Radboud University

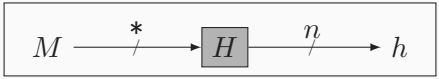
Generic Security of the SAFE API and Its Applications

Dmitry Khovratovich, Mario Marhuenda Beltrán, Bart Mennink

December 8, 2023

ESCADA

Cryptographic Hash Functions



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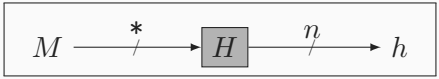
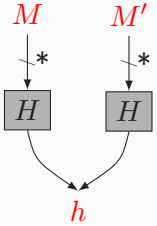


- **Collision resistant**



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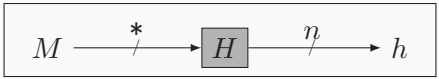

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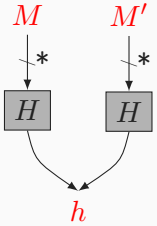
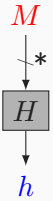
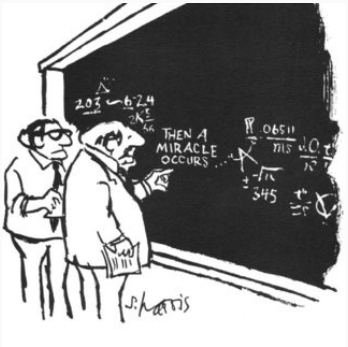
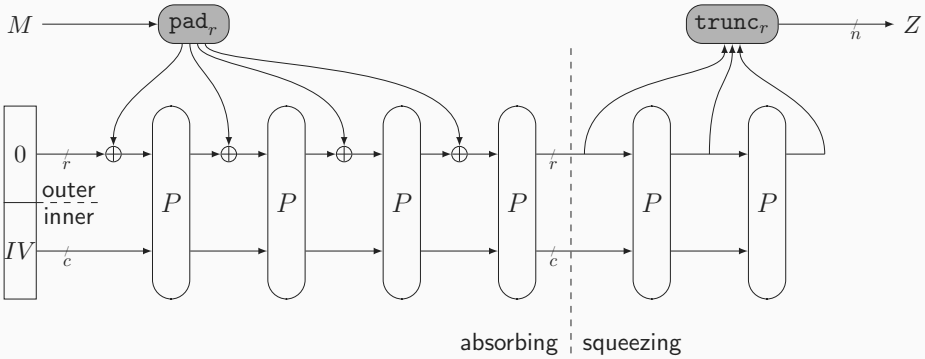




Mathematically easy,
but computationally hard!



Sponge Construction



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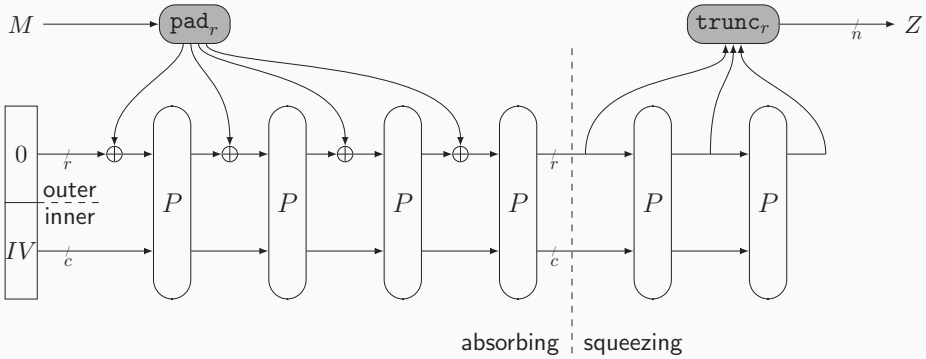

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  - r is the rate.
  - c is the capacity.
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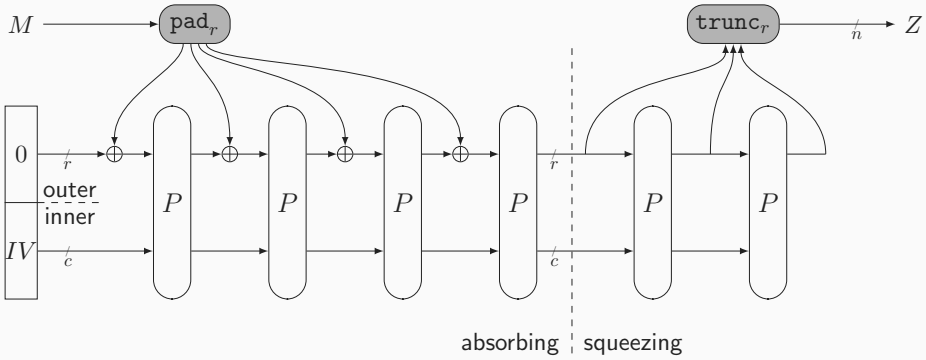

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  - r is the rate.
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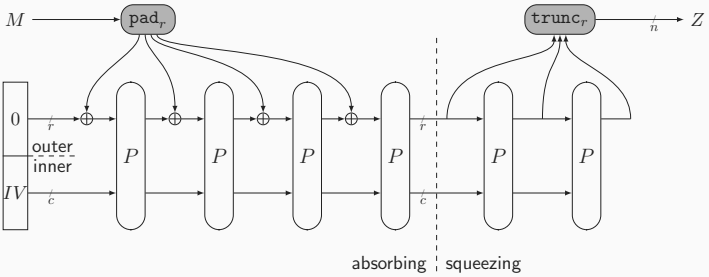
- **Security**: Behaves like RO up to $O(2^{c/2})$ queries [2, 3].

Our objective

A Problem



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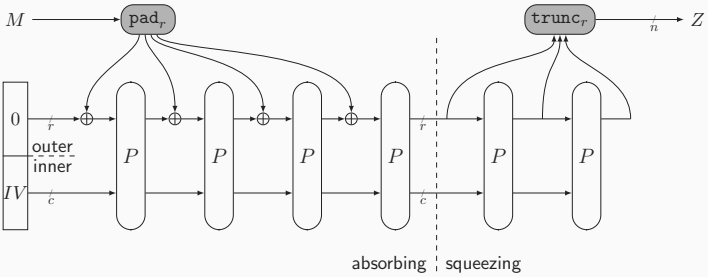

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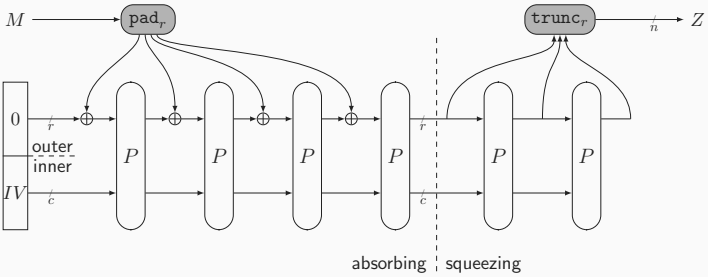

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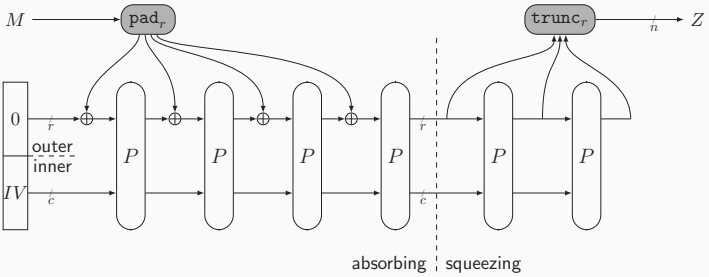

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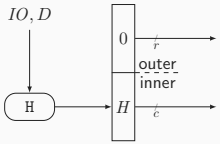
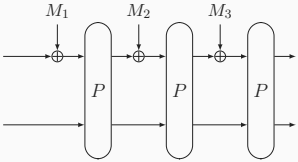
- **Domain separation**.
  - We must make all the absorbs before any squeezing takes place.
  - **Inflexible scheme**.

A Solution: SAFE API [1]

Start



Absorb



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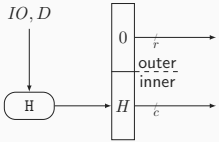

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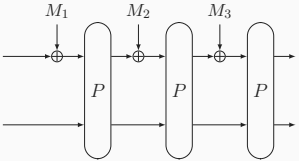
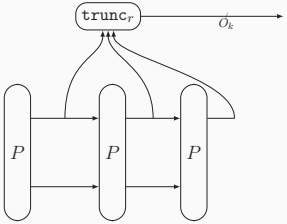


Squeeze



Finish

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- Fix **padding**:
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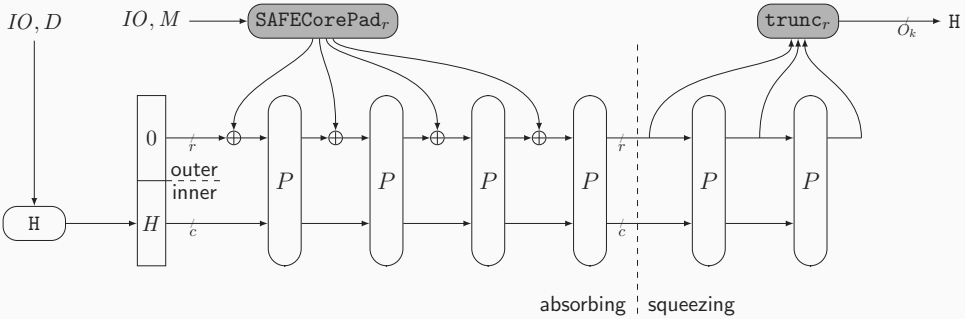
- SAFECore: A variant of the **sponge**.
  - **Security**: Behaves like RO up to $O(2^{c/2})$ queries.


IO, D
IO, M
SAFECorePad_r
trunc_r
H
O_k
H
0
H
r
c
outer
inner
P
P
P
P
P
P
r
c
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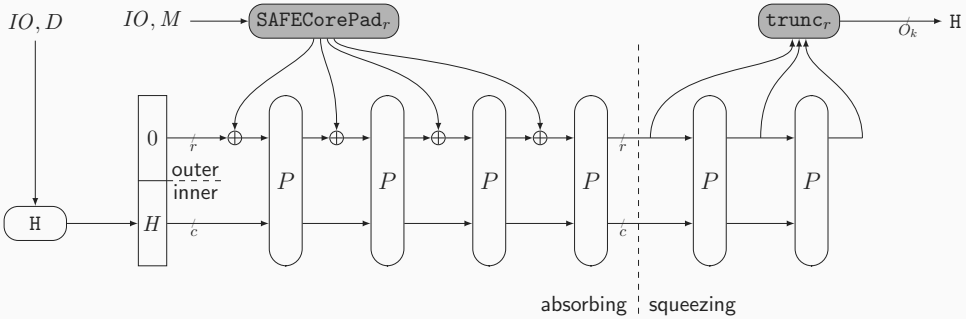

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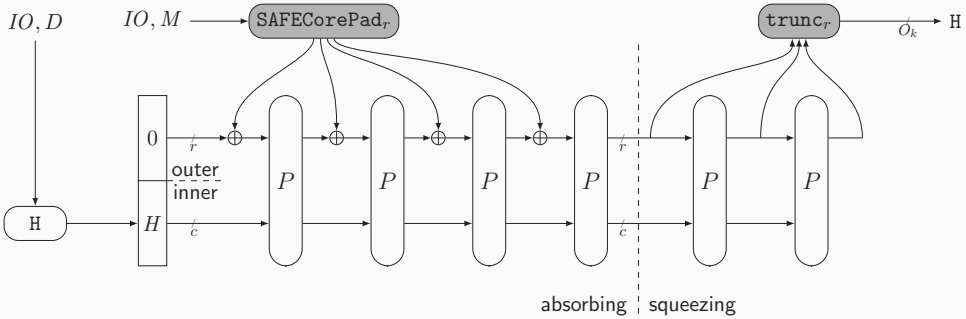

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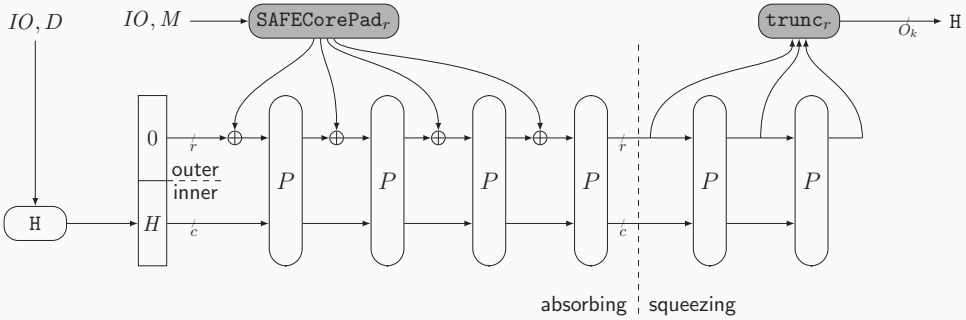

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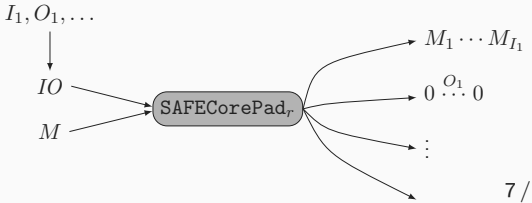
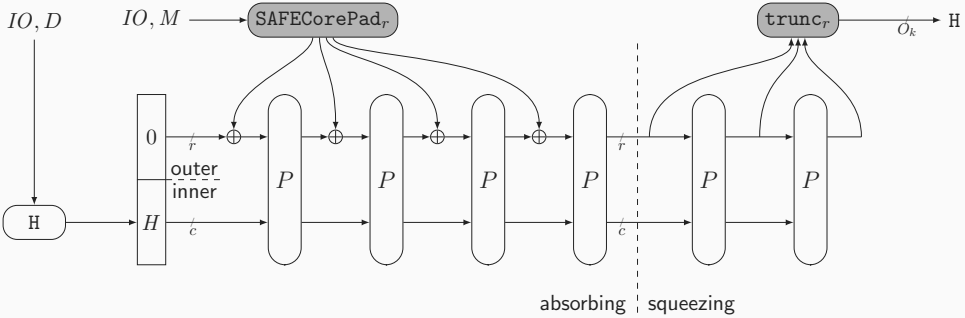


SAFECore: Security



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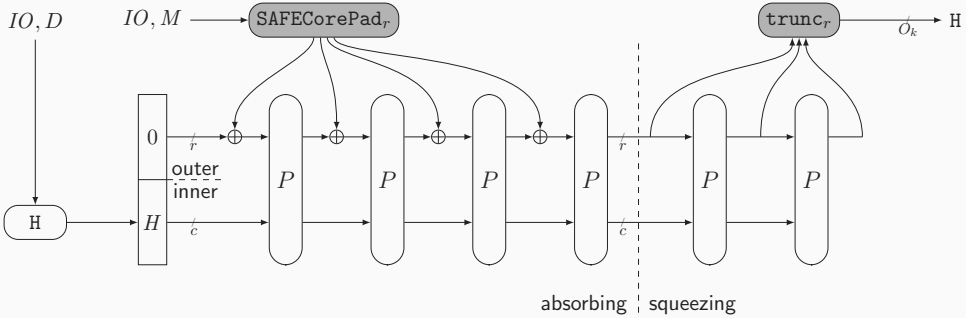

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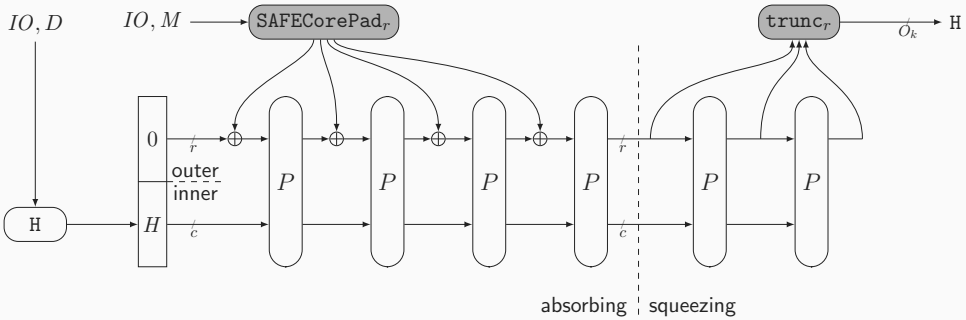
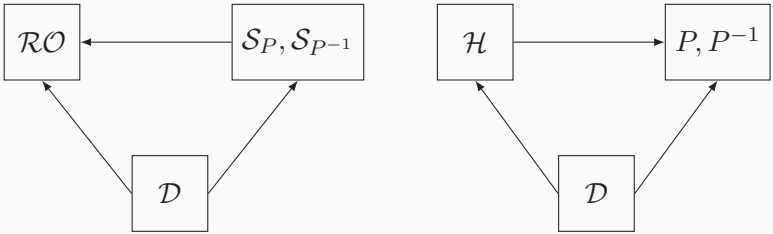


- State of the art does not cover security of SAFECore.
- **Our contribution**: Thorough analysis of SAFE API. Previous state of the art: No proof for SAFE API.
  - We prove generic security of SAFECore ...
  - ... and apply it to SAFE API.

Security

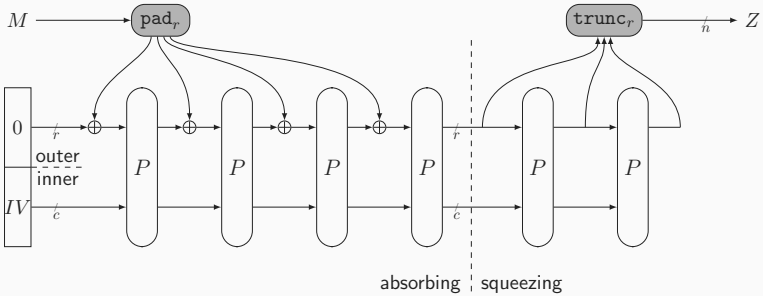
Indifferentiability framework



Ideal world

Real world

Indifferentiability of the Sponge



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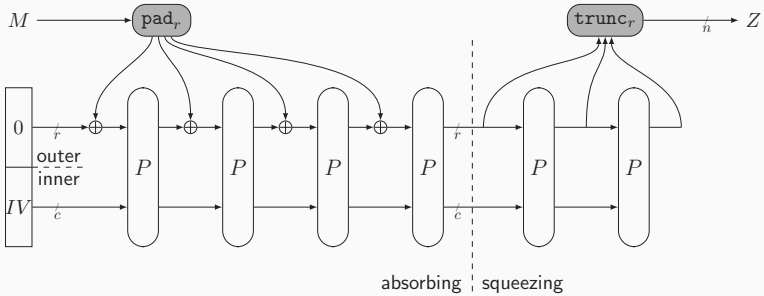

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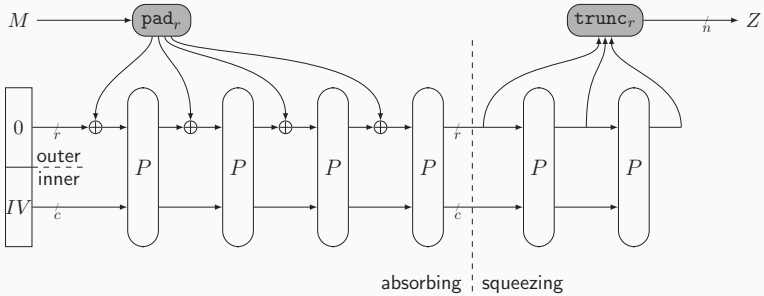

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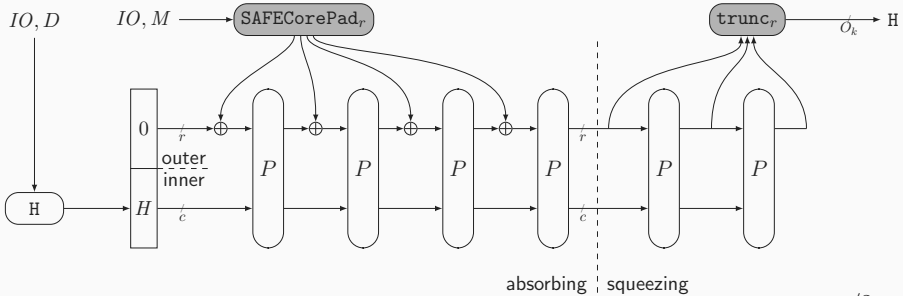

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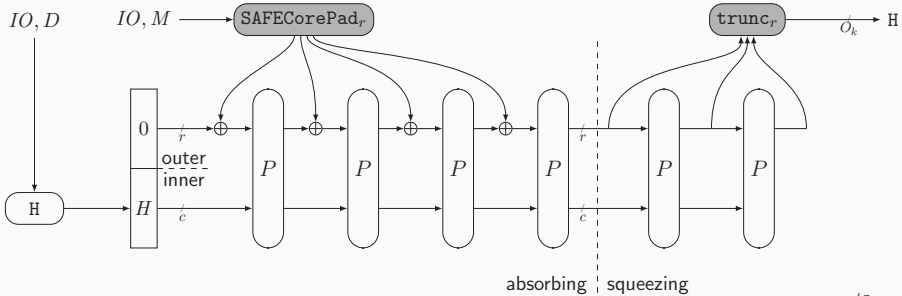

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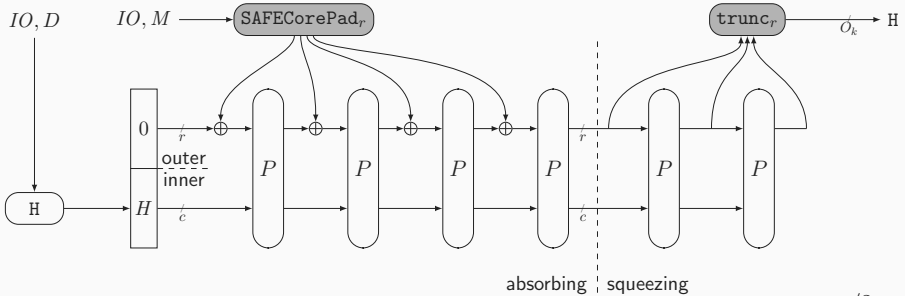

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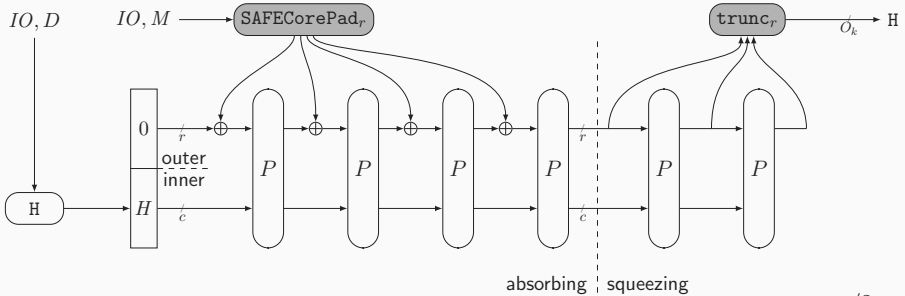

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  - Now $\mathcal{D}$ can win the indifferentiability game.
    - $O(2^{c/2})$ queries to P.
  - **A new attack**: Collision in the inner part by querying $\mathcal{H}$.
  - We lost nothing because we already had this bound in the sponge.

Applications

Applications of SAFE API

- Plain hashing.
- Commitment schemes.
- Interactive protocols.
- Merkle trees.
- Zero Knowledge proofs: SNARKs.
- Lattice cryptography.
- ZKVMs.
- Verifiable encryption.

Applications of SAFE API: Commitment Scheme

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- Generic security of SAFECore implies security of commitment scheme.

Conclusion

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- Allows for more efficient hashing in finite fields.
  - Requires the use of a **another hash function**.
- Generic security bound is **the same** as normal sponge.

Thank you for your attention!

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