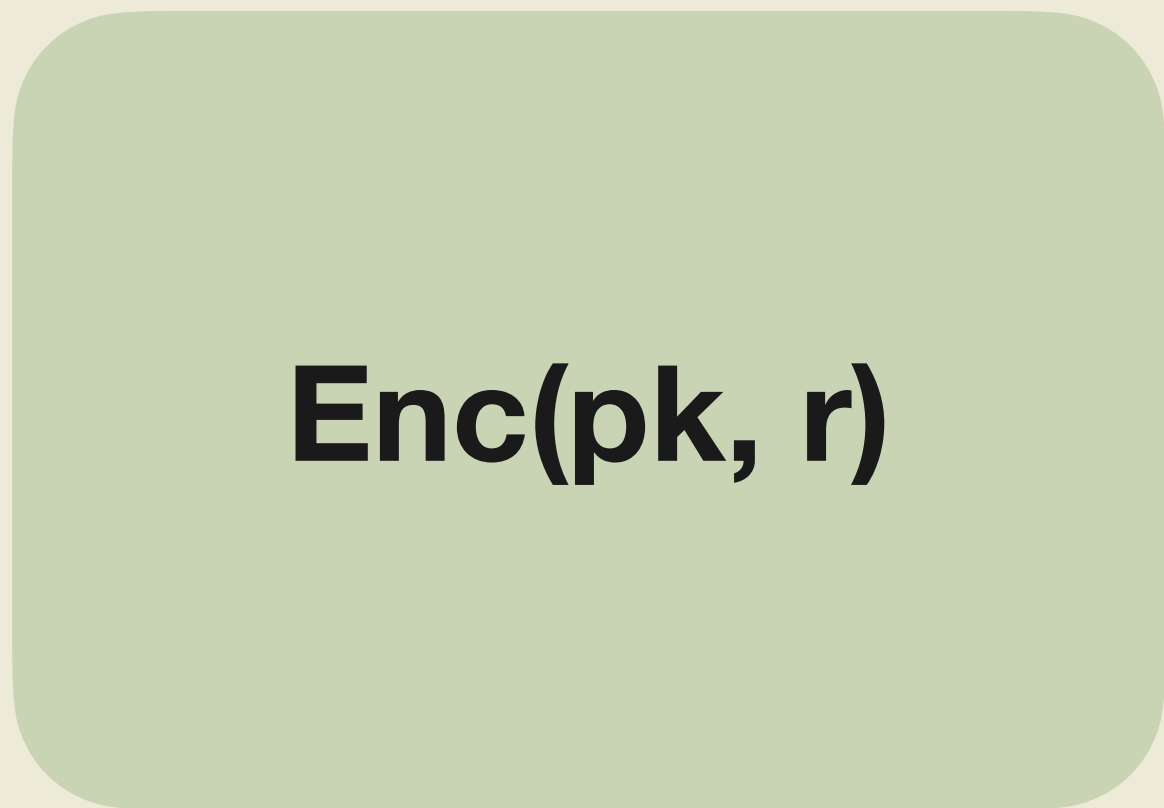
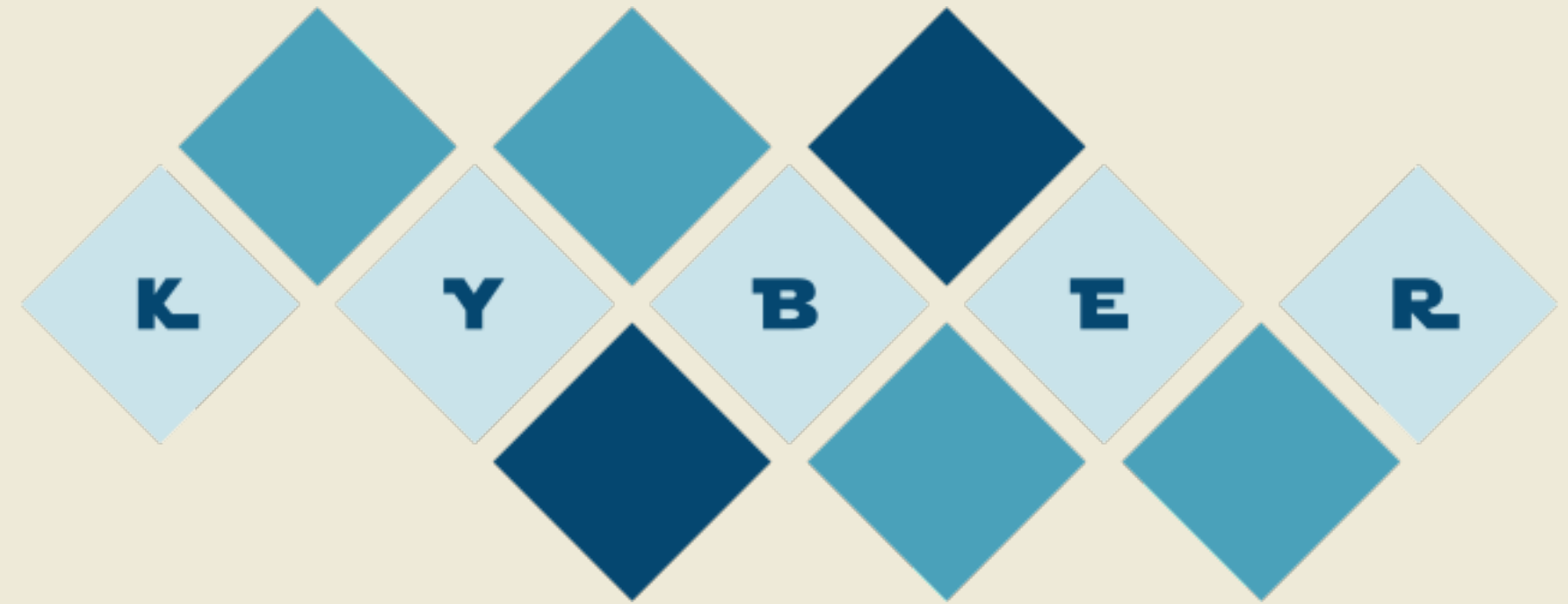
Construction time



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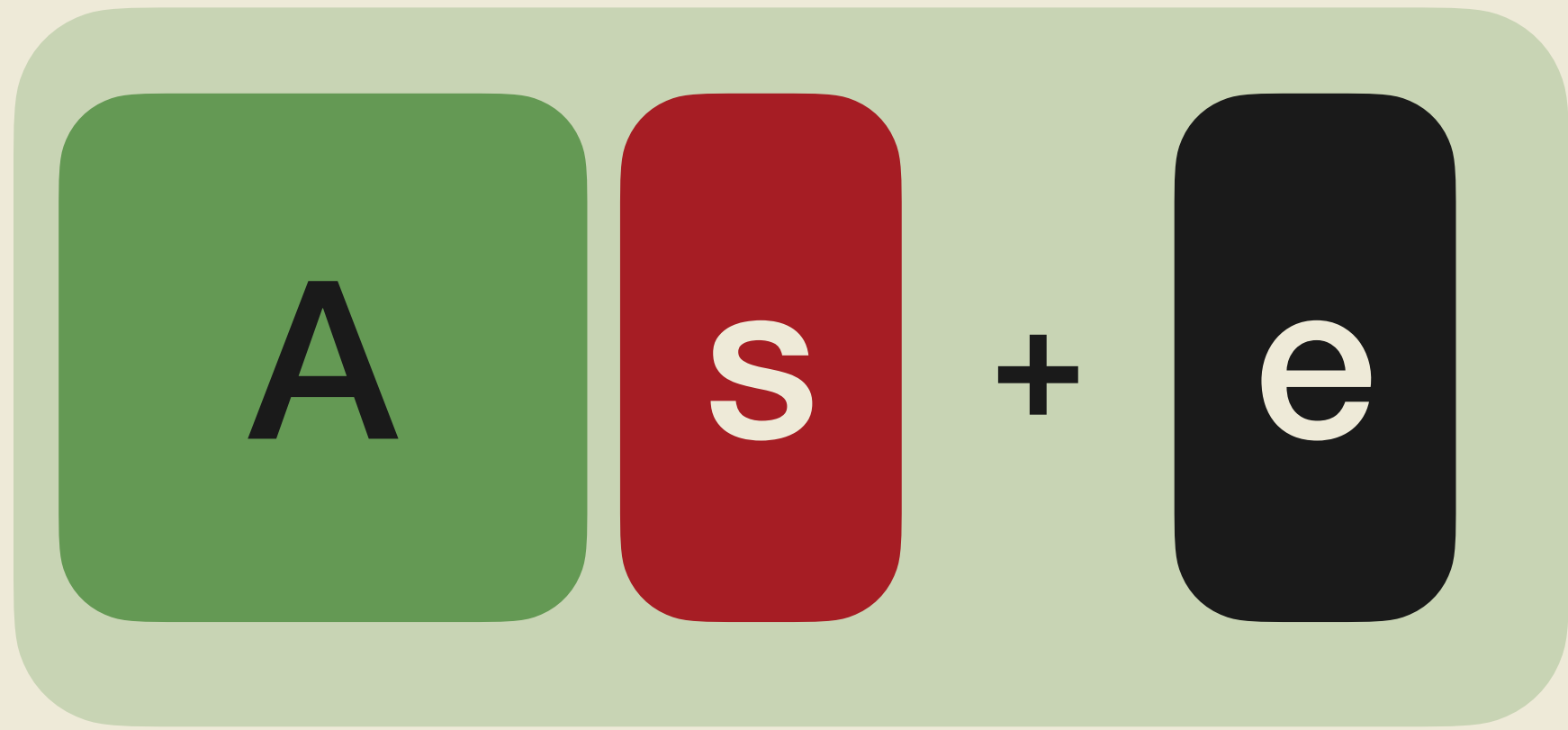
sk



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UPKE

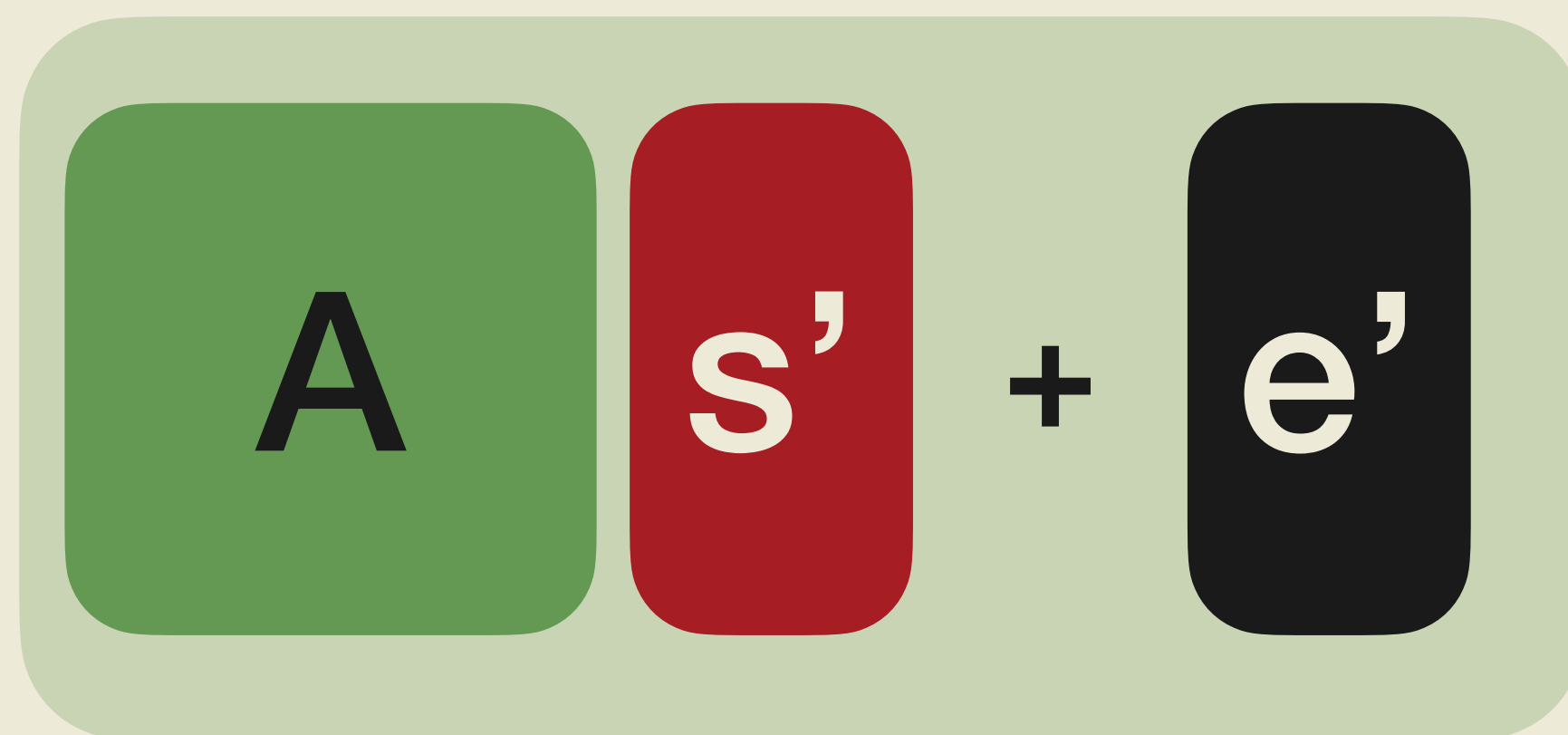
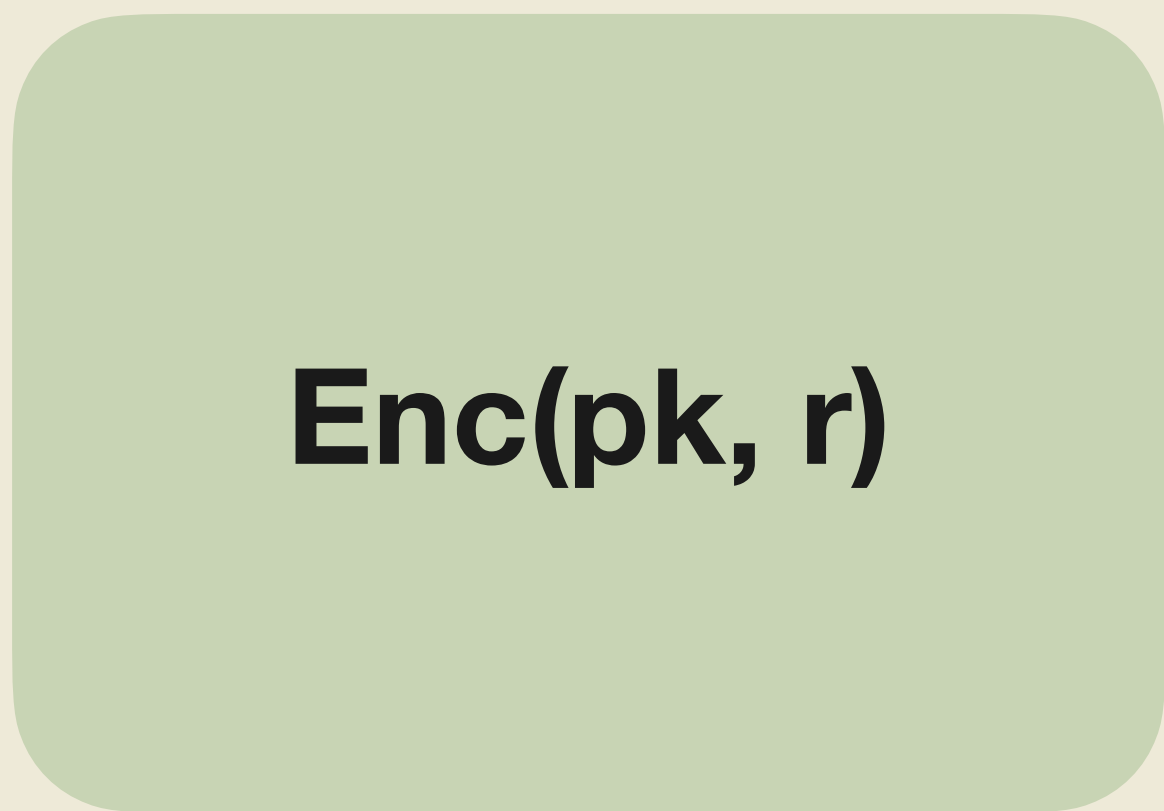
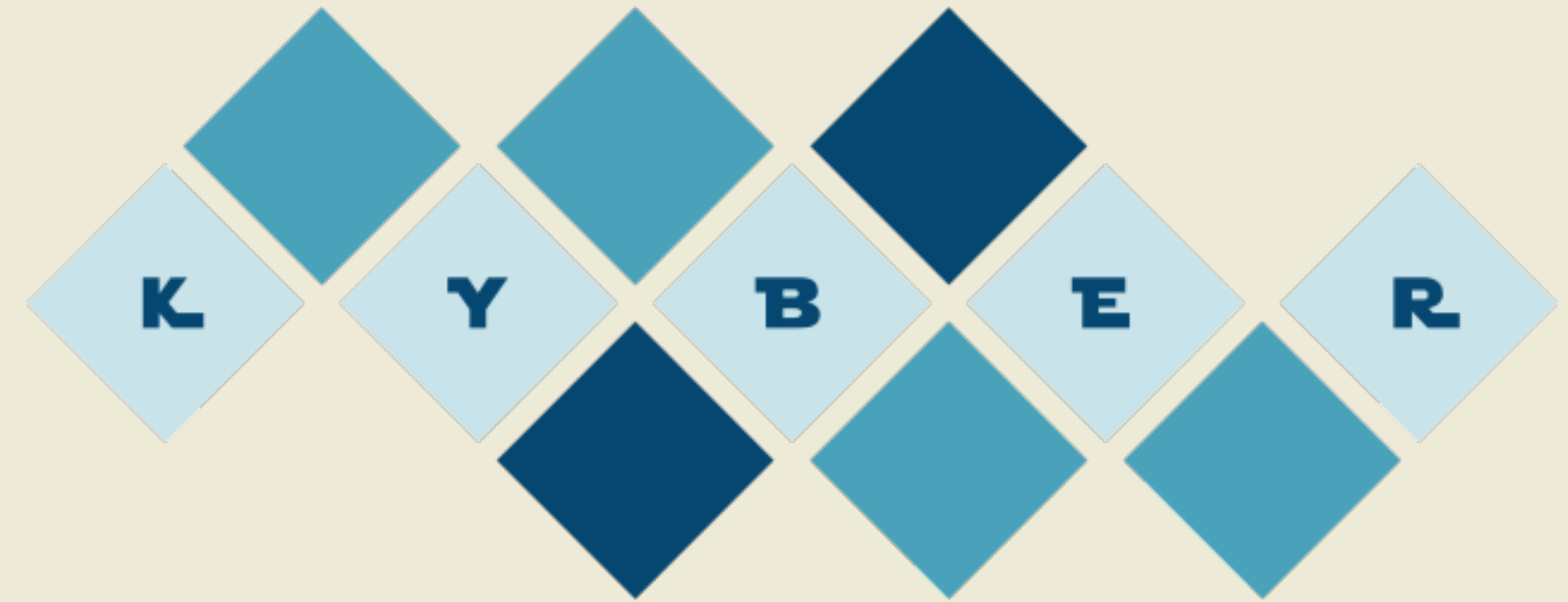
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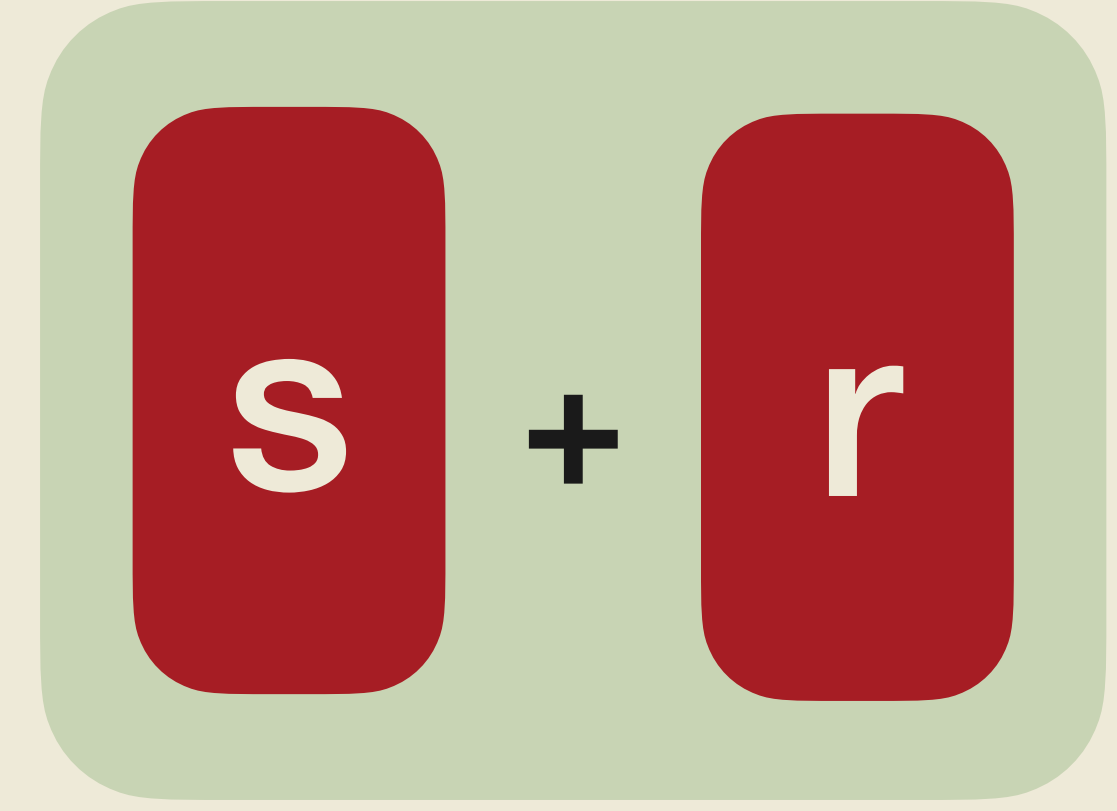
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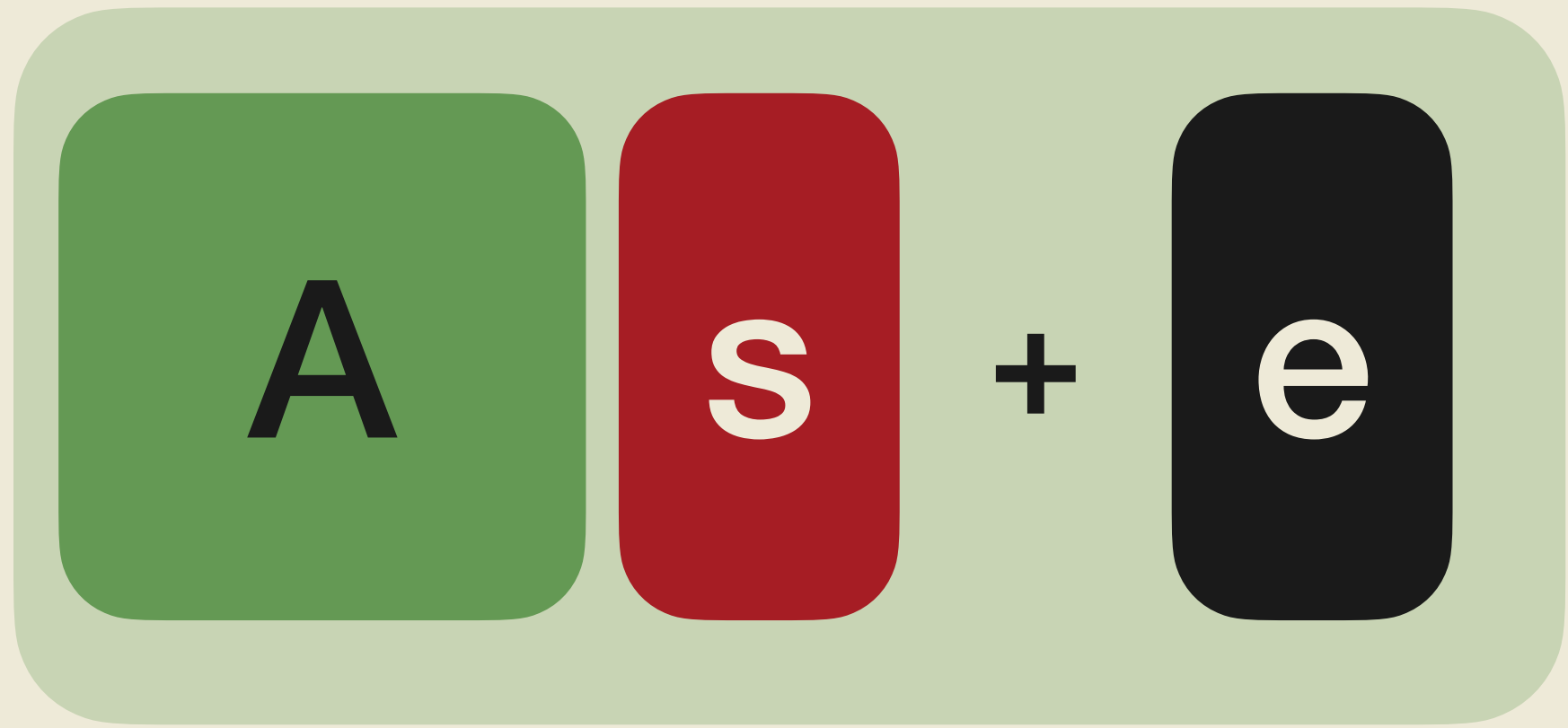
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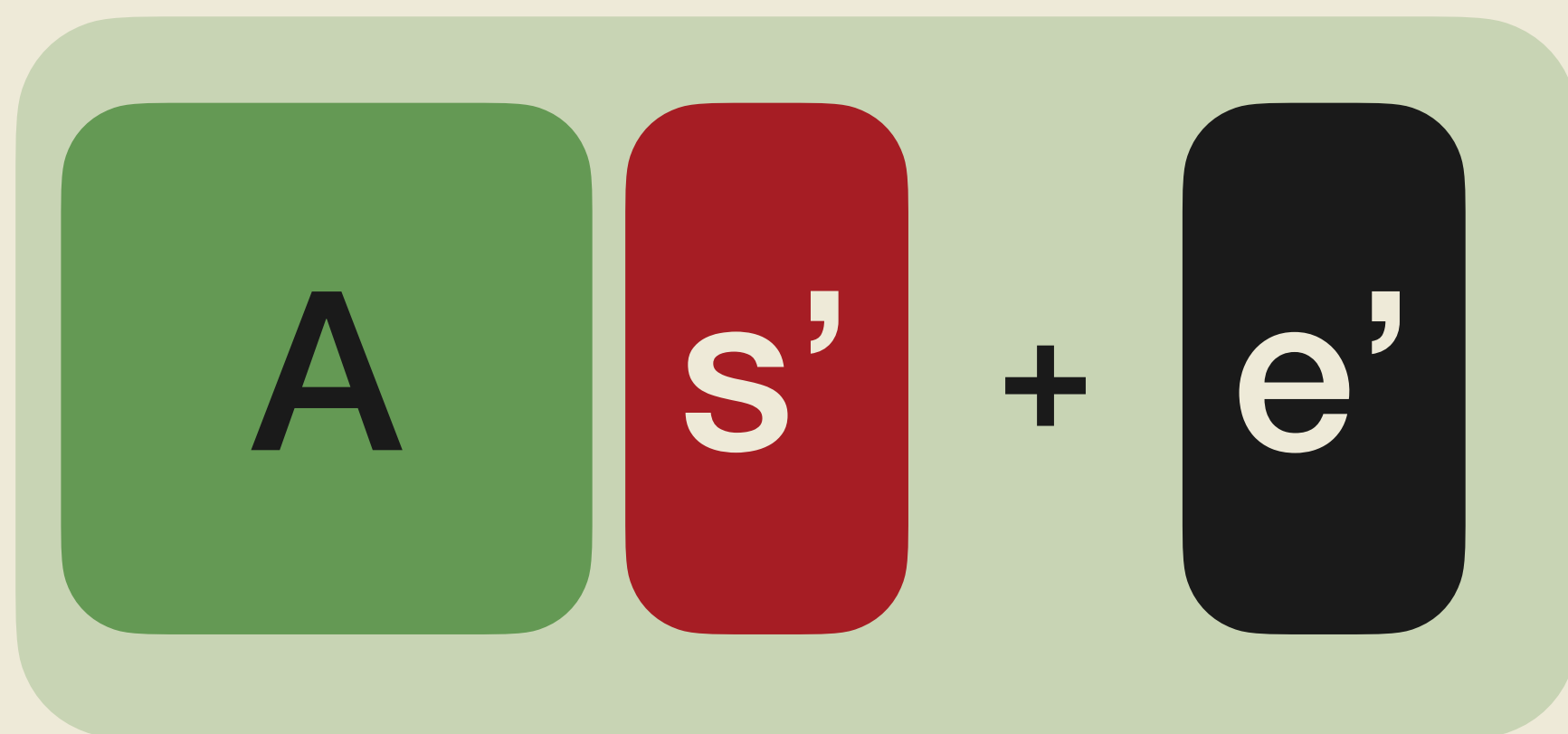
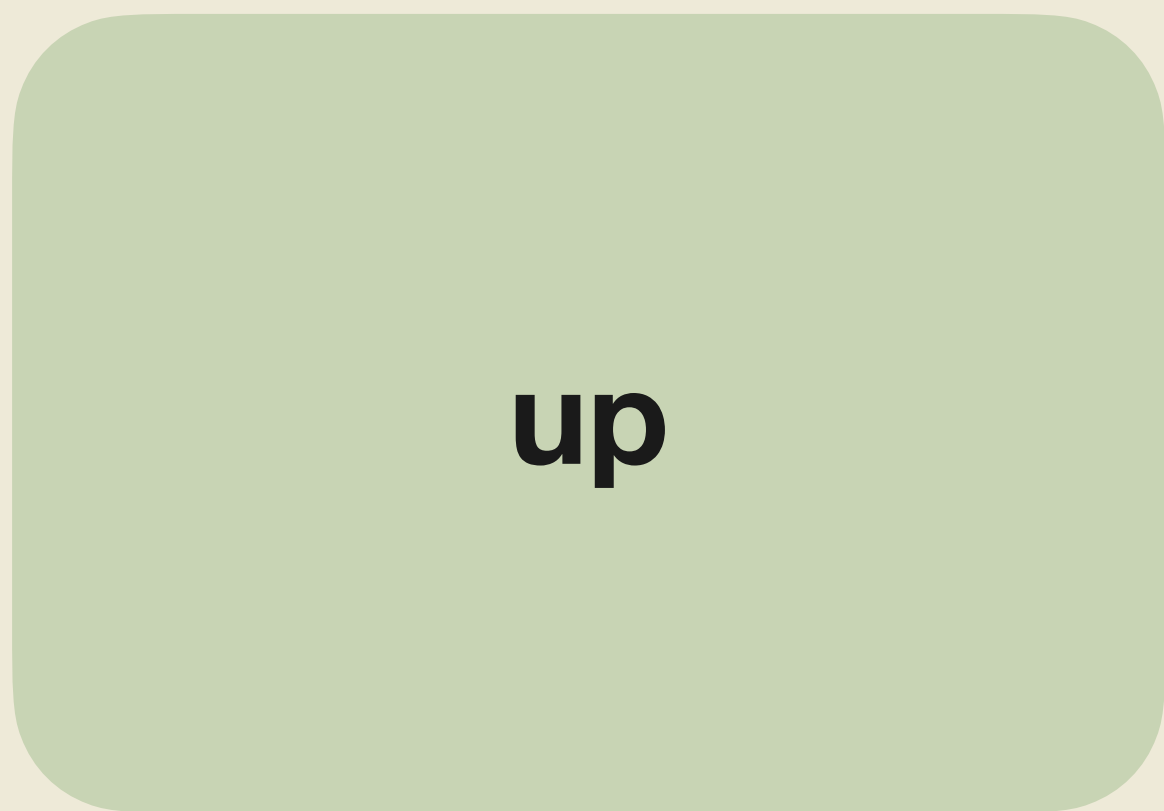
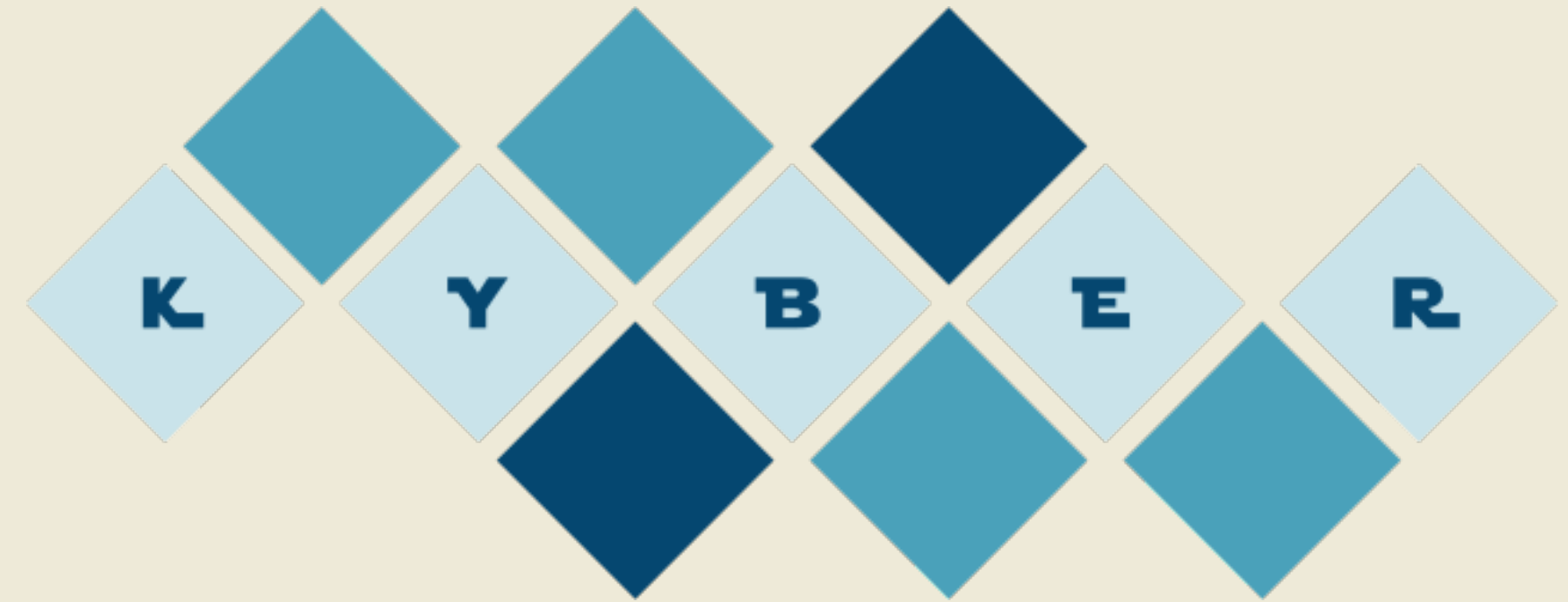
Construction time



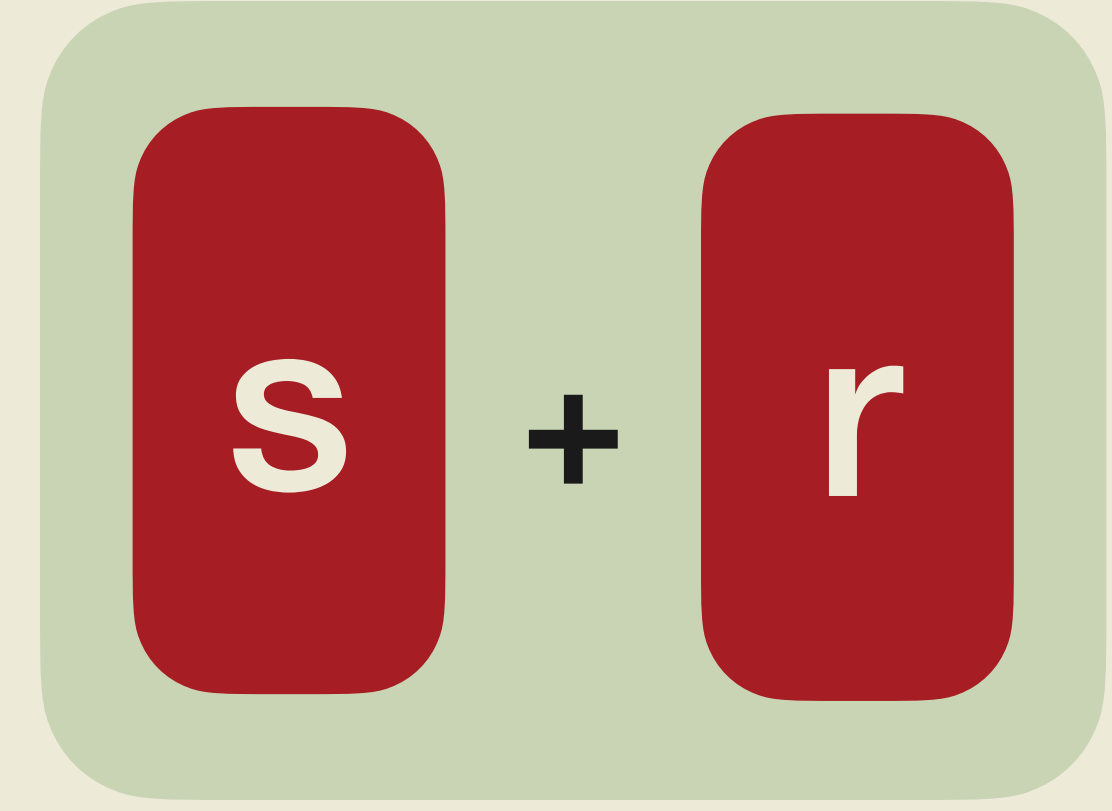
pk



sk



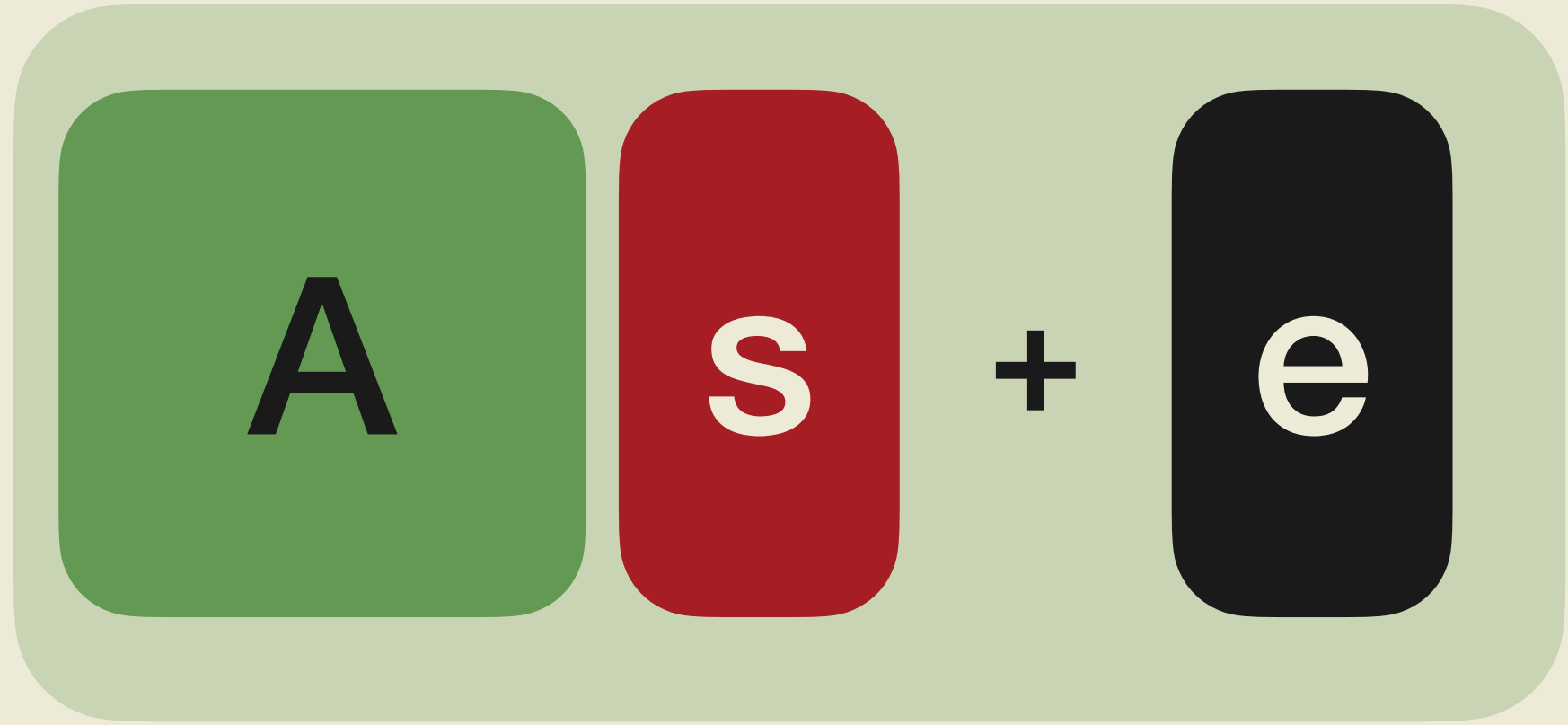
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UPKE

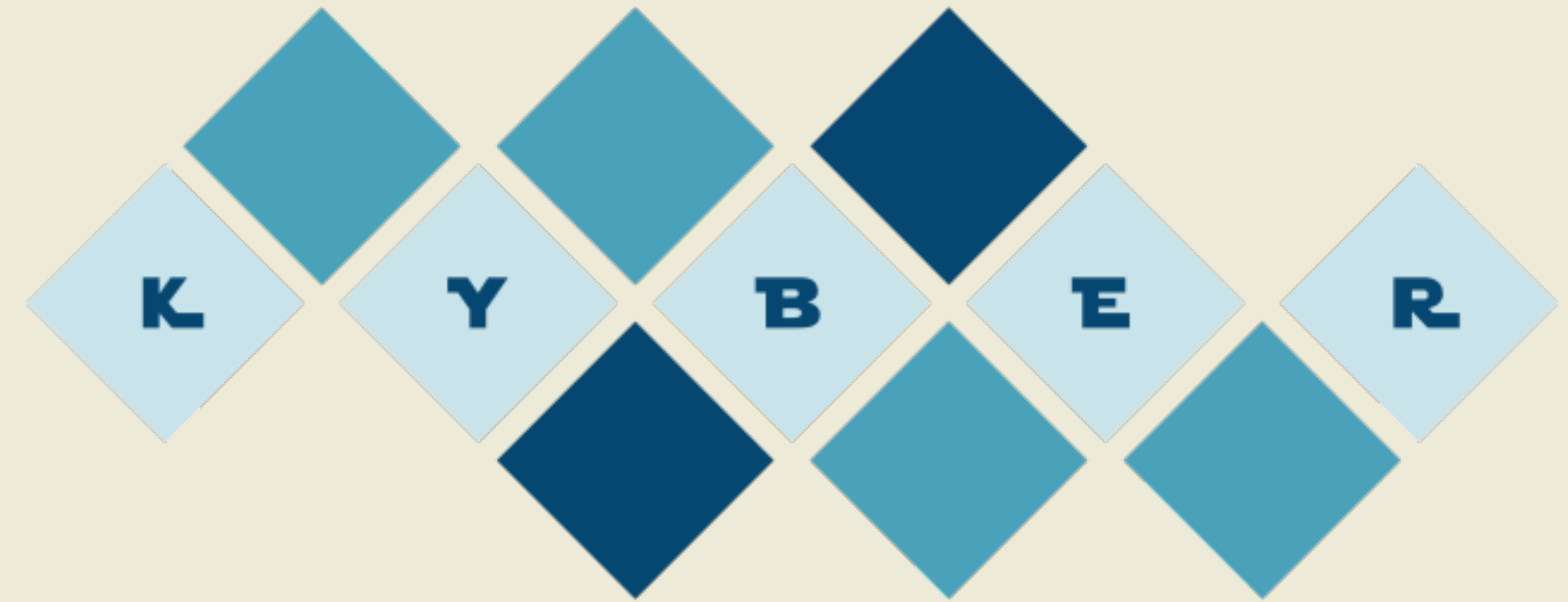
Construction time



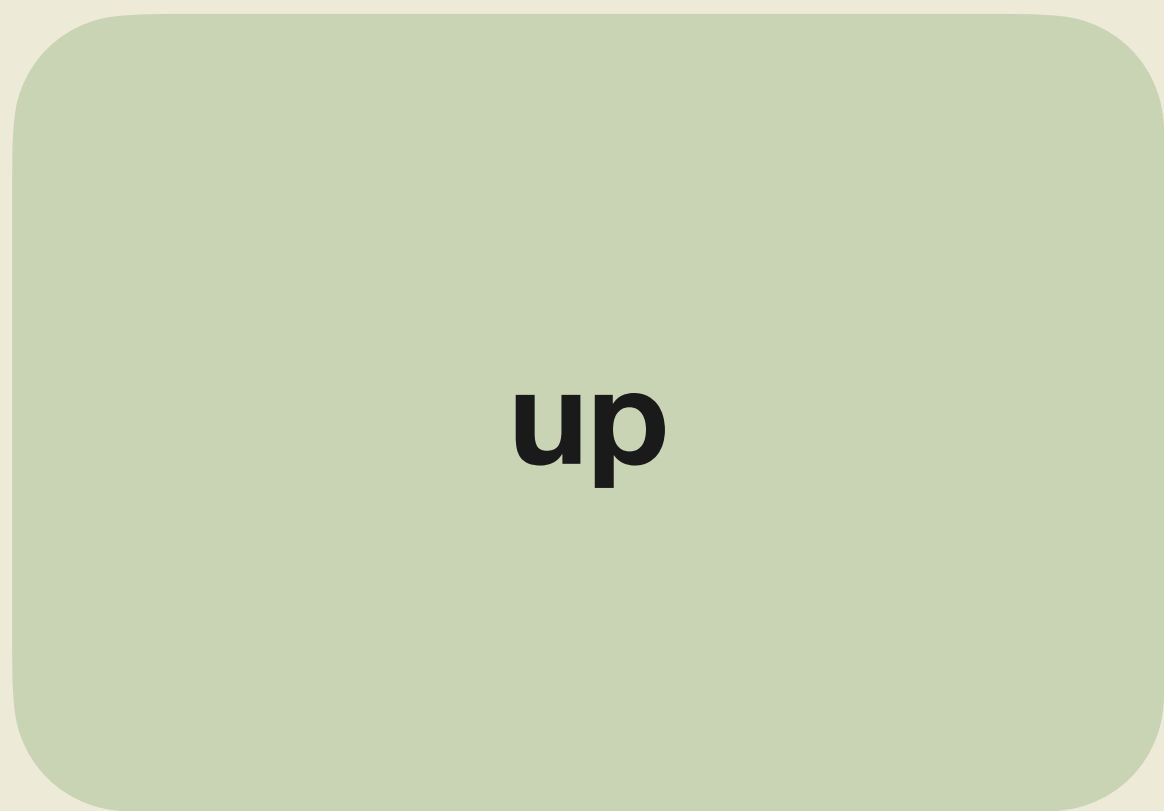
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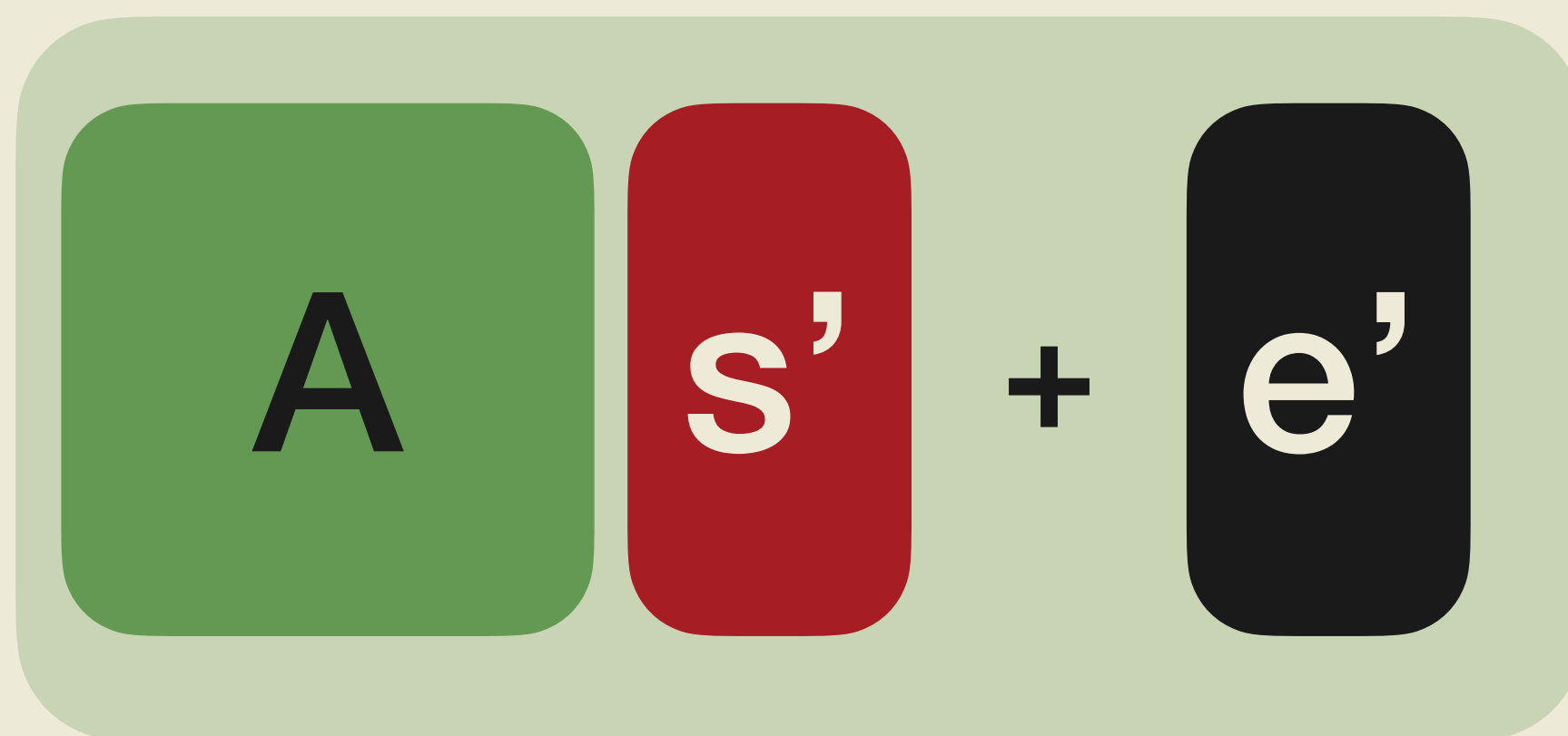
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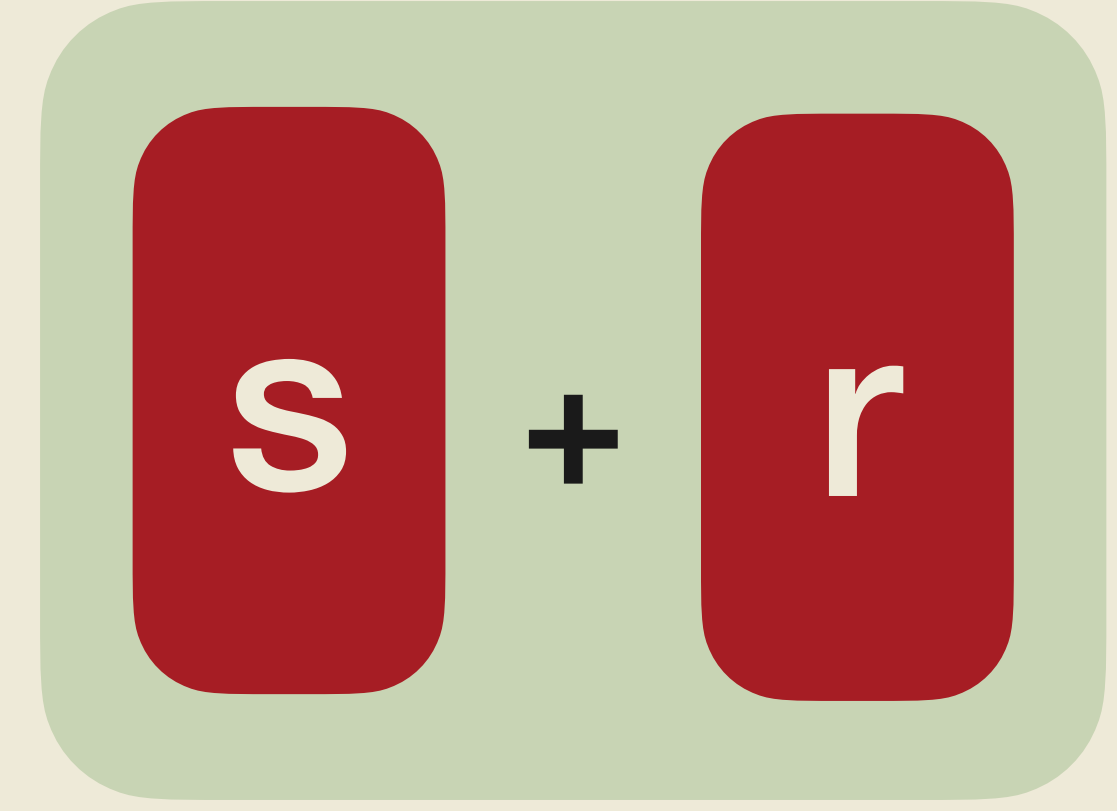
Done !



up



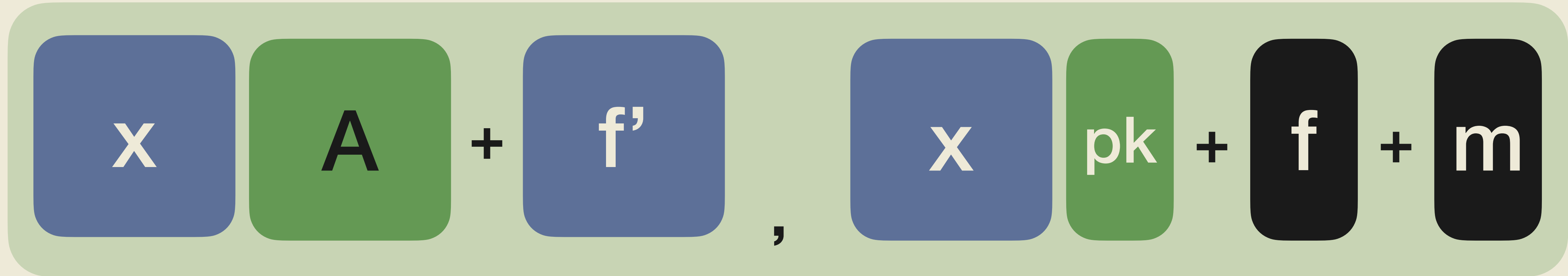
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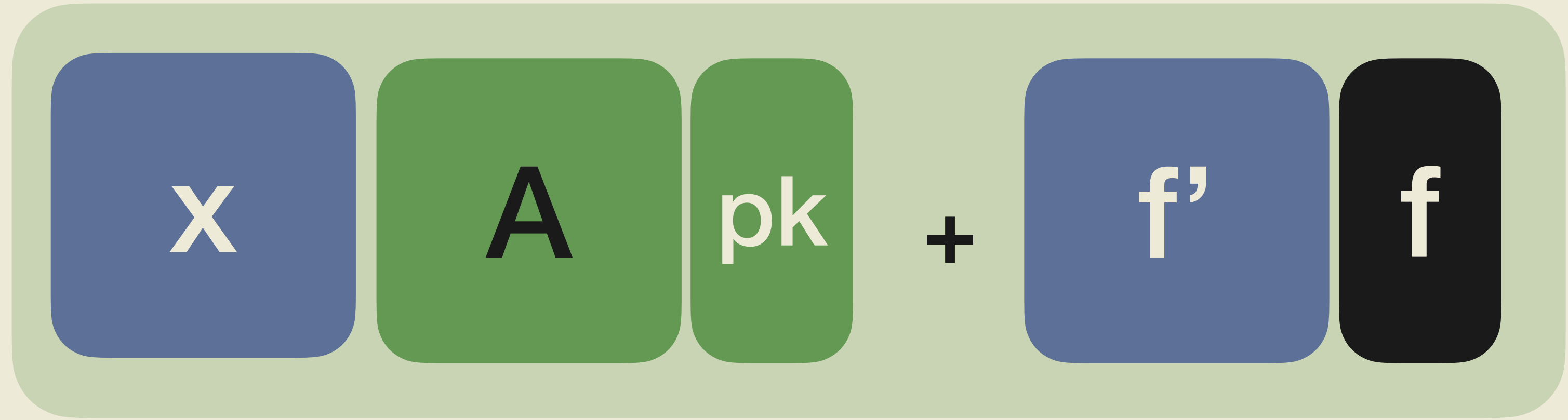
UPKE

Construction time



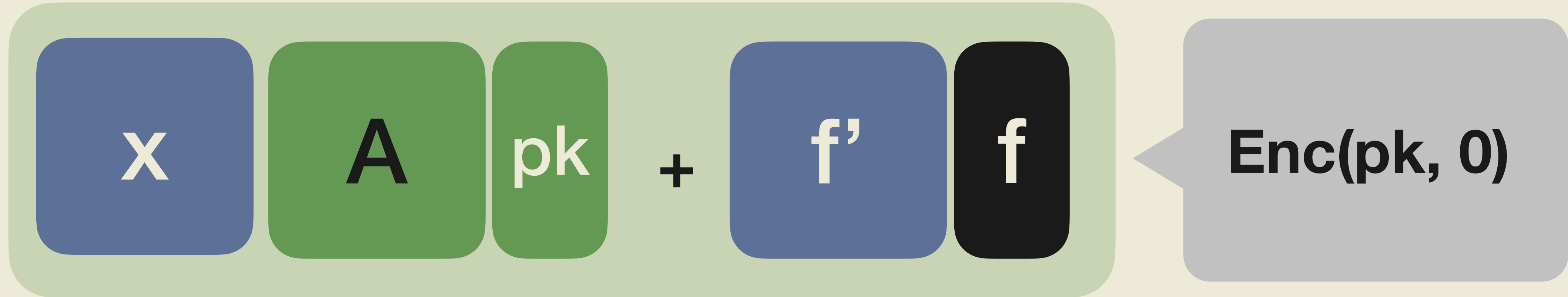
UPKE

Construction time



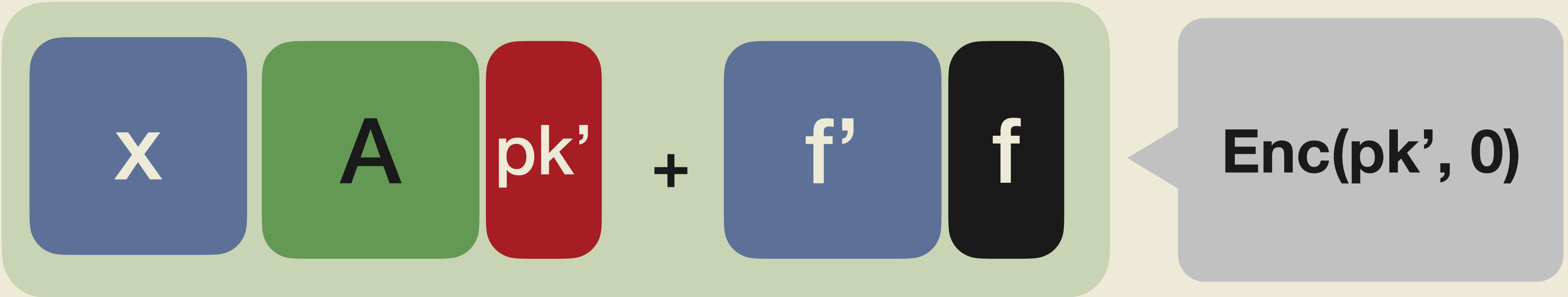
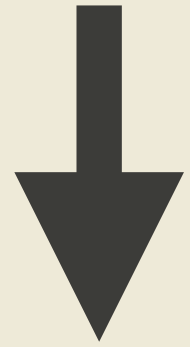
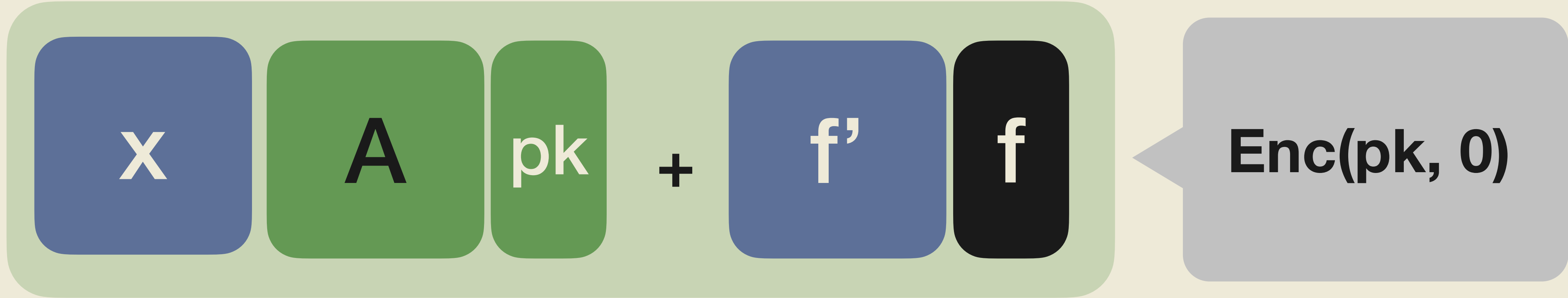
UPKE

Construction time



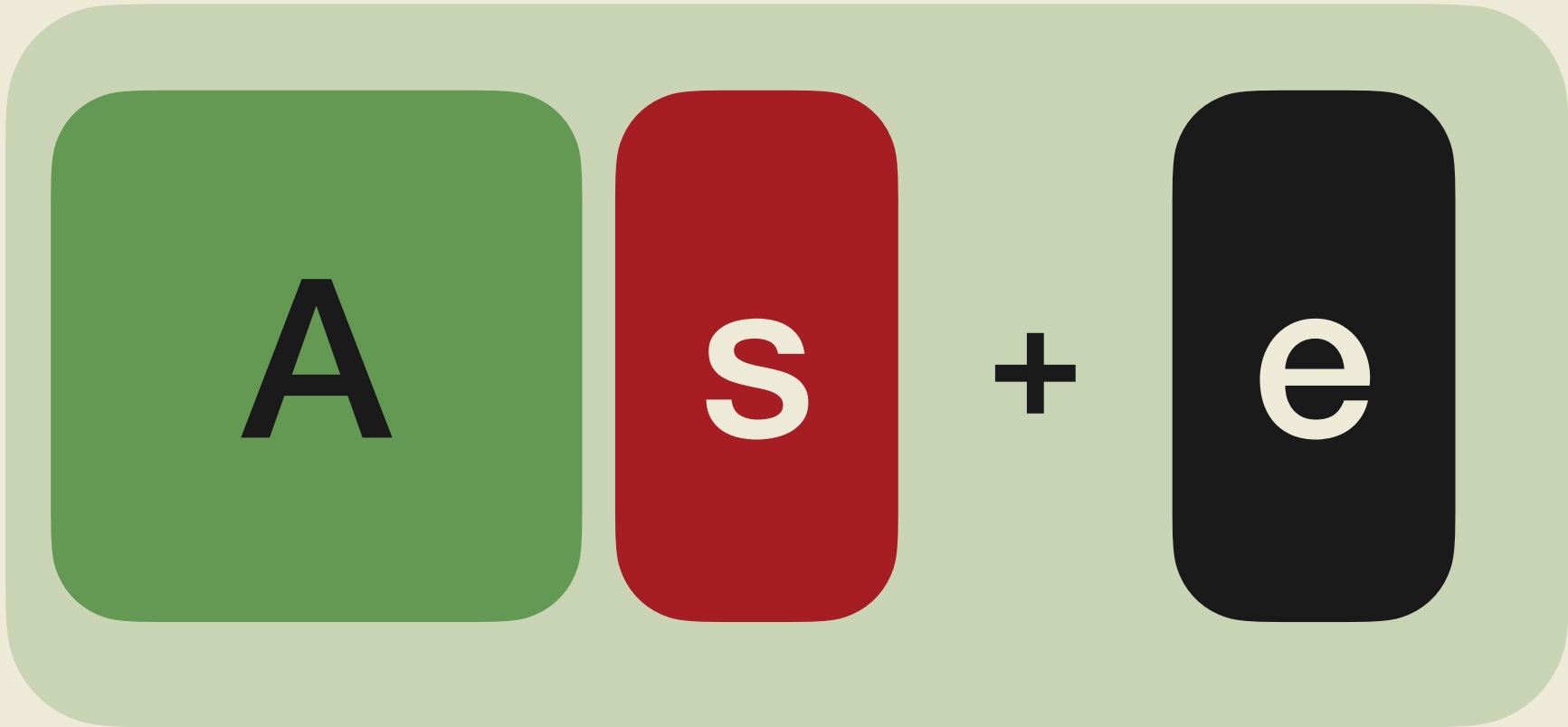
UPKE

Construction time



UPKE

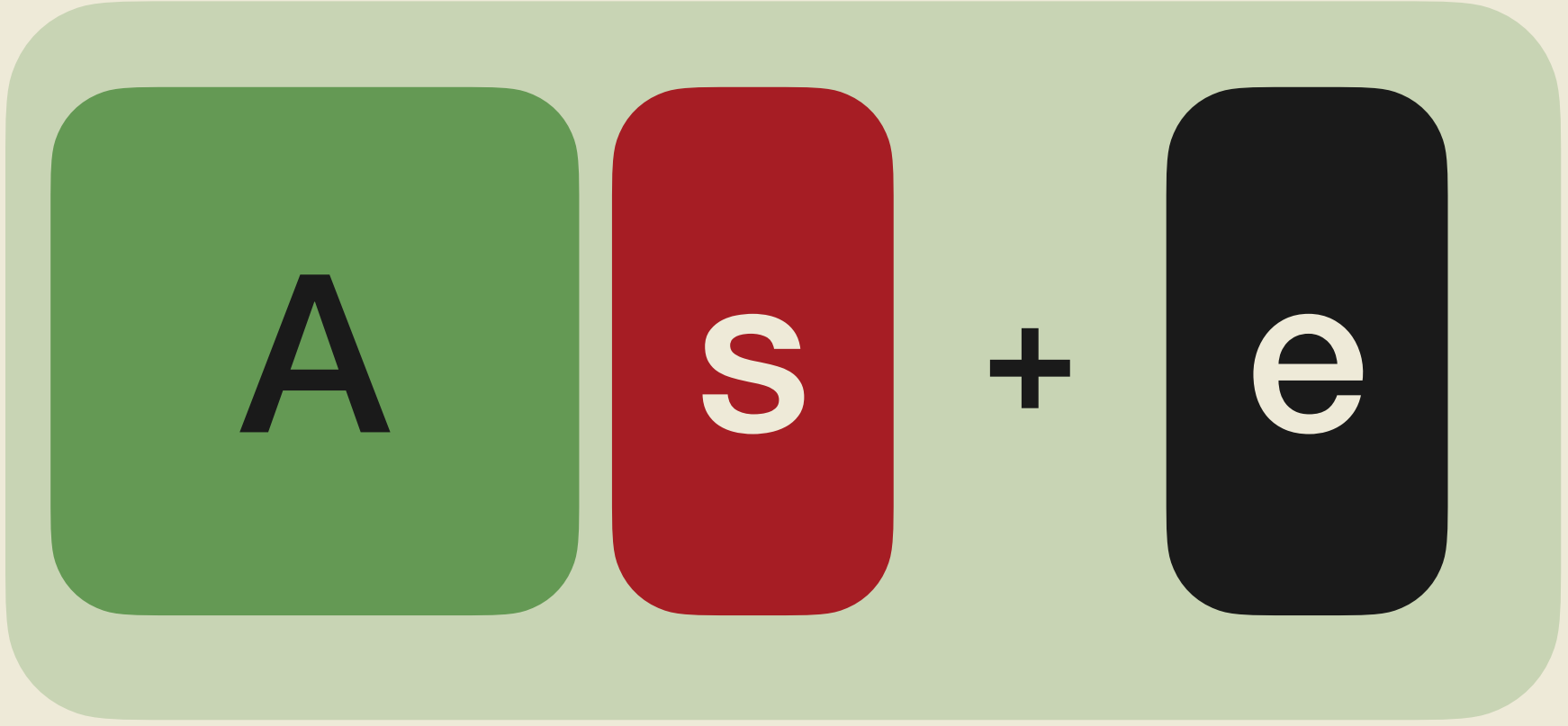
Assumption time



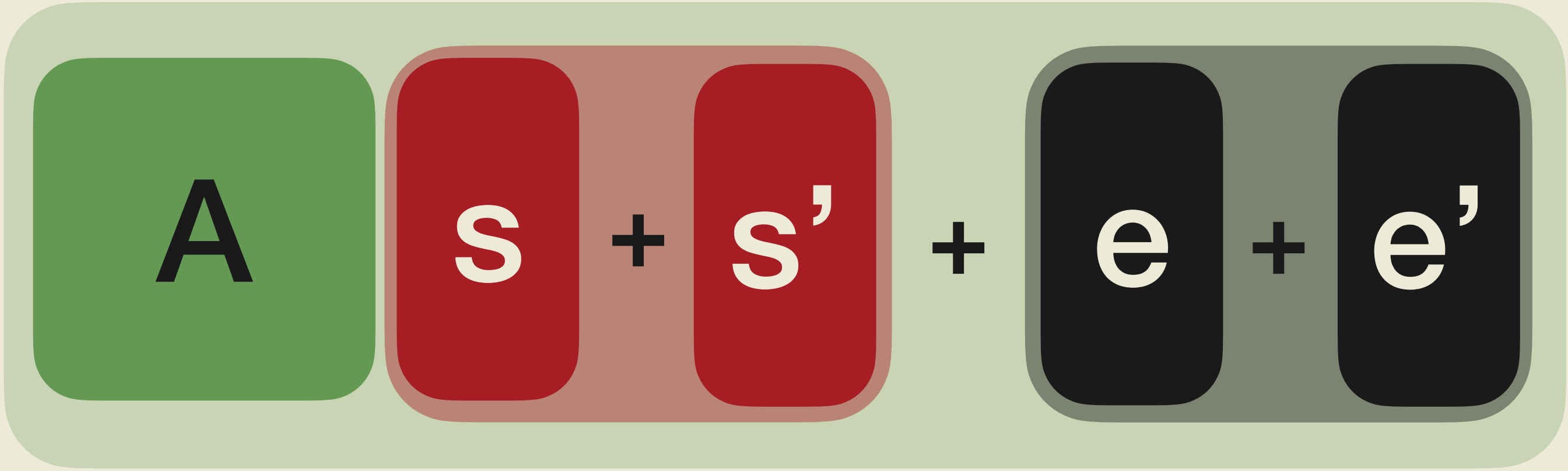
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UPKE

Assumption time



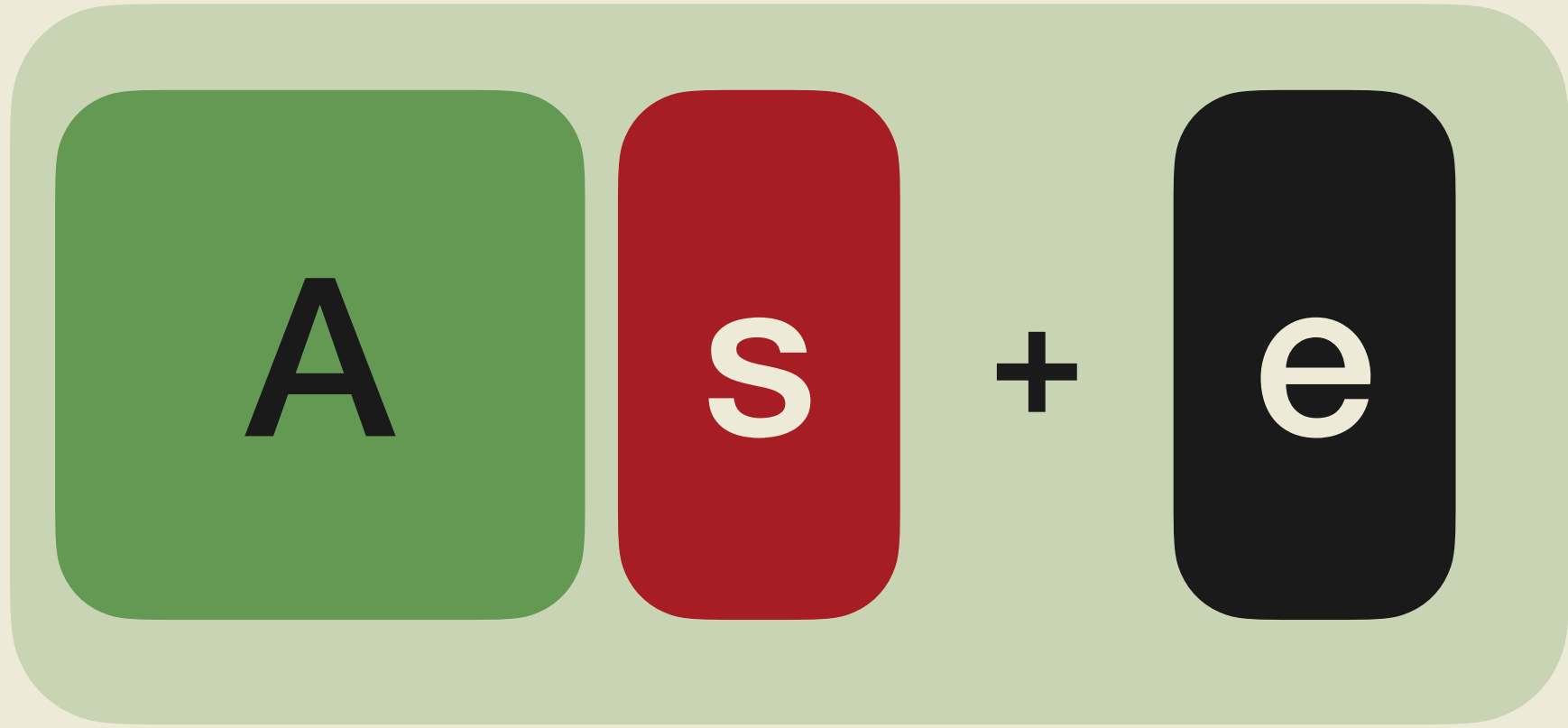
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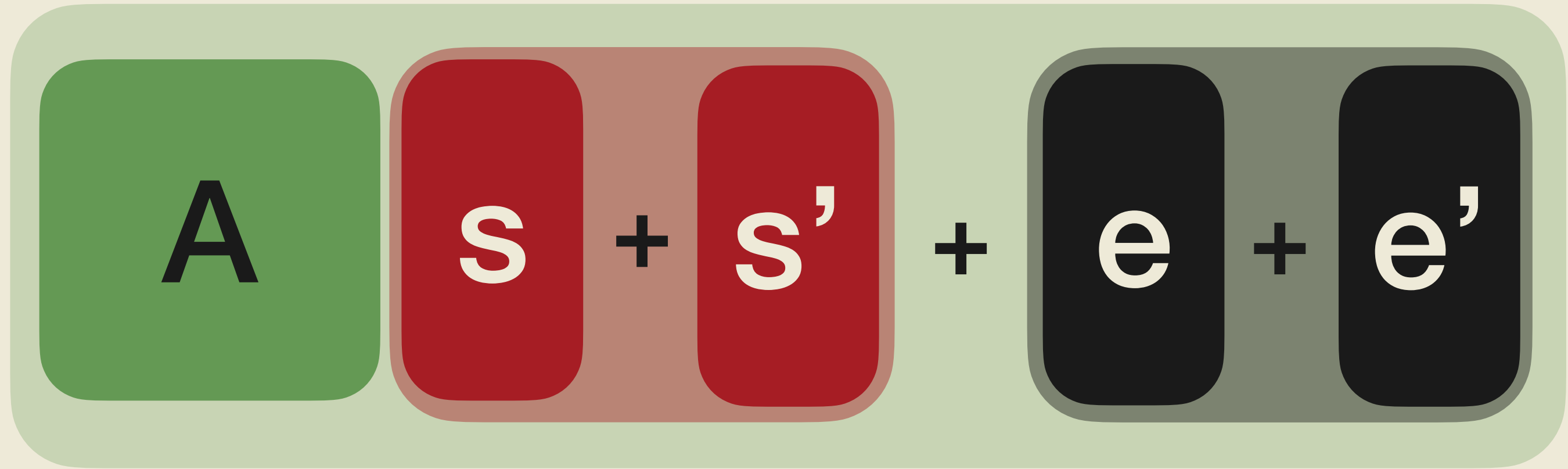
pk'

UPKE

Assumption time



pk

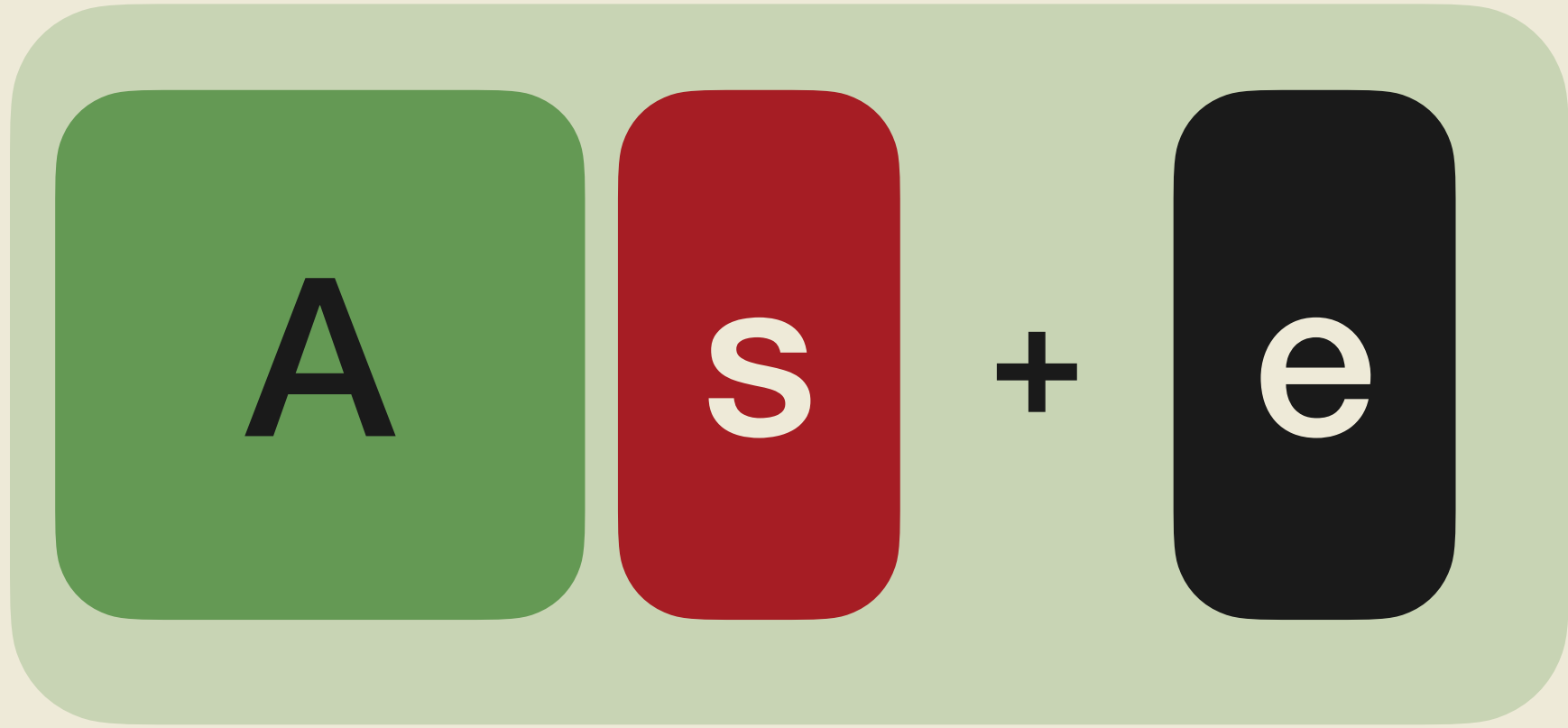


pk'

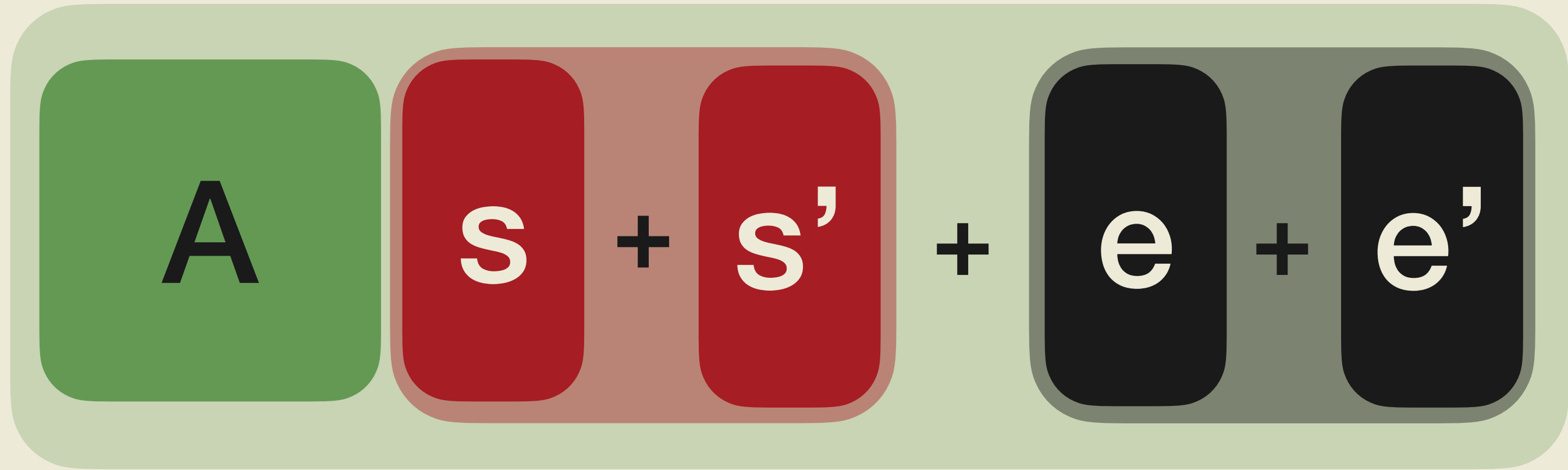
LWE Sample

UPKE

Assumption time



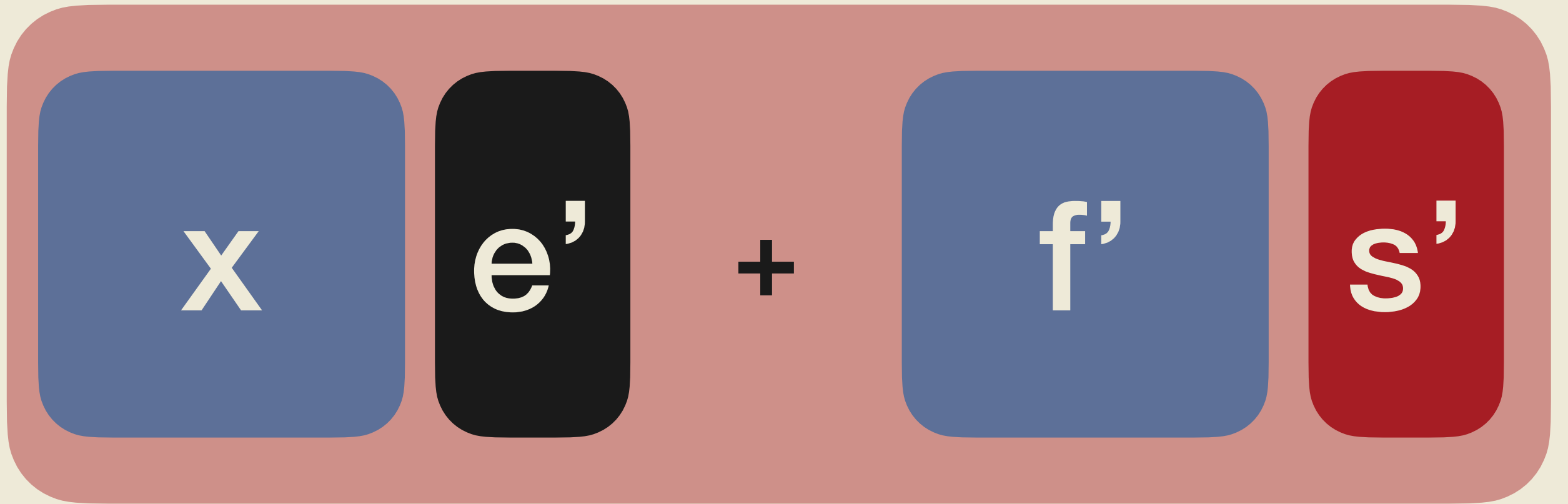
pk



Cross terms

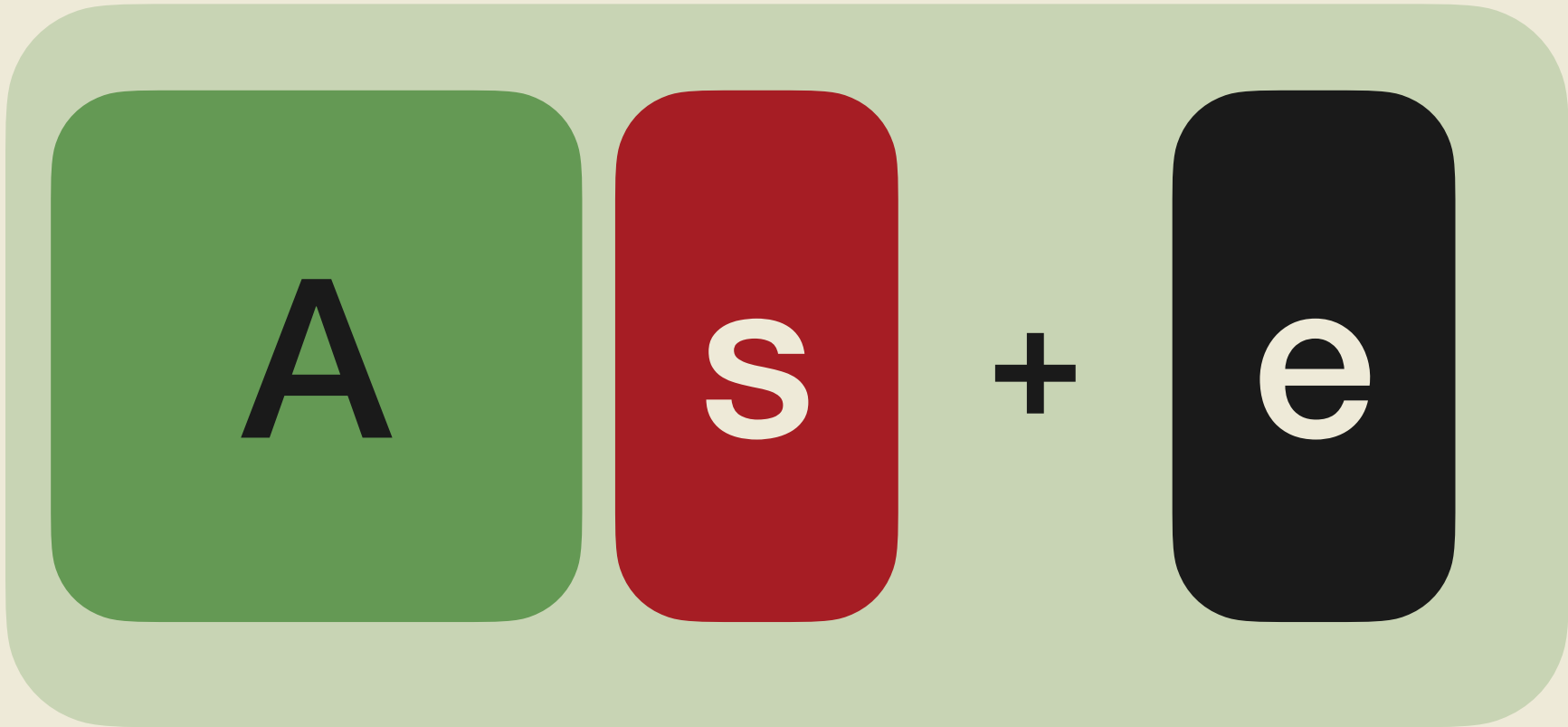


+



UPKE

Assumption time



pk

LWE Sample

+

Very small

**Cross terms - with flooding
Dodis et al. TCC 21**

UPKE

Assumption time



LWE + Hint variant

UPKE

Assumption time



LWE + Hint variant

$$\text{Hint} = z \cdot e \quad \text{Extended LWE}$$

UPKE

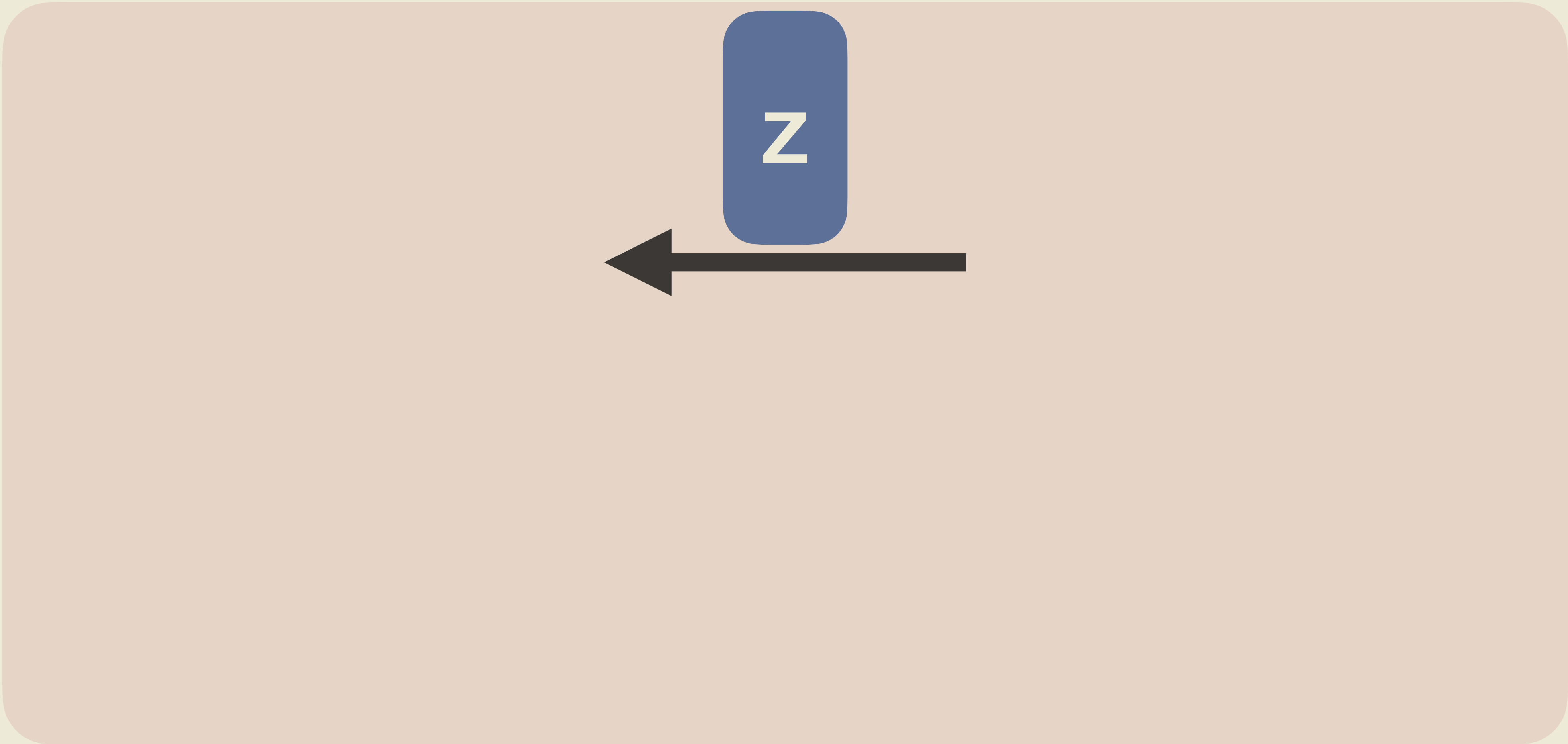
Assumption time



Extended LWE

UPKE

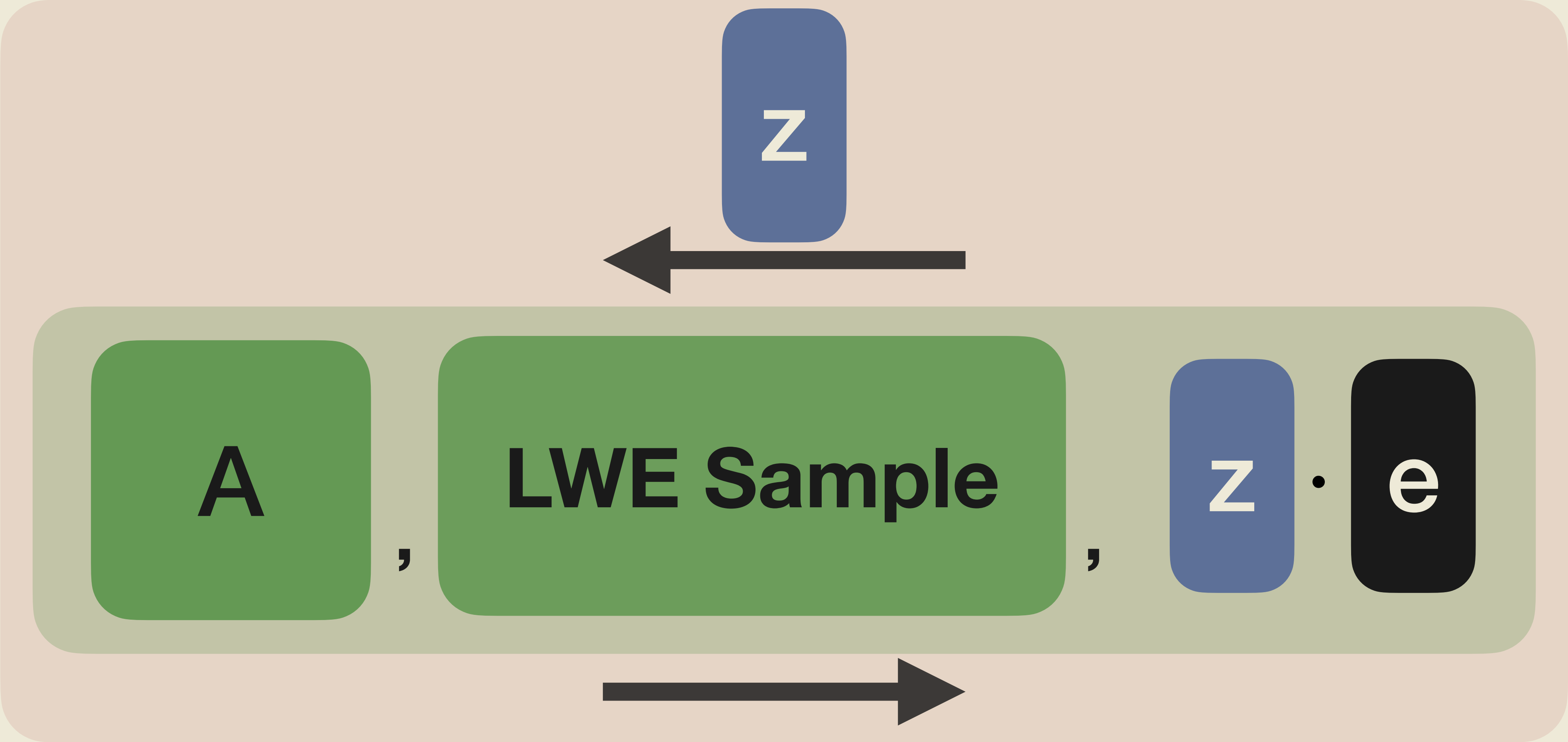
Assumption time



Extended LWE

UPKE

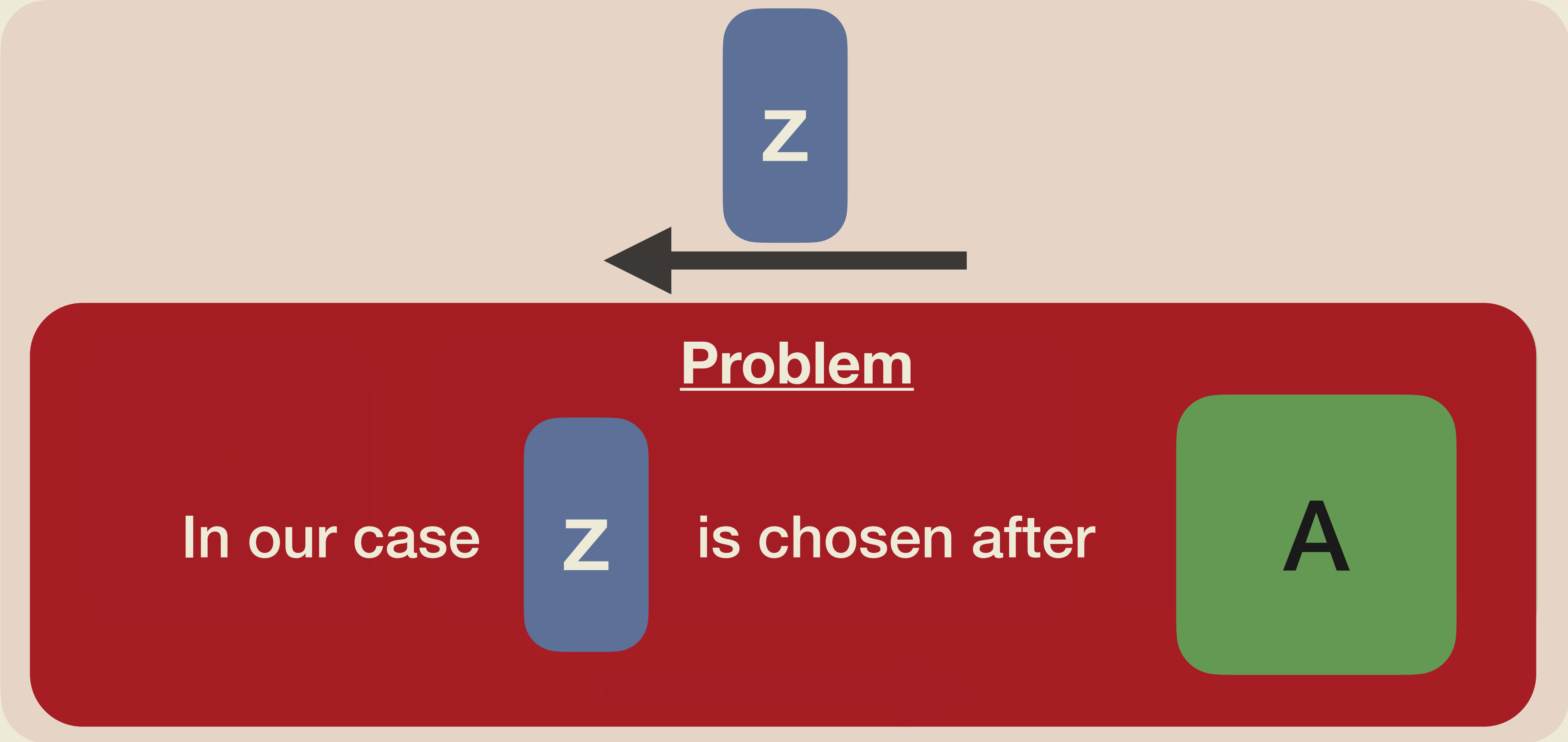
Assumption time



Extended LWE

UPKE

Assumption time



Extended LWE

UPKE

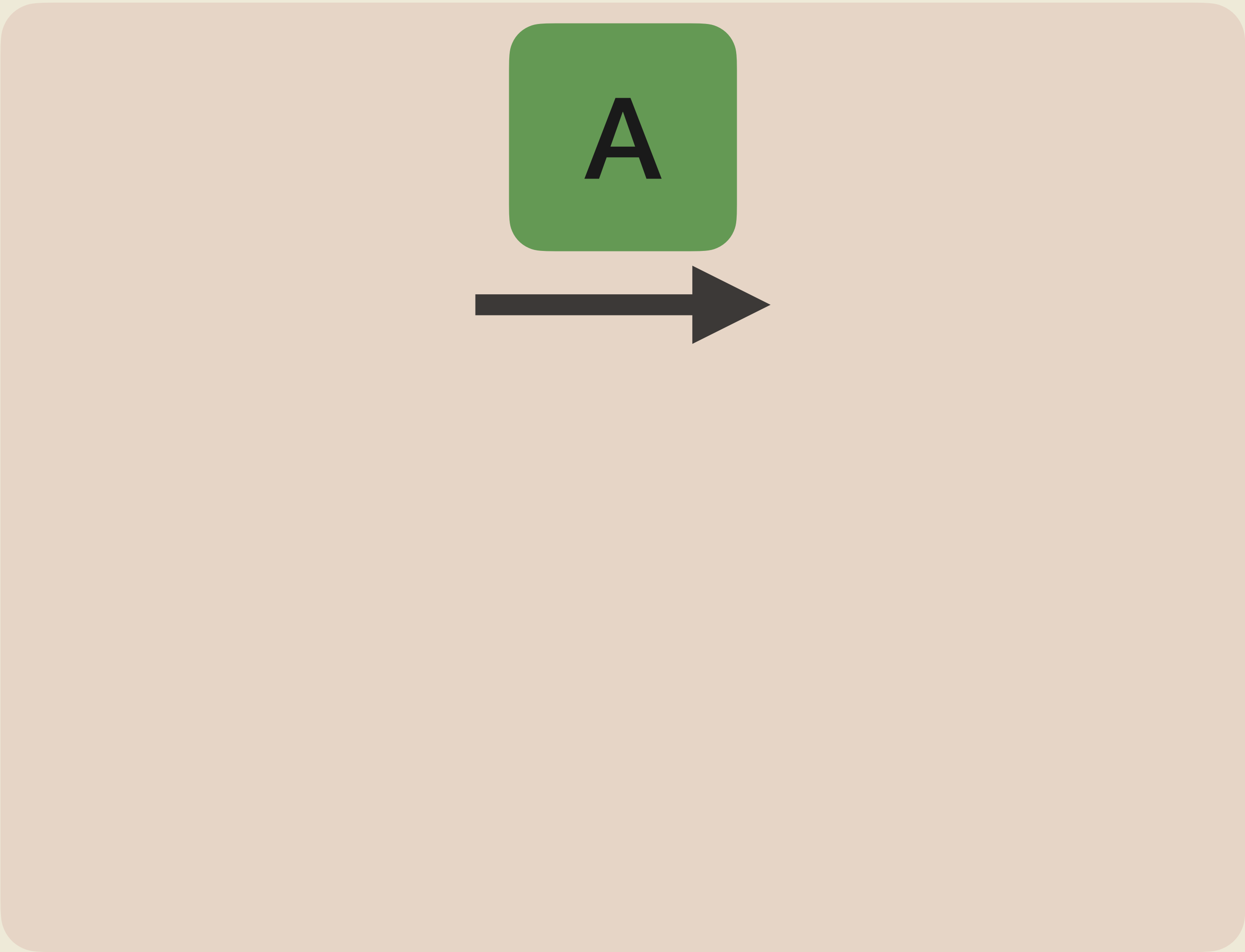
Assumption time



What we need

UPKE

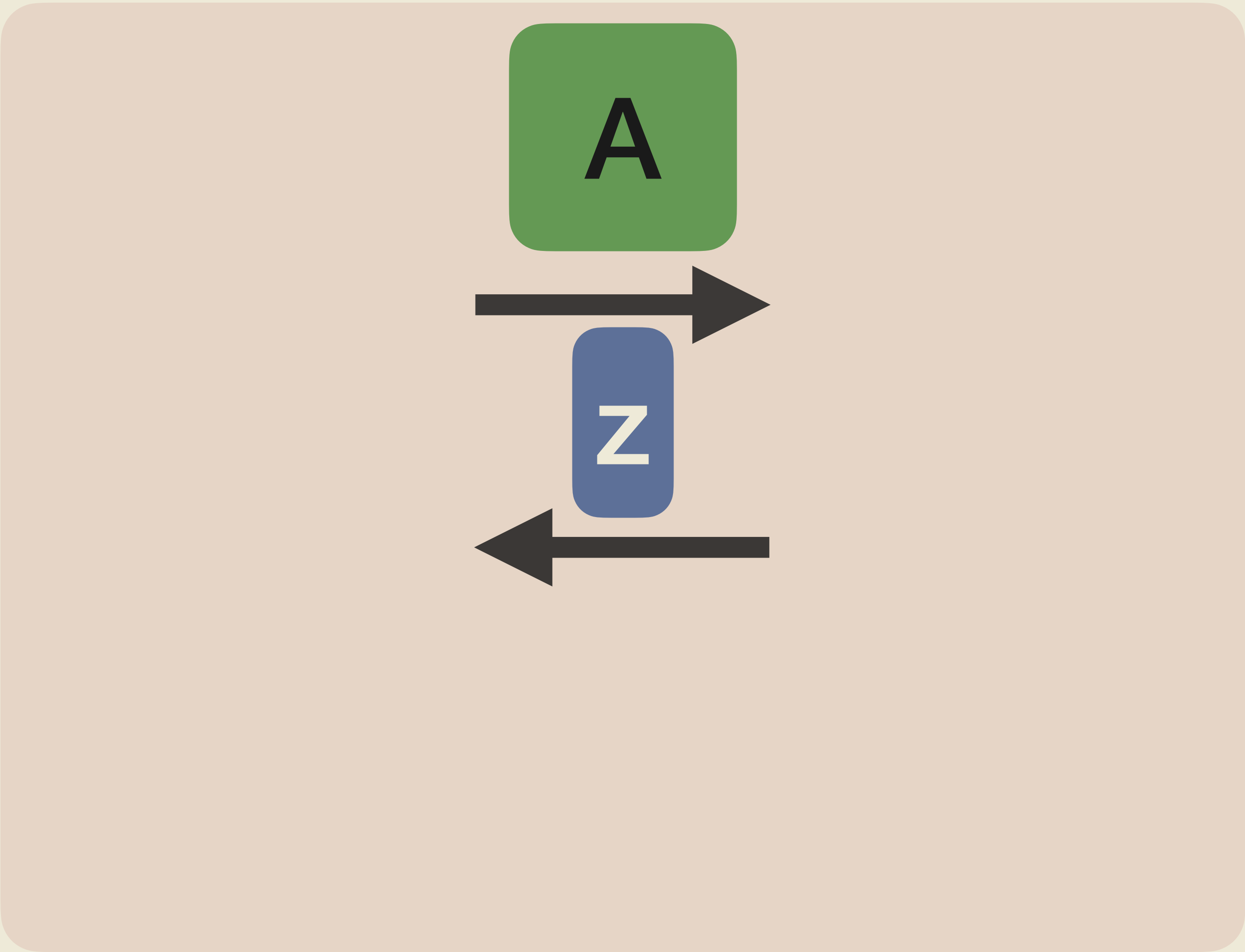
Assumption time



What we need

UPKE

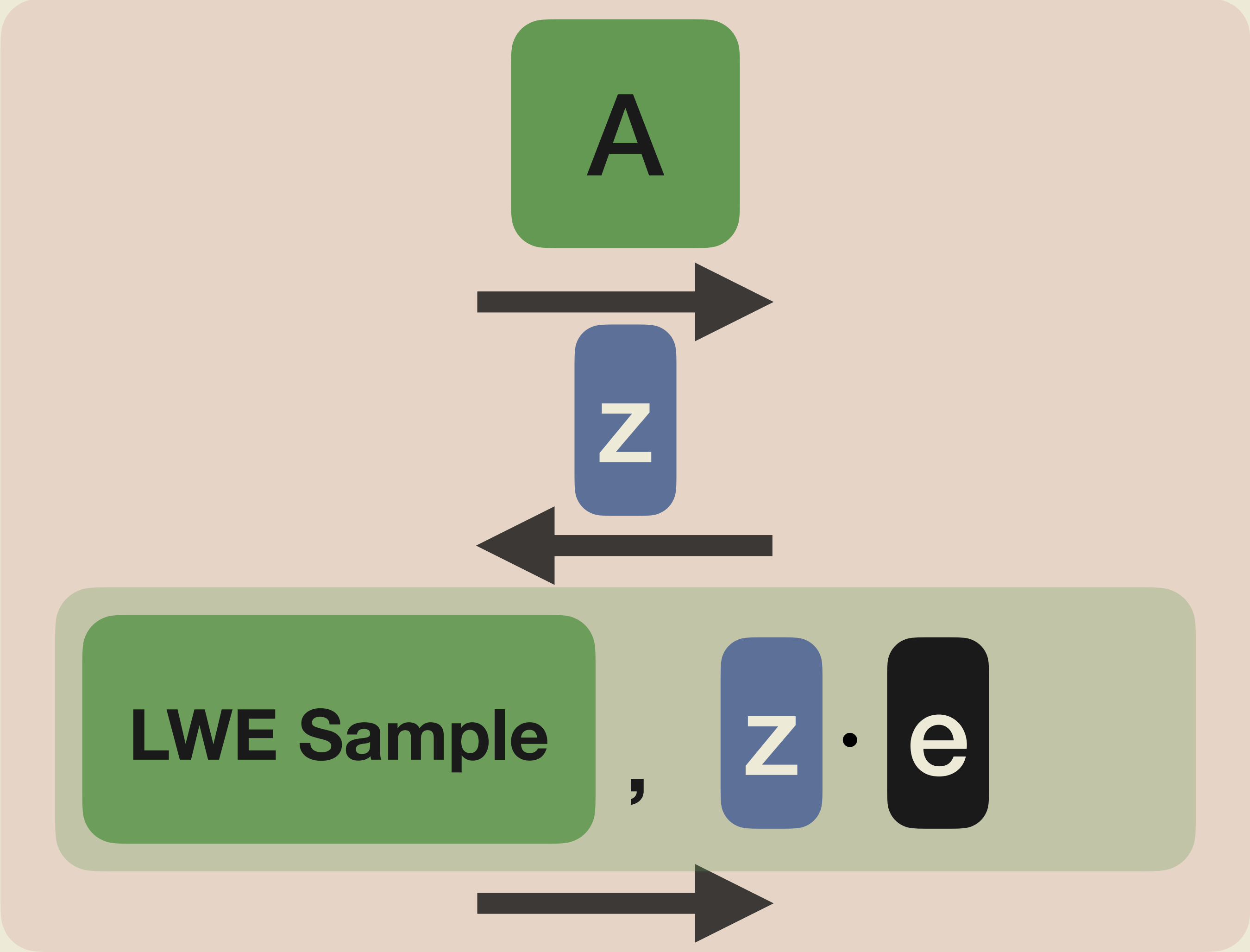
Assumption time



What we need

UPKE

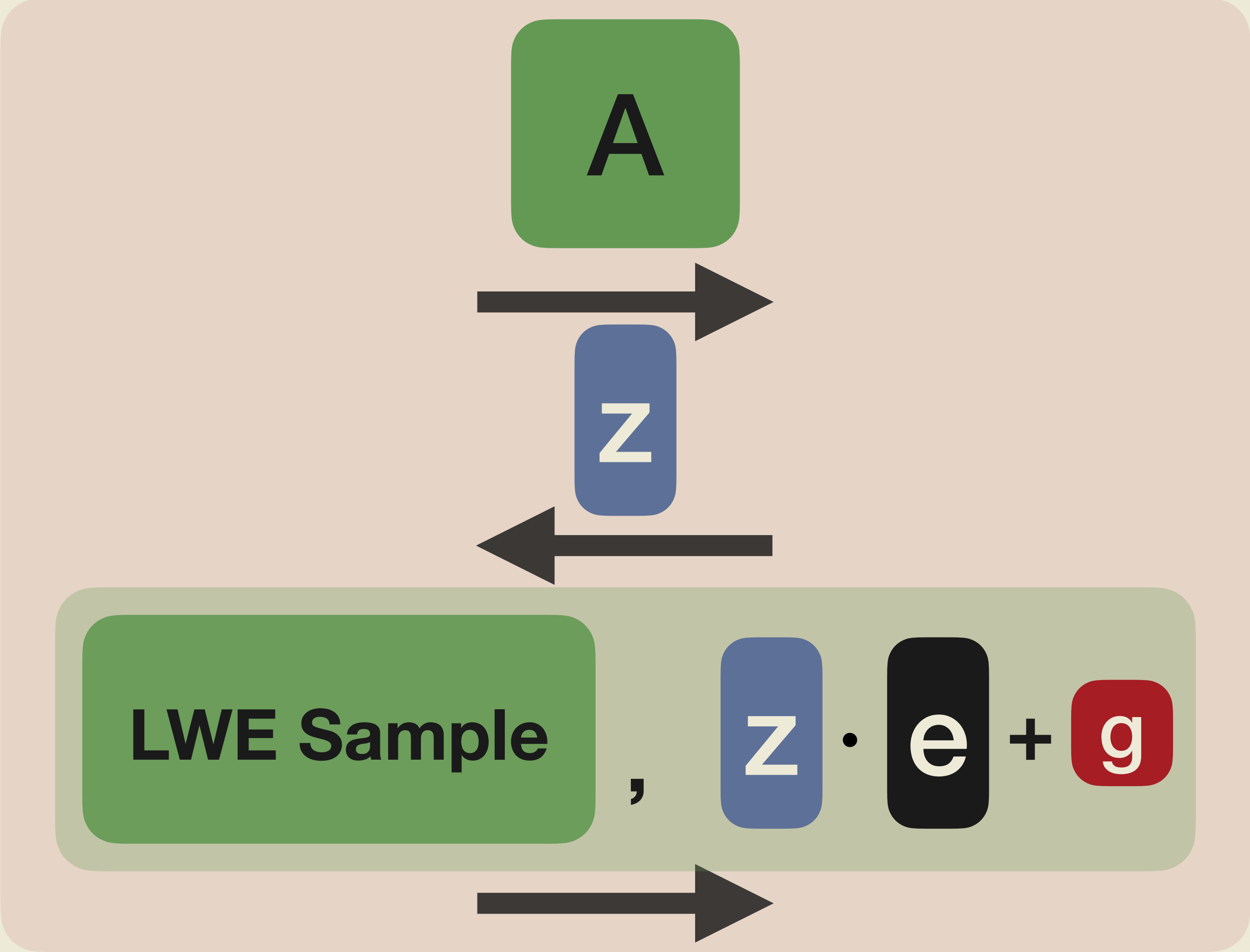
Assumption time



What we need

UPKE

Assumption time

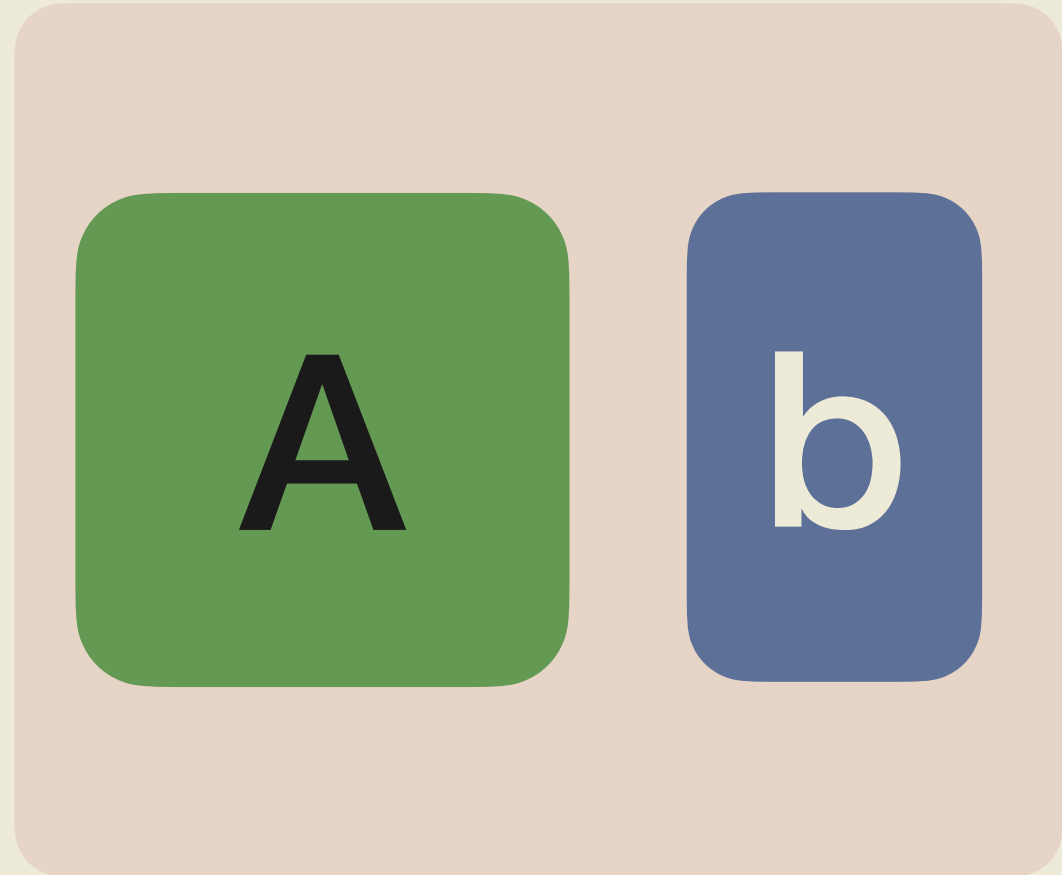


Adaptive extended LWE

Reduction

UPKE

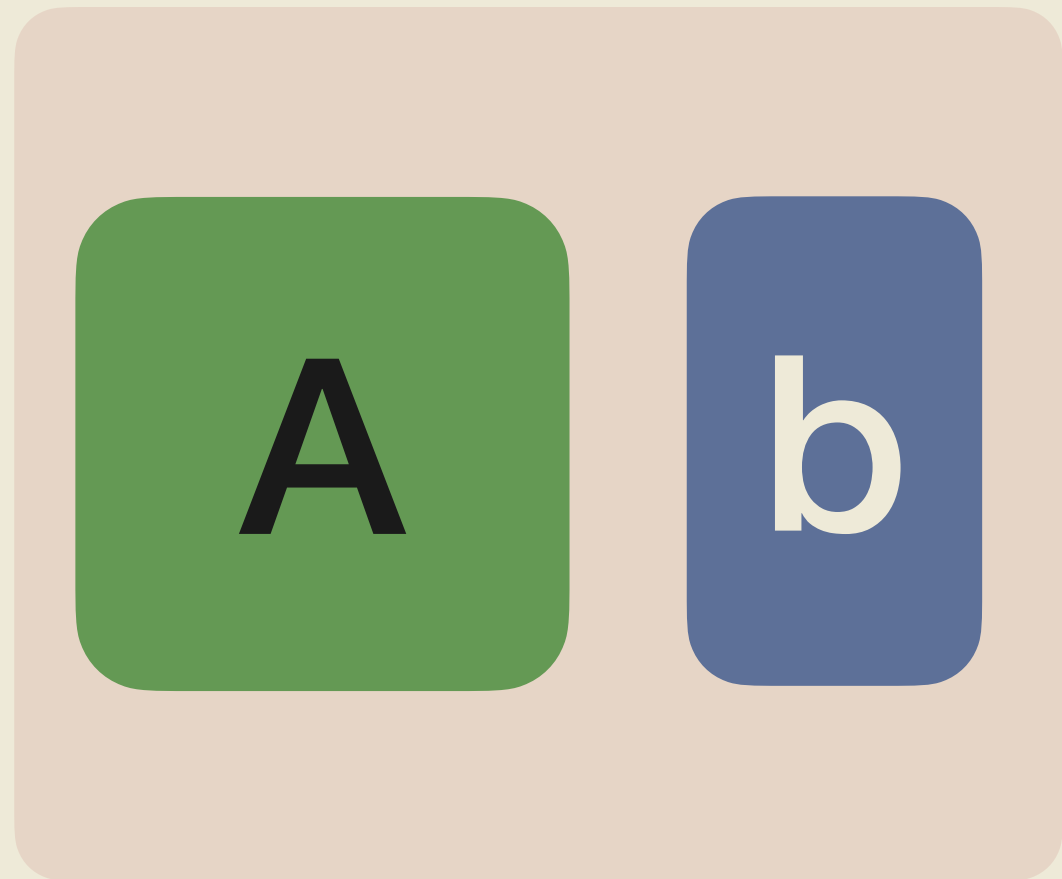
Assumption time - Reduction



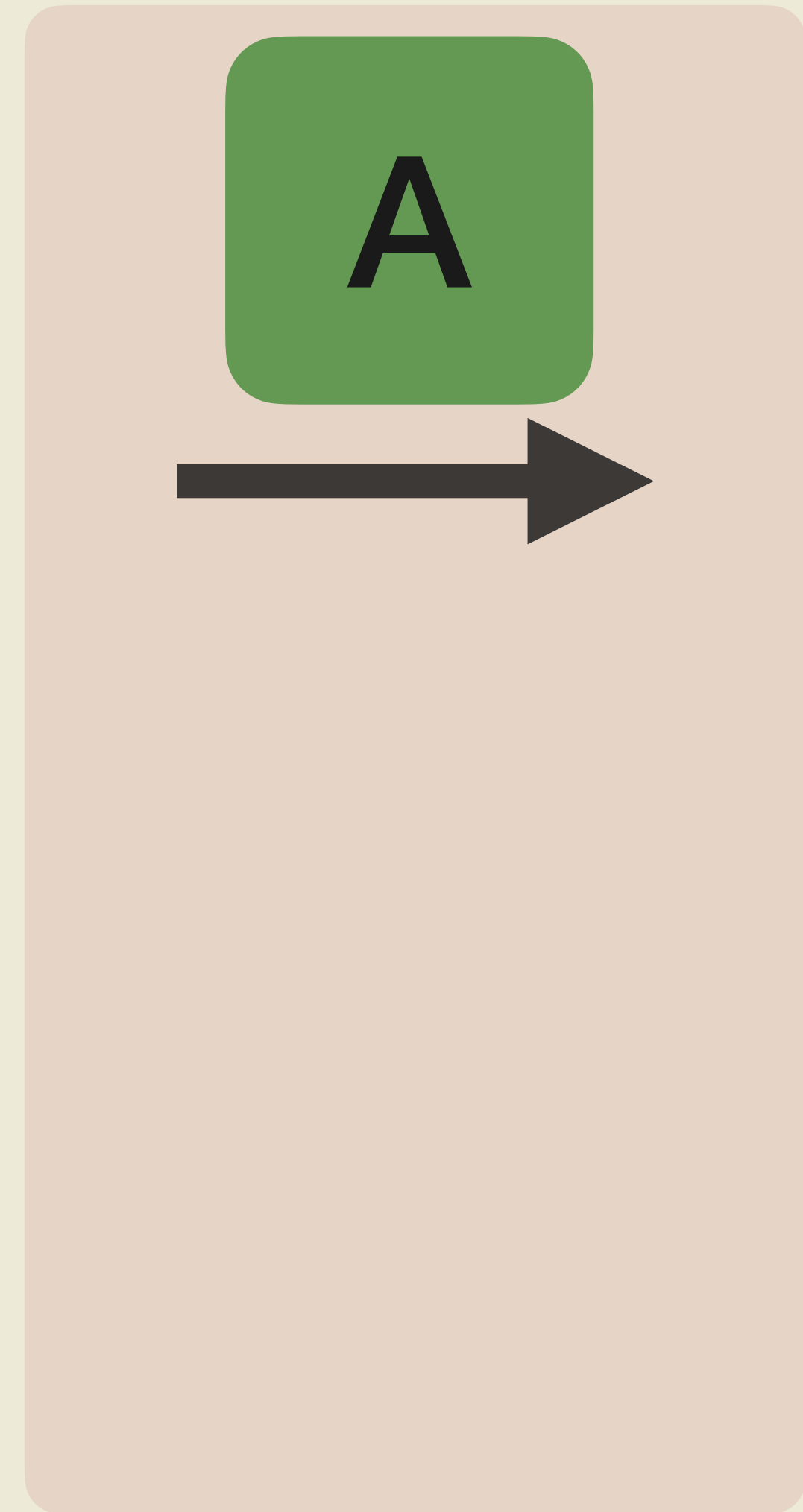
LWE

UPKE

Assumption time - Reduction



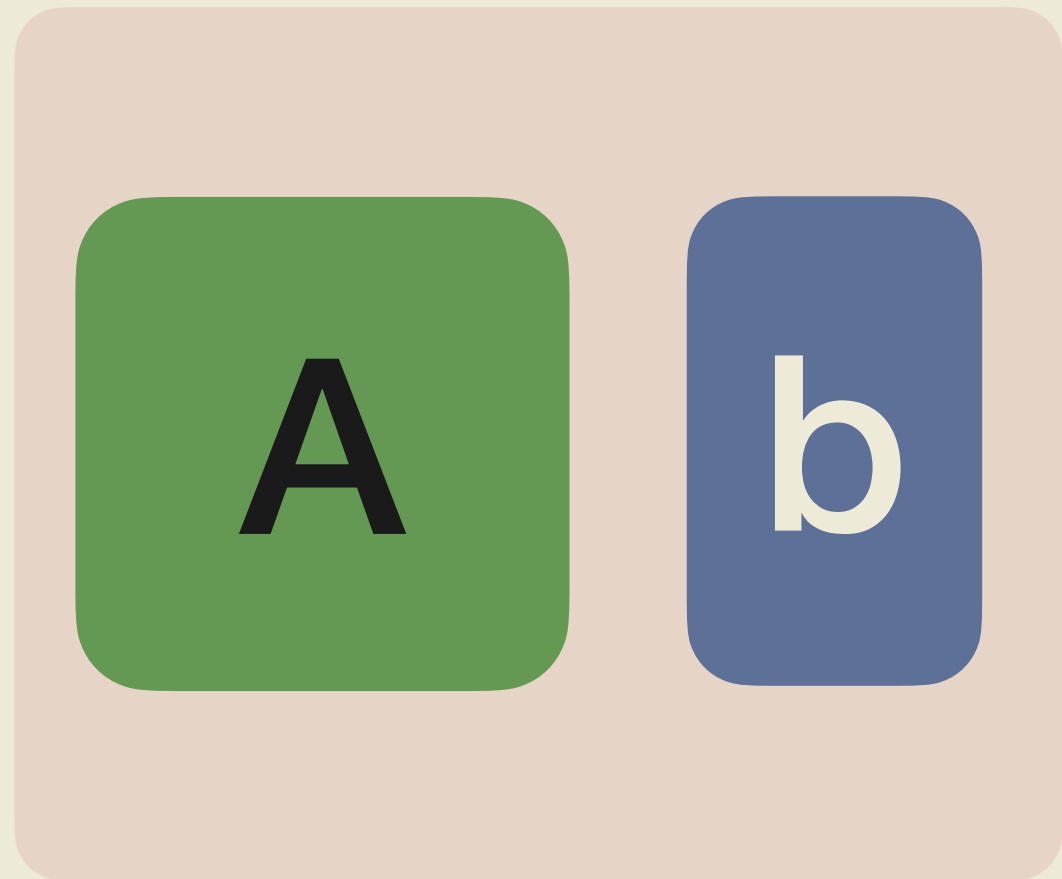
LWE



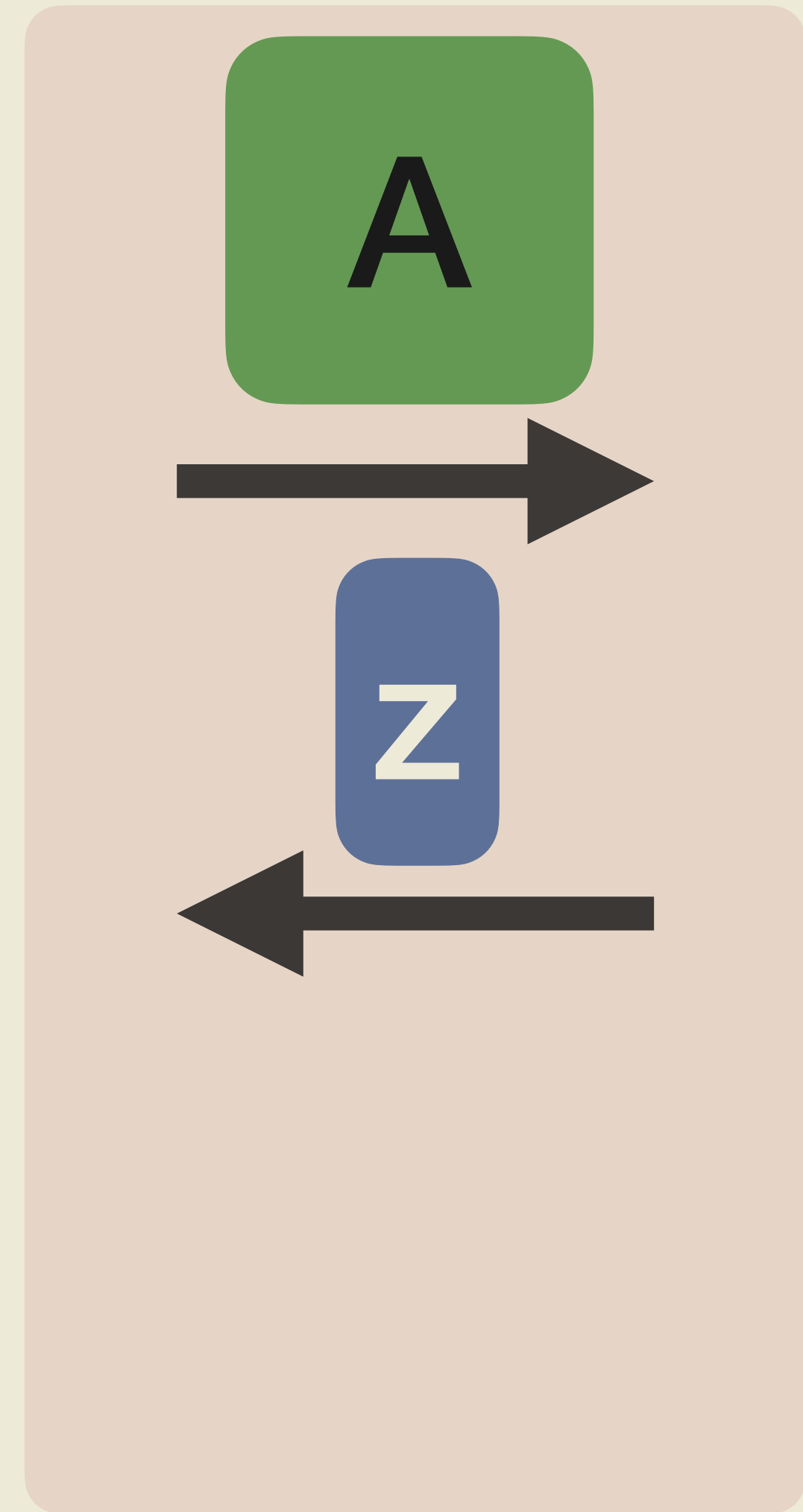
AextLWE

UPKE

Assumption time - Reduction



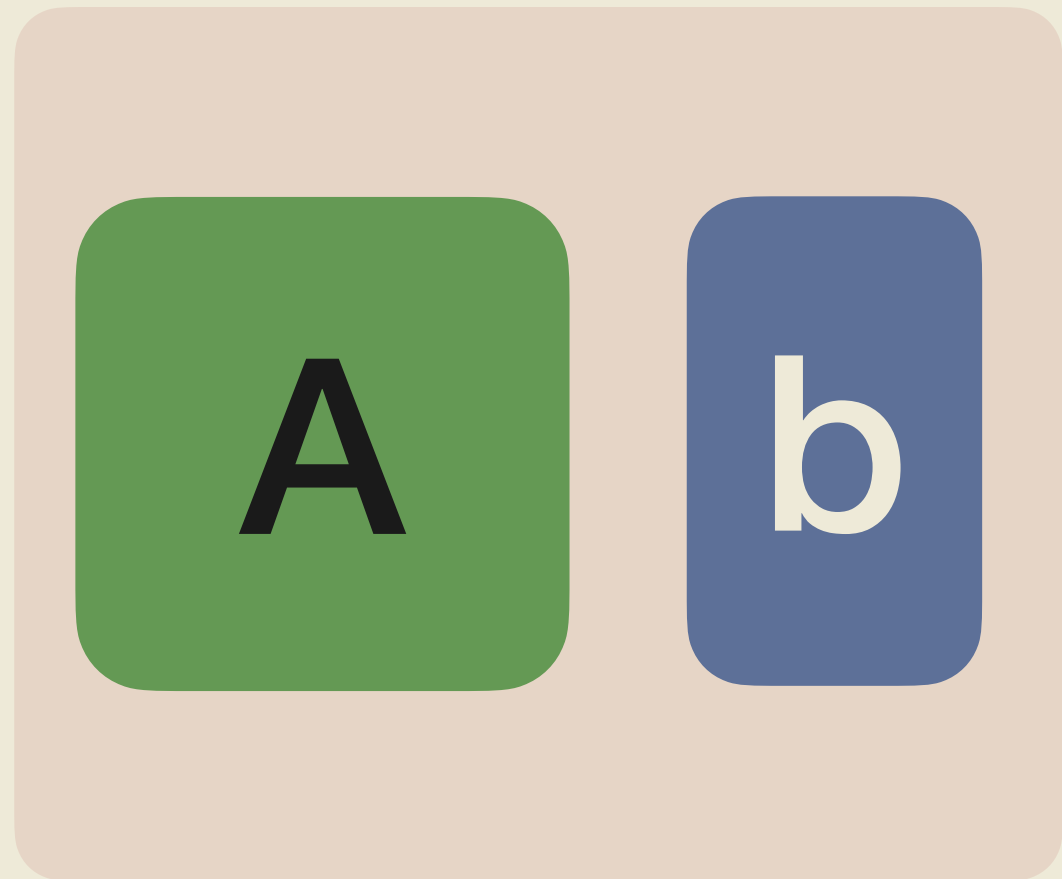
LWE



AextLWE

UPKE

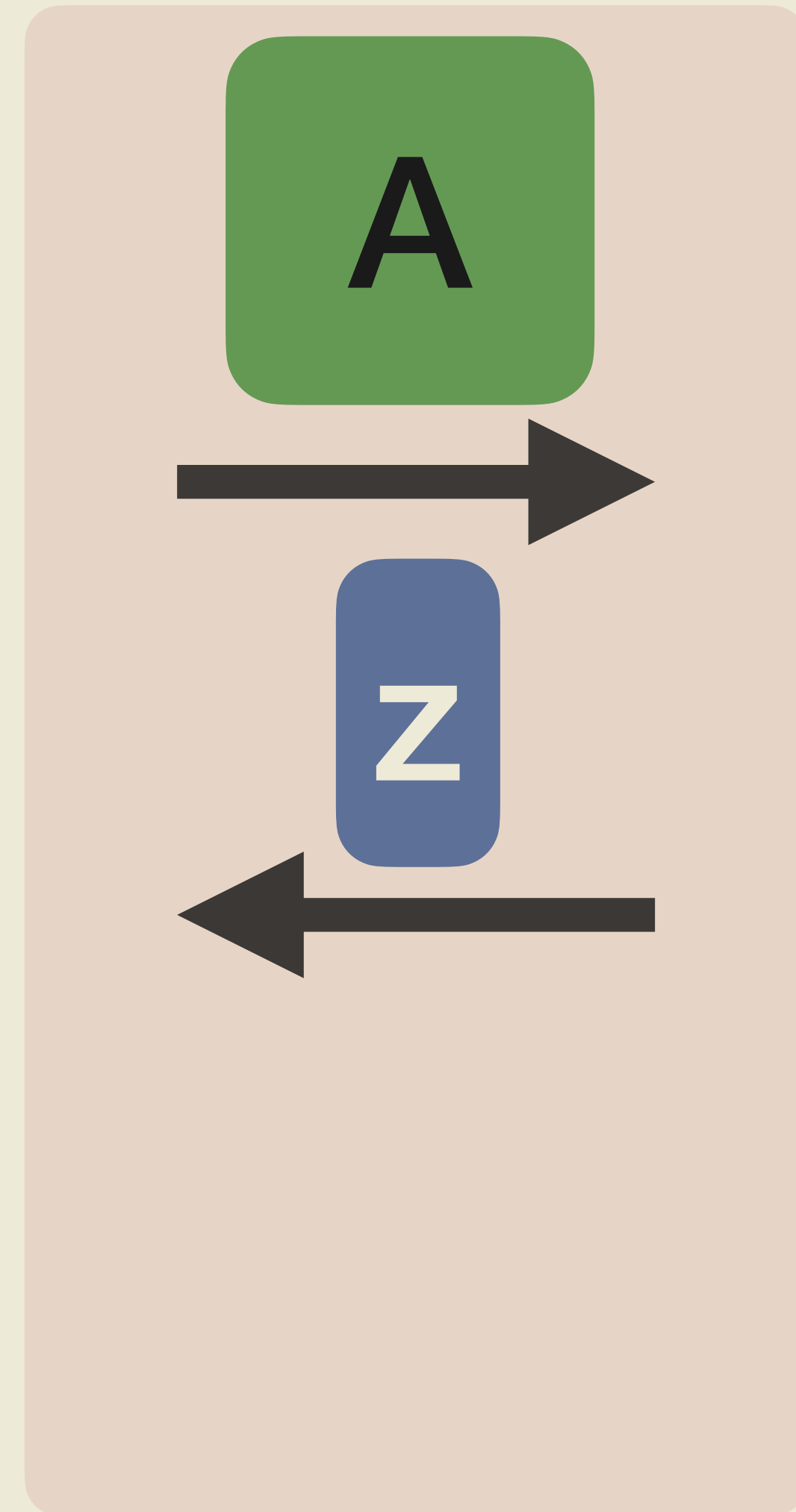
Assumption time - Reduction



LWE

$$b' = b + A s' + e'$$

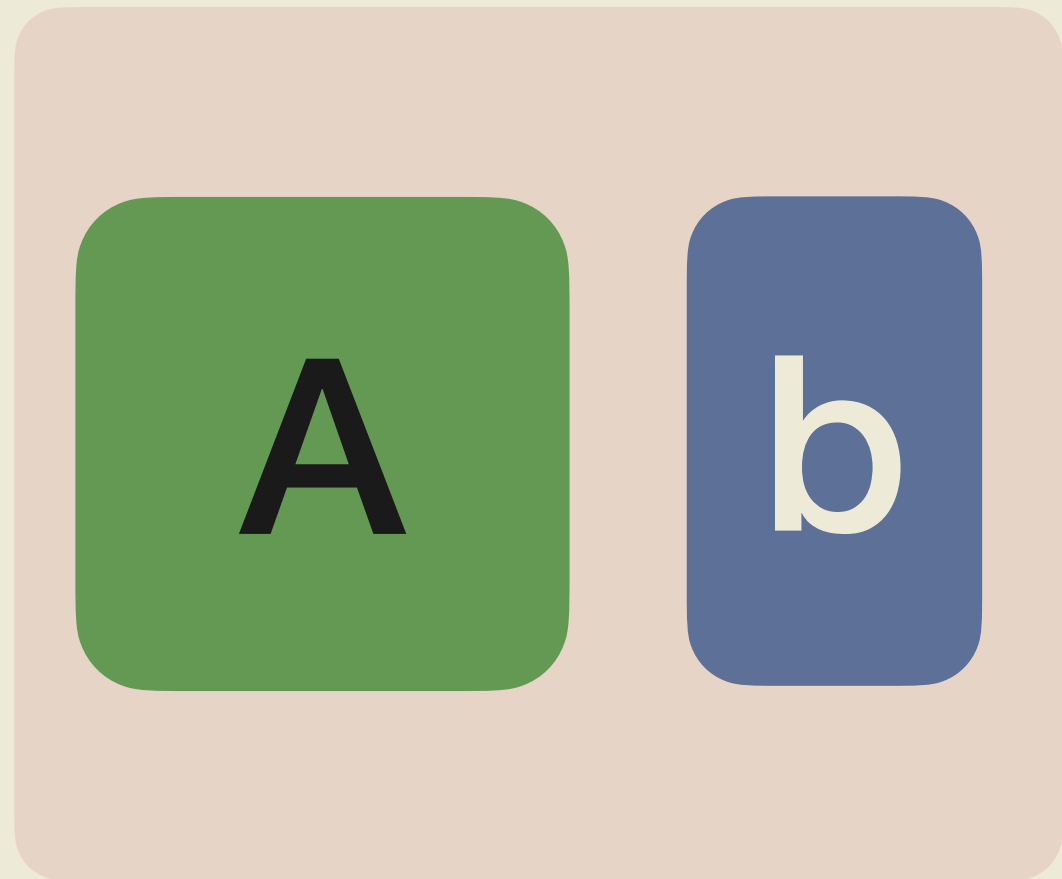
The equation is displayed on a light beige background. The terms are represented by colored rounded shapes: 'b' is a blue rounded rectangle, 'A' is a green rounded square, 's'' is a red rounded rectangle, and 'e'' is a black rounded rectangle.



AextLWE

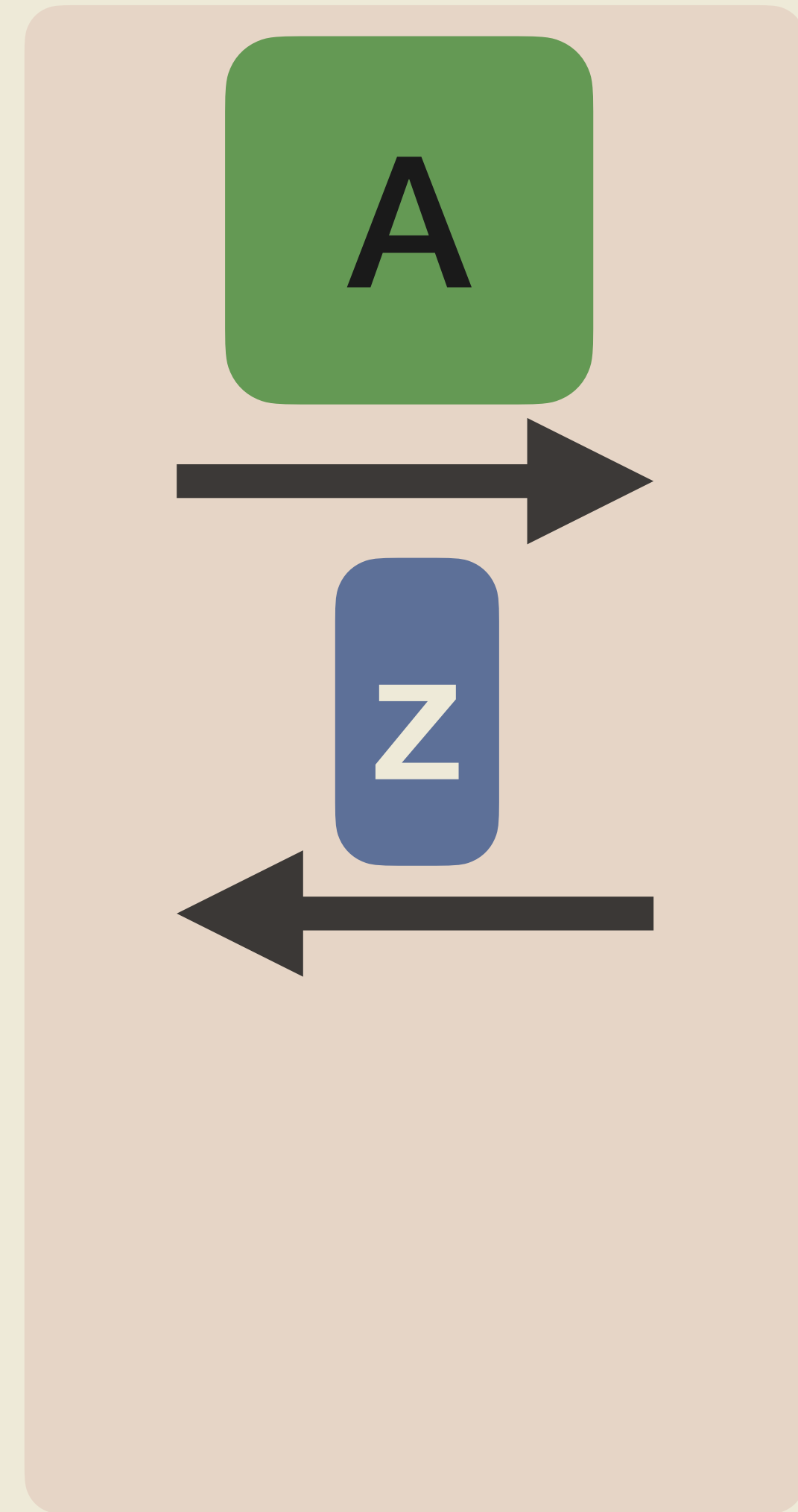
UPKE

Assumption time - Reduction



LWE

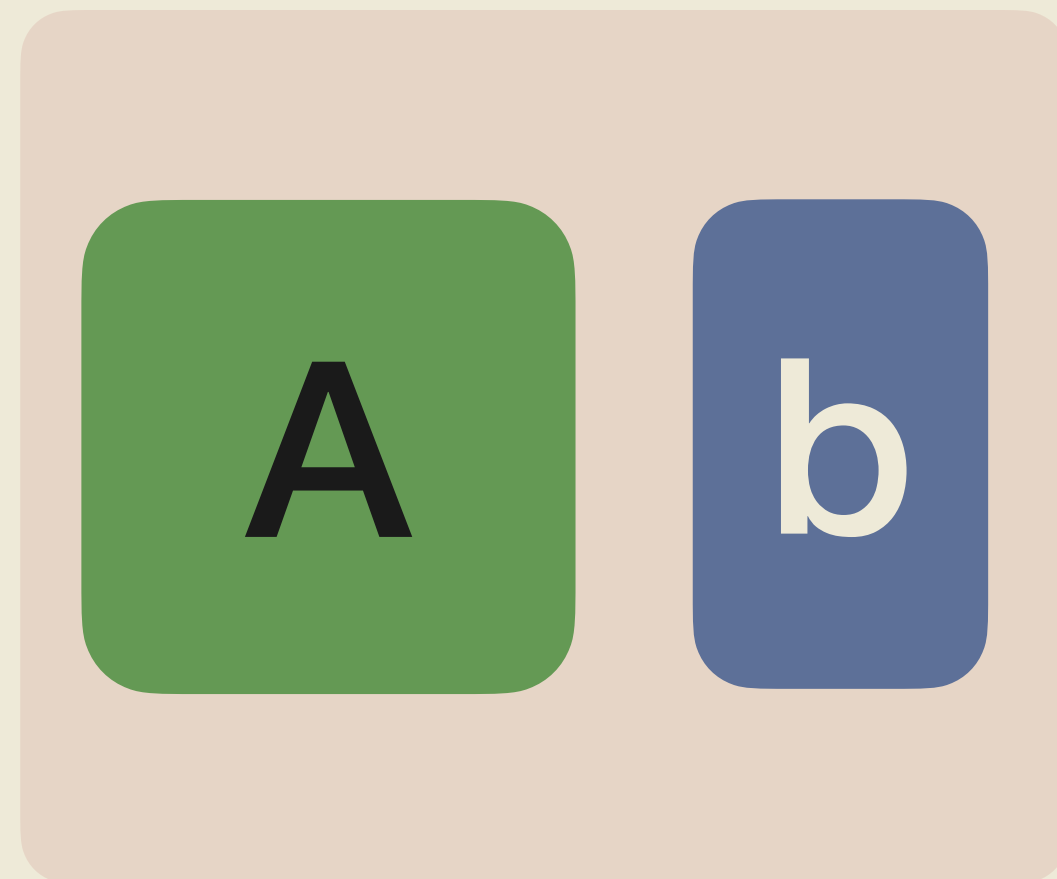
$$b' = b + A s' + e'$$
$$h = e' \cdot z + g'$$



AextLWE

UPKE

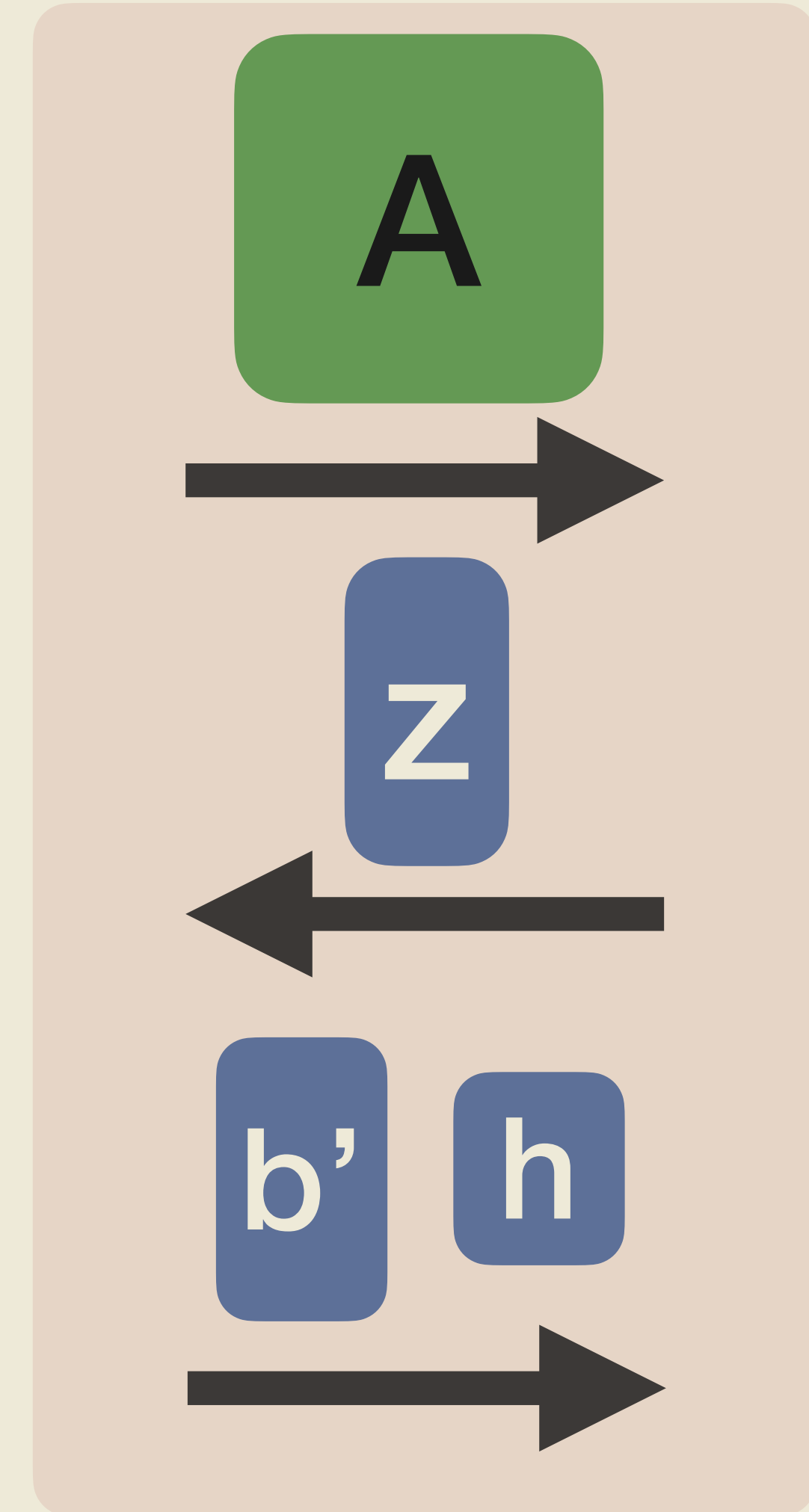
Assumption time - Reduction



LWE

$$b' = b + A s' + e'$$
$$h = e' \cdot z + g'$$

The diagram shows two equations. The first equation, $b' = b + A s' + e'$, uses a blue box for b' , a blue box for b , a green box for A , a red box for s' , and a black box for e' . The second equation, $h = e' \cdot z + g'$, uses a blue box for h , a black box for e' , a blue box for z , and a black box for g' .



AextLWE

UPKE

Assumption time - Reduction

$$b' = b + A s' + e'$$

$$h = e' \cdot z + g'$$

UPKE

Assumption time - Reduction

$$b' = b + A s' + e'$$
$$h = e' \cdot z + g'$$

h is a hint on **e'**
should be of **e + e'**

UPKE

Assumption time - Reduction

$$b' = b + A s' + e'$$

$$h = e + e' \cdot z - e \cdot z + g'$$

h is a hint on **e'**

should be of **e + e'**

UPKE

Assumption time - Reduction

$$b' = b + A s' + e'$$
$$h = e + e' \cdot z + g$$

$$g = -e \cdot z + g'$$

UPKE

Assumption time - Reduction

$$b' = A s + s' + e + e'$$
$$h = e + e' \cdot z + g$$

$$g = -e \cdot z + g'$$

UPKE

Assumption time - Reduction

$$b' = A s' + s' + e + e'$$

$$h = e' + e \cdot z + g$$

$$g = -e \cdot z + g'$$

Needs to be a spherical Gaussian

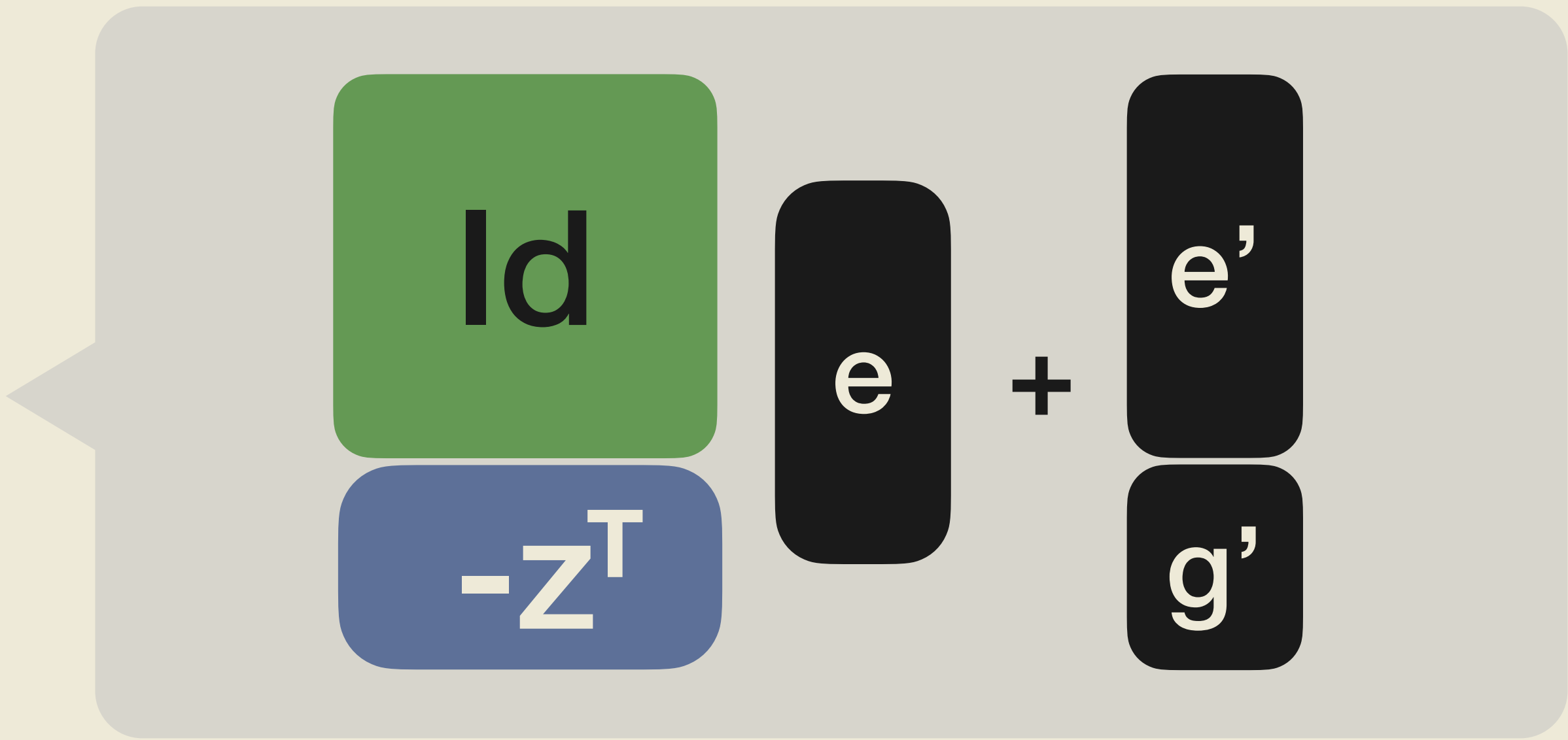
UPKE

Assumption time - Reduction

$$b' = A s' + s' + \begin{matrix} e \\ + \\ e' \end{matrix}$$

$$h = e' + e \cdot z + g$$

$$g = -e \cdot z + g'$$



UPKE

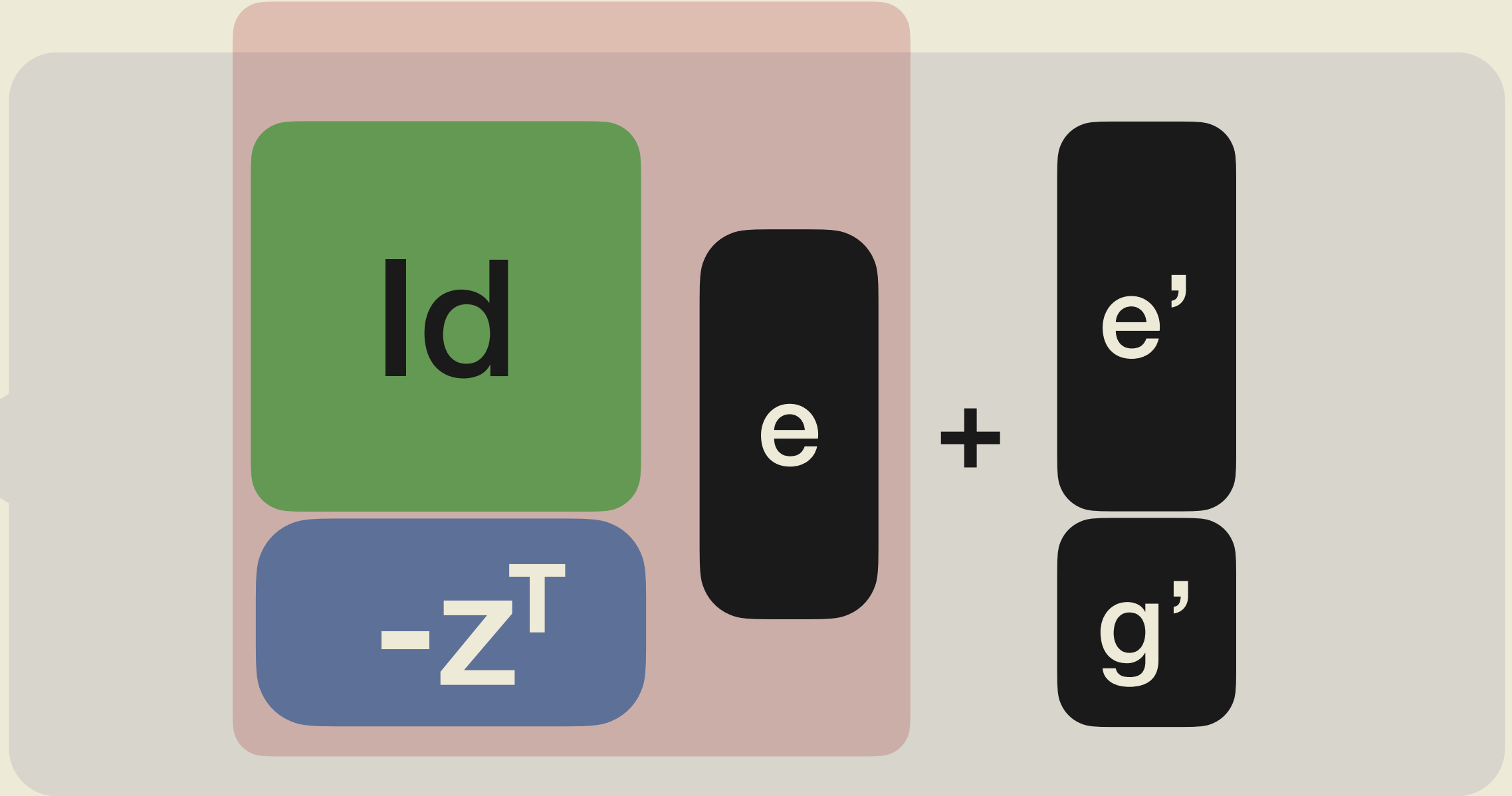
Assumption time - Reduction

$$b' = A s' + s' + \begin{matrix} e \\ + \\ e' \end{matrix}$$

$$h = e' + e \cdot z + g$$

$$g = -e \cdot z + g'$$

No control



UPKE

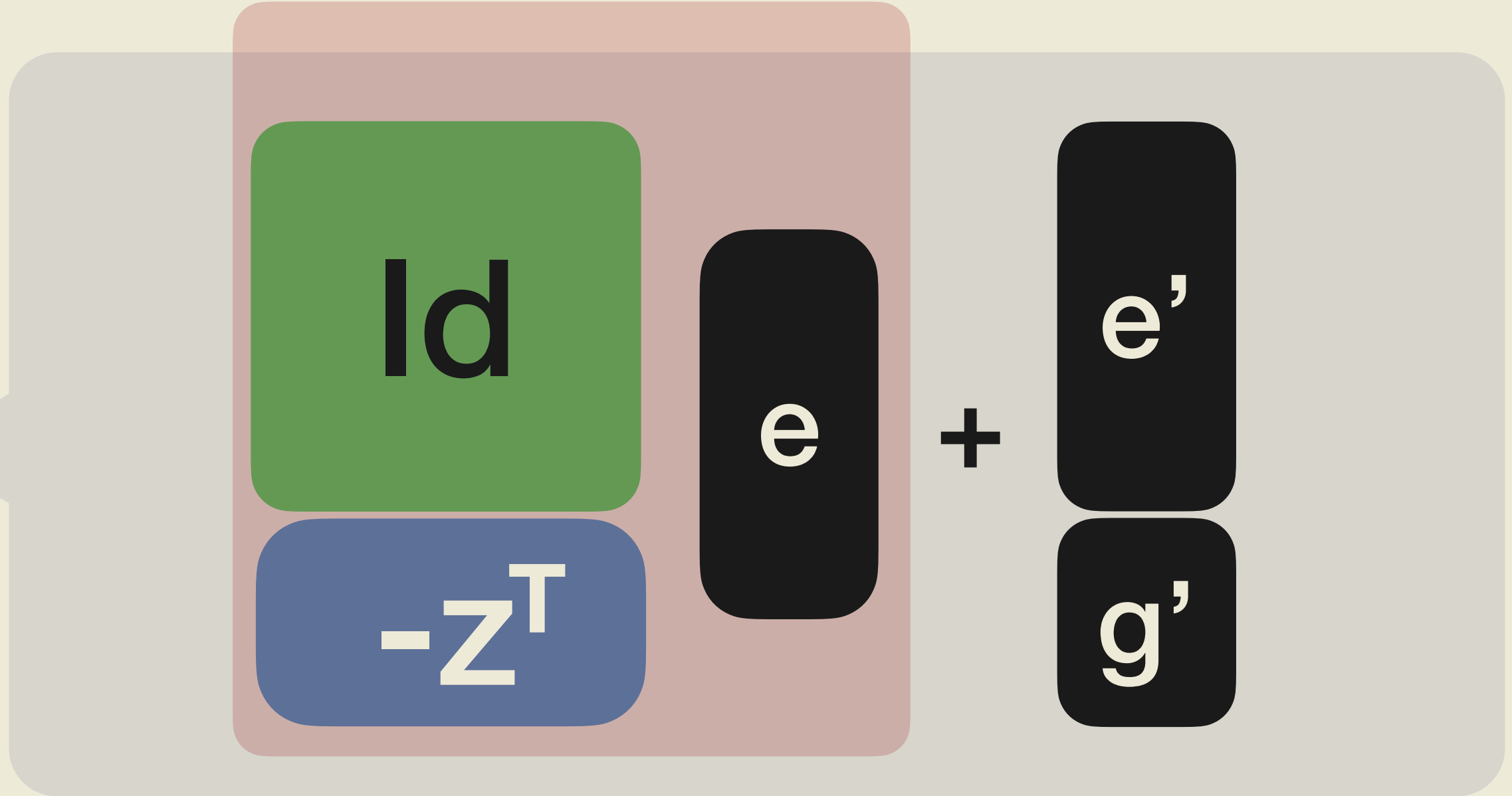
Assumption time - Reduction

$$b' = A s' + s' + \begin{bmatrix} e \\ e' \end{bmatrix}$$

$$h = e' + e \cdot z + g$$

$$g = -e \cdot z + g'$$

Biased Gaussian



UPKE

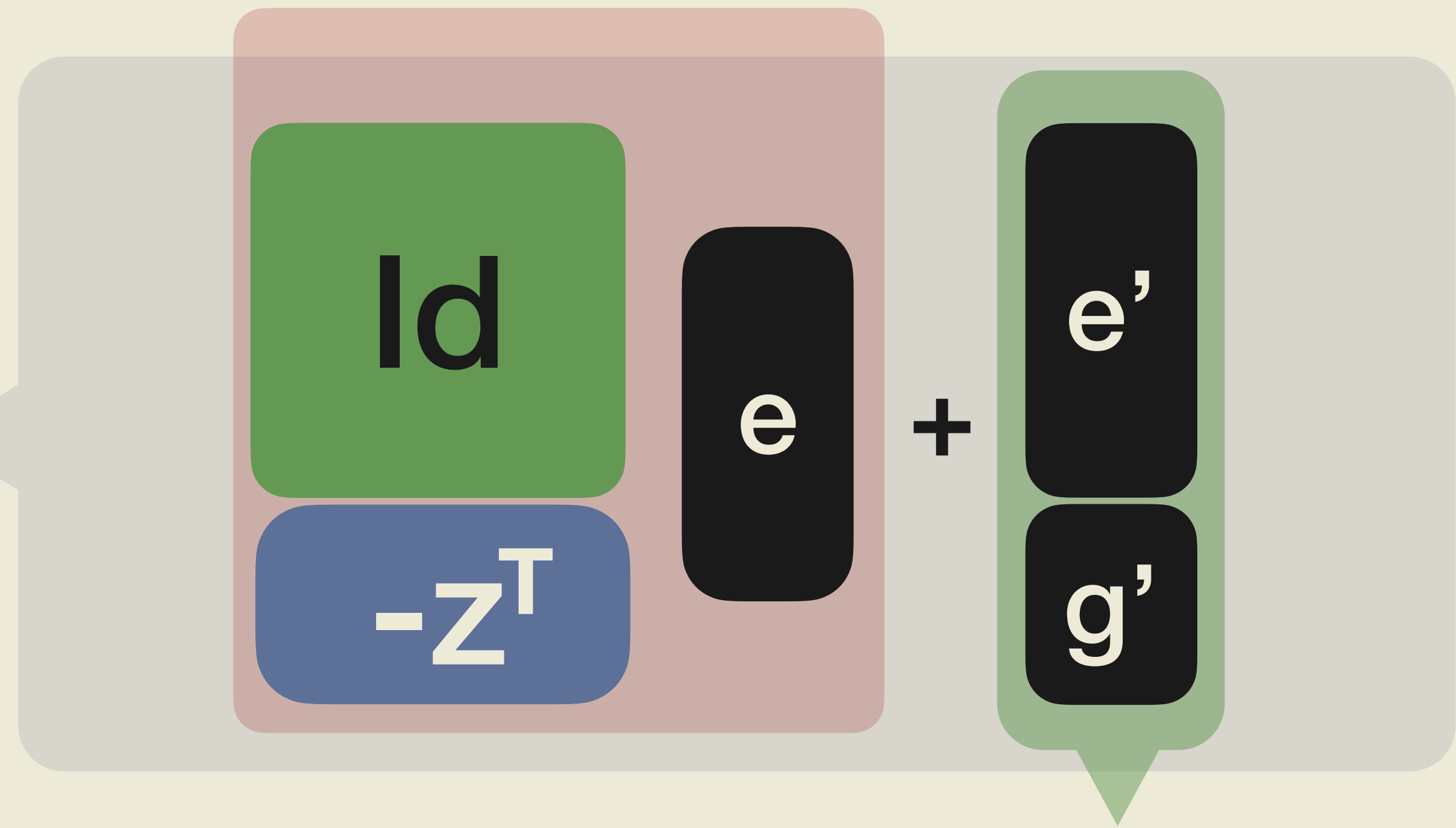
Assumption time - Reduction

$$b' = A s' + s' + \begin{matrix} e \\ + \\ e' \end{matrix}$$

$$h = e' + e \cdot z + g$$

$$g = -e \cdot z + g'$$

Biased Gaussian



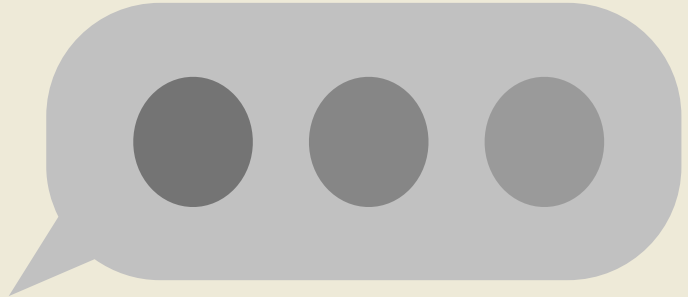
Sample to correct bias

Sizes

Concrete Sizes

| KYBER | ct | up |
|------------------|--------|-------|
| 0 update | 0.8 KB | |
| 2^{10} updates | 3.0 KB | 12 KB |
| 2^{15} updates | 5.8 KB | 12 KB |

Other Contributions



Other Contributions

An FO transform for UPKE



Other Contributions

An FO transform for UPKE

CU transform



Other Contributions

An FO transform for UPKE

CU transform

Thanks ! See ia.cr/2023/1400 for more details