Just How Fair is an Unreactive World?

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Yibin Yang, Georgia Tech

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Fair

Fair Secure Multiparty Computation

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Let’s recall what MPC is

Fair Secure Multiparty Computation

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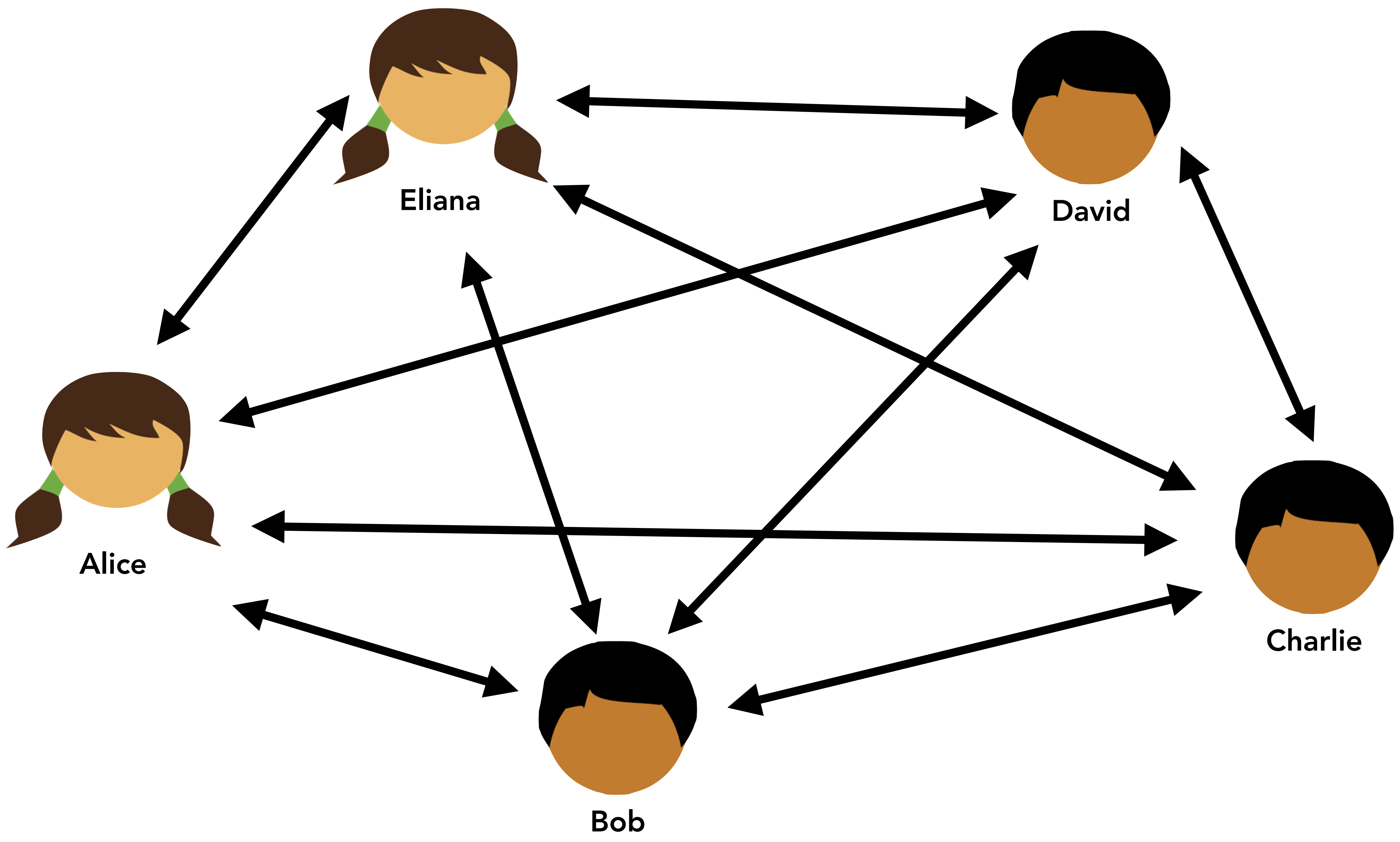
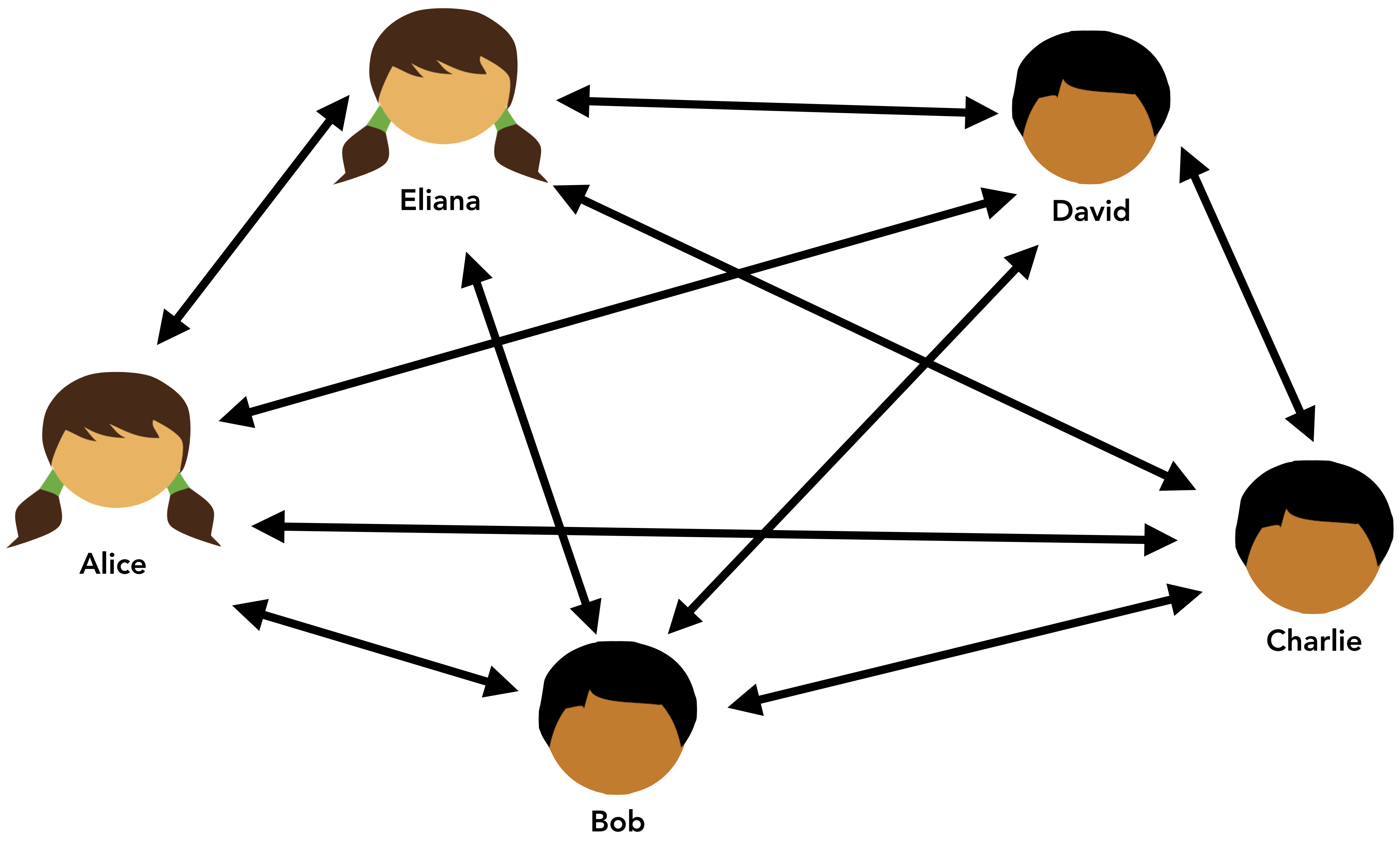


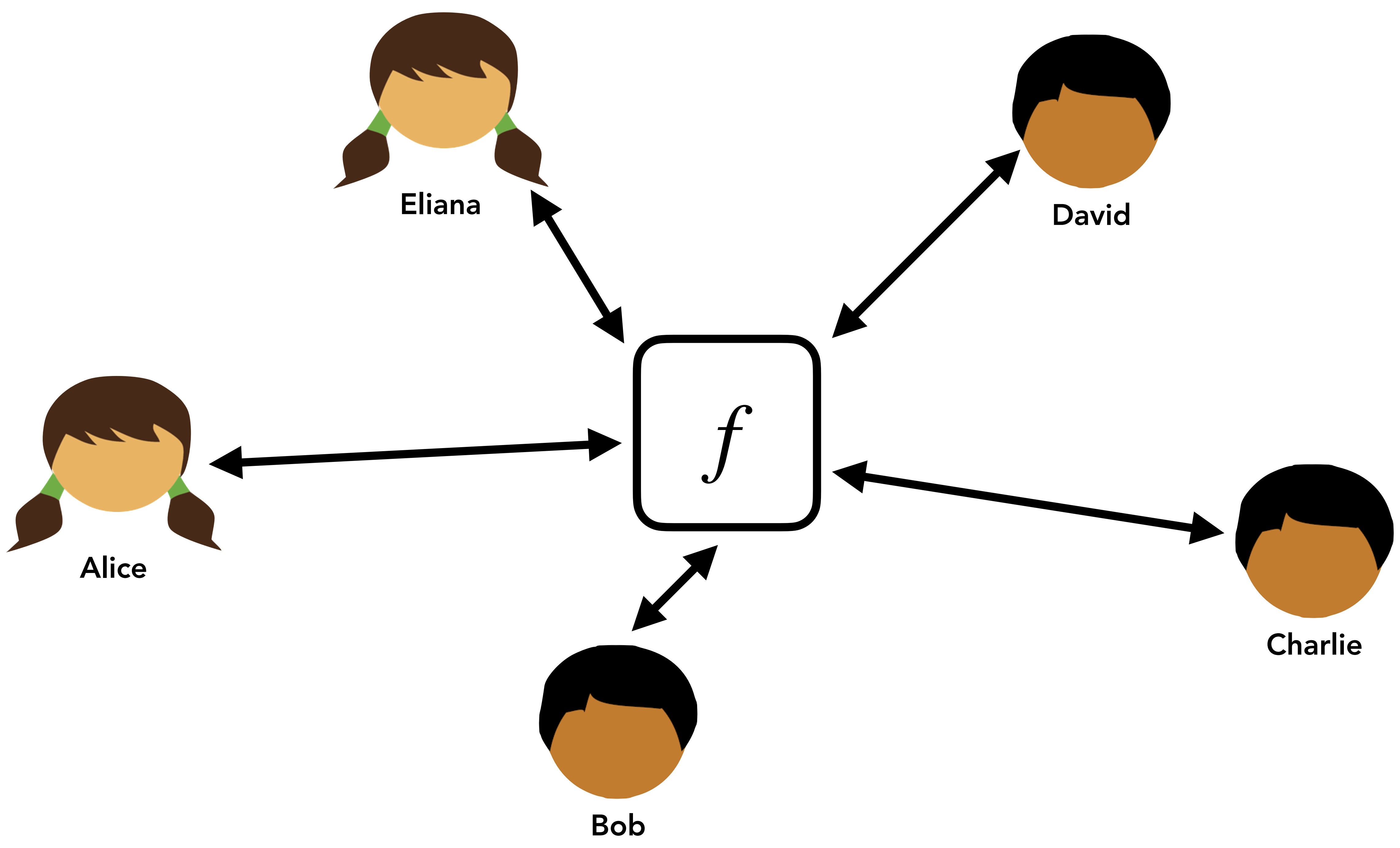
Fair Secure Multiparty Computation

Let's recall what MPC is

Real World



Ideal World



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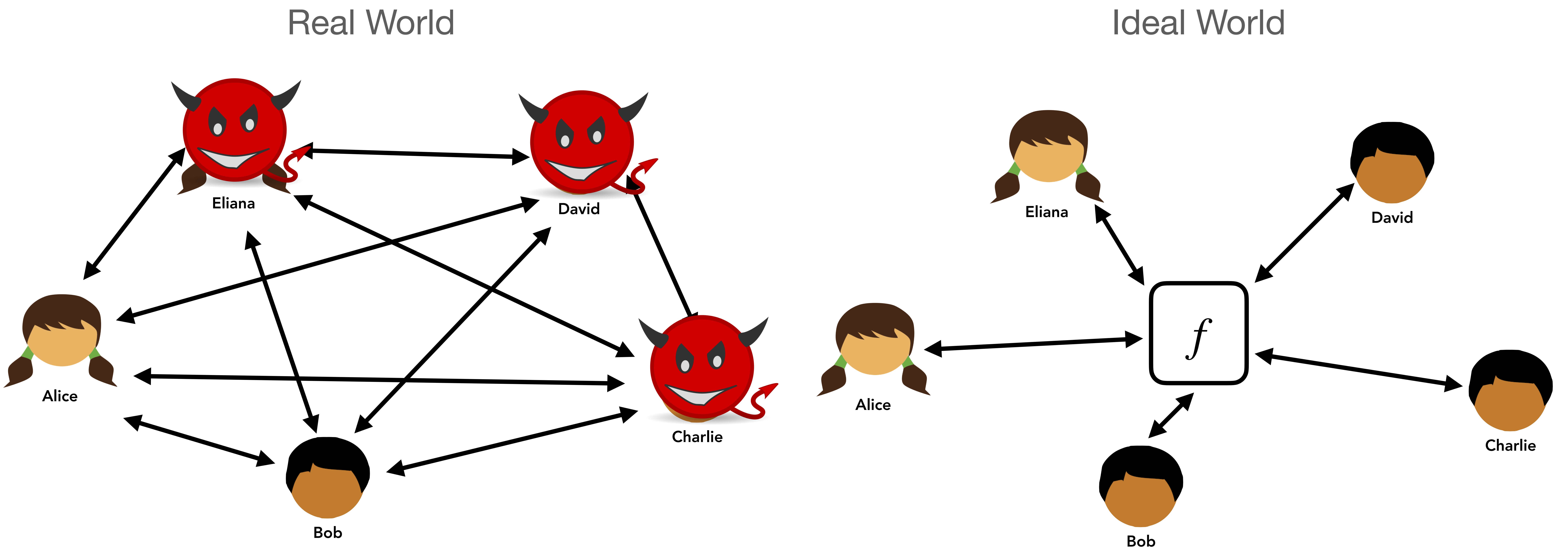

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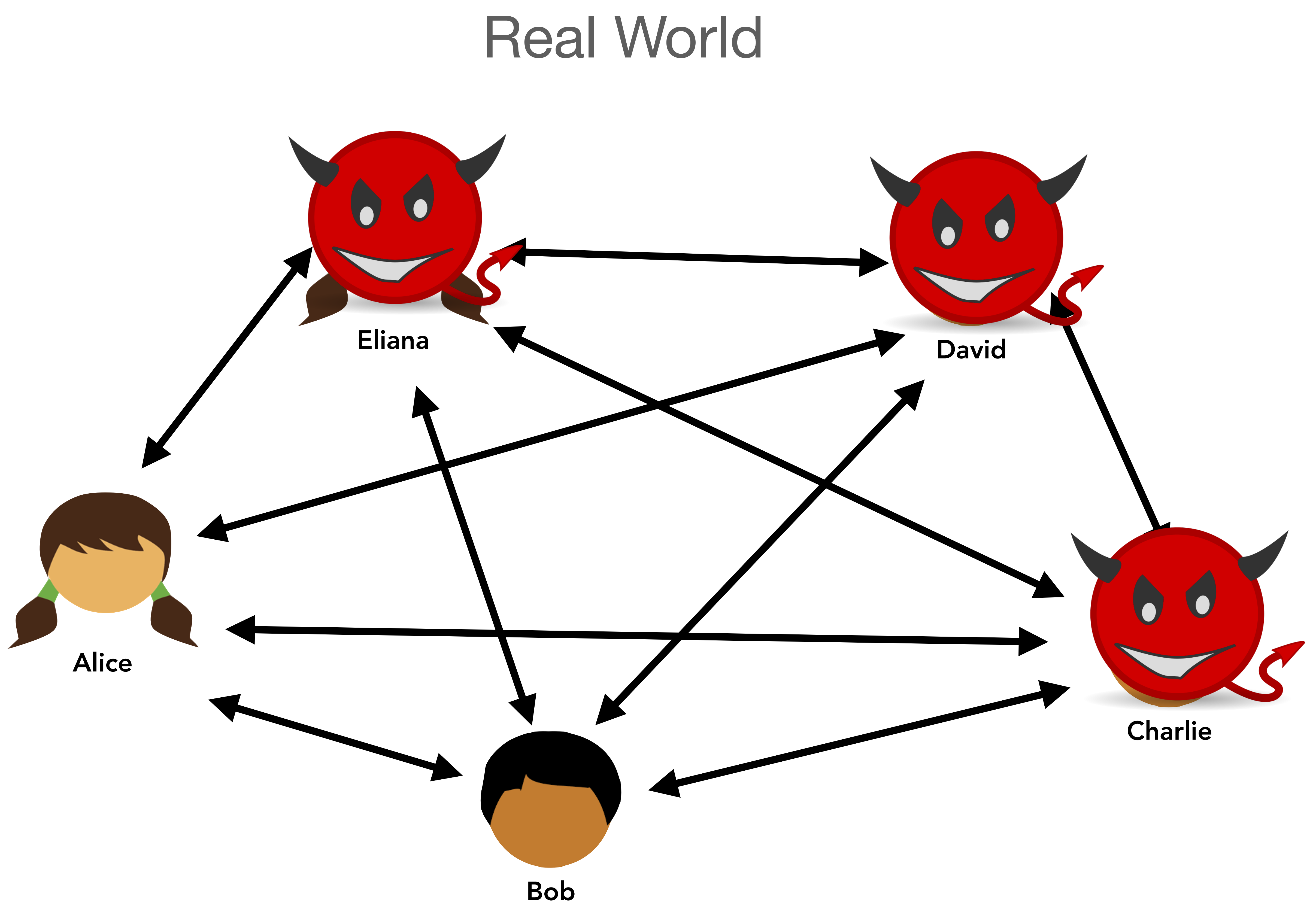

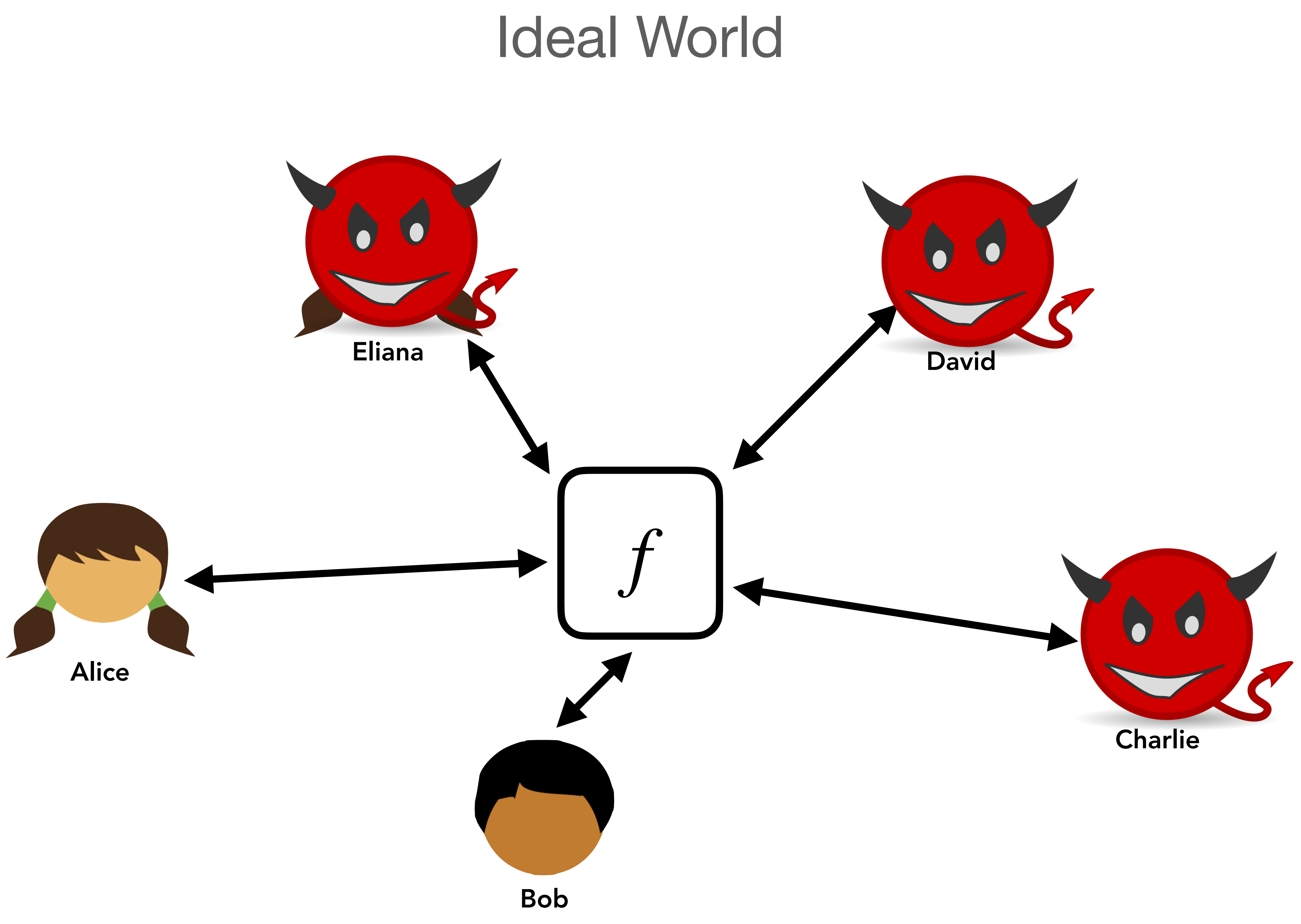

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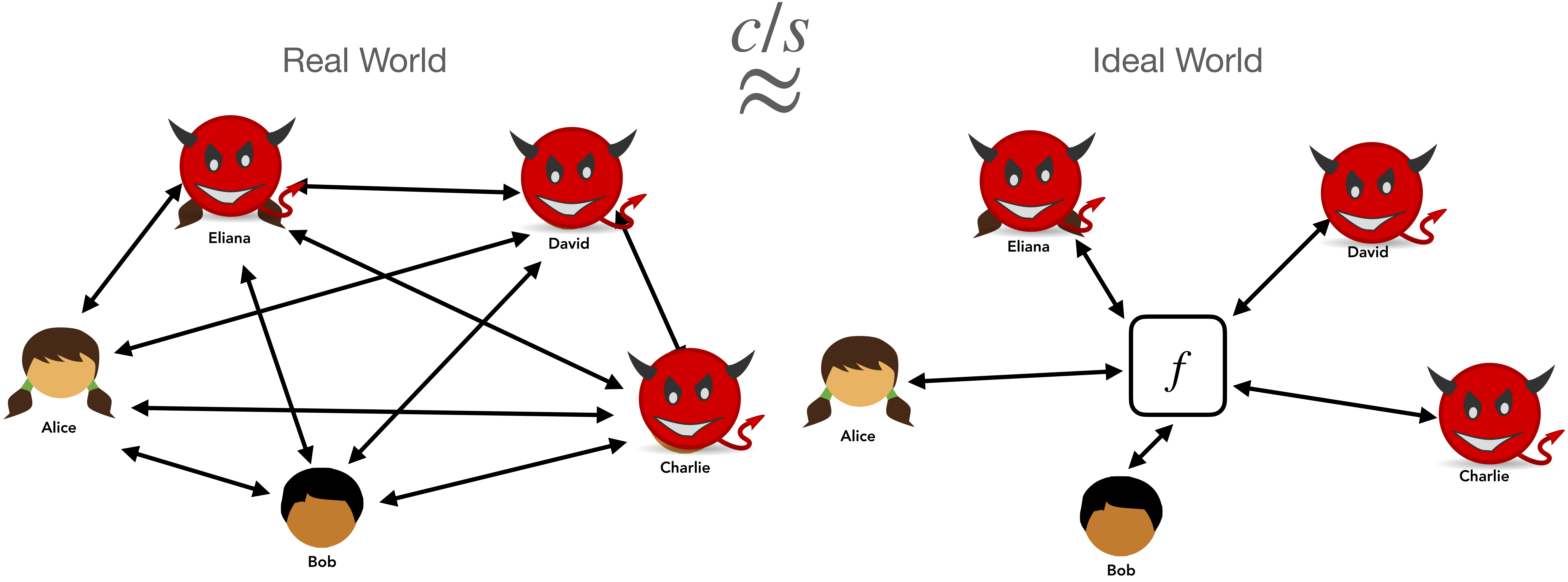

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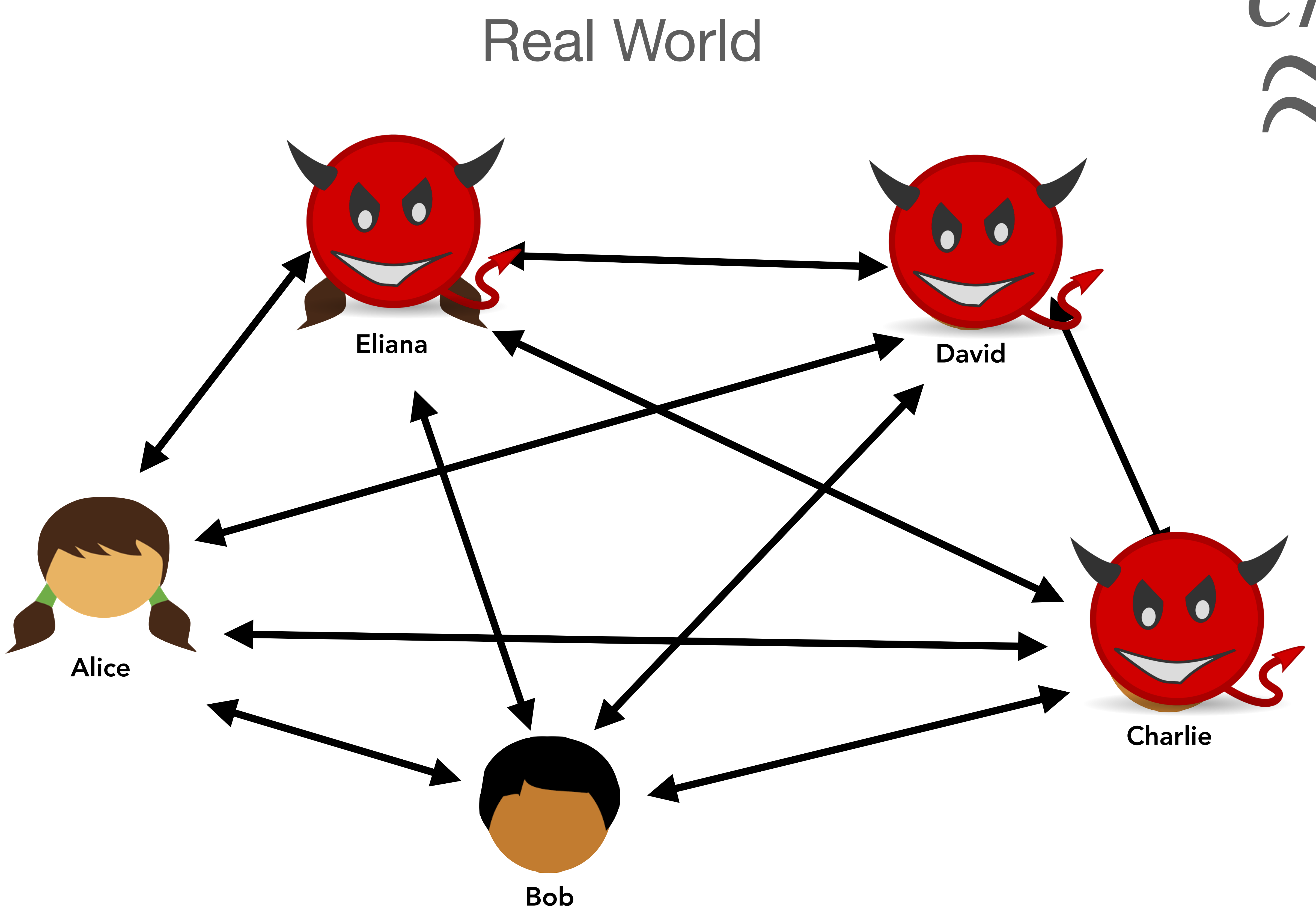
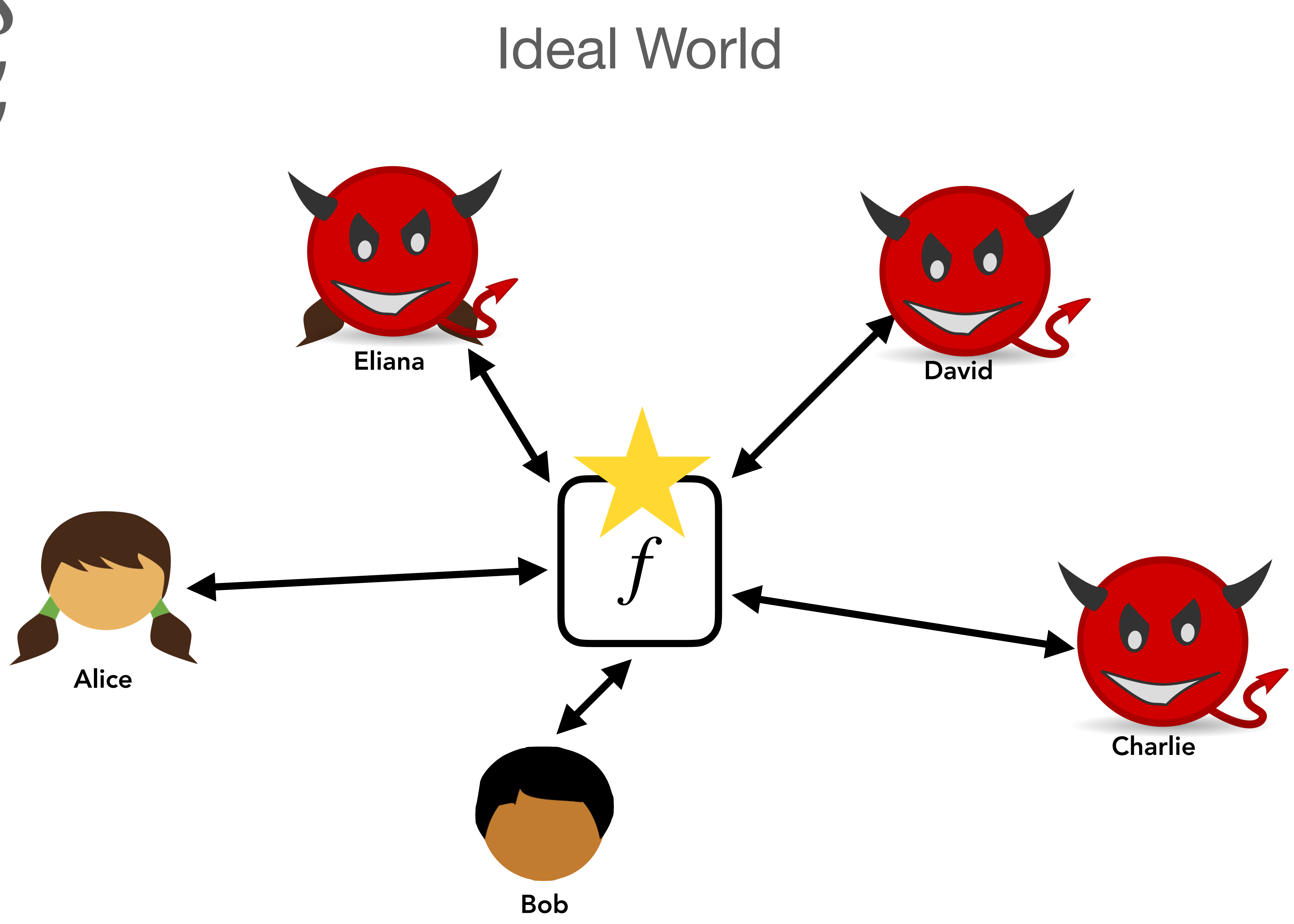


$\overset{c/s}{\approx}$



We consider ***active, malicious majority***

Different Security Levels

Security with Abort v.s. Fairness v.s. Guaranteed Output Delivery

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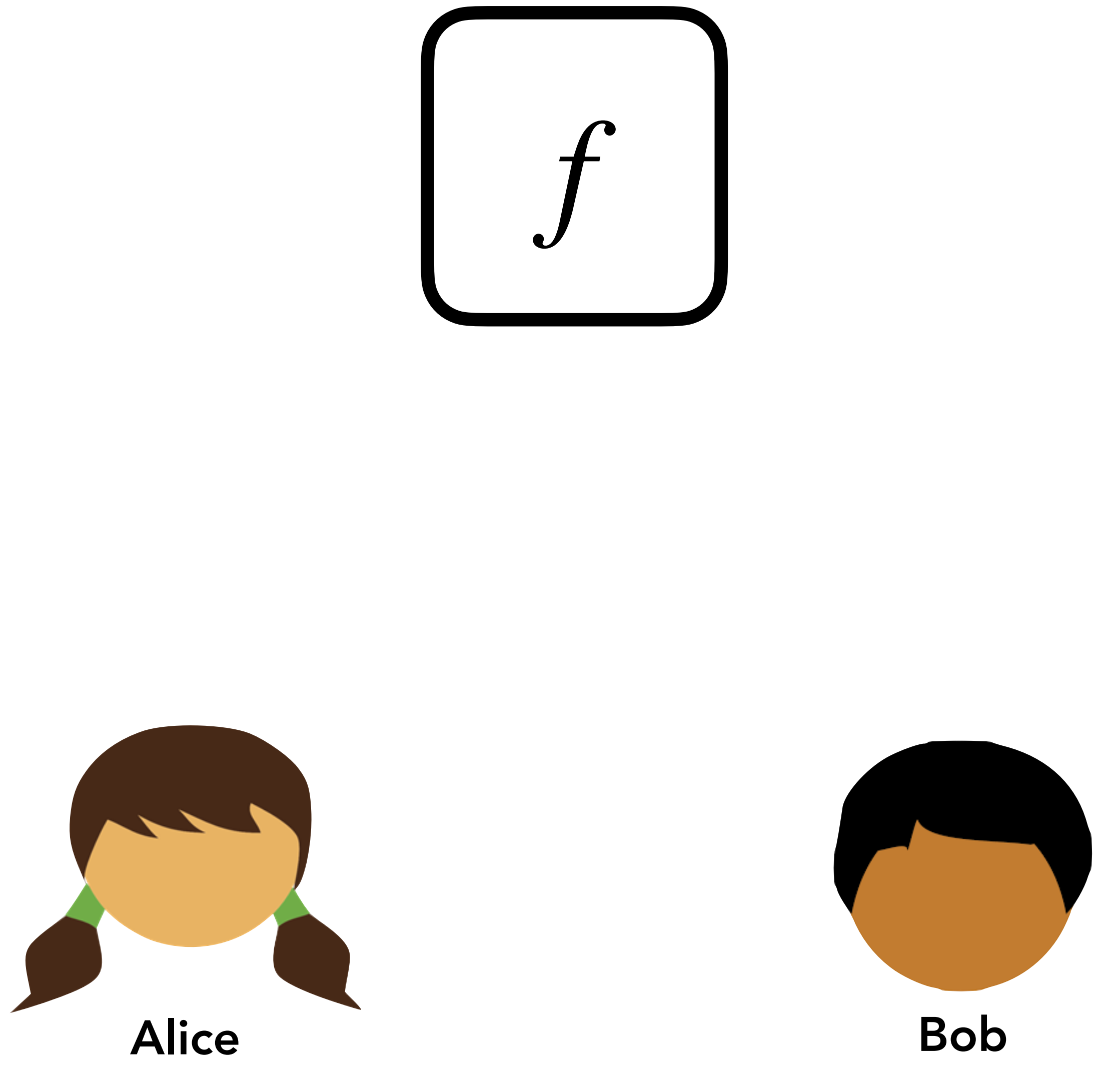
Security with Abort



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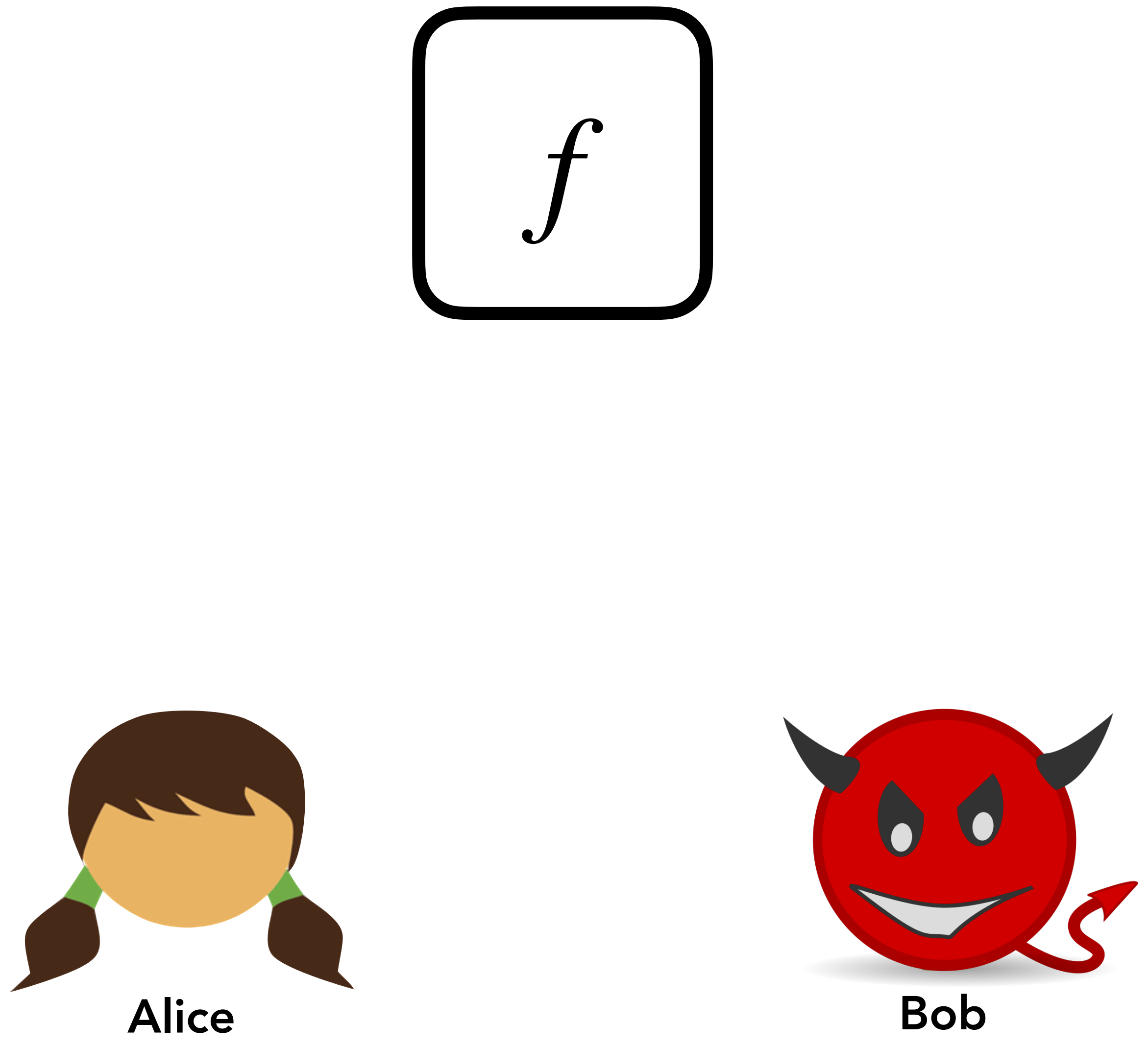

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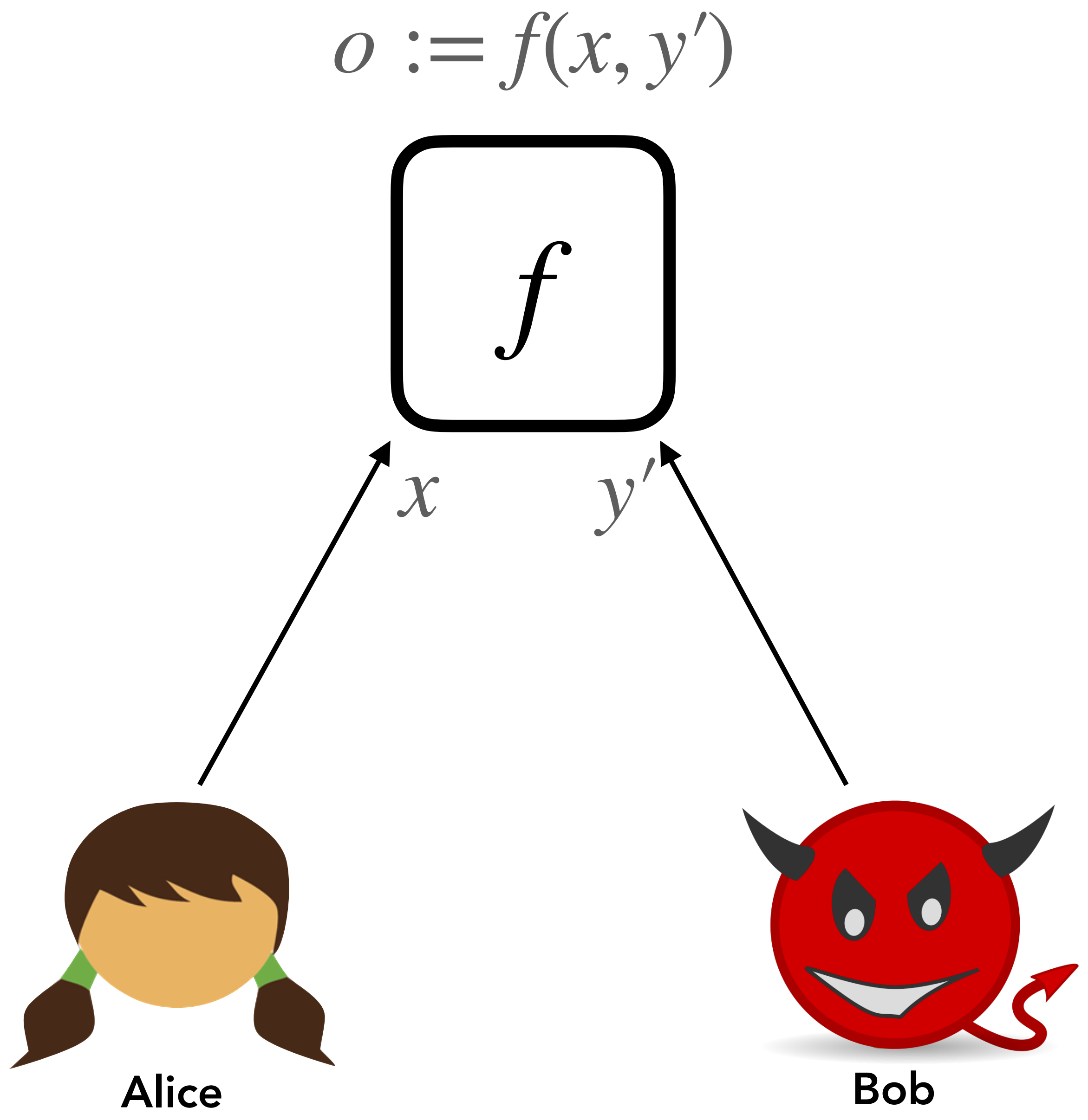

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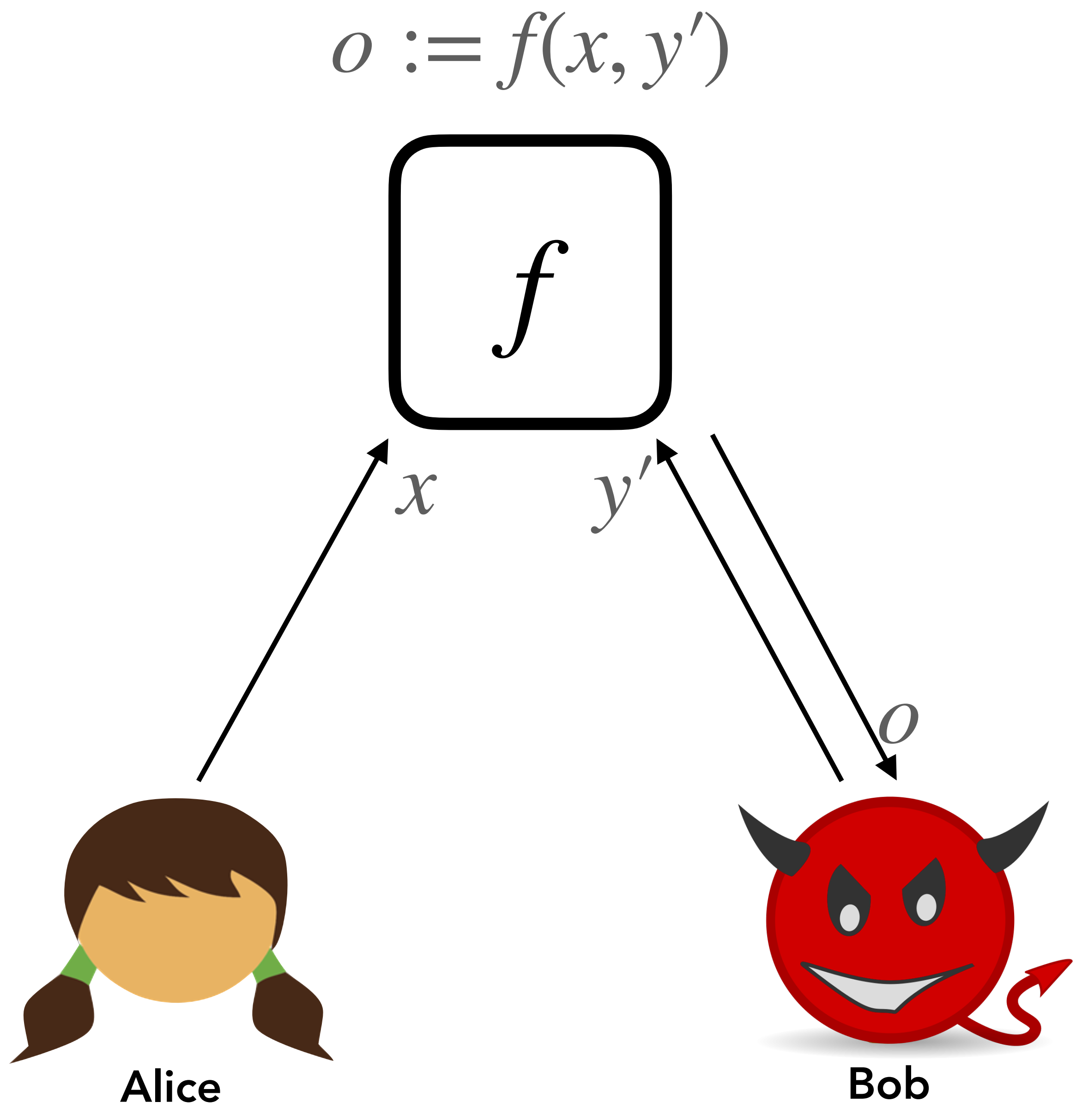

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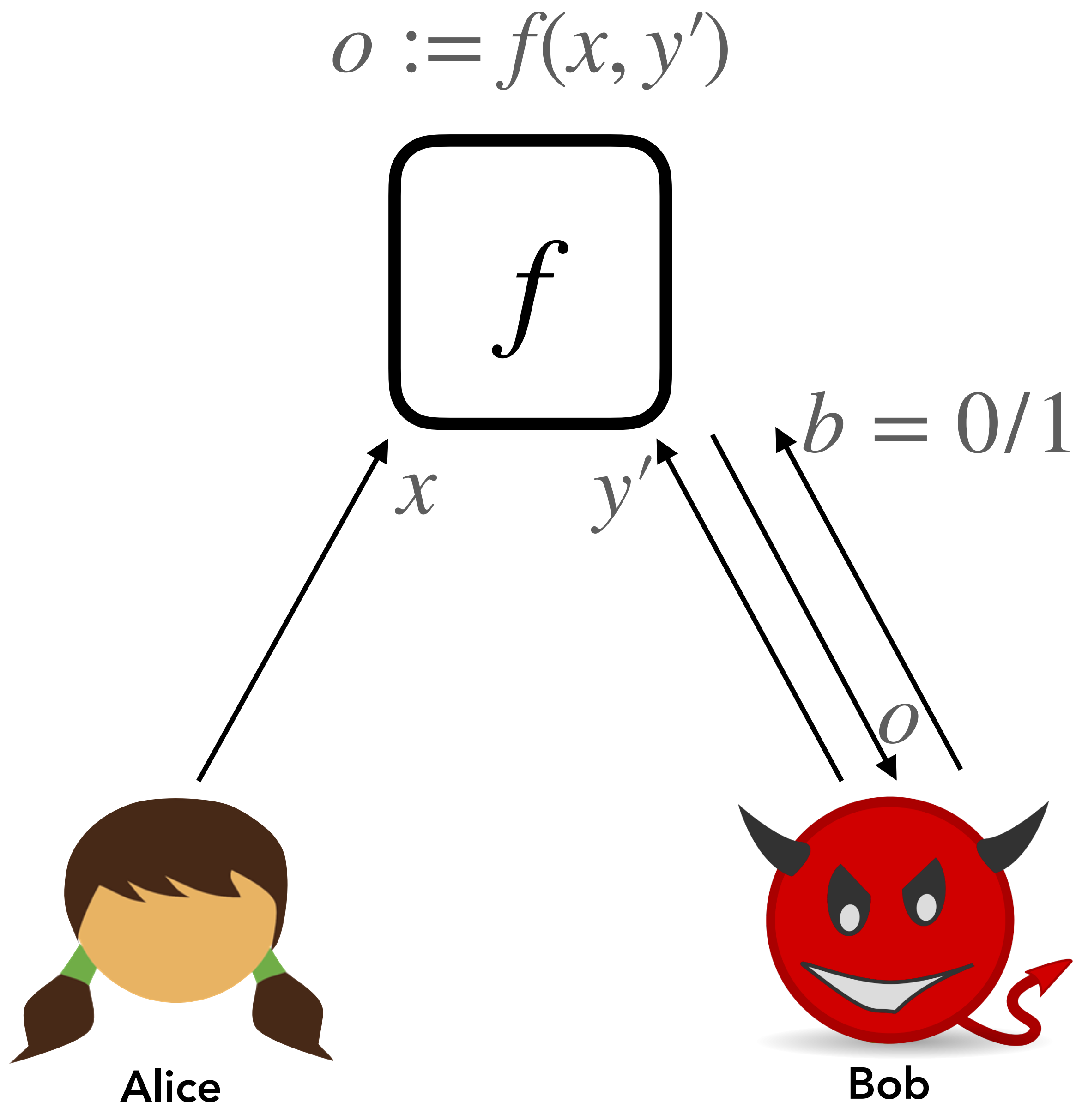

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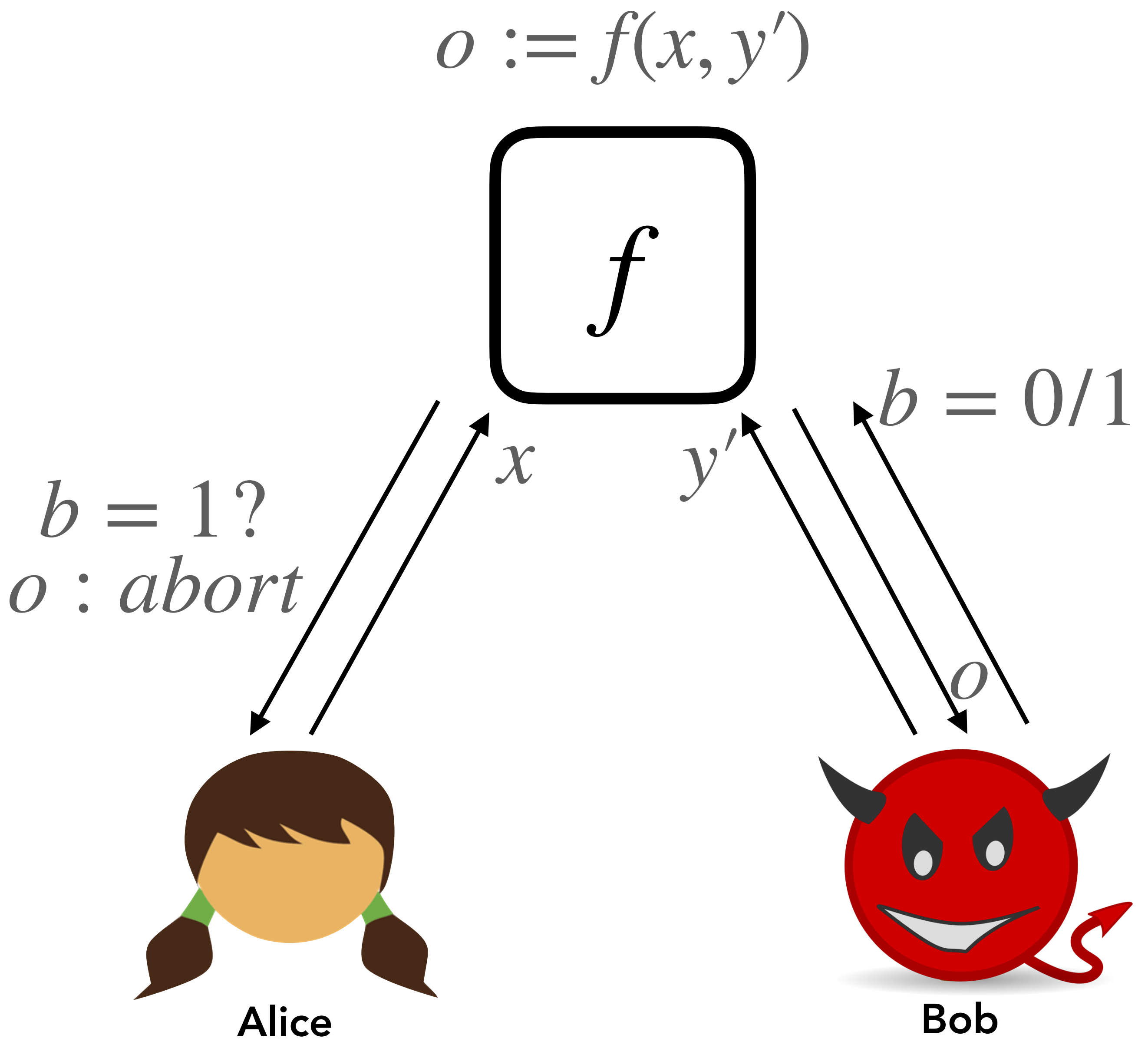

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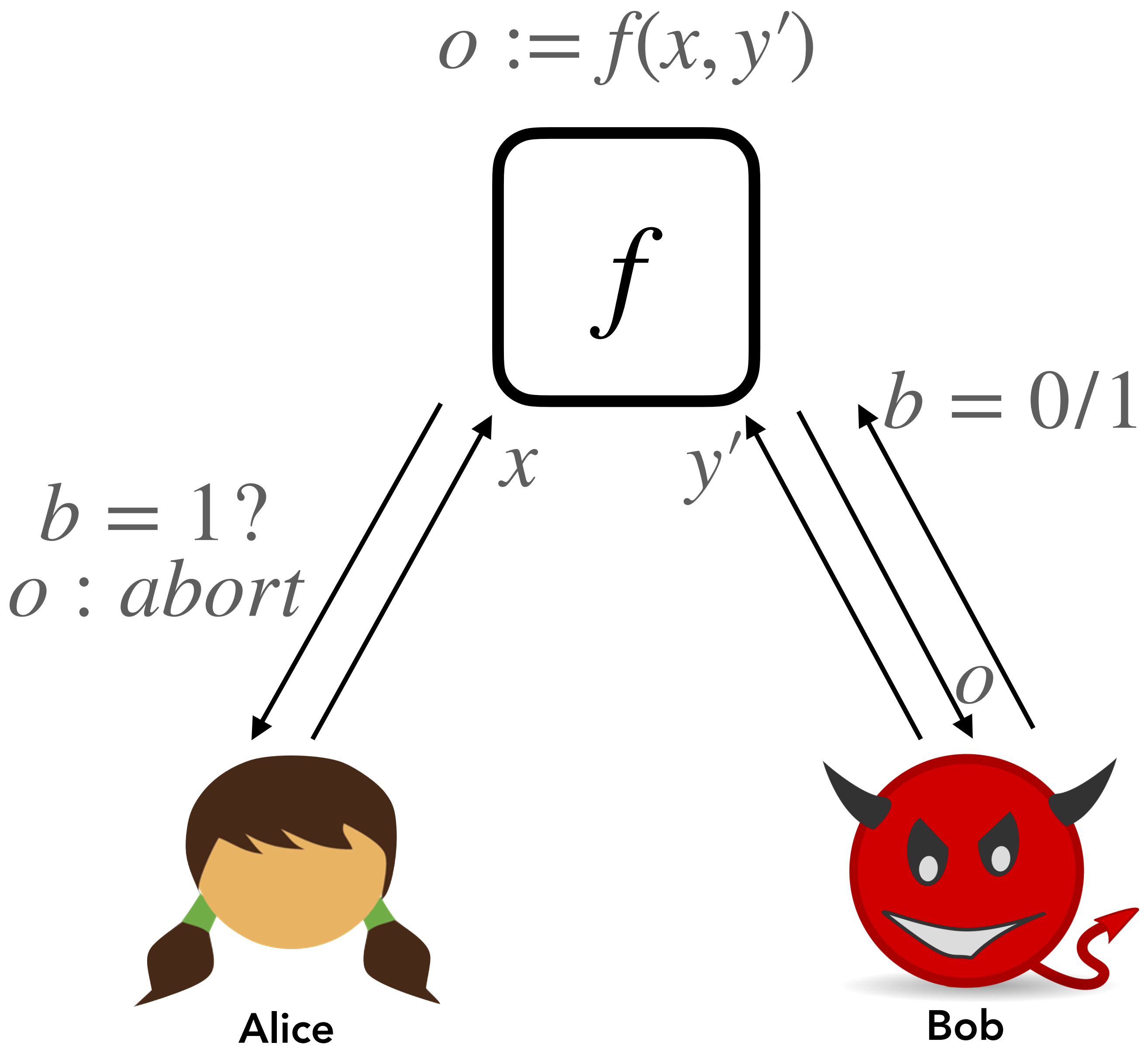
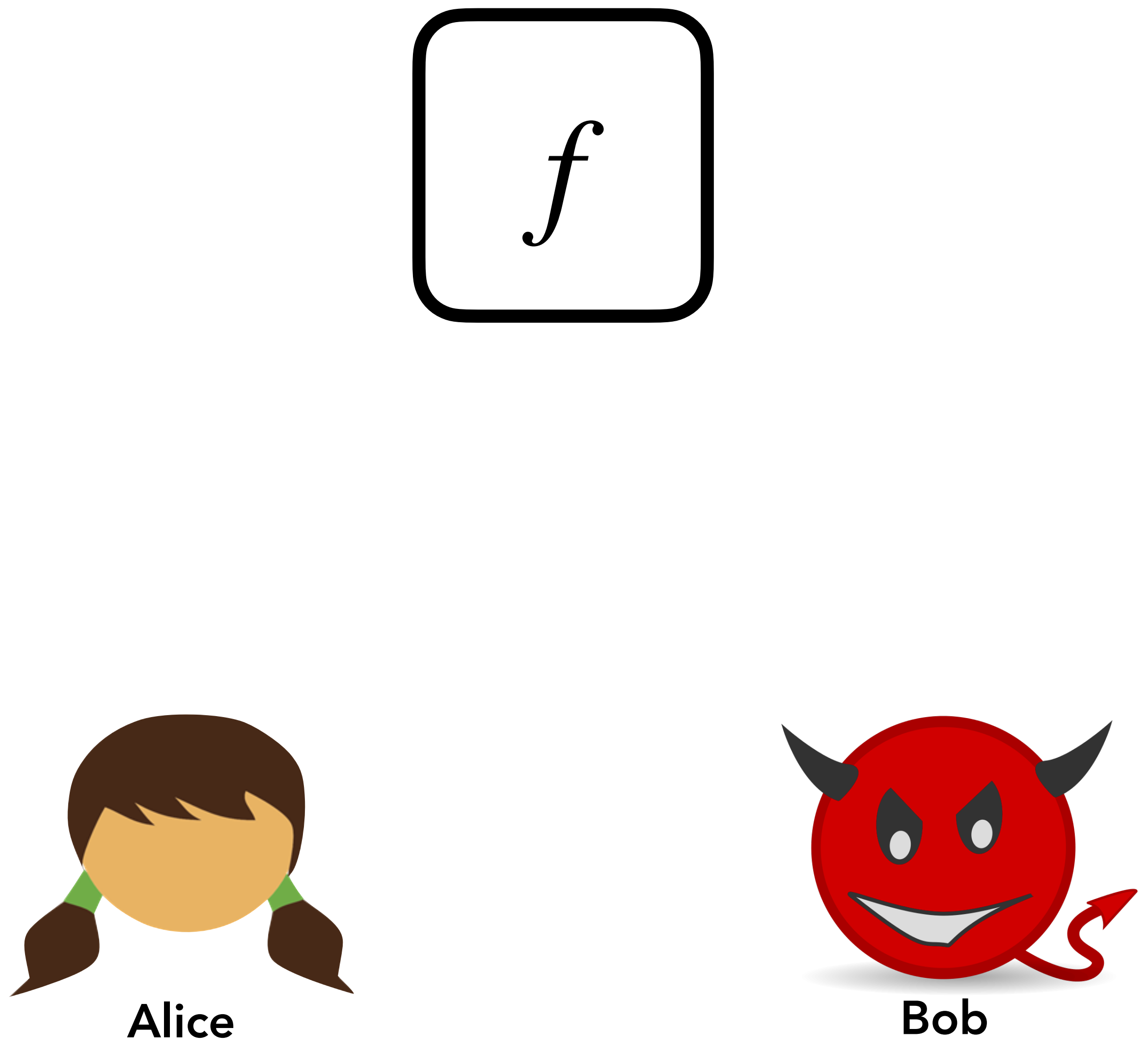


Guaranteed Output Delivery



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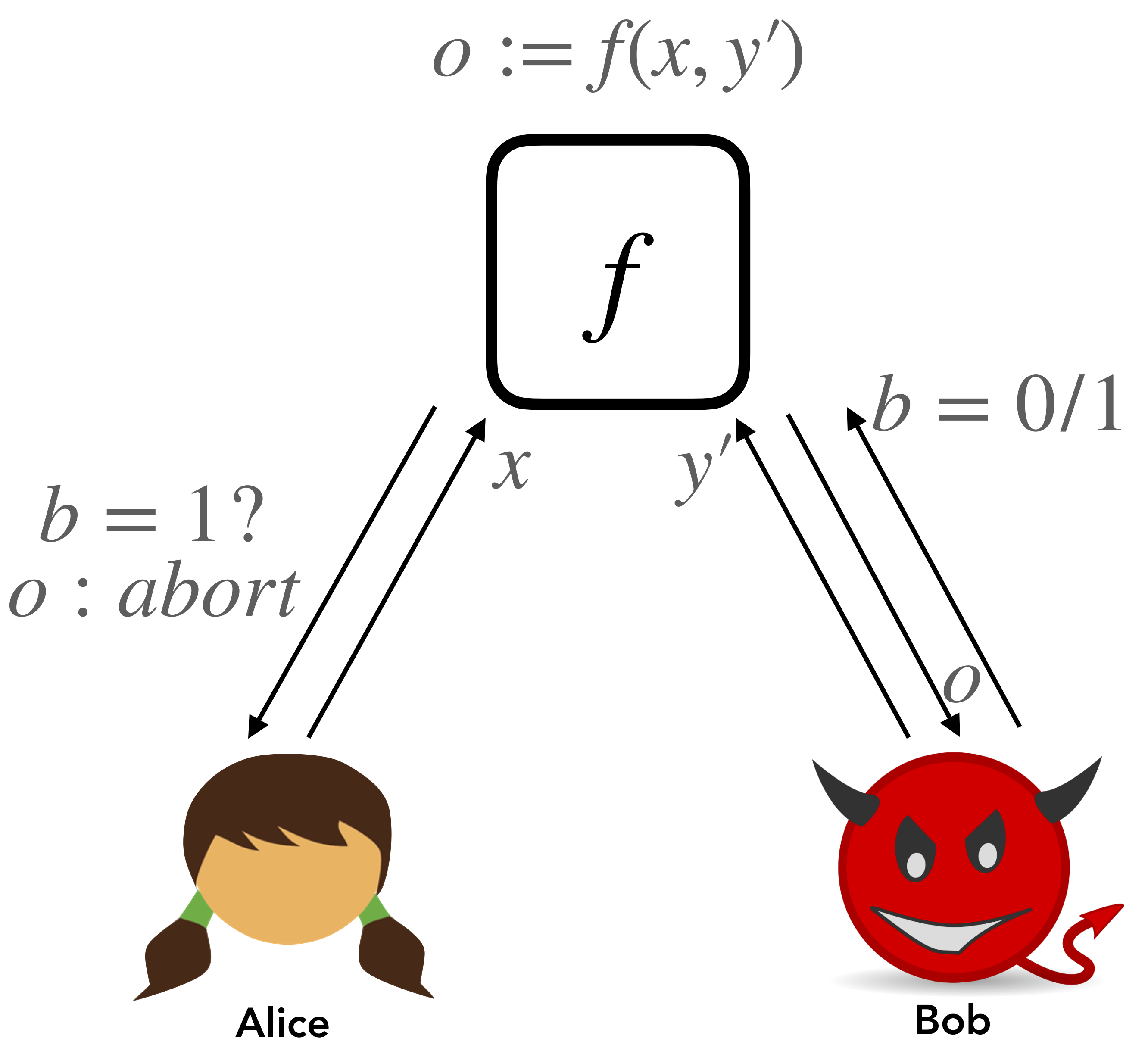

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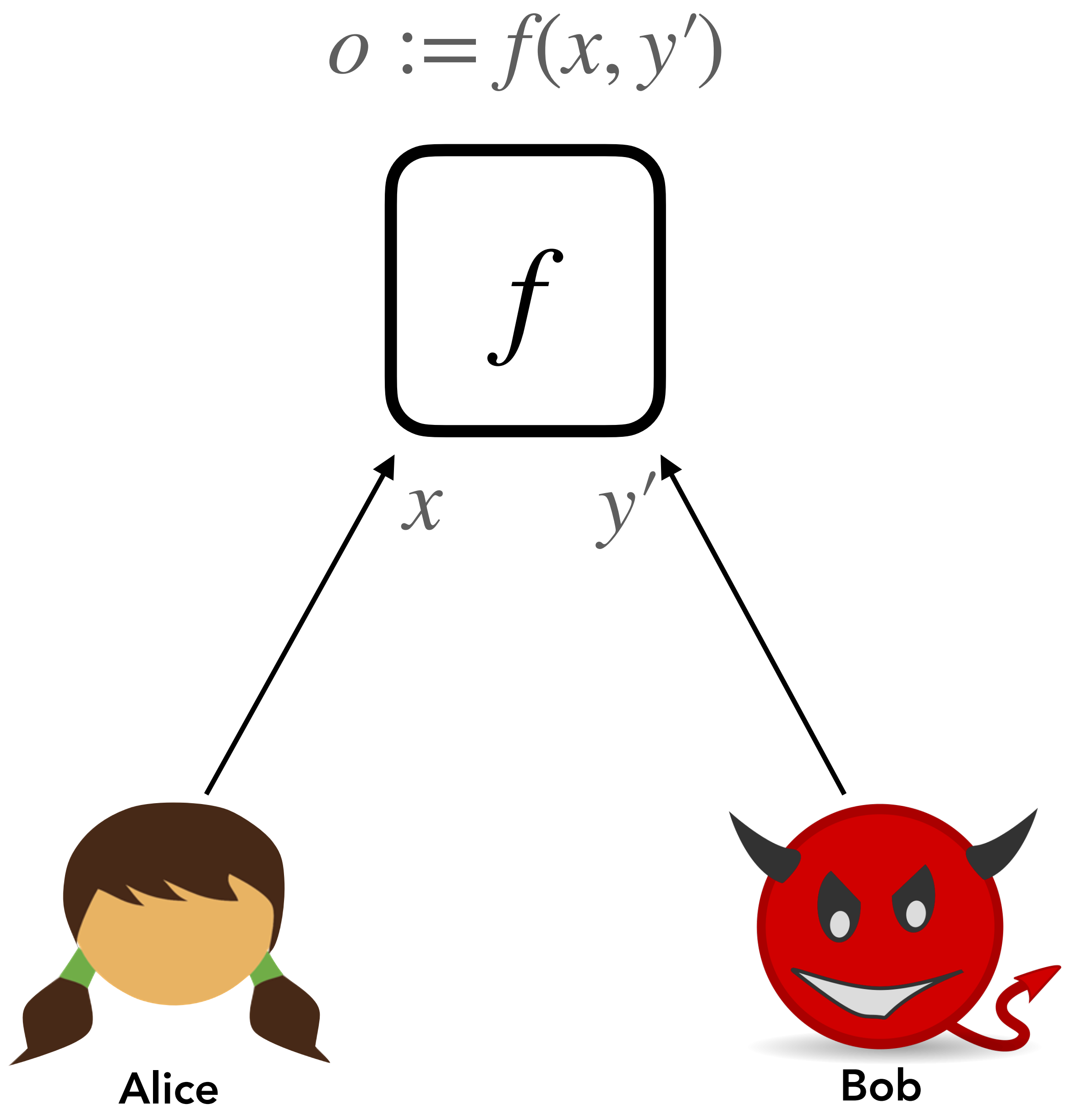

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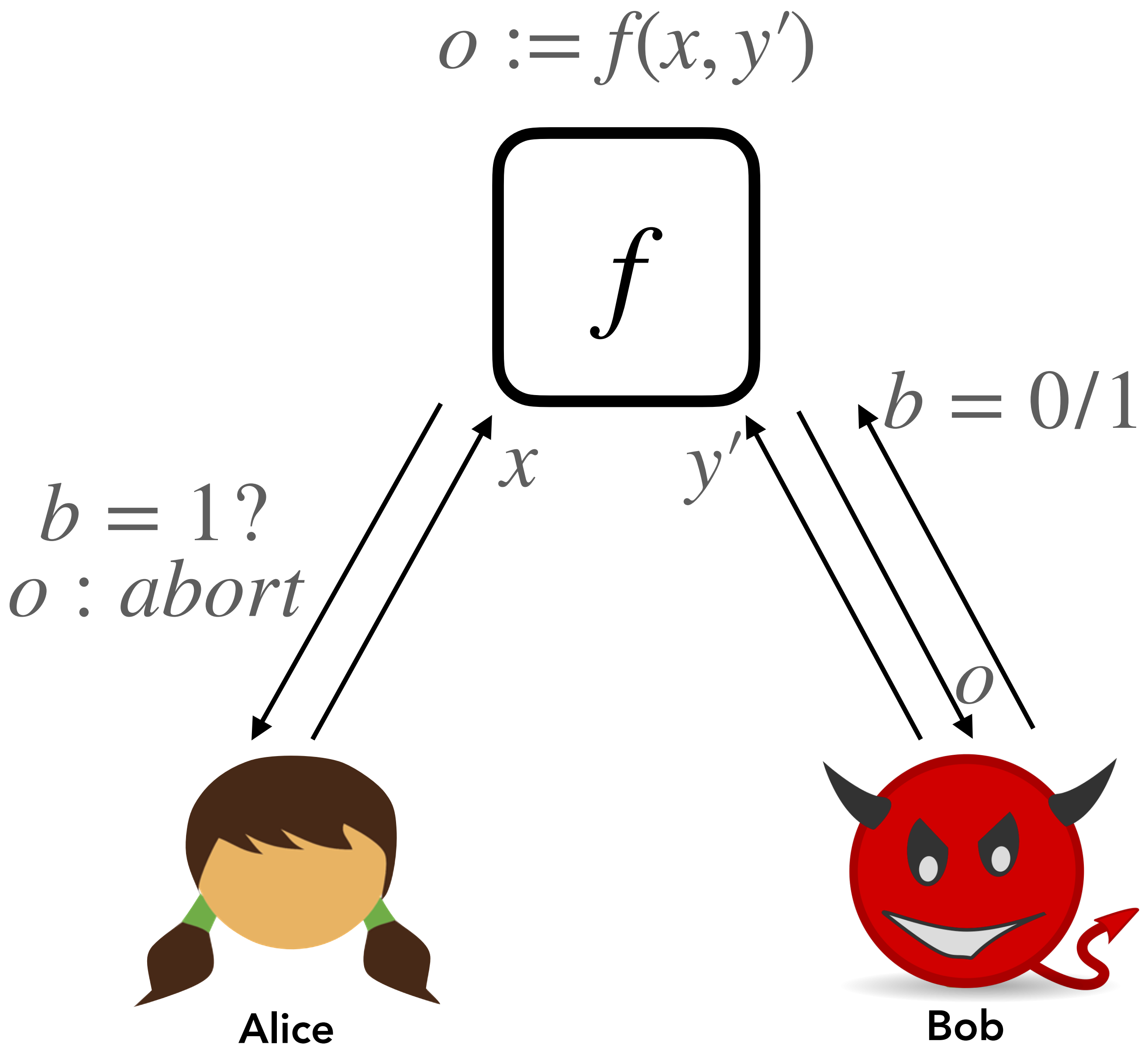

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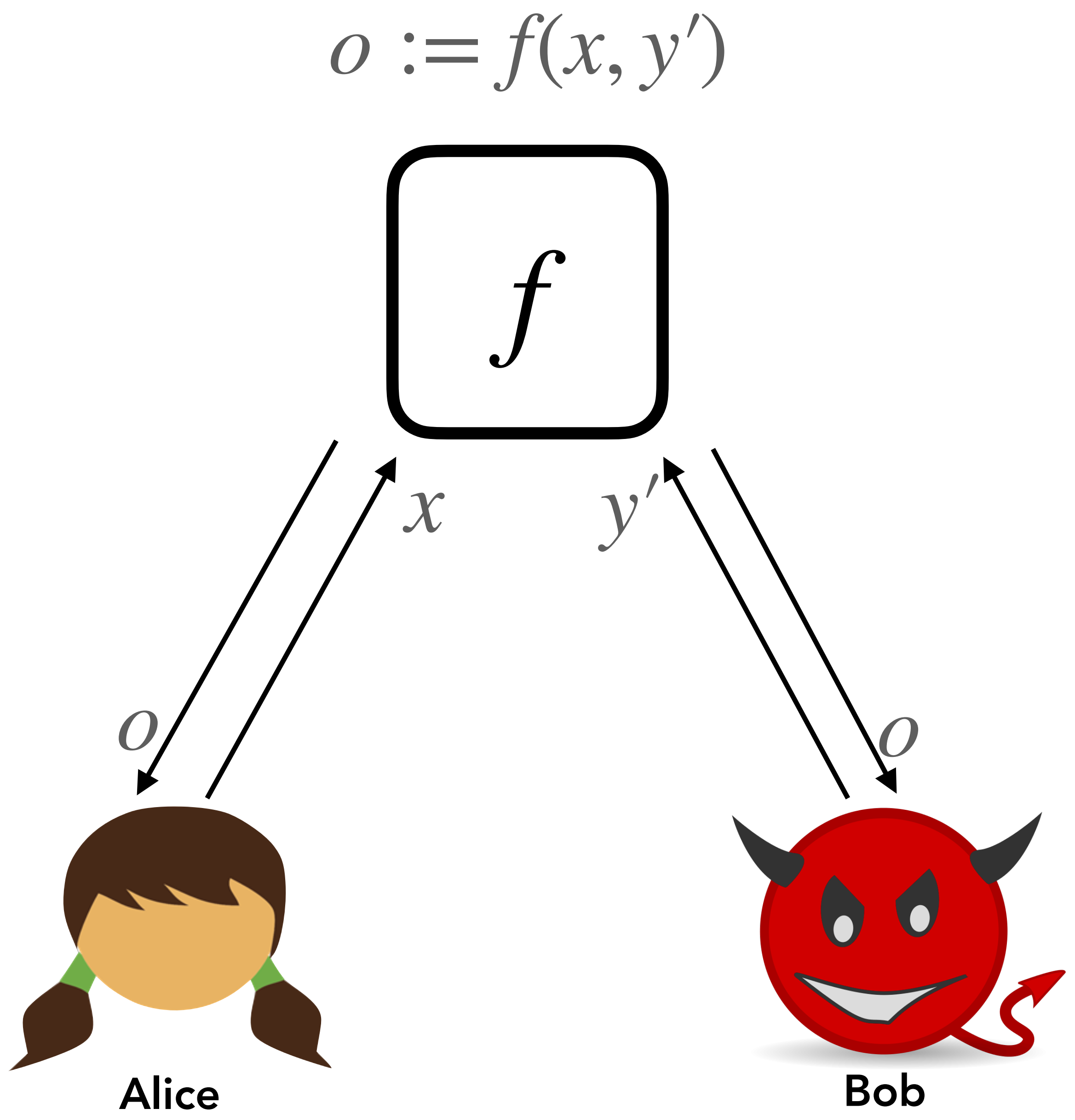

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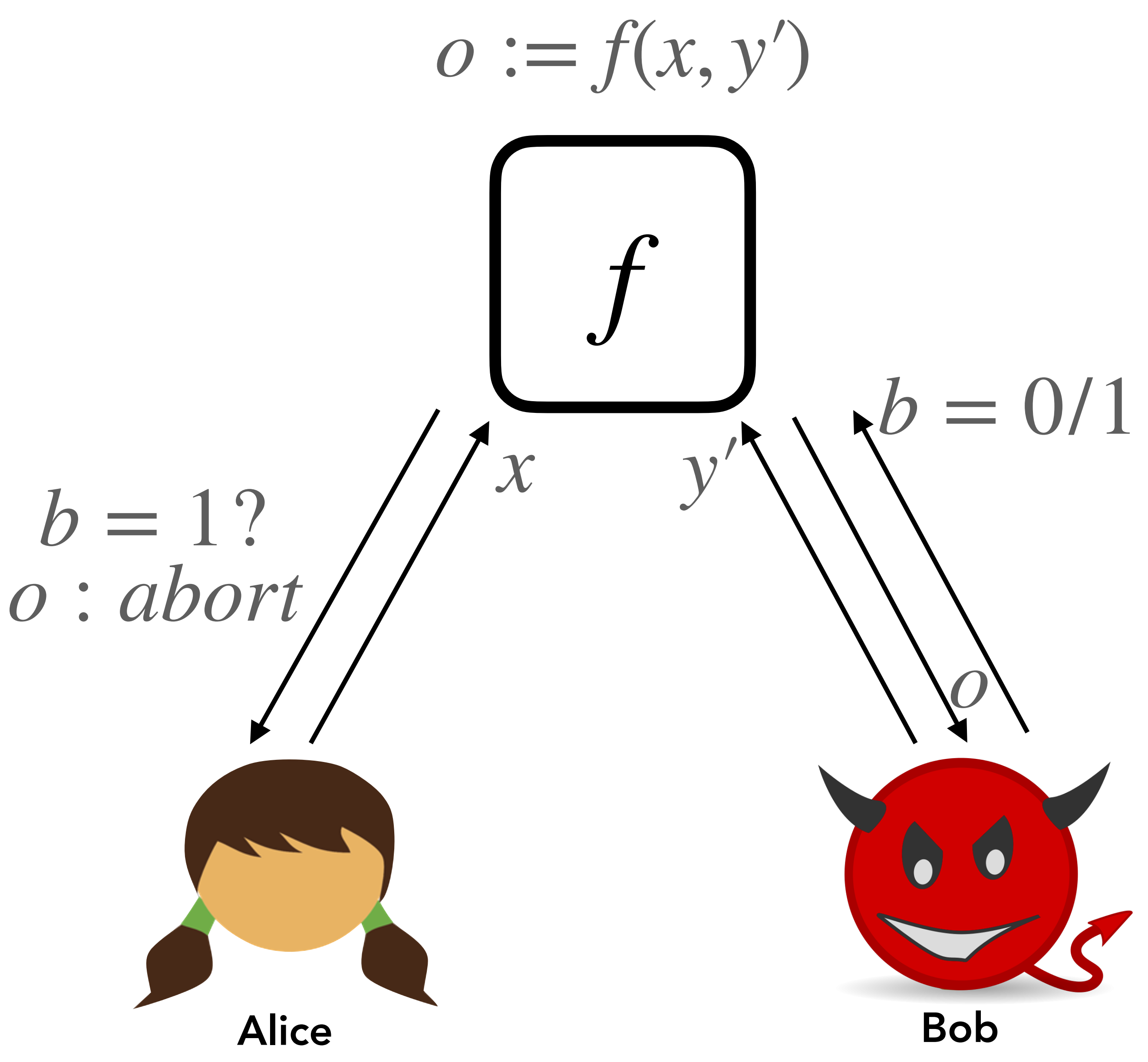
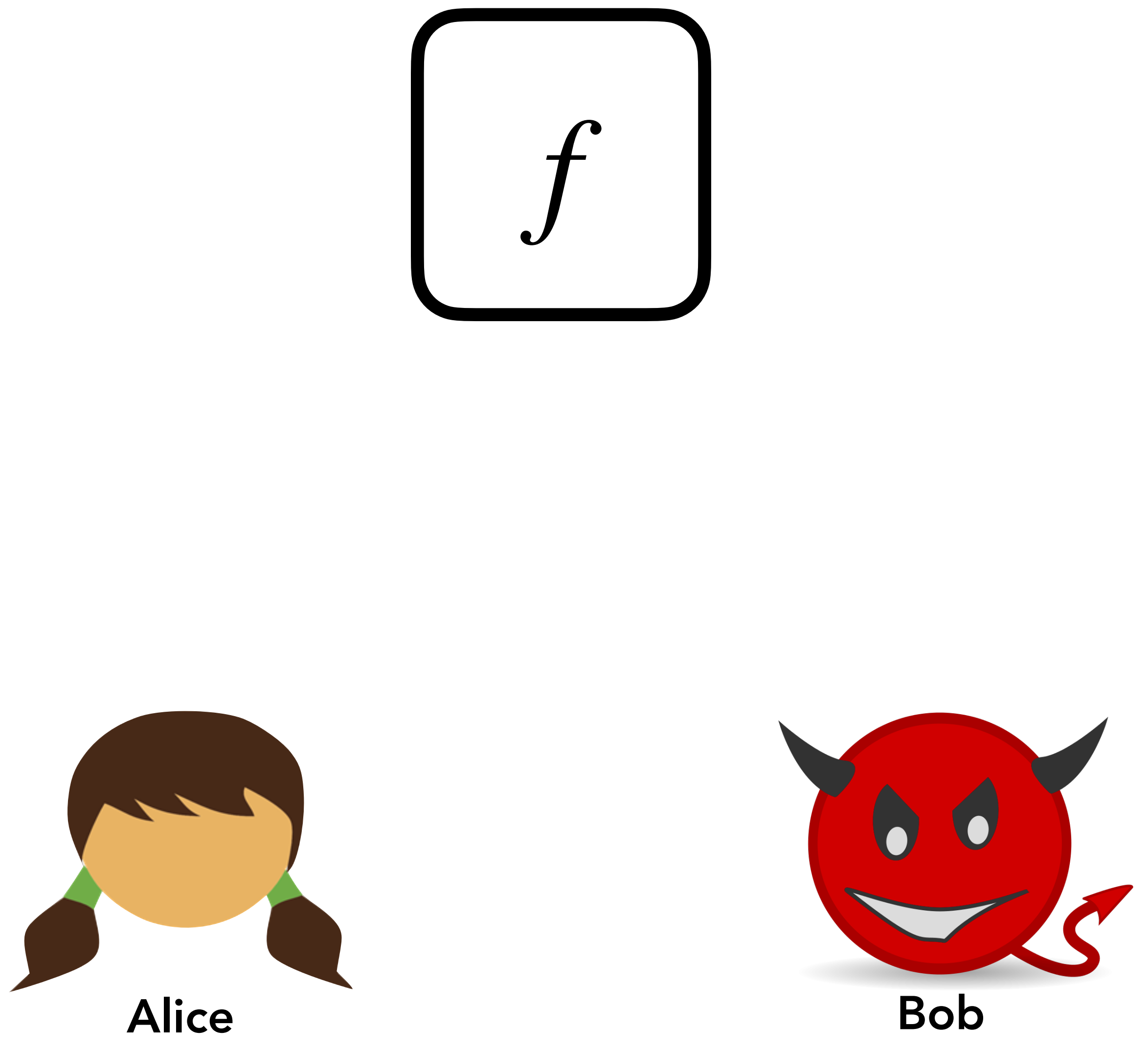
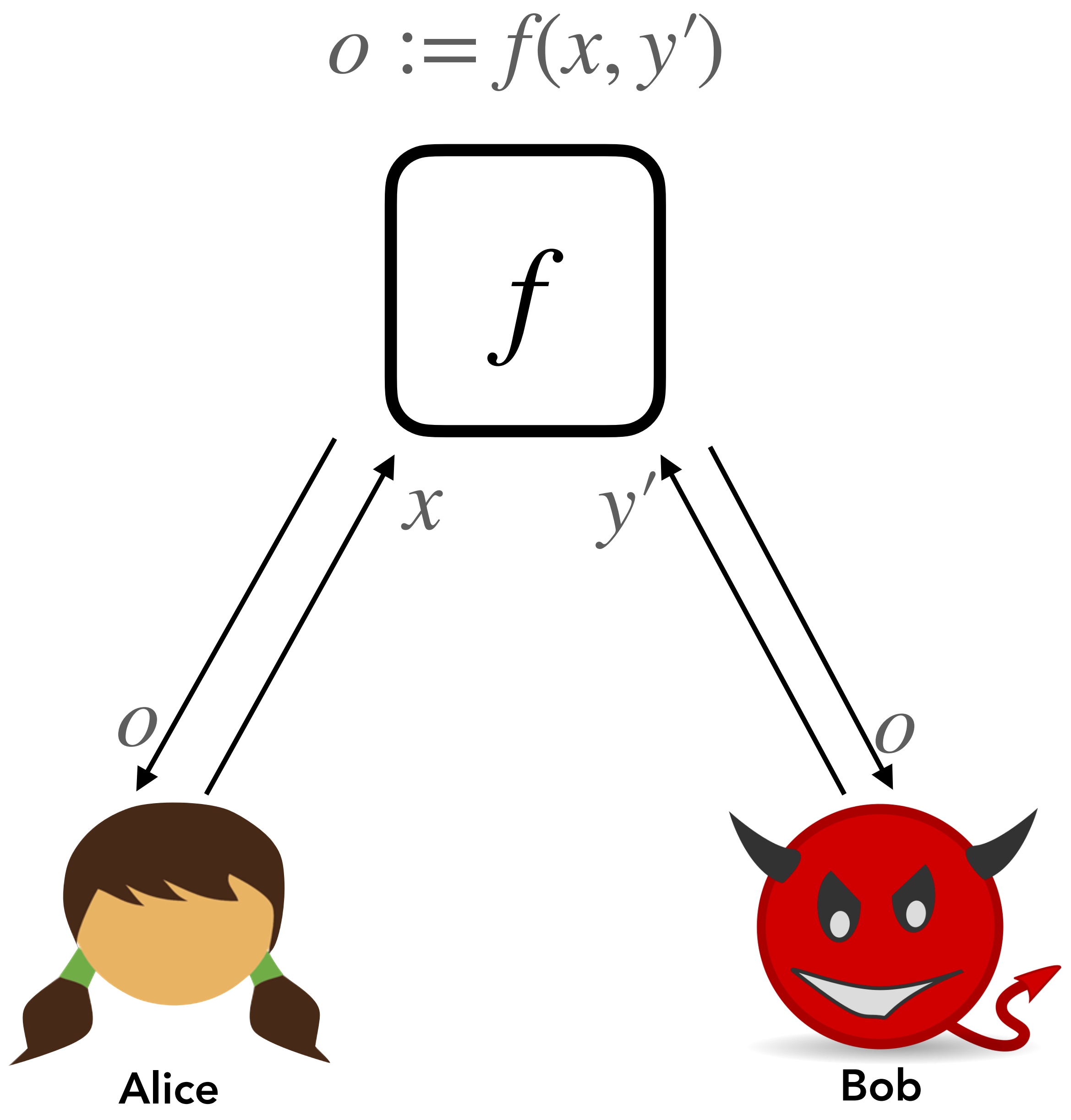


Fairness



Guaranteed Output Delivery



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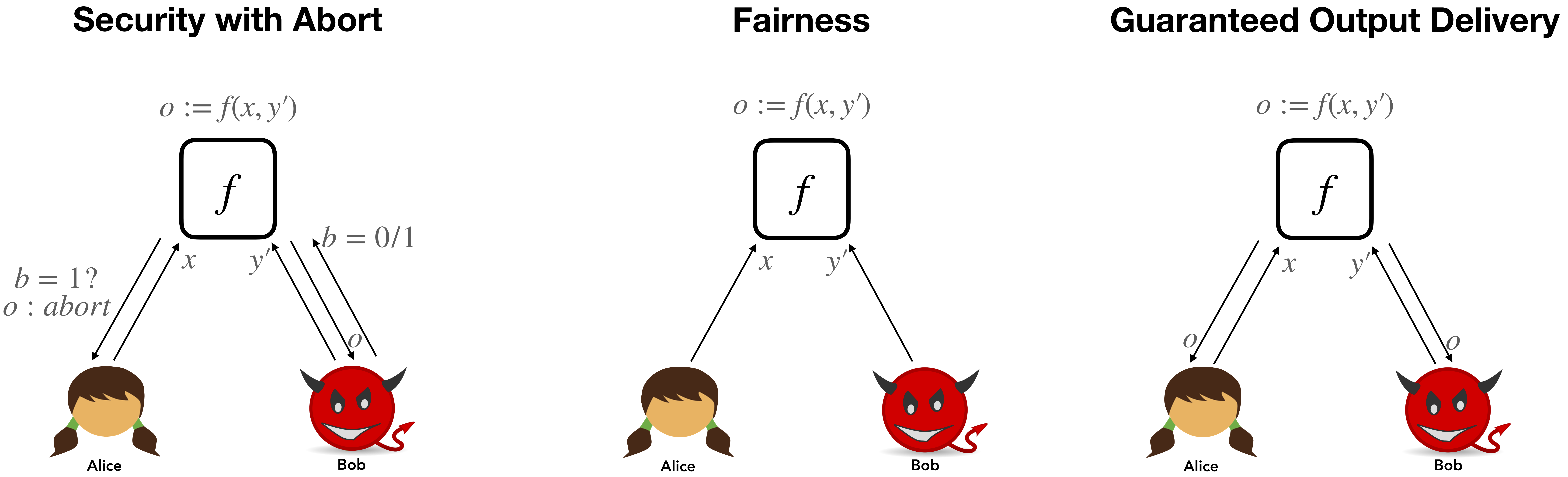

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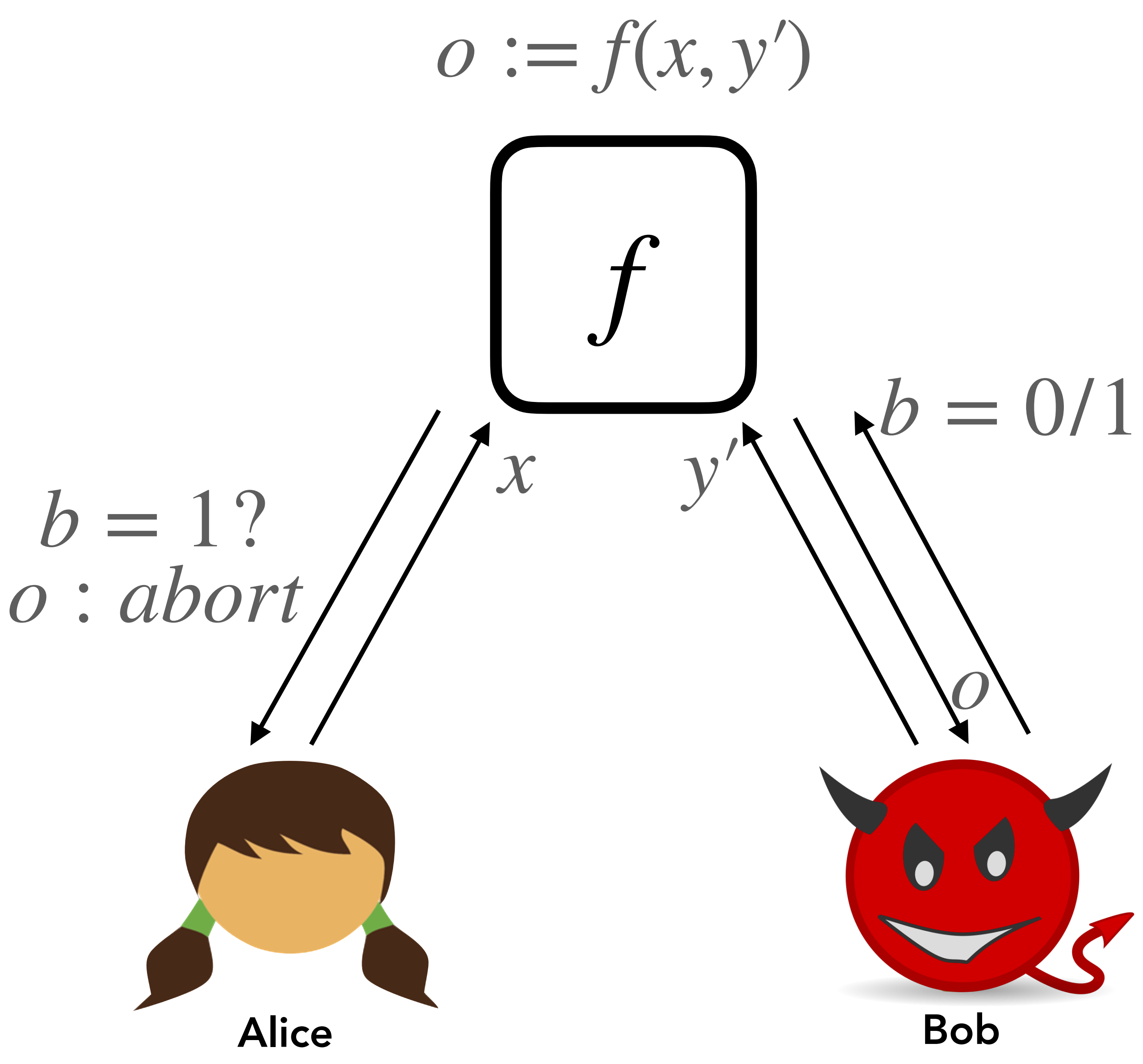

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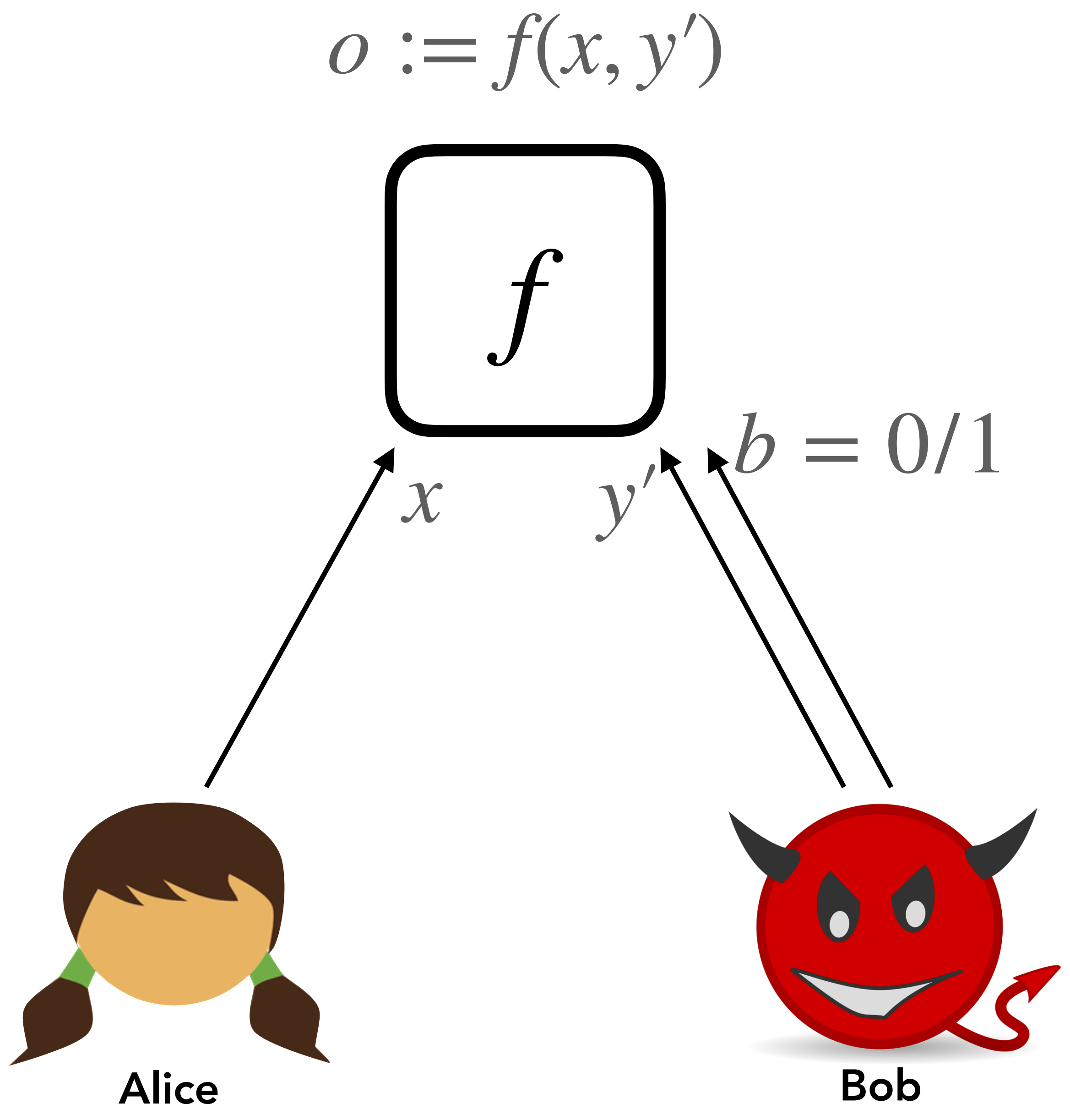
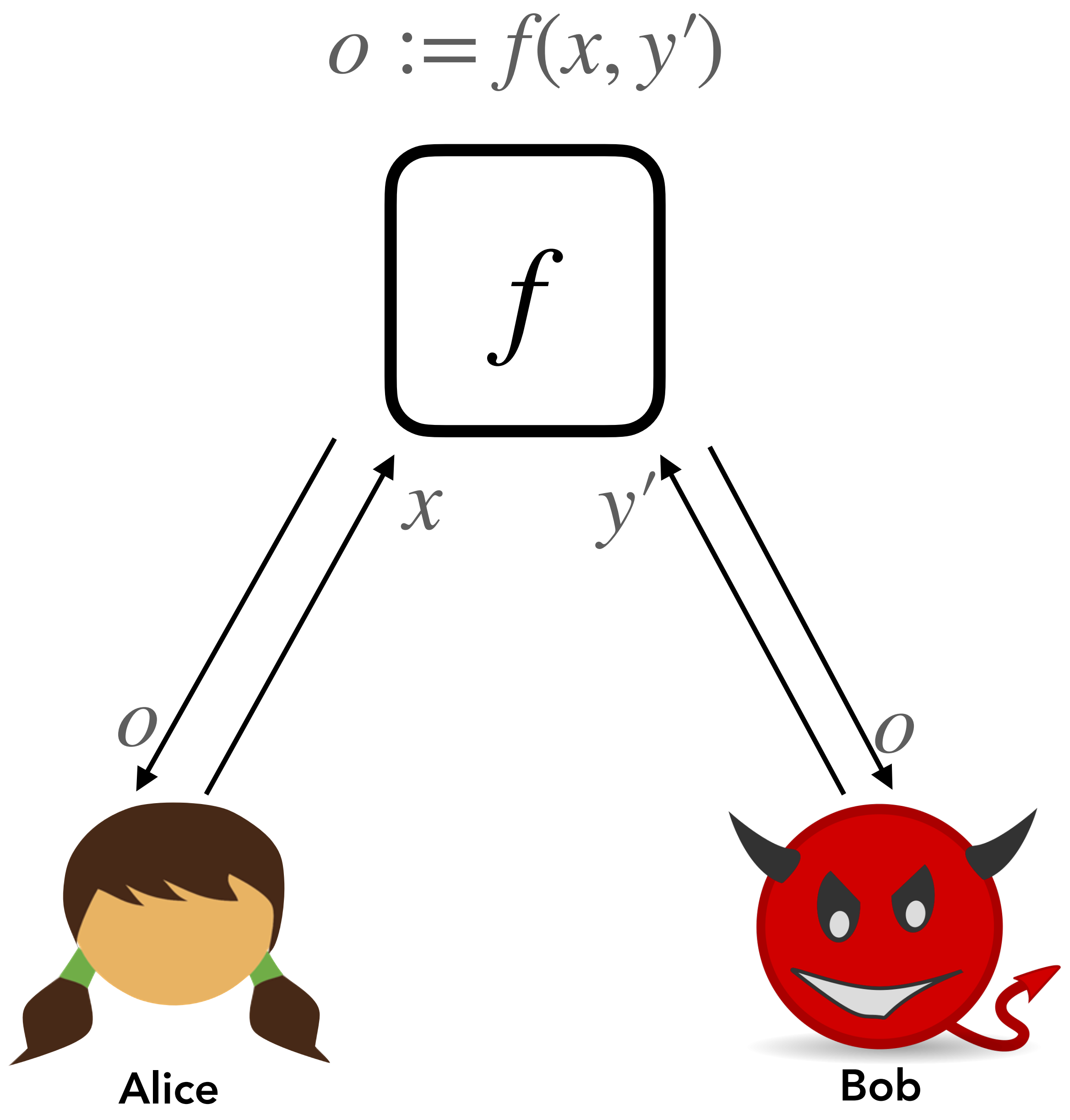


Guaranteed Output Delivery



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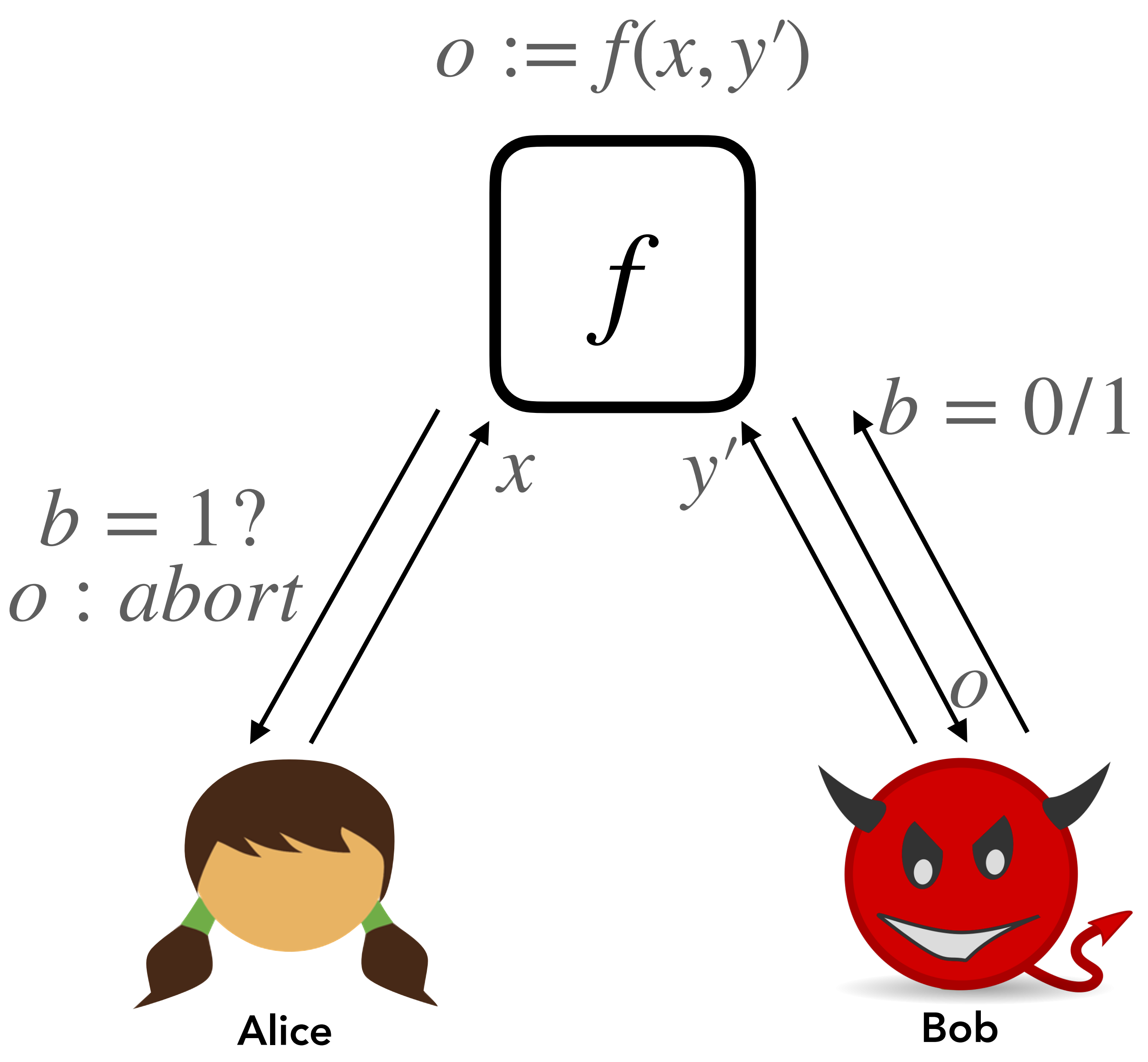

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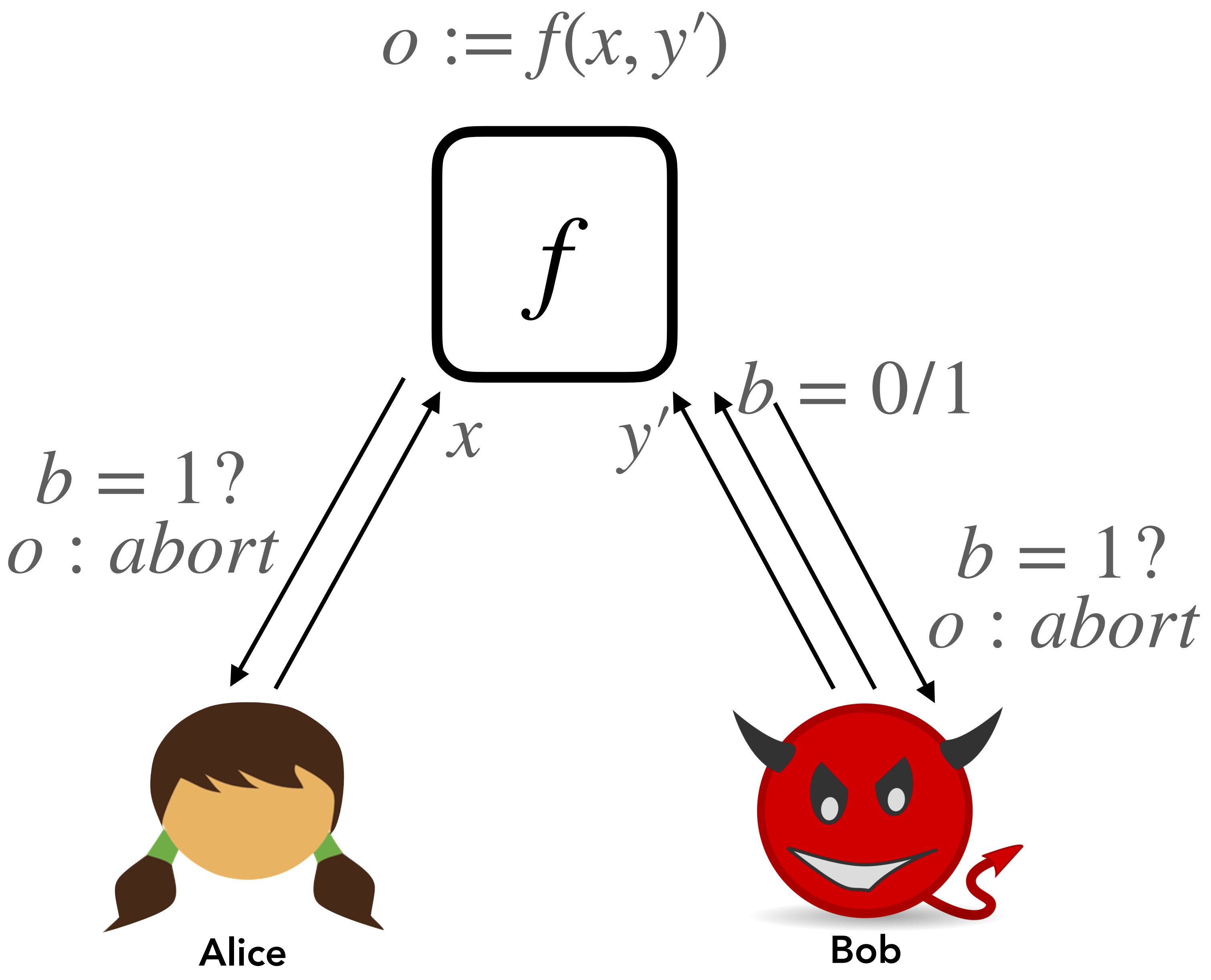

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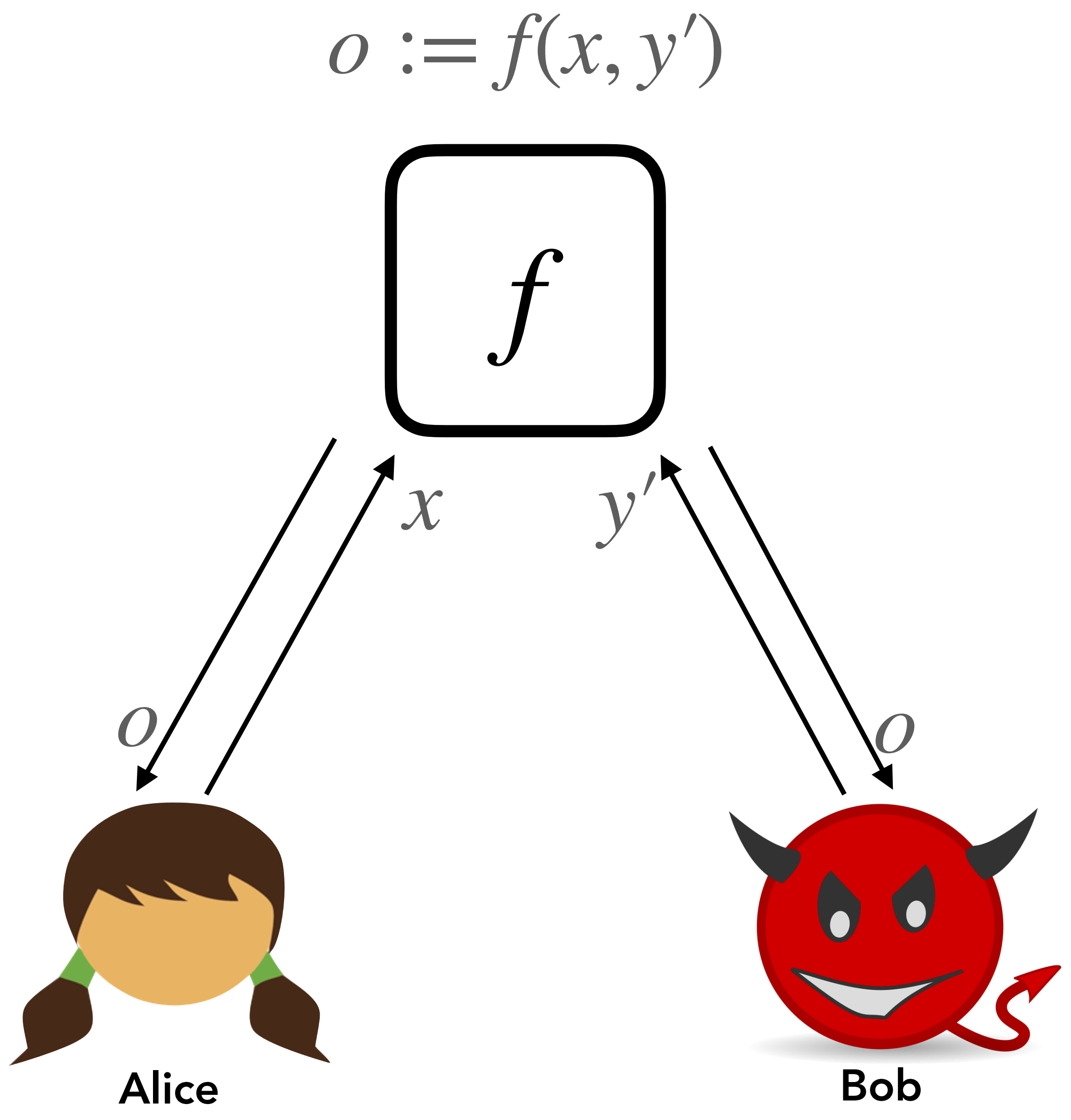

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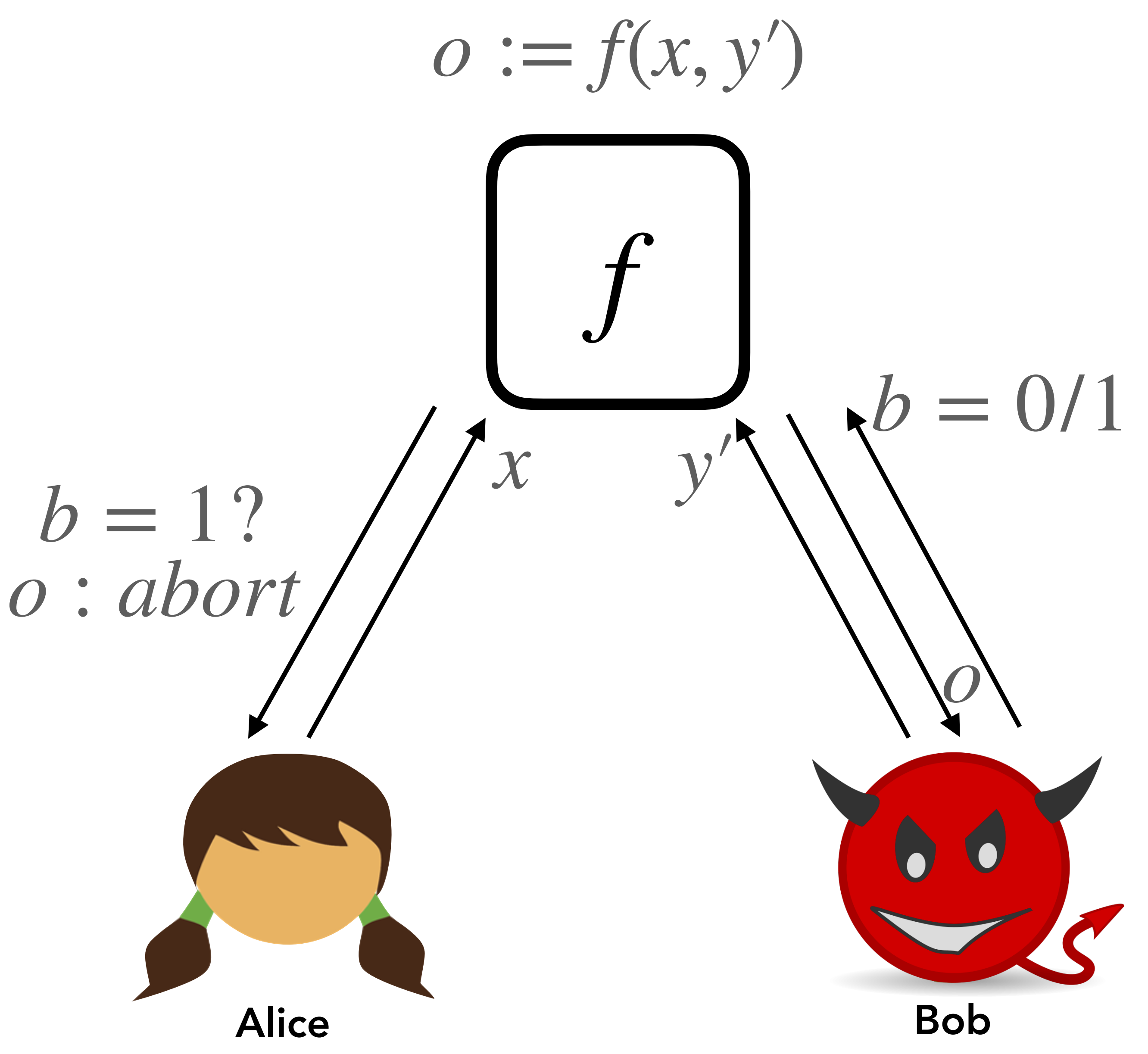

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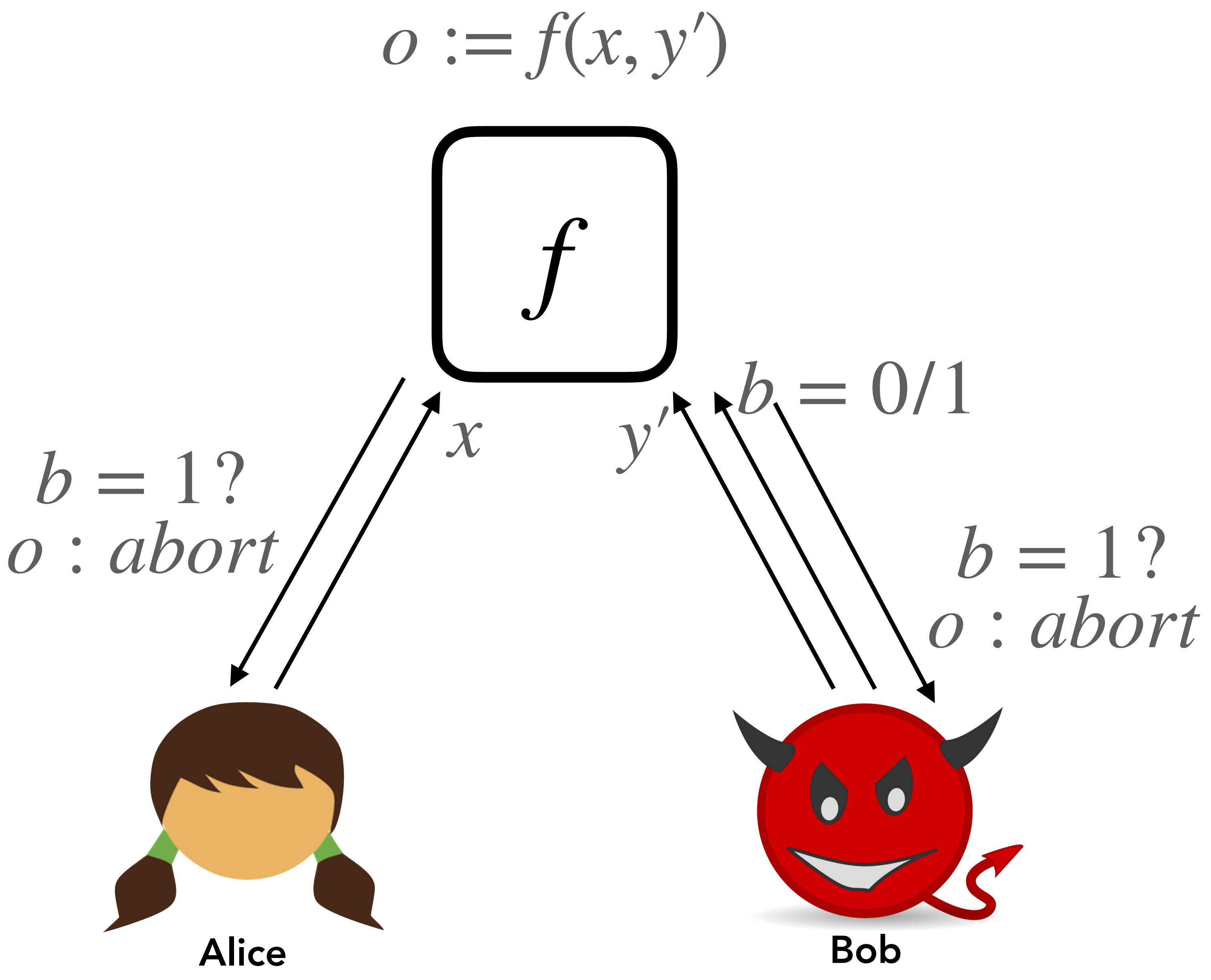

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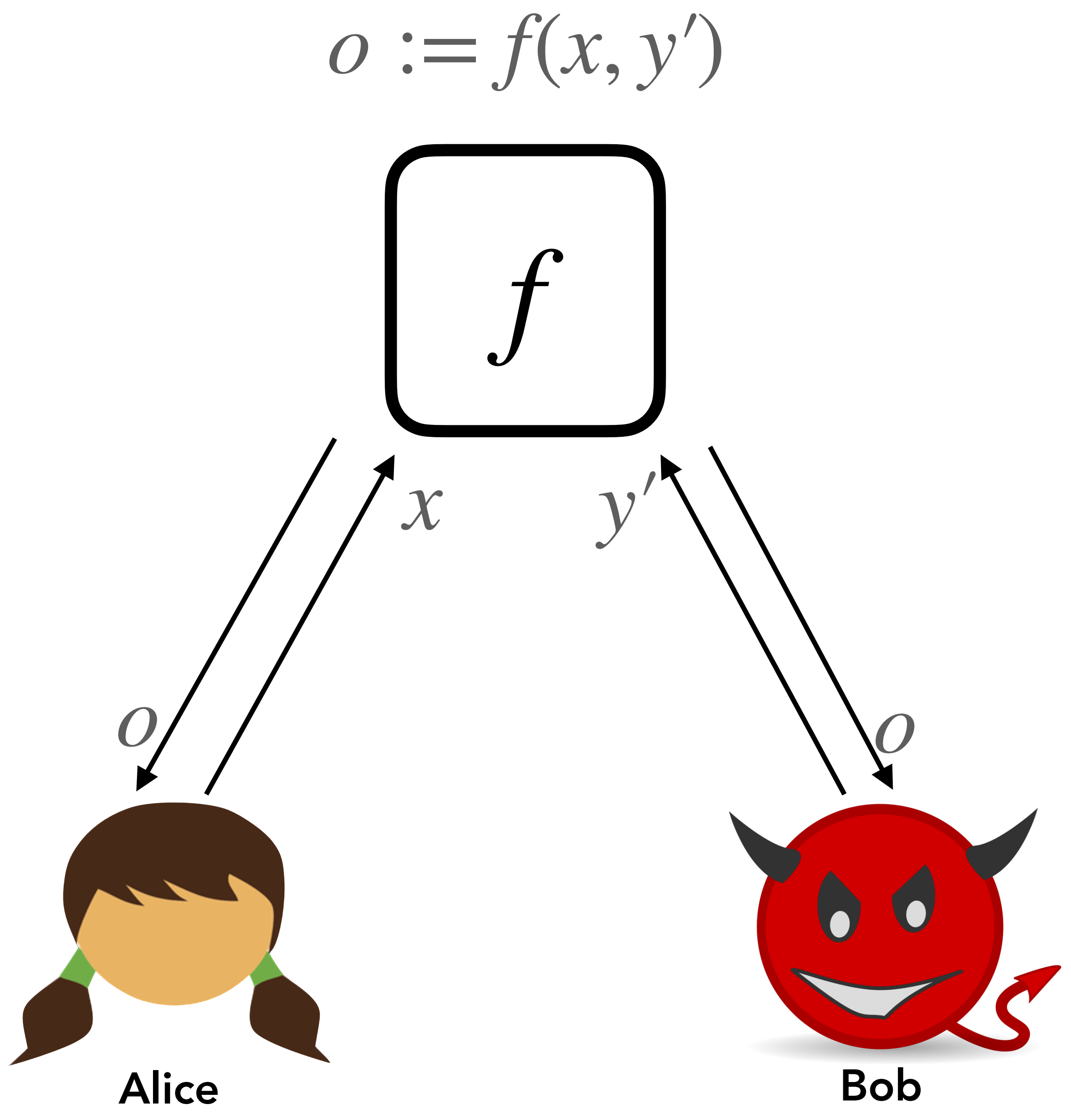

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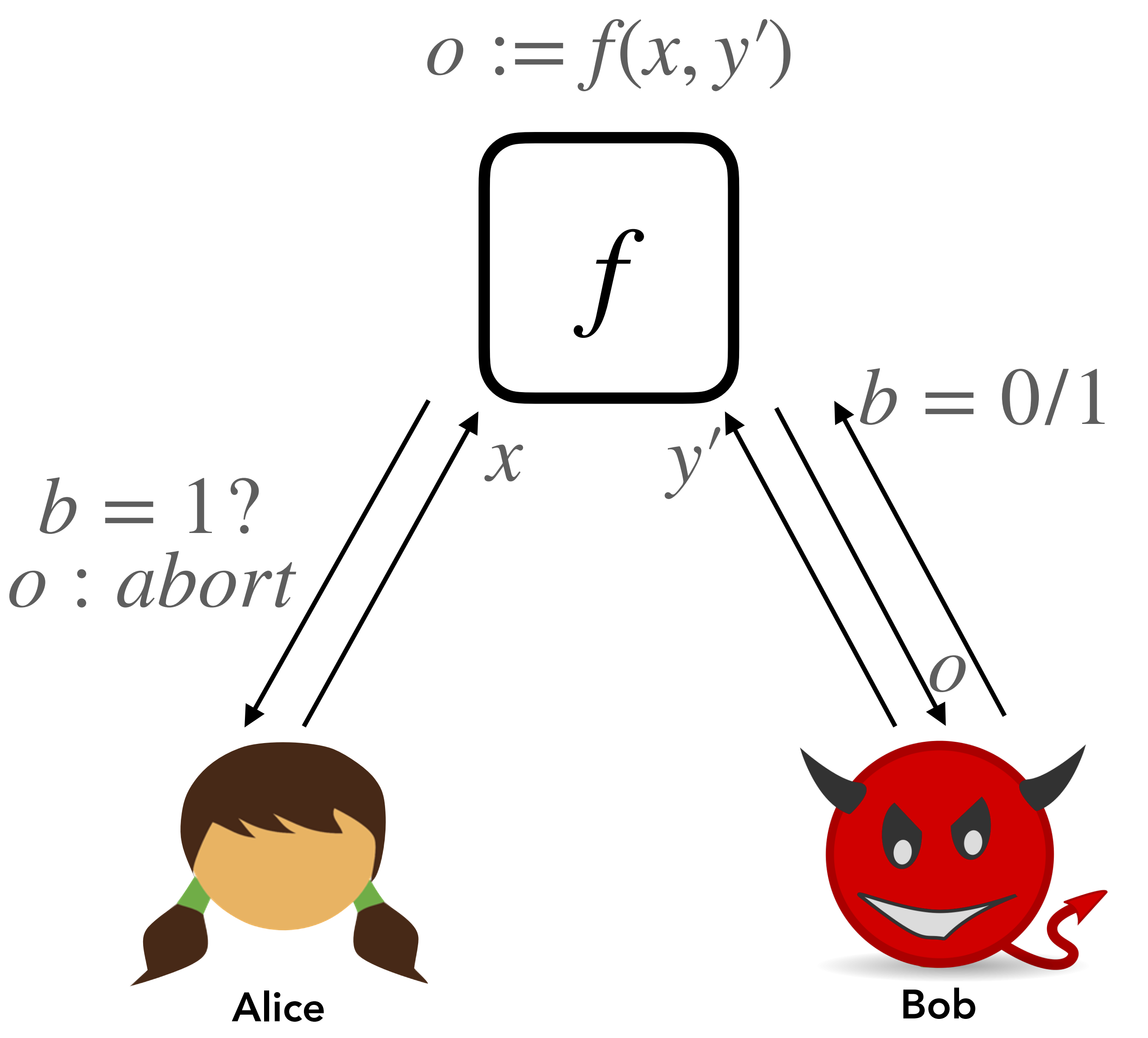
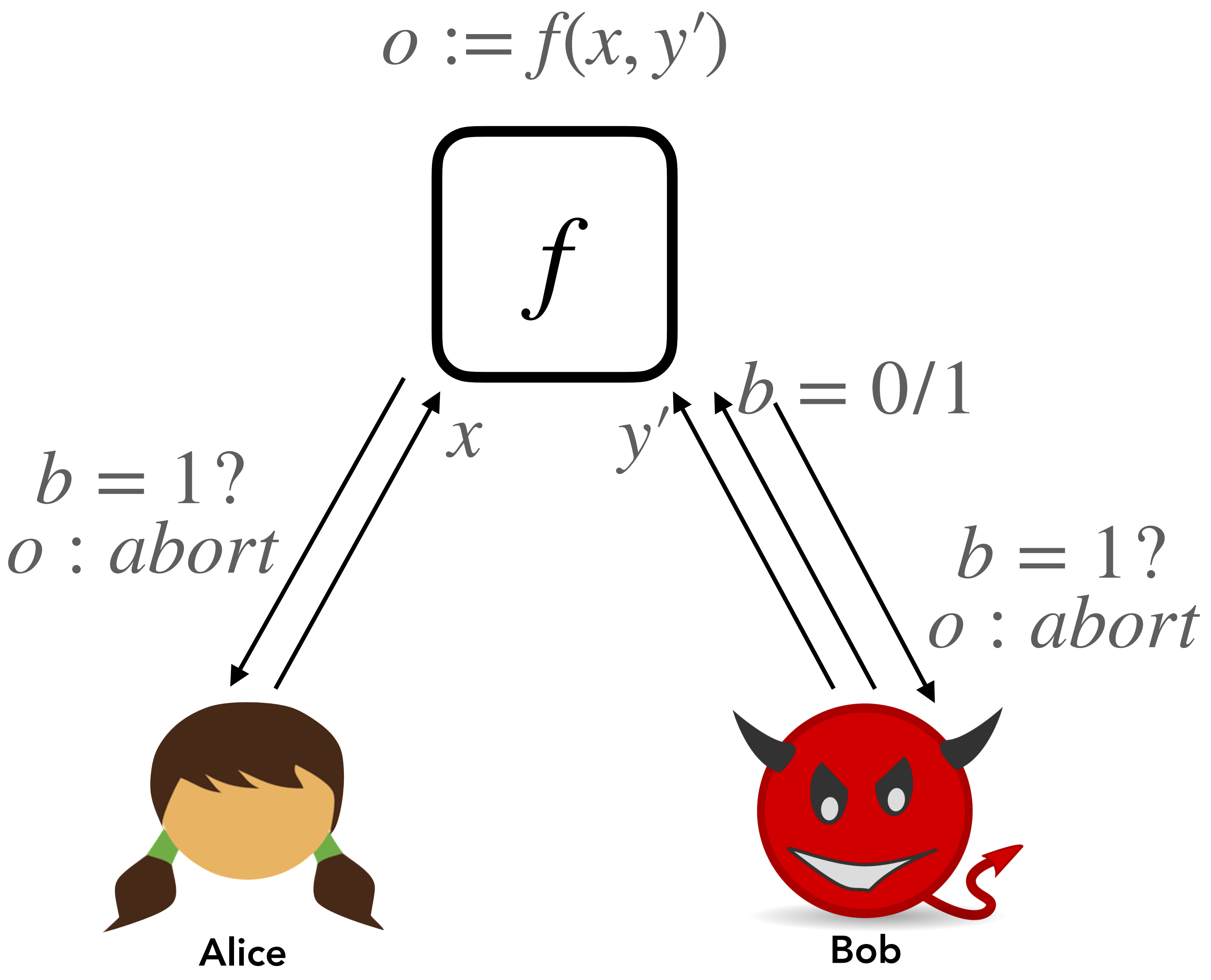
Different Security Levels

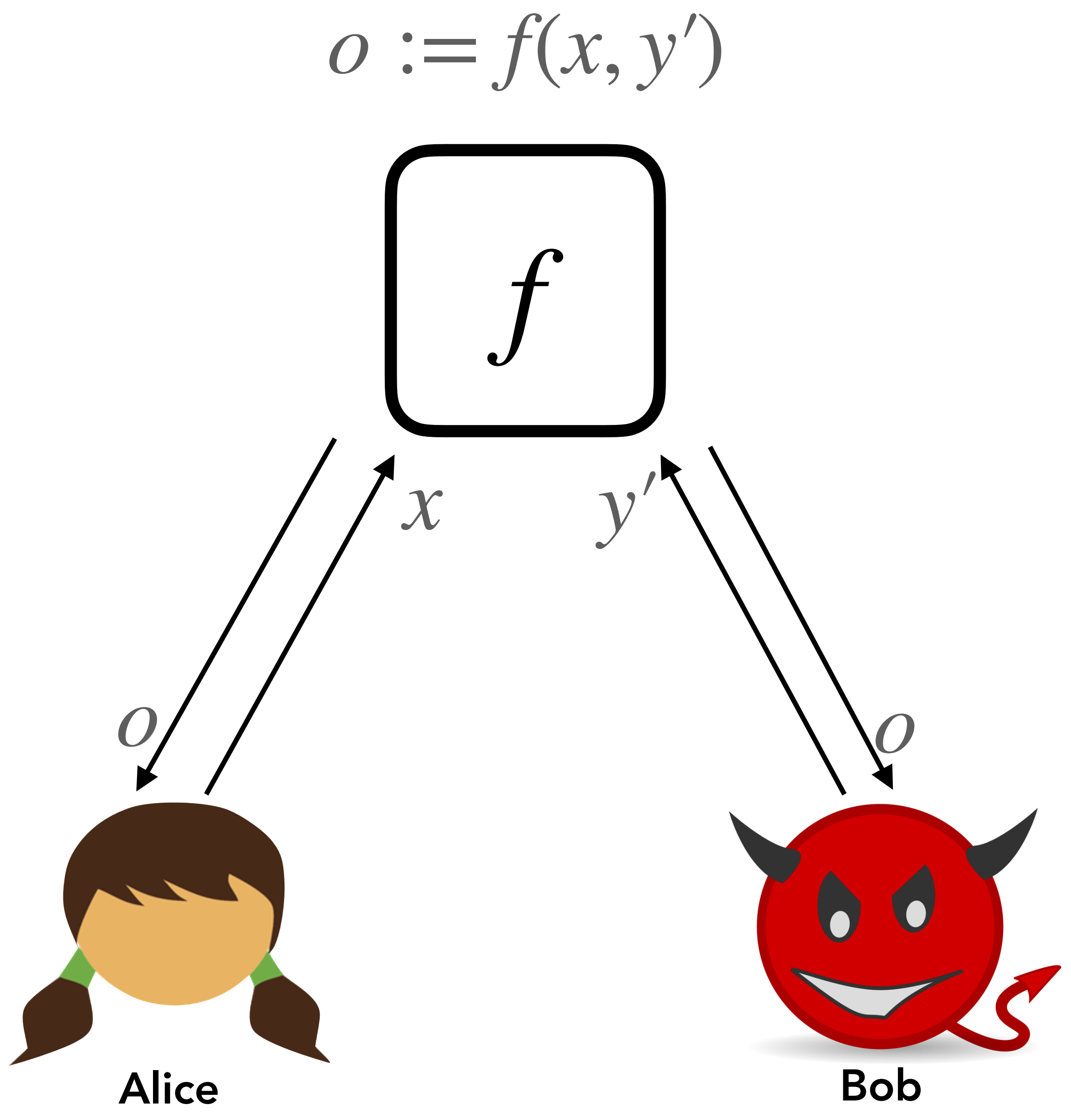
Security with Abort v.s. Fairness v.s. Guaranteed Output Delivery

[Cleve86]

Security with Abort	Fairness	Guaranteed Output Delivery
✓	✗	✗







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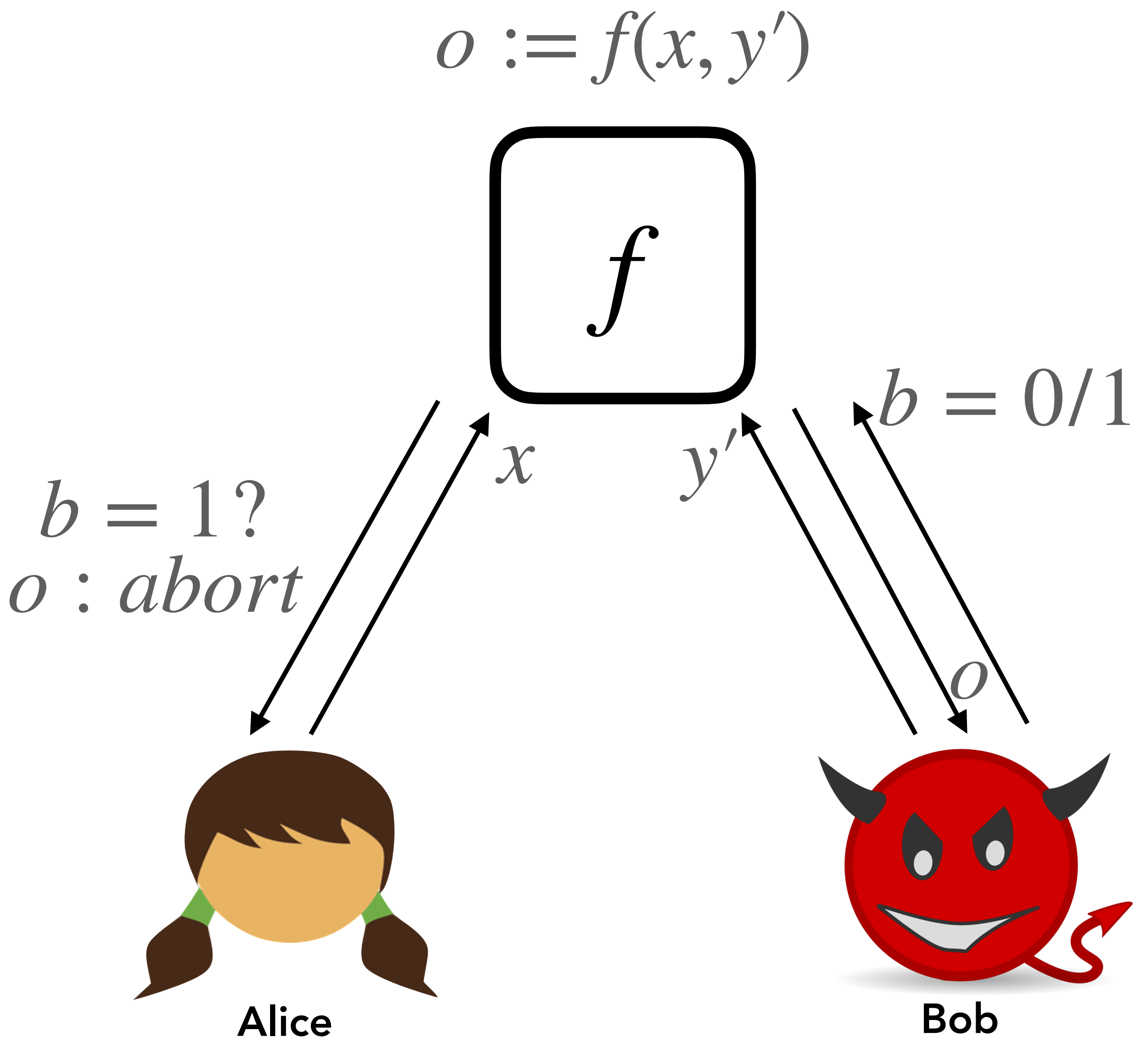
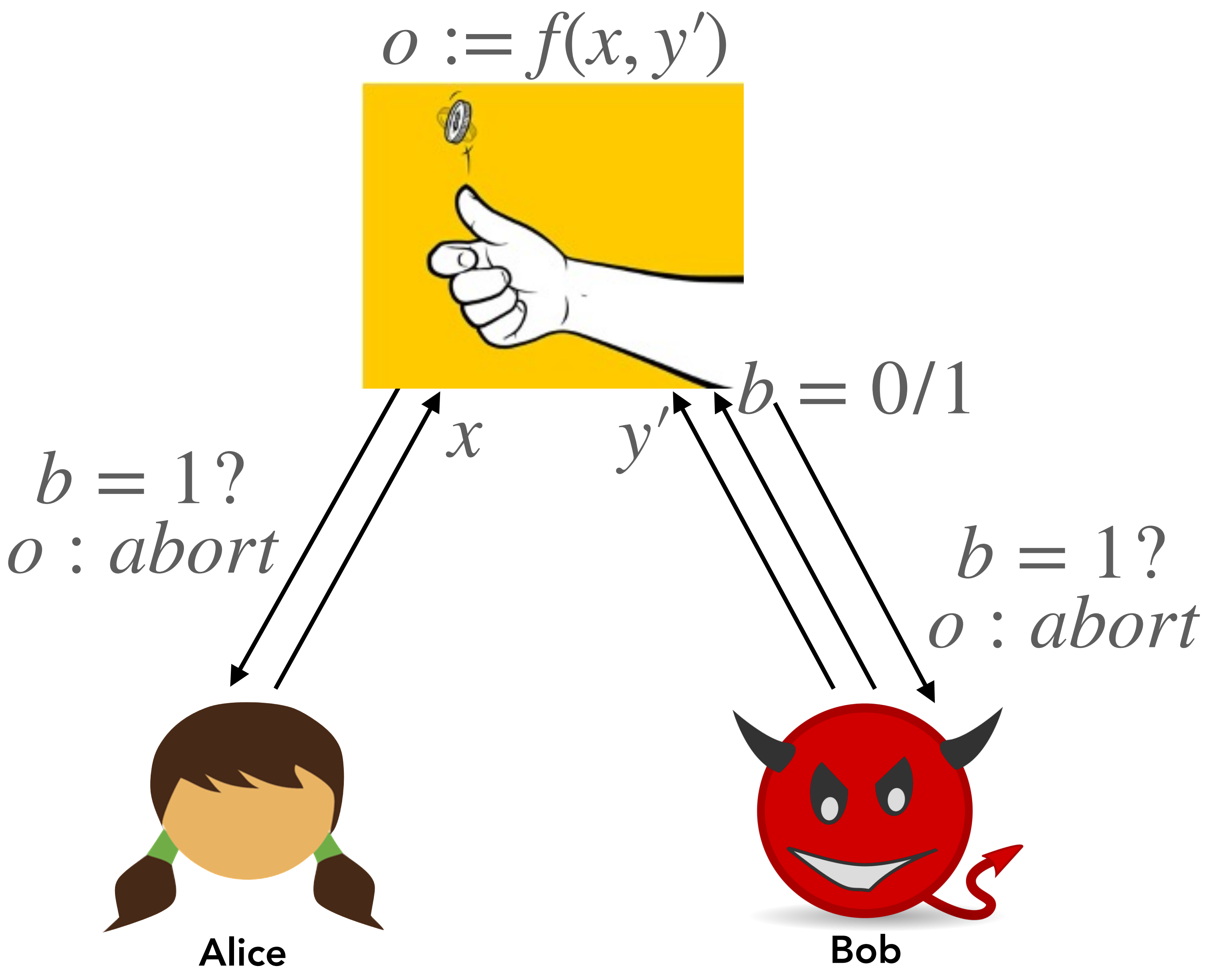
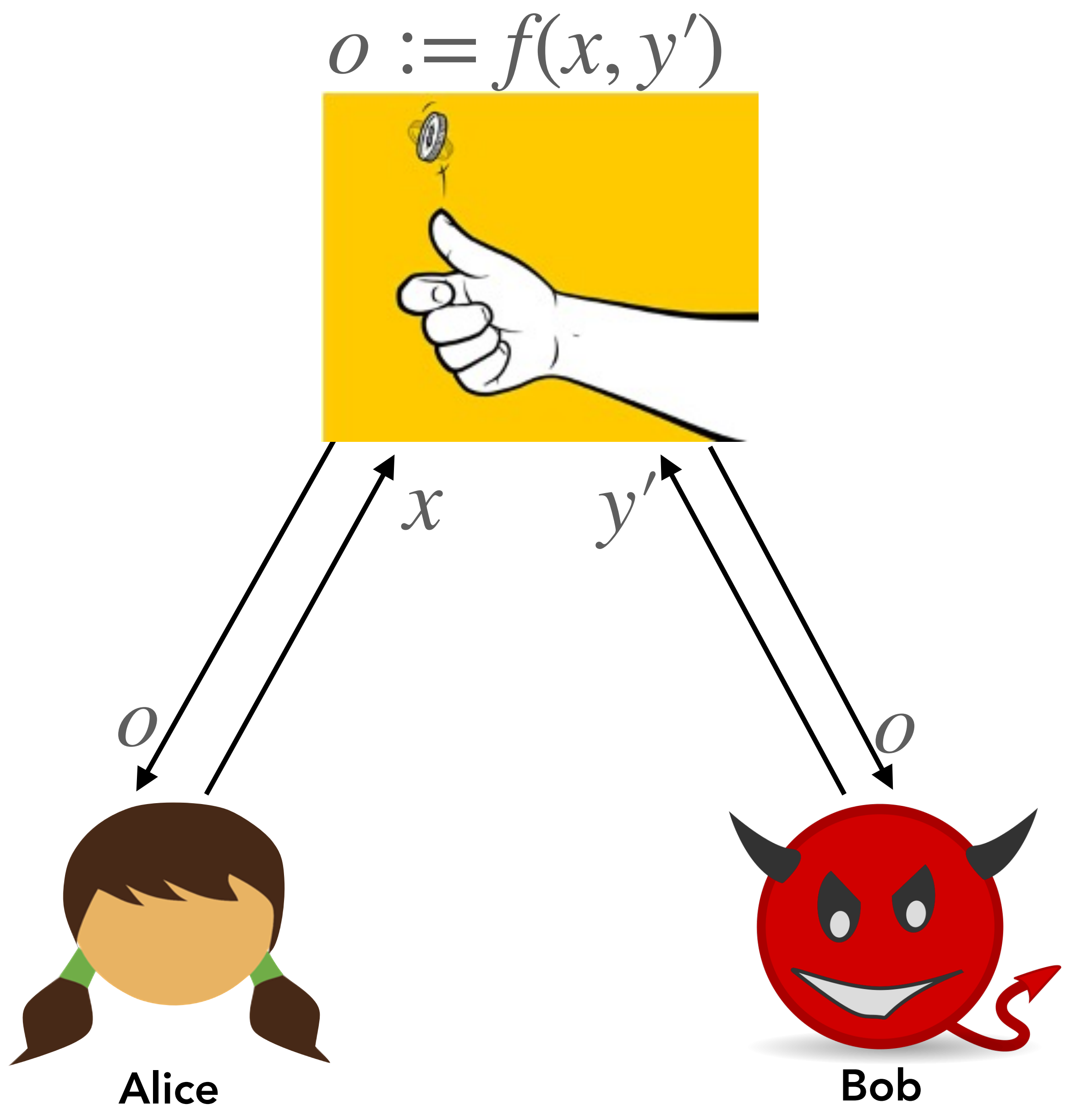
[Cleve86]

Security with Abort | **Fairness** | **Guaranteed Output Delivery**

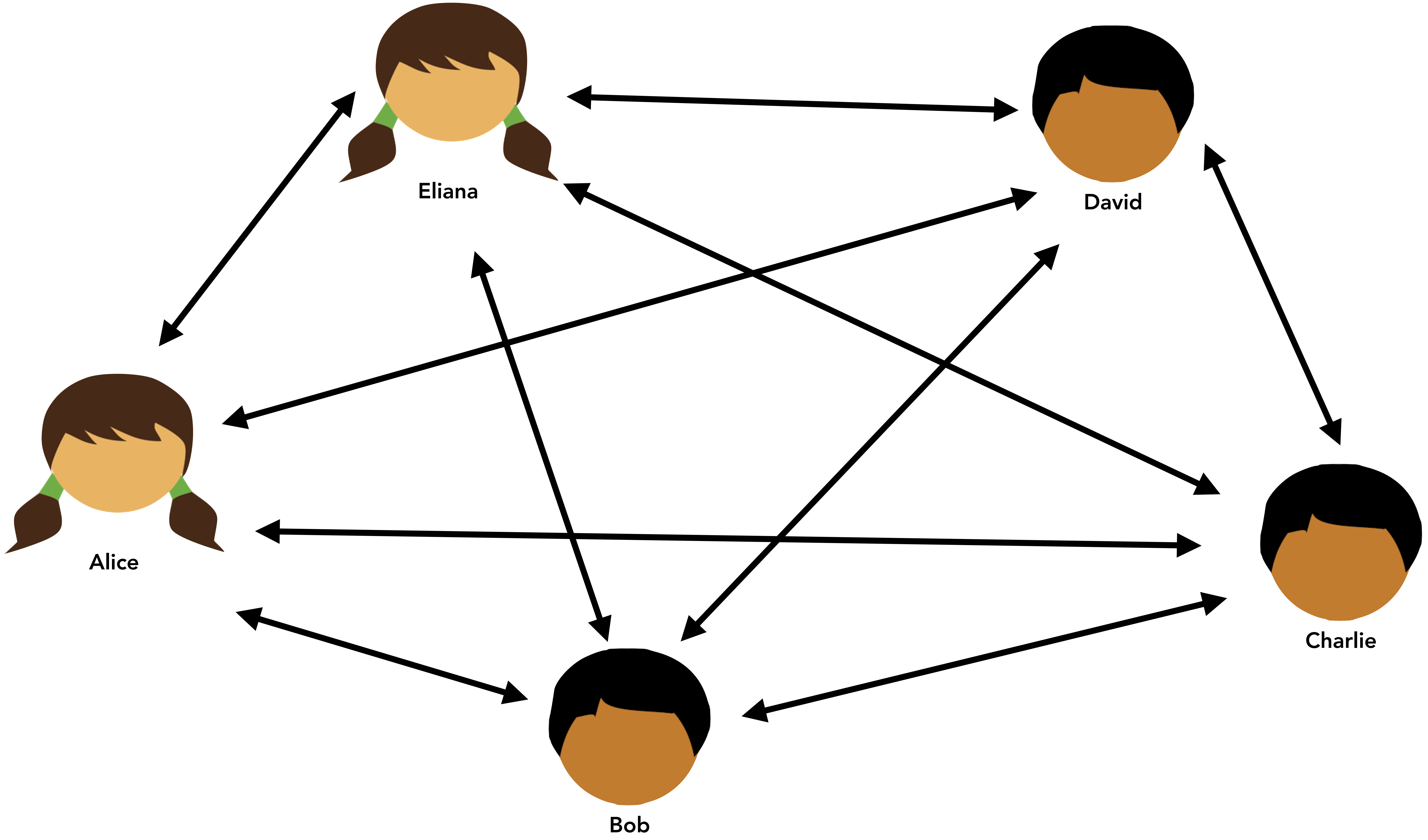






[Cleve86]'s Impossibility

More general



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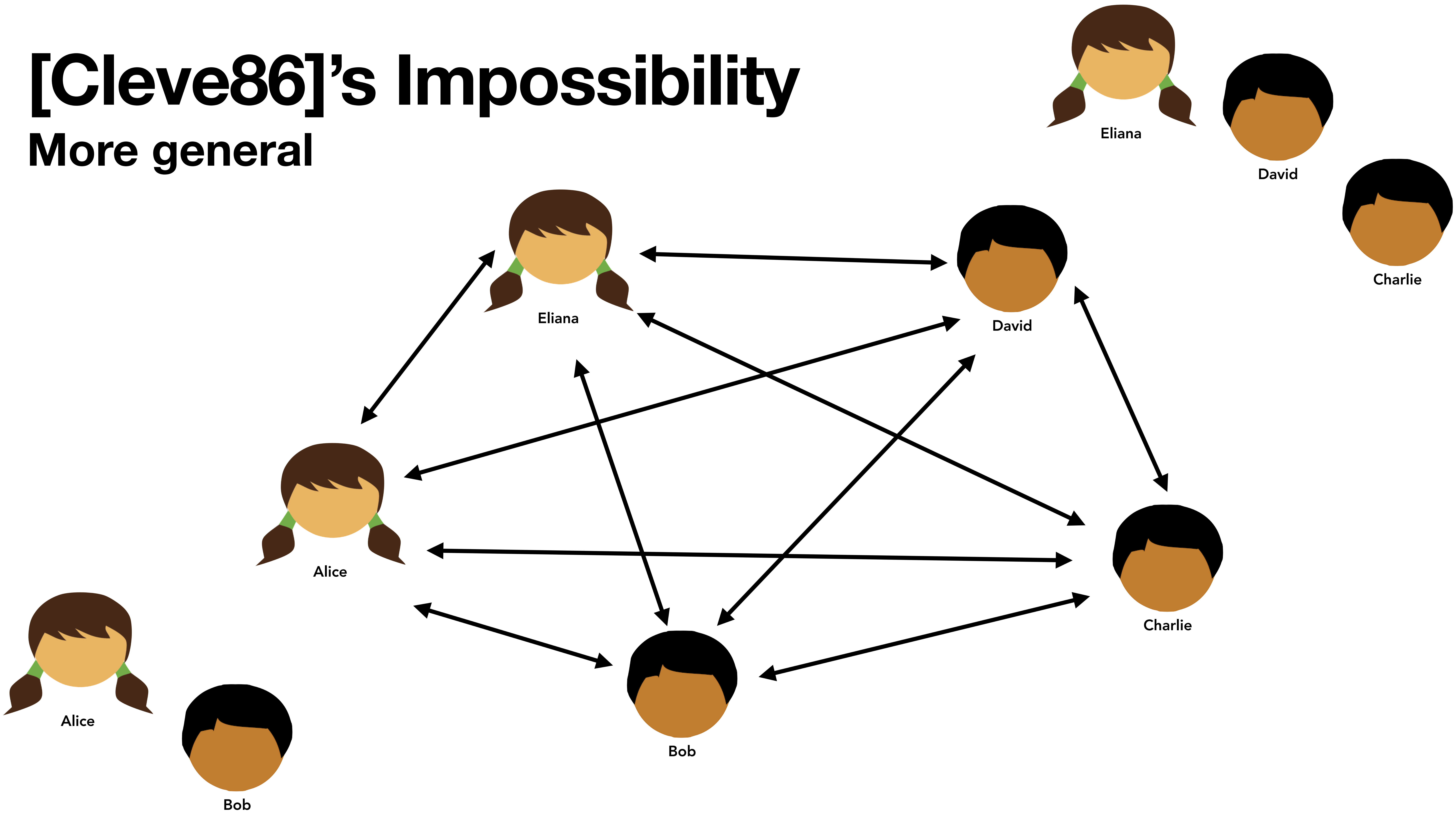

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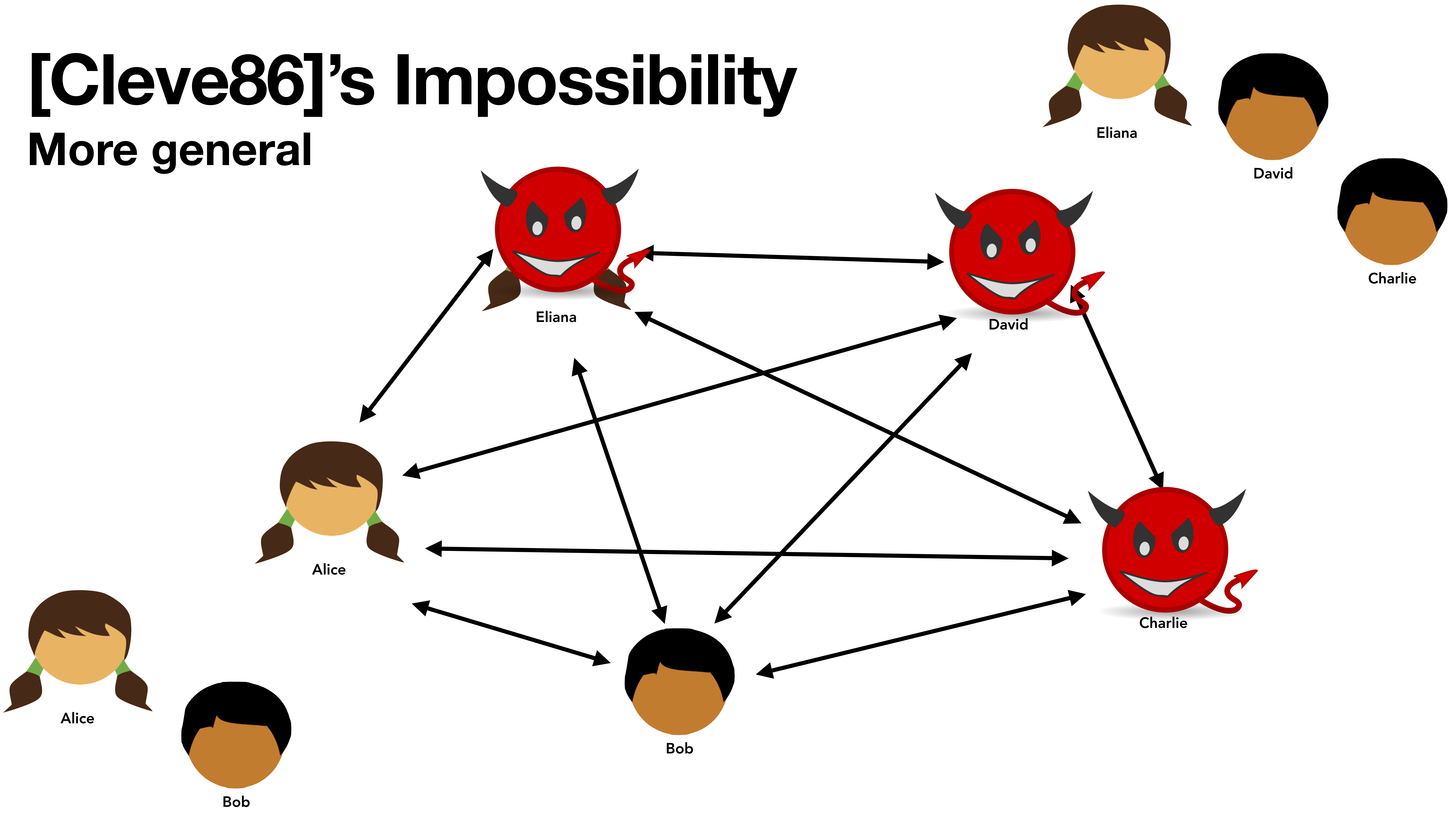

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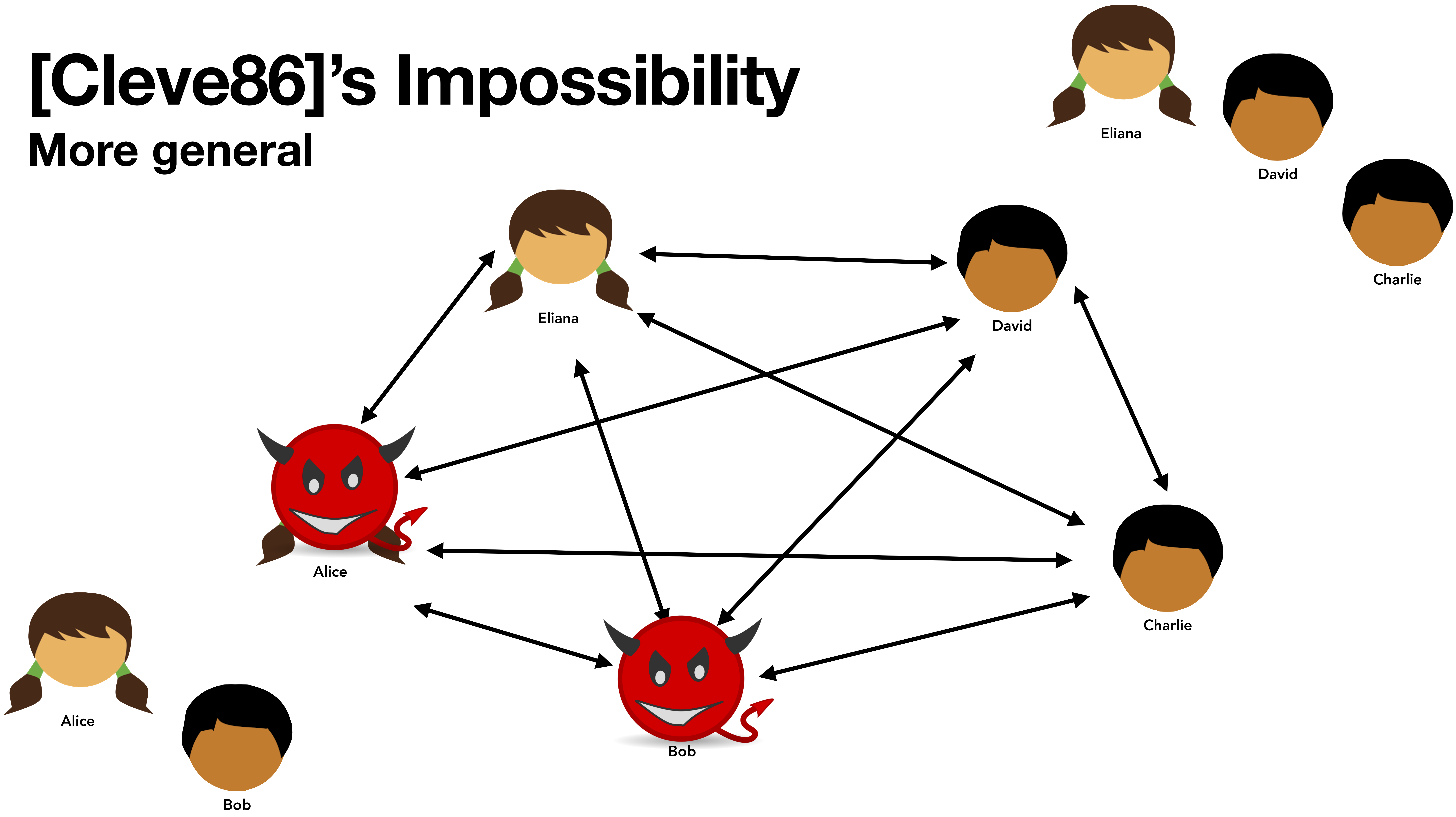

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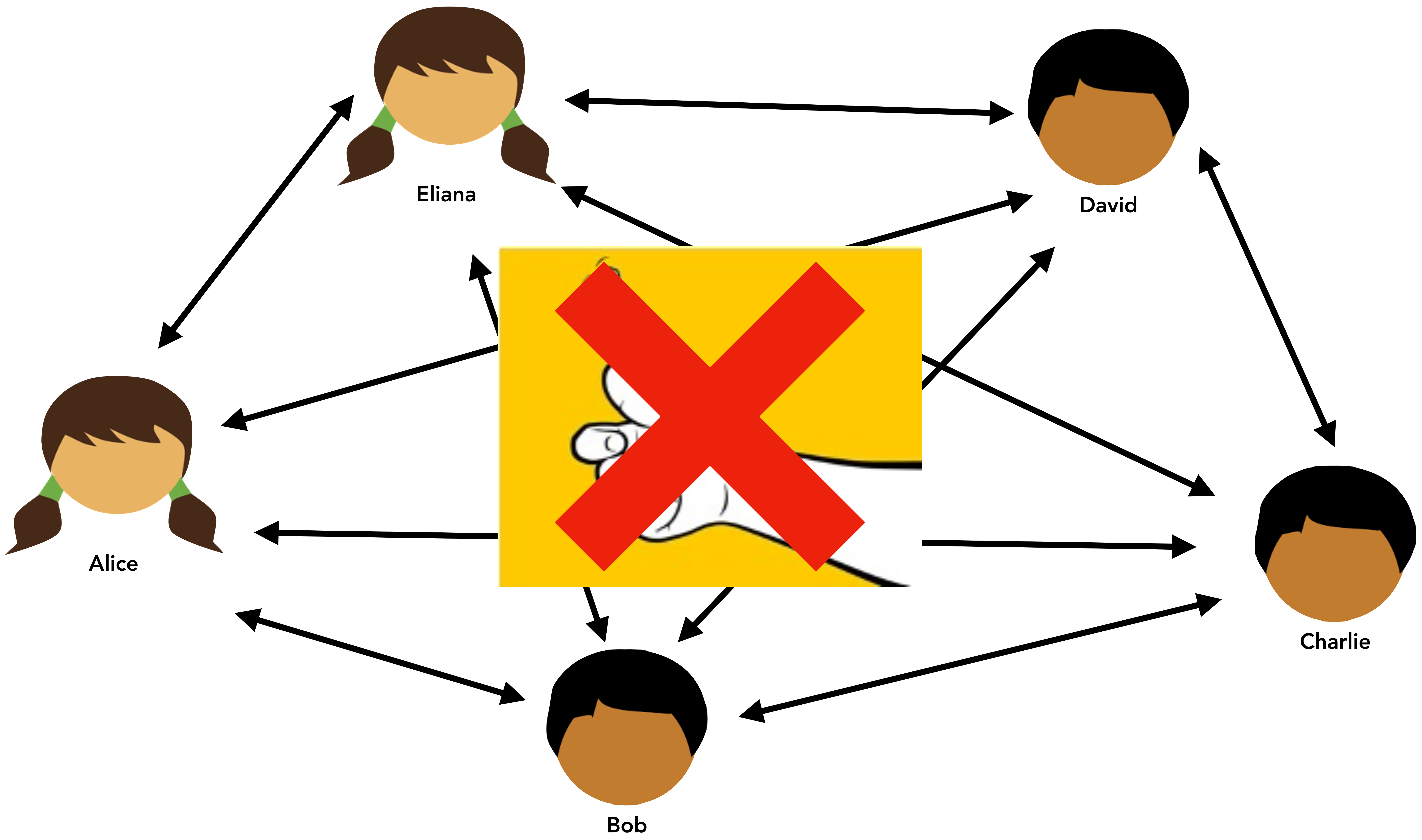

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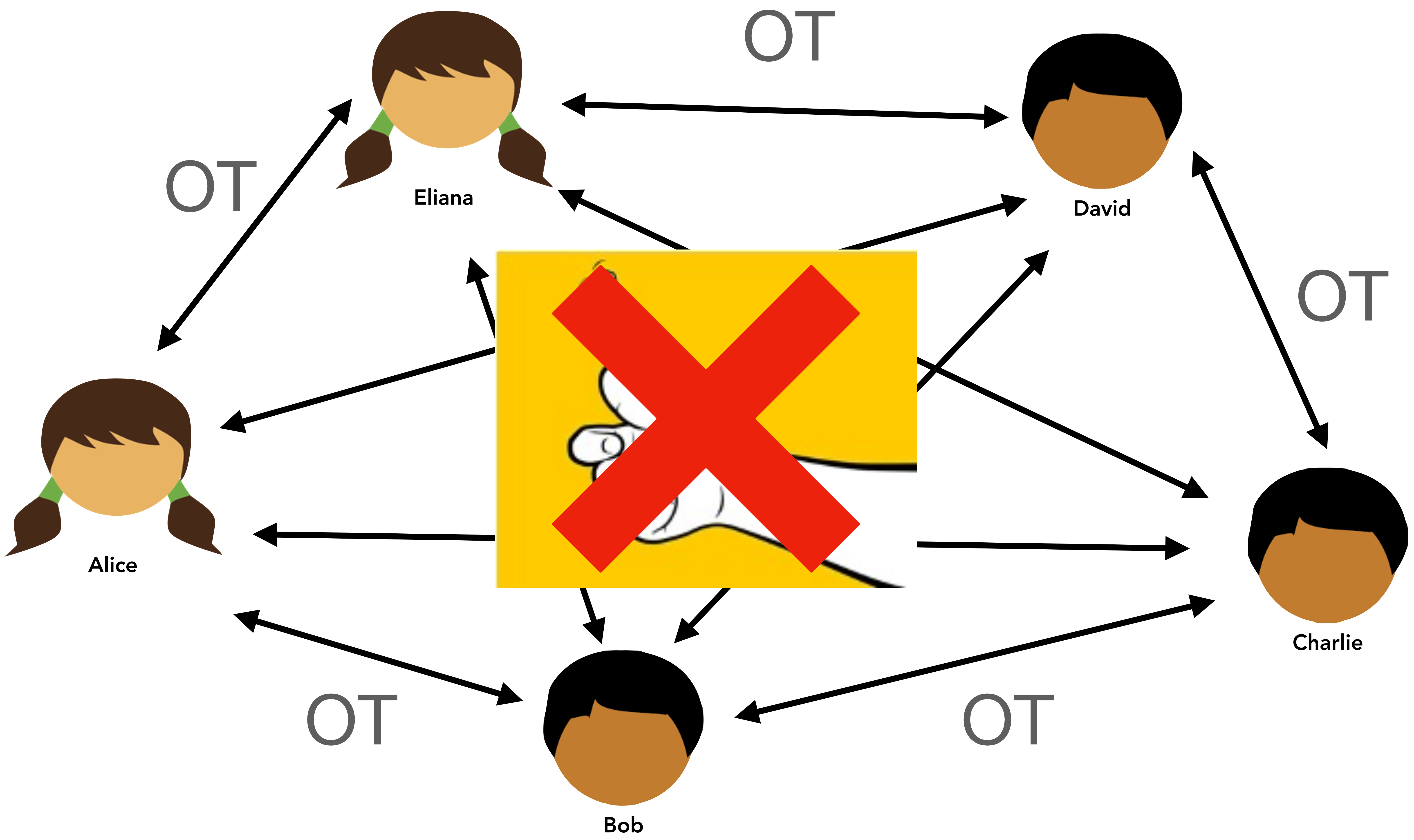
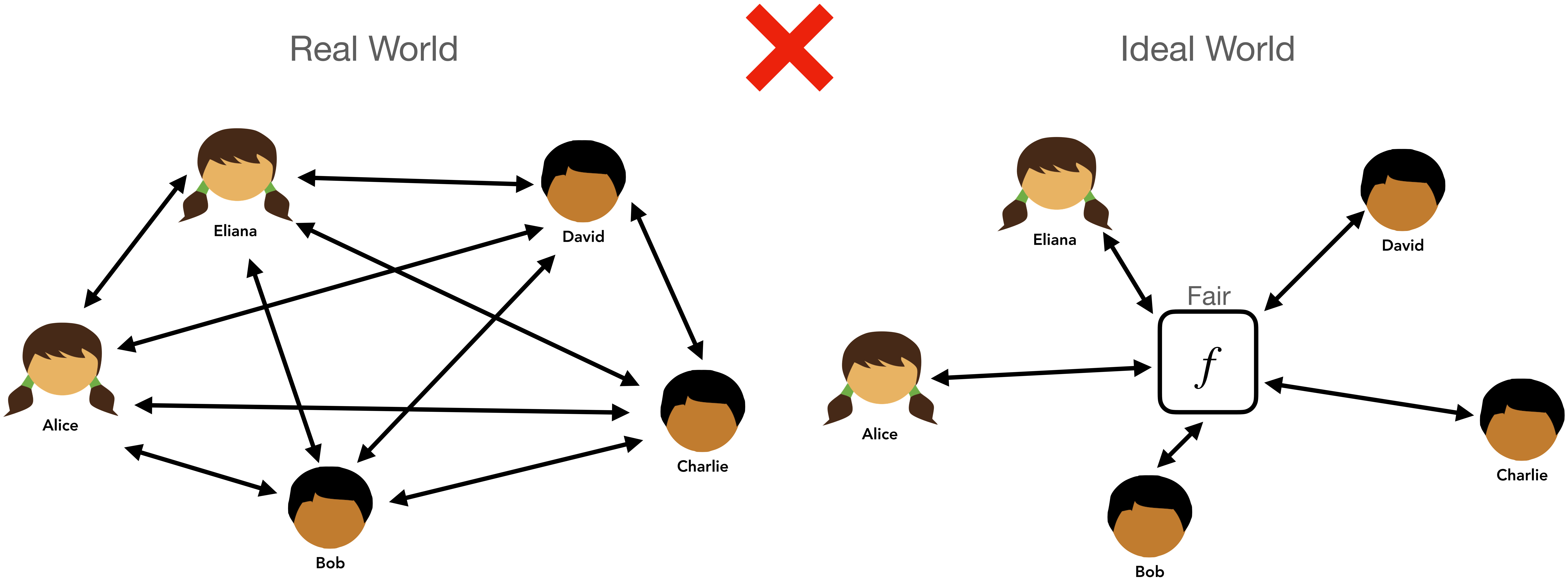
More general

Any one-way channels, e.g, OTs



Bypassing [Cleve86] to Achieve Fair MPC

Via augmenting “stronger” communication channels



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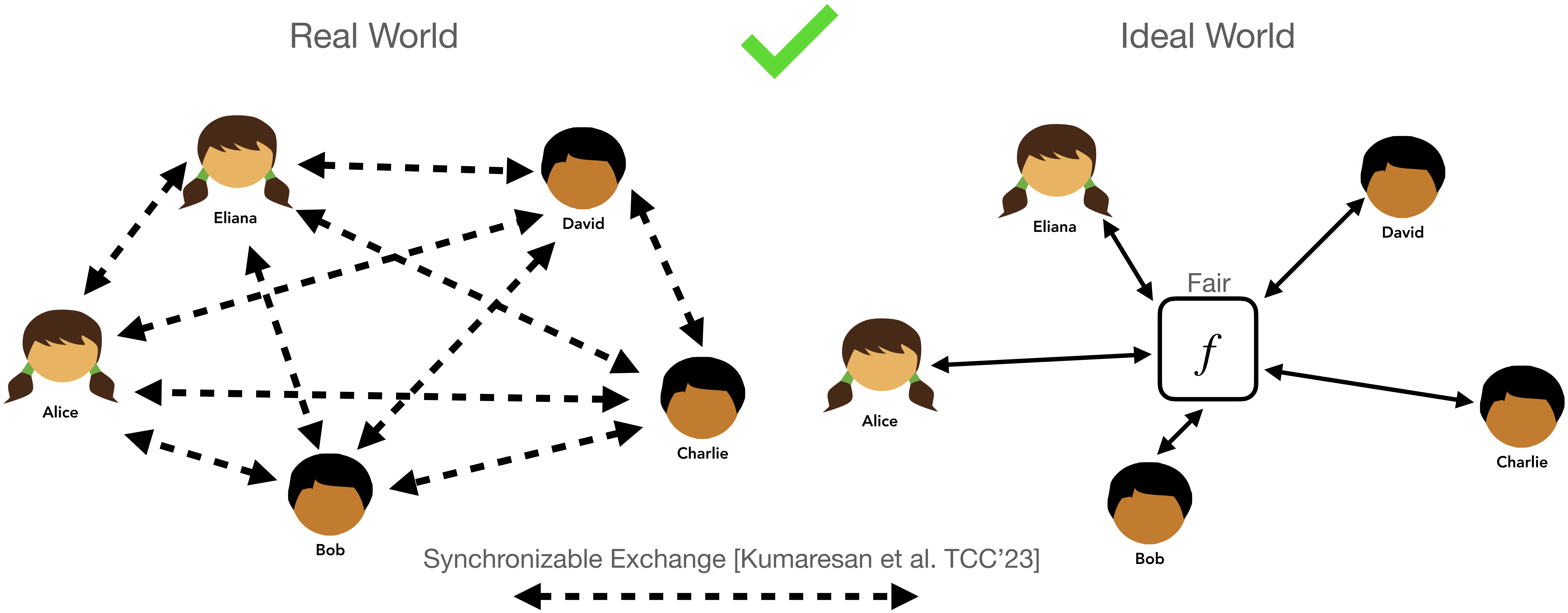
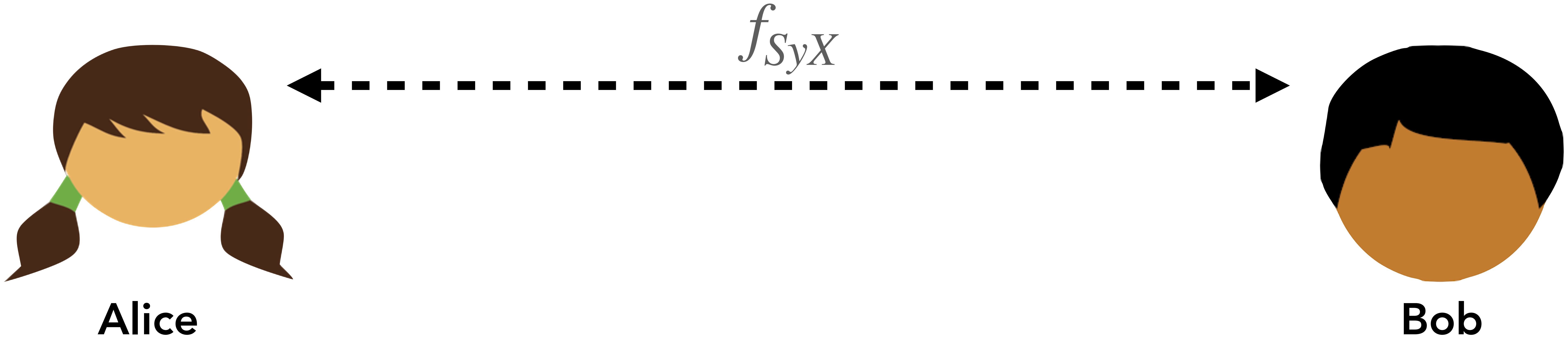


Synchronizable Exchange

[Kumaresan et al. TCC'23]



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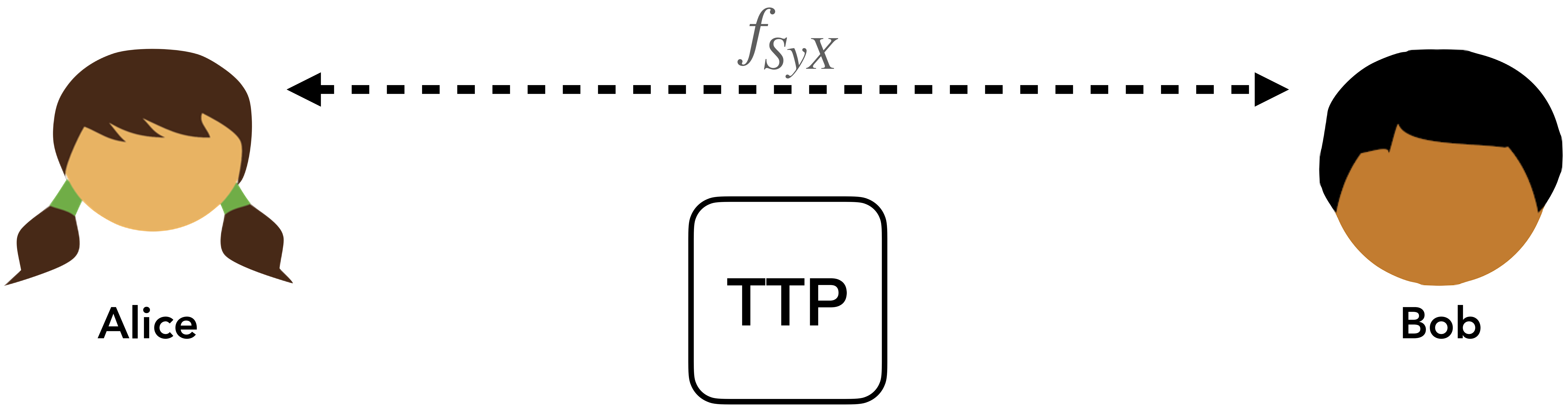

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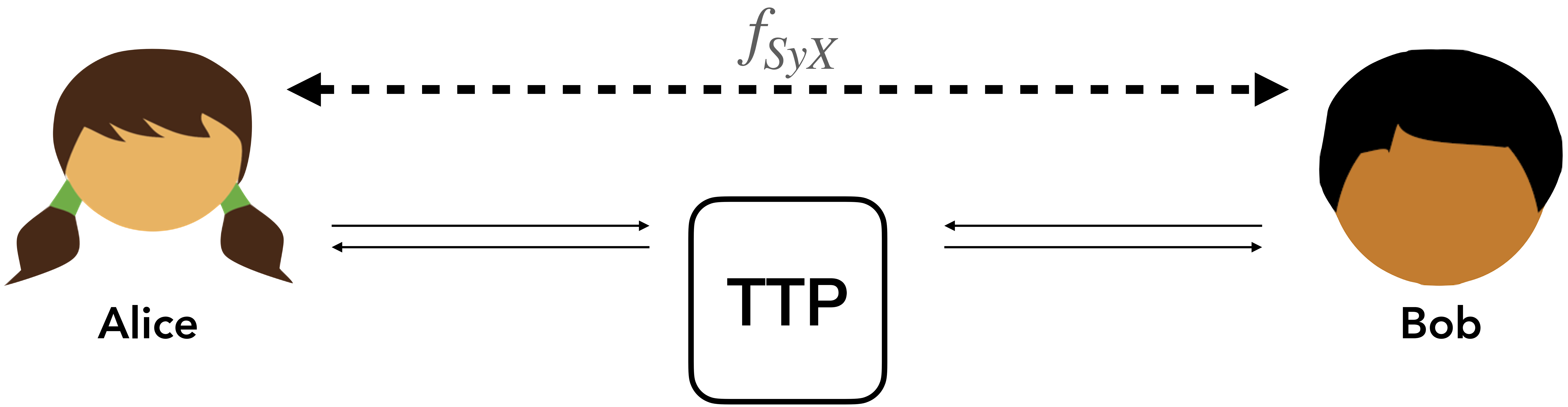

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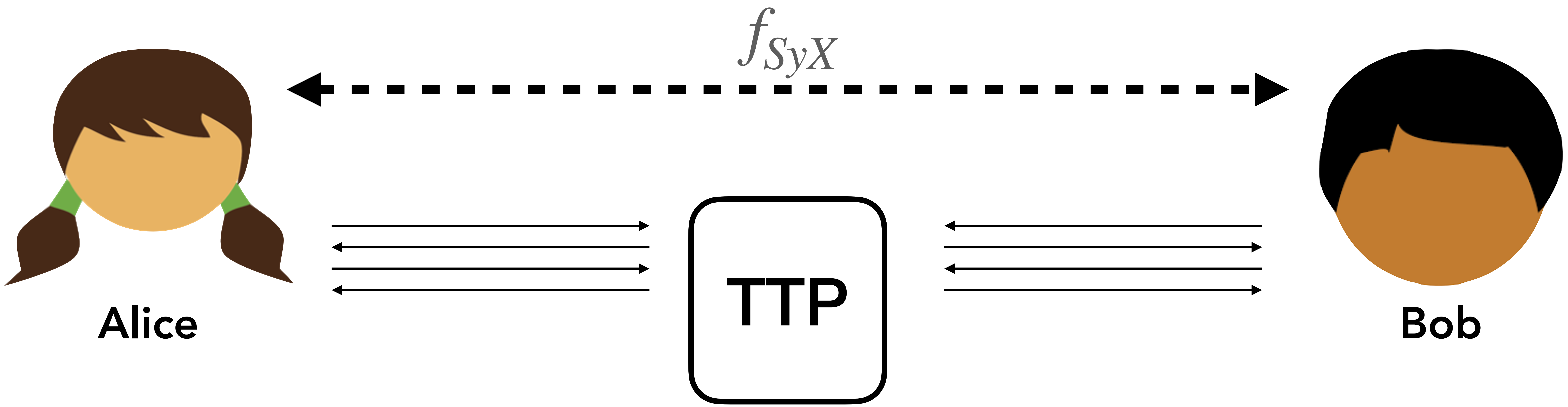

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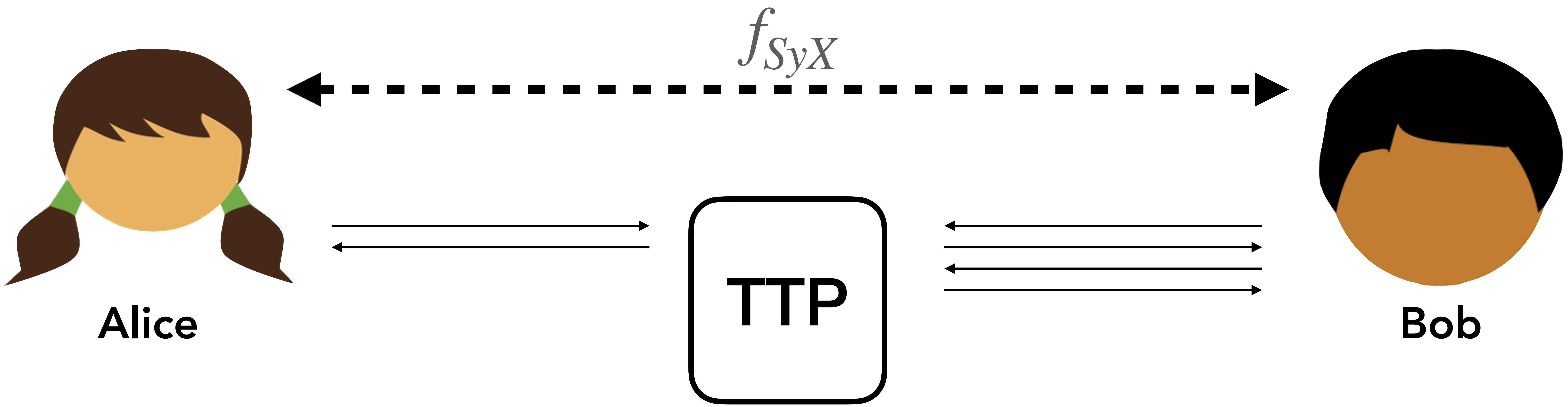

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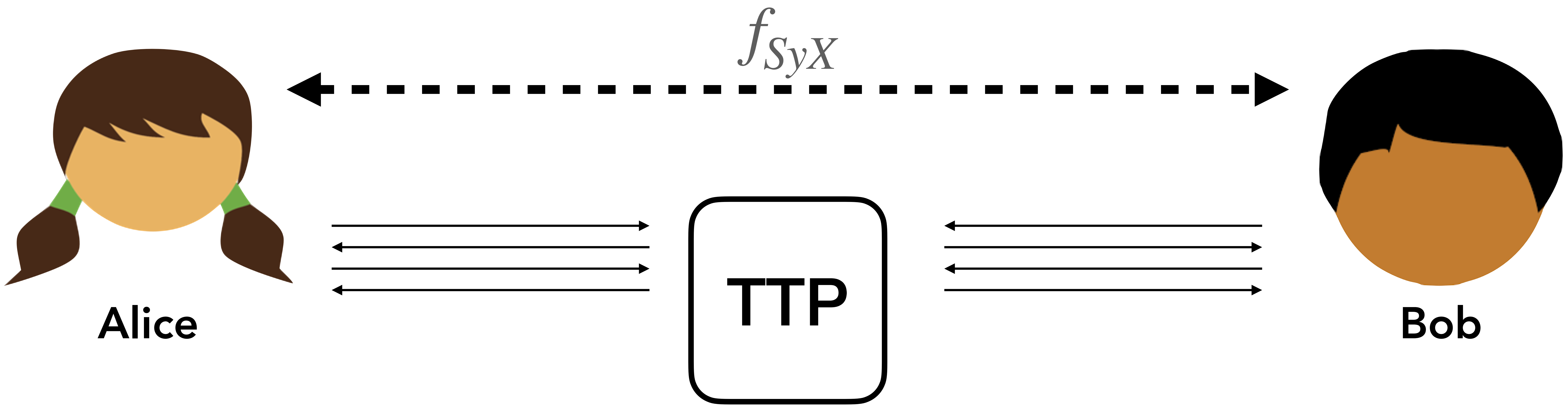
No longer a one-way channel



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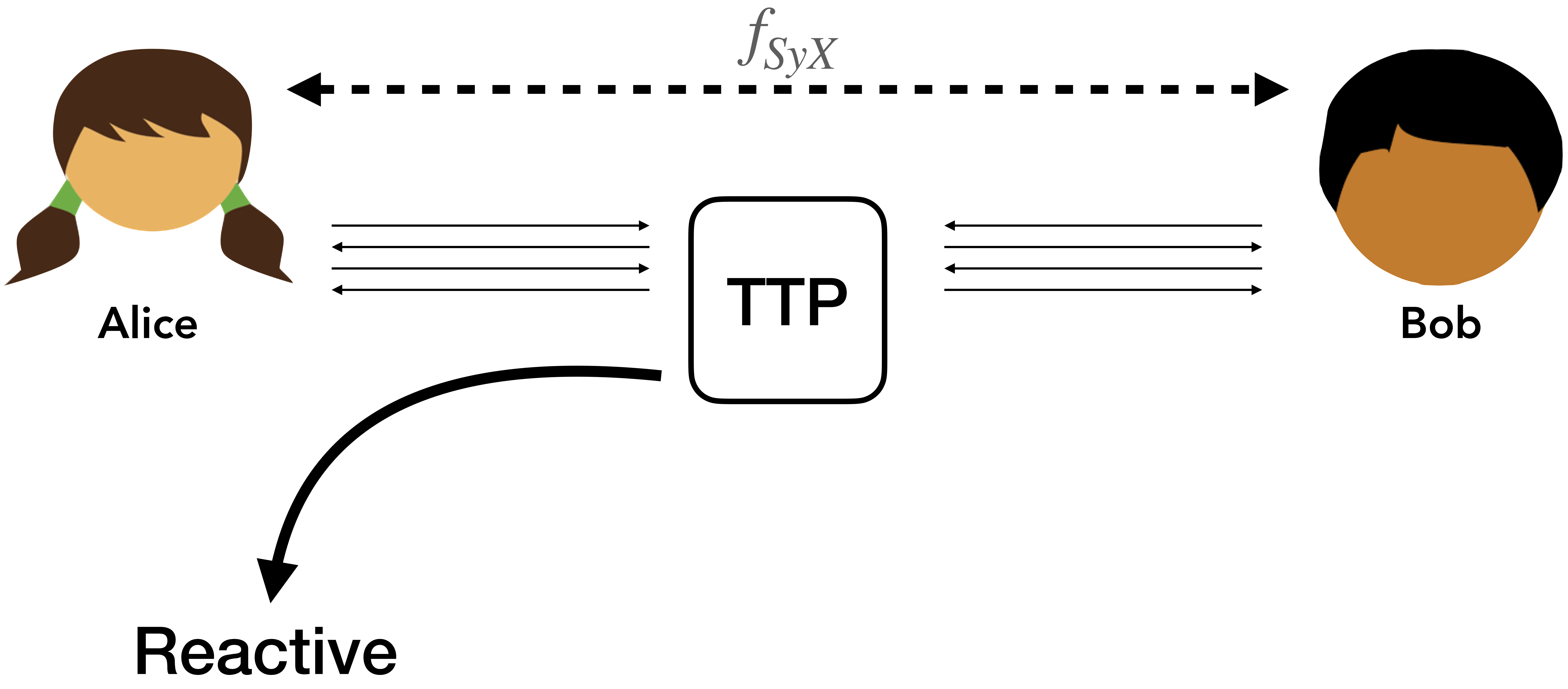

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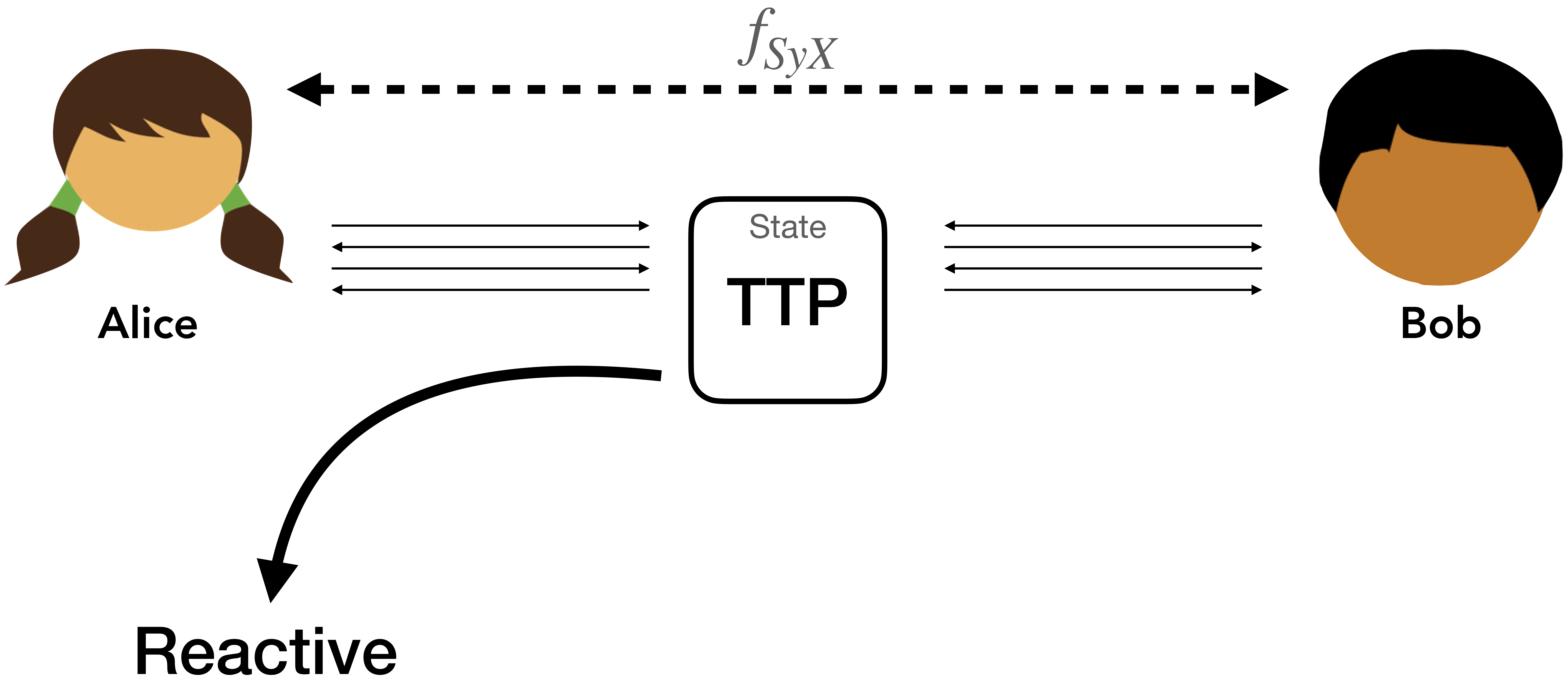

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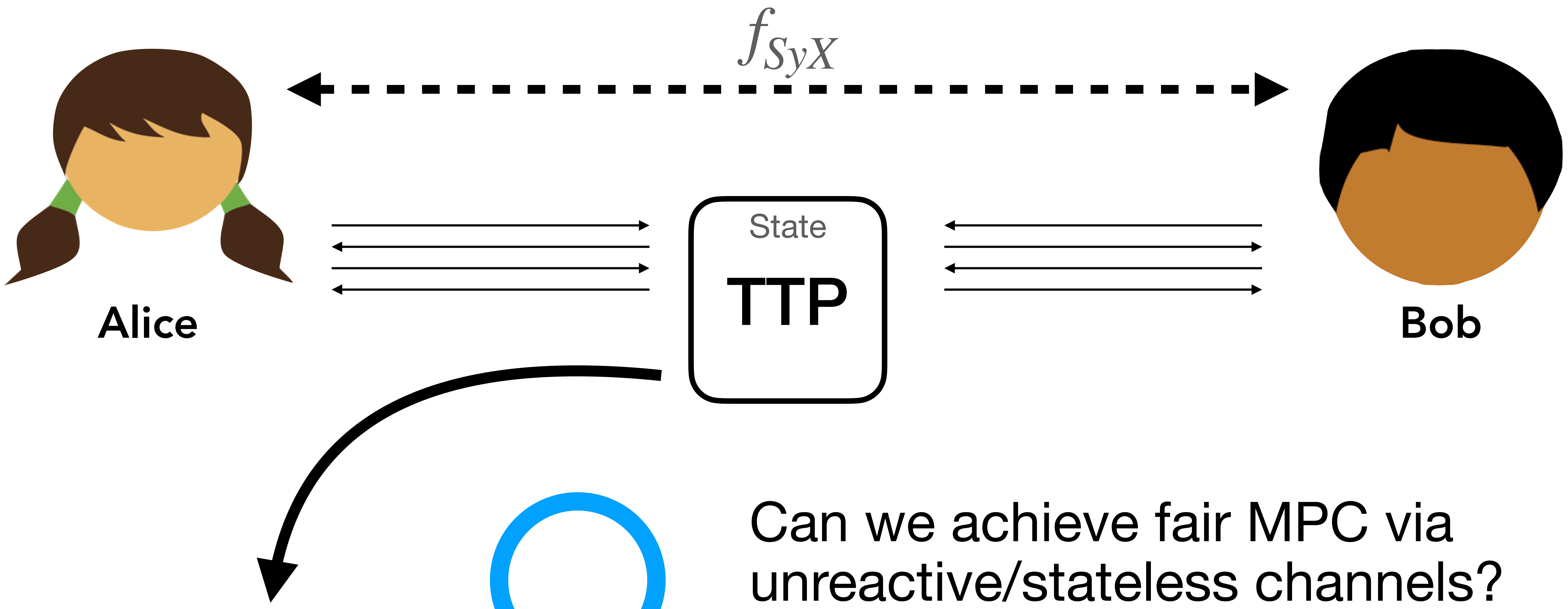

Reactive

Can we achieve fair MPC via unreactive/stateless channels?

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$t =$ #corruption, $n =$ #party

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Table 1: Our contributions.

t	Insufficient functionalities for fair coin tossing	Sufficient functionalities for fair MPC
$t < \frac{n}{2}$	–	Local computation [FGMvR02]
$t = \frac{n}{2}$	Local computation [Cle86]	2-wise fair exchange **[ours]**
$t > \frac{n}{2}$	Arbitrary unreactive t-wise **[ours]**	$(t+1)$-wise fair exchange[a] **[ours]**

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Not very fair ☹

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[Cohen and Lindel, Asiacrypt 14]:

1. Fairness with broadcast →Fairness without broadcast
2. No G.O.D. with broadcast →No fairness (even) with broadcast

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Example: Our Upper Bound

$t = \frac{n}{2}, 2$**-wise fair exchange**

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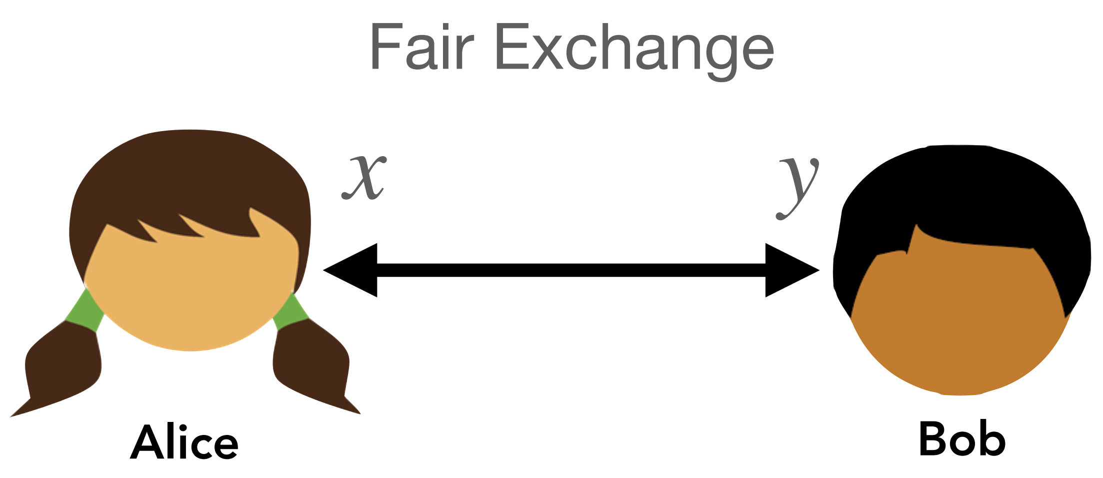

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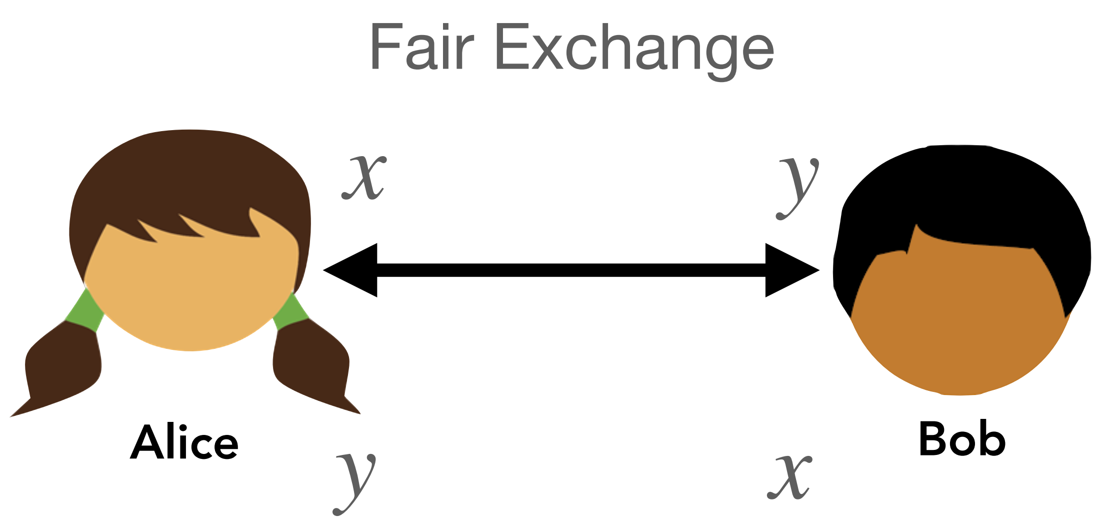
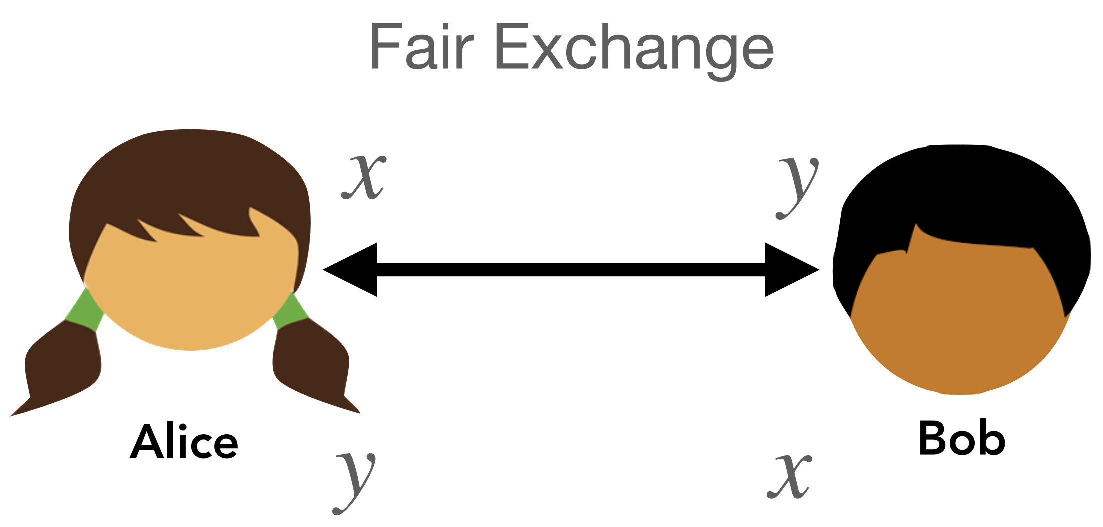
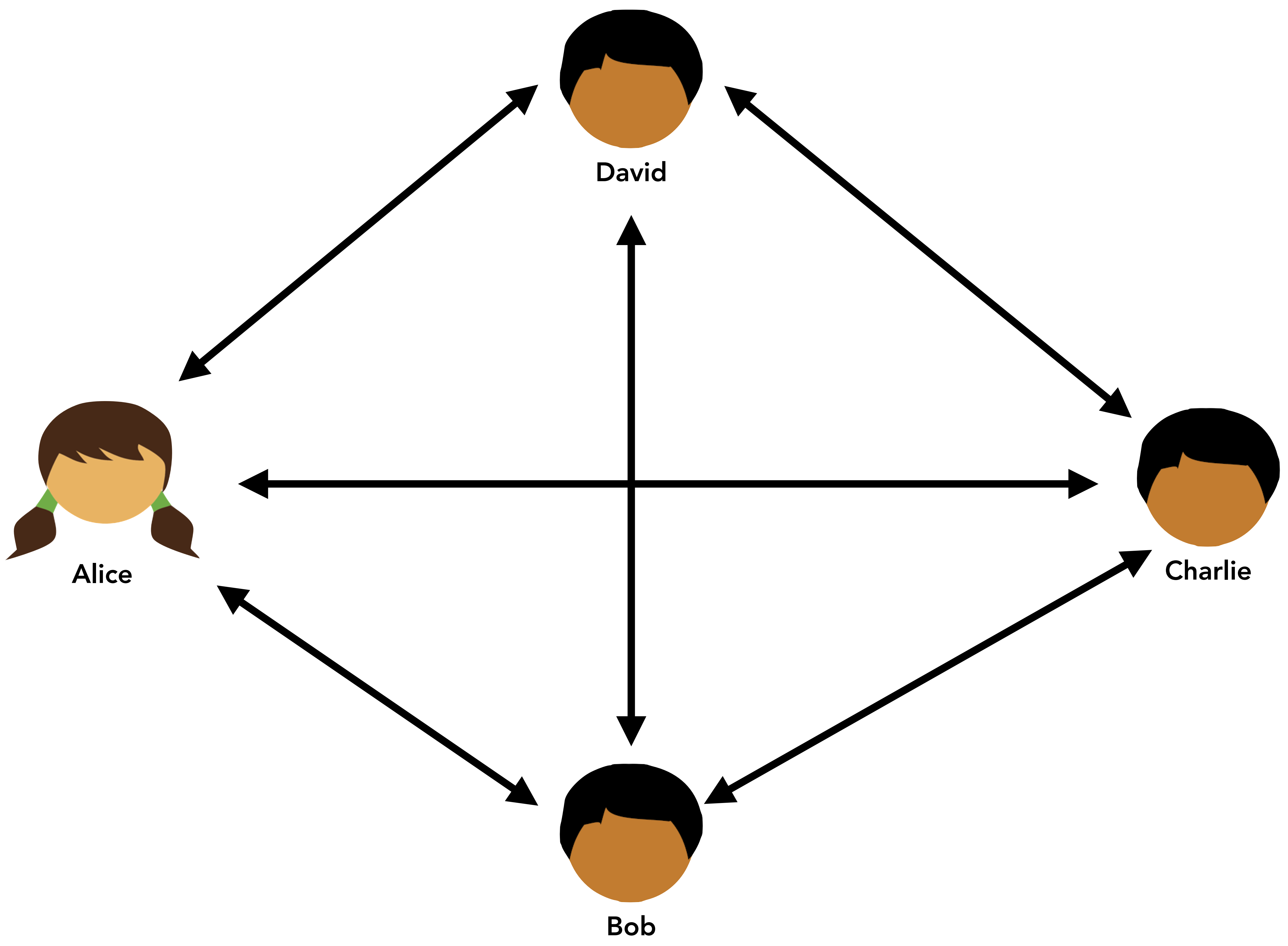


Example: Our Upper Bound

$t = \frac{n}{2}$, 2-wise fair exchange, $n = 4$





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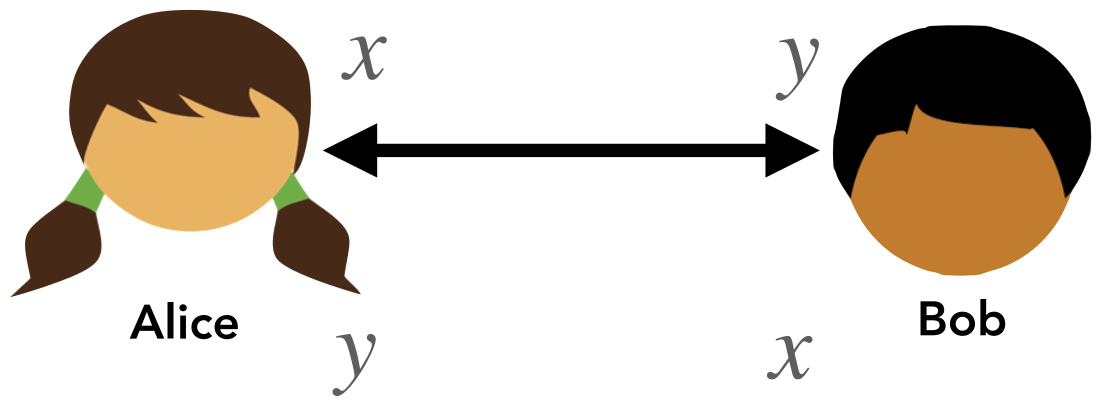

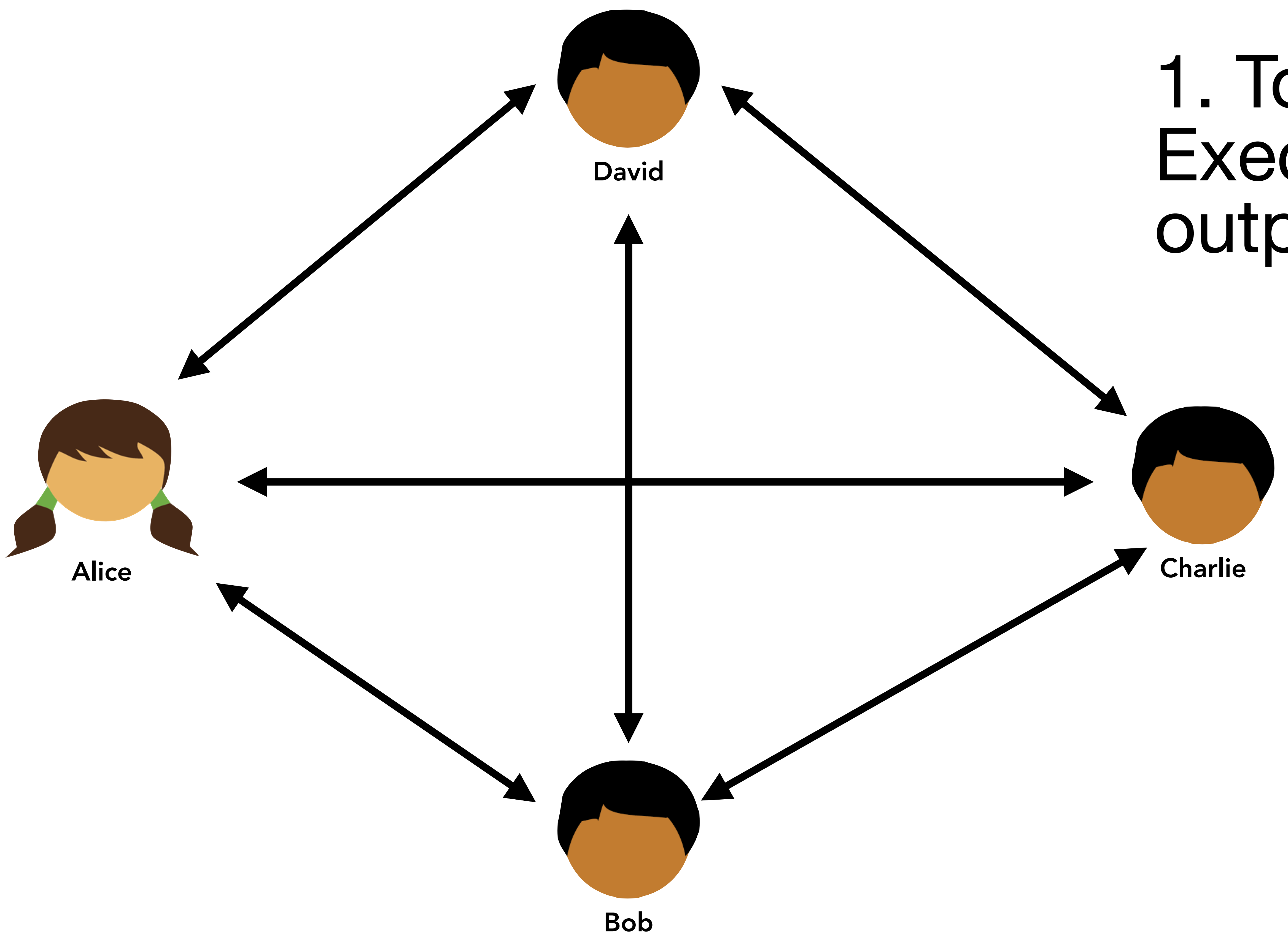

1. To compute f:
Execute some SwA MPC,
output 3-out-of-4 SS.

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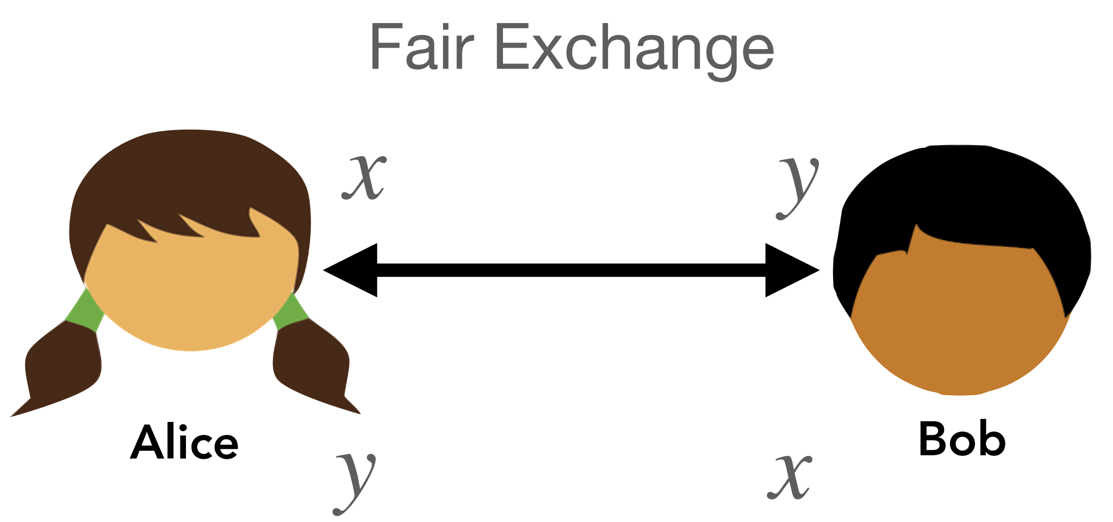

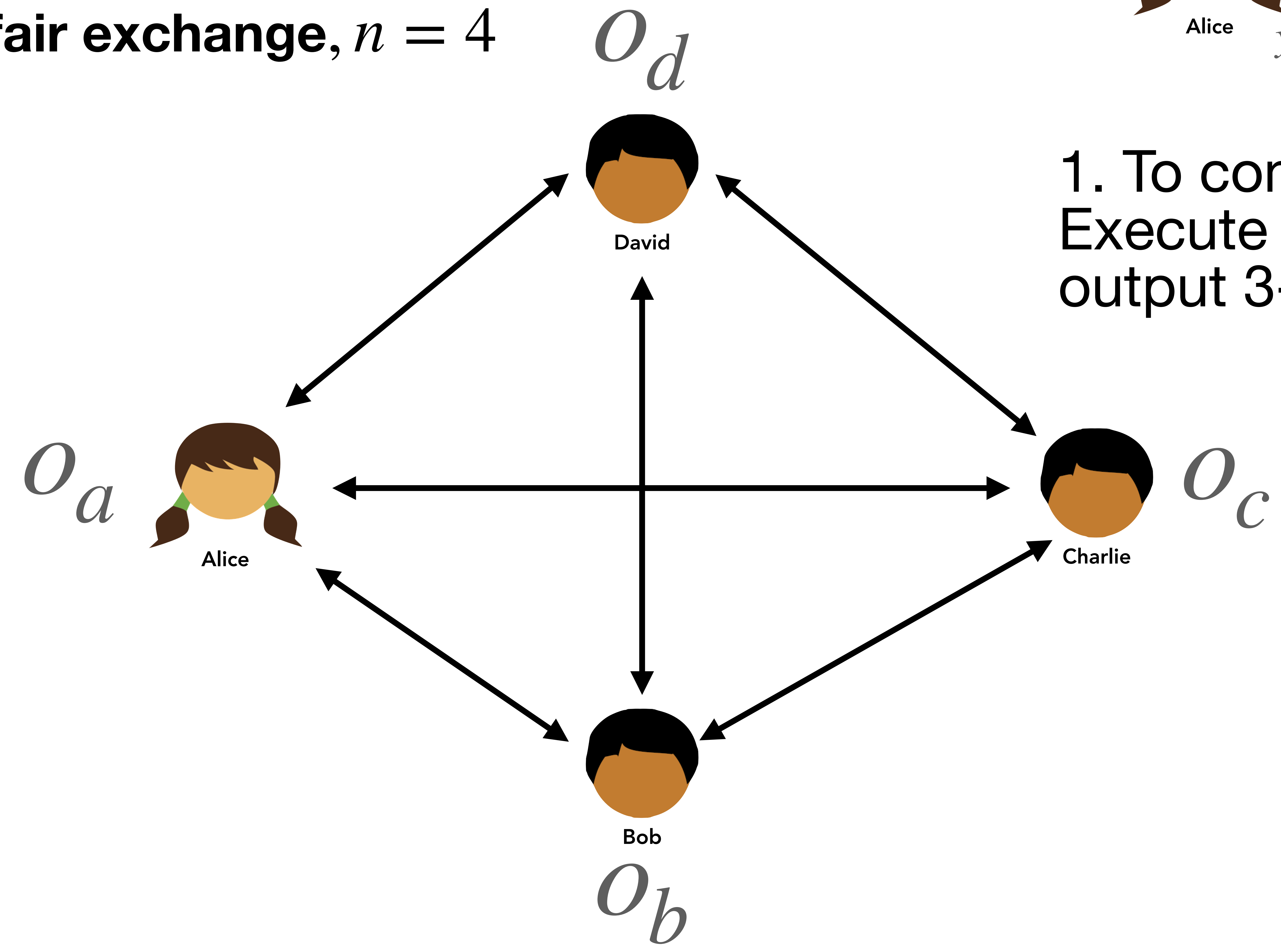

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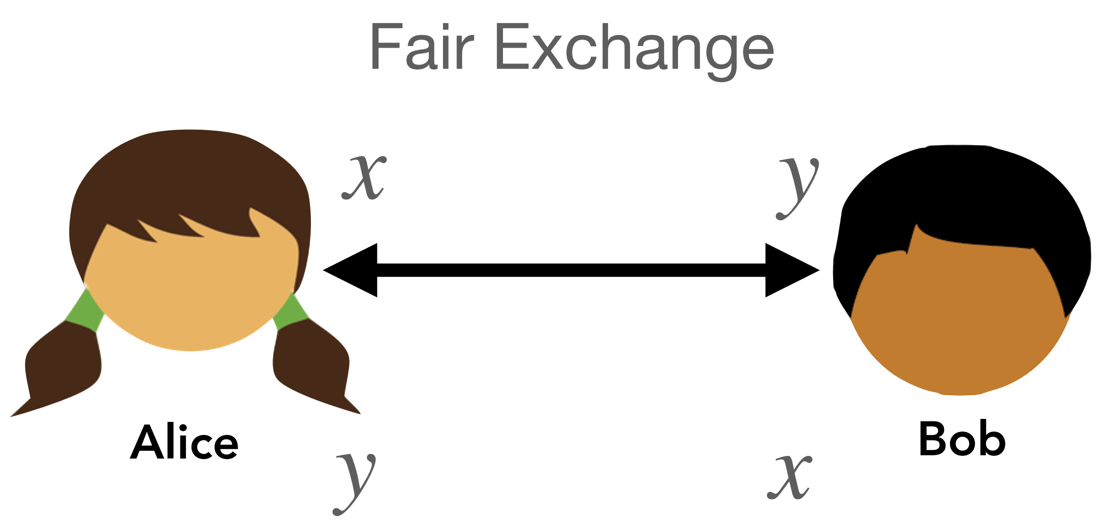

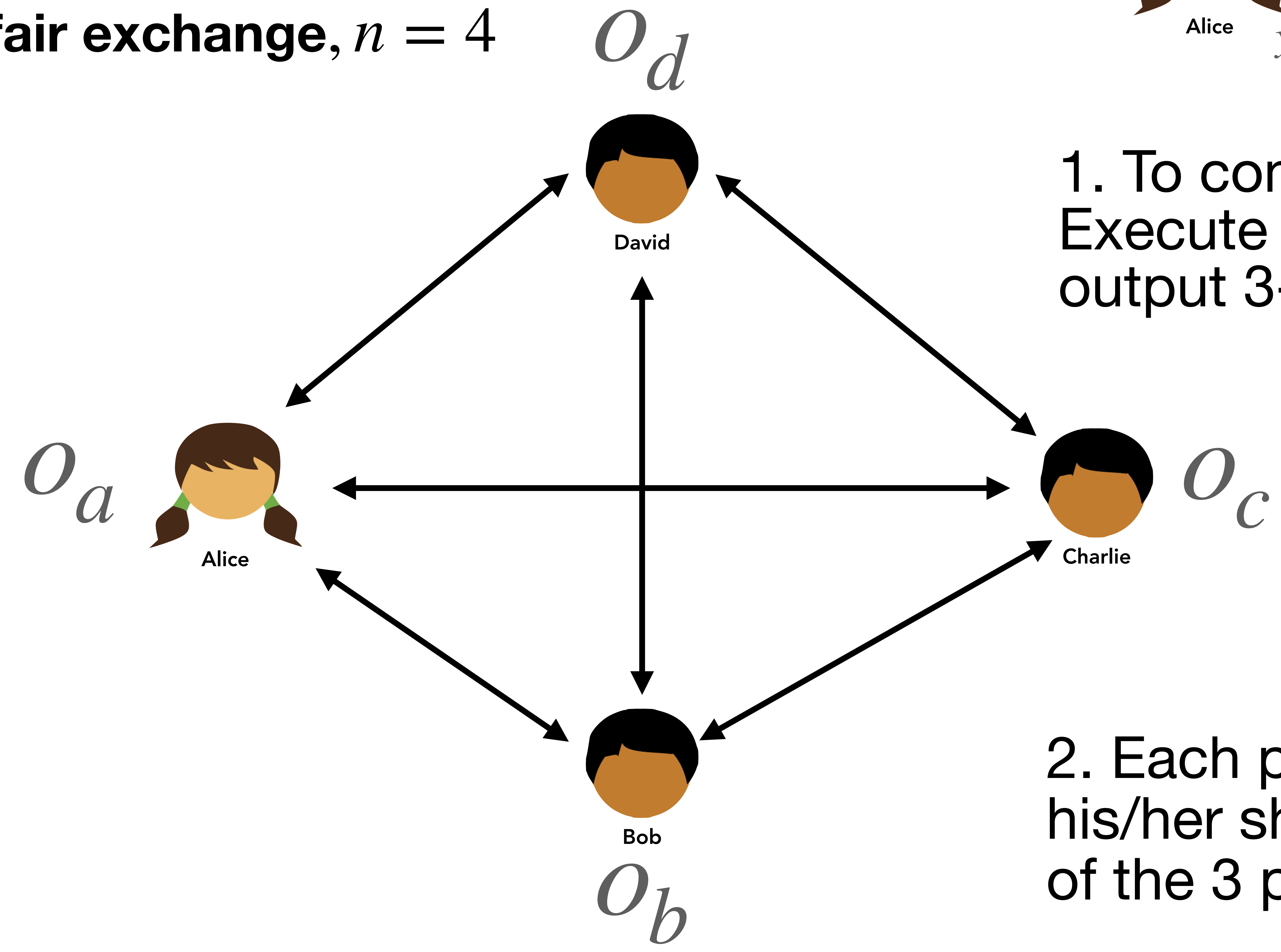

1. To compute f: Execute some SwA MPC, output 3-out-of-4 SS.

2. Each party exchanges his/her share with the rest of the 3 parties.

Example: Our Upper Bound

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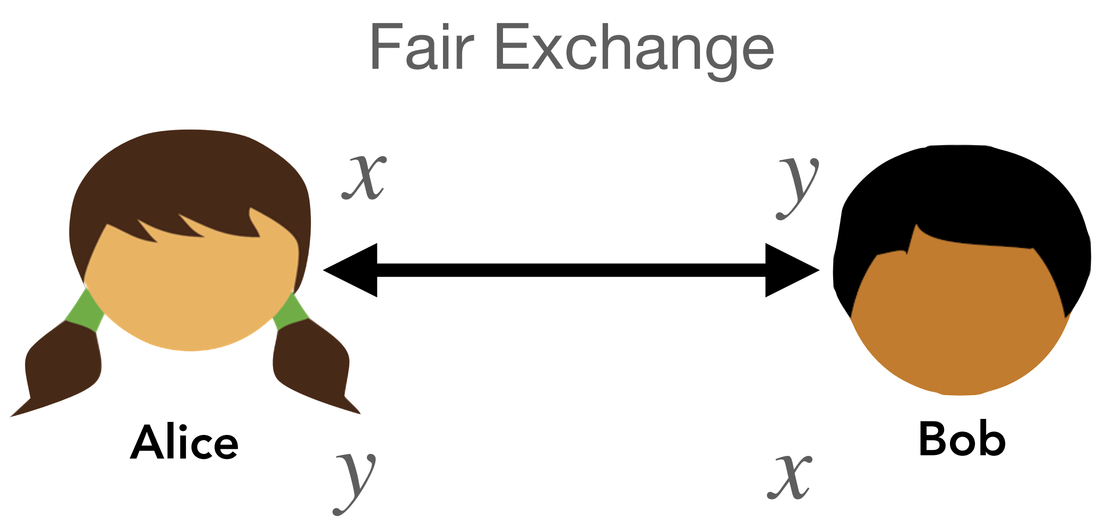

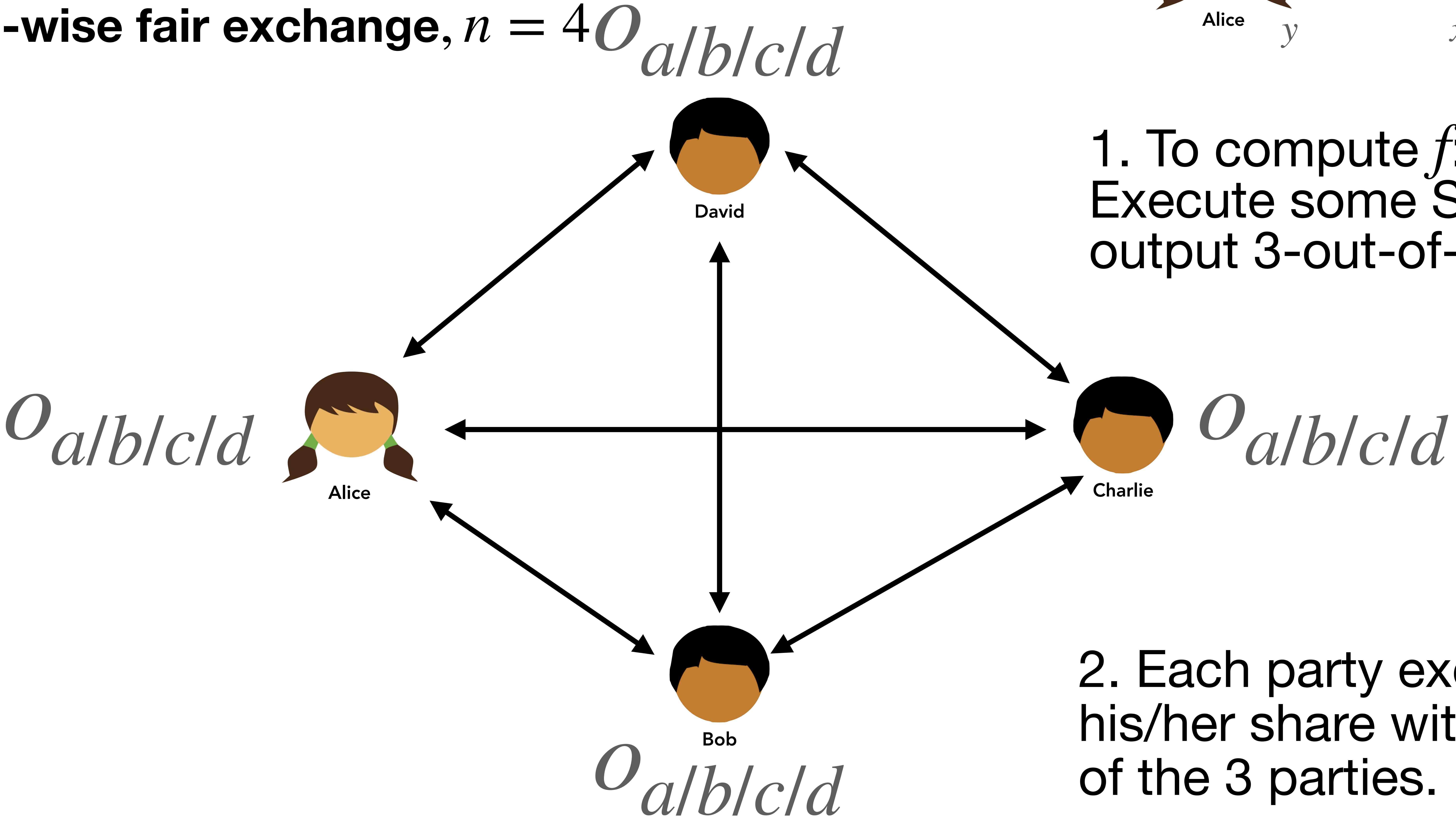

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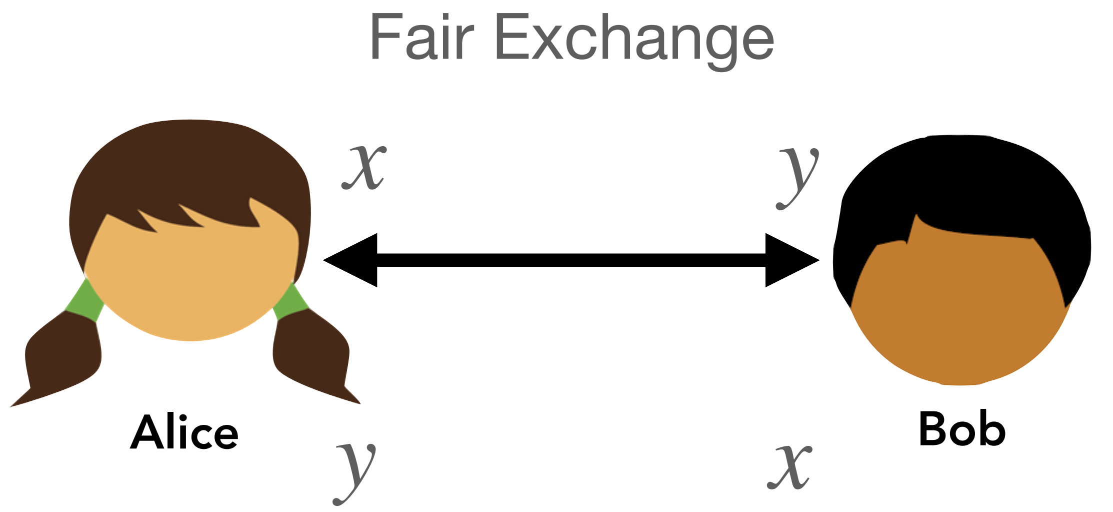

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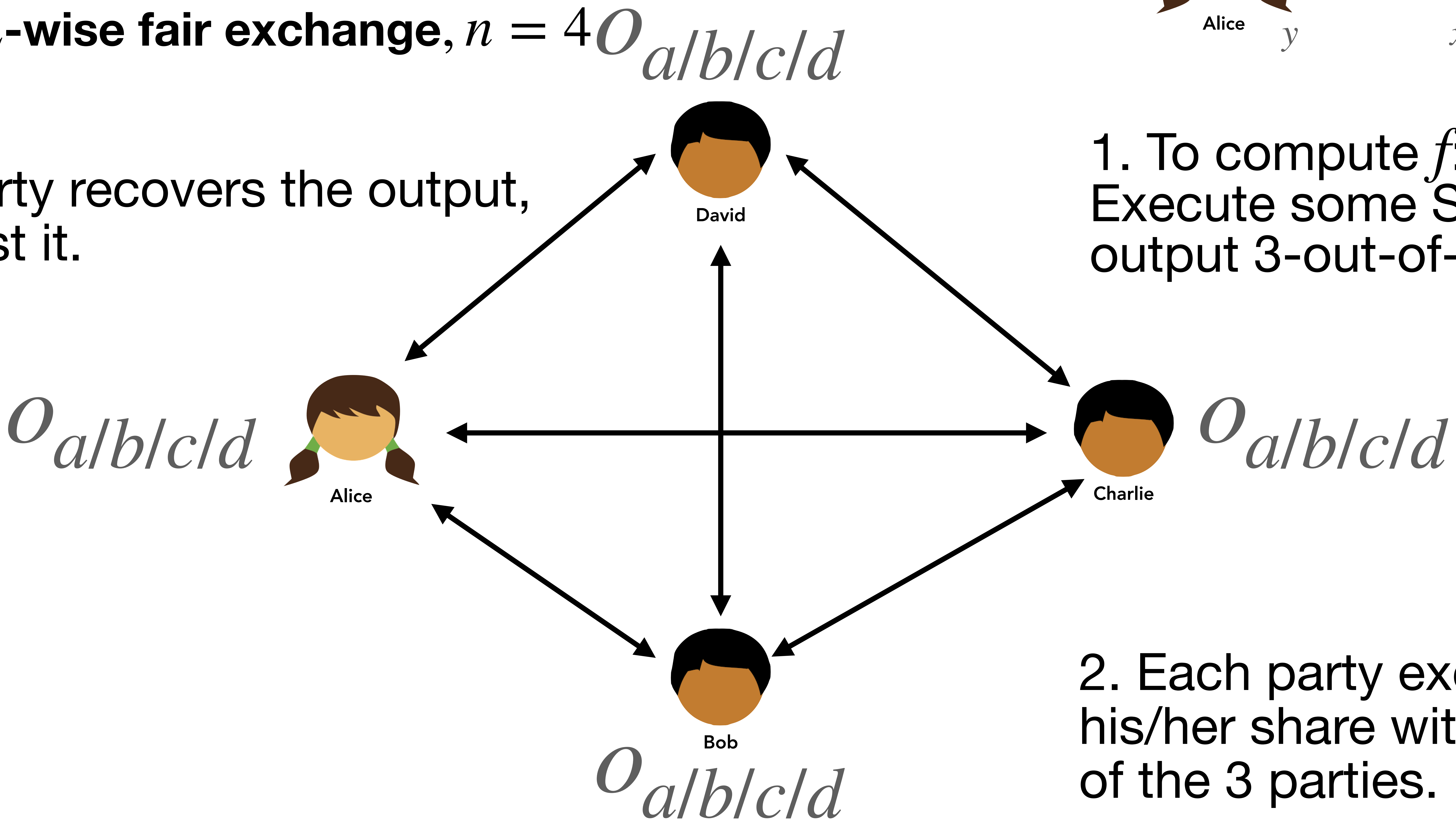

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3. If a party recovers the output, broadcast it.

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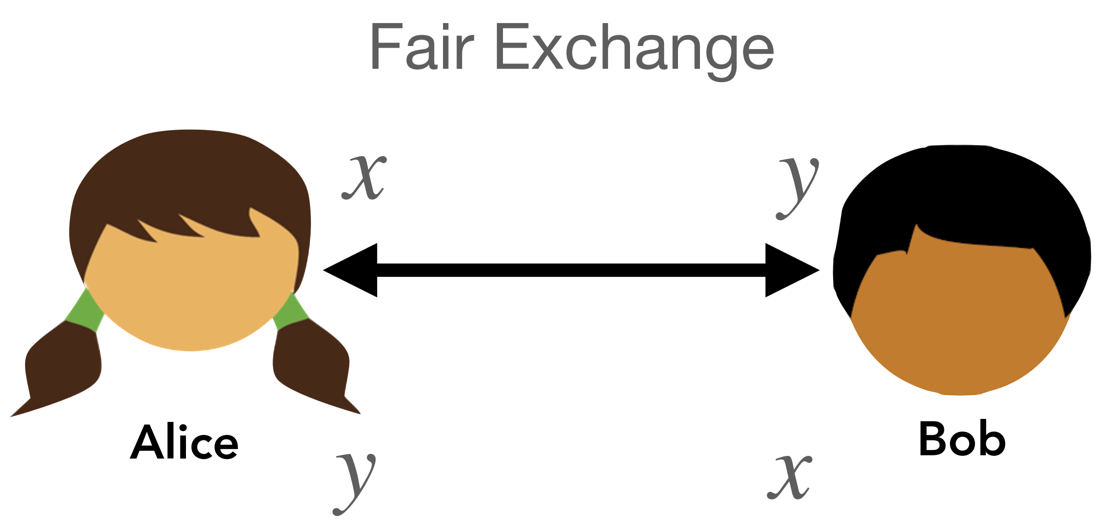
$t = \frac{n}{2}$, 2-wise fair exchange, $n = 4$

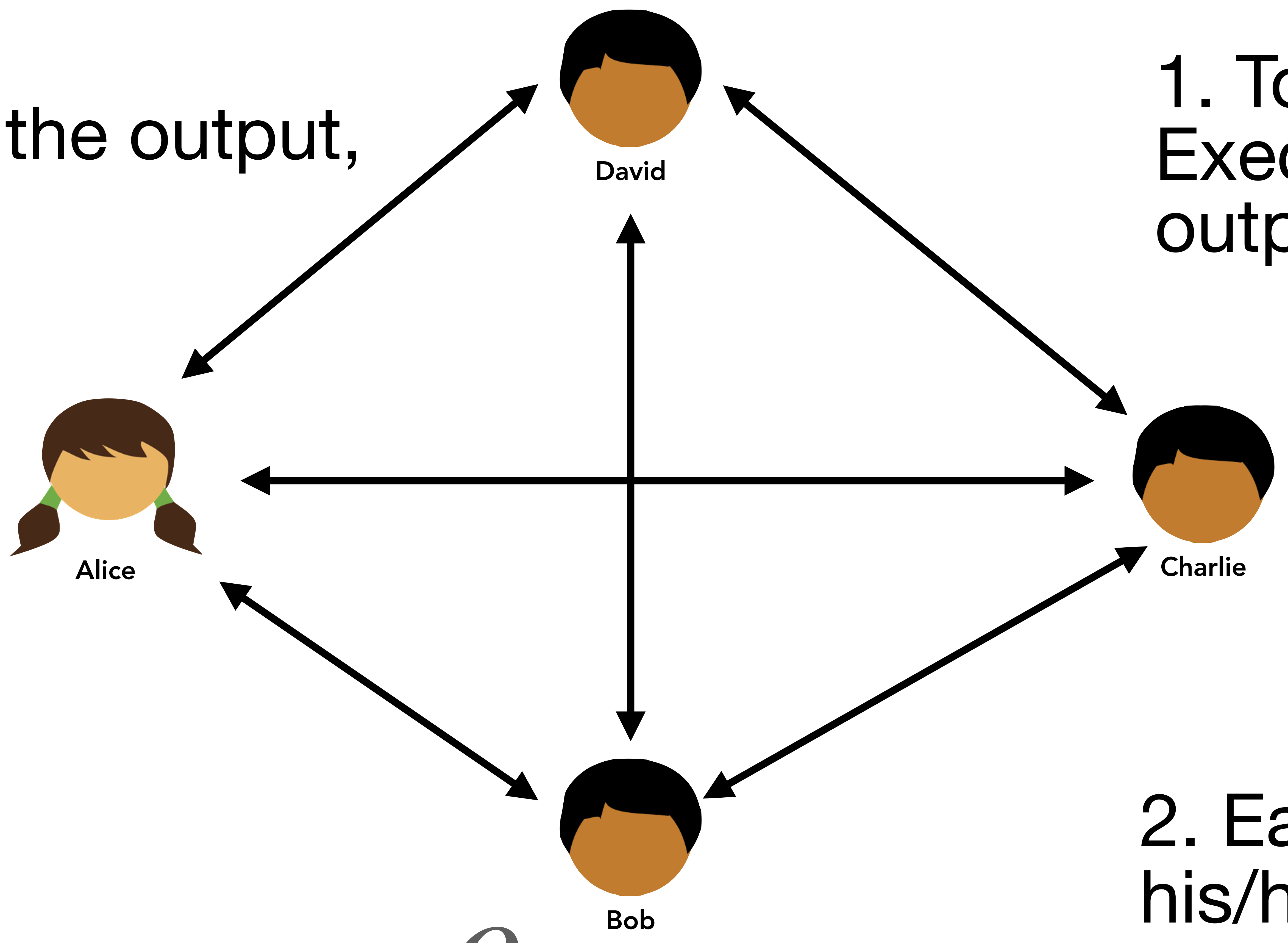


1. To compute f: Execute some SwA MPC, output 3-out-of-4 SS.

2. Each party exchanges his/her share with the rest of the 3 parties.

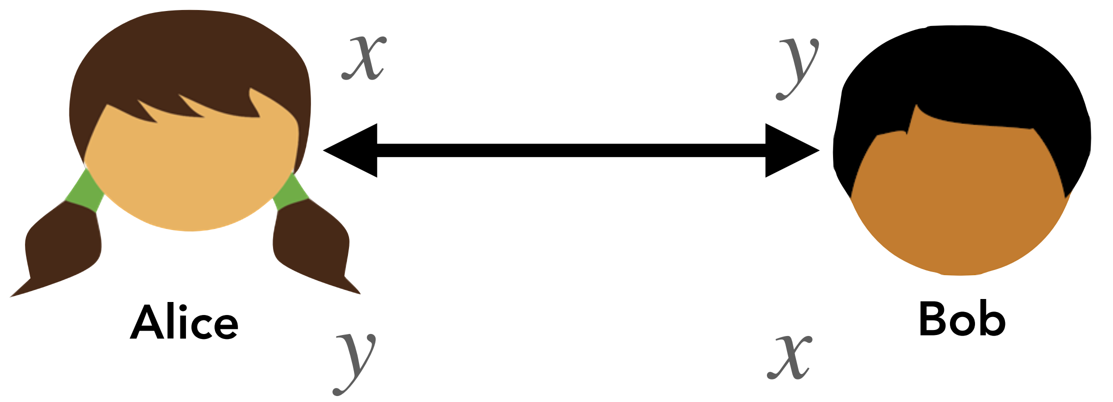
3. If a party recovers the output, broadcast it.

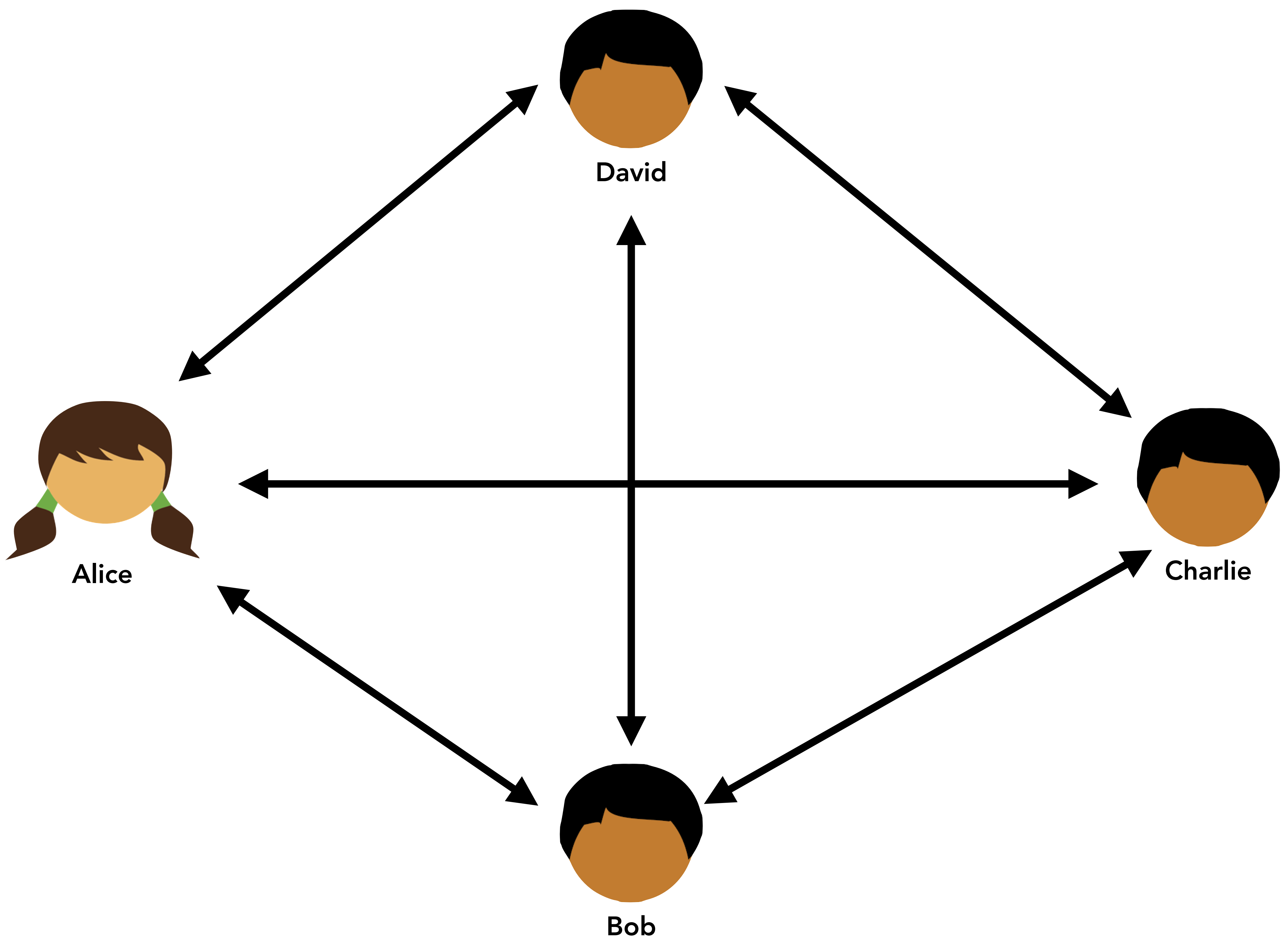
4. Output the result.



Why Fair?

$t = \frac{n}{2}$, 2-**wise fair exchange,** $n = 4$





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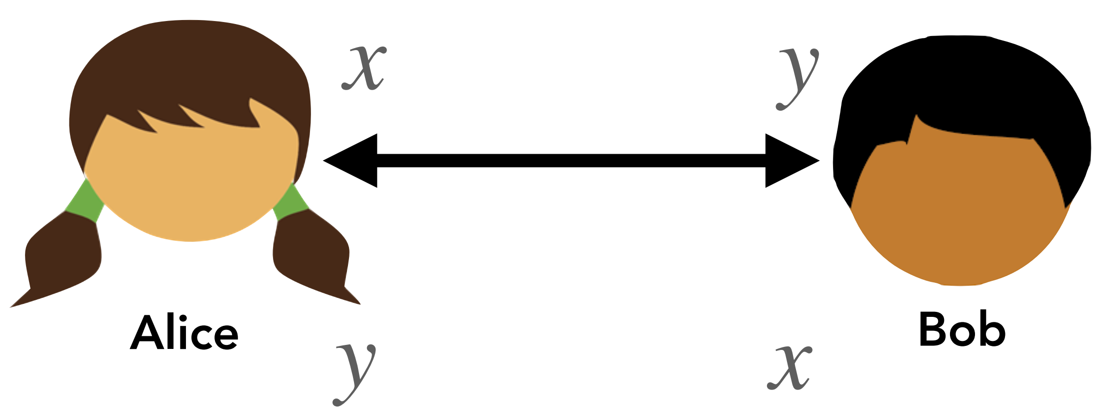

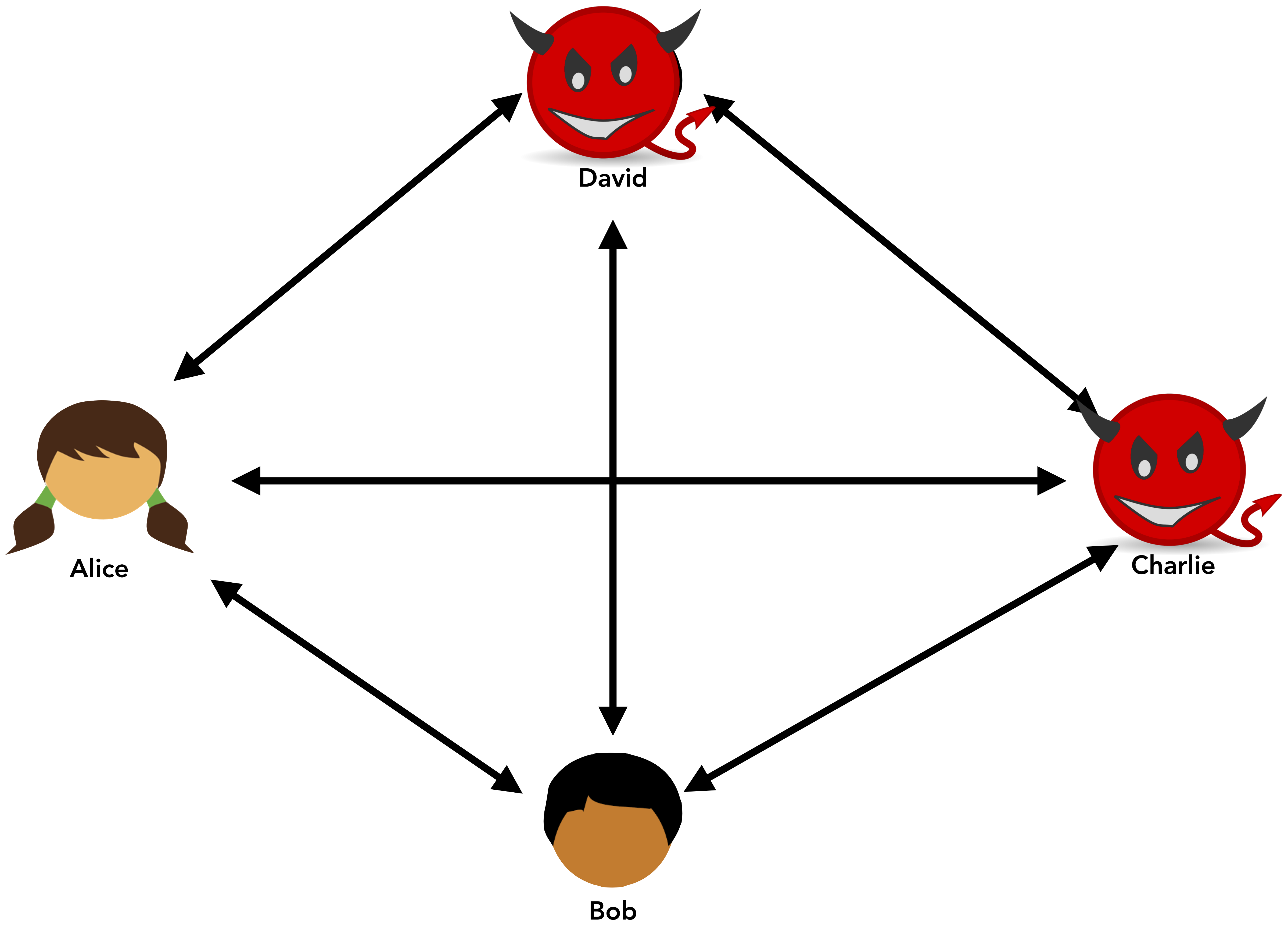

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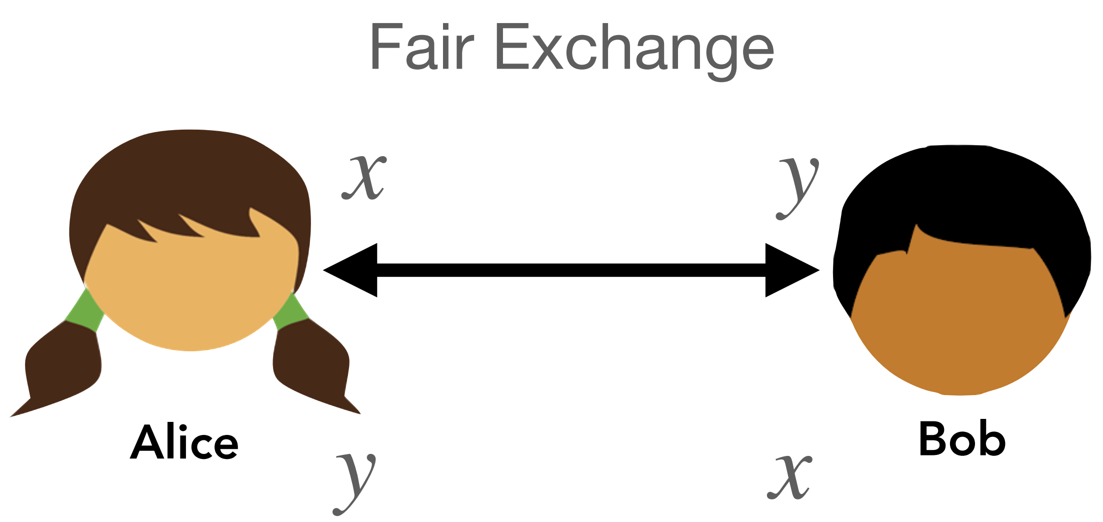

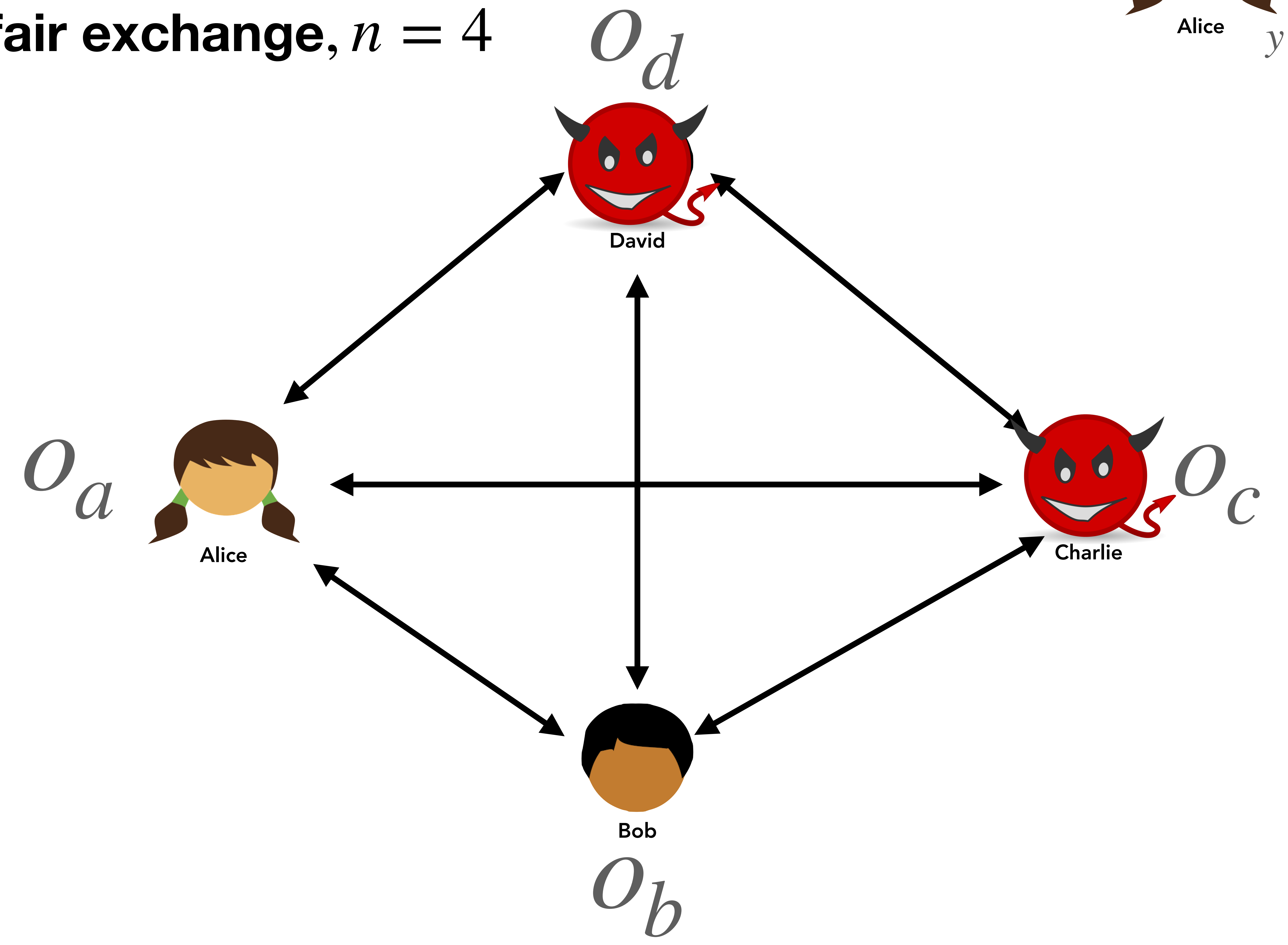

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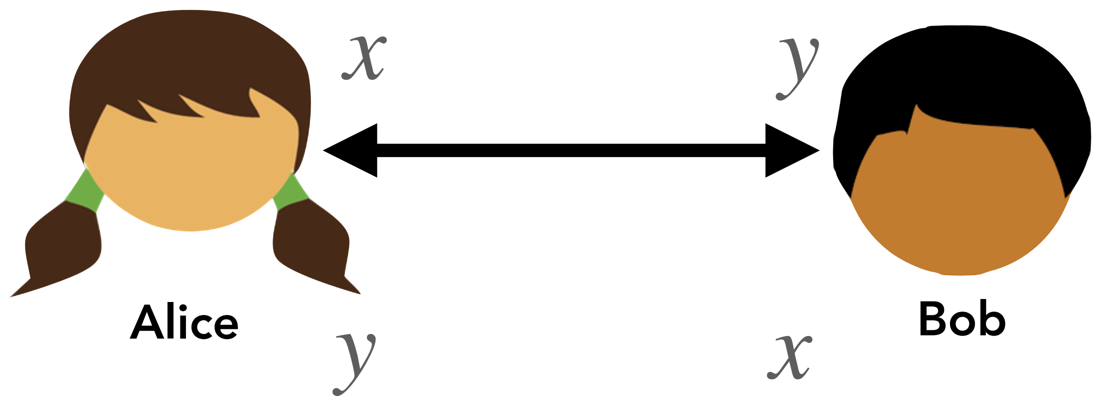
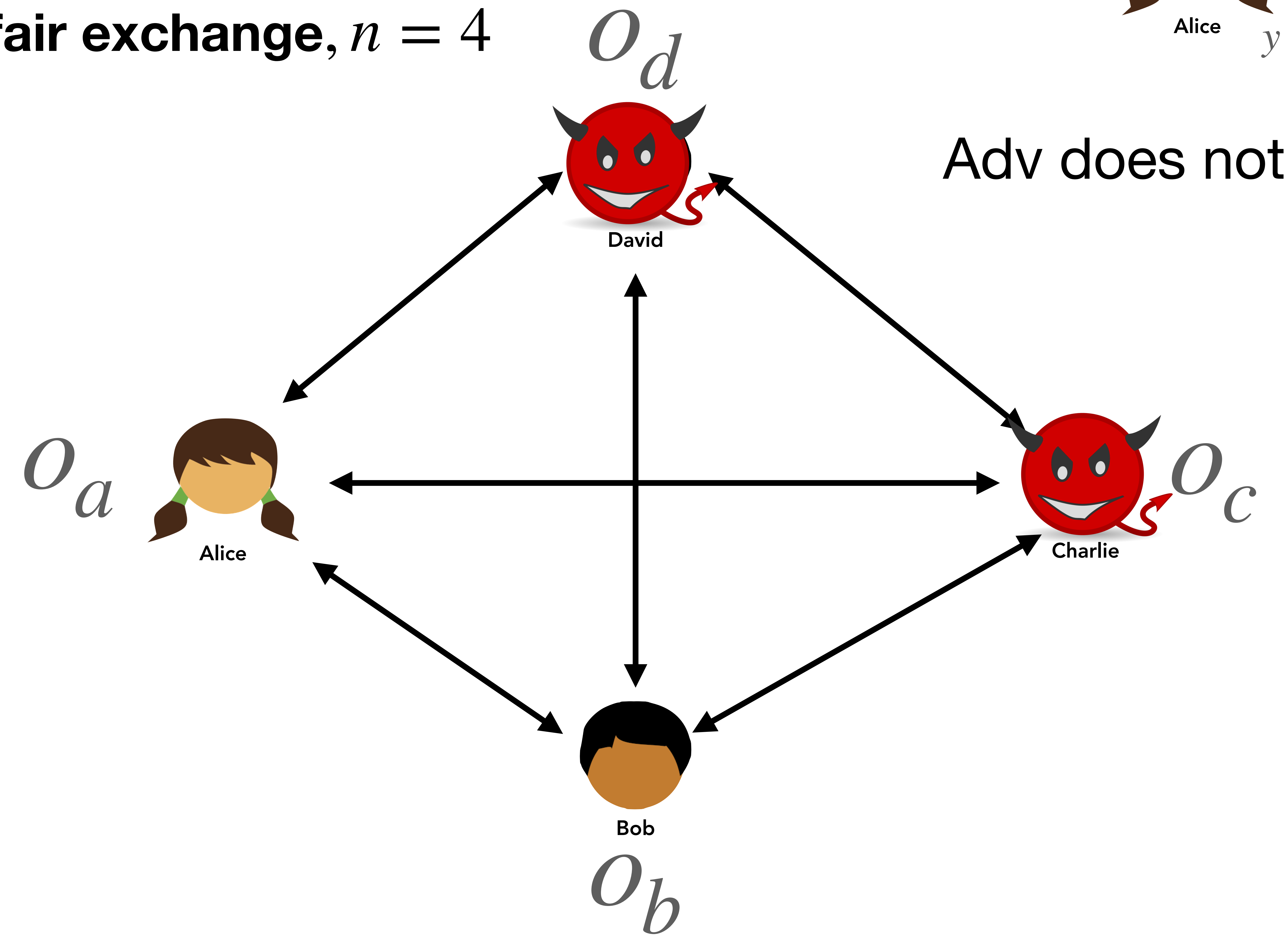


Adv does not learn the output.



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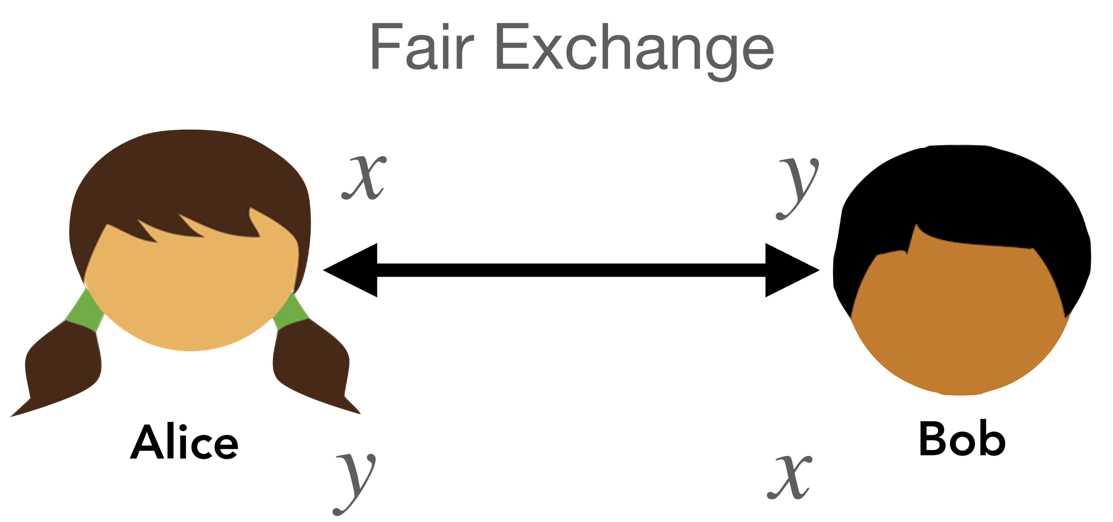
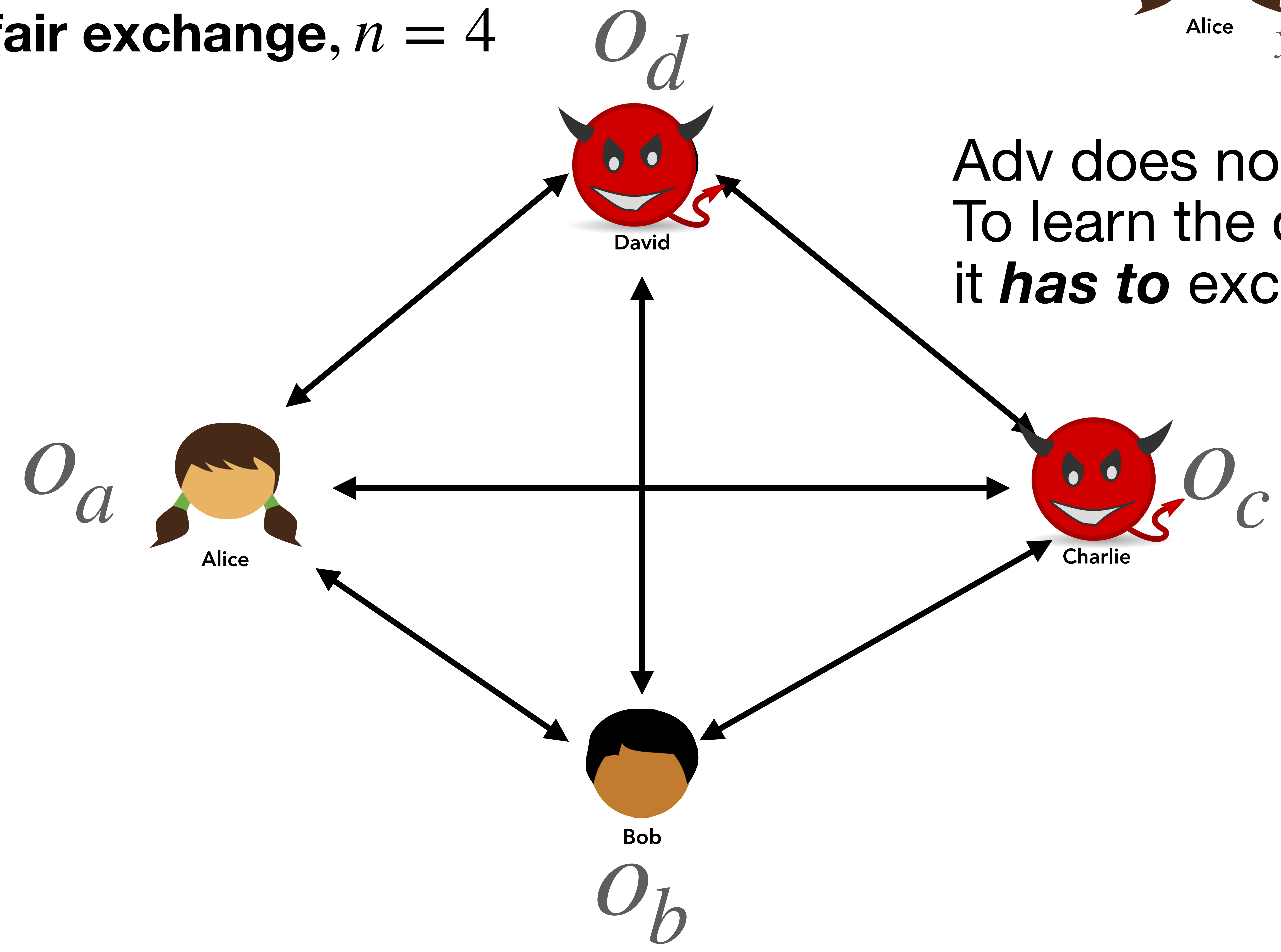


Adv does not learn the output.
To learn the output,
it ***has to*** exchange.



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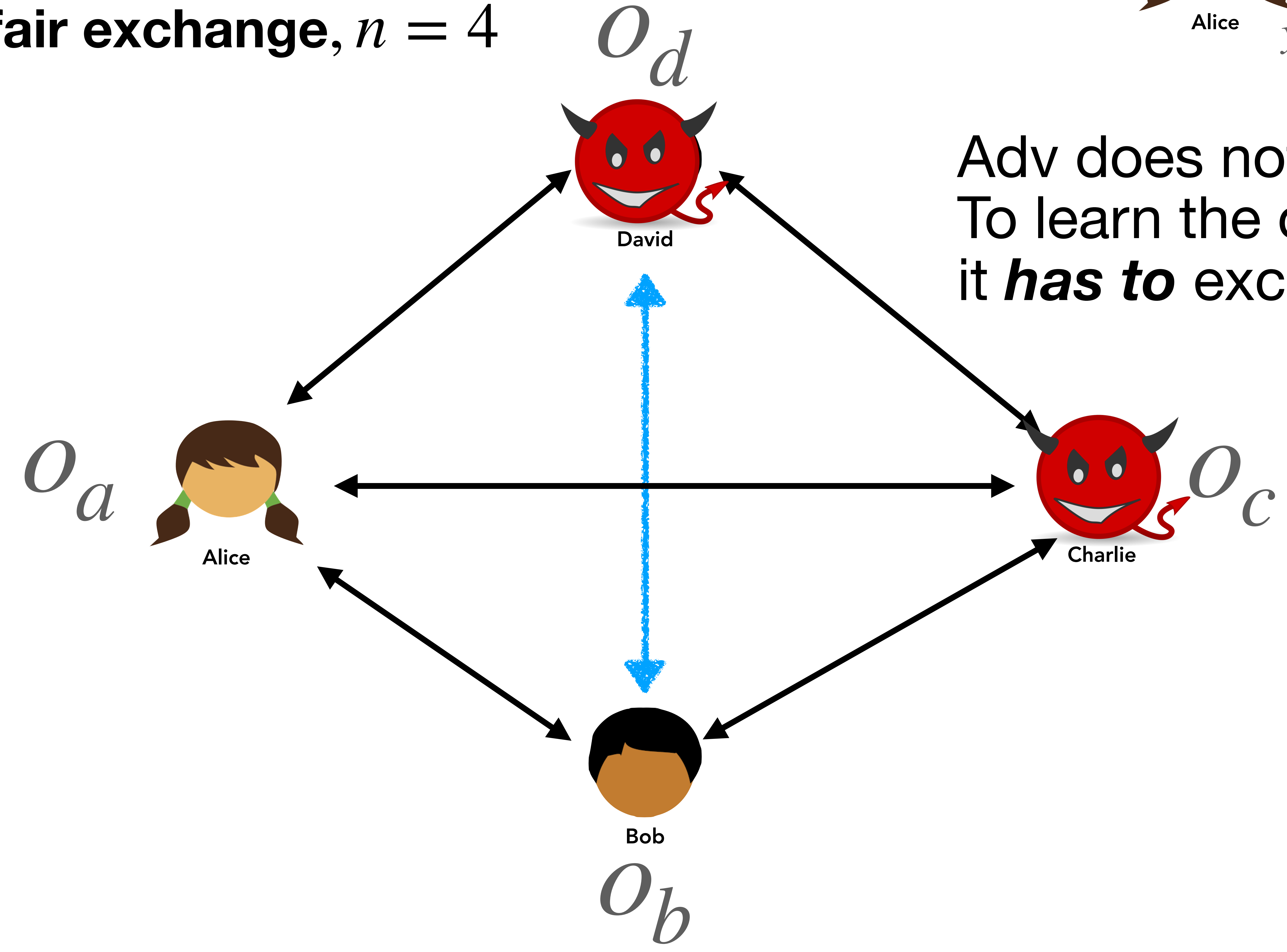

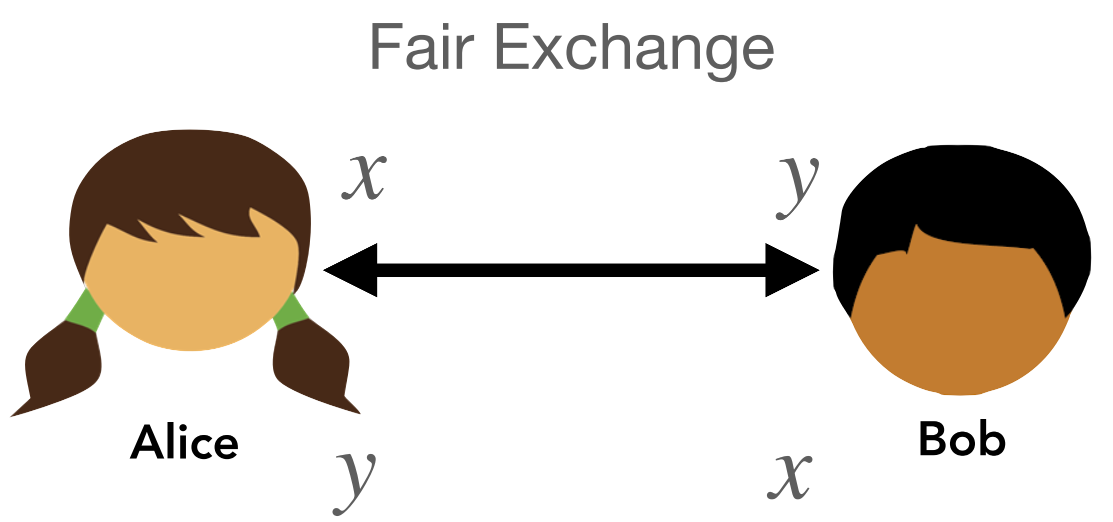

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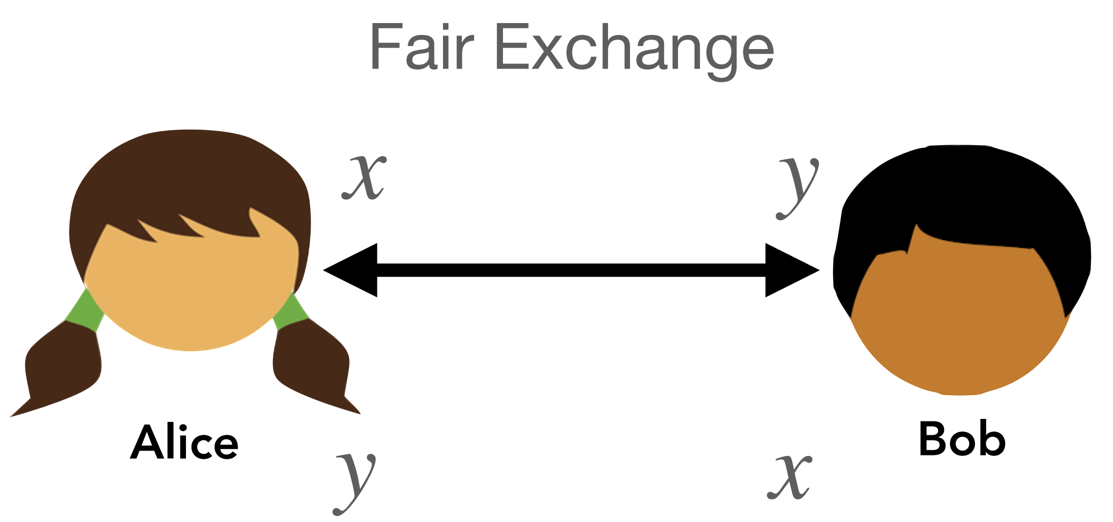

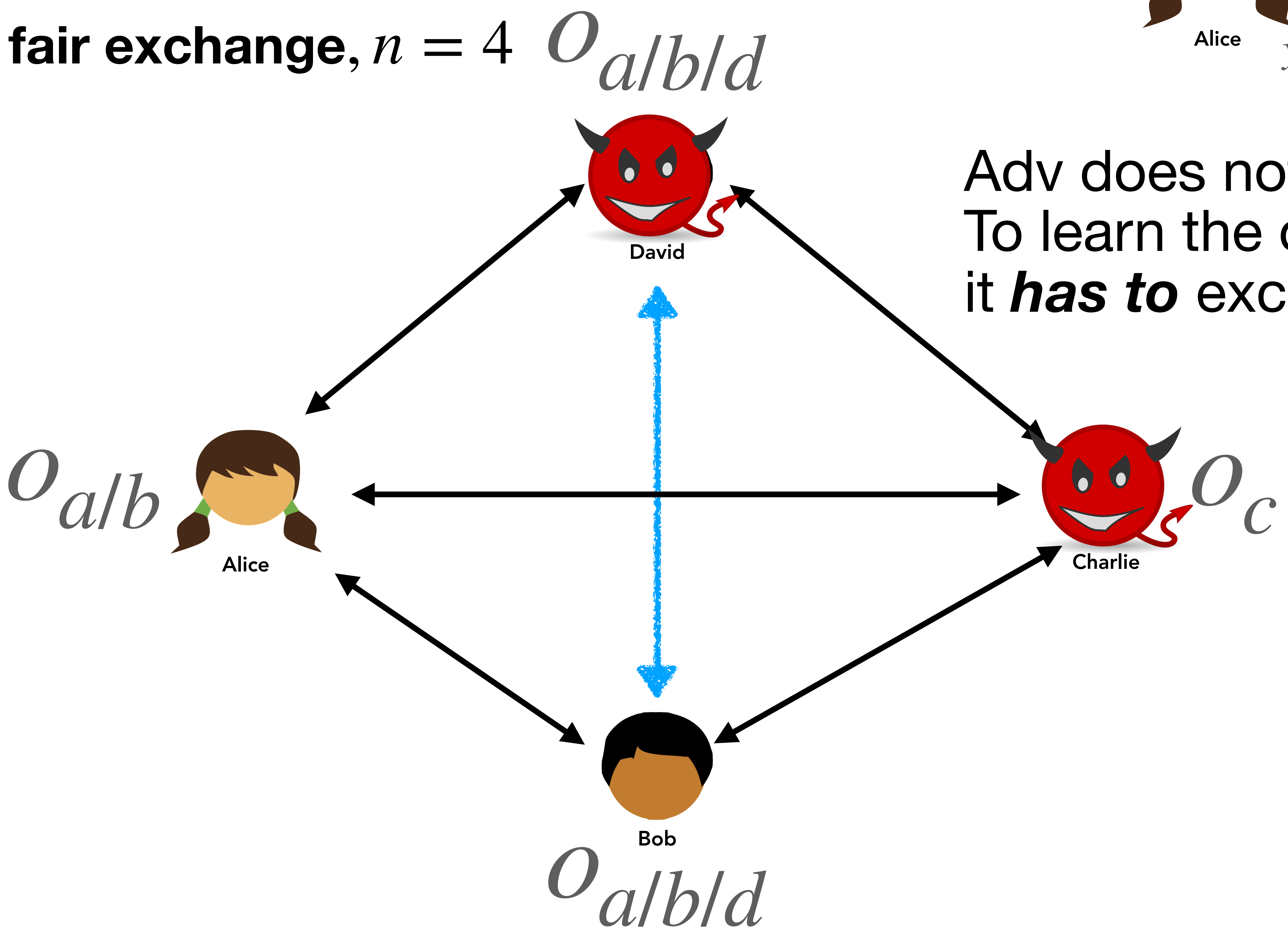

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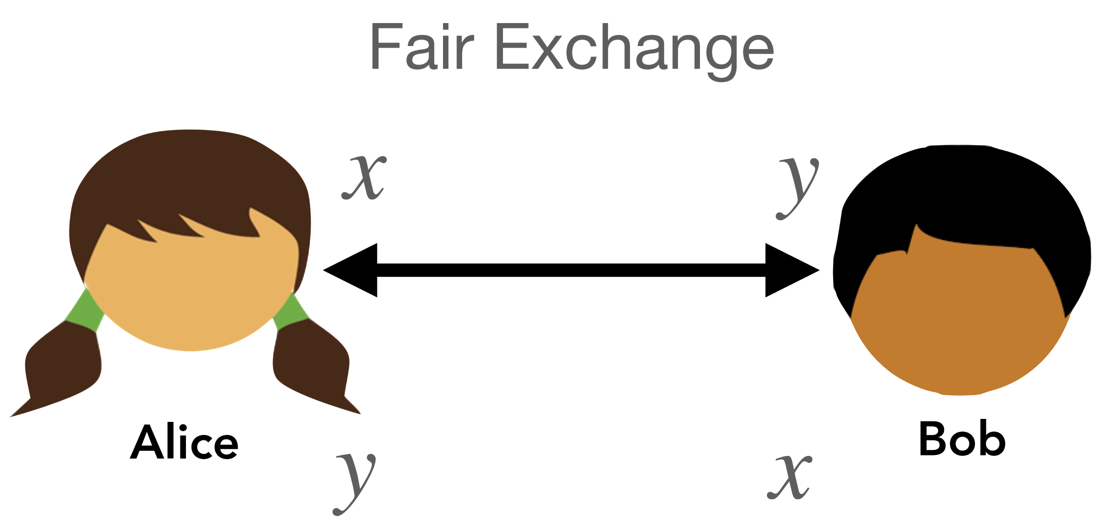

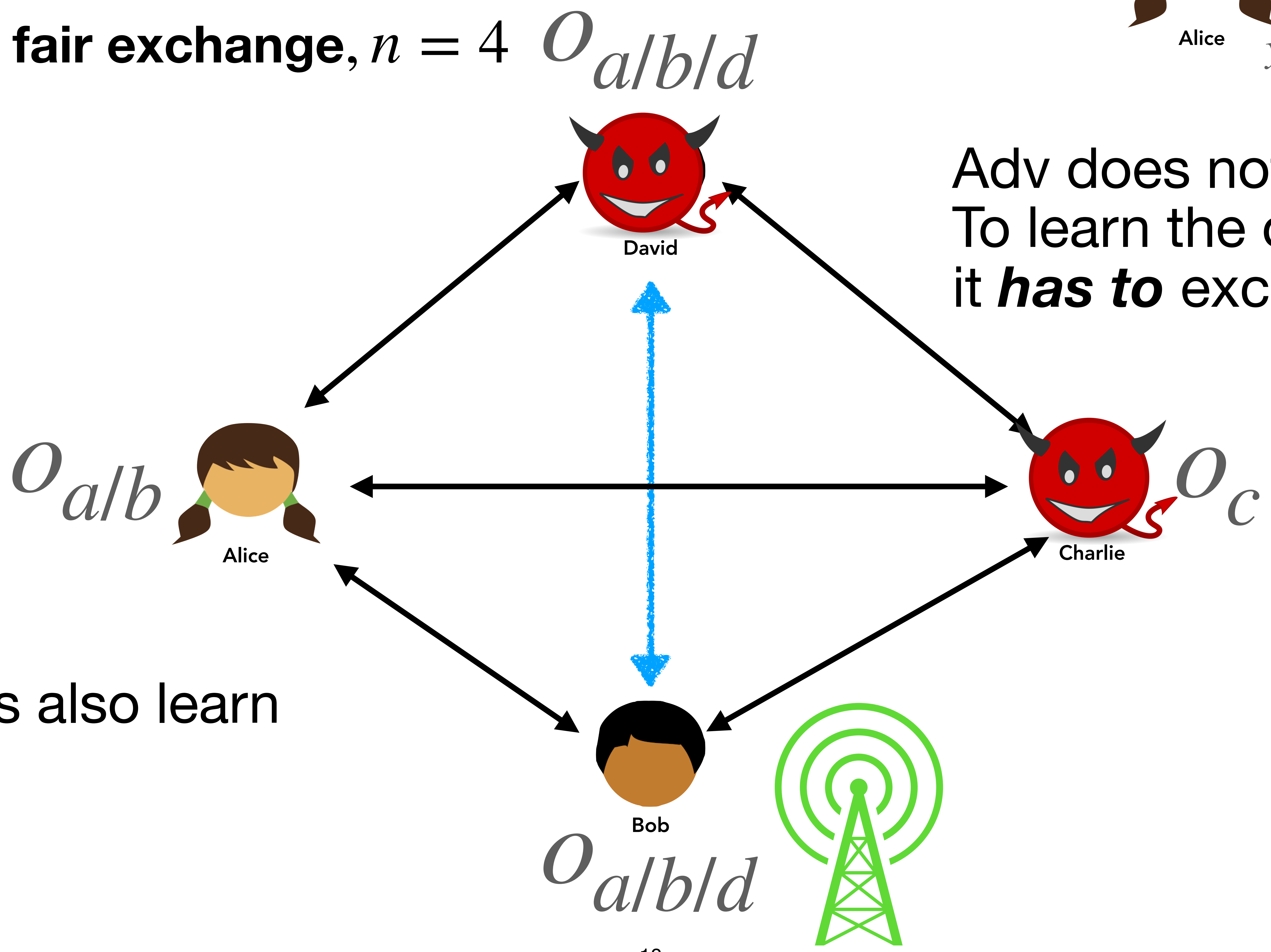

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Honest parties also learn the output.

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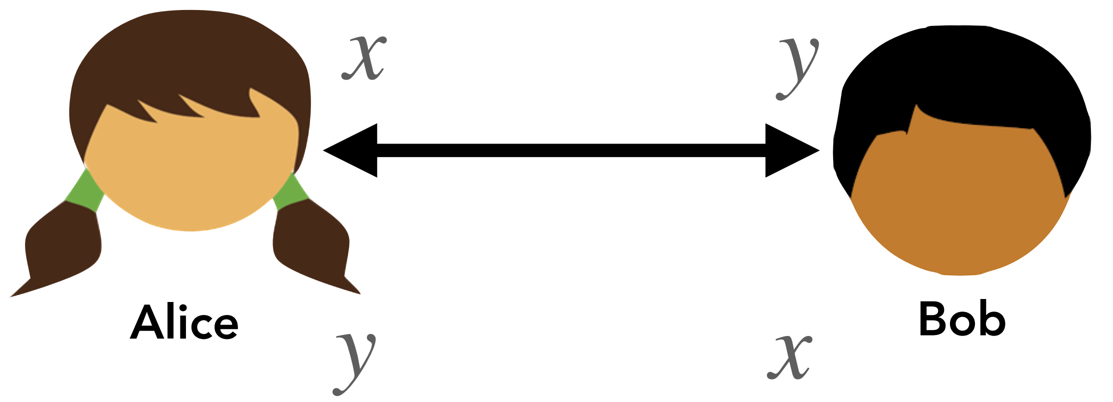
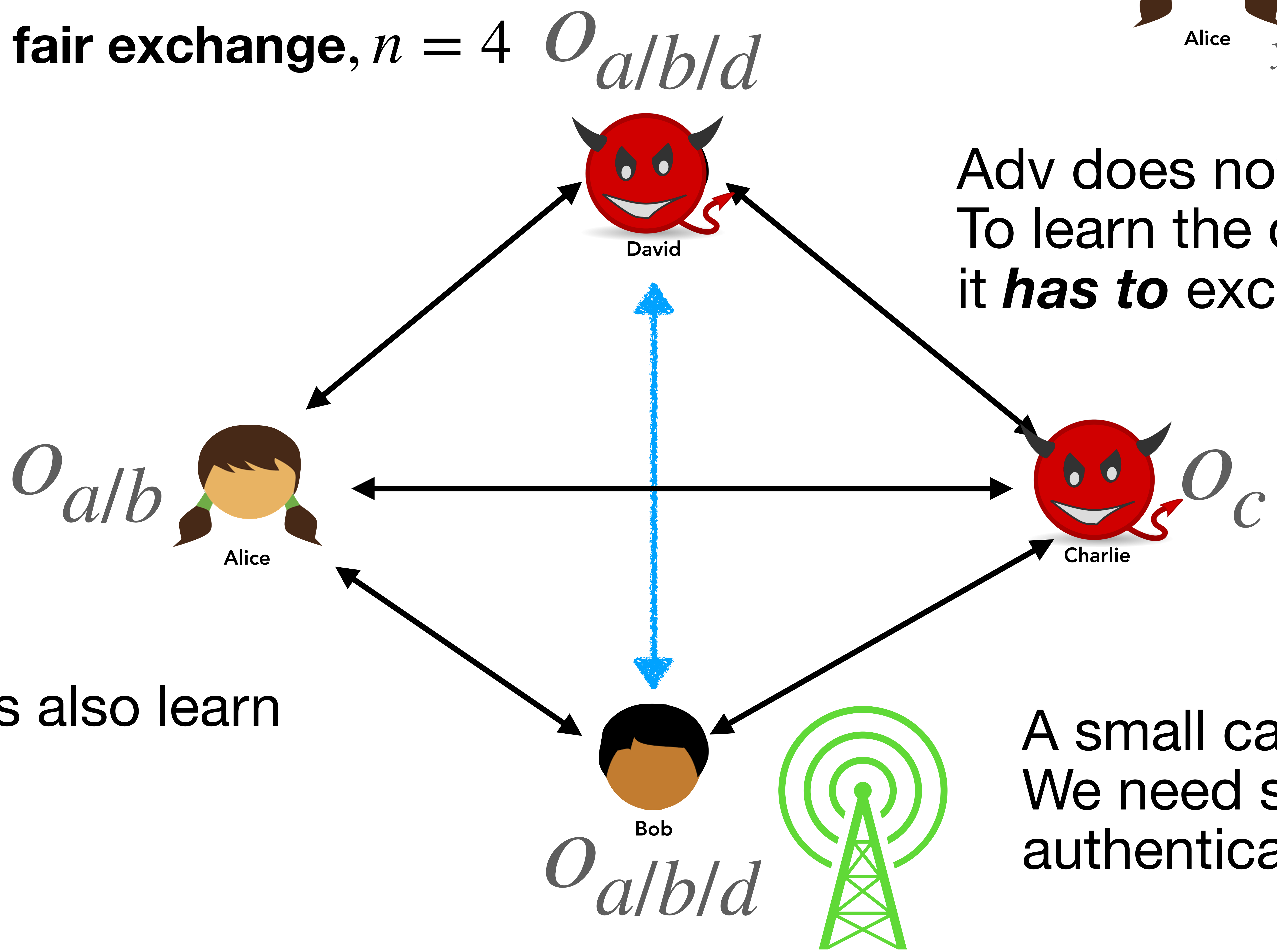
Fair Exchange





Adv does not learn the output.
To learn the output,
it ***has to*** exchange.

Honest parties also learn the output.

A small caveat:
We need some authentication mechanism.

Just How Fair is an Unreactive World?

We completely address this question.
t = #corruption, n = #party

Not very fair ☹

Table 1: Our contributions.

t	Insufficient functionalities for fair coin tossing	Sufficient functionalities for fair MPC
$t < \frac{n}{2}$	–	Local computation [FGMvR02]
$t = \frac{n}{2}$	Local computation [Cle86]	2-wise fair exchange **[ours]**
$t > \frac{n}{2}$	Arbitrary unreactive t-wise **[ours]**	$(t+1)$-wise fair exchange[a] **[ours]**

[Cohen and Lindel, Asiacrypt 14]:

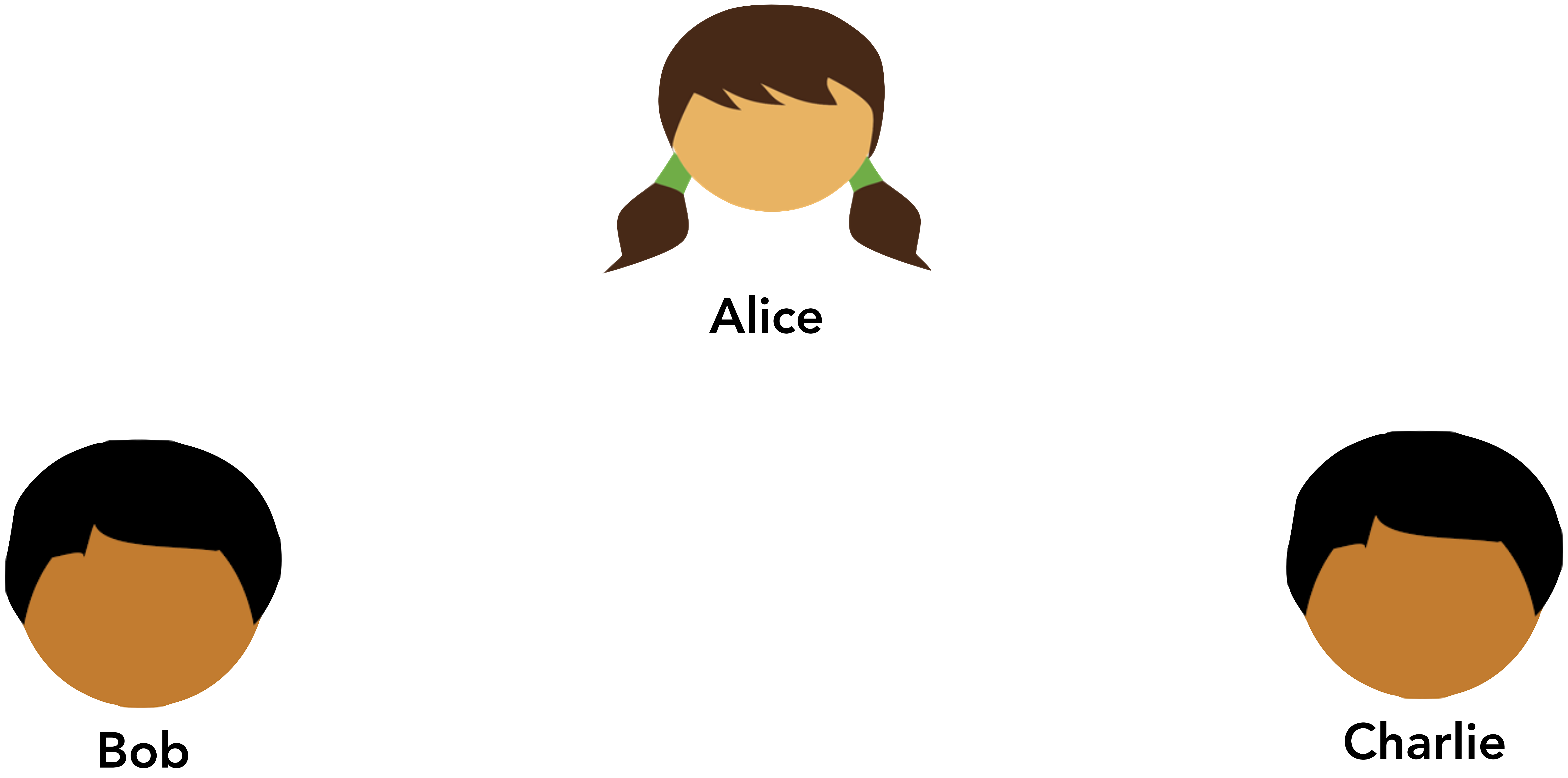
1. Fairness with broadcast →Fairness without broadcast
2. No G.O.D. with broadcast →No fairness (even) with broadcast

Example: Our Lower Bound

$n = 3, t = 2,$ **any** 2**-wise unreactive functionality**

Example: Our Lower Bound

$n = 3$, $t = 2$, any 2-wise unreactive functionality



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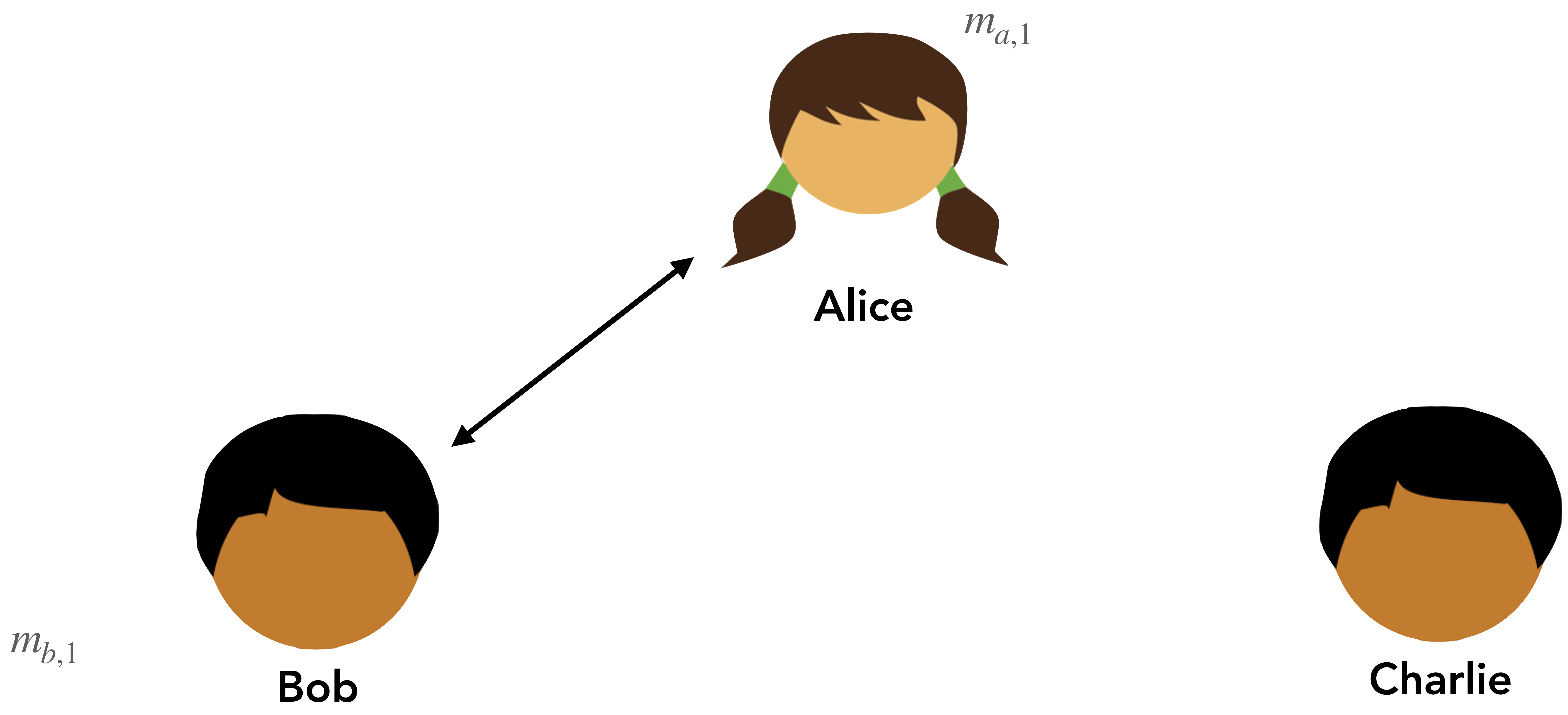

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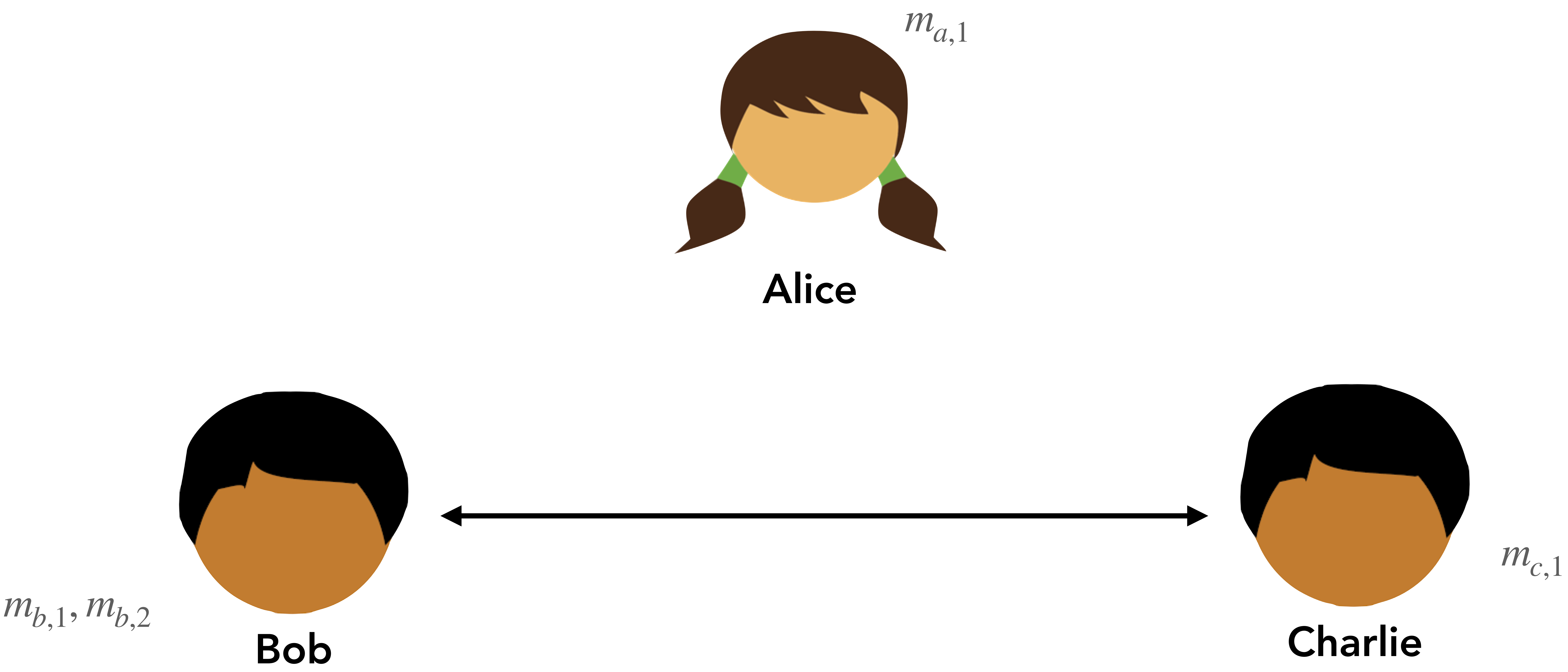

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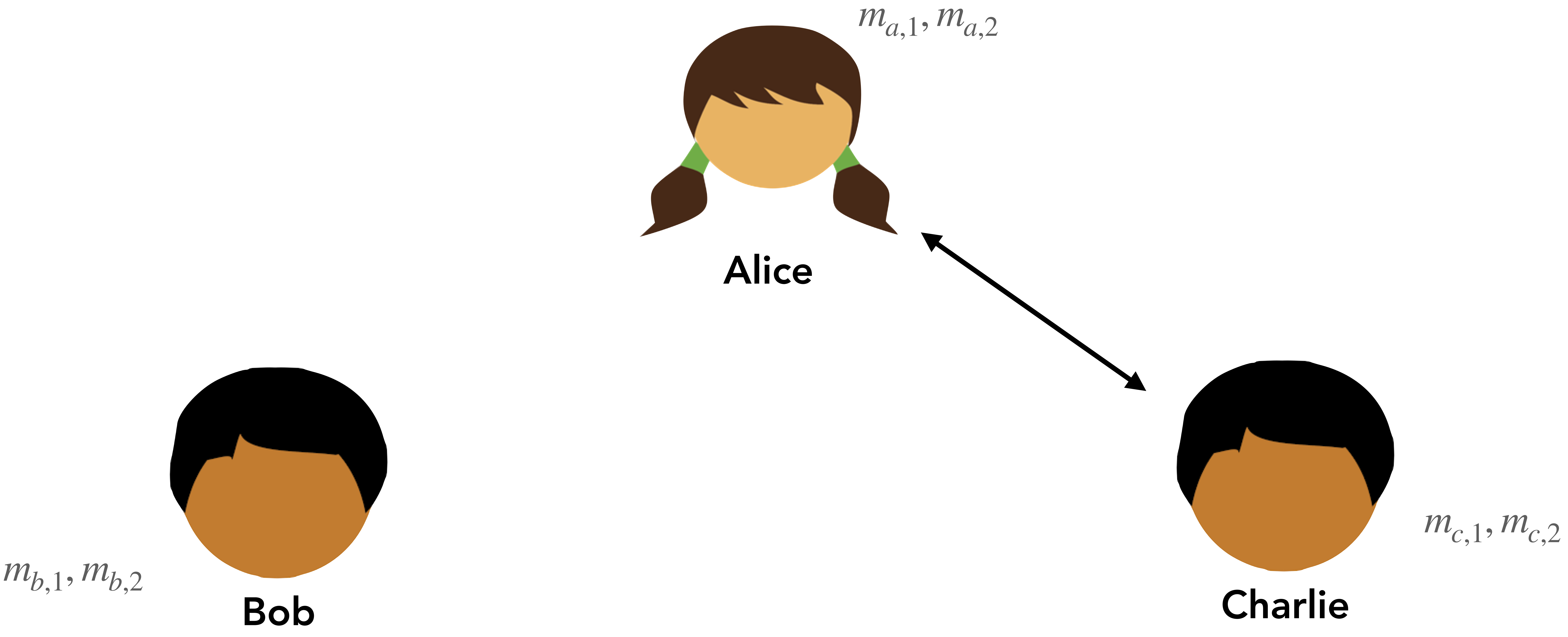

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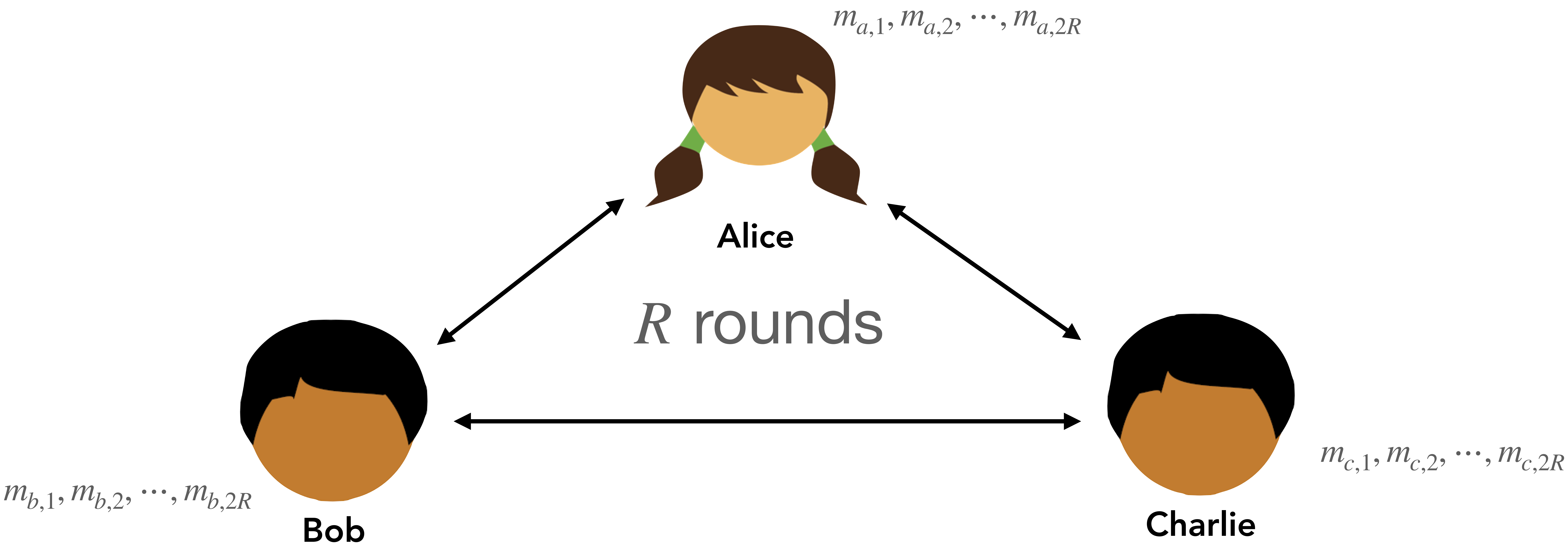

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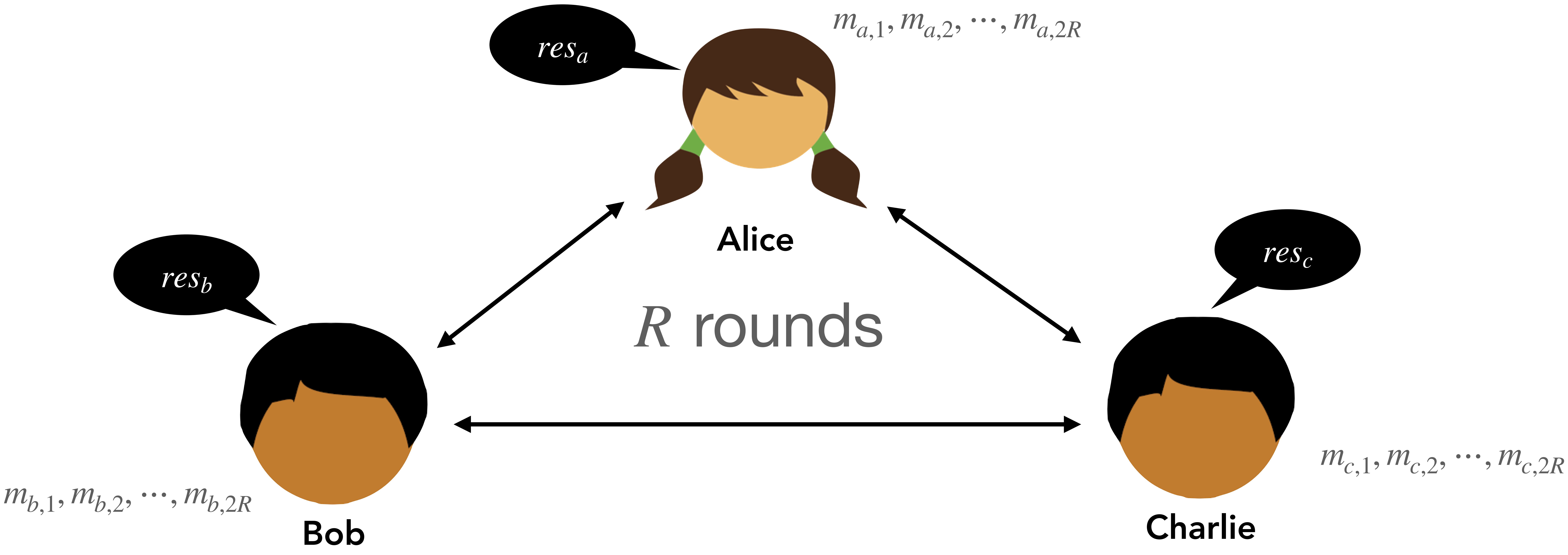
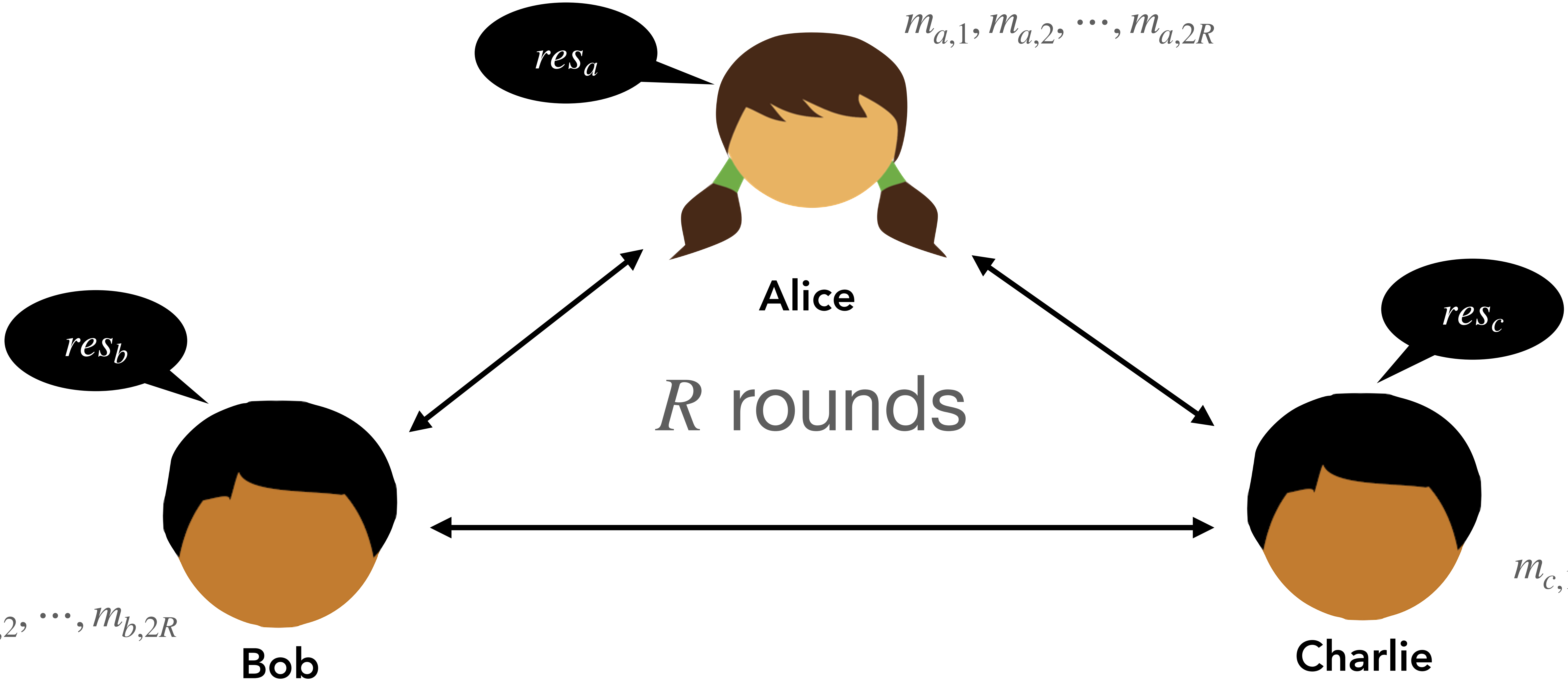


Example: Our Lower Bound

$n = 3, t = 2,$ **any** 2**-wise unreactive functionality**

$$\Pr[res_a = res_b = res_c] = 1$$

$$\Pr[res_a / res_b / res_c = 0] = \frac{1}{2}$$



Example: Our Lower Bound

$n = 3, t = 2,$ **any** 2**-wise unreactive functionality**

$$\Pr[\overbrace{res_a = res_b = res_c}^{res}] = 1$$

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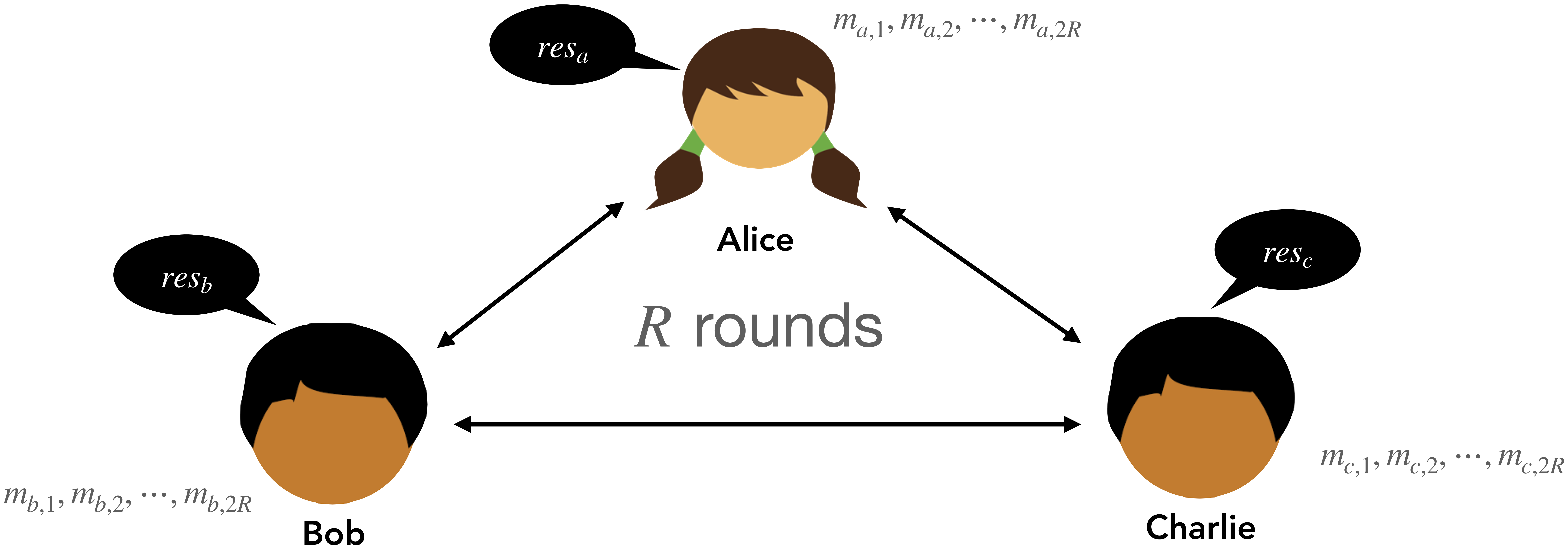

Example: Our Lower Bound

Predictor — Known as “backup” coin

$$\Pr[\overbrace{res_a = res_b = res_c}^{res}] = 1$$

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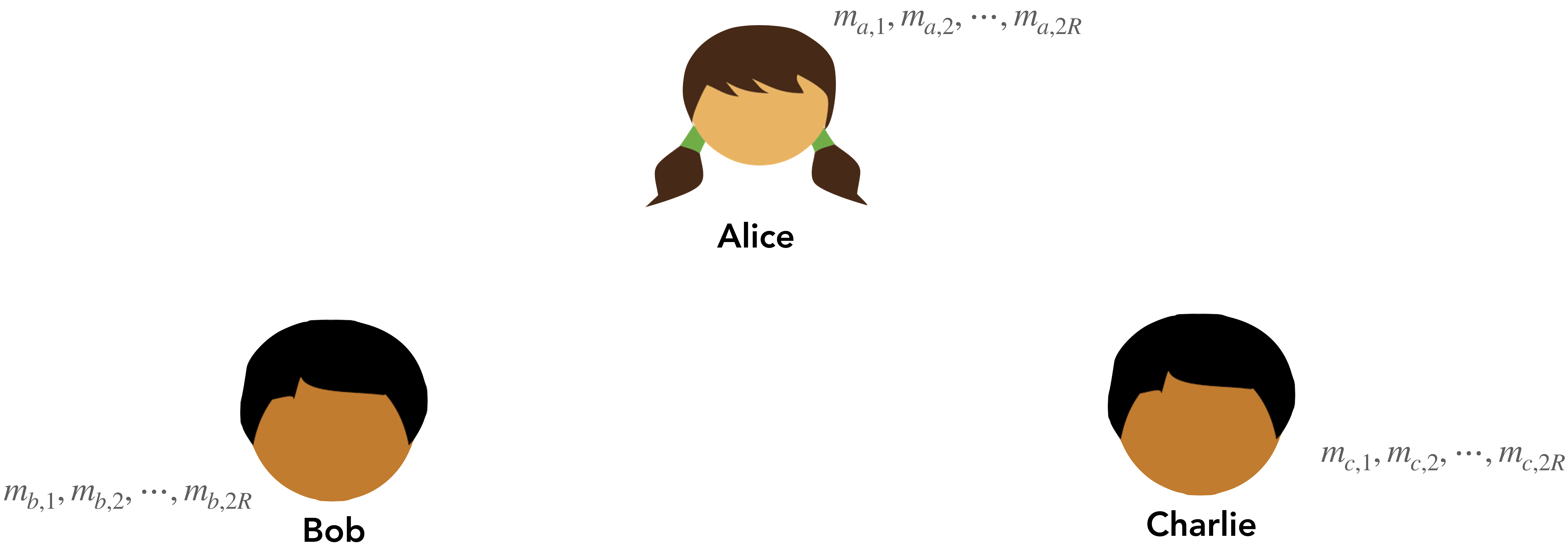

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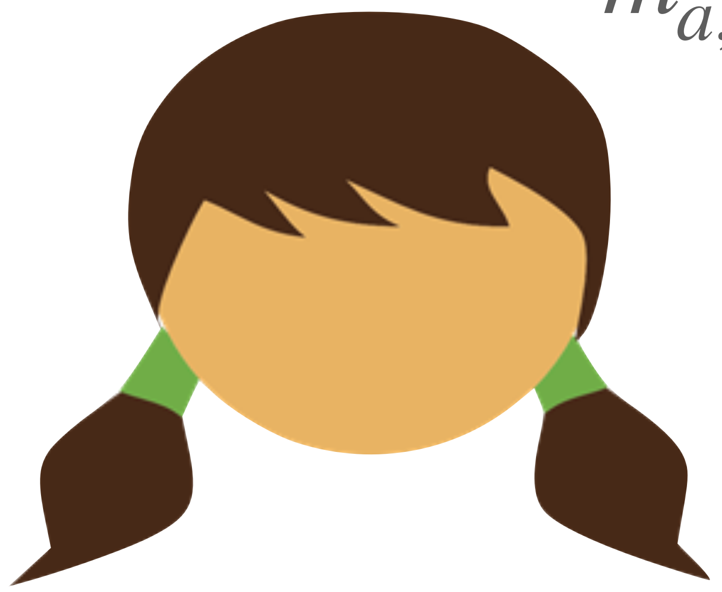
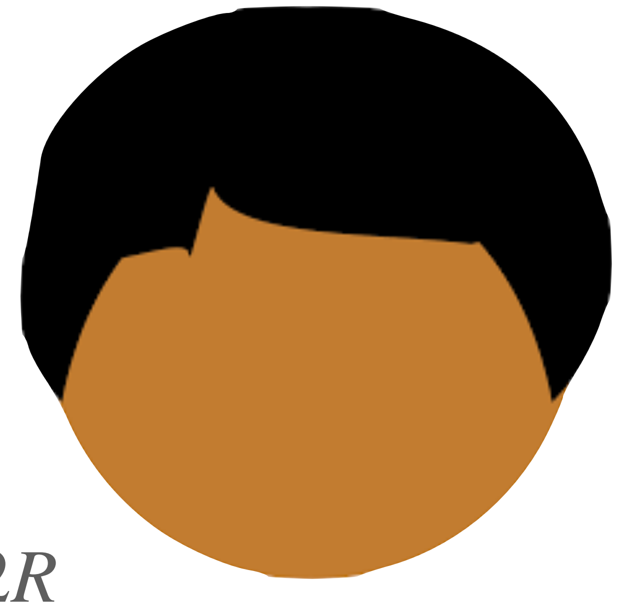
$m_{a,1}, m_{a,2}, \cdots, m_{a,2R}$

$\Pi_{a,0}, \Pi_{a,1}, \Pi_{a,2}, \cdots, \Pi_{a,2R}$



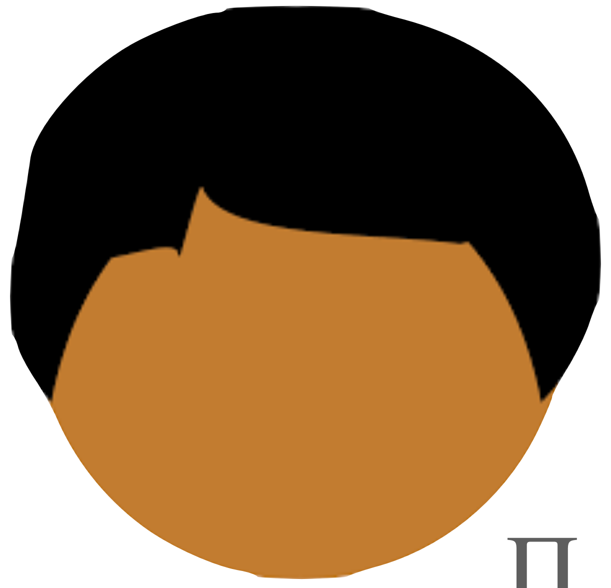
Alice



$m_{b,1}, m_{b,2}, \cdots, m_{b,2R}$

$\Pi_{b,0}, \Pi_{b,1}, \Pi_{b,2}, \cdots, \Pi_{b,2R}$

Bob



$m_{c,1}, m_{c,2}, \cdots, m_{c,2R}$

$\Pi_{c,0}, \Pi_{c,1}, \Pi_{c,2}, \cdots, \Pi_{c,2R}$

Charlie

Example: Our Lower Bound

Predictor — Known as “backup” coin

$$\Pr[\overbrace{res_a = res_b = res_c}^{res}] = 1$$

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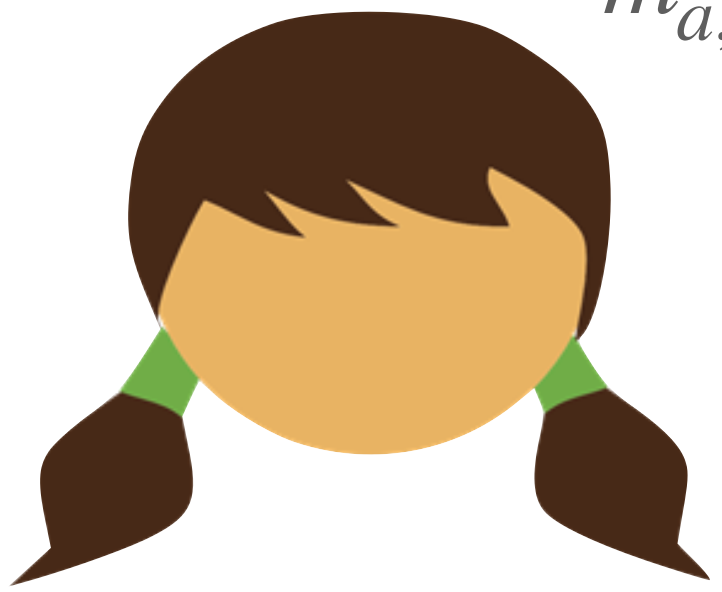


$m_{a,1}, m_{a,2}, \cdots, m_{a,2R}$

$\Pi_{a,0}, \Pi_{a,1}, \Pi_{a,2}, \cdots, \Pi_{a,2R}$

Alice

Predictability

$\Pr[\Pi_{a,0} = res] = \frac{1}{2}$

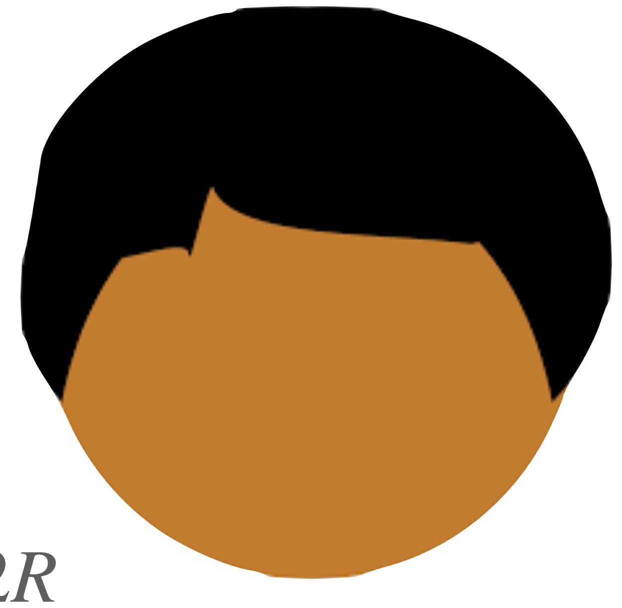
$\Pr[\Pi_{a,2R} = res] = 1$

$\Pr[\Pi_{b,0} = res] = \frac{1}{2}$

$\Pr[\Pi_{b,2R} = res] = 1$

$\Pr[\Pi_{c,0} = res] = \frac{1}{2}$

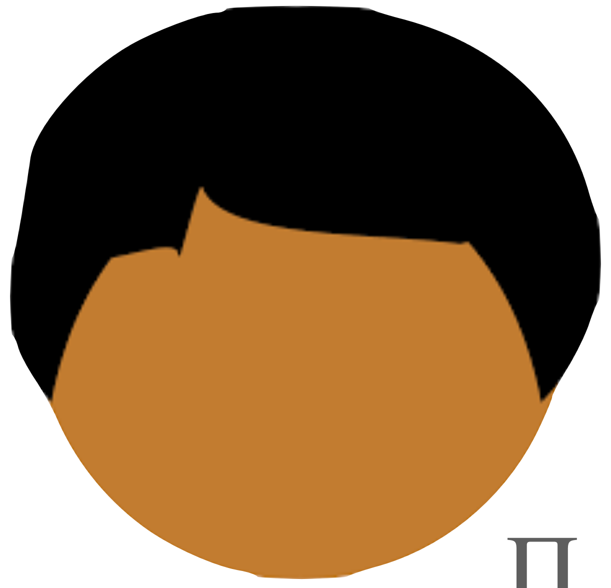
$\Pr[\Pi_{c,2R} = res] = 1$



$m_{b,1}, m_{b,2}, \cdots, m_{b,2R}$

$\Pi_{b,0}, \Pi_{b,1}, \Pi_{b,2}, \cdots, \Pi_{b,2R}$

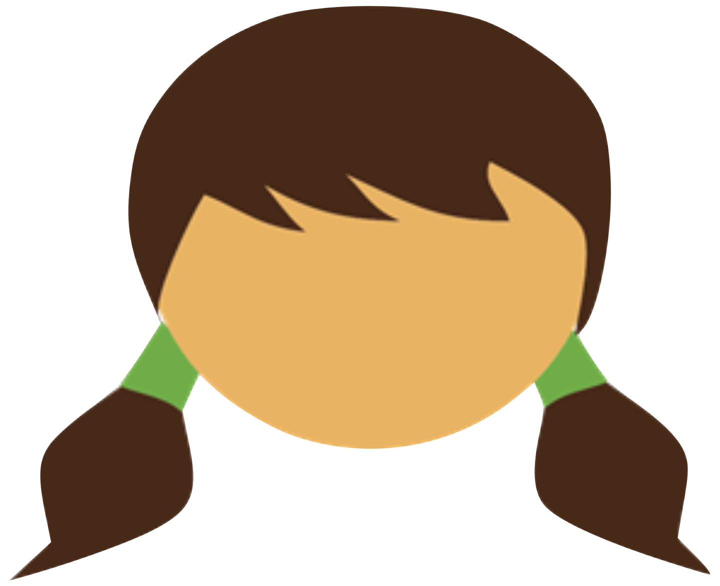
Bob



$m_{c,1}, m_{c,2}, \cdots, m_{c,2R}$

$\Pi_{c,0}, \Pi_{c,1}, \Pi_{c,2}, \cdots, \Pi_{c,2R}$

Charlie



Alice

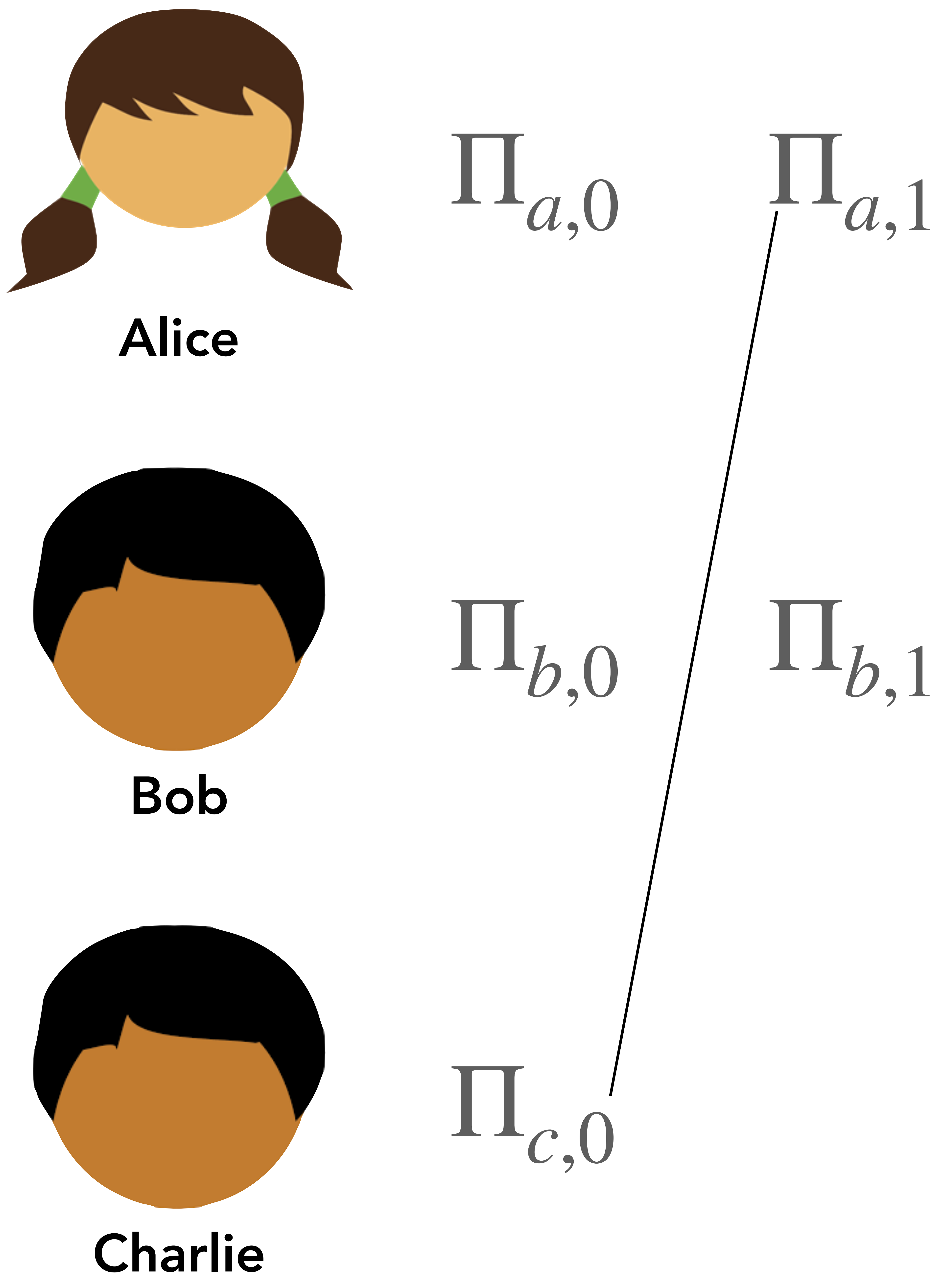
$$\Pi_{a,0}$$

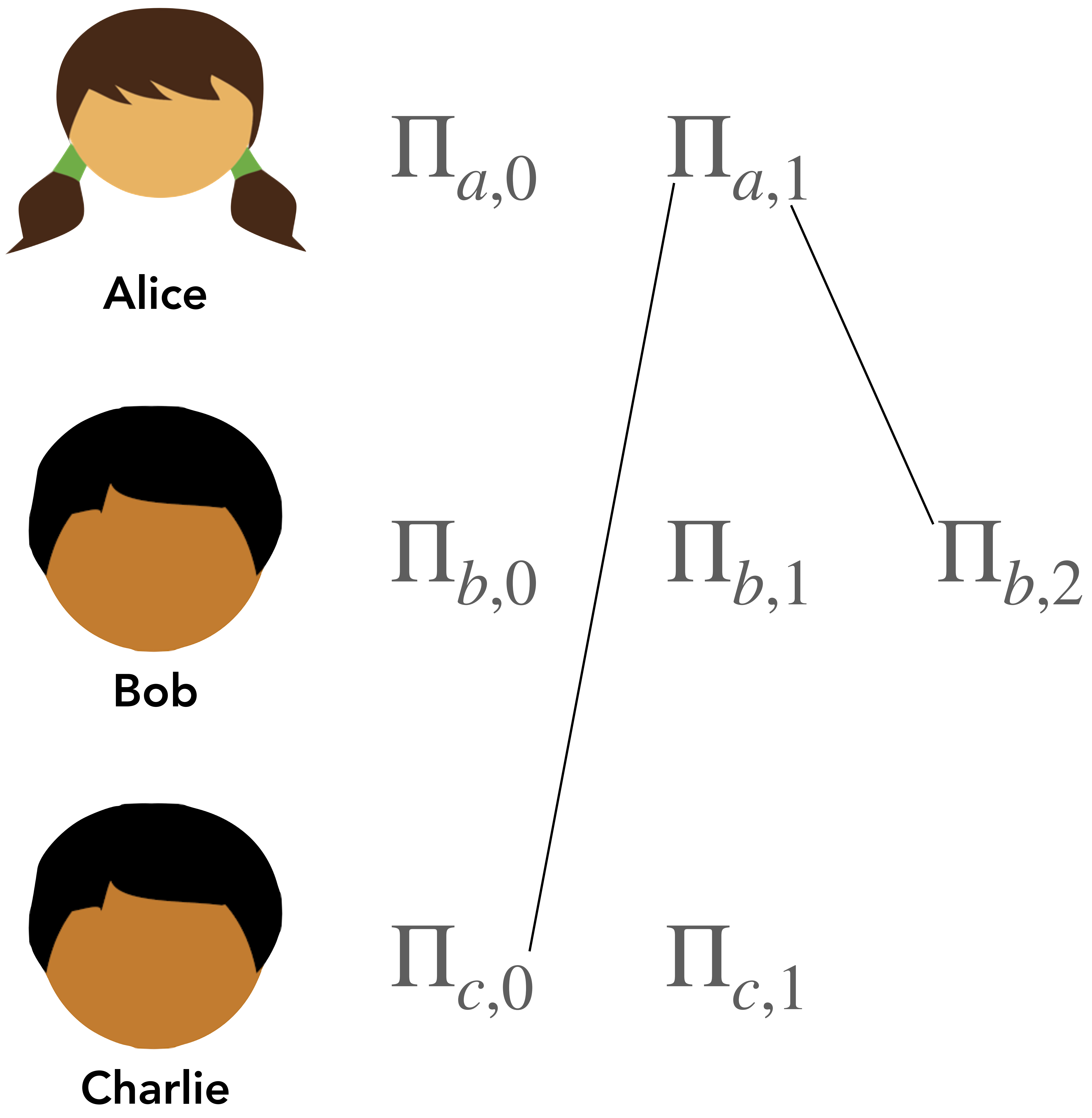
Bob

$$\Pi_{b,0}$$

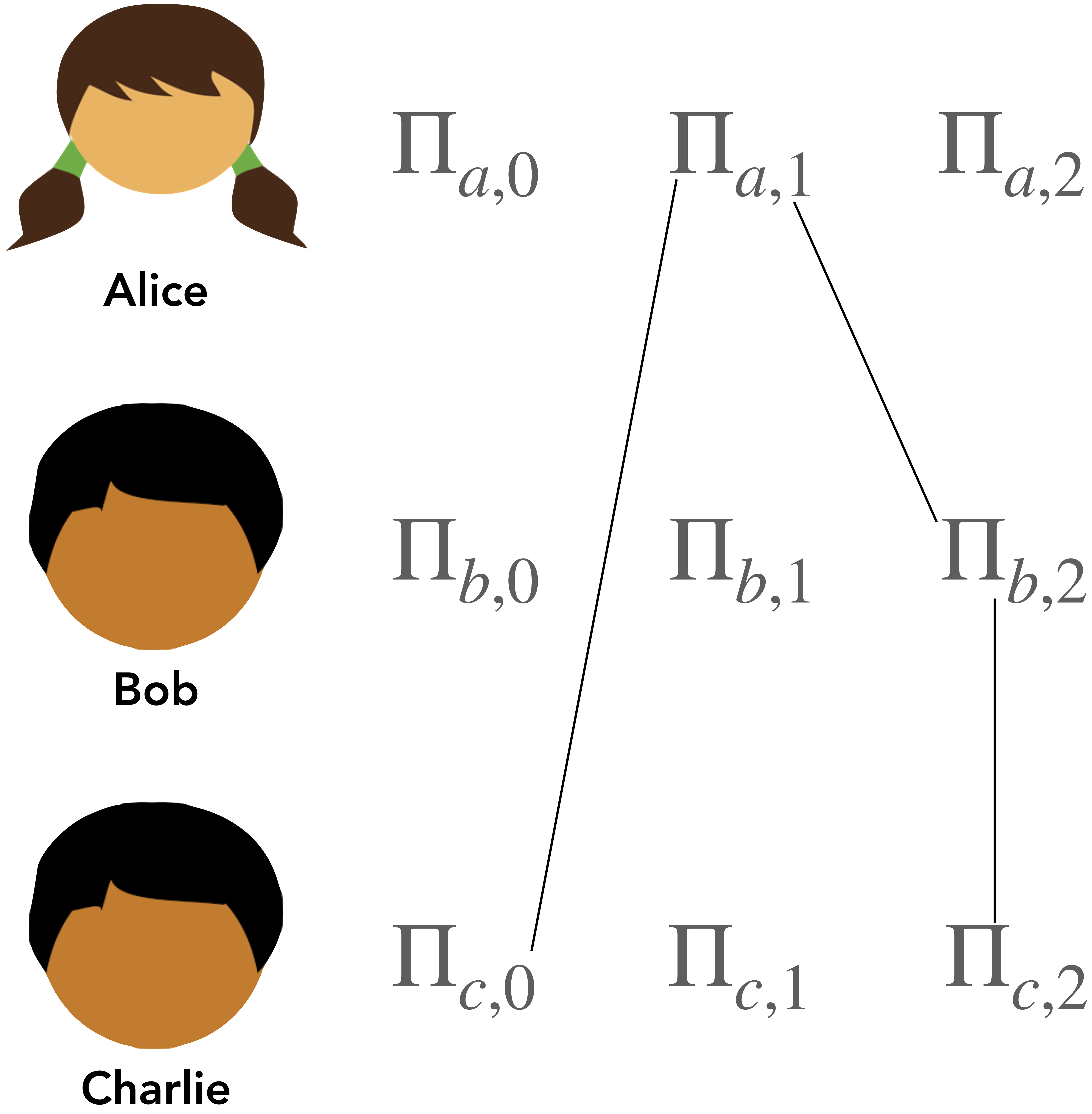
Charlie

$$\Pi_{c,0}$$


Alice
Bob
Charlie
$\Pi_{a,0}$
$\Pi_{a,1}$
$\Pi_{b,0}$
$\Pi_{b,1}$
$\Pi_{c,0}$



Alice
Bob
Charlie
$\Pi_{a,0}$
$\Pi_{a,1}$
$\Pi_{b,0}$
$\Pi_{b,1}$
$\Pi_{b,2}$
$\Pi_{c,0}$
$\Pi_{c,1}$


Alice
Bob
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$\Pi_{a,0}$
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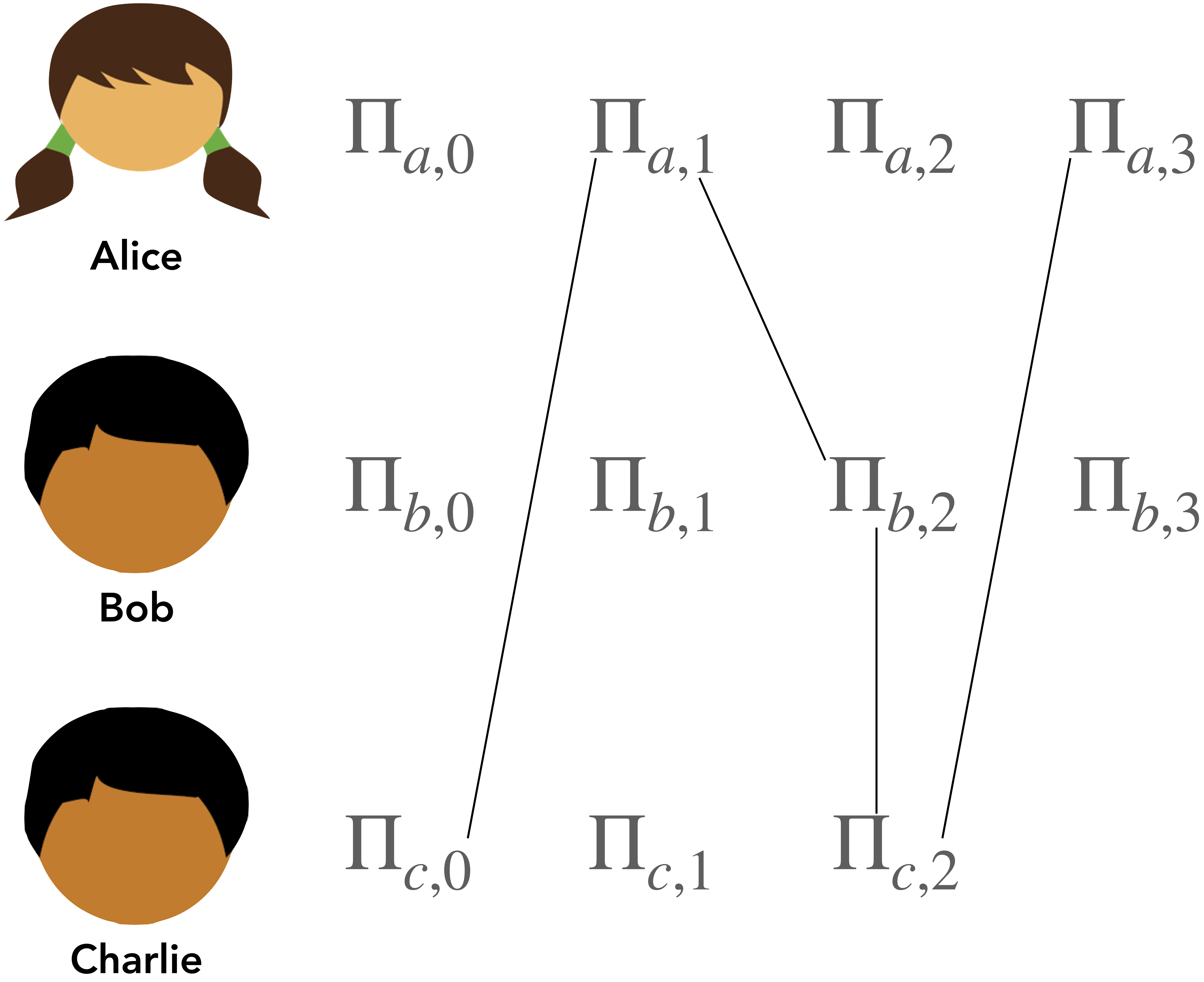
Alice
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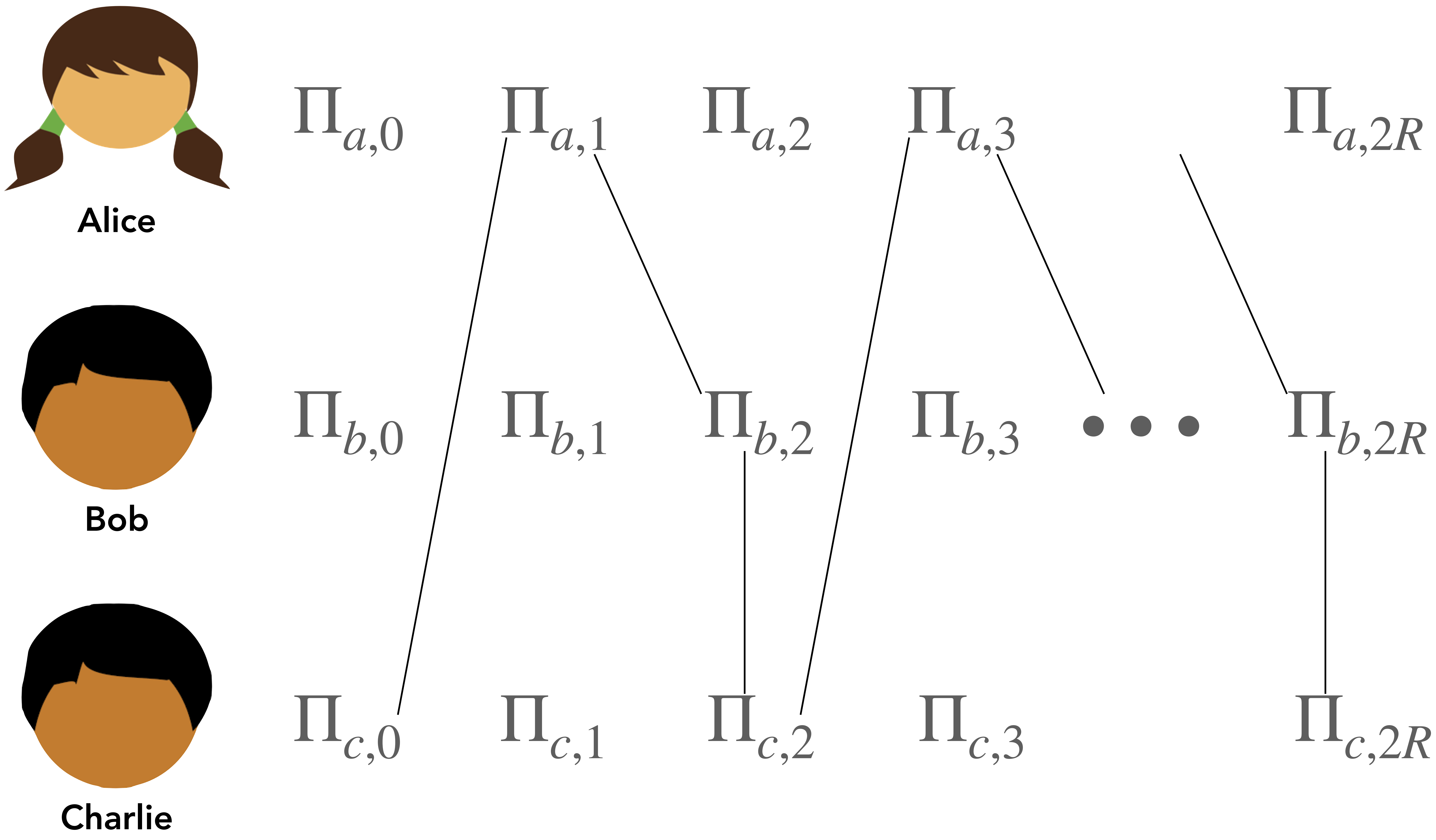
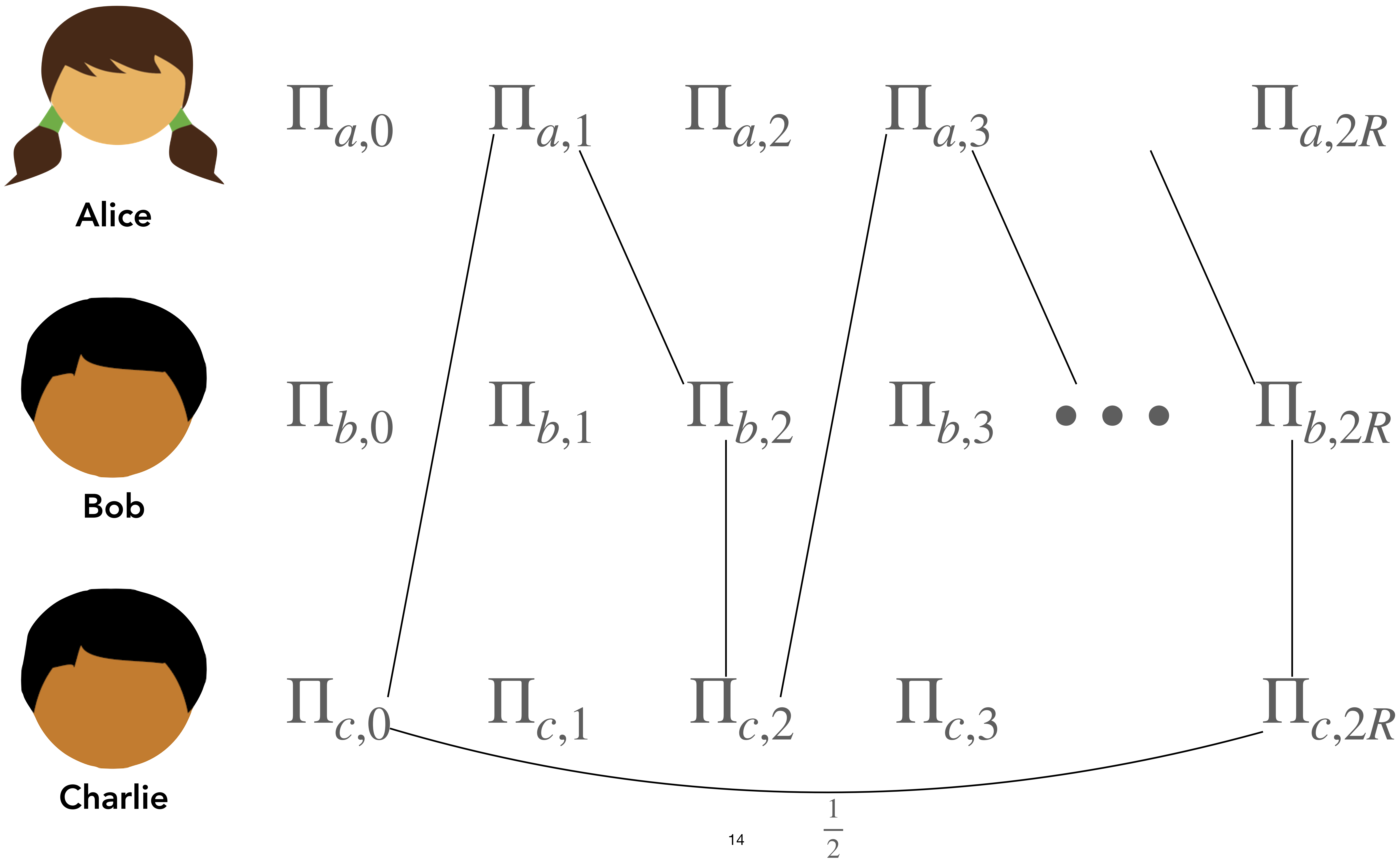
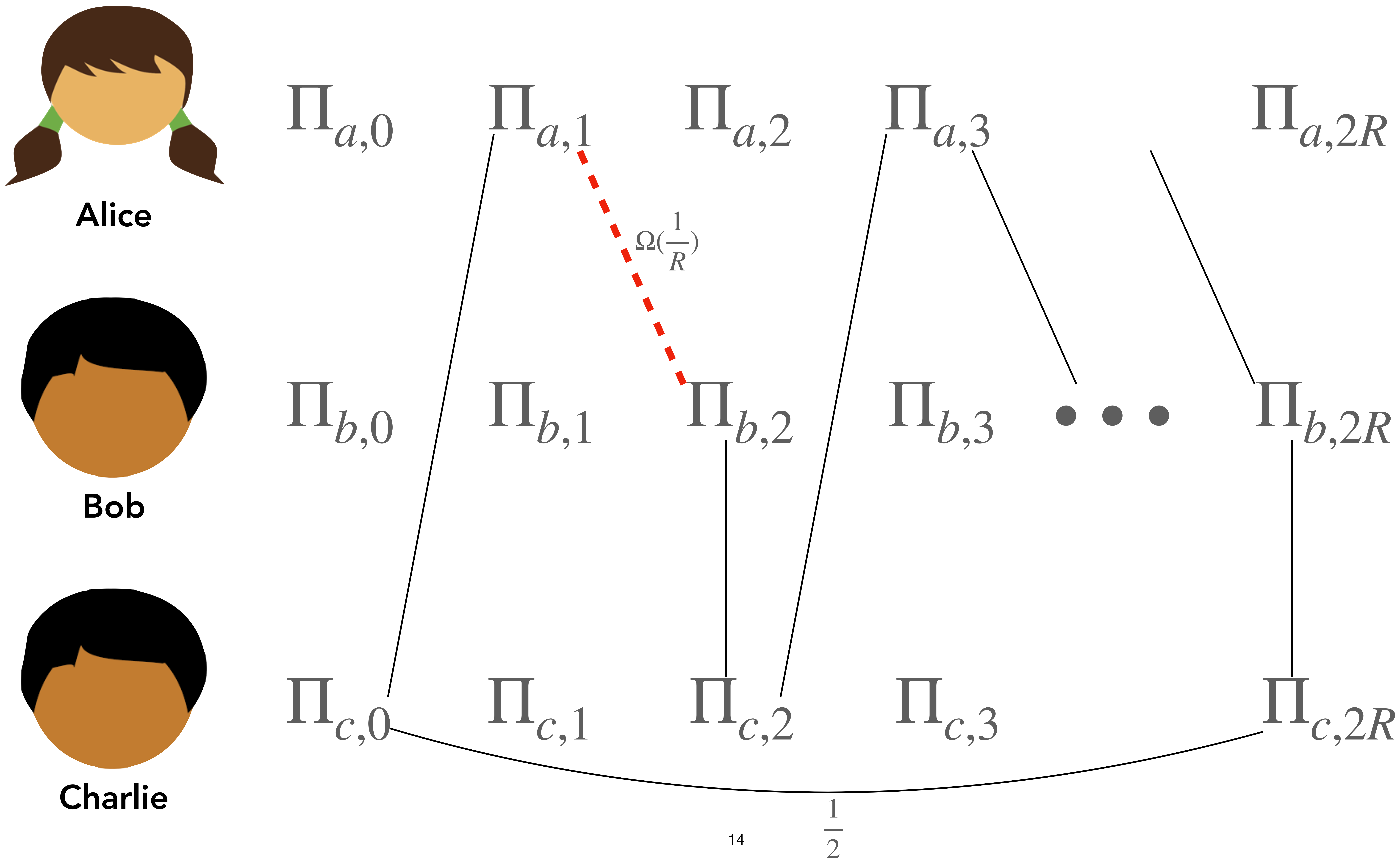
Charlie
$\Pi_{a,0}$ $\Pi_{a,1}$ $\Pi_{a,2}$ $\Pi_{a,3}$
$\Pi_{b,0}$ $\Pi_{b,1}$ $\Pi_{b,2}$ $\Pi_{b,3}$
$\Pi_{c,0}$ $\Pi_{c,1}$ $\Pi_{c,2}$



Alice
Bob
Charlie
$\Pi_{a,0}$ $\Pi_{a,1}$ $\Pi_{a,2}$ $\Pi_{a,3}$ $\Pi_{a,2R}$
$\Pi_{b,0}$ $\Pi_{b,1}$ $\Pi_{b,2}$ $\Pi_{b,3}$ $\Pi_{b,2R}$
$\Pi_{c,0}$ $\Pi_{c,1}$ $\Pi_{c,2}$ $\Pi_{c,3}$ $\Pi_{c,2R}$



Alice
Bob
Charlie
$\Pi_{a,0}$ $\Pi_{a,1}$ $\Pi_{a,2}$ $\Pi_{a,3}$ $\Pi_{a,2R}$
$\Pi_{b,0}$ $\Pi_{b,1}$ $\Pi_{b,2}$ $\Pi_{b,3}$ $\cdots$ $\Pi_{b,2R}$
$\Pi_{c,0}$ $\Pi_{c,1}$ $\Pi_{c,2}$ $\Pi_{c,3}$ $\Pi_{c,2R}$
$\frac{1}{2}$


Alice
Bob
Charlie
$\Pi_{a,0}$
$\Pi_{a,1}$
$\Pi_{a,2}$
$\Pi_{a,3}$
$\Pi_{a,2R}$
$\Omega(\frac{1}{R})$
$\Pi_{b,0}$
$\Pi_{b,1}$
$\Pi_{b,2}$
$\Pi_{b,3}$
$\Pi_{b,2R}$
$\Pi_{c,0}$
$\Pi_{c,1}$
$\Pi_{c,2}$
$\Pi_{c,3}$
$\Pi_{c,2R}$
$\frac{1}{2}$

Future Work

Does an unreactive world enable more fair functionalities?

Can we fairly toss a coin that agrees with $\frac{1}{2} + \text{non-negl}(\lambda)$ probability?

How to instantiate our upper bound protocols?

Q/A

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