Adaptive Distributional Security for Garbling Schemes with O(|x|) Online Complexity

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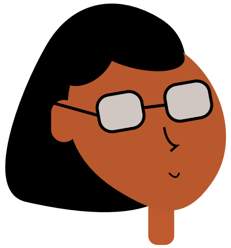
CIPHERA

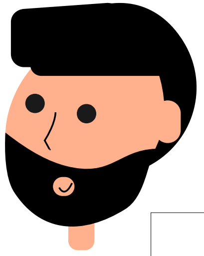
Aalto University

THE UNIVERSITY of EDINBURGH

Preprocessing Multiparty Computation: Offline Garbling

Evaluator





Garbler

$C(\cdot, x_G)$

Preprocessing Multiparty Computation: Offline Garbling

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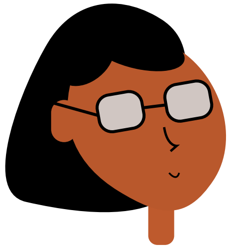

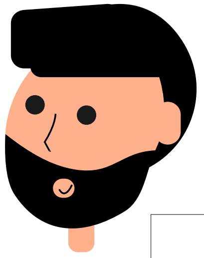

Garbler

$C(\cdot, x_G)$

$$\tilde{C}, K \leftarrow \mathsf{GCircuit}(C)$$

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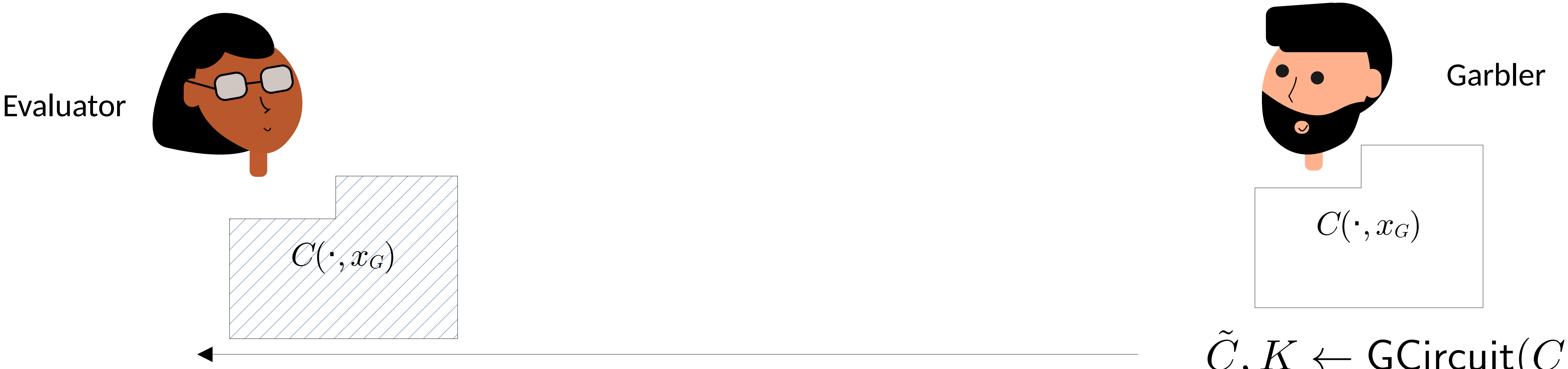
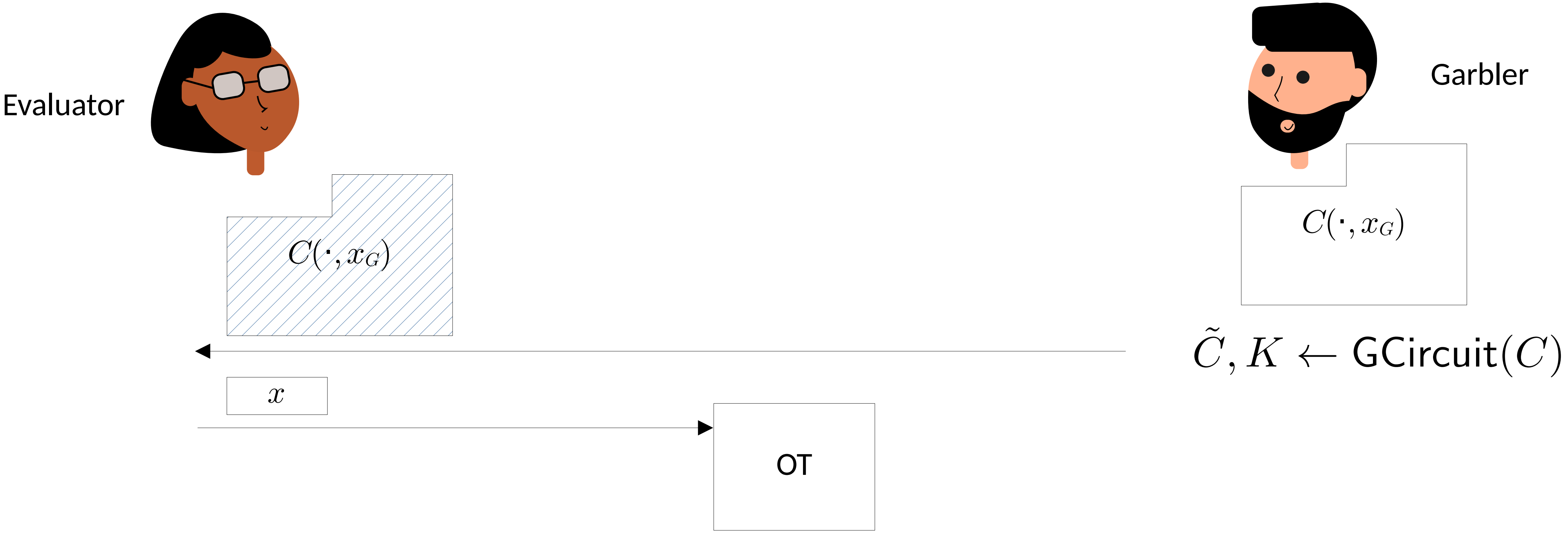


$$\tilde{C}, K \leftarrow \mathsf{GCircuit}(C)$$

Preprocessing Multiparty Computation: Offline Garbling



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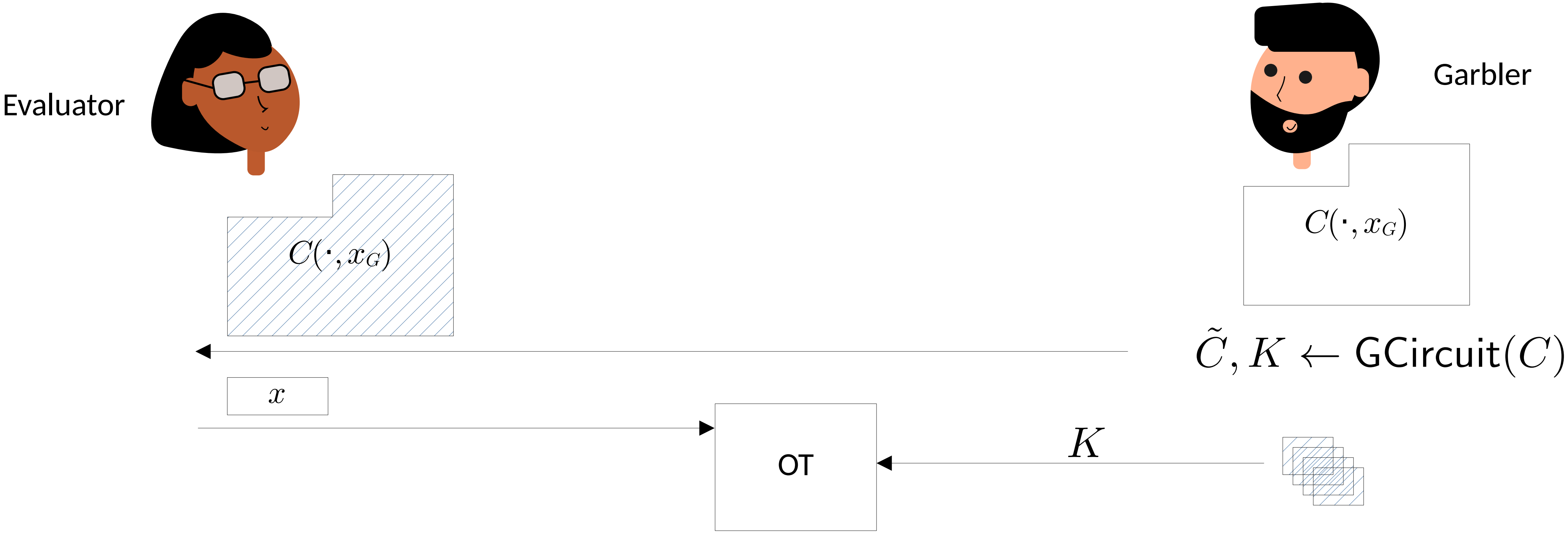

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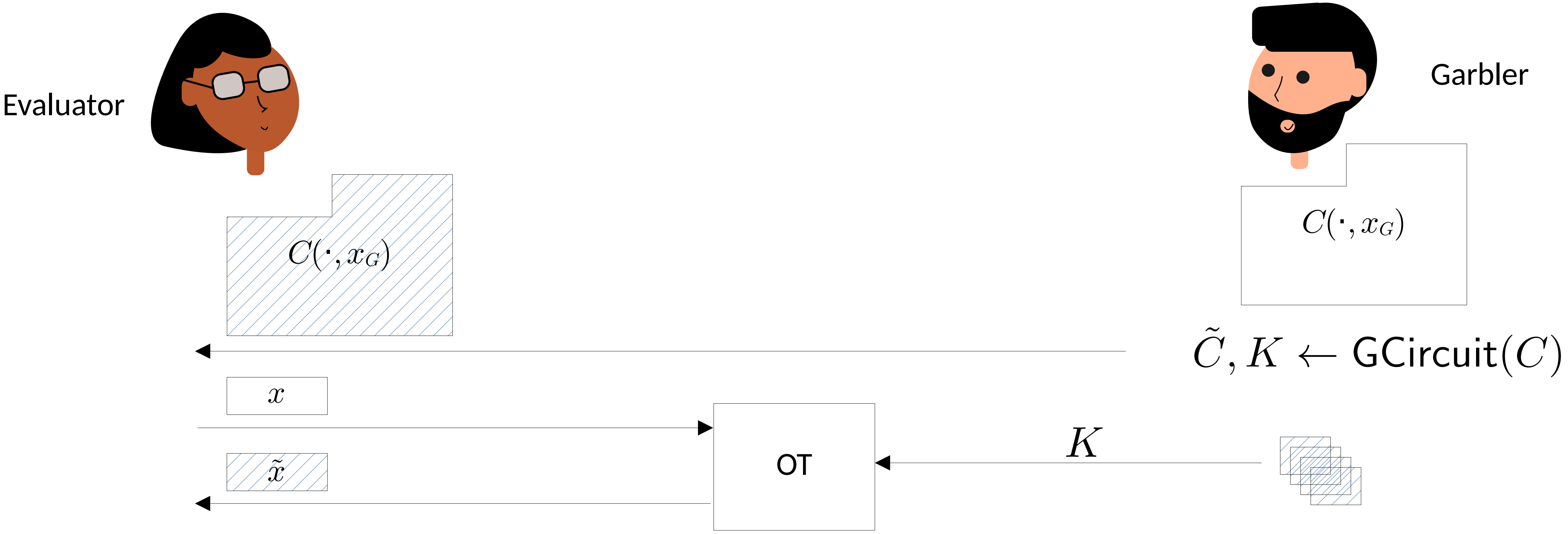

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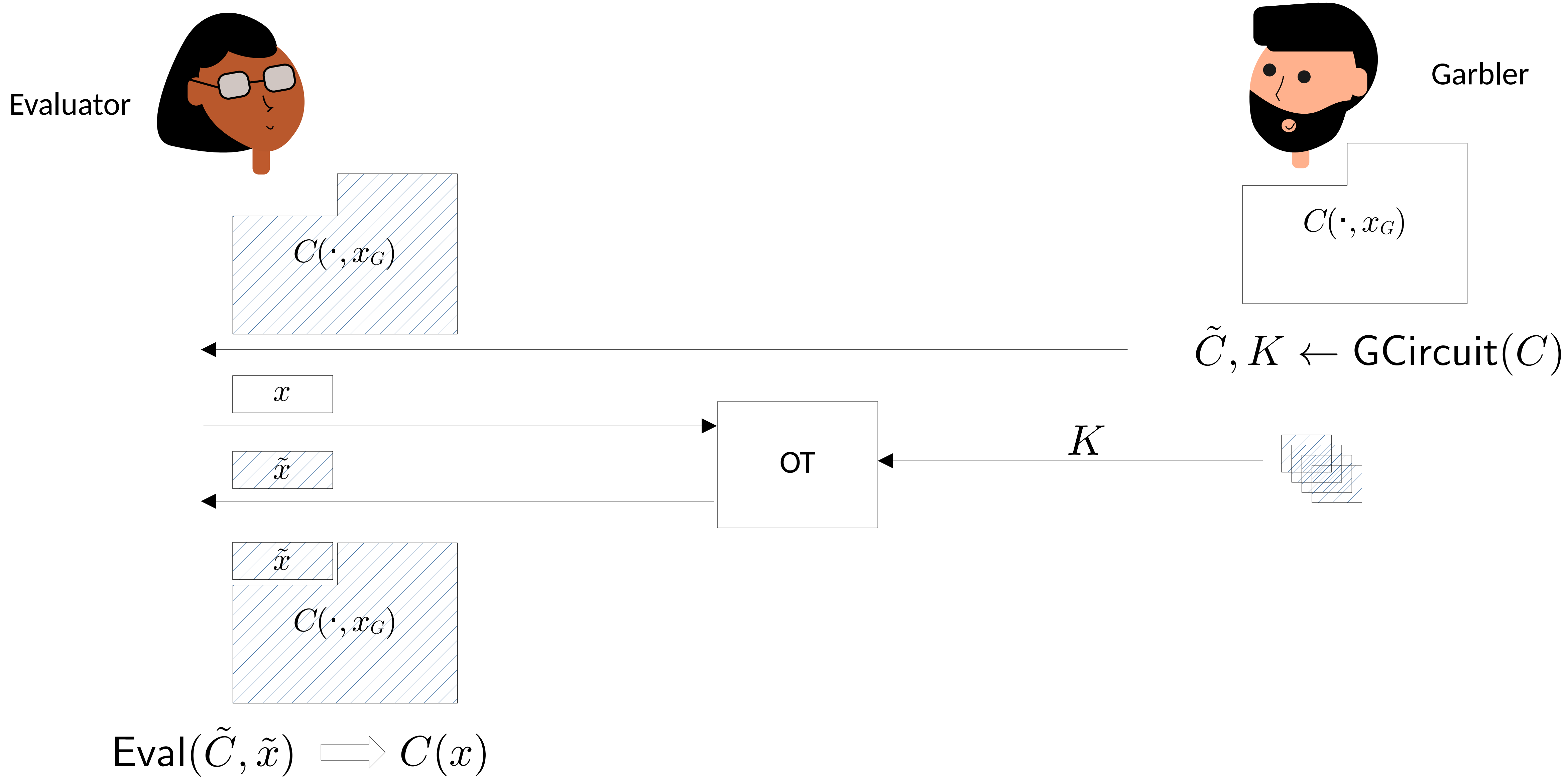
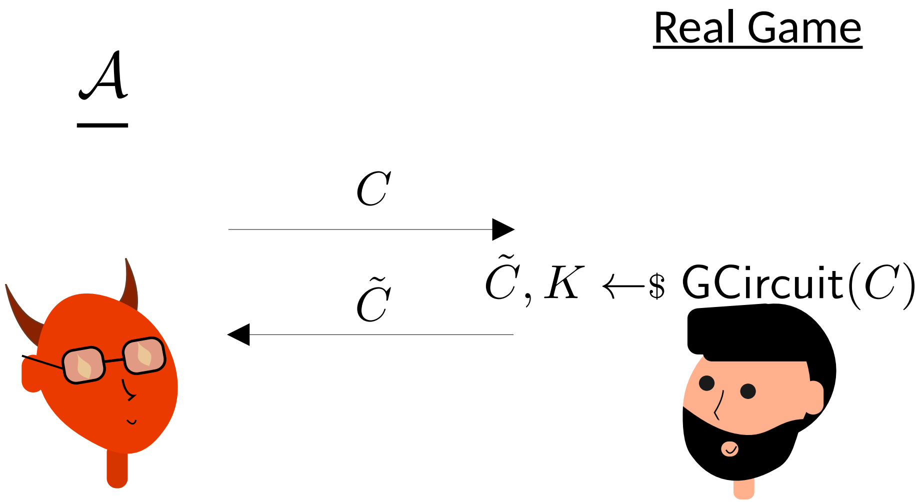


Adaptive Simulation Security (SIM) [BHR'12]

(Defined by Bellare, Hoang and Rogaway **[BHR'12]**)



Adaptive Simulation Security (SIM) [BHR'12]

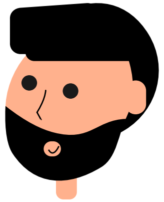
<u>Real Game</u>

$\underline{\mathcal{A}}$

$$\xrightarrow{\quad C \quad}$$

$$\xleftarrow{\quad \tilde{C} \quad}$$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$



Adaptive Simulation Security (SIM) [BHR'12]

Real Game

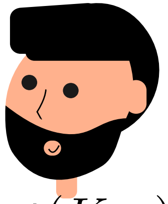
$\underline{\mathcal{A}}$

$\xrightarrow{\quad C \quad}$

$\xleftarrow{\quad \tilde{C} \quad}$ $\tilde{C}, K \leftarrow\$ \ \mathsf{GCircuit}(C)$

$\xrightarrow{\quad x \quad}$

$\xleftarrow{\quad \tilde{x} \quad}$ $\tilde{x} \leftarrow \mathsf{GInput}(K, x)$



Adaptive Simulation Security (SIM) [BHR'12]

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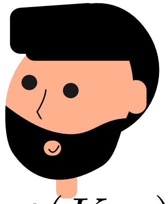
$\xrightarrow{C}$

$\xleftarrow{\tilde{C}}$ $\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$



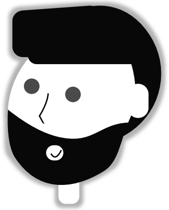
$\xrightarrow{x}$

$\xleftarrow{\tilde{x}}$ $\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

<u>Ideal Game</u> (with ***Simulator***)

$\xrightarrow{C}$

$\underline{\mathcal{S}}$



Adaptive Simulation Security (SIM) [BHR'12]

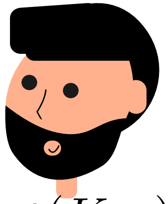
<u>Real Game</u>

$\underline{\mathcal{A}}$

$\xrightarrow{C}$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

$\xleftarrow{\tilde{C}}$



$\xrightarrow{x}$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

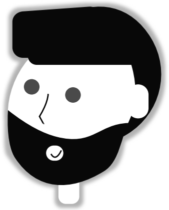
$\xleftarrow{\tilde{x}}$

<u>Ideal Game</u> (with ***Simulator***)

$\xrightarrow{C}$

$\xrightarrow{\Phi(C)}$

$\underline{\mathcal{S}}$



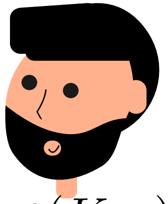
Adaptive Simulation Security (SIM) [BHR'12]

<u>Real Game</u>

$\underline{\mathcal{A}}$

C →

← $\tilde{C}$ $\quad$ $\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

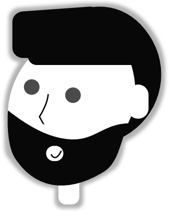


x →

← $\tilde{x}$ $\quad$ $\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

<u>Ideal Game</u> (with ***Simulator***)

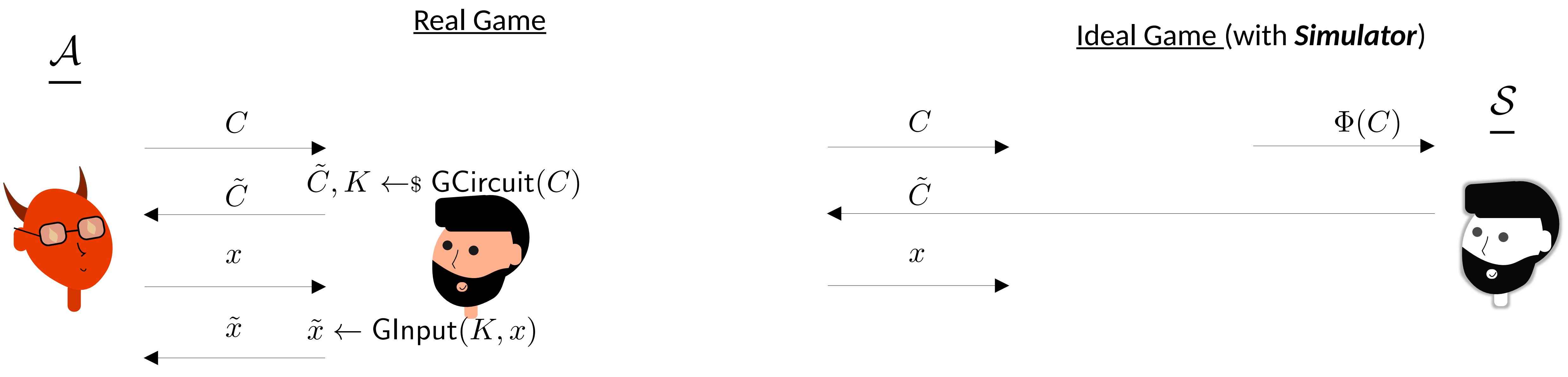
$\underline{\mathcal{S}}$



C → $\qquad$ $\Phi(C)$ →

← $\tilde{C}$

Adaptive Simulation Security (SIM) [BHR’12]



Adaptive Simulation Security (SIM) [BHR'12]

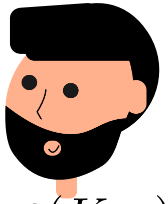
Real Game

$\underline{\mathcal{A}}$

C

$\tilde{C}$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$



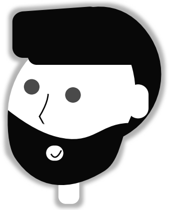
x

$\tilde{x}$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

Ideal Game (with ***Simulator***)

$\underline{\mathcal{S}}$



C

$\Phi(C)$

$\tilde{C}$

x

$C(x)$

Adaptive Simulation Security (SIM) [BHR'12]

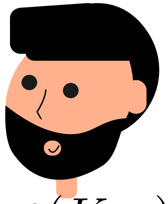
Real Game

$\mathcal{A}$

$C \longrightarrow$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

$\longleftarrow \tilde{C}$



$x \longrightarrow$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

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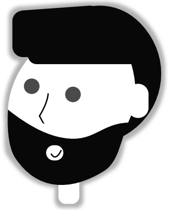
Ideal Game (with ***Simulator***)

$\mathcal{S}$



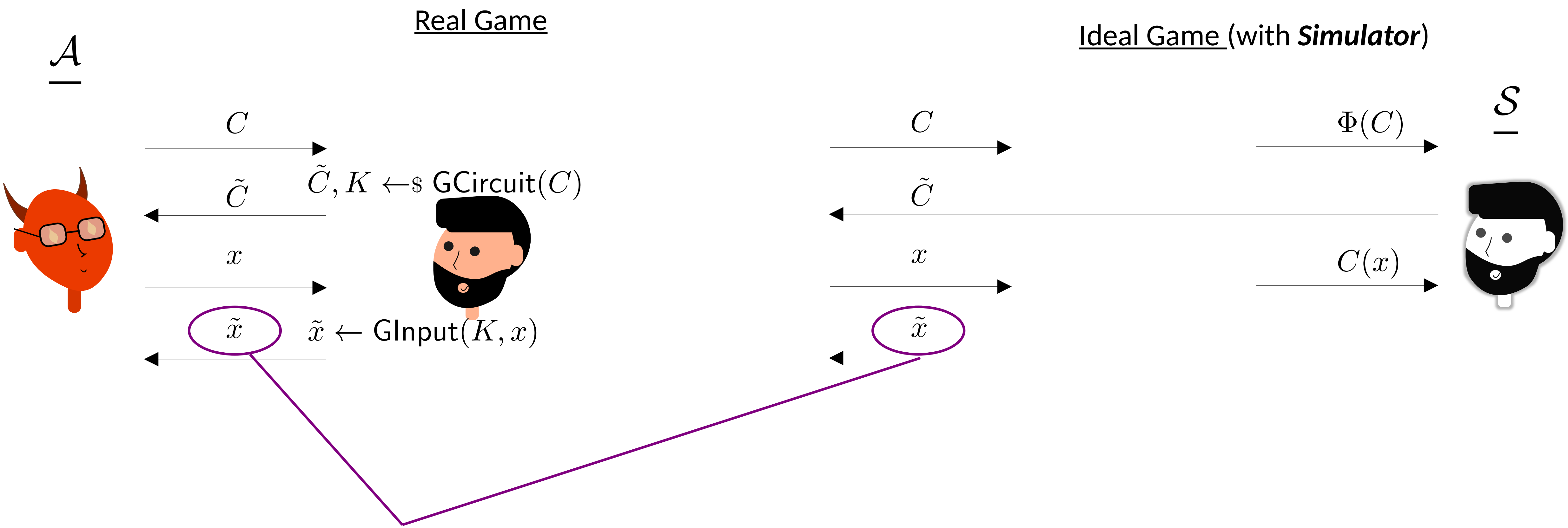
$C \longrightarrow$ $\quad \Phi(C) \longrightarrow$

$\longleftarrow \tilde{C}$

$x \longrightarrow$ $\quad C(x) \longrightarrow$

$\longleftarrow \tilde{x}$

Adaptive Simulation Security (SIM) [BHR'12]



The size of $\tilde{x}$, $|\tilde{x}|$ is known as the ***online complexity***

Adaptive Simulation Security (SIM) [BHR'12]

Real Game

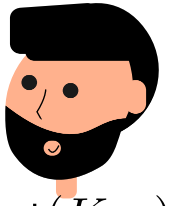
$\underline{\mathcal{A}}$

C $\longrightarrow$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

$\longleftarrow$ $\tilde{C}$

x $\longrightarrow$



$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

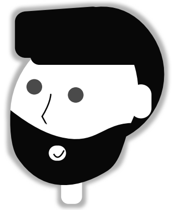
$\longleftarrow$ $\tilde{x}$

Ideal Game (with ***Simulator***)

C $\longrightarrow$

$\Phi(C)$ $\longrightarrow$

$\underline{\mathcal{S}}$



$\longleftarrow$ $\tilde{C}$

x $\longrightarrow$

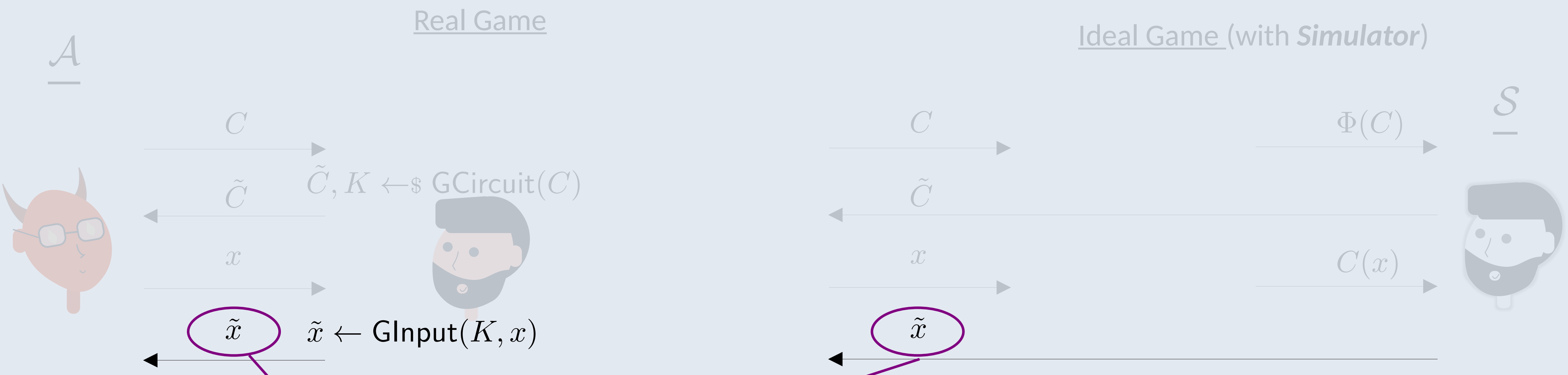
$C(x)$ $\longrightarrow$

$\longleftarrow$ $\tilde{x}$

The size of $\tilde{x}$, $|\tilde{x}|$ is known as the ***online complexity***

Ideally, $|\tilde{x}| = O(|x|)$

Adaptive Simulation Security (SIM) [BHR'12]



Unfortunately: Applebaum, Ishai, Kushilevitz, Waters [**AIKW'13**] show in general SIM requires $|\tilde{x}| = O(|x| + |y|)$.

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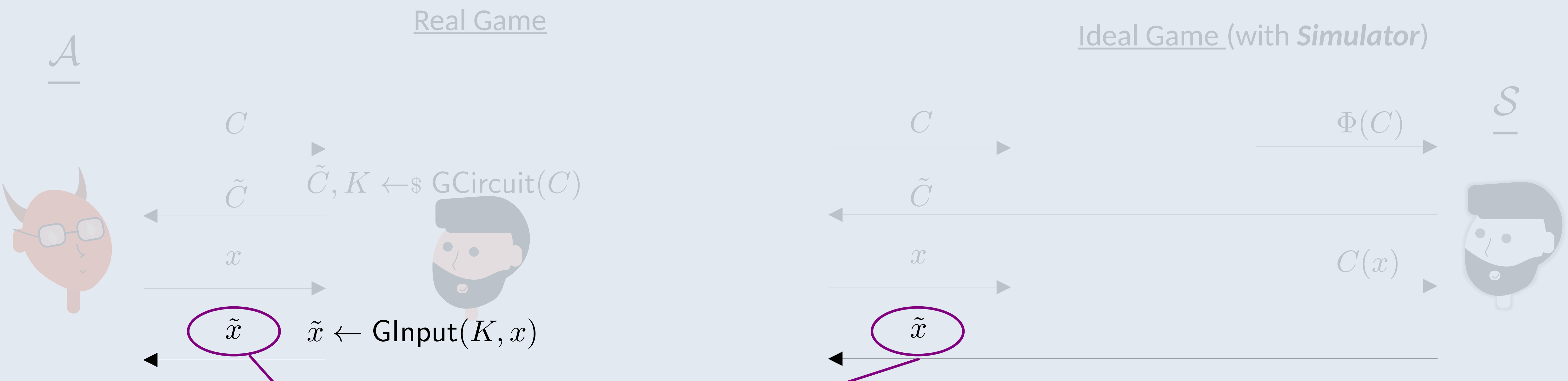

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In general, *pseudorandom* circuits such as PRGs, PRFs, etc have this requirement for simulation based MPC [**Hubacek-Wichs'14**]*.

*We give a slightly better lower bound for garbling schemes.

Adaptive Indistinguishability Security (IND) [BHR'12]

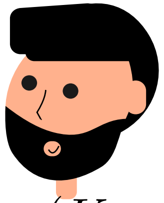
$\underline{\mathcal{A}}$

$\xrightarrow{C_0, C_1}$

$\xleftarrow{\tilde{C}}$ $\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C_b)$

$\xrightarrow{x_0, x_1}$

$\xleftarrow{\tilde{x}}$ $\tilde{x} \leftarrow \mathsf{GInput}(K, x_b)$



Requirement: $C_0(x_0) = C_1(x_1)$

Adaptive Indistinguishability Security (IND) [BHR'12]

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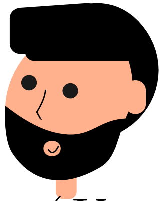


$\xrightarrow{x_0, x_1}$

$\xleftarrow{\tilde{x}}$ $\quad \tilde{x} \leftarrow \mathsf{GInput}(K, x_b)$

Requirement: $C_0(x_0) = C_1(x_1)$

Seems ***too*** strong for *cryptographic circuits*
i.e. only puncturable primitives.

Adaptive Indistinguishability Security (IND) [BHR'12]

Can we define a notion that
- Does not suffer from SIM lower bounds, and IND weakness?

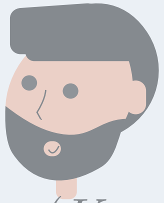
- Garbling cryptographic circuits with long outputs?

$\underline{\mathcal{A}}$

C_0, C_1

$\tilde{C}, K \leftarrow\$ \ \mathsf{GCircuit}(C_b)$

$\tilde{C}$



x_0, x_1

$\tilde{x} \leftarrow \mathsf{GInput}(K, x_b)$

$\tilde{x}$

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$\mathcal{A}$

In fact, we want a *distributional* notion (*Idea*: Limit adversarial knowledge).

This has been a successful approach for defining things like:

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This has been a successful approach for defining things like:

1. Deterministic Public Key Encryption (Bellare, Boldyreva, O'Neill [**BBO'07**]):
   (Cannot allow adversary to pick the whole message)

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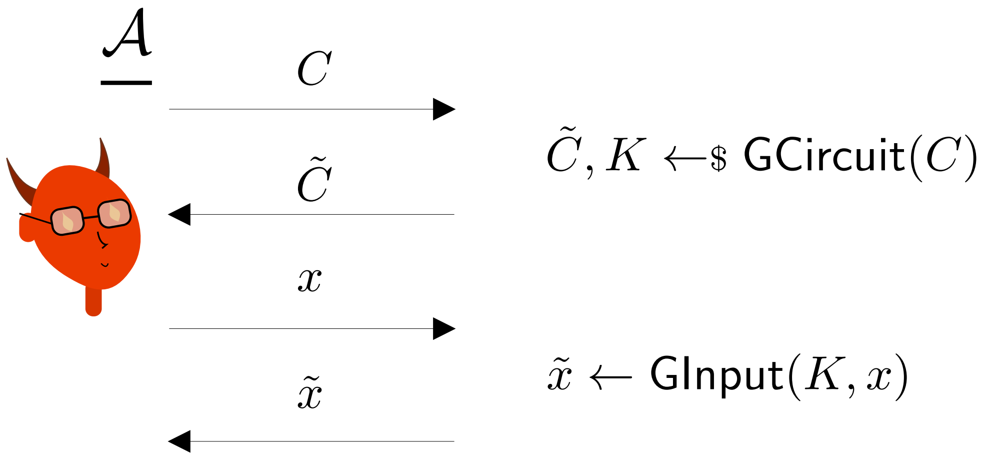
This has been a successful approach for defining things like:

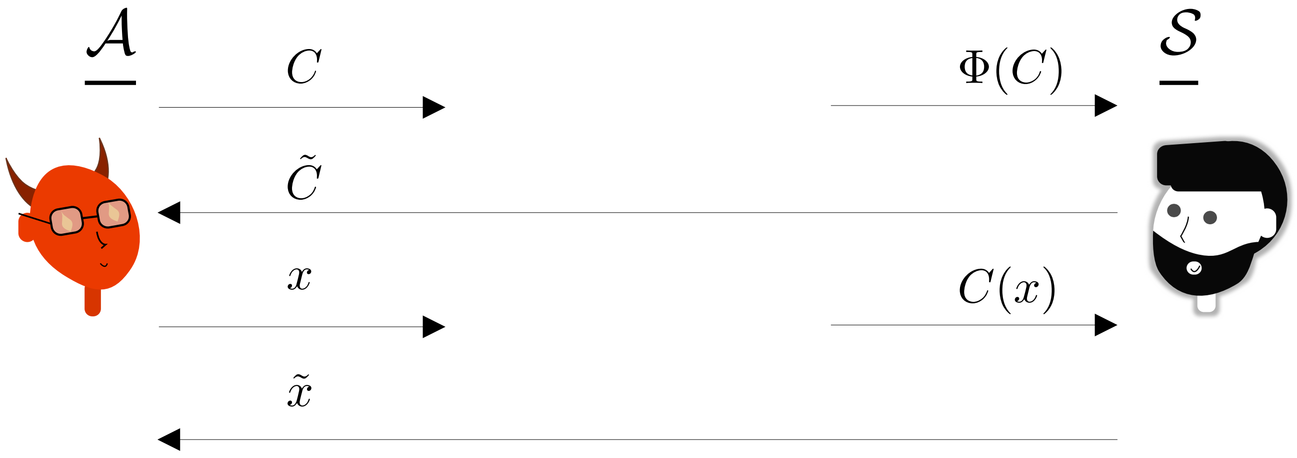
1. Deterministic Public Key Encryption (Bellare, Boldyreva, O'Neill [**BBO'07**]):
   (Cannot allow adversary to pick the whole message)

2. Distributional Zero Knowledge [**Goldreich'93, DNRS'99, JKKR'17, Khurana'21**]
   - Random statement instead of *any* statement.

Adaptive Distributional Simulation Security (DSIM)

Recall SIM notion:





Adaptive Distributional Simulation Security (DSIM)

Our DSIM notion:

$\underline{\mathcal{A}}$

$\xrightarrow{C}$

$\xleftarrow{\tilde{C}}$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

$\xrightarrow{x}$

$\xleftarrow{\tilde{x}}$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

$\underline{\mathcal{A}}$

$\xrightarrow{C}$

$\xleftarrow{\tilde{C}}$

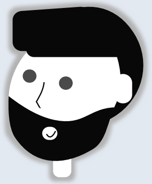
$\xrightarrow{x}$

$\xleftarrow{\tilde{x}}$

$\xrightarrow{\Phi(C)}$

$\xrightarrow{C(x)}$

$\underline{\mathcal{S}}$



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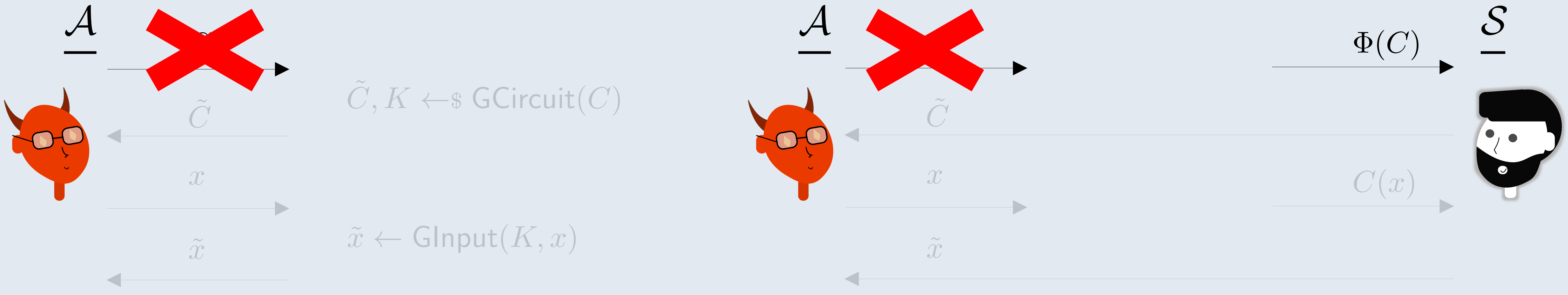
Our DSIM notion:



Adversary no longer chooses the circuit C

Adaptive Distributional Simulation Security (DSIM)

Our DSIM notion:

$\underline{\mathcal{A}}$

$\tilde{C}$

x

$\tilde{x}$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

$\underline{\mathcal{A}}$

$\tilde{C}$

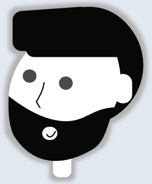
x

$\tilde{x}$

$\Phi(C)$

$C(x)$

$\underline{\mathcal{S}}$



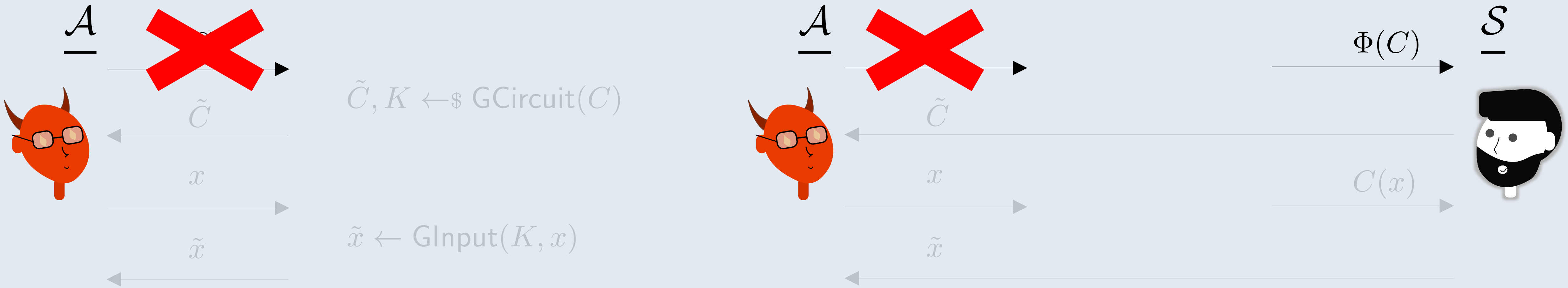
Adversary no longer chooses the circuit C

Instead, C comes from a sampler:

$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow_{\$} \mathsf{Sam}(1^{\lambda})$

Adaptive Distributional Simulation Security (DSIM)

Our DSIM notion:



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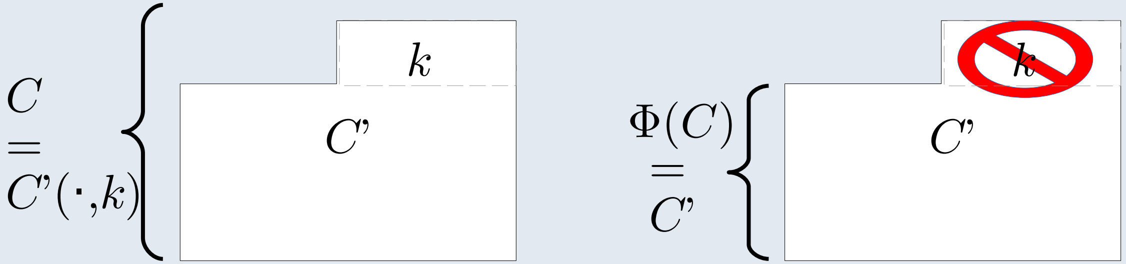

Adaptive Distributional Simulation Security (DSIM)

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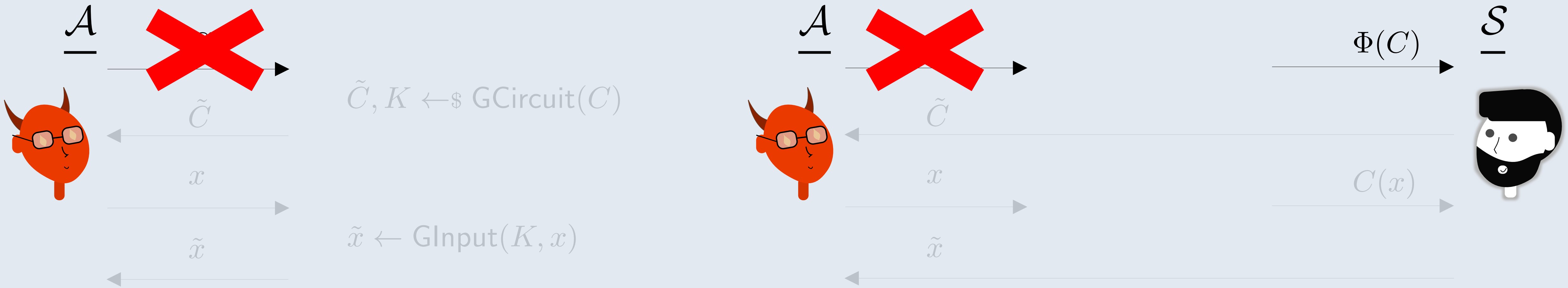

Adversary no longer chooses the circuit C

Instead, C comes from a sampler:
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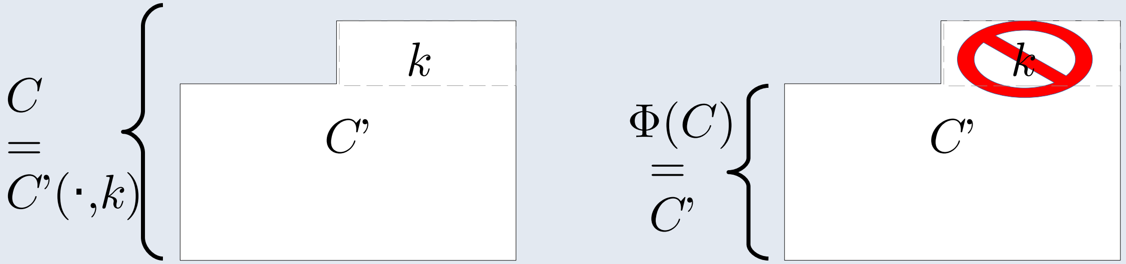
Think of $C = C'(\cdot, k)$ with public part $\Phi(C) = C'$ and secret k.



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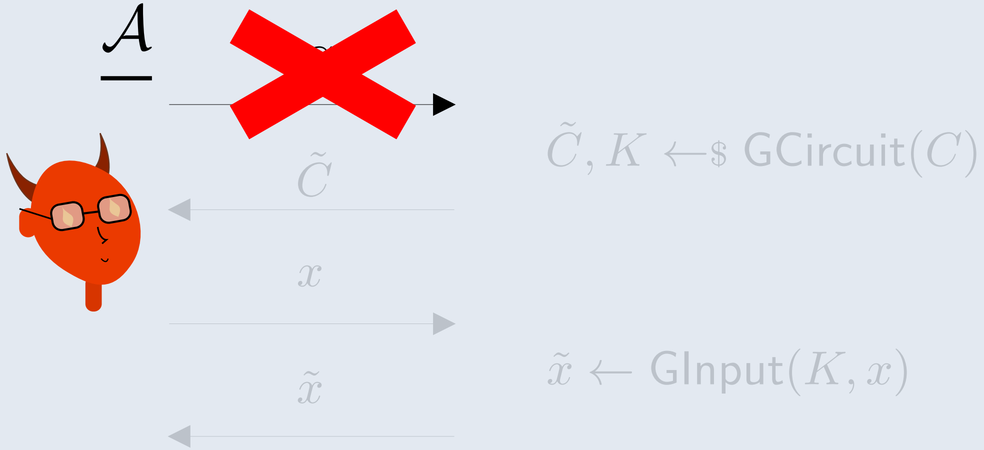

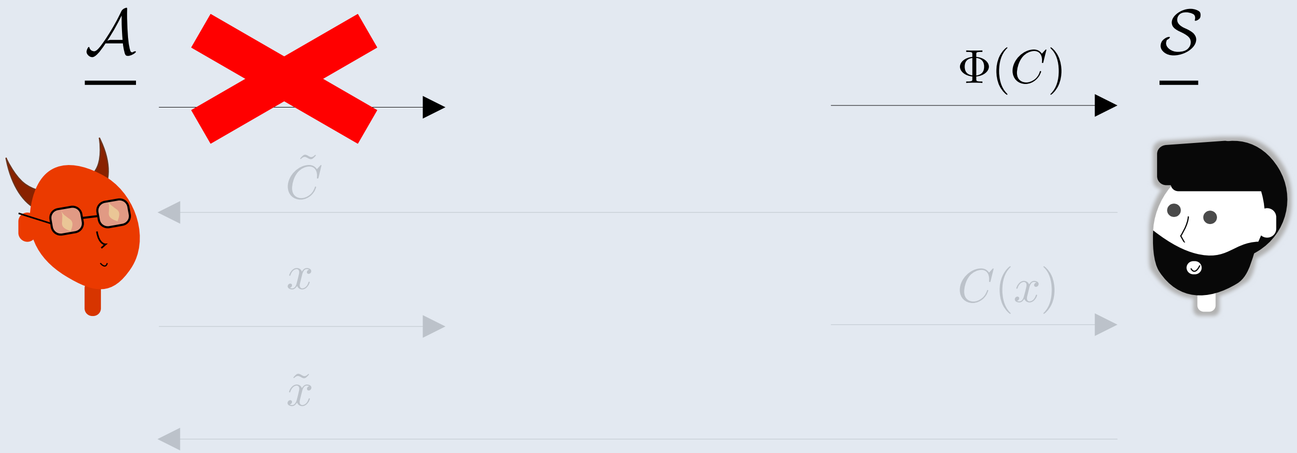

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Instead, C comes from a sampler:

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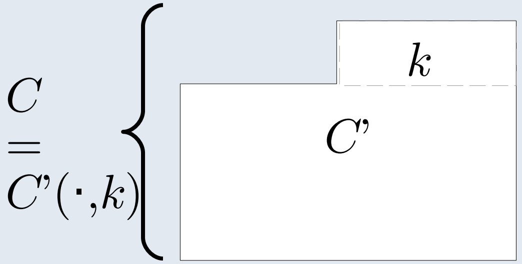
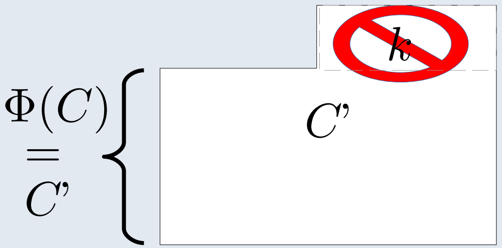
Think of $C = C'(\cdot, k)$ with public part $\Phi(C) = C'$ and secret k.

C is "cryptographic":

If k unknown, $C(x,k) \approx C(0^{|x|},k)$





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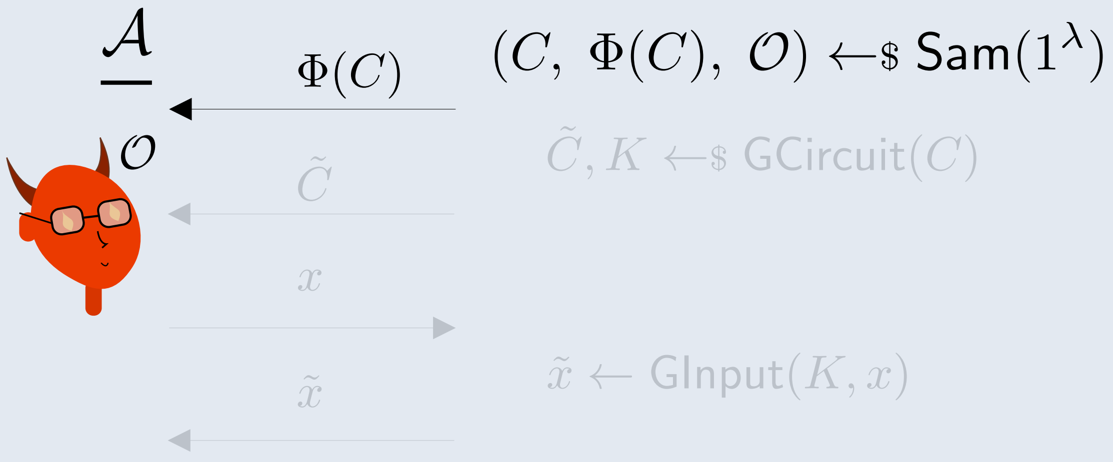

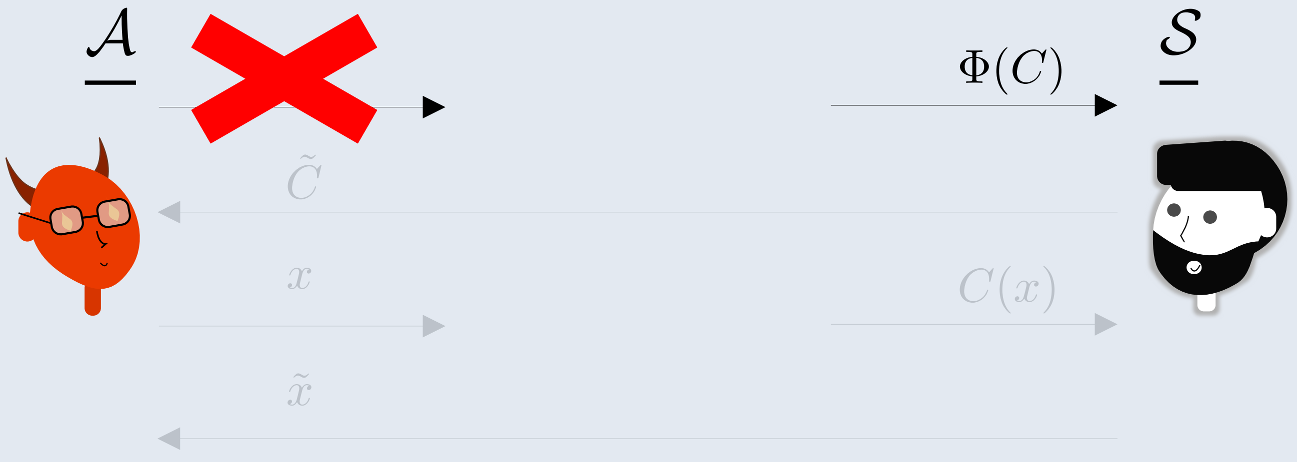

Instead, C comes from a sampler:

$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow\!\$\ \mathsf{Sam}(1^\lambda)$

Adaptive Distributional Simulation Security (DSIM)

Our DSIM notion:

$\underline{\mathcal{A}}$

$\Phi(C)$

$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow_{\$} \mathsf{Sam}(1^\lambda)$

$\mathcal{O}$

$\tilde{C}$

$\tilde{C}, K \leftarrow_{\$} \mathsf{GCircuit}(C)$

x

$\tilde{x}$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

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$\underline{\mathcal{A}}$ $\qquad$ $\underline{\mathcal{S}}$

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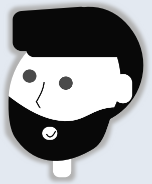

$\tilde{C}$

x $\qquad$ $C(x)$

$\tilde{x}$

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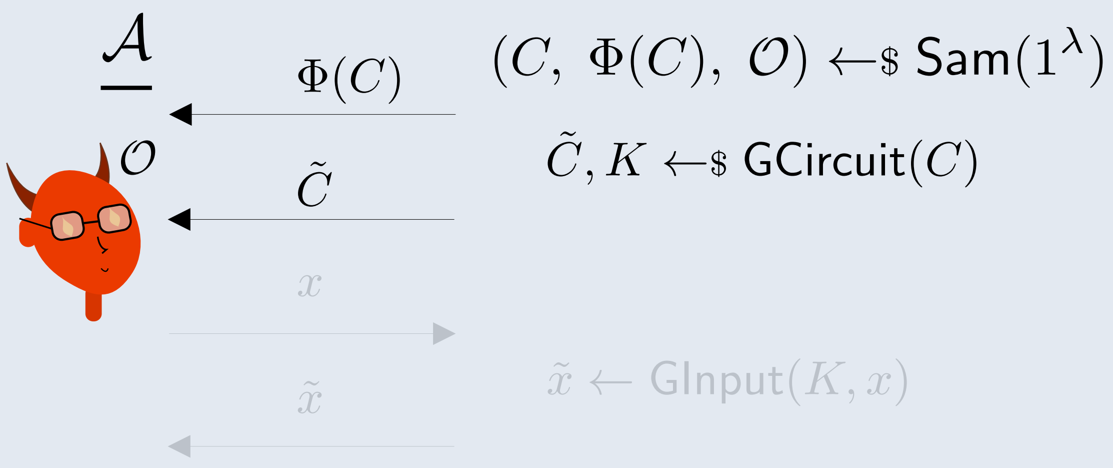

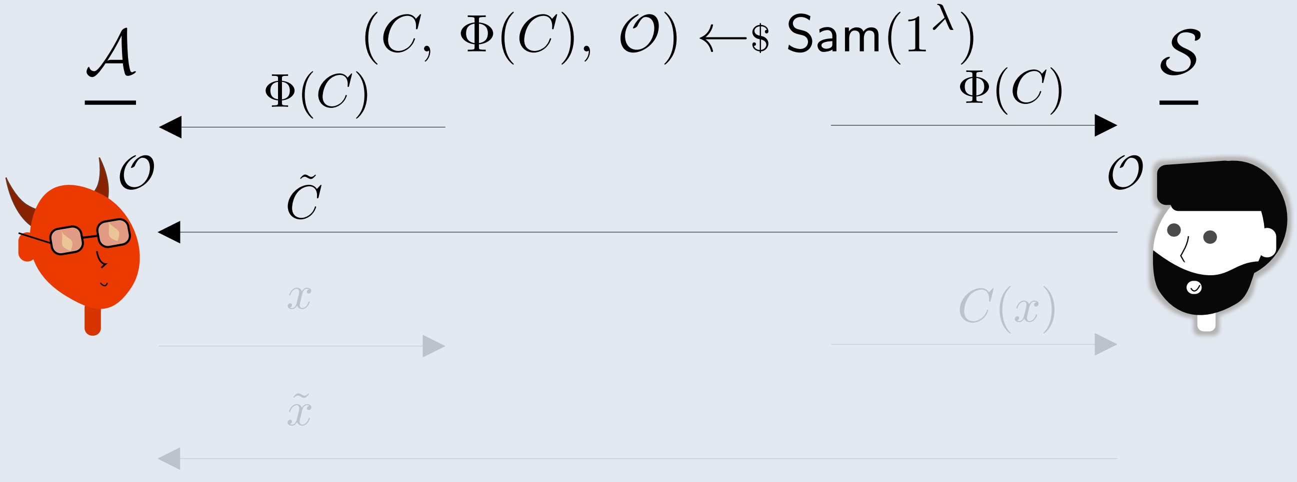

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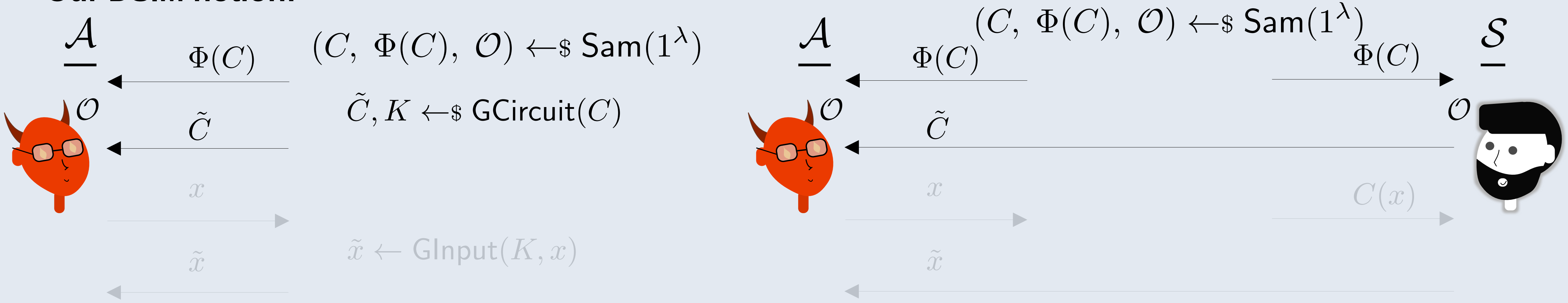

Next we make the simulator's *consistency* requirement flexible: (via leakage function $\Lambda(C,x)$)

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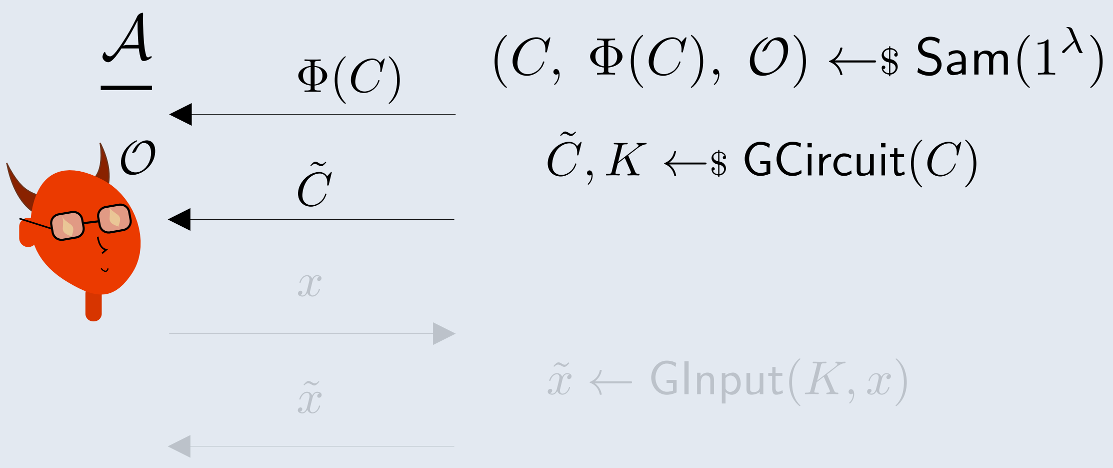

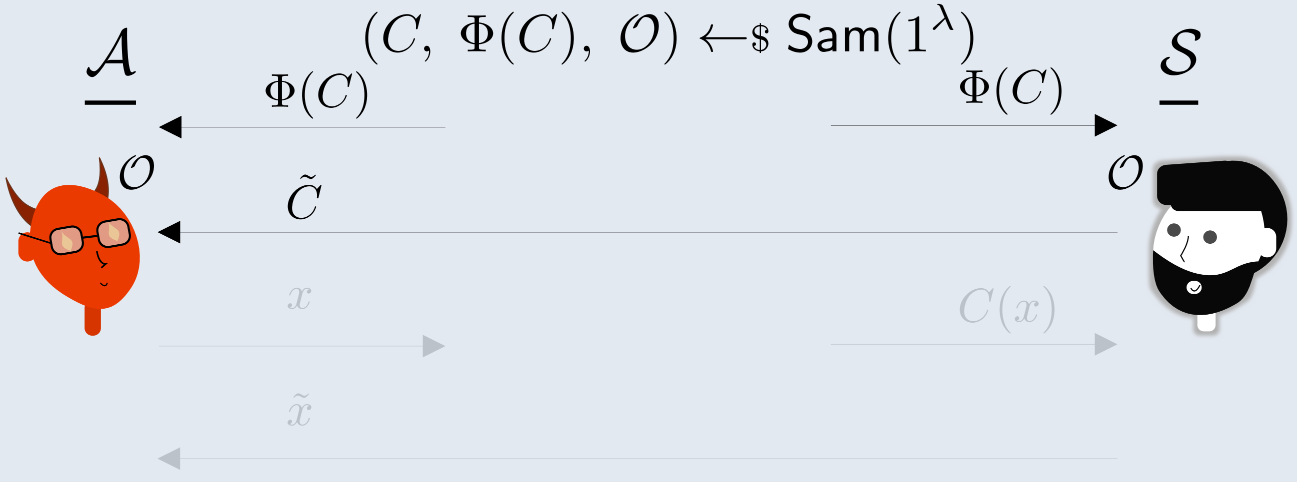

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In the extreme case (SIM) the simulator is given $\Lambda(C,x) = C(x)$ since it needs to produce the exact same output.

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$\tilde{C}$

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$\tilde{x}$

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$\mathcal{O}$

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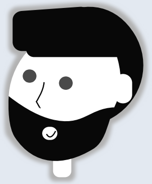
$\Phi(C)$

$\Phi(C)$

$\underline{\mathcal{S}}$

$\mathcal{O}$



$\tilde{C}$

x

$C(x)$

$\tilde{x}$

Next we make the simulator's *consistency* requirement flexible: (via leakage function $\Lambda(C,x)$)

In the extreme case (SIM) the simulator is given $\Lambda(C,x) = C(x)$ since it needs to produce the exact same output.

In the other extreme we can make $\Lambda(C,x) = \emptyset$ for *pseudorandom outputs*

Adaptive Distributional Simulation Security (DSIM)

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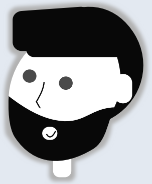

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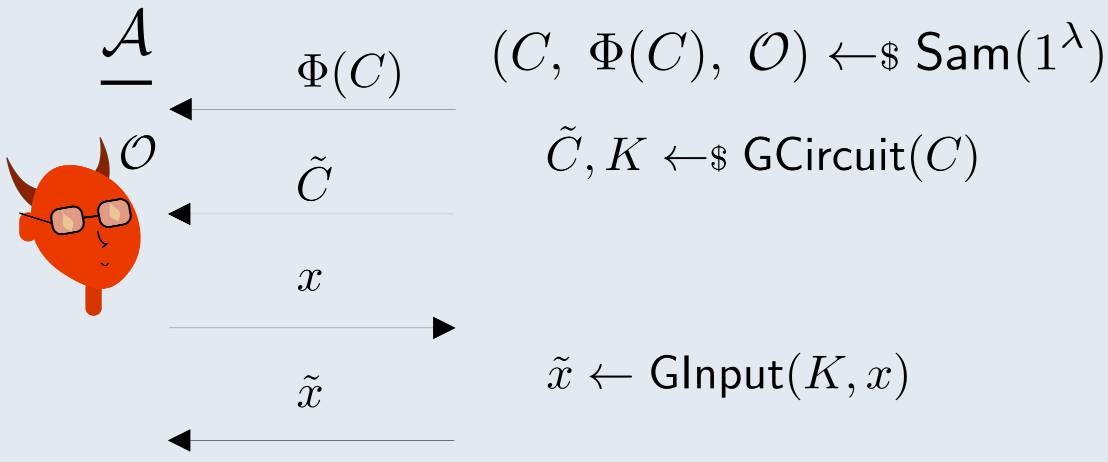

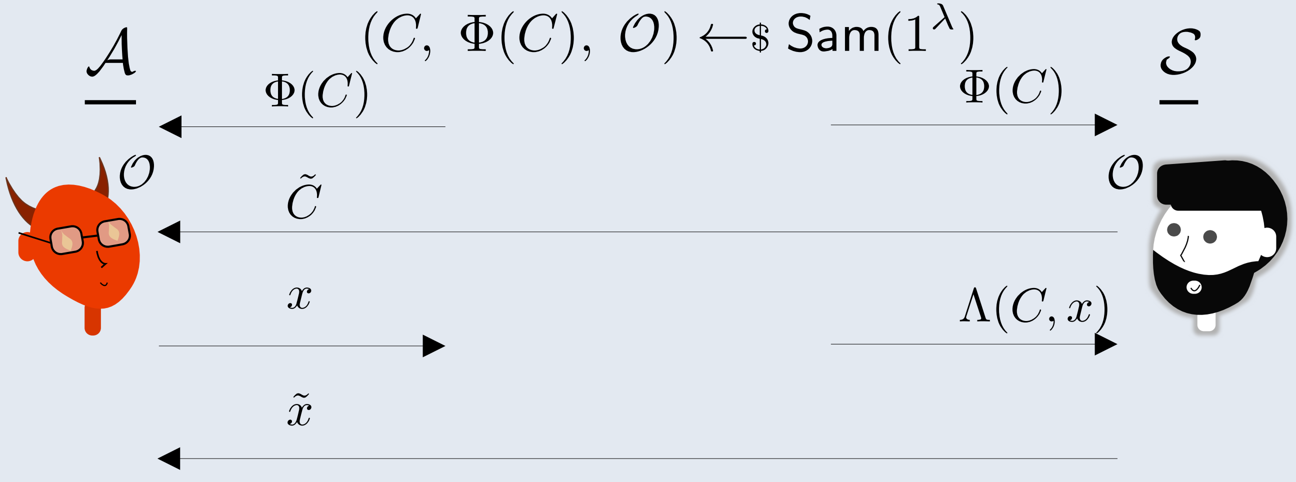

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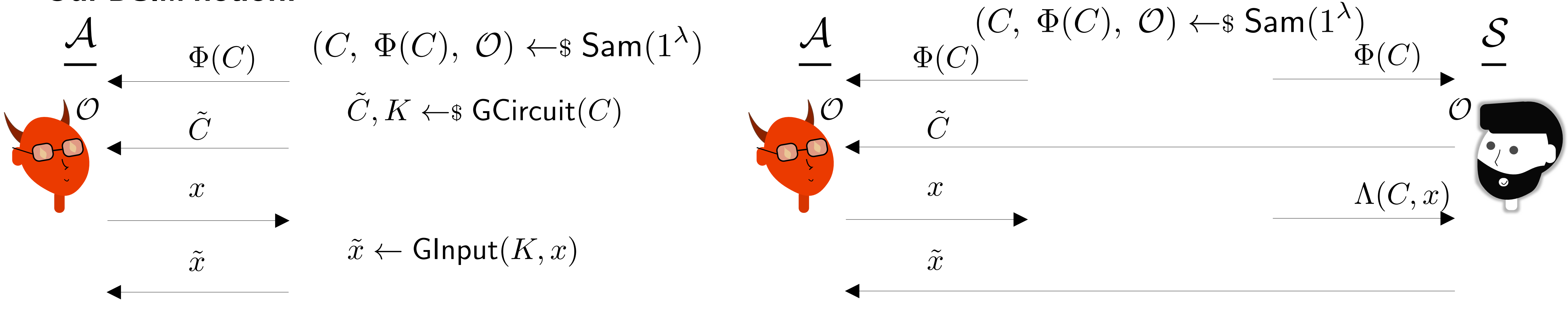

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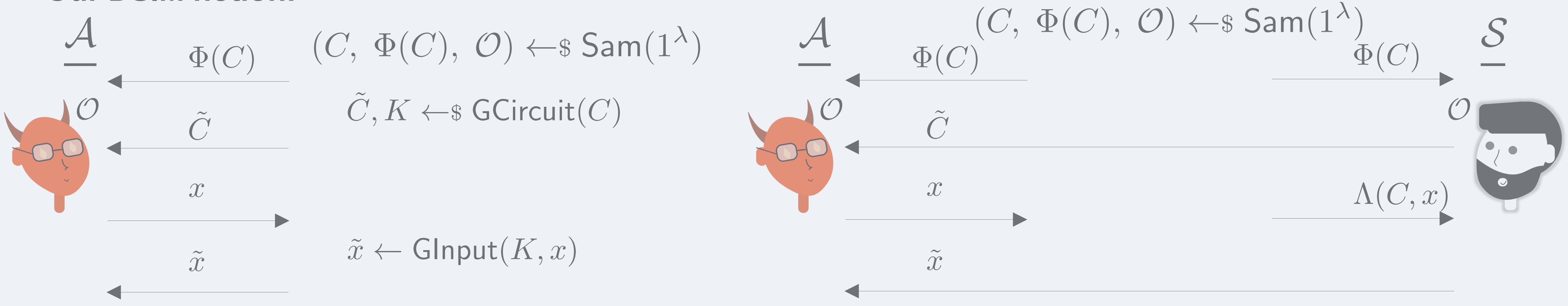
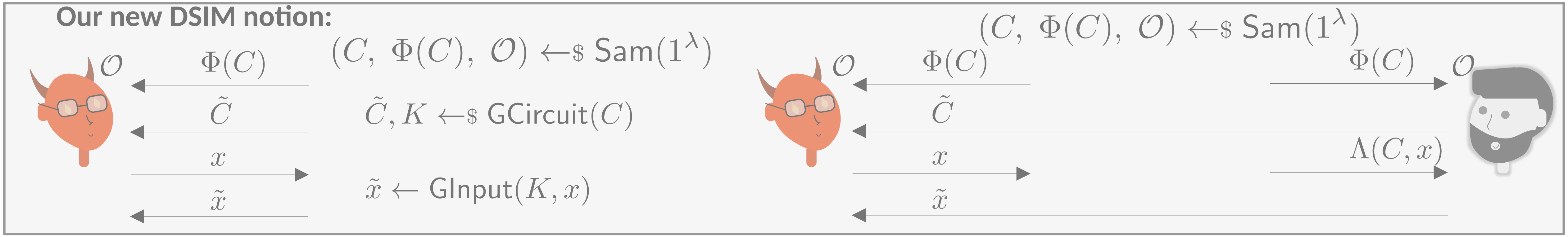


Let's look at an example application of DSIM, where we want to garble a length expanding cryptographic (encryption) circuit.

Example: Distributed Symmetric Encryption (DSE) from DSIM



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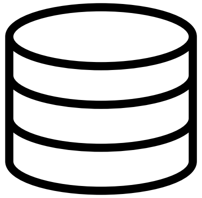

We want to store some encrypted data $enc(k, m)$ in some external database.

Our new DSIM notion:

$\mathcal{O}$

$\xleftarrow{\Phi(C)}$ $(C,\ \Phi(C),\ \mathcal{O}) \leftarrow\$\ \mathsf{Sam}(1^\lambda)$

$\xleftarrow{\tilde{C}}$ $\tilde{C}, K \leftarrow\$\ \mathsf{GCircuit}(C)$

$\xrightarrow{x}$

$\xleftarrow{\tilde{x}}$ $\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow\$\ \mathsf{Sam}(1^\lambda)$

$\mathcal{O}$

$\xleftarrow{\Phi(C)}$ $\xrightarrow{\Phi(C)}$ $\mathcal{O}$

$\xleftarrow{\tilde{C}}$

$\xrightarrow{x}$ $\xrightarrow{\Lambda(C, x)}$

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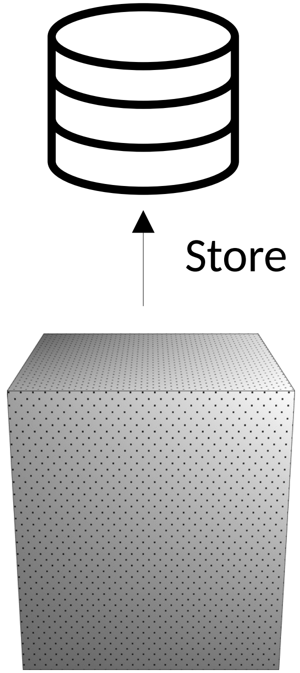

We want to store some encrypted data $enc(k, m)$ in some external database.

Store $enc(k, m)$.

Server keeps k.

Our new DSIM notion:

$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow_\$ \mathsf{Sam}(1^\lambda)$

$\mathcal{O}$ $\xleftarrow{\Phi(C)}$

$\xleftarrow{\tilde{C}}$ $\qquad \tilde{C}, K \leftarrow_\$ \mathsf{GCircuit}(C)$

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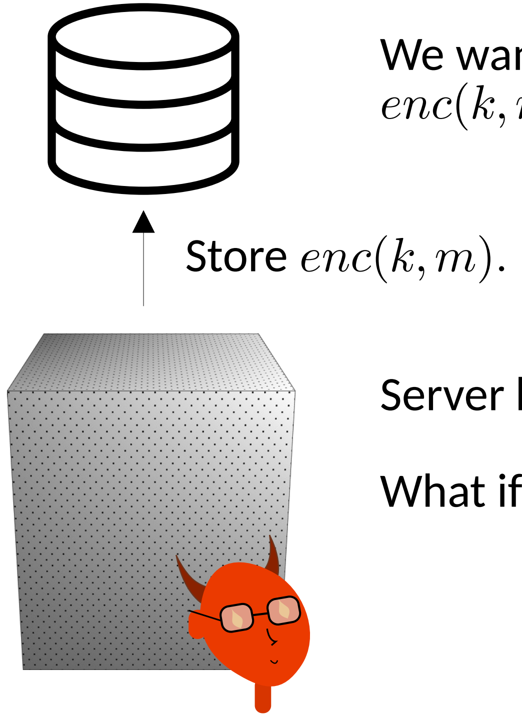
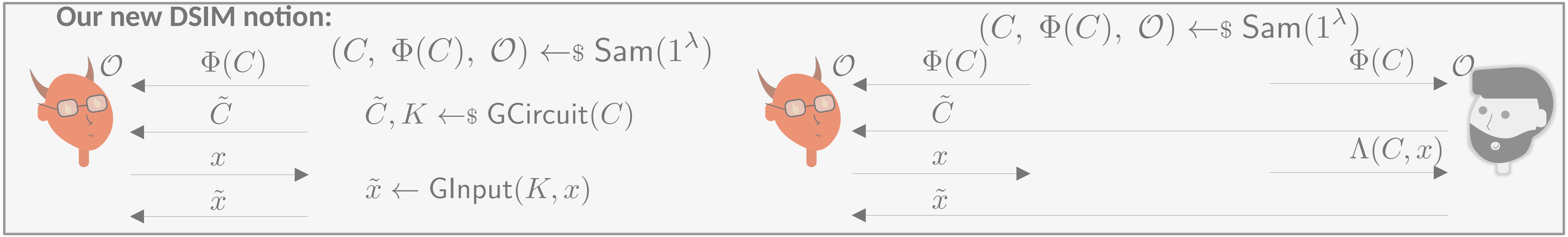


We want to store some encrypted data $enc(k, m)$ in some external database.

Server keeps k.

What if it gets corrupted?



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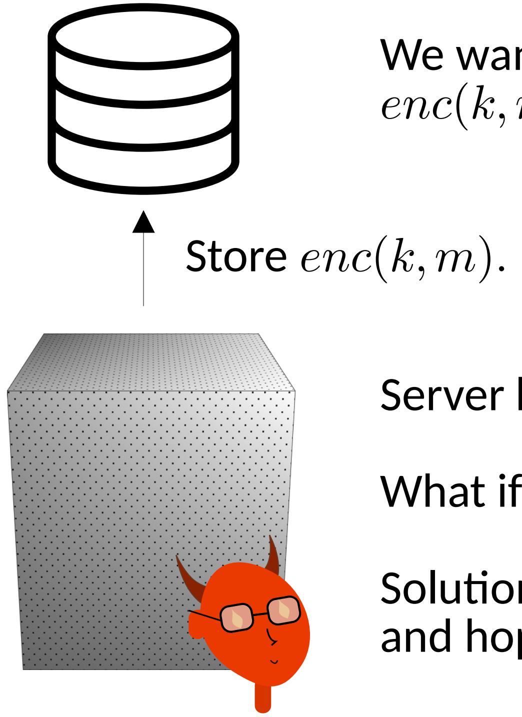


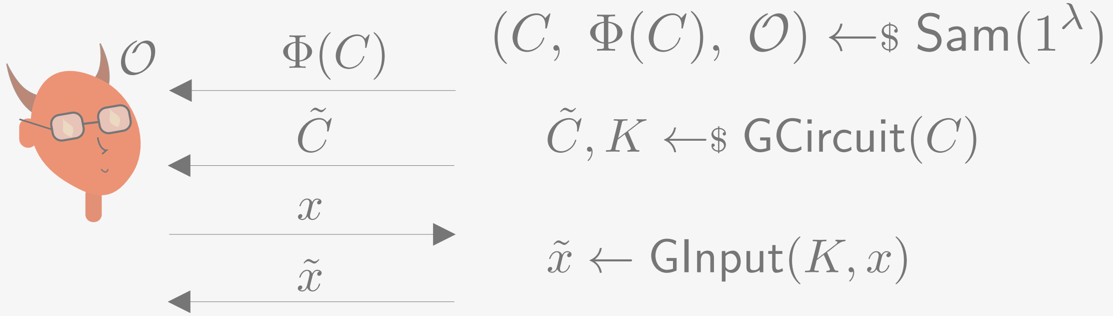
We want to store some encrypted data $enc(k, m)$ in some external database.

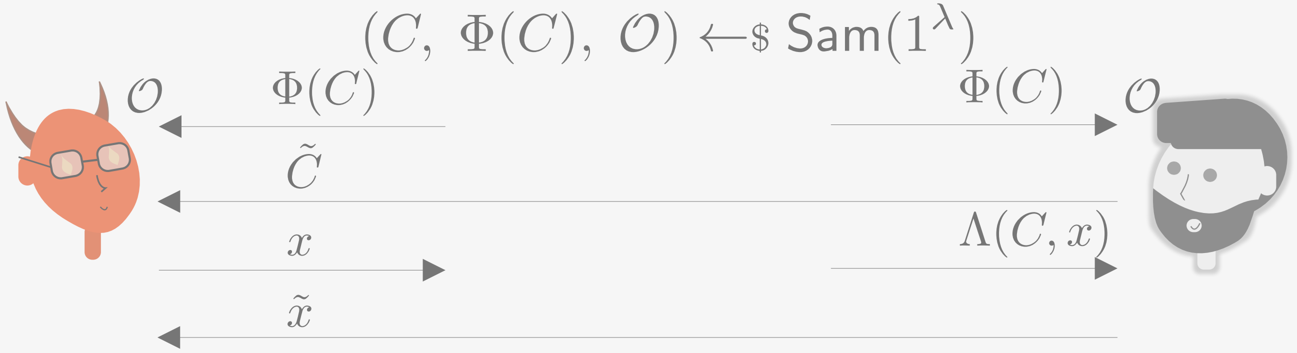
Server keeps k.

What if it gets corrupted?

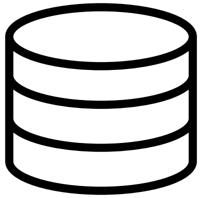
Solution: ***Secret share key*** k to many servers and hope that not all of them get corrupted.

Our new DSIM notion:





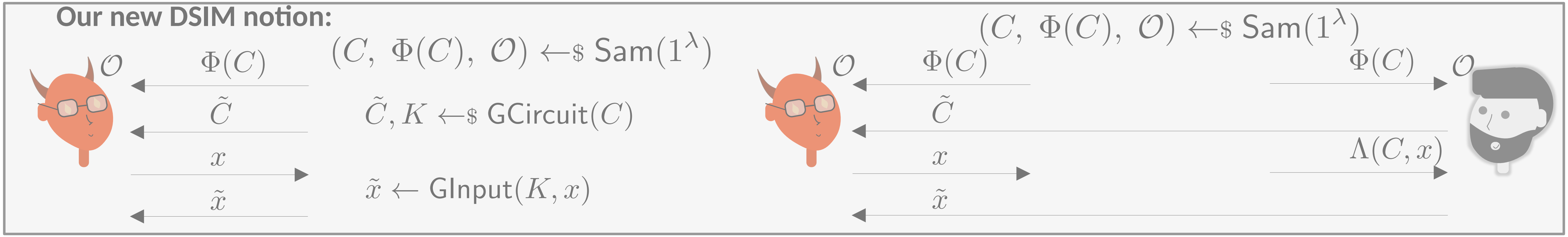
2-Party DSE Protocol



Key share k_1

($k = k_1 \oplus k_2$ secret shared in a setup phase)

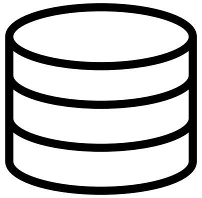
Key share k_2

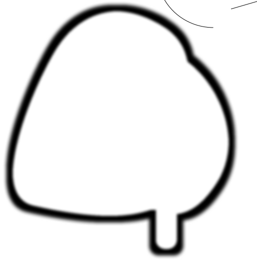


2-Party DSE Protocol

*Randomness made implicit to avoid clutter

Store $enc(k_1 \oplus k_2, m)$.





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$\mathcal{O}$

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$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow_\$ \mathsf{Sam}(1^\lambda)$

$\tilde{C}$

$\tilde{C}, K \leftarrow_\$ \mathsf{GCircuit}(C)$

x

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

$\tilde{x}$

$(C,\ \Phi(C),\ \mathcal{O}) \leftarrow_\$ \mathsf{Sam}(1^\lambda)$

$\mathcal{O}$

$\Phi(C)$

$\Phi(C)$

$\mathcal{O}$

$\tilde{C}$

x

$\Lambda(C, x)$

$\tilde{x}$

Goal: Store $enc(k_1 \oplus k_2, m)$.

2-Party DSE Protocol

Key share k_2

Key share k_1

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Goal: Store $enc(k_1 \oplus k_2, m)$.

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Key share k_2

Key share k_1

$$\underbrace{enc(k_1 \oplus \cdot, \cdot)}_{C}$$

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Goal: Store $enc(k_1 \oplus k_2, m)$.

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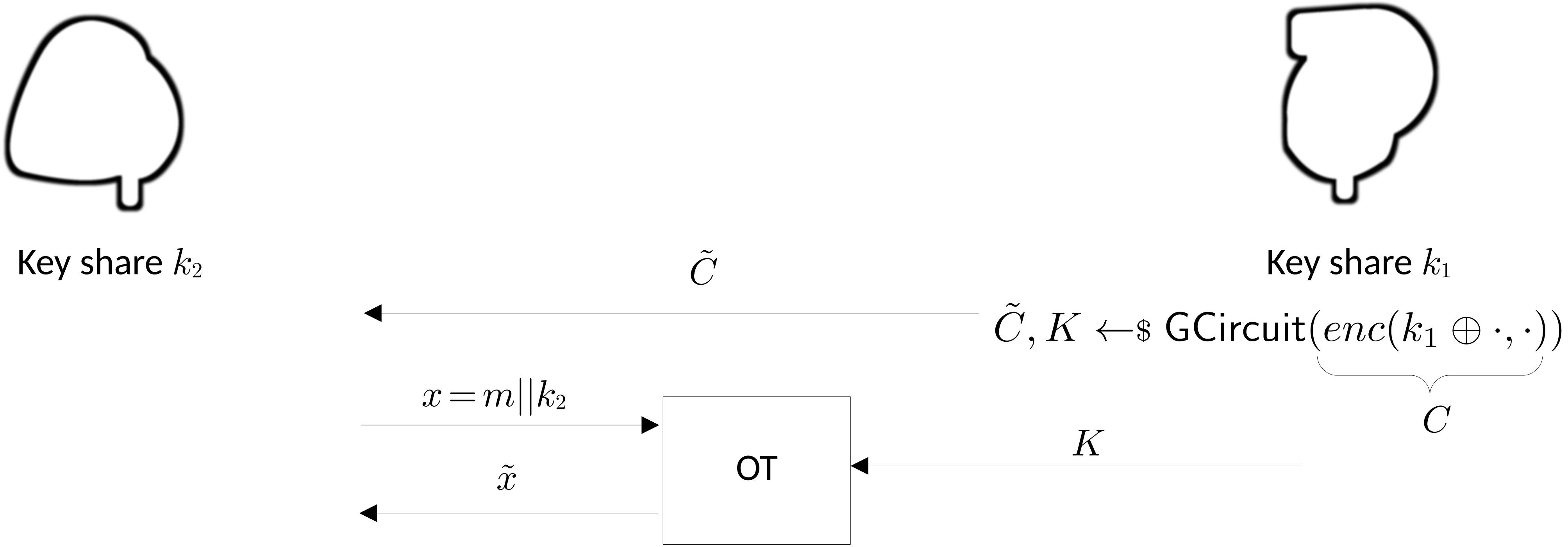
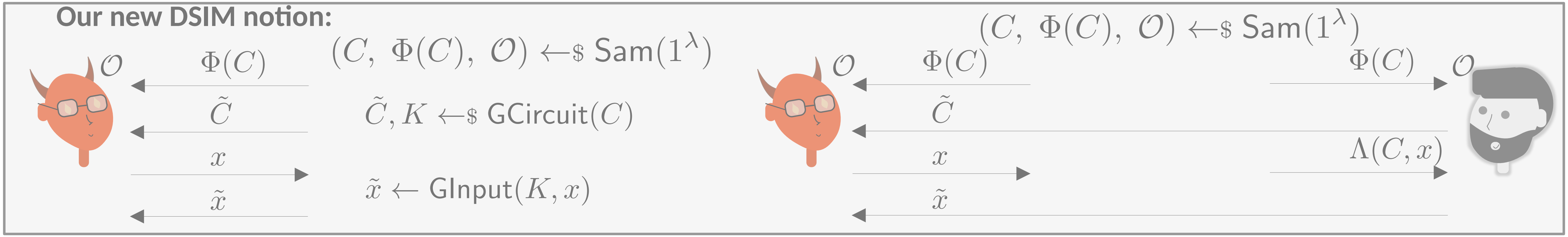
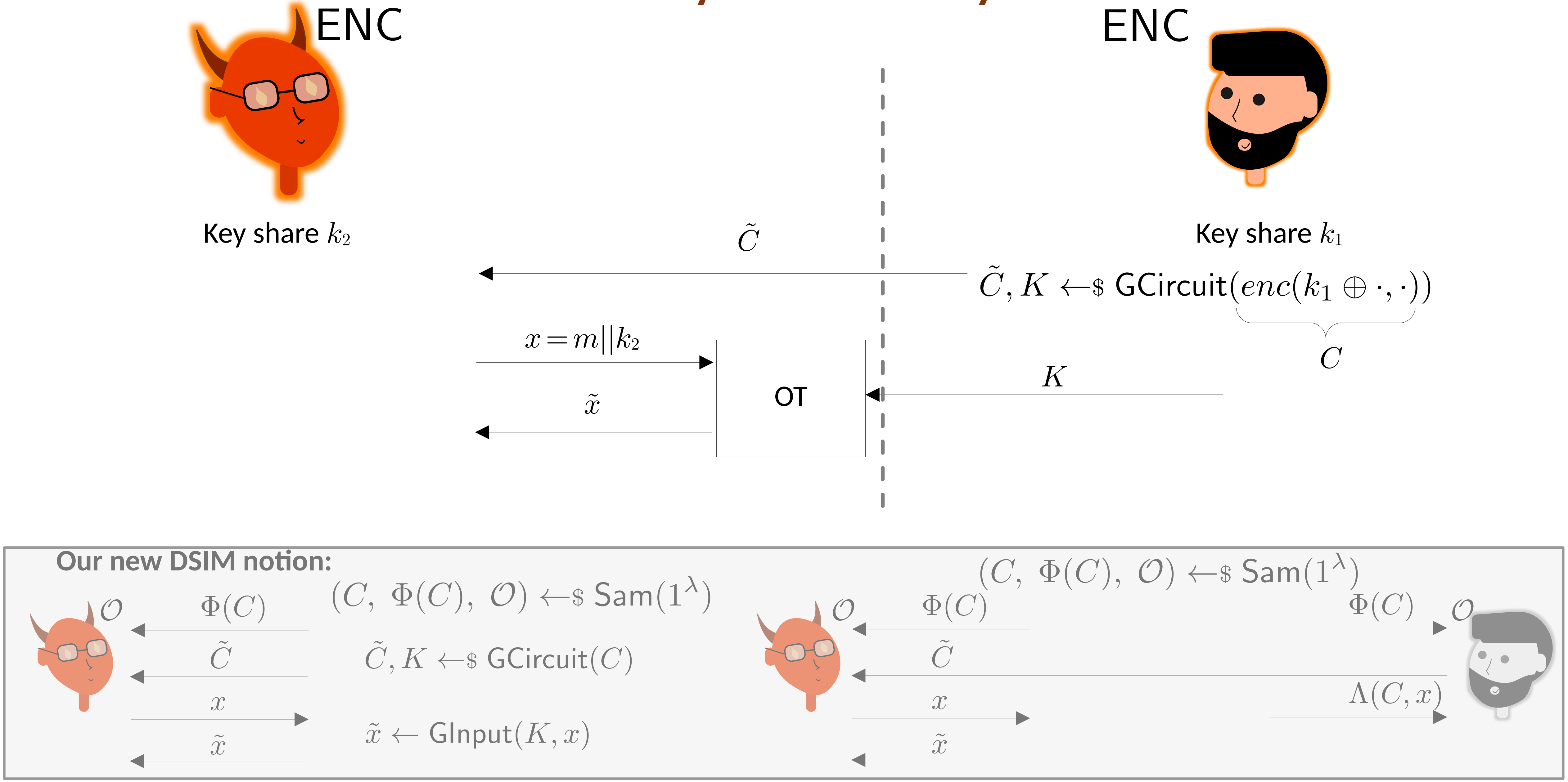




2-Party DSE: Security



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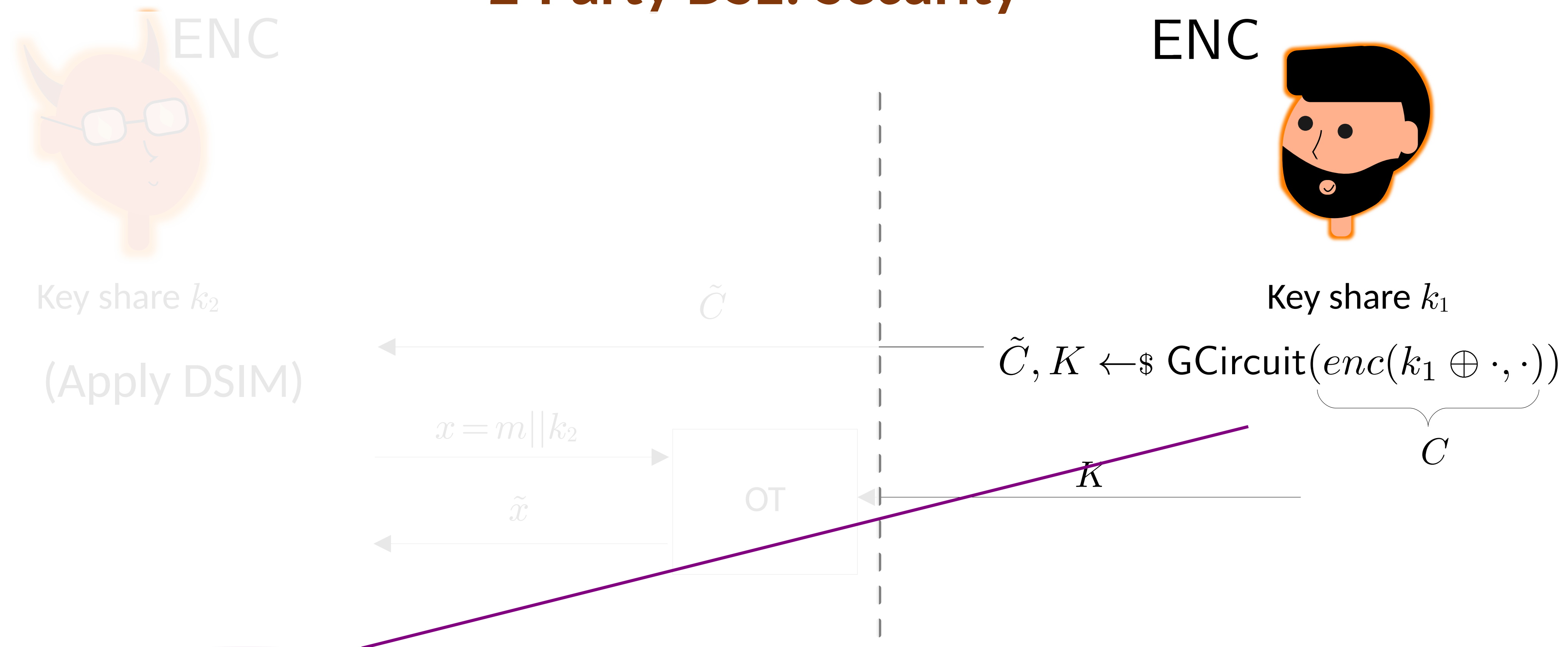

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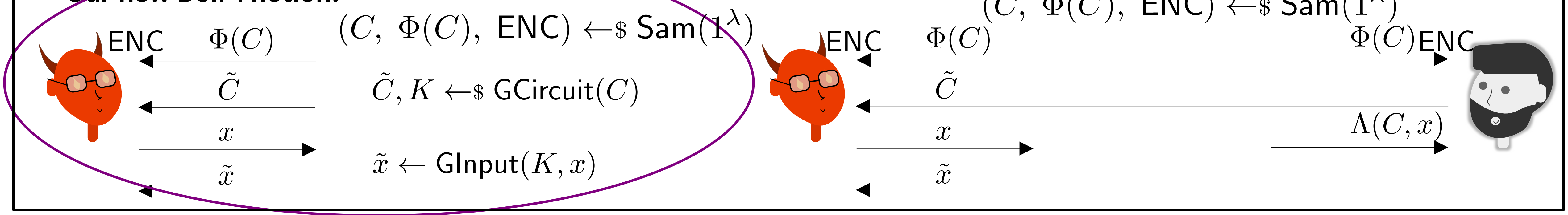


2-Party DSE: Security

ENC

Key share k_2

(Apply DSIM)

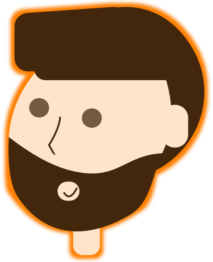
$\tilde{C}$

$x = m || k_2$

$\tilde{x}$

OT

ENC



Key share k_1

$\tilde{C}, K \leftarrow_\$ \mathcal{S}^{\mathsf{ENC}}(\Phi(C))$

K

Our new DSIM notion:

ENC

$\Phi(C)$

$\tilde{C}$

x

$\tilde{x}$

$(C,\ \Phi(C),\ \mathsf{ENC}) \leftarrow_\$ \mathsf{Sam}(1^\lambda)$

$\tilde{C}, K \leftarrow_\$ \mathsf{GCircuit}(C)$

$\tilde{x} \leftarrow \mathsf{GInput}(K, x)$

$(C,\ \Phi(C),\ \mathsf{ENC}) \leftarrow_\$ \mathsf{Sam}(1^\lambda)$

ENC

$\Phi(C)$

$\tilde{C}$

x

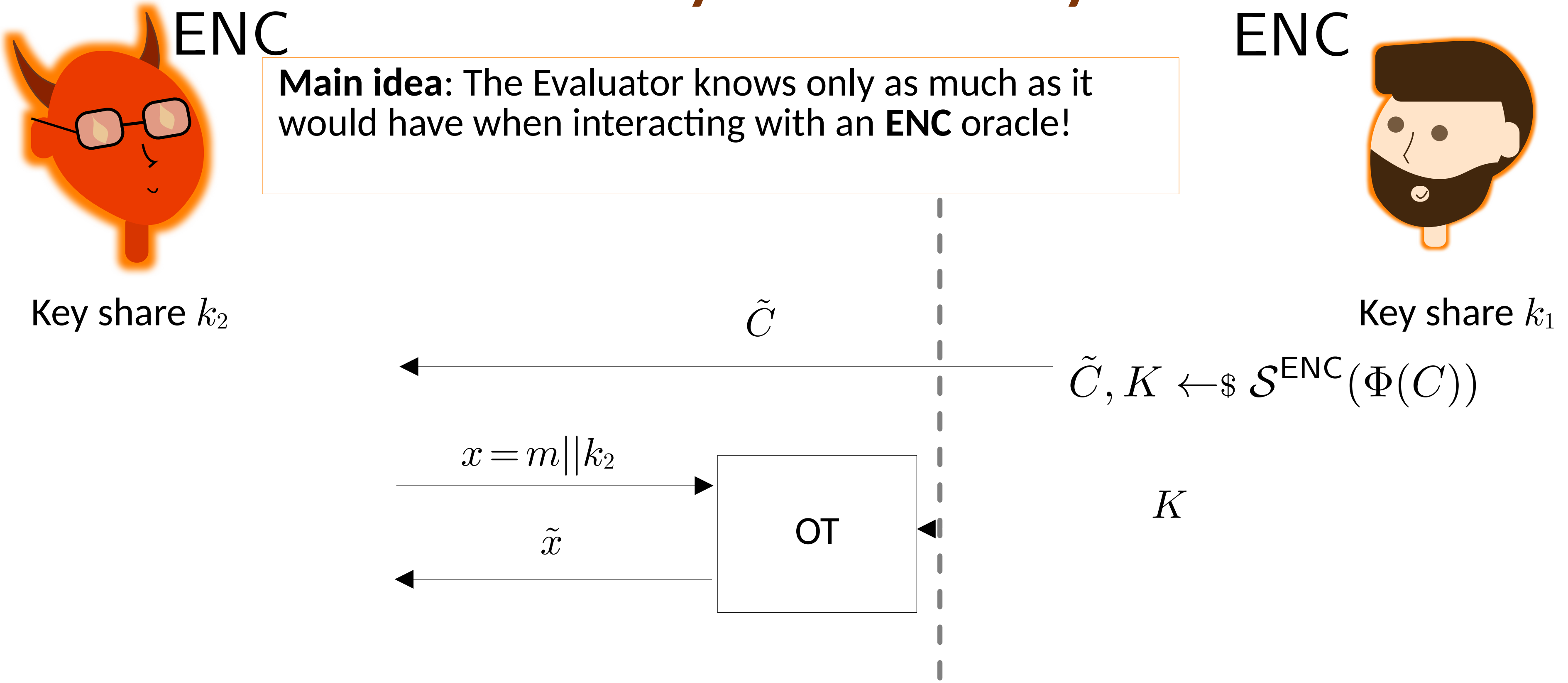
$\tilde{x}$

$\Phi(C)$ ENC

$\Lambda(C, x)$

2-Party DSE: Security

Main idea: The Evaluator knows only as much as it would have when interacting with an **ENC** oracle!



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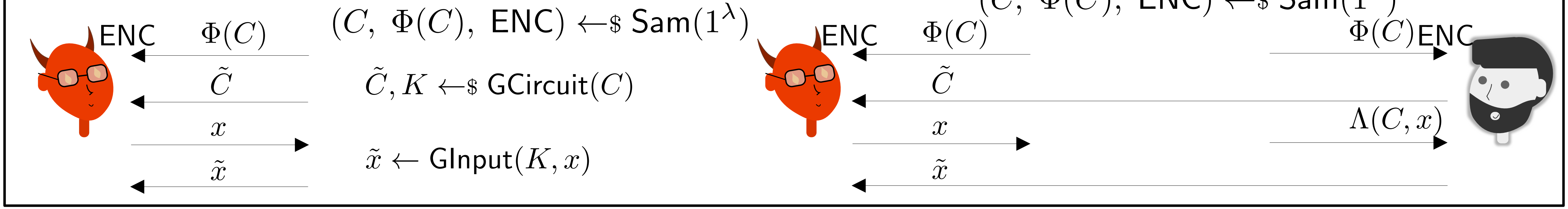

Summary of Results (Check out our paper! 😎)

1. DSIM Definition

2. Application of DSIM to DSE

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4. We observe that the *online complexity* of the Jafargholi-Scafuro-Wichs [**JSW'17**] construction for IND can be improved: $O(|x|)$ instead of $O(|x|+d)$.

5. Tighter online complexity lower bounds for SIM but for garbling schemes (improvement of the Hubacek-Wichs [**HW'14**] bound for MPC) via pseudo-entropy.

Open Questions

- Can we construct DSIM (from reasonable, standard assumptions)?
  In particular can we construct DSIM[**NC0**]?

- Would DSIM have applications beyond MPC?