

# Registered (Inner-Product) Functional Encryption

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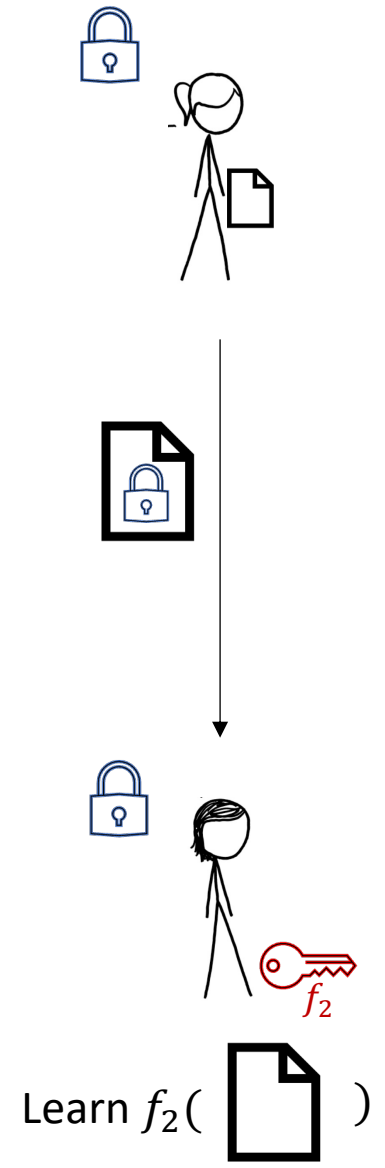
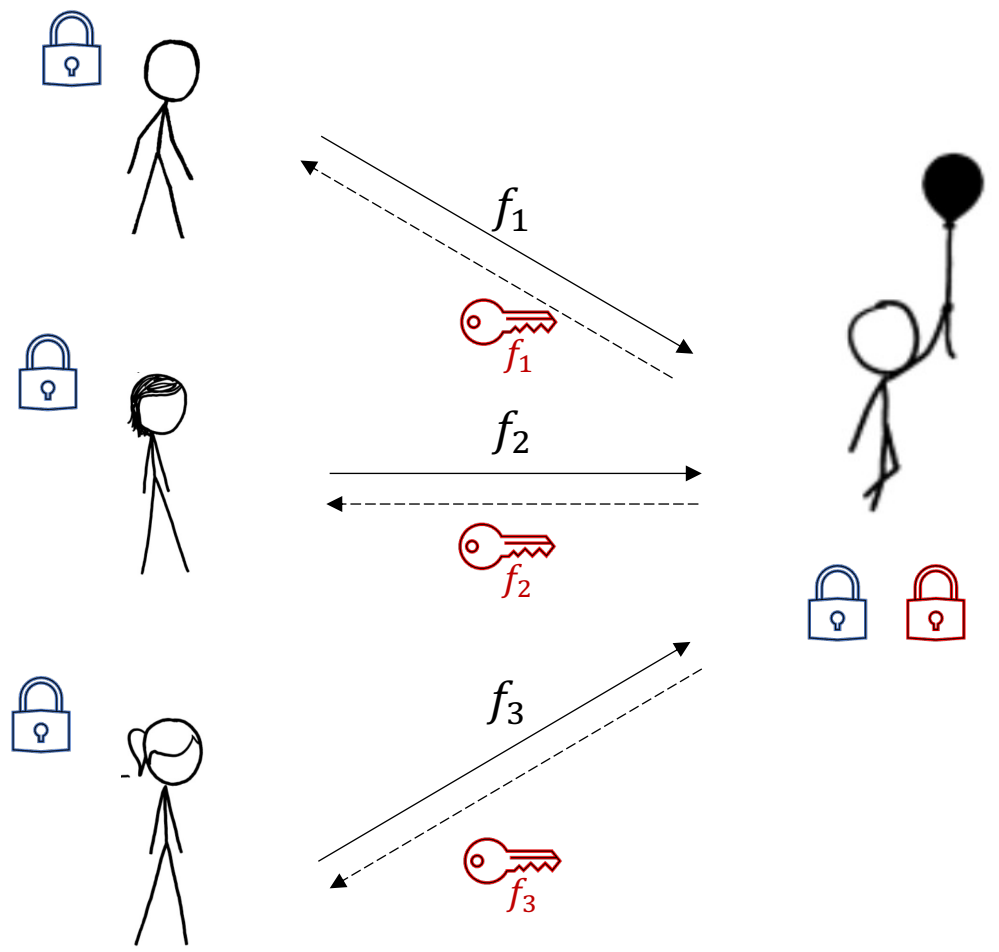
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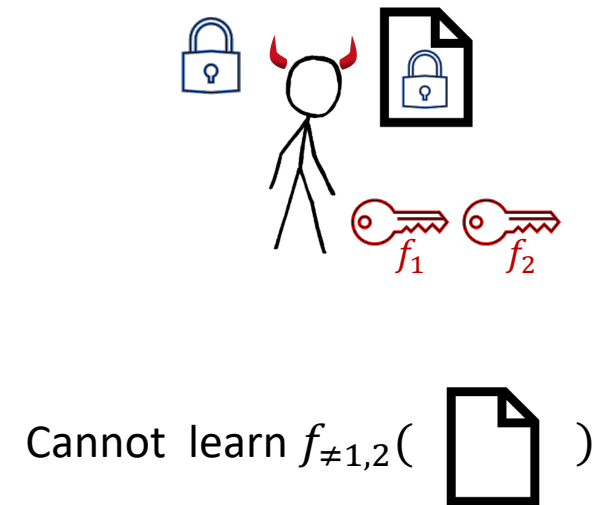
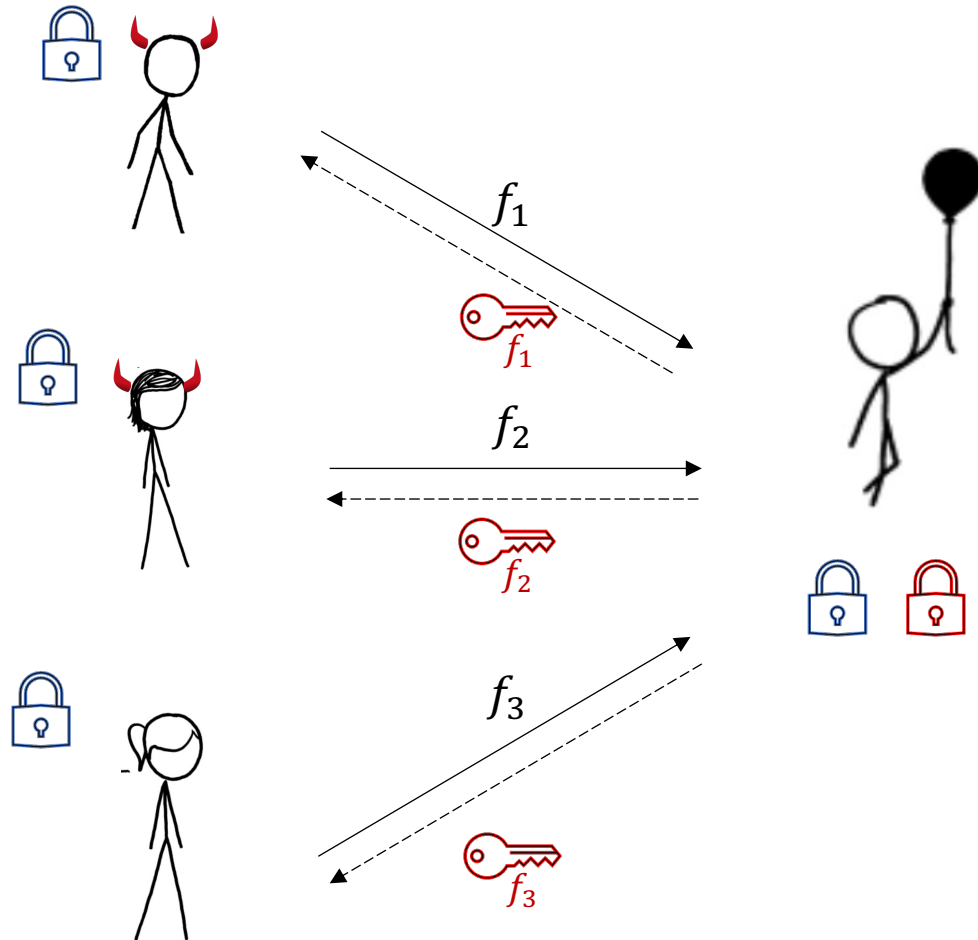
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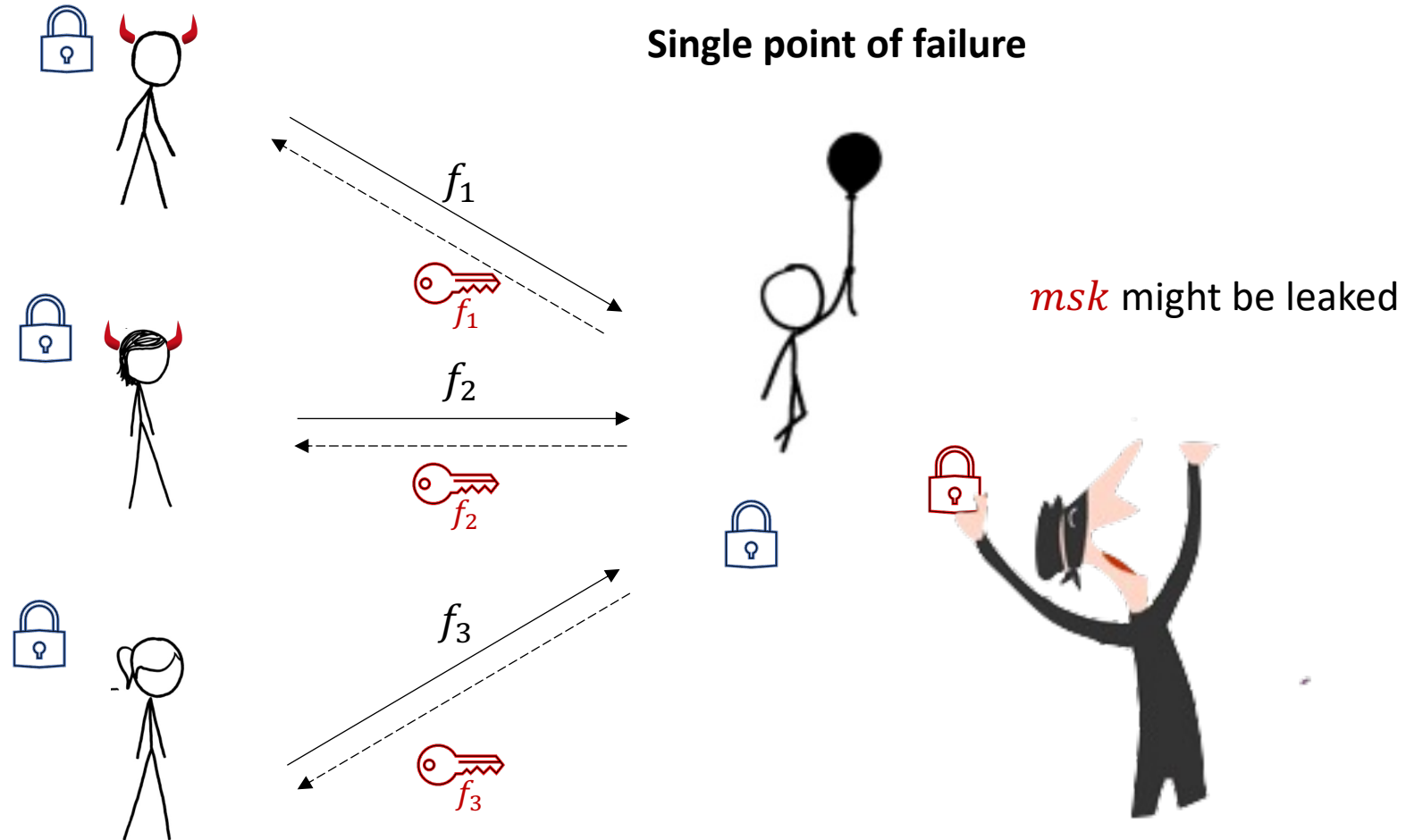
# Functional Encryption



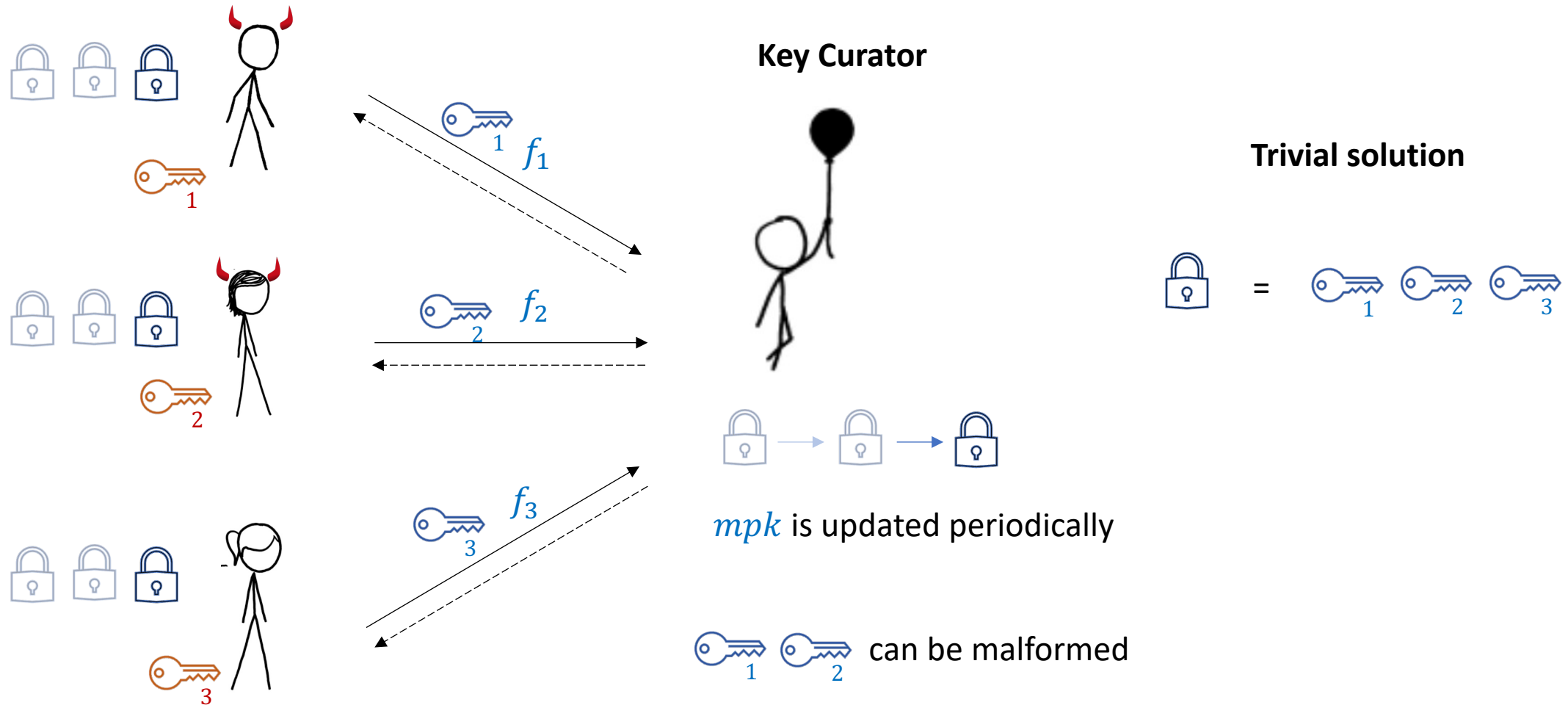
# Functional Encryption



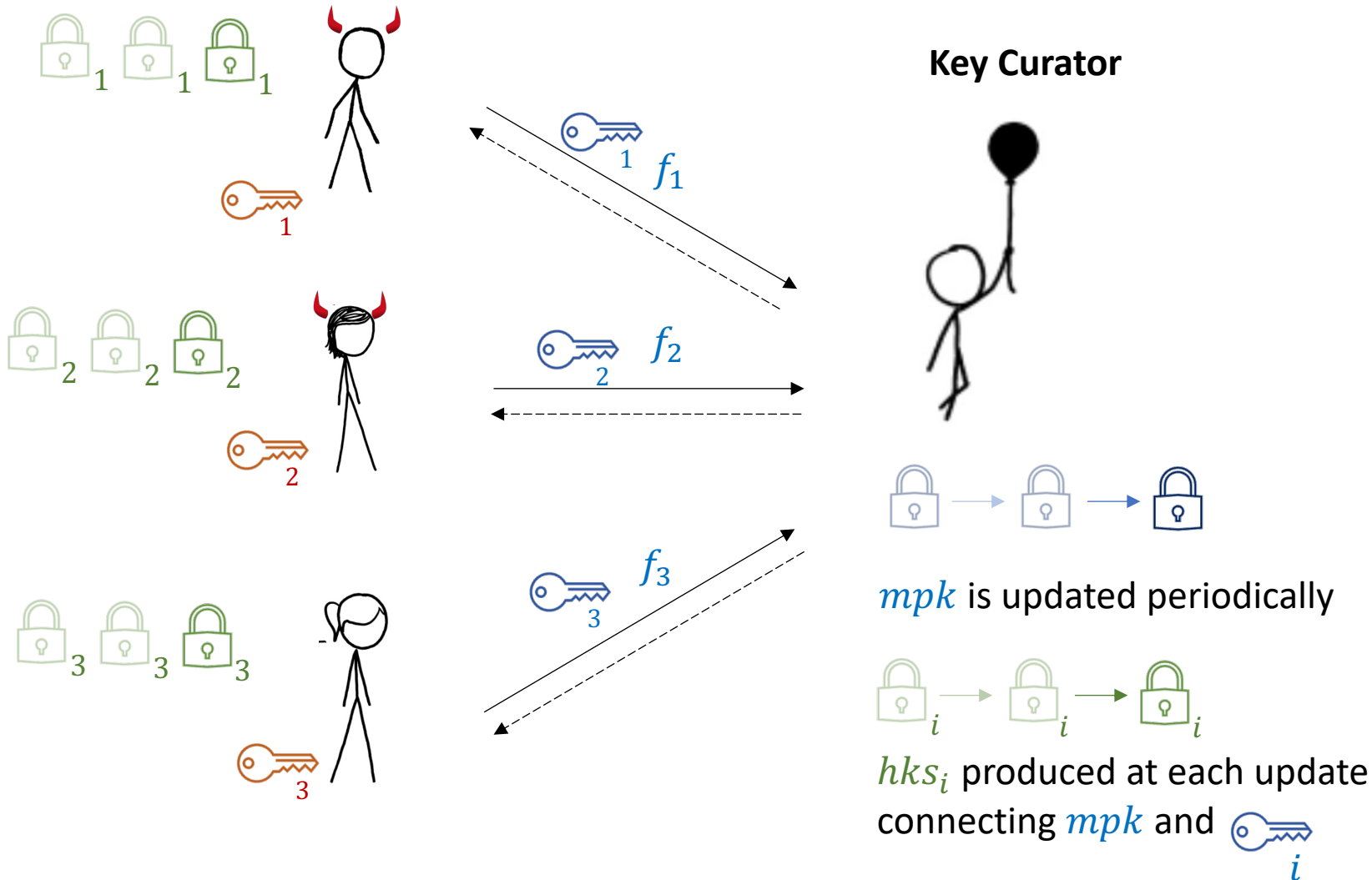
# Key-Escrow Problem



# Registered Functional Encryption



# Registered Functional Encryption



## Requirements:

$L = \#Users$

$|crs|, |hks_i|, |mpk|$   
= **polylog** in  $L$

Keygen and registration  
run in **polylog** time in  $L$

$\#Updates = O(\log L)$

# Our Contributions

- *Registered (attribute-hiding) Inner Product Encryption*  
from prime order groups in the bilinear GGM. Recasted in RFE as:

$$f_{\mathbf{x}}(m, \mathbf{y}) = \begin{cases} m & \text{if } \langle \mathbf{x}, \mathbf{y} \rangle = 0 \\ \perp & \text{otherwise} \end{cases}$$

- *Registered Functional Encryption*  
from iO and SSB hash functions for generic functionalities and large universe of users

**Similar blueprint of [HLWW22] Registered Attribute-Based Encryption**

# Comparison with [HLWW22] RABE:

- *Our RIPE*

PROS:

- Large function space:  
n-size vectors
- Strong attribute-hiding:
  - CPA-2-sided security

CONS:

- Inner-Product
- Pairings of prime order +  
GGM

- *[HLWW22] RABE*

CONS:

- Small attribute space
- Attributes in clear

PROS:

- LSSS policies
- Pairings of composite order

Both CONS:

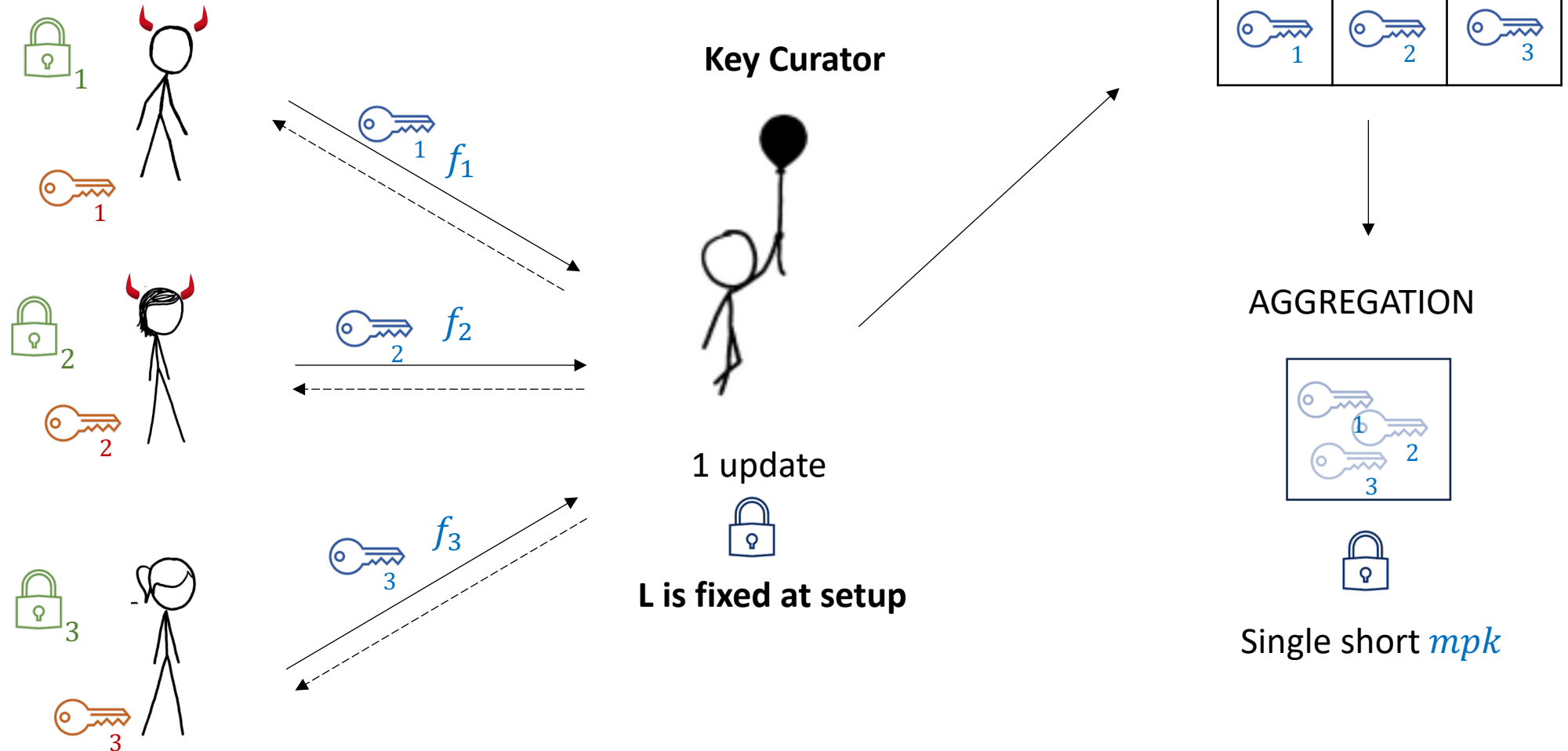
- Require a bounded number of users
- CRS, Kgen and registration runtime dependent on L



# Comparison with [HLWW22] RABE:

Reference	Type	CRS size	Keygen runtime	Registration key runtime	Master public key size	Helper dec. key size	# Updates	Unbounded users	BB	Assumptions
[GHMR18]	IBE	$O(1)$	$O(1)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$O(\log L)$	✓	✗	iO + SSB
[GHMR18]	IBE	$O(1)$	$O(1)$	$O(L)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$O(\log L)$	✓	✗	CDH/LWE
[GHM <sup>+</sup> 19]	Anon. IBE	$O(1)$	$O(1)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$O(\log L)$	✓	✗	CDH/LWE
[GV20]	IBE	$O(1)$	$O(1)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$O(\log L)$	✓	✗	CDH/LWE
[CES21]	IBE	$O(1)$	$O(1)$	$\text{poly}(\log L)$	$O(\sqrt{L})$	$\text{poly}(\log L)$	$O(\log L)$	✓	✗	CDH/LWE
[GKMR22]	IBE $O(1)$ -size ciphertexts	$O(\sqrt{L})$	$O(\sqrt{L})$	$O(\sqrt{L})$	$O(\sqrt{L})$	$O(\sqrt{L})$	$O(\sqrt{L})$	✗	✓	Pairings of Prime Order
[GKMR22]	IBE $O(\log L)$ -size ciphertexts	$O(\sqrt{L})$	$O(\sqrt{L})$	$O(\sqrt{L} \log L)$	$O(\sqrt{L} \log L)$	$O(\log L)$	$O(\log L)$	✗	✓	Pairings of Prime Order
[DKL <sup>+</sup> 23]	IBE	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$O(L)$	$\text{poly}(\log L)$	$\text{poly}(\log L)$	$O(\log L)$	✓	✓	LWE
[HLWW22]	ABE small attribute space $\mathcal{U}$ LSSS policies	$L^2 \cdot \text{poly}( \mathcal{U} , \log L)$	$L \cdot \text{poly}( \mathcal{U} , \log L)$	$L \cdot \text{poly}( \mathcal{U} , \log L)$	$ \mathcal{U}  \cdot \text{poly}(\log L)$	$ \mathcal{U}  \cdot \text{poly}(\log L)$	$O(\log L)$	✗	✓	Pairings of Composite Order
[HLWW22]	ABE large attribute space $\mathcal{U}$ arbitrary policies	$O(1)$	$O(1)$	$O(L)$	$O(1)$	$O(1)$	$O(\log L)$	✓	✗	iO + SSB
Ours §6	Inner-Product PE large function space $\mathcal{F}$ $n$ -size vectors	$n \cdot L^2 \cdot \text{poly}(\log L)$	$L \cdot \text{poly}(\log L)$	$n \cdot L^2 \cdot \text{poly}(\log L)$	$n \cdot \text{poly}(\log L)$	$n \cdot \text{poly}(\log L)$	$O(\log L)$	✗	✓	Pairings of Prime Order + GGM
Ours §B	FE large function space $\mathcal{F}$ arbitrary functions	$O(1)$	$O(1)$	$O(L)$	$O(1)$	$O(1)$	$O(\log L)$	✓	✗	iO + SSB

# Slotted RFE



Slightly modified compiler of [HLWW22] to make L independent with log updates

# Slotted RIPE (Single slot)

## CRS

- Prime order  $q$ :

$$\mathcal{G} = \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, q, g_1, g_2, e$$

- General params:

$$h = g_1^\beta \quad Z = e(g_1, g_2)^\alpha$$

- Slot-specific:

$$A = g_2^t \quad B = g_2^\alpha A^\beta = g^{\alpha+\beta t}$$

- Key-specific:

$$U_w = g_1^{u_w} \text{ for each } w \in [n + 1]$$

## Key registration




  $pk = U_{n+1}^{-sk} \mathbf{x} = (x_1, \dots, x_n)$

---

## Key aggregation:

$$pk \cdot \prod_{w=1}^n U_w^{-x_w}$$

# Slotted RIPE (Single slot)



$$\begin{aligned}
 & \mathcal{G} \quad Z = e(g_1, g_2)^\alpha \\
 & U_w = g_1^{u_w} \quad h = g_1^\beta \quad pk \cdot \prod_{w=1}^n U_w^{-x_w} \quad \text{lock icon} \quad A = g_2^t \quad B = g_2^\alpha A^\beta = g^{\alpha+\beta t}
 \end{aligned}$$

**Enc** ( $m, \mathbf{y} = (y_1, \dots, y_n)$ ):

$$C_1 = m \cdot Z^s$$

$$C_2 = g_1^s$$

$$C_{3,w} = h^{y_w \cdot r + s} \cdot U_w^{-z}, \forall w \in [n]$$

$$C_{3,n+1} = h^s \cdot U_{n+1}^{-z}$$


$$C_{3,n+2} = h^s \cdot \left( pk \prod_{w=1}^n U_w^{-x_w} \right)$$

**Dec** ( $C_1, C_2, C_{3,w}, C_{3,n+1}, sk, \mathbf{x}$ ):


$$m = \frac{C_1}{e(C_2, B)} \cdot e(h^s, A)$$

$$h^s = \left( C_{3,n+2} \cdot C_{3,n+1}^{sk} \cdot \prod_{w=1}^n C_{3,w}^{x_w} \right)^{(1+sk+\sum x_w)^{-1}}$$

# Slotted RIPE (Single slot)



$$Z = e(g_1, g_2)^\alpha$$

$$U_w = g_1^{u_w} \quad h = g_1^\beta \quad pk \cdot \prod_{w=1}^n U_w^{-x_w}$$


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$$C_{3,n+1} = h^s \cdot U_{n+1}^{-z}$$

$$C_{3,n+2} = h^s \cdot (pk \prod_{w=1}^n U_w^{-x_w})$$

**Dec** ( $C_1, C_2, C_{3,w}, C_{3,n+1}, sk, \mathbf{x}$ ):


$$m = \frac{C_1}{e(C_2, B)} \cdot e(\boxed{h^s}, A)$$

$$\prod_{w=1}^n C_{3,w}^{x_w} = \prod_{w=1}^n h^{x_w (y_w \cdot r)} \quad \prod_{w=1}^n h^{x_w s} \quad \prod_{w=1}^n U_w^{-z \cdot x_w}$$


$$C_{3,n+1}^{sk} = h^{s \cdot sk} \quad U_{n+1}^{-z \cdot sk}$$

$$C_{3,n+2} = h^s \quad U_{n+1}^{z \cdot sk} \quad \prod_{w=1}^n U_w^{z \cdot x_w}$$

# Slotted RIPE (Single slot)



$$Z = e(g_1, g_2)^\alpha$$

$$U_w = g_1^{u_w} \quad h = g_1^\beta \quad pk \cdot \prod_{w=1}^n U_w^{-x_w}$$


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$$m = \frac{C_1}{e(C_2, B)} \cdot e(h^s, A)$$

$$\prod_{w=1}^n C_{3,w}^{x_w} = \prod_{w=1}^n h^{x_w (y_w \cdot r)}$$

$$C_{3,n+1}^{sk} = h^{s \cdot sk}$$

$$C_{3,n+2} = h^s$$


$$U_{n+1}^{-z \cdot sk}$$

$$U_{n+1}^{z \cdot sk}$$


$$\prod_{w=1}^n h^{x_w s} \quad \prod_{w=1}^n U_w^{-z \cdot x_w}$$

$$\prod_{w=1}^n U_w^{z \cdot x_w}$$

# Slotted RIPE (Single slot)



$$Z = e(g_1, g_2)^\alpha$$

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$$m = \frac{C_1}{e(C_2, B)} \cdot e(h^s, A)$$

$$\prod_{w=1}^n C_{3,w}^{x_w} = \prod_{w=1}^n h^{x_w (y_w \cdot r)}$$

$$C_{3,n+1}^{sk} = h^{s \cdot sk}$$


$$C_{3,n+2} = h^s$$

$$\frac{U_{n+1}^{-z \cdot sk}}{U_{n+1}^{z \cdot sk}}$$


$$\prod_{w=1}^n h^{x_w s}$$

$$\frac{\prod_{w=1}^n U_w^{-z \cdot x_w}}{\prod_{w=1}^n U_w^{z \cdot x_w}}$$

# Slotted RIPE (Single slot)



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$$C_{3,n+1}^{sk} = h^{s \cdot sk} \cdot U_{n+1}^{-z \cdot sk}$$

$$C_{3,n+2} = h^s \cdot U_{n+1}^{z \cdot sk}$$

$$\prod_{w=1}^n h^{x_w s}$$


$$\prod_{w=1}^n U_w^{-z \cdot x_w}$$

$$U_{n+1}^{z \cdot sk}$$


$$\prod_{w=1}^n U_w^{z \cdot x_w}$$



# Slotted RIPE (Single slot)



$$Z = e(g_1, g_2)^\alpha$$

$$U_w = g_1^{u_w} \quad h = g_1^\beta \quad pk \cdot \prod_{w=1}^n U_w^{-x_w}$$


$$A = g_2^t \quad B = g_2^\alpha A^\beta = g^{\alpha+\beta t}$$

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$$C_{3,n+2} = h^s \cdot \left( pk \prod_{w=1}^n U_w^{-x_w} \right)^{-z}$$

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$$m = \frac{C_1}{e(C_2, B)} \cdot e(h^s, A)$$

$$\prod_{w=1}^n C_{3,w}^{x_w} = \prod_{w=1}^n h^{x_w (y_w \cdot r)}$$


$$C_{3,n+1}^{sk} = h^{s \cdot sk} \cdot U_{n+1}^{-z \cdot sk}$$

$$C_{3,n+2} = h^s \cdot U_{n+1}^{z \cdot sk}$$


$$\prod_{w=1}^n h^{x_w s}$$

$$\prod_{w=1}^n U_w^{-z \cdot x_w} \cdot \prod_{w=1}^n U_w^{z \cdot x_w}$$

# Slotted RIPE (Single slot)



$$Z = e(g_1, g_2)^\alpha$$

$$U_w = g_1^{u_w} \quad h = g_1^\beta \quad pk \cdot \prod_{w=1}^n U_w^{-x_w}$$


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$$m = \frac{C_1}{e(C_2, B)} \cdot e(h^s, A)$$

$$\prod_{w=1}^n C_{3,w}^{x_w} = \prod_{w=1}^n h^{x_w (y_w \cdot r)}$$

$$\prod_{w=1}^n \boxed{h^{x_w s}} \prod_{w=1}^n U_w^{-z \cdot x_w}$$

$$C_{3,n+1}^{sk} = \boxed{h^{s \cdot sk}} U_{n+1}^{-z \cdot sk}$$

$$C_{3,n+2} = h^s U_{n+1}^{z \cdot sk}$$

$$\prod_{w=1}^n U_w^{z \cdot x_w}$$

# Slotted RIPE (2 slots) IDEA:

## CRS Generation:

$A_1, B_1$

$A_2, B_2$


$\{U_{w,1}\}$

$\{U_{w,2}\}$

$\{U_{w,1} \cdot U_{w,2}\}$

## Key Generation:

  $pk_1 = U_{n+1}^{-sk_1}$        $\mathbf{x}_1 = (x_{1,1}, \dots, x_{n,1})$

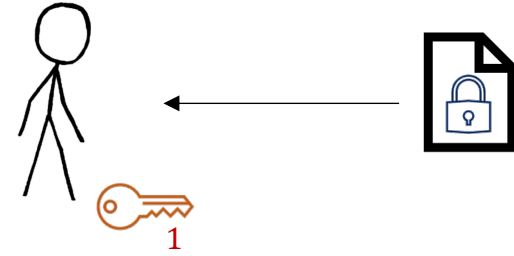
  $pk_2 = U_{n+1}^{-sk_2}$        $\mathbf{x}_2 = (x_{1,2}, \dots, x_{n,2})$

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## Key Aggregation:

$$pk_1 \cdot pk_2 \cdot \prod U_{w,1}^{x_{w,1}} \prod U_{w,2}^{x_{w,2}}$$

# Slotted RIPE (2 slots)



$$\prod_{w \in [n]} C_{3,w}^{x_{w,1}} = \prod_{w \in [n]} h^{(y_w \cdot r + s) \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,1}^{-z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{-z \cdot x_{w,1}}$$

$$C_{3,n+1}^{\text{sk}_1} = h^{s \cdot \text{sk}_1} \cdot U_{n+1,1}^{-z \cdot \text{sk}_1} \cdot U_{n+1,2}^{-z \cdot \text{sk}_1}$$

$$C_{3,n+2} = h^s \cdot U_{n+1,1}^{z \cdot \text{sk}_1} \cdot U_{n+1,2}^{z \cdot \text{sk}_2} \cdot \prod_{w=1}^n U_{w,1}^{z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{z \cdot x_{w,2}}$$

# Slotted RIPE (2 slots)

## CRS Generation:

$$A_1, B_1 \quad A_2, B_2$$

$$\{W_{1,2,w} = A_1^{u_{2,w}}\} \quad \{W_{2,1,w} = A_2^{u_{1,w}}\}$$

$$\{U_{w,1} = g^{u_{1,w}}\} \quad \{U_{w,2} = g^{u_{2,w}}\}$$

$$\{U_{w,1} \cdot U_{w,2}\}$$

## Key Generation:

$$\text{key}_1 \quad pk_1 = U_{n+1}^{-sk_1} \quad x_1 = (x_{1,1}, \dots, x_{n,1})$$

$$\{W_{2,1,w}^{sk_1}\}$$

$$\text{key}_2 \quad pk_2 = U_{n+1}^{-sk_2} \quad x_2 = (x_{1,2}, \dots, x_{n,2})$$

$$\{W_{1,2,w}^{sk_2}\}$$

## Key Aggregation:

$$\text{lock} \quad pk_1 \cdot pk_2 \cdot \prod U_{w,1}^{x_{w,1}} \prod U_{w,2}^{x_{w,2}}$$



$$W_{2,1,n+1}^{sk_1} \cdot W_{1,2,n+1}^{sk_2}$$

# Slotted RIPE (2 slots)

$$\begin{array}{c}
 \text{lock} \\
 \{U_{w,1} \cdot U_{w,2}\} \\
 Z
 \end{array}
 \begin{array}{c}
 h \\
 pk_1 \cdot pk_2 \cdot \prod U_{w,1}^{x_{w,1}} \prod U_{w,2}^{x_{w,2}} \\
 Z
 \end{array}
 \begin{array}{c}
 A_1, B_1 \\
 \text{lock}_1 \\
 W_{2,1,n+1}^{sk_1} \cdot W_{1,2,n+1}^{sk_2}
 \end{array}$$

$$\prod_{w \in [n]} C_{3,w}^{x_{w,1}} = \prod_{w \in [n]} h^{(y_w \cdot r + s) \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,1}^{-z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{-z \cdot x_{w,1}}$$

$$C_{3,n+1}^{sk_1} = h^{s \cdot sk_1} \cdot U_{n+1,1}^{-z \cdot sk_1} \cdot U_{n+1,2}^{-z \cdot sk_1}$$

$$C_{3,n+2} = h^s \cdot U_{n+1,1}^{z \cdot sk_1} \cdot U_{n+1,2}^{z \cdot sk_2} \cdot \prod_{w=1}^n U_{w,1}^{z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{z \cdot x_{w,2}}$$

$$C_4 = g_1^z$$

# Slotted RIPE (2 slots)



$$\left\{ W_{1,2,w} = A_1^{u_{2,w}/\gamma} \right\} \quad \left\{ W_{2,1,w} = A_2^{u_{1,w}/\gamma} \right\}$$



$$\{U_{w,1} \cdot U_{w,2}\}$$

$h$

$$pk_1 \cdot pk_2 \cdot \prod U_{w,1}^{x_{w,1}} \prod U_{w,2}^{x_{w,2}}$$



$A_1, B_1$

$$\Gamma = g_1^\gamma$$

$Z$

$$W_{2,1,n+1}^{sk_1} \cdot W_{1,2,n+1}^{sk_2}$$

$$\prod_{w \in [n]} C_{3,w}^{x_{w,1}} = \prod_{w \in [n]} h^{(y_w \cdot r + s) \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,1}^{-z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{-z \cdot x_{w,1}}$$

$$C_{3,n+1}^{sk_1} = h^{s \cdot sk_1} \cdot U_{n+1,1}^{-z \cdot sk_1} \cdot U_{n+1,2}^{-z \cdot sk_1}$$

$$C_{3,n+2} = h^s \cdot U_{n+1,1}^{z \cdot sk_1} \cdot U_{n+1,2}^{z \cdot sk_2} \cdot \prod_{w=1}^n U_{w,1}^{z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{z \cdot x_{w,2}}$$

$$C_4 = \Gamma^z$$

# Slotted RIPE (2 slots)



$$\{W_{1,2,w} = A_1^{u_{2,w}/\gamma}\} \quad \{W_{2,1,w} = A_2^{u_{1,w}/\gamma}\}$$



$$\{U_{w,1} \cdot U_{w,2}\}$$

$h$

$$pk_1 \cdot pk_2 \cdot \prod U_{w,1}^{x_{w,1}} \prod U_{w,2}^{x_{w,2}}$$



$A_1, B_1$

$$\Gamma = g_1^\gamma$$

$Z$

$$W_{2,1,n+1}^{sk_1} \cdot W_{1,2,n+1}^{sk_2}$$

$$\prod_{w \in [n]} C_{3,w}^{x_{w,1}} = \prod_{w \in [n]} h^{(y_w \cdot r + s) \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,1}^{-z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{-z \cdot x_{w,1}}$$

$$C_{3,n+1}^{sk_1} = h^{s \cdot sk_1} \cdot U_{n+1,1}^{-z \cdot sk_1} \cdot U_{n+1,2}^{-z \cdot sk_1}$$

$$C_{3,n+2} = h^s \cdot U_{n+1,1}^{z \cdot sk_1} \cdot U_{n+1,2}^{-z \cdot sk_2} \cdot \prod_{w=1}^n U_{w,1}^{z \cdot x_{w,1}} \cdot \prod_{w=1}^n U_{w,2}^{z \cdot x_{w,2}}$$

$$C_4 = \Gamma^z$$



# Conclusions

- Registered RFE Definition
- Registered IPE from pairings in the GGM
- Registered RFE for P/poly and unbounded users from iO and SSB hash functions
- Open problems
  - RFE from any compact and polynomially-hard FE
  - RFE for specialized function classes from weaker assumptions
  - Prove our pairing-based RIPE in the standard model



Thank you for your attention!

<https://ia.cr/2023/395>