## Two-Round Concurrent 2PC from Sub-Exponential LWE

Behzad Abdolmaleki ${ }^{1}$, Saikrishna Badrinarayanan, ${ }^{2}$ Rex Fernando ${ }^{3}$, Giulio Malavolta, ${ }^{4,5}$ Ahmadreza Rahimi ${ }^{5}$, and Amit Sahai ${ }^{6}$

1. University of Sheffield, UK
2. LinkedIn, USA
3. Carnegie Mellon University, USA
4. Bocconi University, Italy
5. Max Planck Institute for Security and Privacy, Germany
6. UCLA, USA

## Problem Statement of Concurrent 2PC

-Def.:Two-party computation (2PC) protocols where both parties receive output $\mathrm{z}=\mathrm{f}(\mathrm{x}, \mathrm{y})$


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- Formally: simulation-based security.


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Can we achieve two-round concurrently secure two-party computation under simple, post-quantum assumptions, in the plain model?

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## Impossibility Result:


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## Overcome the above mentioned impossibility results:

- The bounded concurrent model [Pass04],
- In the multiple ideal-query model [GoyJai13],
- input-indistinguishable computation [MicPas06].
-And an standard relaxation of simulation security: the notion of super-polynomial simulation, or SPS [Pass03]. (which is widely used to circumvent many lower-bound results)


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[GGJS12, KMO14]: Constant-round protocols (approximately 20 rounds). [GKP17] : 5 rounds with SPS security from standard sub-exponential assumptions.
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[BGJKS17]: Concurrent MPC in four-round with SPS security.
[ABGKM21] : Two-round MPC with standalone security in the plain model assuming subexponential NIWI arguments, the subexponential SXDH assumption, and the existence of non-interactive NMC
[FJK22]: Concurrent two-round MPC protocol, assuming subexponential quantum hardness of LWE, subexponential classical hardness of SXDH, the existence of a subexponentially-secure (classicallyhard) iO , and time-lock puzzles

## Standard Simulation-Real Ideal Paradigm

## Real

Ideal

## Standard Simulation-Real Ideal Paradigm

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Computationally Indistinguishable

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- The existence of a one-round NMC. Instead, we are able to use the two-round NMCs of [KhuSah17], which is instantiable from sub-exponential LWE.
- The existence of non-interactive witness indistinguishable arguments or time-lock puzzles.


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## The Applications:

1) The first two-round PAKE scheme in the plain model, resolving a longstanding open problem in the area
2) The first concurrent 2PC for quantum functionalities (in the plain model) with classical inputs and outputs

## Observarion: The 2PC Construction

Observarion from [ABGKM21] -The need for Verifiability

$Z=f(x, y)$

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the OT1 message will be used by party $P_{\text {_ }}$ i in reconstructing its own output

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the OT1 message will be used by party P_i in reconstructing its own output


How do we prevent the adversary from learning $f(x, y)$, then?

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Done!

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## Thanks

