Zero-Knowledge Functional Elementary Databases

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December 5, 2023

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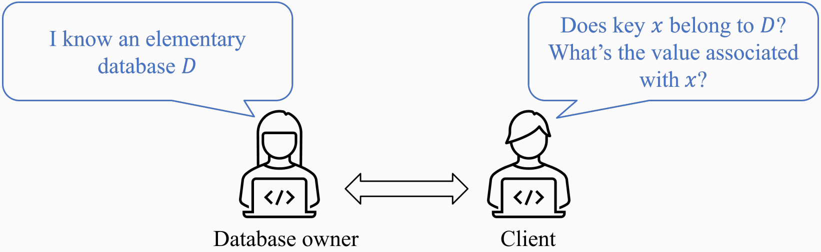
Backgroud

Zero-Knowledge Elementary Databases

Consider the following scenario:

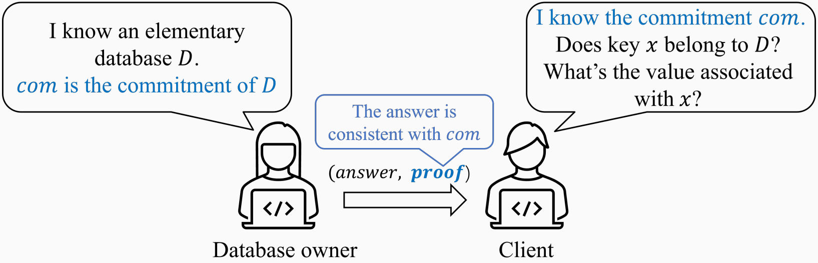
Let $D = \{(x, v)\}$ be an elementary database $((x, v) \in D, (x, v') \in D \Rightarrow v = v')$.



- The database owner cannot answer the queries inconsistently.
- The client cannot learn extra knowledge.

Zero-Knowledge Elementary Databases

Zero-Knowledge Elementary Databases (ZK-EDB):



A ZK-EDB consists of four algorithms (Setup, Com, Prove, Verify):

- Soundness: The database owner/committer cannot answer the same queries inconsistently.
- Zero-knowledge: The commitment and proof will not reveal any extra knowledge, **including the size of** D. The size of D is not contained in the input of simulator.

Zero-Knowledge Elementary Databases

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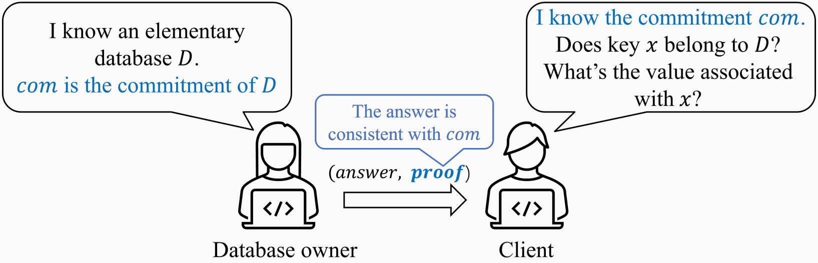

Application: End-to-end encrypted communication (E2EE) systems

Provide an auditable and queryable directory of their users' public keys (Key Transparent system).

The Quries of ZK-EDB

Most constructions:

- Follow the paradigm of Chase et al.
- Only support membership queries.

Libert et al.'s zero-knowledge expressive elementary databases:

- Modify Chase et al.'s paradigm.
- Support range queries over keys and/or values.

Question:
Can we construct ZK-EDB supporting richer queries?

Difficulties

A naive attempt:

Commitment + zk-SNARKs

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Commitment + zk-SNARKs

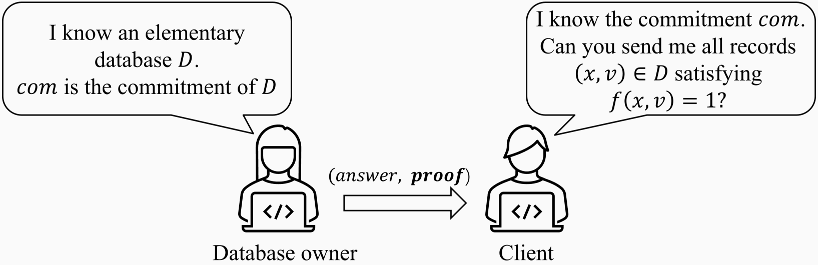
However, this attempt would fail due to the potential revelation of the database size.

- Almost all zk-SNARKs expose the length of the witness.
- For generalize functional query, the witness must include all records in database to ensure the correctness of query.

Our Contributions

Our Contributions

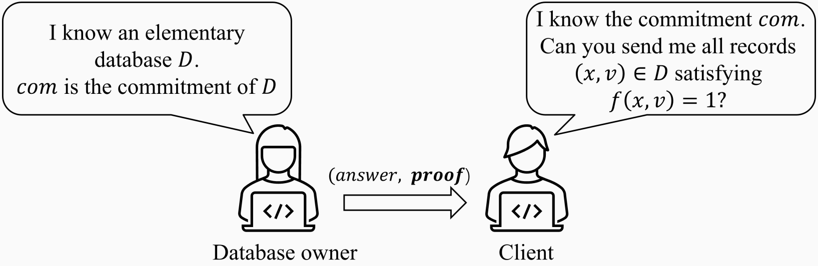
Zero-Knowledge Functional Elementary Databases (ZK-FEDB)



- Allow **the most generalize functional queries**: For any Boolean circuit f, clients can query that: "Send me all records $(x, v) \in D$ satisfying $f(x, v) = 1$."
- Function Binding (Soundness) and Zero-Knowledge.

Our Contributions

Zero-Knowledge Functional Elementary Databases (ZK-FEDB)



Construction based on unknown-order group.

- Proof size: $O(|(x, v)| + |f|)$ (independent of $|D|$)
- Secure in the random oracle model and generic group model.

Technique Contributions

Our technical constribution is two-fold.

- **A new variant of zero-knowledge sets (ZKS)**: Support combined operations queries on committed sets.
- **A new transformation technique**: Transform the query of Boolean circuit into a query of combined operations on related sets.

Note.
ZKS: the "set" version of ZK-EDB, committing sets rather than databases.
Combined operation: a "circuit" with gates "intersection", "union" and "set-difference".

Zero-Knowledge Sets with Set-Operation Queries

Start from RSA Accumulators

RSA Accumulator

- g: The ganerator of an unknown-order group.
- Commitment of set $S = \{x_i\}_{i \in [m]}$:

$$\mathsf{C} = \mathsf{g}^{\prod_{i \in [m]} p_i}$$

  where $p_i = \mathcal{H}_{prime}(x_i)$ is a prime.
- Membership proof of $x_j \in S$: g_j satisfying $\mathsf{g}_j^{p_j} = \mathsf{C}$.
- Non-membership proof of $x \notin S$: (a, b) satisfying $\mathsf{C}^a \mathsf{g}^{b\mathcal{H}_{prime}(x)} = \mathsf{g}$.

A pair of membership proof and non-membership proof of same element can be used to break strong RSA assumption.

Basic Set Operations

Basic Set Relation:
"Intersection, Union, Set-Defference"

$\Downarrow$

Simpler set relations:

- Disjoint relation
$$\{(J_0, J_1) | J_0 \cap J_1 = \emptyset\}$$
- Union among disjoint relation
$$\left\{ (U, J_0, J_1) \middle| \begin{array}{l} U = J_0 \cup J_1 \wedge \\ J_0 \cap J_1 = \emptyset \end{array} \right\}$$
.

Basic Set Relation on Commitments:
"Intersection, Union, Set-Defference"

$\Downarrow$

Group Element Relations:

- Co-prime relation
$$\left\{ (\mathrm{C}_1, \mathrm{C}_2) \middle| \begin{array}{r} \exists a, b \in \mathbb{Z} \; s.t. \\ \gcd(a, b) = 1 \wedge \\ (\mathrm{C}_1, \mathrm{C}_2) = (\mathrm{g}^a, \mathrm{g}^b) \end{array} \right\}.$$
- DDH tuples relation

Zero-Knowledge Sets

RSA accumulators can be convert into ZKS by adding randomness r to provide privacy.

$$\mathrm{g}^{\prod_{i\in[m]} p_i} \Rightarrow \mathrm{g}^{r\cdot\prod_{i\in[m]} p_i}.$$

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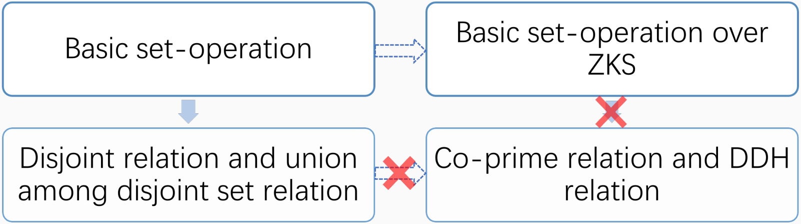

Zero-Knowledge Sets

Key observation:
In ZKS commitment, randomness is sampled from small and bounded range of $[0, B]$.

- Let A, B be disjoint sets, $g^{r \cdot \mathcal{H}_{prime}(A)}, g^{r' \cdot \mathcal{H}_{prime}(B)}$ are their ZKS commitments.

$$\gcd(r \cdot \mathcal{H}_{prime}(A), r' \cdot \mathcal{H}_{prime}(B)) = \gcd(r, r') \text{ is small}$$

- Let A, B be disjoint sets, $U = A \cup B$, $g^{r \cdot \mathcal{H}_{prime}(A)}, g^{r' \cdot \mathcal{H}_{prime}(B)}$, $g^{r'' \cdot \mathcal{H}_{prime}(U)}$ are their ZKS commitments.

$$(g^{r \cdot \mathcal{H}_{prime}(A)}, g^{r' \cdot \mathcal{H}_{prime}(B)}, g^{r'' \cdot \mathcal{H}_{prime}(U)}) \text{ is close to a DDH-tuple}$$

We call above two relations as pseudo-coprime relation and pseudo-DDH relation.

Zero-Knowledge Protocol

Tools:

1. Schnorr's Σ-protocol for bounded discrete-log:

$$\mathcal{R}_{boundedDL} = \{(\mathsf{u}, \mathsf{w}, T; x) | \mathsf{u}^x = \mathsf{w} \wedge |x| \leq T\}$$

2. (A new variant of) Boneh et al.'s ZK-argument for multidimensional discrete-log.

$$\mathcal{R}_{multiDL} = \{(\{\mathsf{u}_i\}_{i\in[n]}, \mathsf{w}; \{x_i\}_{i\in[n]}) | \Pi_{i\in[n]} \mathsf{u}_i^{x_i} = \mathsf{w}\}$$

Note: Both of above protocols only achieve a weak soundness due to that "Computing $\mathsf{g}^{\frac{1}{a}}$ in an unknown-order group is hard". Luckily, it is sufficient for our construction.

Zero-Knowledge Protocol for Pseudo-Coprime Relation

Prove pseudo-coprime relation:

$$\{(u, v, T; a_1, a_2) | u = g^{a_1}, v = g^{a_2}, \gcd(a_1, a_2) < T\}$$

⇕ Equivalence

Find a, b and prove:

$$u^a \cdot v^b = g^{\gcd(a_1,a_2)} \wedge \gcd(a_1, a_2) < T$$

⇓ Add randomness $r < B$ to mask $g^{\gcd(a_1,a_2)}$.

ZK protocol

Send $g^{r\cdot\gcd(a_1,a_2)}$ and prove:

$$u^{ra} \cdot v^{rb} = g^{r\cdot\gcd(a_1,a_2)} \wedge r \cdot \gcd(a_1, a_2) < BT$$

- $u^{ra} \cdot v^{rb} = g^{r\cdot\gcd(a_1,a_2)}$: ZK for multi-DL
- $r \cdot \gcd(a_1, a_2) < BT$: Schnorr's Σ-protocol

- Only achieve a weak soundness. (The GCD of exponents might be larger than T, however, it is still bounded by a proper upper bound.)
- One can use the Fiat-Shamir heuristic to obtain the non-interactive version.

Zero-Knowledge Protocol for Pseudo-DDH Relation

Prove pseudo-DDH relation:
$u = g^{a_1 x}, v = g^{a_2 y}, w = g^{a_3 xy} \wedge a_1, a_2, a_3 < T$

Choose randomness $r_1, r_2 < B$ to generate a close DDH-tuple: $(u' = g^{r_1 x}, v' = g^{r_2 y},\ w' = g^{r_1 r_2 xy})$

ZK protocol

Send (u', v', w') and prove:

(u', v', w') is a DDH-tuple $\wedge$ (u', v', w') and (u, v, w) is close

Boneh et al's PoDDH — Schnorr's Σ-protocol

- Only achieve a weak soundness. (That is, the statement might not close to DDH-tuple as we required, however, it is still close enough.)
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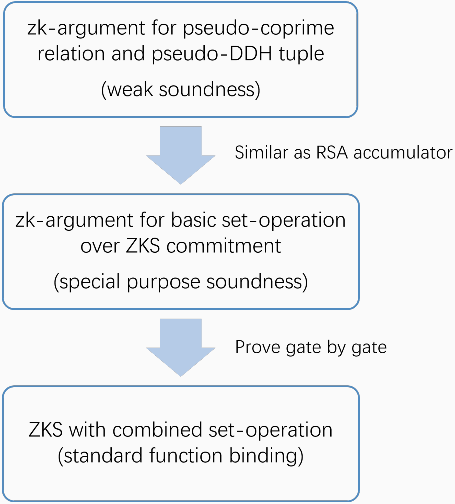
Zero-Knowledge Sets



From Boolean Circuit Queries to Set-Operation

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Our goal:

Query of Boolean circuit f over a set S
(requesting $S_{output} := \{x | x \in S \wedge f(x) = 1\}$)

$$\Downarrow$$

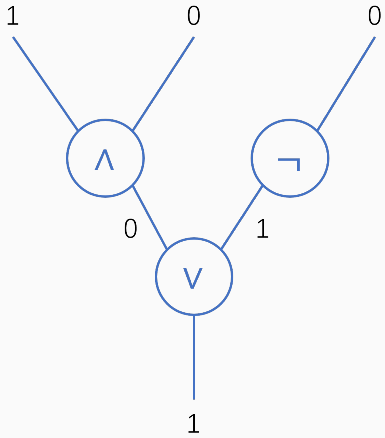
Query of combined operations $\mathcal{Q}$ on related sets
$S_i^b := \{x | x \in S \wedge \text{ the i-th bit of ``}x\text{'' is } b\}$
(requesting $S_{output} := \mathcal{Q}(\{S_i^b\})$)

From Boolean Circuit Queries to Set-Operation

Let f be a Boolean circuit.

- When running f on an input, each wire in f has a value.

Example:



From Boolean Circuit Queries to Set-Operation

Let f be a Boolean circuit.

- When running f on an input, each wire in f has a value.
- When running f on a set S, according the value of wire, each wire i can be associated with two subsets $\{S_i^b\}_{b\in\{0,1\}}$. That is, $S_i^b := \{x | x \in S \wedge \text{ the value of i-th wire of } f(x) \text{ is } b\}$

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Key Observation:

- For each input wire i, $S_i^b = \{x | x \in S \wedge \text{ the i-th bit of ``}x\text{'' is } b\}$.

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Key Observation:

- For each input wire i, $S_i^b = \{x | x \in S \wedge \text{ the i-th bit of "}x\text{" is } b\}$.
- For the output wire *output*, the second associated set S_{output}^1, is exactly the answer of the query of Boolean circuit f.

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- For any AND gate in f with input wires a, b and output wire c, $S_c^0 = S_a^0 \cup S_b^0$ and $S_c^1 = S_a^1 \cap S_b^1$.

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- For any OR gate with input wires a, b and output wire c, $S_c^0 = S_a^0 \cap S_b^0$ and $S_c^1 = S_a^1 \cup S_b^1$.

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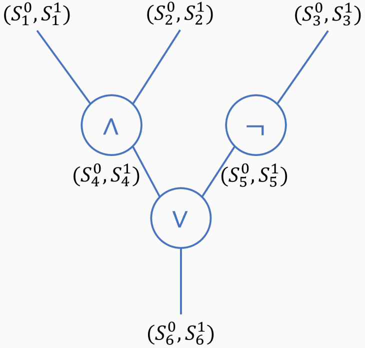
- For any OR gate with input wires a, b and output wire c, $S_c^0 = S_a^0 \cap S_b^0$ and $S_c^1 = S_a^1 \cup S_b^1$.
- For any NOT gate with input wire a and output wire b, $S_b^0 = S_a^1$ and $S_b^1 = S_b^0$.

From Boolean Circuit Queries to Set-Operation

Example: $f(x) = \bar{x}_1 \wedge \bar{x}_2 \vee (\neg \bar{x}_3)$ where $x = \bar{x}_1 \| \bar{x}_2 \| \bar{x}_3 \in \{0,1\}^3$



$$S_4^0 = S_1^0 \cup S_2^0$$
$$S_4^1 = S_1^1 \cap S_2^1$$
$$S_5^0 = S_3^1$$
$$S_5^1 = S_3^0$$
$$S_6^0 = S_4^0 \cap S_5^0 = S_1^0 \cup S_2^0 \cap S_3^1$$
$$S_6^1 = S_4^1 \cup S_5^1 = S_1^1 \cap S_2^1 \cup S_3^0$$

Output set:

$$S_{output} = S_6^1 = S_1^1 \cap S_2^1 \cup S_3^0$$

Zero-Knowledge Functional Elementary Databases

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Setup(1^λ): Genrate using public parameters.

Commit(D): Let $S_i^b := \{x | (x, v) \in D \wedge \text{the i-th bit of "} x \| v \text{" is } b\}$

1. Use ZKS scheme to commit all S_i^b.
2. Use ZK-EDB scheme to commit D.

Prove(com, τ, f, D_{output}): Transform f into combined operation $\mathcal{Q}$,

1. Prove that for each $(x, v) \in D_{output}$ and each i, $x \in S_i^{\bar{x}_i}$ and $x \notin S_i^{1-\bar{x}_i}$.
   Showing the correctness of S_b^i.
2. Prove that $\{x | (x, v) \in D_{output}\} = \mathcal{Q}(S_1^0, S_1^1, \cdots)$.
   Showing the correctness of function.
3. Prove that for each $(x, v) \in D_{output}$, $(x, v) \in D$ through ZK-EDB.
   Showing the validness of associated value v.

Verify(com, f, D_{output}, π): Check the correctness of proofs.

Performance

Performance of our ZK-FEDB[1]:

	Prover's work	Verifier's work	Communication
Commit	$O(\ell\|D\|)\text{EXT} + O(\|D\|)h$	N/A	$O(\ell)\mathbb{G}$
Query	$O(\ell\|D\| + \|D\|\|f\|)\text{EXT}$ $+O(\|D\| + \ell + \|f\|)h$	$O(\ell + \|f\|)\text{EXT}$ $+O(\|D_{output}\| + \ell + \|f\|)h$	$O(\ell + \|f\|)\mathbb{G}$

where ℓ is the bit length of record, $|D|$ and $|D_{output}|$ denote the size of committed database and output database respectly, $|f|$ is the size of query function, $\mathbb{G}$ represents a group element, h denotes hashing to a prime and EXT is a λ-bit exponentiation.

[1]Utilizing our ZKS scheme and ZK-EDB scheme (constructed in the full version of our paper), and applying the standard batching technique.

Thank you for your attention