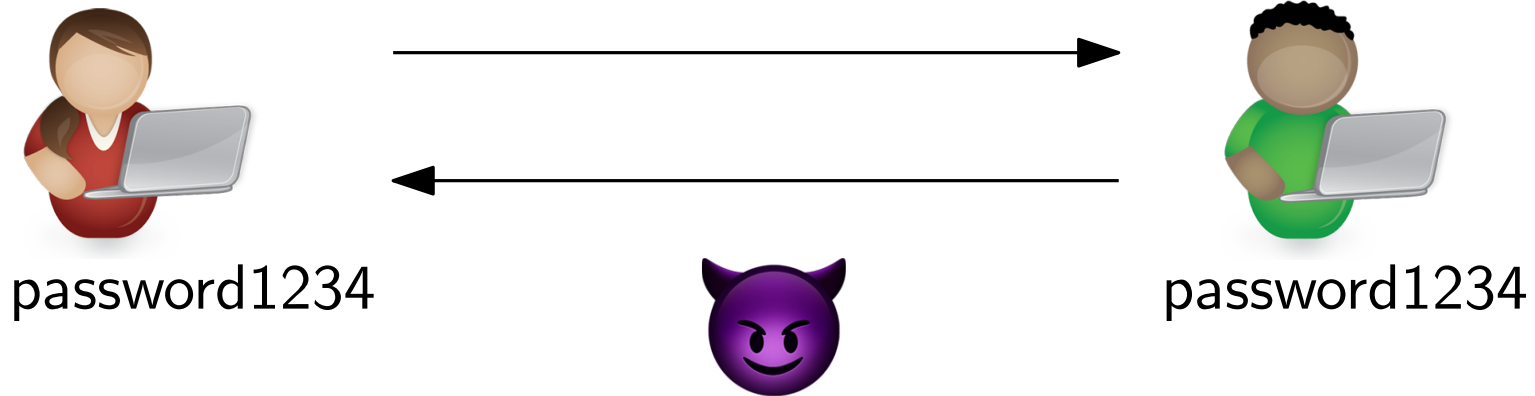


Generalized Fuzzy Password-Authenticated Key Exchange from Error Correcting Codes

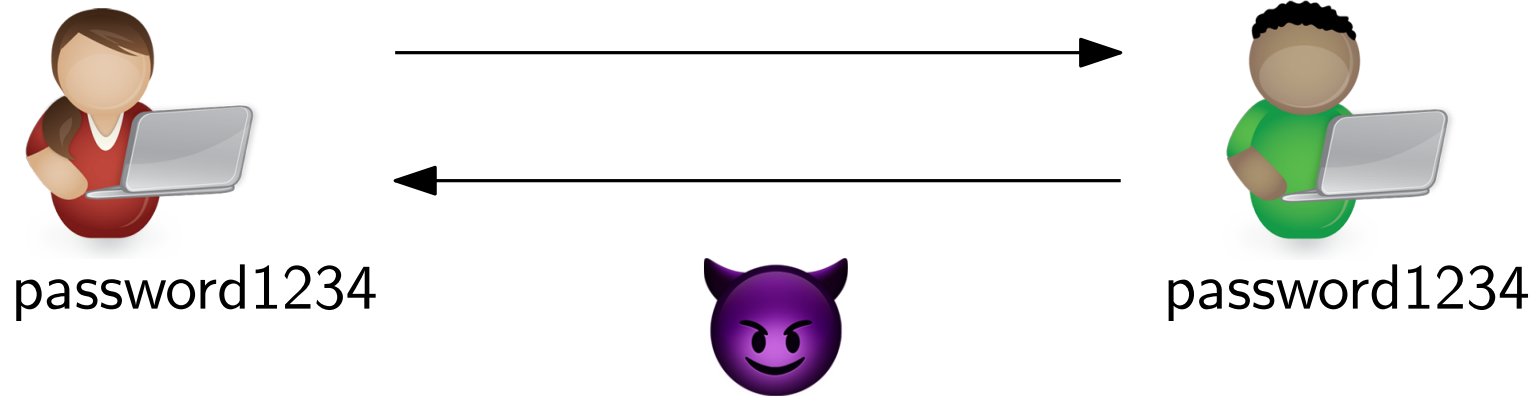
Jonathan Bootle, Sebastian Faller, Julia Hesse, Kristina Hostáková, [Johannes Ottenhues](#)

December 05, 2023

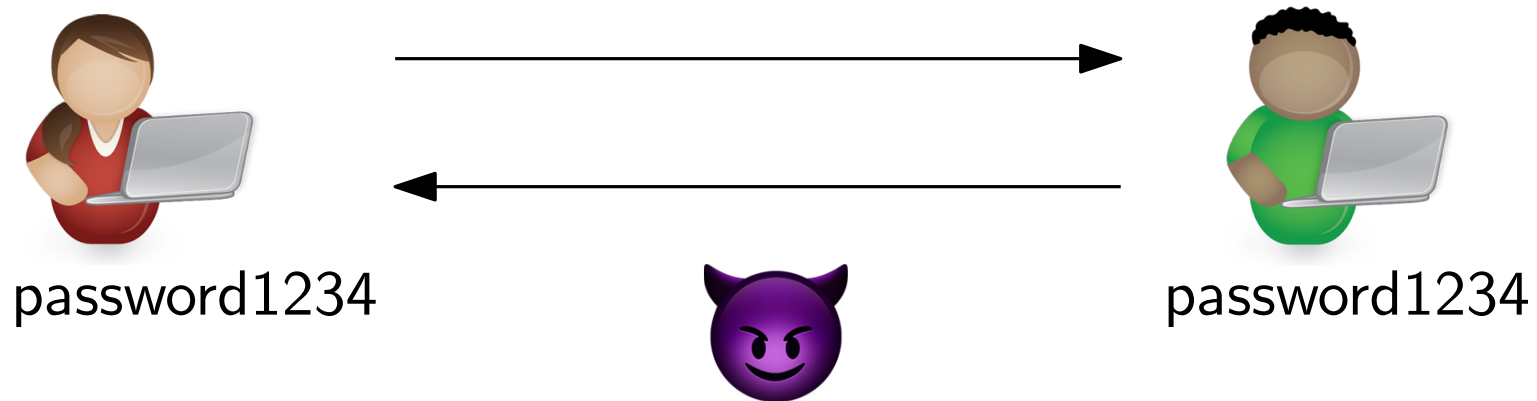
Password Authenticated Key Exchange (PAKE)



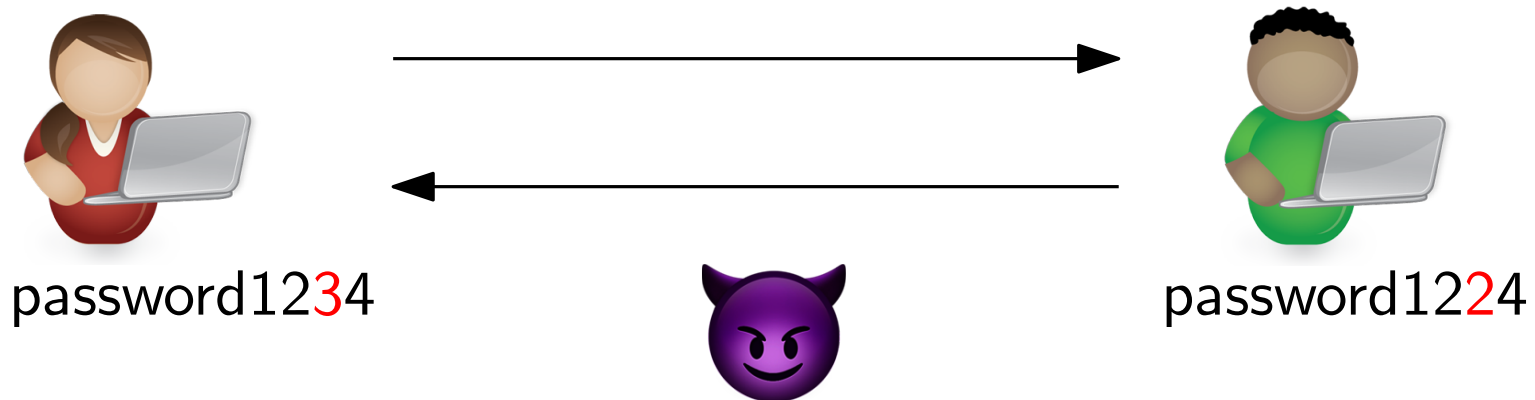
Password Authenticated Key Exchange (PAKE)



Password Authenticated Key Exchange (PAKE)

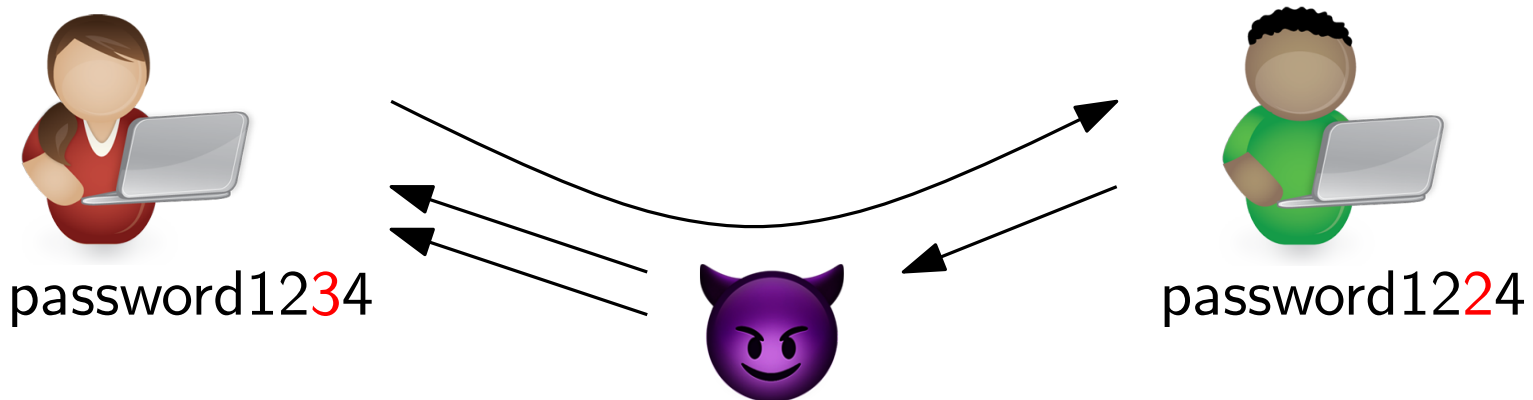


Fuzzy PAKE



- Typo tolerance
- Biometric readings as passwords
- Pairing of nearby IoT devices with sensor data as passwords

Fuzzy PAKE - Attack Model



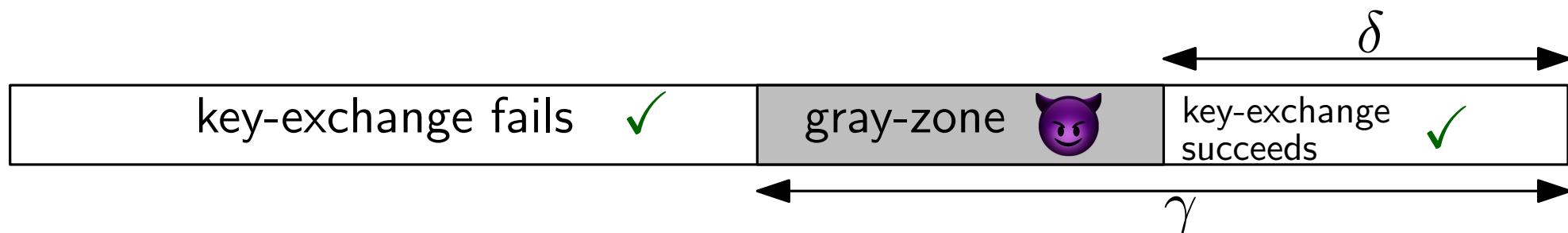
- Same session-key, if and only if the passwords are close
- \mathcal{A} should not learn anything about the passwords
- No offline attacks
- \mathcal{A} should only have one password guess per session

Existing Approaches from [DHP⁺18]

- Garbled circuit based protocol for arbitrary distances
- Robust secret sharing based, for Hamming distance

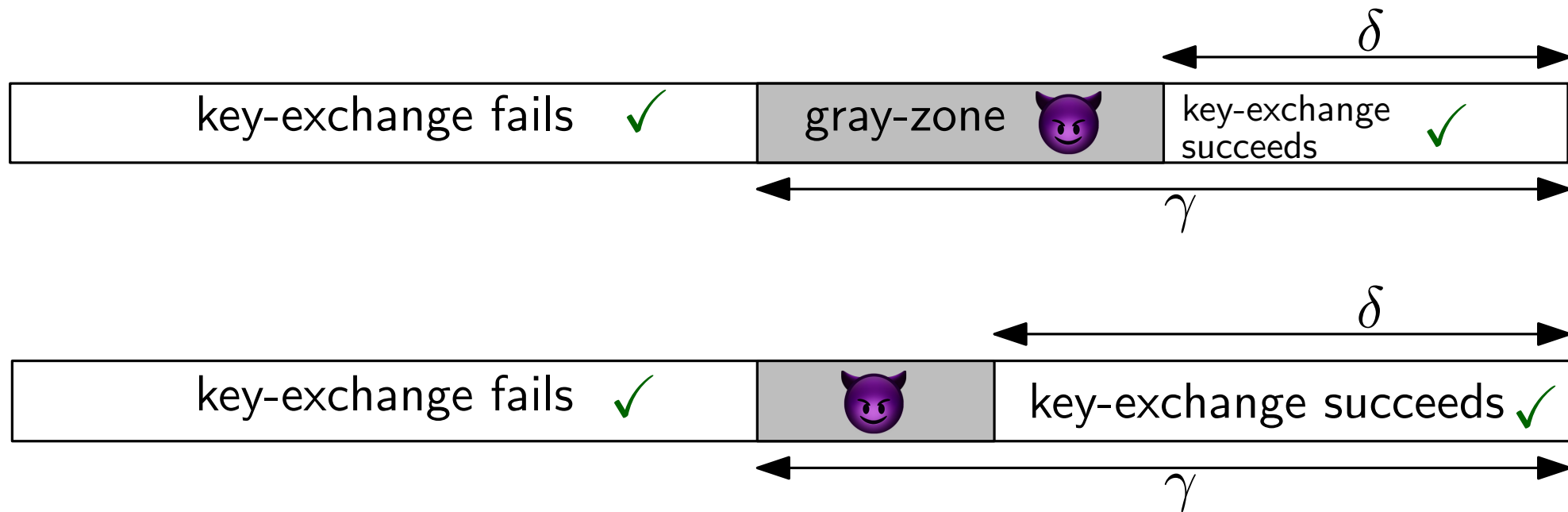
Existing Approaches from [DHP⁺18]

- Garbled circuit based protocol for arbitrary distances
- Robust secret sharing based, for Hamming distance



Existing Approaches from [DHP⁺18]

- Garbled circuit based protocol for arbitrary distances
- Robust secret sharing based, for Hamming distance



Intuition of the fPAKE-RSS protocol

pw = 0110

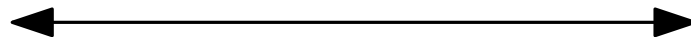


pw' = 0100



Intuition of the fPAKE-RSS protocol

pw = 0110


$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$


pw' = 0100


$$\begin{pmatrix} k_1 \\ k_2 \\ k'_3 \\ k_4 \end{pmatrix}$$

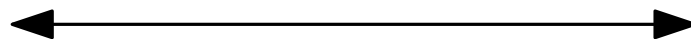
Intuition of the fPAKE-RSS protocol

pw = 0110



$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$



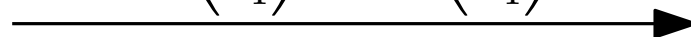
pw' = 0100



$$\begin{pmatrix} k_1 \\ k_2 \\ k'_3 \\ k_4 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c'_3 \\ c_4 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$

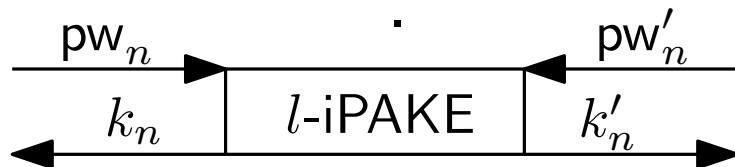


Attack on the fPAKE-RSS protocol

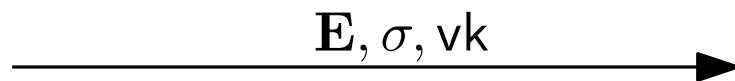
$\text{pw} = \text{pw}_1, \dots, \text{pw}_n$



⋮



$\mathbf{K} := (k_1, \dots, k_n)$
choose U at random
 $\mathbf{C} \leftarrow \text{Encode}(U)$
 $\mathbf{E} := \mathbf{C} + \mathbf{K}$
 $\sigma \leftarrow \text{Sign}(\text{sk}, \mathbf{E})$
session-key := U



$\text{pw}' = \text{pw}'_1, \dots, \text{pw}'_n$



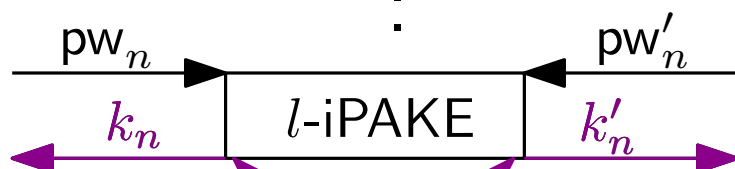
$\mathbf{K}' := (k_1, \dots, k_n)$
verify signature
 $\mathbf{C}' := \mathbf{E} - \mathbf{K}'$
 $U' \leftarrow \text{Decode}(\mathbf{C}')$
session-key := U'

Attack on the fPAKE-RSS protocol

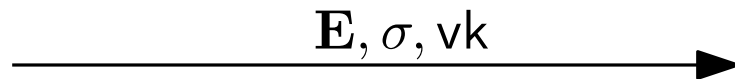
$\text{pw} = \text{pw}_1, \dots, \text{pw}_n$



⋮



$\mathbf{E}, \sigma, \text{vk}$



$\text{pw}' = \text{pw}'_1, \dots, \text{pw}'_n$



$\mathbf{K} := (k_1, \dots, k_n)$
 choose U at random
 $\mathbf{C} \leftarrow \text{Encode}(U)$
 $\mathbf{E} := \mathbf{C} + \mathbf{K}$
 $\sigma \leftarrow \text{Sign}(\text{sk}, \mathbf{E})$
 session-key := U

$\mathbf{K}' := (k_1, \dots, k_n)$
 verify signature
 $\mathbf{C}' := \mathbf{E} - \mathbf{K}'$
 $U' \leftarrow \text{Decode}(\mathbf{C}')$
 session-key := U'

Summary of the Attack

- Requirements:
 - Both parties run the protocol
 - Both parties use the same password
 - The attacker actively interferes in the protocol
- Effect:
 - The attacker learns one bit of their choice of the password

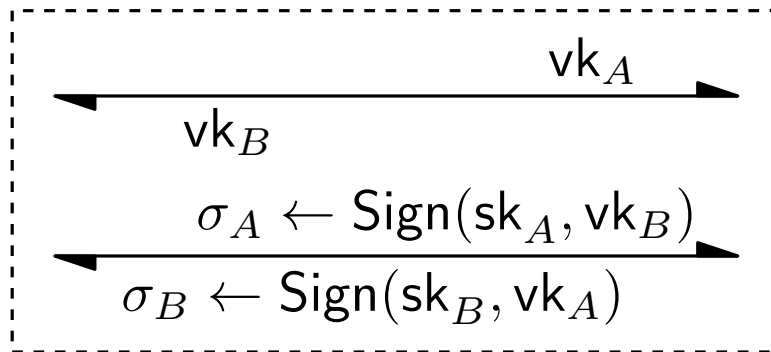
Variations of the attack also work when the passwords are not identical

Overview of our fuzzy PAKE protocol

$\text{pw} = \text{pw}_1, \dots, \text{pw}_n$



$\text{pw}' = \text{pw}'_1, \dots, \text{pw}'_n$

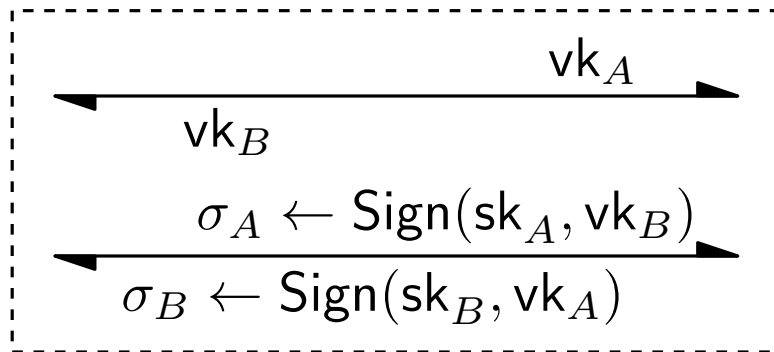


Overview of our fuzzy PAKE protocol

$\text{pw} = \text{pw}_1, \dots, \text{pw}_n$



$\text{pw}' = \text{pw}'_1, \dots, \text{pw}'_n$



$\mathbf{C} \leftarrow \text{Encode}(\text{secret})$

$\mathbf{E} := \text{blind}(\mathbf{C})$

$h := H(\mathbf{C})$

\mathbf{E}, h

$\mathbf{C}' := \text{unblind}(\mathbf{E})$

$L \leftarrow \text{ListDecode}(\mathbf{C}')$

identify correct \mathbf{C} via h

$\text{secret} := \text{Decode}(\mathbf{C})$

Our fuzzy PAKE protocol

$pw = pw_1, \dots, pw_n$



$pw' = pw'_1, \dots, pw'_n$



Our fuzzy PAKE protocol

$\text{pw} = \text{pw}_1, \dots, \text{pw}_n$



$\text{pw}' = \text{pw}'_1, \dots, \text{pw}'_n$



⋮



$\mathbf{K} := (k_1, \dots, k_n)$

$\mathbf{K}' := (k_1, \dots, k_n)$

Our fuzzy PAKE protocol

$$pw = pw_1, \dots, pw_n$$



$$pw' = pw'_1, \dots, pw'_n$$



⋮



$$\mathbf{K} := (k_1, \dots, k_n)$$

$$\mathbf{K}' := (k'_1, \dots, k'_n)$$

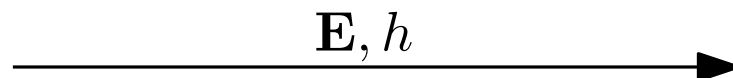
$$\mathbf{s} := (s_1, \dots, s_k) \xleftarrow{\$} \mathbb{F}_q^k$$

$$\mathbf{C} \leftarrow \text{Encode}(\mathbf{s})$$

$$\mathbf{E} := \mathbf{C} \oplus \mathbf{K}$$

$$h := H_0(\mathbf{C})$$

$$\text{session-key} := H_1(s_1)$$

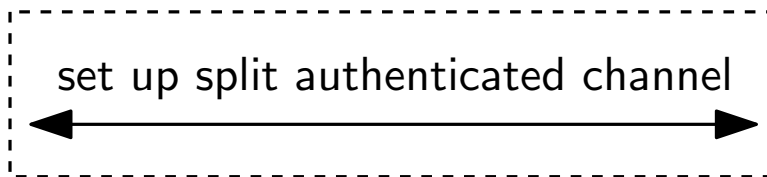


Our fuzzy PAKE protocol

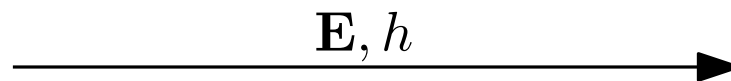
$$pw = pw_1, \dots, pw_n$$



$$pw' = pw'_1, \dots, pw'_n$$



⋮



$$\mathbf{K} := (k_1, \dots, k_n)$$

$$s := (s_1, \dots, s_k) \xleftarrow{\$} \mathbb{F}_q^k$$

$$\mathbf{C} \leftarrow \text{Encode}(s)$$

$$\mathbf{E} := \mathbf{C} \oplus \mathbf{K}$$

$$h := H_0(\mathbf{C})$$

$$\text{session-key} := H_1(s_1)$$

$$\mathbf{K}' := (k_1, \dots, k_n)$$

$$\mathbf{C}' := \mathbf{E} \oplus \mathbf{K}'$$

$$(\mathbf{C}^1, \dots, \mathbf{C}^l) \leftarrow \text{ListDecode}(\mathbf{C}')$$

for $j \in \{1, \dots, l\}$:

if $H_0(\mathbf{C}^j) = h$:

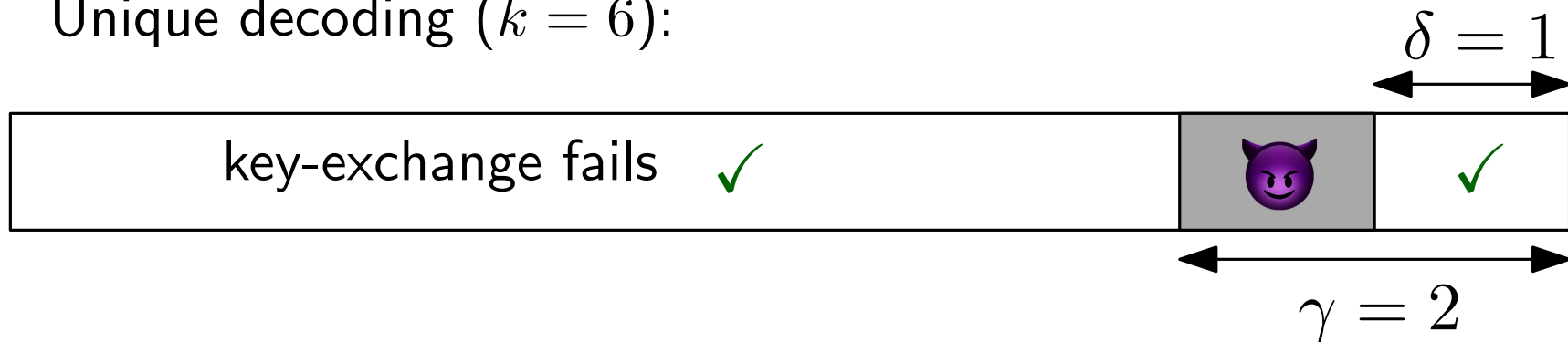
set $s' := \text{Decode}(\mathbf{C}^j)$

$$\text{session-key} := H_1(s'_1)$$

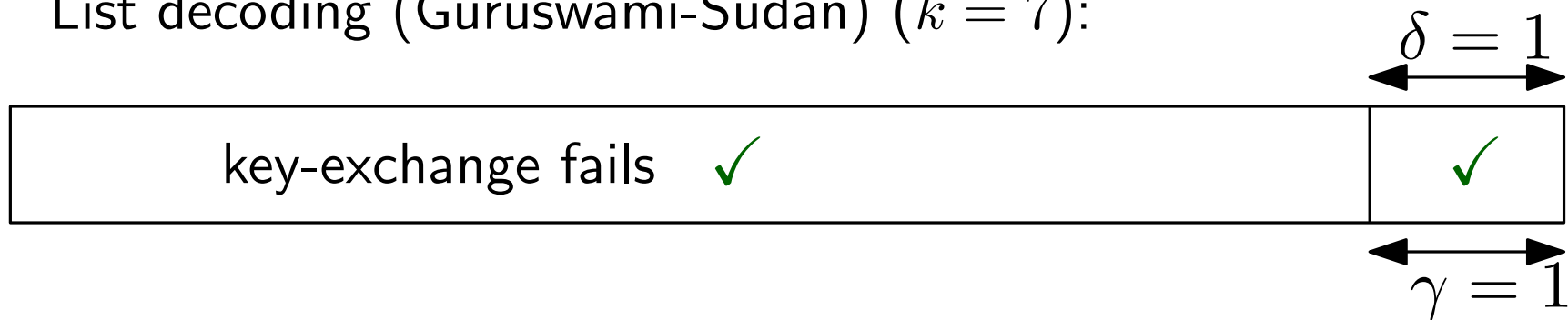
Improvements through list decoding

Correcting 1 error of an 8 character password (with an $[n, k]$ Code)

Unique decoding ($k = 6$):



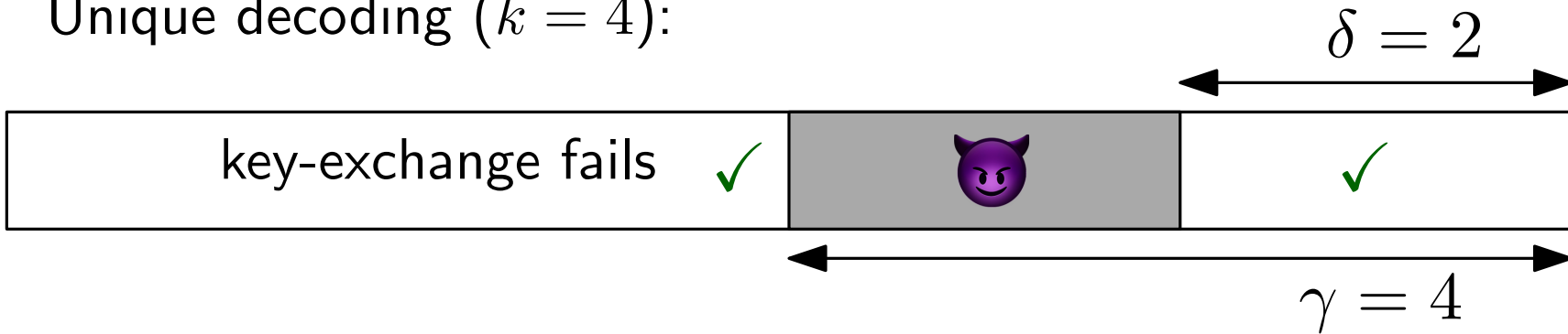
List decoding (Guruswami-Sudan) ($k = 7$):



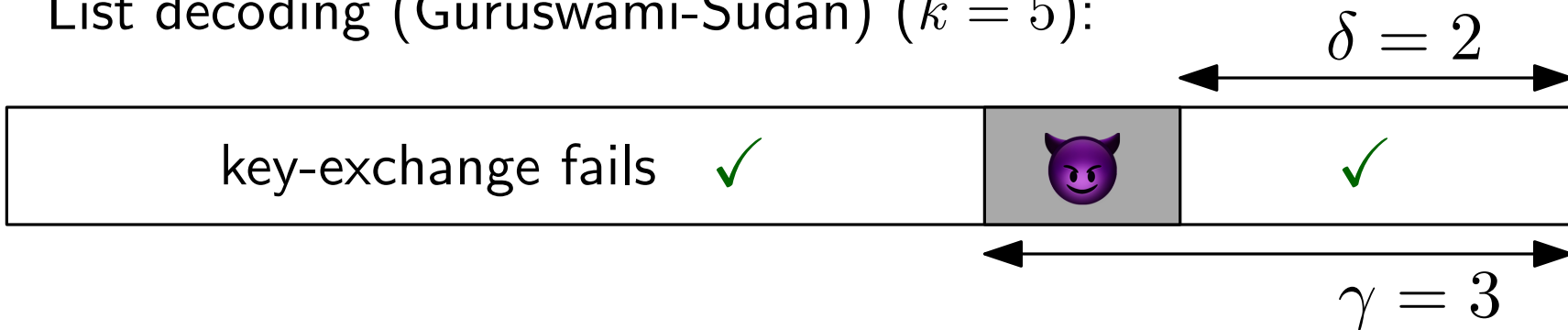
Improvements through list decoding

Correcting 2 error of an 8 character password (with an $[n, k]$ Code)

Unique decoding ($k = 4$):



List decoding (Guruswami-Sudan) ($k = 5$):



Paper at ia.cr/2023/1415