



# Cuckoo Commitments: Registration-Based Encryption and Key-Value Map Commitments for Large Spaces

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DIENS, CNRS, Inria, université PSL  
& Cosmian



# Outline

*Motivation: Registration-Based Encryption*

*Our Contributions*

*Our RBE compiler*

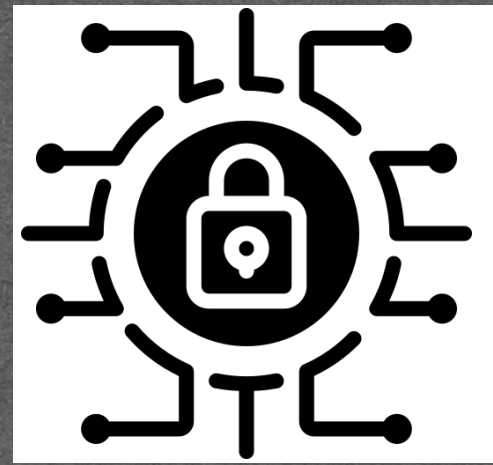
*Our KVC compiler*

*Conclusions and Open Problems*

*Motivation: Registration-Based Encryption*

# Identity-Based Encryption [Sha84], [BF01]

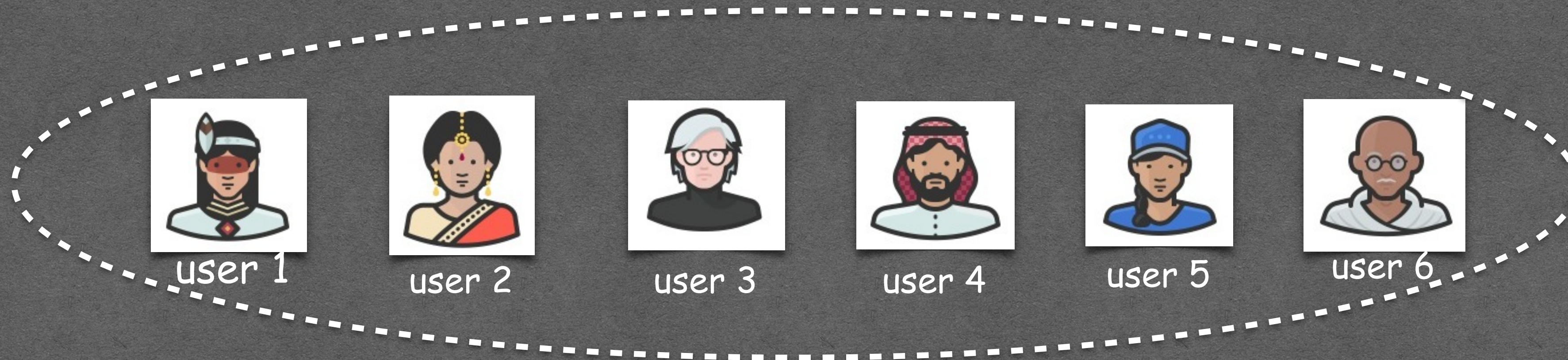
Private Key Generator



Encryptor

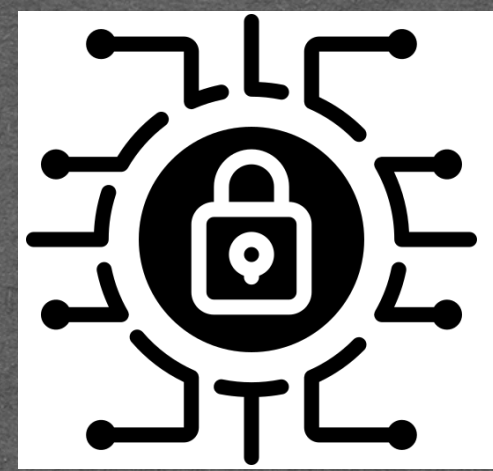


Decryptors



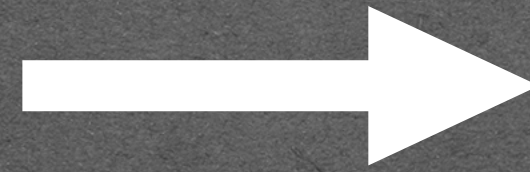
# Identity-Based Encryption [Sha84], [BF01]

## Private Key Generator



1

$\text{Setup}(1^\lambda, n) \rightarrow (\text{msk}, \text{mpk})$

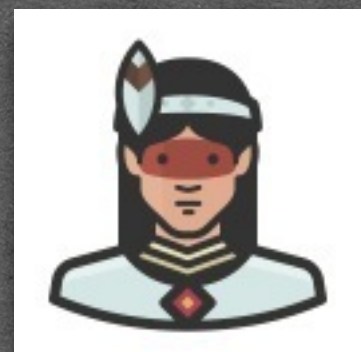


mpk

## Encryptor



## Decryptors



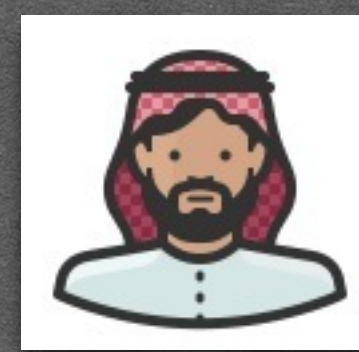
user 1



user 2



user 3



user 4



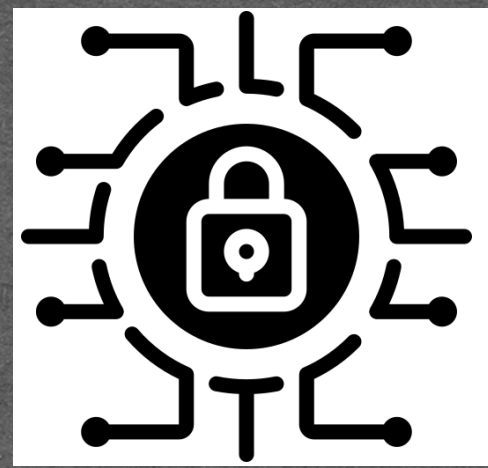
user 5



user 6

# Identity-Based Encryption [Sha84], [BF01]

## Private Key Generator



1  $\text{Setup}(1^\lambda, n) \rightarrow (\text{msk}, \text{mpk})$

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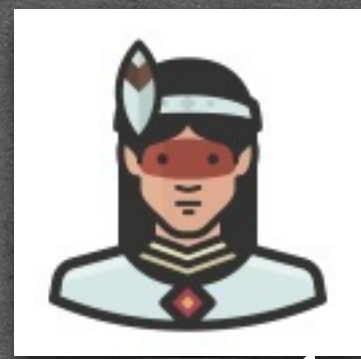
## Encryptor



2  $\text{KeyGen}(\text{msk}, i) \rightarrow \text{sk}_i$

$\text{sk}_1$   $\text{sk}_2$  ...  $\text{sk}_n$

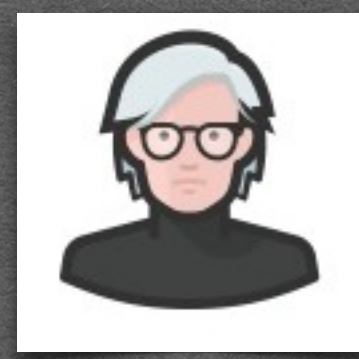
## Decryptors



user 1



user 2



user 3



user 4



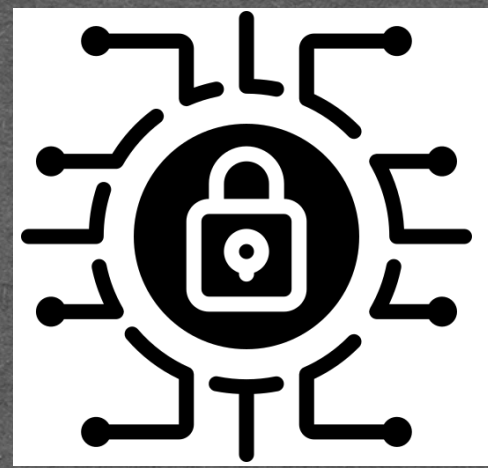
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# Identity-Based Encryption [Sha84], [BF01]

## Private Key Generator



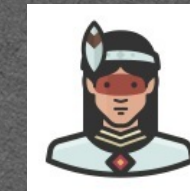
1  $\text{Setup}(1^\lambda, n) \rightarrow (\text{msk}, \text{mpk})$

$\text{mpk}$

## Encryptor



message  $m$ ,  
Identity  $\text{id}$



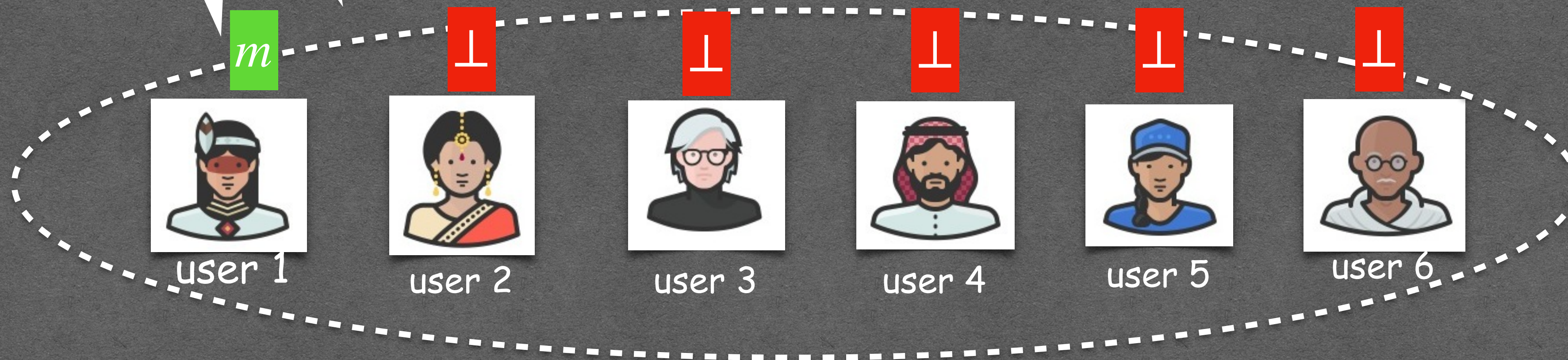
2  $\text{KeyGen}(\text{msk}, i) \rightarrow \text{sk}_i$

$\text{sk}_1$   $\text{sk}_2$  ...  $\text{sk}_n$

3  $\text{Encrypt}(\text{mpk}, \text{id}, m) \rightarrow \text{ct}$

$\text{ct}$

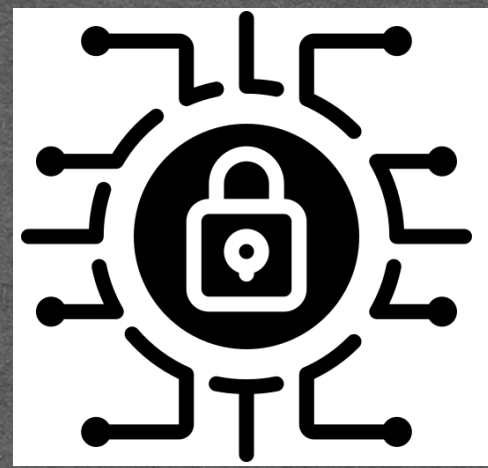
## Decryptors



4  $\text{Decrypt}(\text{sk}_{\text{id}}, \text{id}, \text{ct}) \rightarrow m$

# Identity-Based Encryption [Sha84], [BF01]

## Private Key Generator



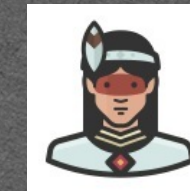
1  $\text{Setup}(1^\lambda, n) \rightarrow (\text{msk}, \text{mpk})$

mpk

## Encryptor



message  $m$ ,  
Identity  $\text{id}$



2  $\text{KeyGen}(\text{msk}, i) \rightarrow \text{sk}_i$

$\text{sk}_1$

$\text{sk}_2$

$\dots \text{sk}_n$

3  $\text{Encrypt}(\text{mpk}, \text{id}, m) \rightarrow \text{ct}$

ct

**Non-triviality:**

$|\text{mpk}| \ll n$

## Decryptors

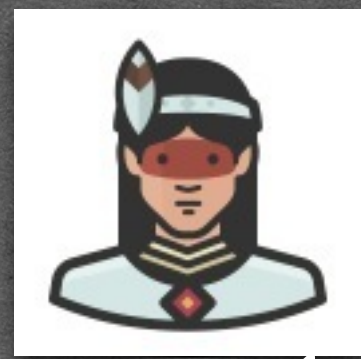
$m$

$\perp$

$\perp$

$\perp$

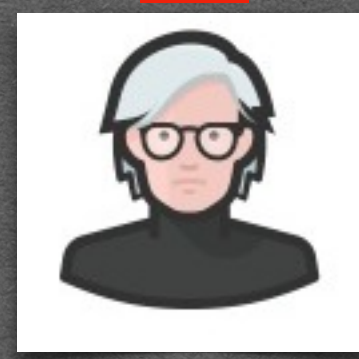
$\perp$



user 1



user 2



user 3



user 4



user 5

**Security:**

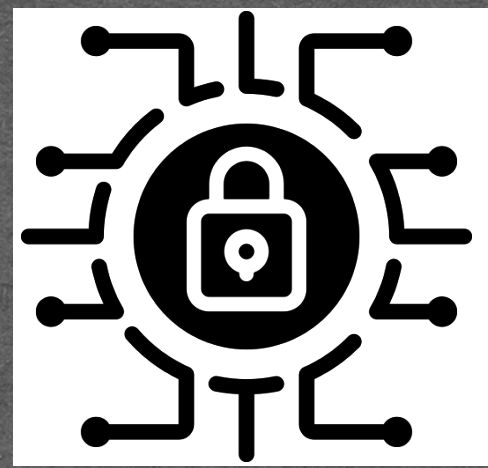
Even if all users  $\neq \text{id}$  collude  
ct is semantically secure

4  $\text{Decrypt}(\text{sk}_{\text{id}}, \text{id}, \text{ct}) \rightarrow m$



# The key-escrow problem

## Private Key Generator



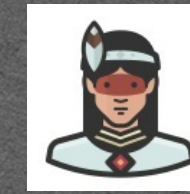
1  $\text{Setup}(1^\lambda, n) \rightarrow (\text{msk}, \text{mpk})$

$\text{mpk}$

## Encryptor



message  $m$ ,  
Identity  $\text{id}$



2  $\text{KeyGen}(\text{msk}, i) \rightarrow \text{sk}_i$

$\text{sk}_1$   $\text{sk}_2$  ...  $\text{sk}_n$

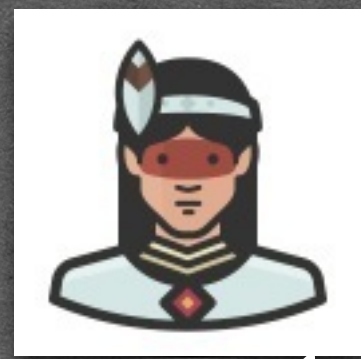
3  $\text{Encrypt}(\text{mpk}, \text{id}, m) \rightarrow \text{ct}$

$\text{ct}$

**Non-triviality:**  
 $|\text{mpk}| \ll n$

## Decryptors

$m$



user 1

$\perp$



user 2

$\perp$



user 3

$\perp$



user 4

$\perp$



user 5

**Security:**  
Even if all users  $\neq \text{id}$  collude  
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4  $\text{Decrypt}(\text{sk}_{\text{id}}, \text{id}, \text{ct}) \rightarrow m$

# The key-escrow problem

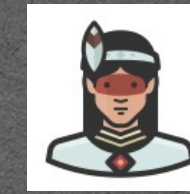
## Private Key Generator



Setup( $1^n$ )

PKG can decrypt all messages!

message  $m$ ,  
Identity  $id$



2 KeyGen( $msk, i$ )  $\rightarrow$   $sk_i$

3 Encrypt( $mpk, id, m$ )  $\rightarrow$   $ct$

**Non-triviality:**  
 $|mpk| \ll n$

$sk_1$   $sk_2$  ...  $sk_n$

$ct$

## Decryptors

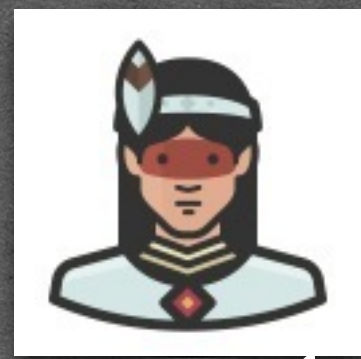
$m$

$\perp$

$\perp$

$\perp$

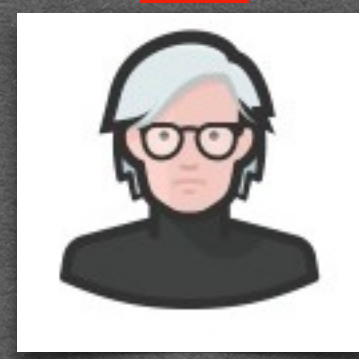
$\perp$



user 1



user 2



user 3



user 4



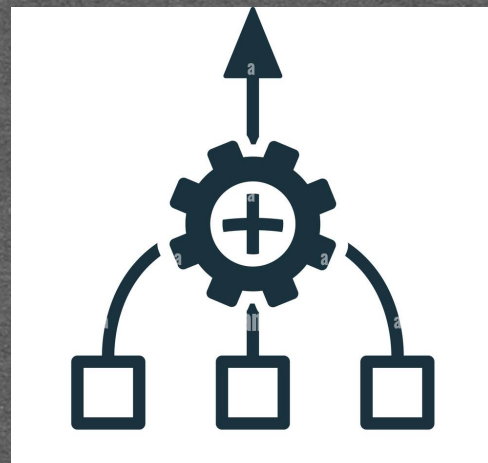
user 5

**Security:**  
Even if all users  $\neq id$  collude  
 $ct$  is semantically secure

4 Decrypt( $sk_{id}, id, ct$ )  $\rightarrow$   $m$

# Registration-Based Encryption [GHMR18]

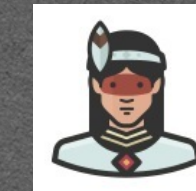
Key Curator



Encryptor



message  $m$ ,  
Identity  $id$



4  $\text{Encrypt}(pp, id, m) \rightarrow ct$

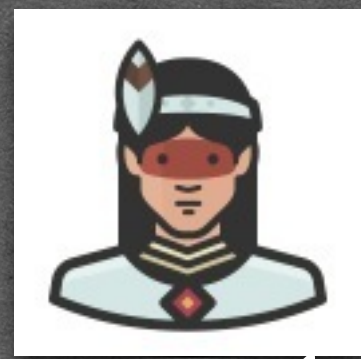
**Non-triviality:**

$$|pp| \ll n$$

$ct$

Decryptors

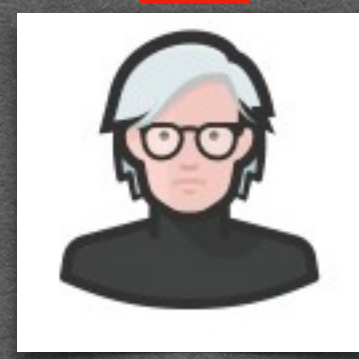
$m$



user 1



user 2



user 3



user 4



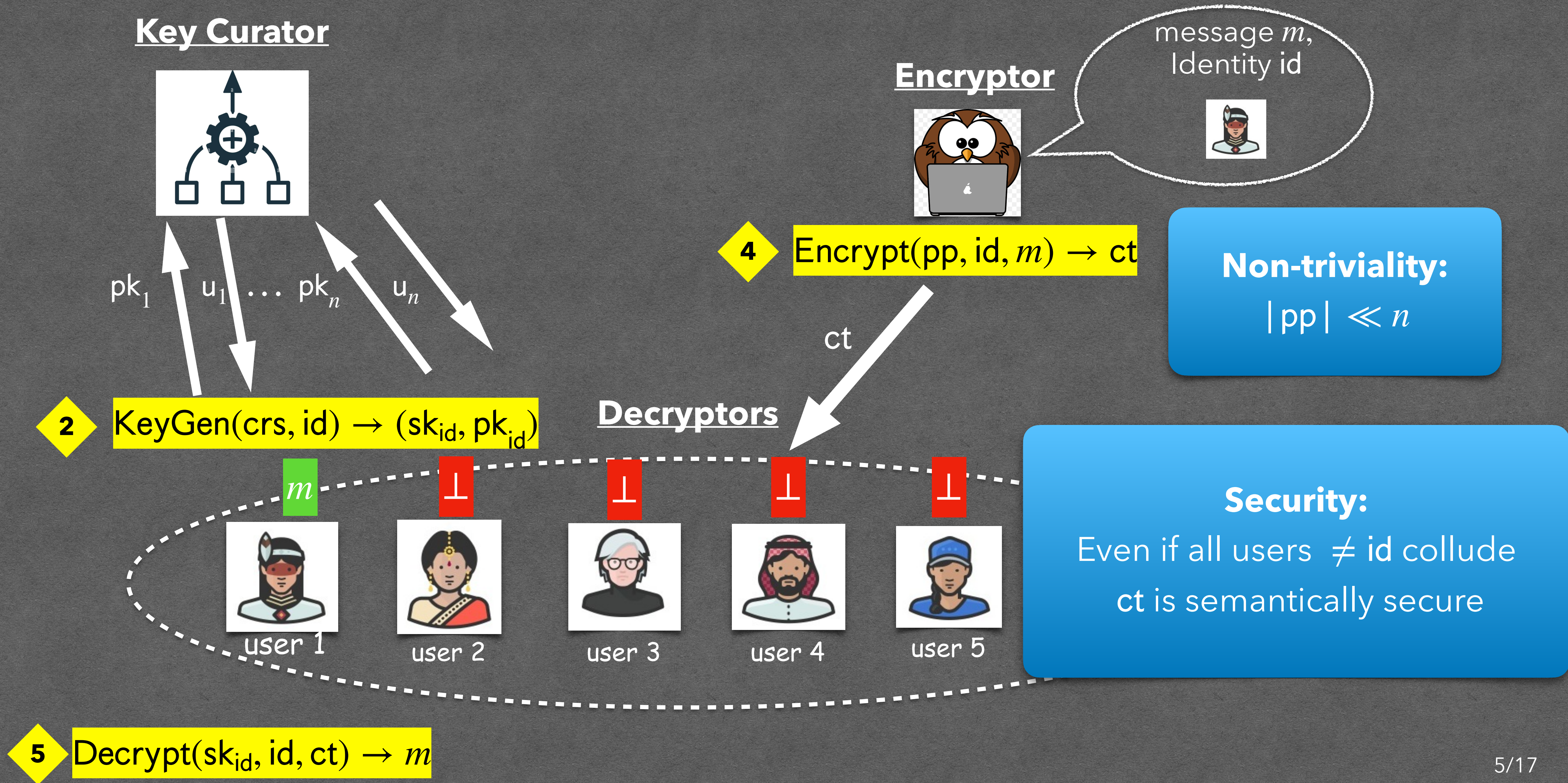
user 5

**Security:**

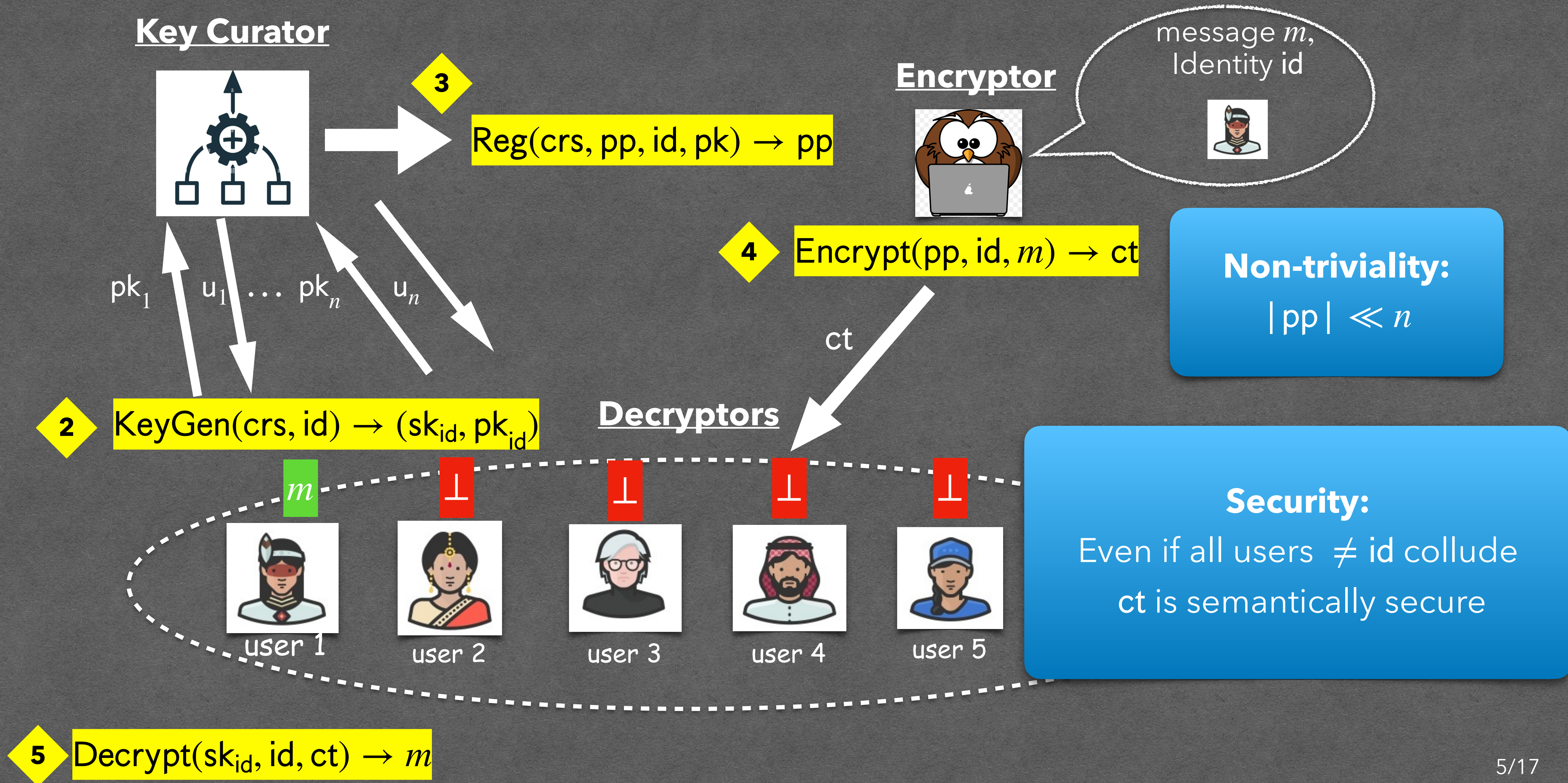
Even if all users  $\neq id$  collude  
 $ct$  is semantically secure

5  $\text{Decrypt}(sk_{id}, id, ct) \rightarrow m$

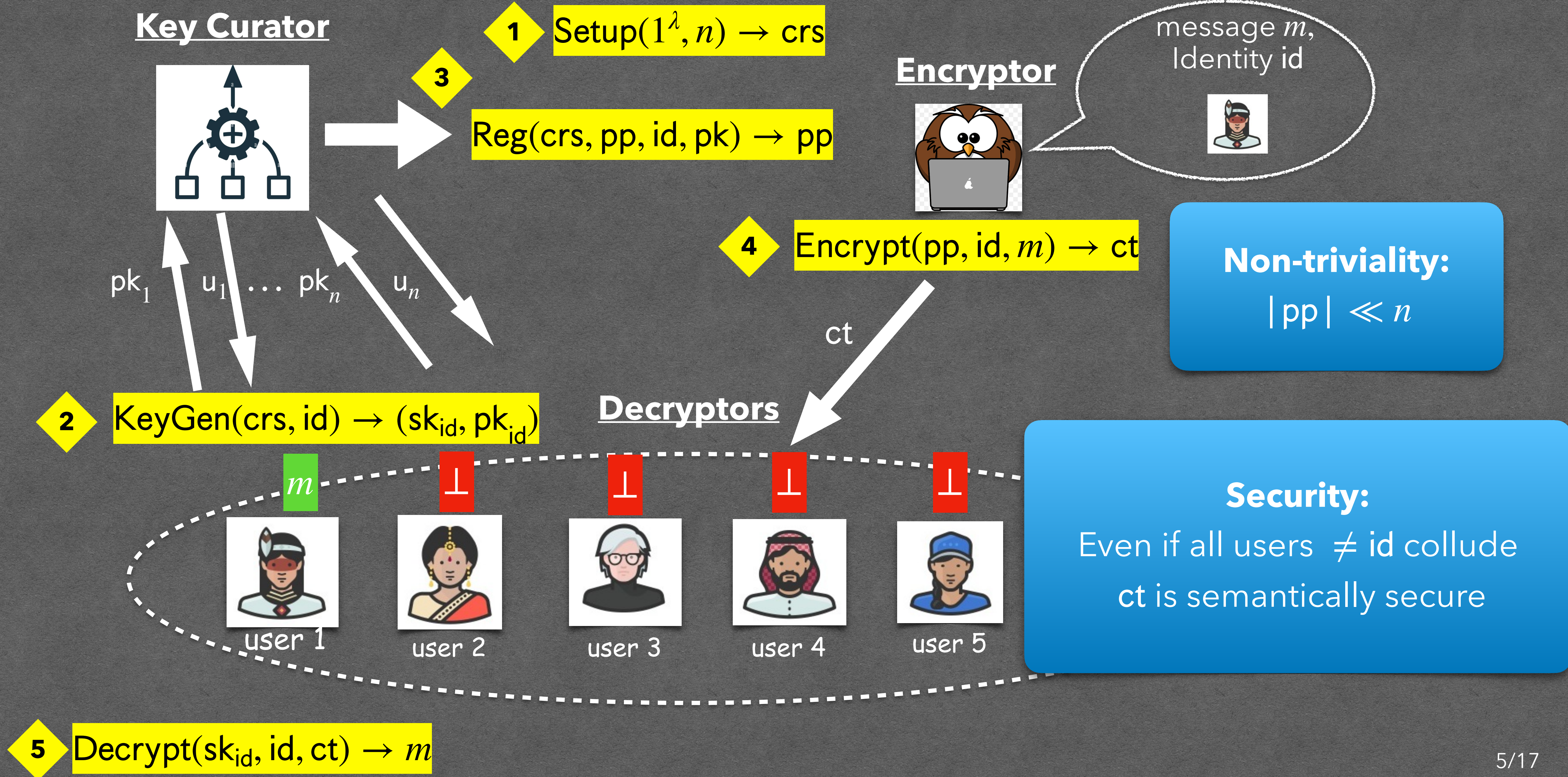
# Registration-Based Encryption [GHMR18]



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# Registration-Based Encryption: Prior constructions

- ❖ Early (non-black box) constructions [GHMR18], [GHMRS19], [GV20], [CES21]

indistinguishability  
Obfuscation

or

Hash-Garbling

[CDGGMP17]  
[DG17]

- ❖ Recent efficient constructions:

	Setting	Efficiency	ID-space
[DKLLMR23]	Lattices	×	Unbounded
[HLWW23]	Pairings (composite)	✓	Unbounded
[GKMR23]	Pairings (prime)	✓✓	poly( $\lambda$ )

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[GKMR23]	Pairings (prime)	✓✓	poly( $\lambda$ )

Our objective: Can we get unbounded identities in [GKMR23]?



# *Our Contributions*

# Our Contributions

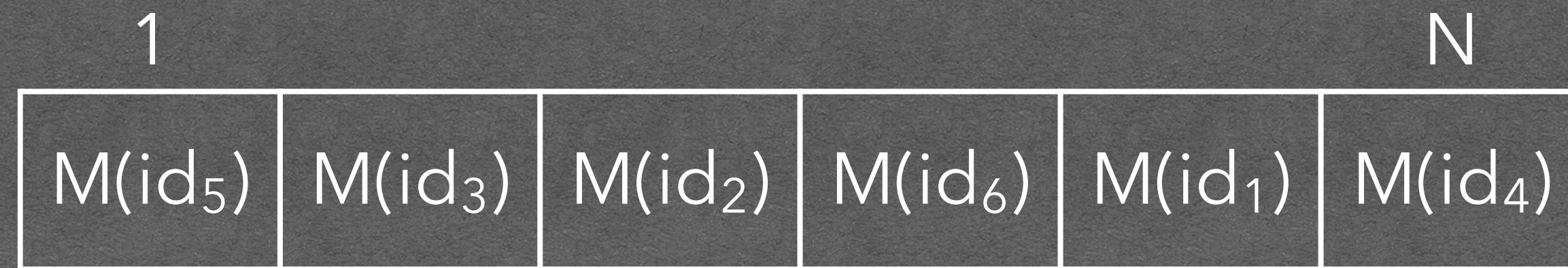
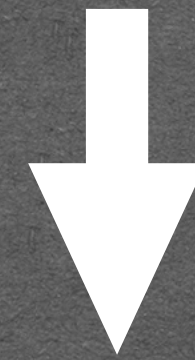
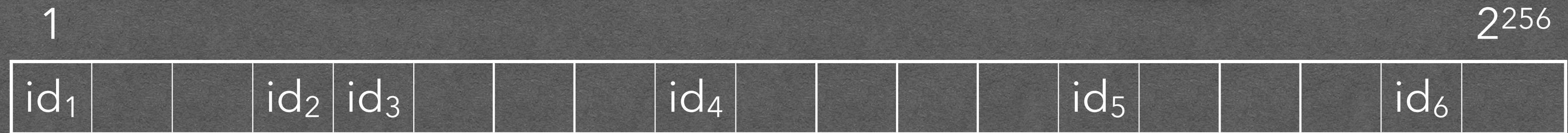
- ★ *Efficient Pairing-based RBE construction with unbounded identities*
- ★ *Generic compiler: Bounded ID-RBE  $\Rightarrow$  Unbounded ID-RBE*
- ★ *Generic compiler: Vector Commitment  $\Rightarrow$  Key-Value Map Commitment*
- ★ *Techniques: Novel use of Cuckoo-Hashing*

*Our RBE compiler*

# Our Core idea

Map the identities to a smaller domain

$$M : \{0,1\}^{2\lambda} \rightarrow [1,n]$$

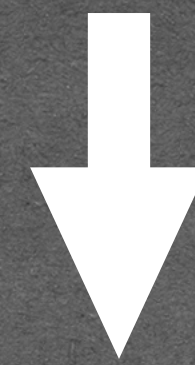


And use an RBE with small identity space

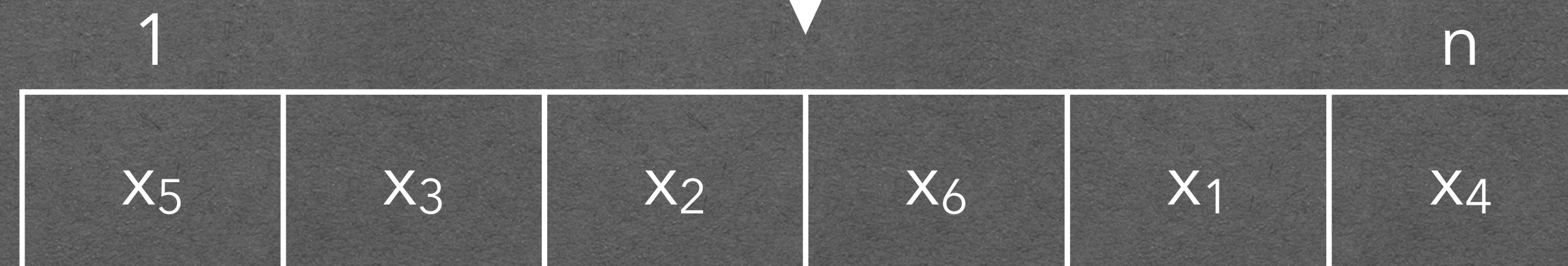
# Cuckoo Hashing [PR04]

## Cuckoo Hashing

Powerful **probabilistic** data structure to map elements from  $[m]$  to elements in  $[n]$ , where  $m > n$  (overcoming collisions)



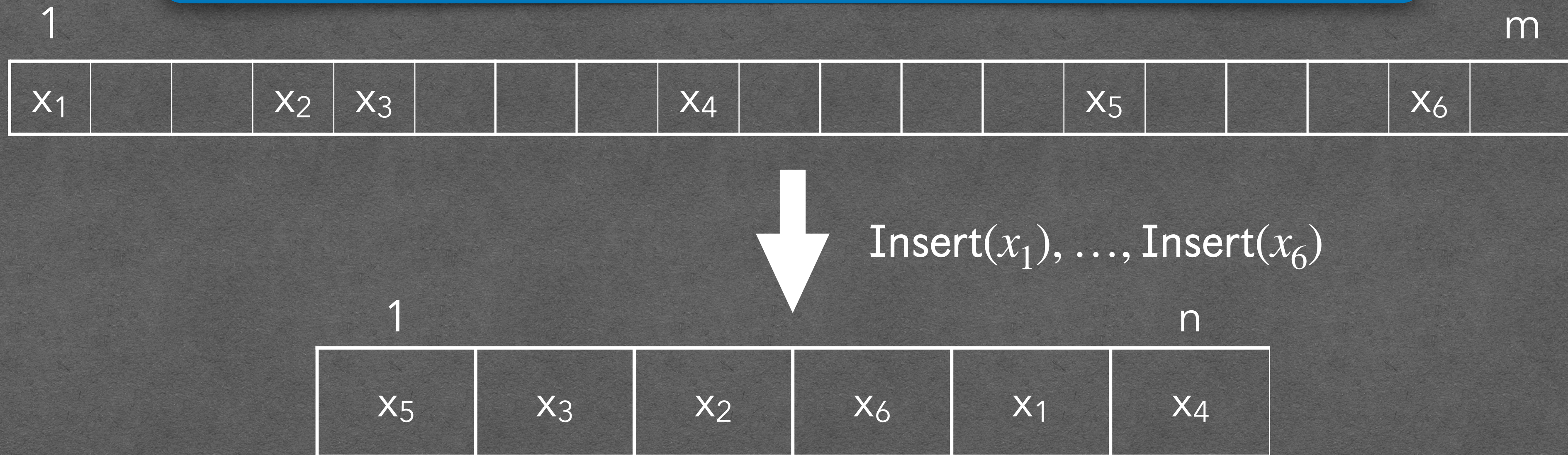
Insert( $x_1$ ), ..., Insert( $x_6$ )



# Cuckoo Hashing [PR04]

## Cuckoo Hashing

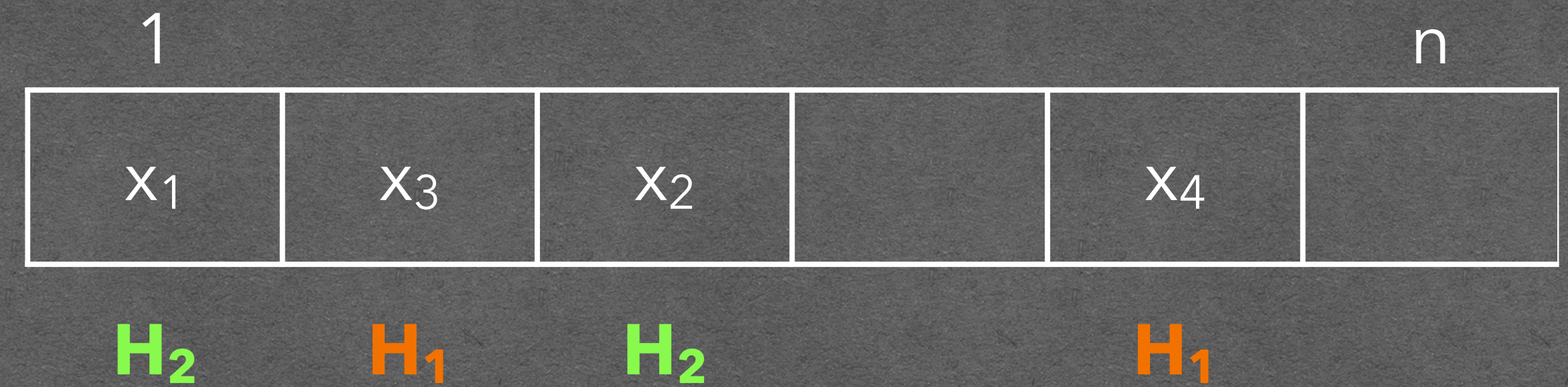
Powerful **probabilistic** data structure to map elements from  $[m]$  to elements in  $[n]$ , where  $m > n$  (overcoming collisions)



- ❖  $\Pr[\text{Collision}] = 0$
- ❖  $\Pr[\text{Insert}(x) = \perp] = \alpha > 0$
- ❖ Uses  $k$  hash functions and evictions.

# Cuckoo Hashing: Example [PR04]

(k=2 hashes)

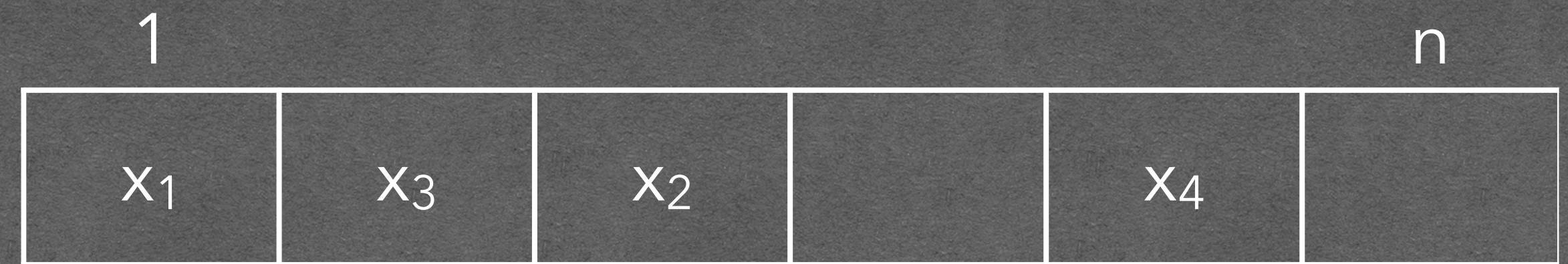


Insert( $x_5$ ):

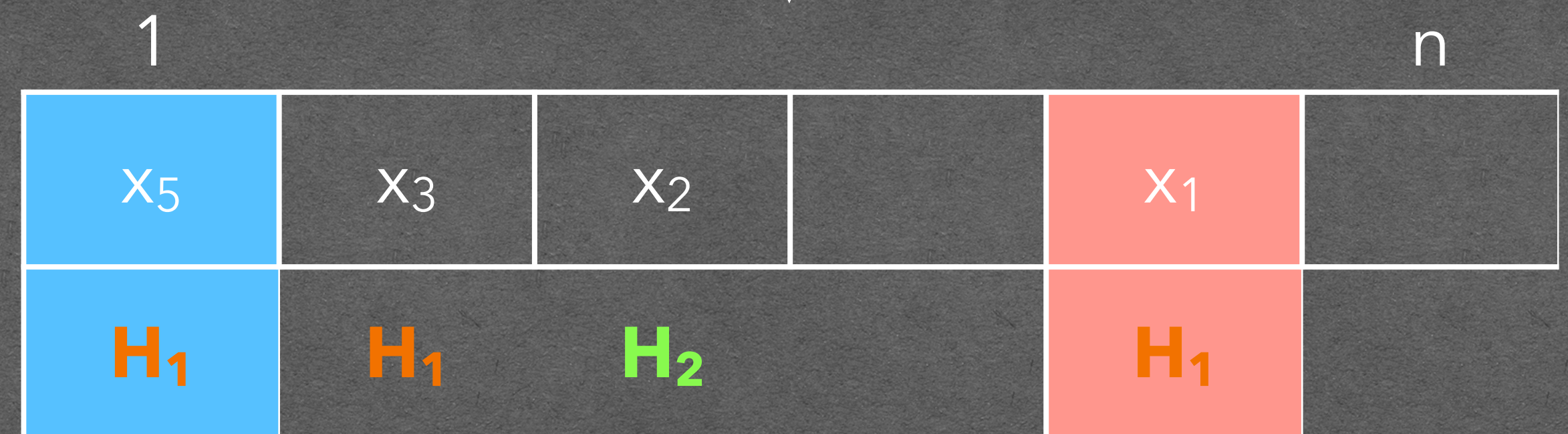
- Puts  $x_5$  in  $H_1(x_5)$
- If  $H_1(x_5)$  is occupied 'evicts' the element
- Continues until there no eviction

# Cuckoo Hashing: Example [PR04]

(k=2 hashes)



$H_2$     $H_1$     $H_2$     $H_1$



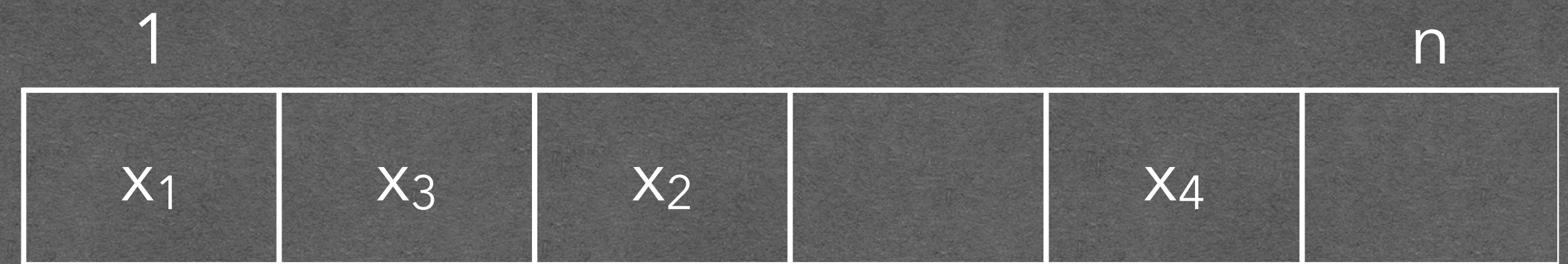
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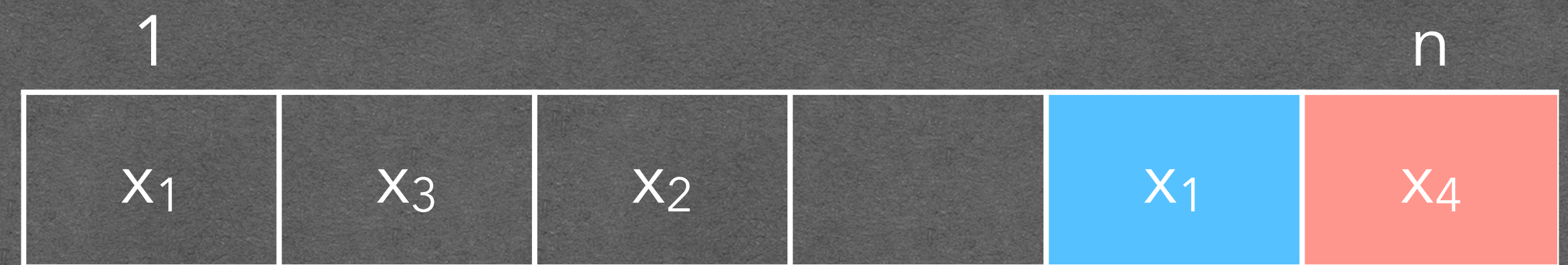
(k=2 hashes)



$H_2$     $H_1$     $H_2$     $H_1$



$H_1$     $H_1$     $H_2$     $H_1$



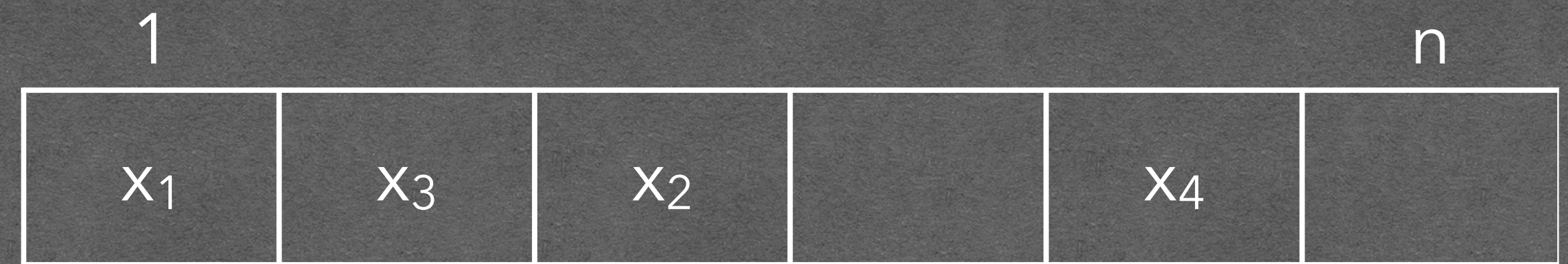
$H_2$     $H_1$     $H_2$     $H_1$     $H_2$

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(k=2 hashes)



$H_2$     $H_1$     $H_2$     $H_1$



$H_1$     $H_1$     $H_2$     $H_1$



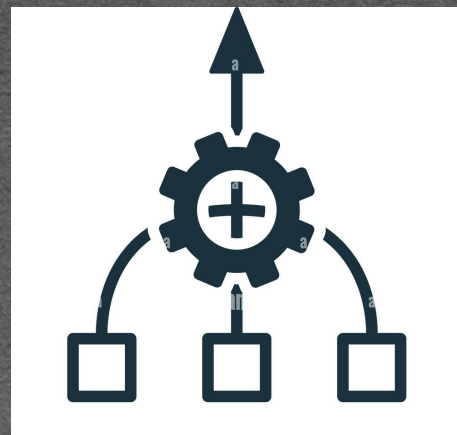
$H_2$     $H_1$     $H_2$     $H_1$     $H_2$

Insert( $x_5$ ):

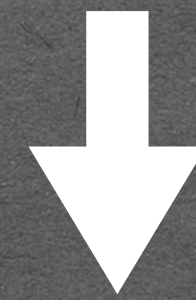
- Puts  $x_5$  in  $H_1(x_5)$
- If  $H_1(x_5)$  is occupied 'evicts' the element
- Continues until there no eviction

$x$  is either in  
 $H_1(x)$  or  $H_2(x)$

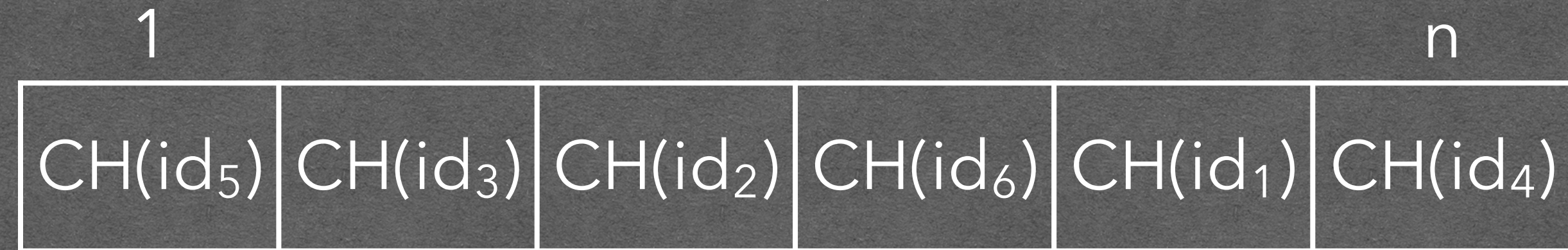
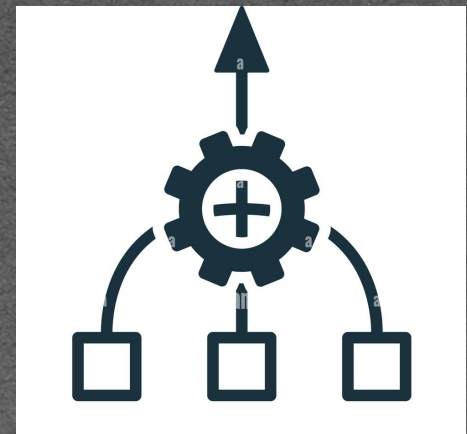
# Cuckoo Hashing in RBE (with small ID-space)



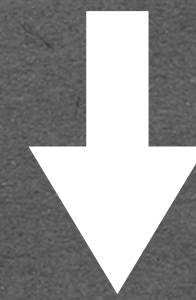
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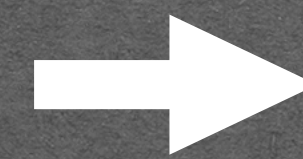
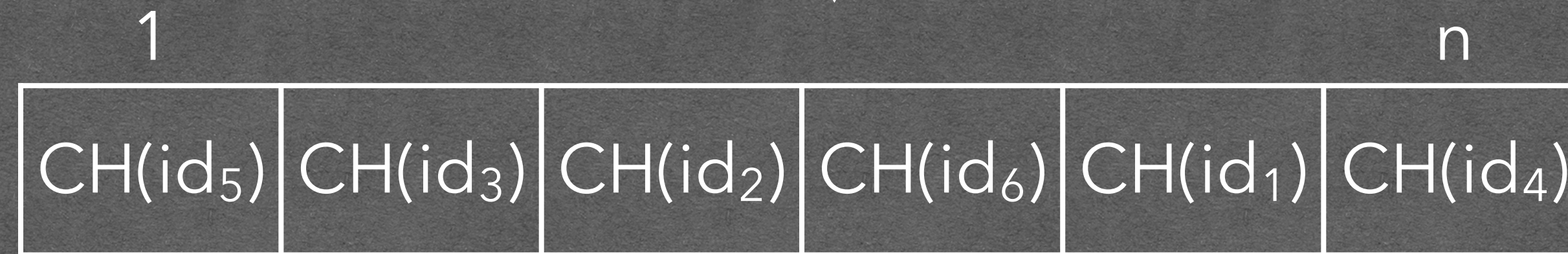
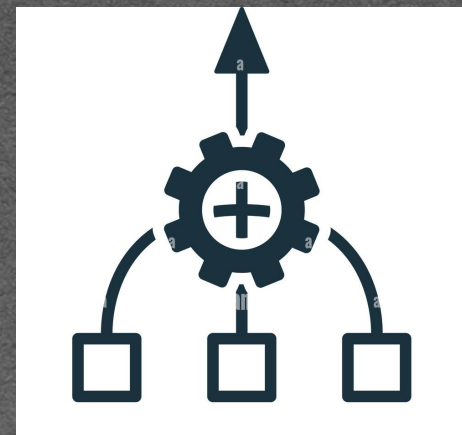
Cuckoo Hashing



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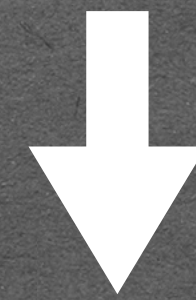
Cuckoo Hashing



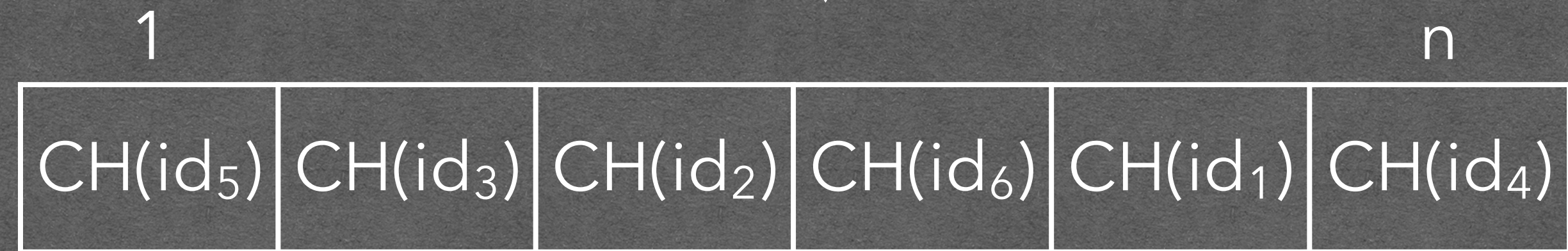
RBE . Reg(..., CH(id), ...)



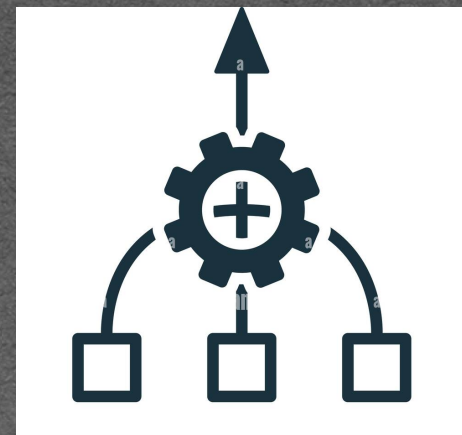
# Cuckoo Hashing in RBE (with small ID-space)



Cuckoo Hashing



→ RBE . Reg(..., CH(id), ...)



RBE . Enc(..., H<sub>1</sub>(id), ...)

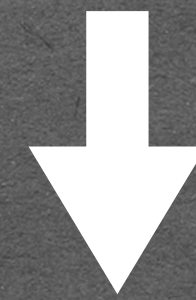
RBE . Enc(..., H<sub>2</sub>(id), ...)

⋮

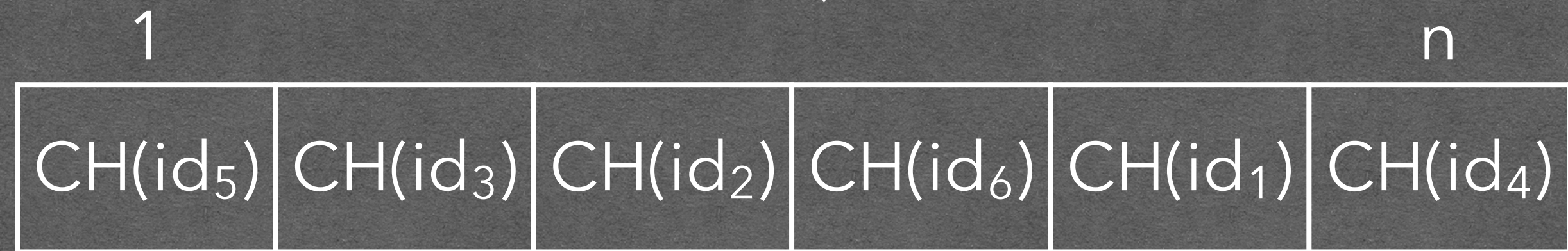
RBE . Enc(..., H<sub>k</sub>(id), ...)



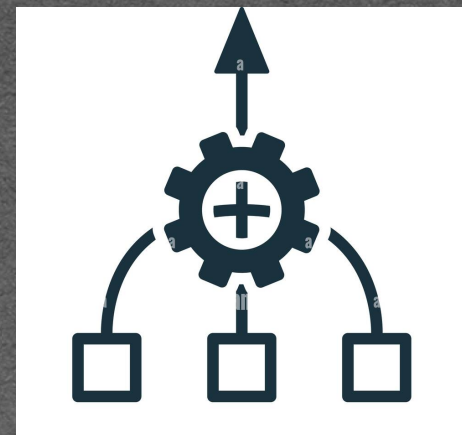
# Cuckoo Hashing in RBE (with small ID-space)



Cuckoo Hashing



RBE . Reg(..., CH(id), ...)



RBE . Enc(..., H<sub>1</sub>(id), ...)

RBE . Enc(..., H<sub>2</sub>(id), ...)

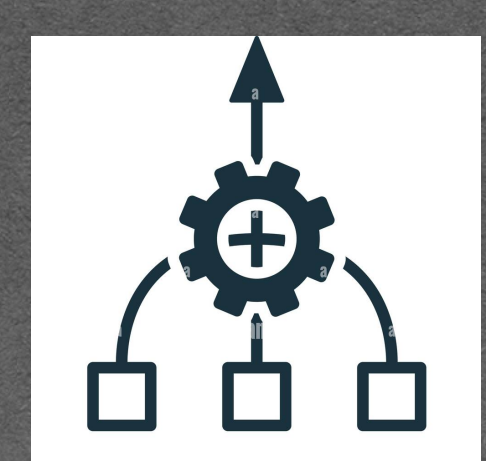
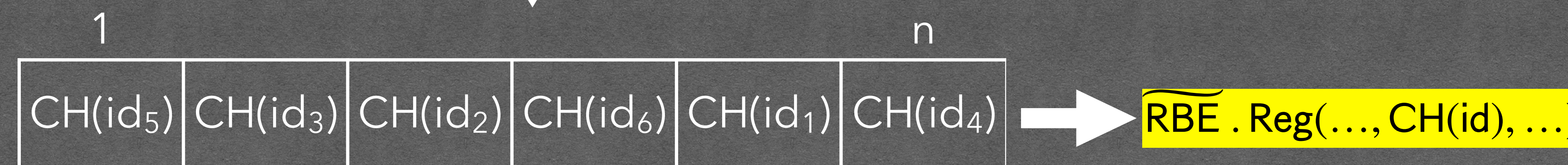
⋮

RBE . Enc(..., H<sub>k</sub>(id), ...)

Not secure yet!



# Cuckoo Hashing in RBE (with small ID-space)



Maybe id'

Only id

Maybe id'

RBE . Enc(..., H<sub>1</sub>(id), ...)

RBE . Enc(..., H<sub>2</sub>(id), ...)

⋮

RBE . Enc(..., H<sub>k</sub>(id), ...)

Not secure yet!



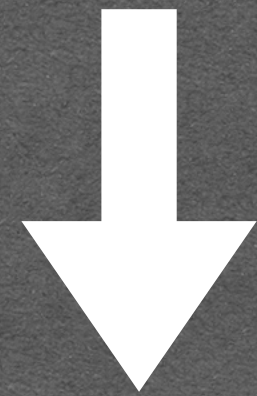
# The missing piece: Witness Encryption for Vector Commitments

$\vec{v} =$

$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	...	$v_{n-1}$	$v_n$
-------	-------	-------	-------	-------	-------	-----	-----------	-------

Vector  
Commitment

Commit



$C$

Open



$v_i$



$\Lambda_i$

Succinctness



$\ll n$

[LY10], [CF13]

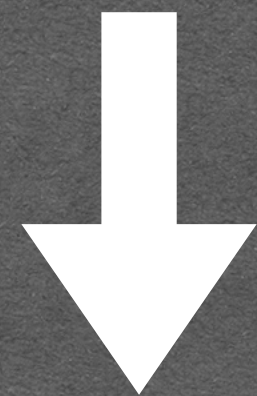
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[LY10], [CF13]

★ *Witness Encryption w.r.t. Vector Commitment (VCWE):*

$\text{Enc}(C, v^*, i, m) \rightarrow \text{ct}$  such that  $\Lambda_i$  decrypts

only if  $\vec{v}[i] = v^*$

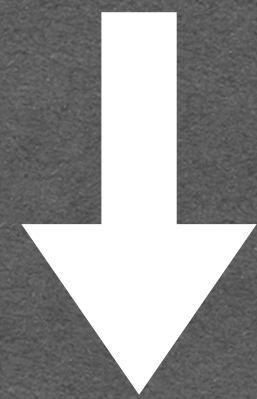
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-------	-------	-------	-------	-------	-------	-----	-----------	-------

Vector  
Commitment

Commit



[LY10], [CF13]



$C$

Open



$v_i$



$\Lambda_i$

Succinctness



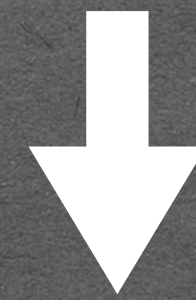
$\ll n$

★ *Witness Encryption w.r.t. Vector Commitment (VCWE):*

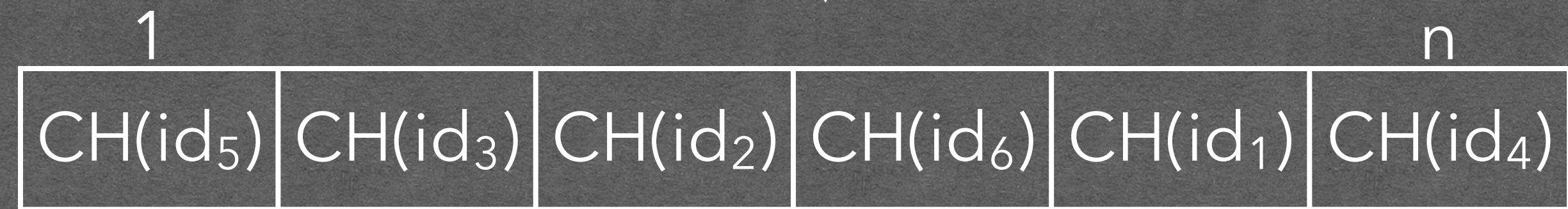
$\text{Enc}(C, v^*, i, m) \rightarrow \text{ct}$  such that  $\Lambda_i$  decrypts only if  $\vec{v}[i] = v^*$

★ *In the paper: Efficient constructions from pairings and from LWE.*

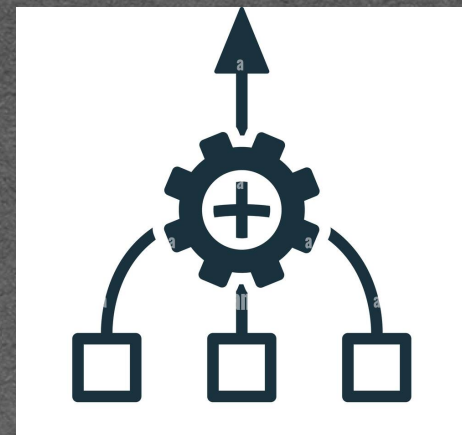
# The final compiler



Cuckoo Hashing



$\widetilde{\text{RBE}} . \text{Reg}(\dots, \text{CH}(\text{id}), \dots)$



$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_1(\text{id}), m_1^{(1)}, \dots)$

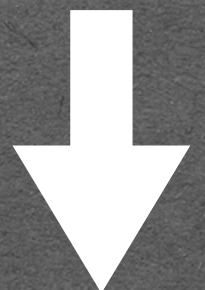
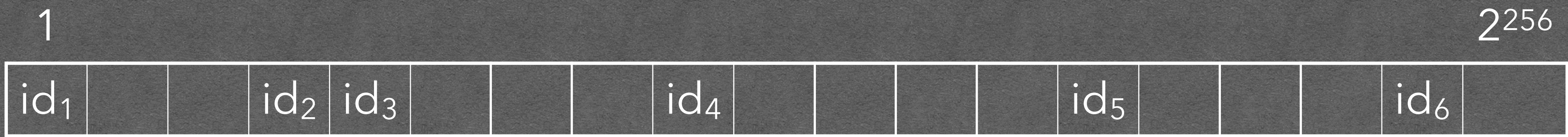
$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_2(\text{id}), m_1^{(2)}, \dots)$

⋮

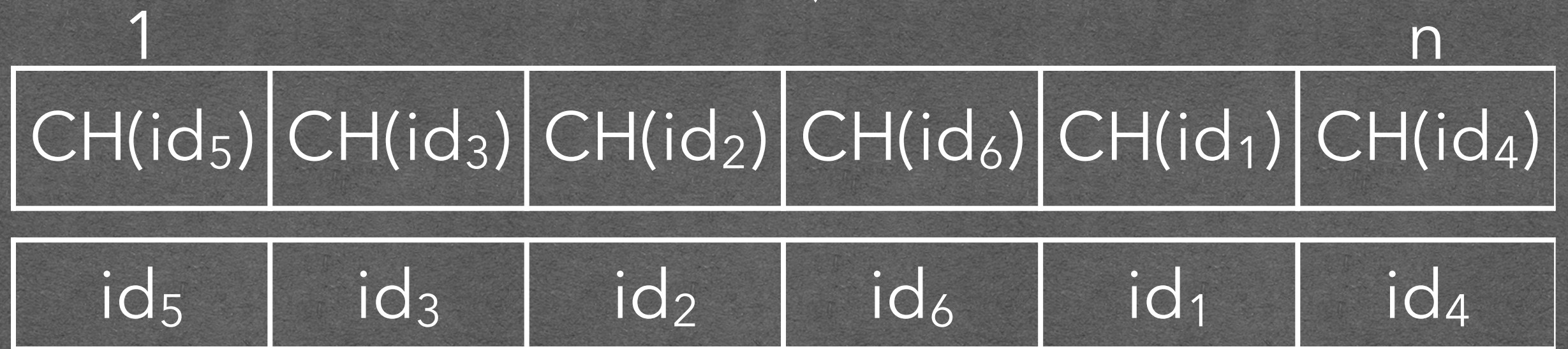
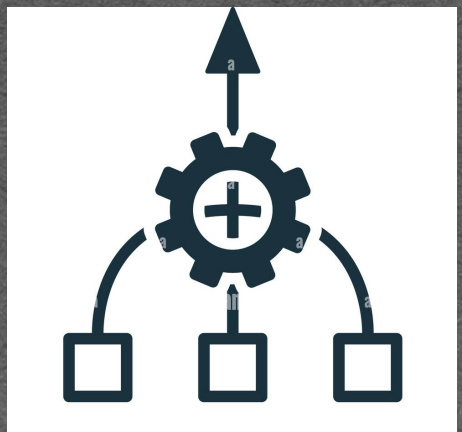
$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_k(\text{id}), m_1^{(k)}, \dots)$



# The final compiler



Cuckoo Hashing



$\widetilde{\text{RBE}} . \text{Reg}(\dots, \text{CH}(\text{id}), \dots)$

$D \leftarrow \text{VC} . \text{Commit}(\dots)$

$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_1(\text{id}), m_1^{(1)}, \dots)$

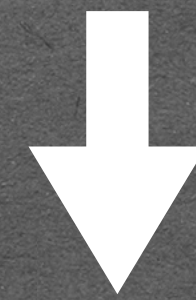
$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_2(\text{id}), m_1^{(2)}, \dots)$

⋮

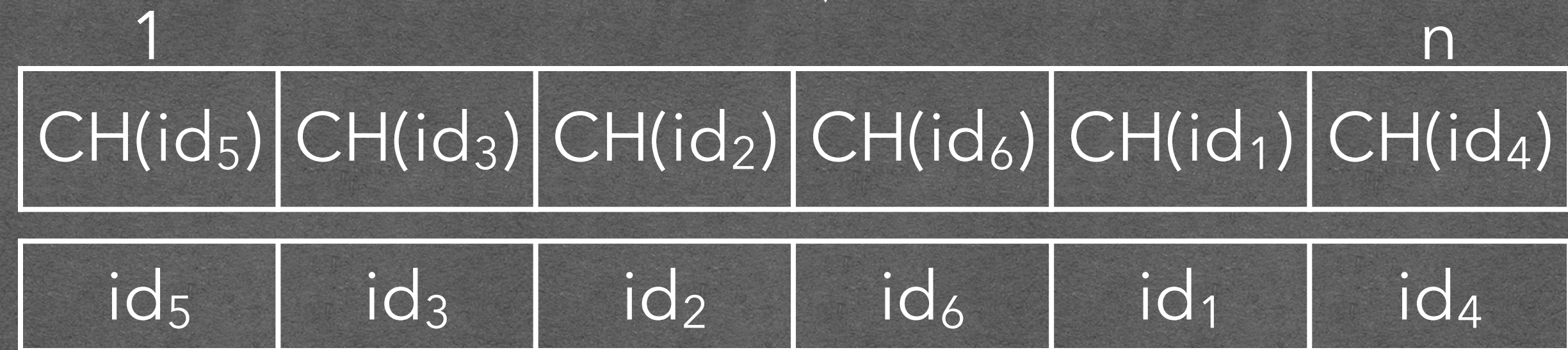
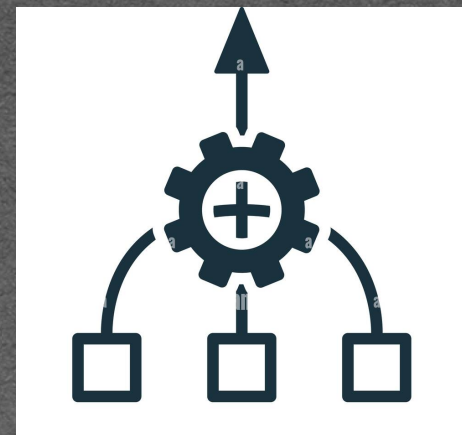
$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_k(\text{id}), m_1^{(k)}, \dots)$



# The final compiler



Cuckoo Hashing



→ RBE . Reg(..., CH(id), ...)

→ D ← VC . Commit(...)

SS . Share( $m$ ) → ( $m_1^{(i)}, m_2^{(i)}$ )

RBE . Enc(..., H<sub>1</sub>(id),  $m_1^{(1)}$ , ...)

VCWE . Enc( $D$ , H<sub>1</sub>(id), id,  $m_2^{(1)}$ , ...)

RBE . Enc(..., H<sub>2</sub>(id),  $m_1^{(2)}$ , ...)

VCWE . Enc( $D$ , H<sub>2</sub>(id), id,  $m_2^{(2)}$ , ...)

⋮

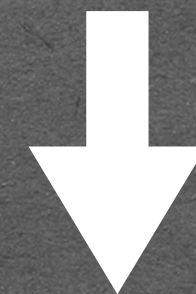
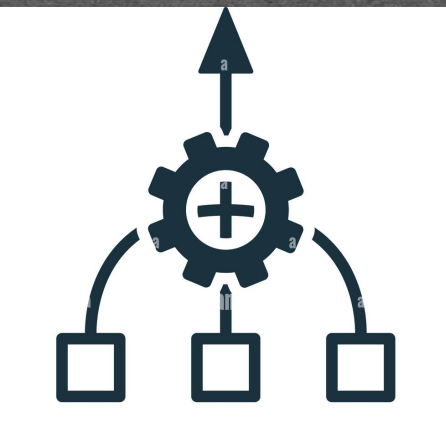
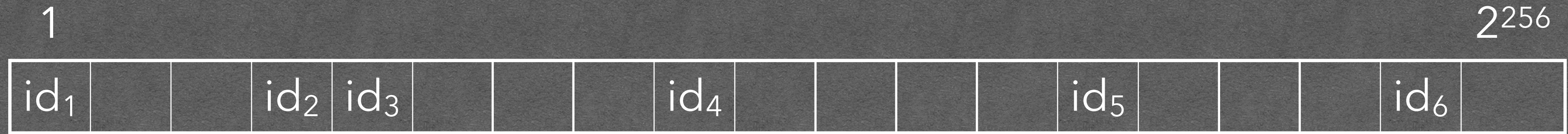
⋮

RBE . Enc(..., H<sub>k</sub>(id),  $m_1^{(k)}$ , ...)

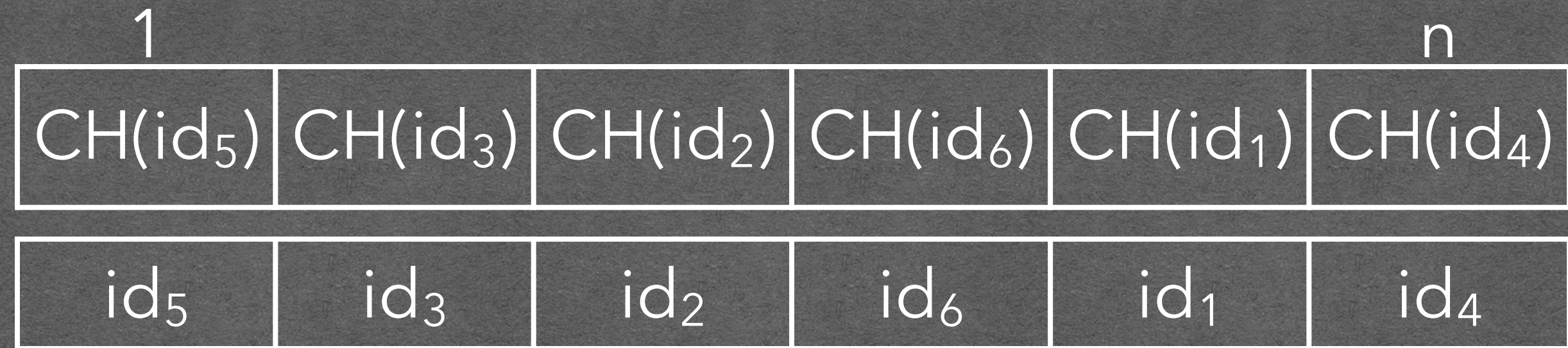
VCWE . Enc( $D$ , H<sub>k</sub>(id), id,  $m_2^{(k)}$ , ...)



# The final compiler



Cuckoo Hashing



$\widetilde{\text{RBE}} . \text{Reg}(\dots, \text{CH}(\text{id}), \dots)$

$D \leftarrow \text{VC} . \text{Commit}(\dots)$

$\text{SS} . \text{Share}(m) \rightarrow (m_1^{(i)}, m_2^{(i)})$

Maybe id'

Only id

Maybe id'



$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_1(\text{id}), m_1^{(1)}, \dots)$

$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_2(\text{id}), m_1^{(2)}, \dots)$

⋮

$\widetilde{\text{RBE}} . \text{Enc}(\dots, H_k(\text{id}), m_1^{(k)}, \dots)$

$\text{VCWE} . \text{Enc}(D, H_1(\text{id}), \text{id}, m_2^{(1)}, \dots)$

$\text{VCWE} . \text{Enc}(D, H_2(\text{id}), \text{id}, m_2^{(2)}, \dots)$

⋮

$\text{VCWE} . \text{Enc}(D, H_k(\text{id}), \text{id}, m_2^{(k)}, \dots)$

Nobody!

Everyone

Nobody!

# Dealing with CH insertion failures

How to deal with  $\Pr[\text{Insert}(\text{id}) = \perp] = \alpha$  ?



# Dealing with CH insertion failures

How to deal with  $\Pr[\text{Insert}(\text{id}) = \perp] = \alpha$ ?

## ★ Solution 1: Cuckoo hashing with a stash

- $k = 2, s = O(\log(n)) \Rightarrow \alpha = \text{negl}(\lambda)$
- Selective adversary (choice of id before seeing  $h_1, h_2$ )

## ★ Solution 2: Robust Cuckoo Hashing [Yeo23]

- $k = \lambda \Rightarrow \alpha = \text{negl}(\lambda)$
- Adaptive adversary

# The resulting RBE schemes

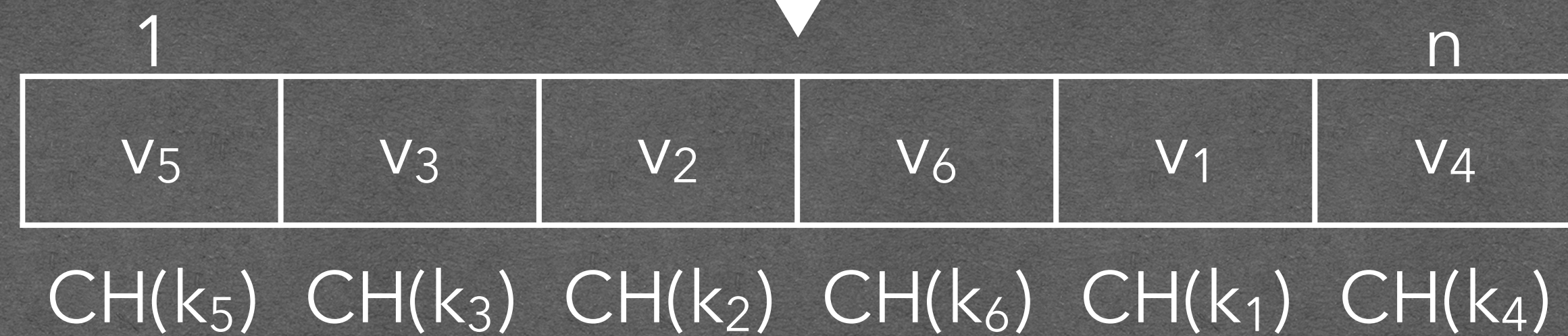
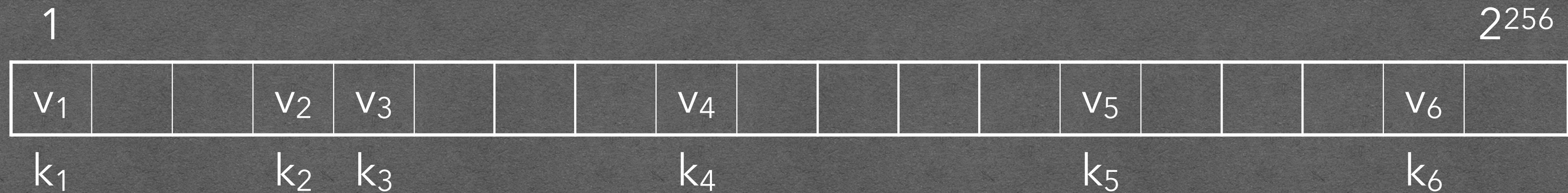
	Setting	$\mathcal{ID}$	Compactness	$ ct $	#updates	$ pp  +  crs $
[HLWW23]	Pairings (C)	$\{0, 1\}^*$	Adaptive	$O(\lambda \log n)$	$\log n$	$O(\lambda n^{2/3} \log n)$
[GKMR22]	Pairings (P)	$[1, n]$	Adaptive	$4 \log n$	$\log n$	$O(\sqrt{n} \log n)$
Ours P1	Pairings (P)	$\{0, 1\}^*$	Adaptive	$6\lambda \log n$	$\log n$	$O(\sqrt{\lambda n} \log n)$
Ours P2	Pairings (P)	$\{0, 1\}^*$	Selective	$12 \log n$	$\log n$	$O(\sqrt{n} \log n)$
[DKL <sup>+</sup> 23]	Lattices	$\{0, 1\}^*$	Adaptive	$(2\lambda + 1) \log n$	$\log n$	$O(\log n)$
Ours L	Lattices	$\{0, 1\}^*$	Selective	$4 \log^2 n$	$\log n$	$O(\log n)$

Pairings:  $|ct|$  measured in group elements

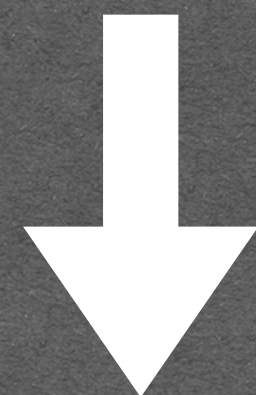
Lattices:  $|ct|$  measured in LWE ciphertexts

*Our KVC compiler*

# Key-Value Map Commitments (KVC) from any Vector Commitment (VC)



VC.Commit



Open  $k$   
→

$v[H_1(k)]$  ,

VC.Open

$v[H_2(k)]$  ,

VC.Open

⋮

KVC: Commit to  $[(k_1, v_1), \dots, (k_n, v_n)]$   
Open any key  $k_i$

# *Conclusions and Open Problems*

# Conclusions and Open problems

## Summary:

- ❖ Efficient pairing-based RBE with unbounded identities
- ❖ Key-Value Map Commitments from any Vector Commitment

## Coclusions:

- ❖ Cuckoo-Hashing very powerful and under-explored (in PKE)

## Open Problems:

- ❖ Pairing-based RBE with unbounded identity space without  $O(\lambda)$  overhead in  $|ct|$
- ❖ Lattice-based RBE with optimal  $O(\log n)$   $|ct|$
- ❖ Pairing-based KVC with  $O(1)$  openings

**Thank you!**