New SIDH Countermeasures for a More Efficient Key Exchange

Andrea Basso, Tako Boris Fouotsa

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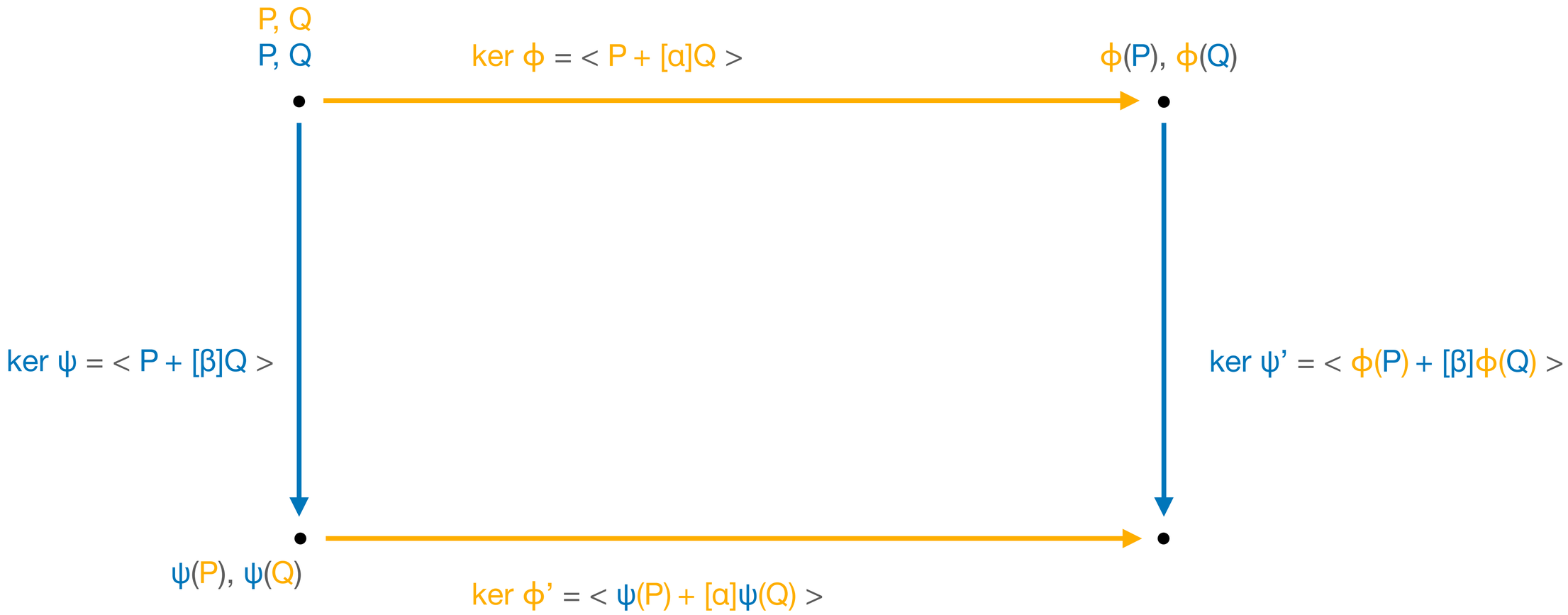
Previously, in isogeny land...



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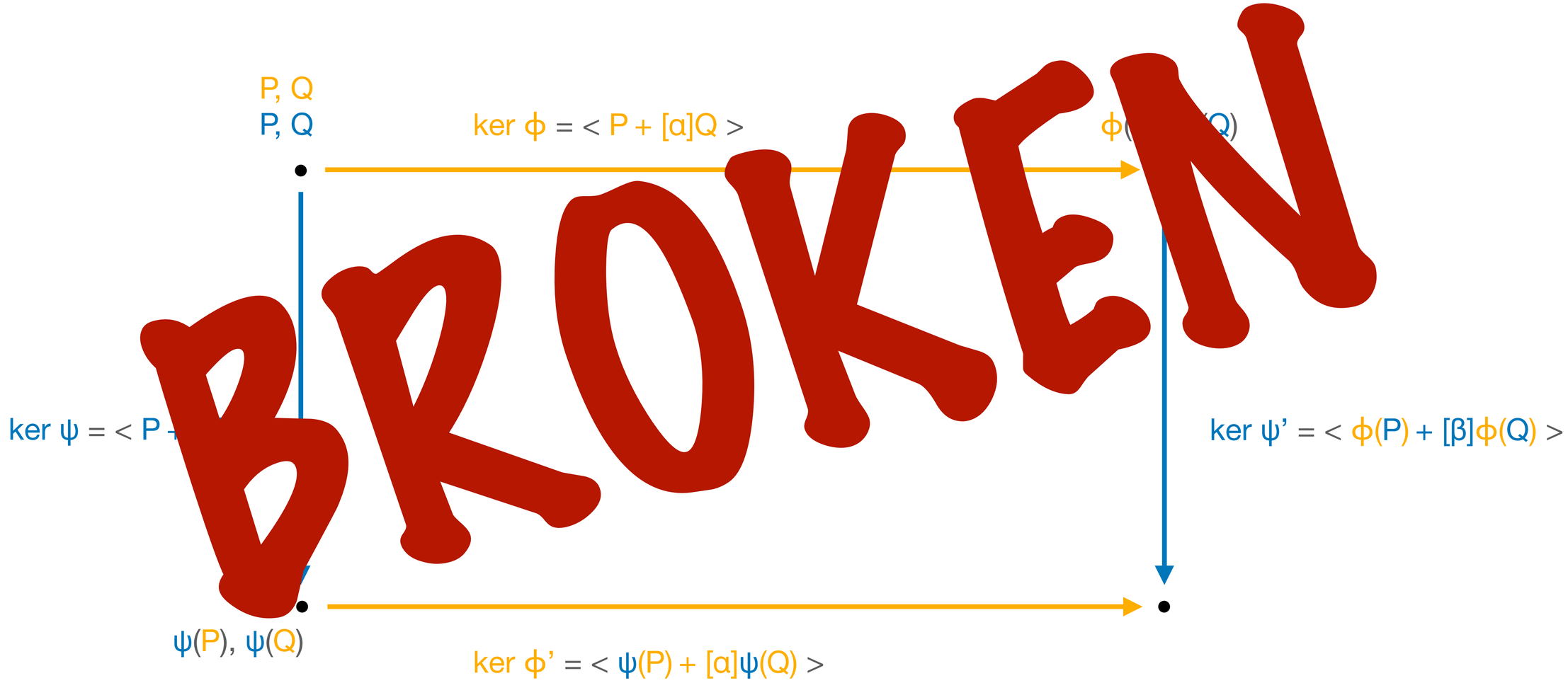

The attacks on SIDH

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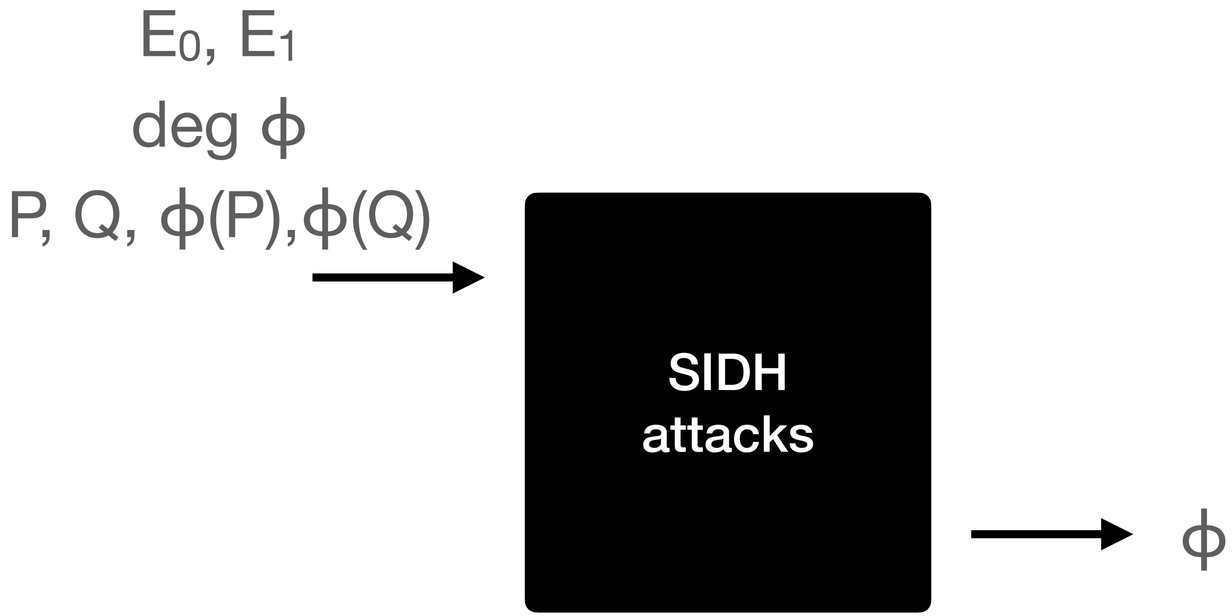

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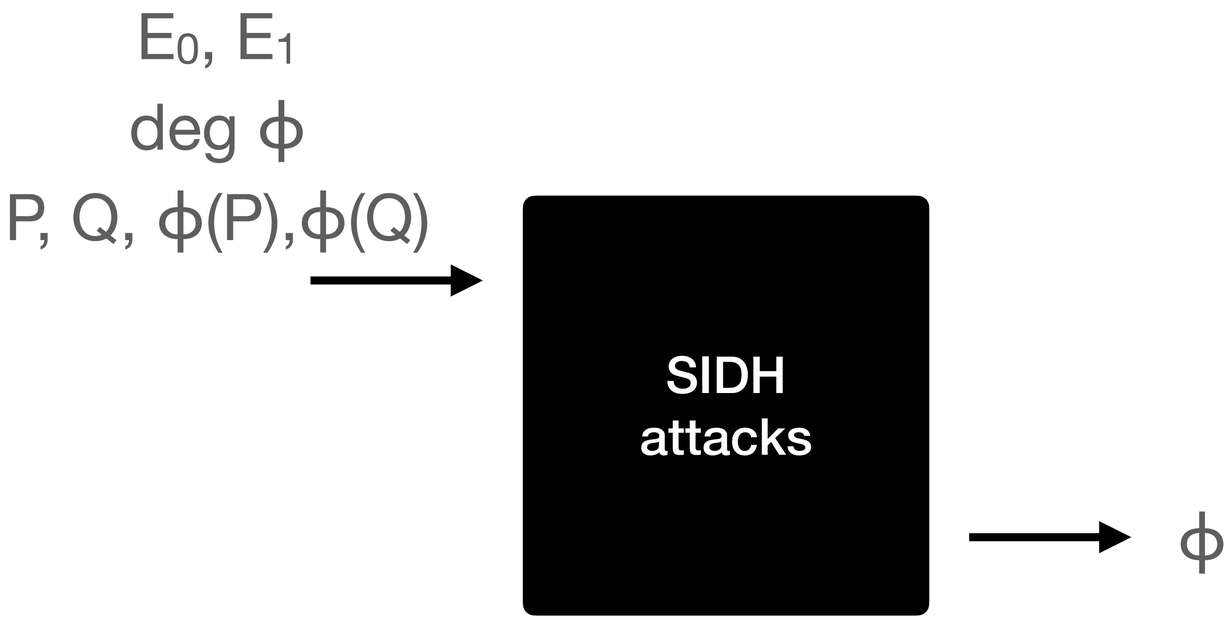

Countermeasures

- Masked degree
- Masked torsion

The attacks on SIDH

E_0, E_1
deg ϕ
P, Q, $\phi(P)$,$\phi(Q)$

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$\phi(P),\phi(Q) \longrightarrow [\alpha]\phi(P), [\alpha]\phi(Q)$

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leaks α^2

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$\Downarrow$

leaks α^2

$\Downarrow$

very large params ($p \approx 2^{6000} - 2^{14,000}$)

A new approach

A new approach

A new approach

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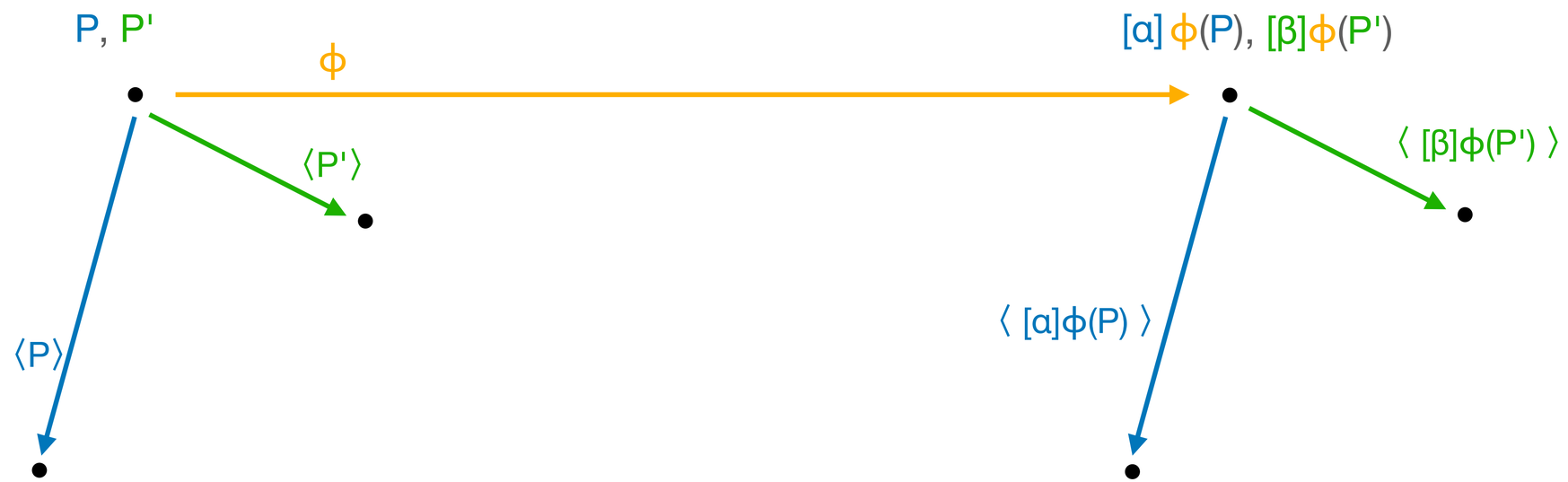
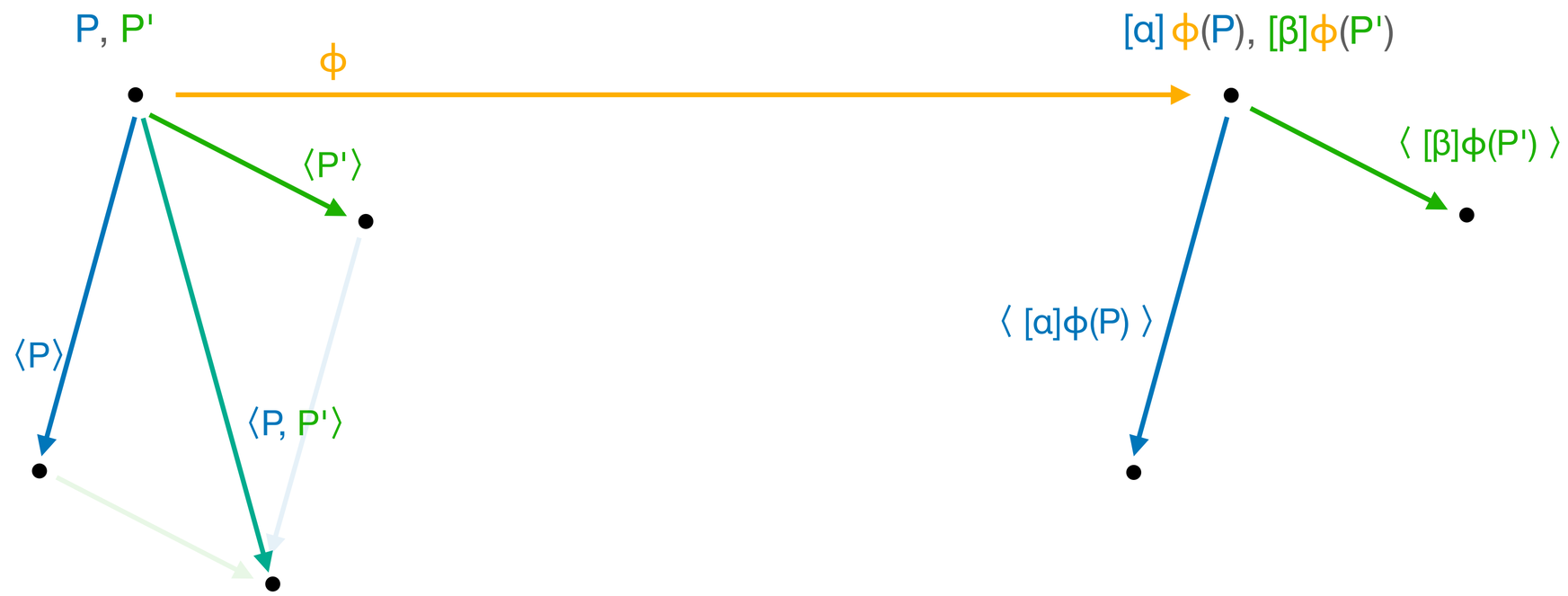


A new approach



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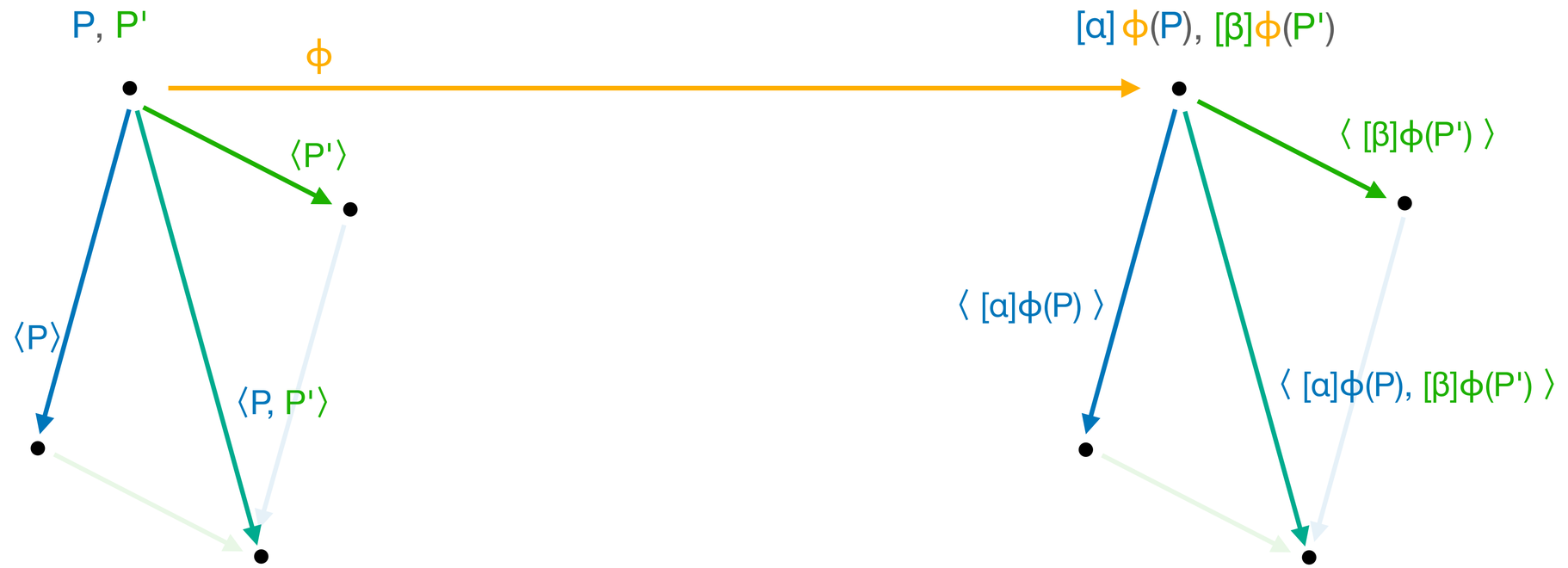

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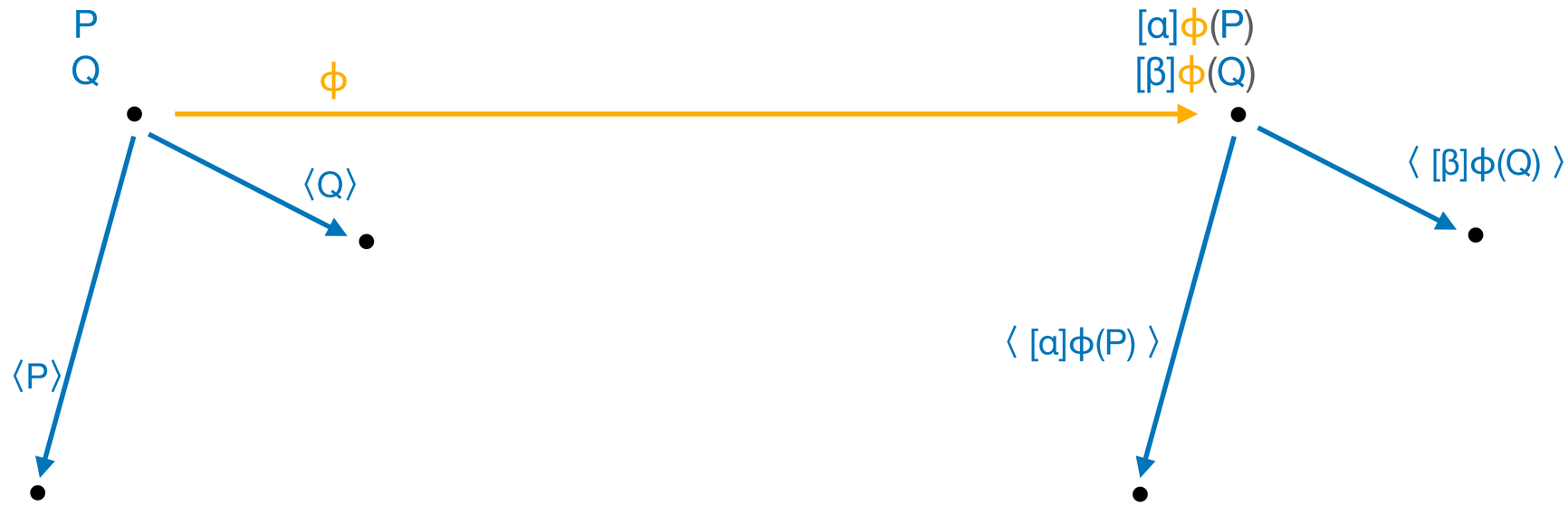

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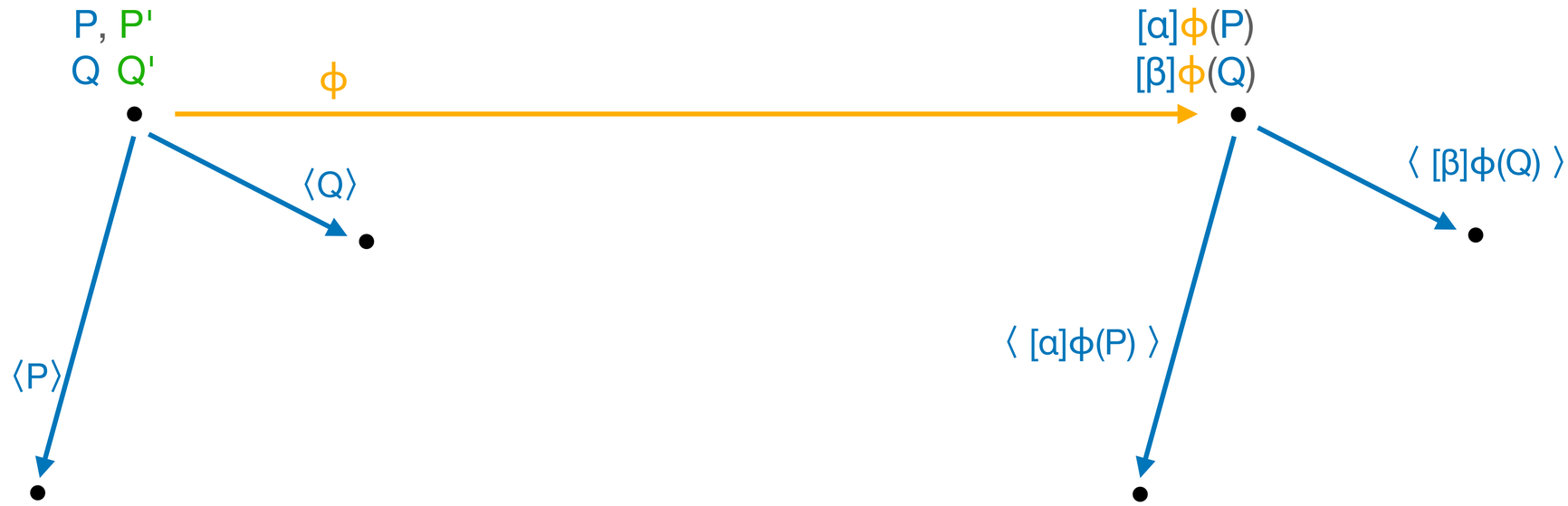

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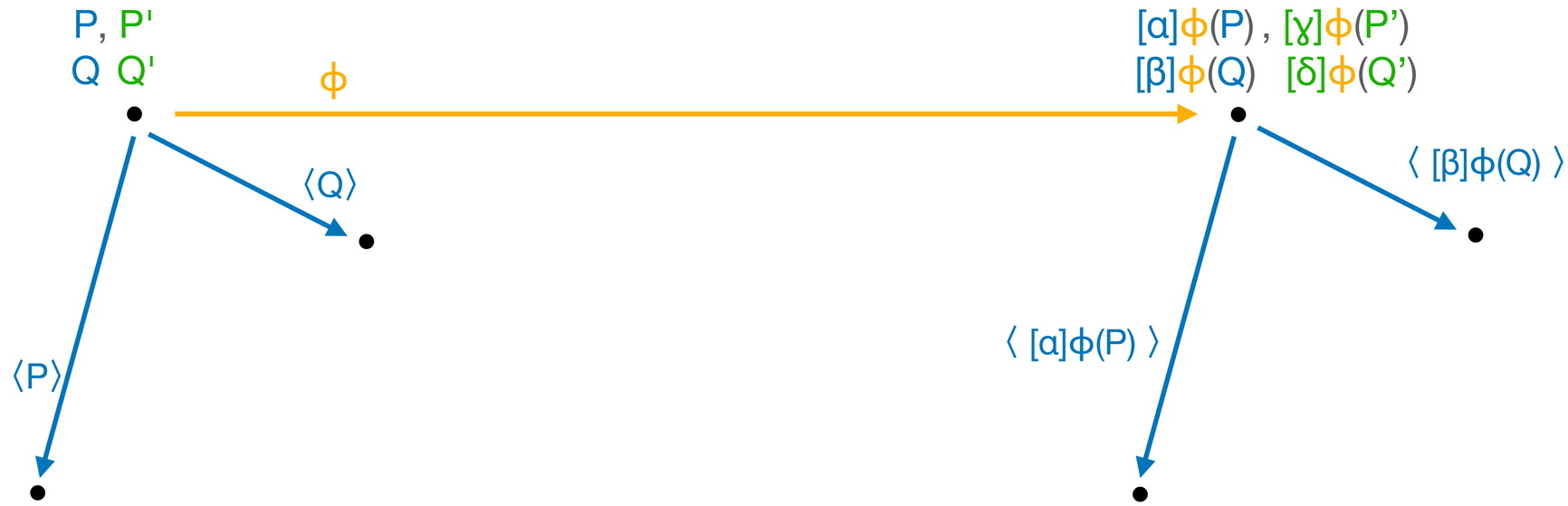

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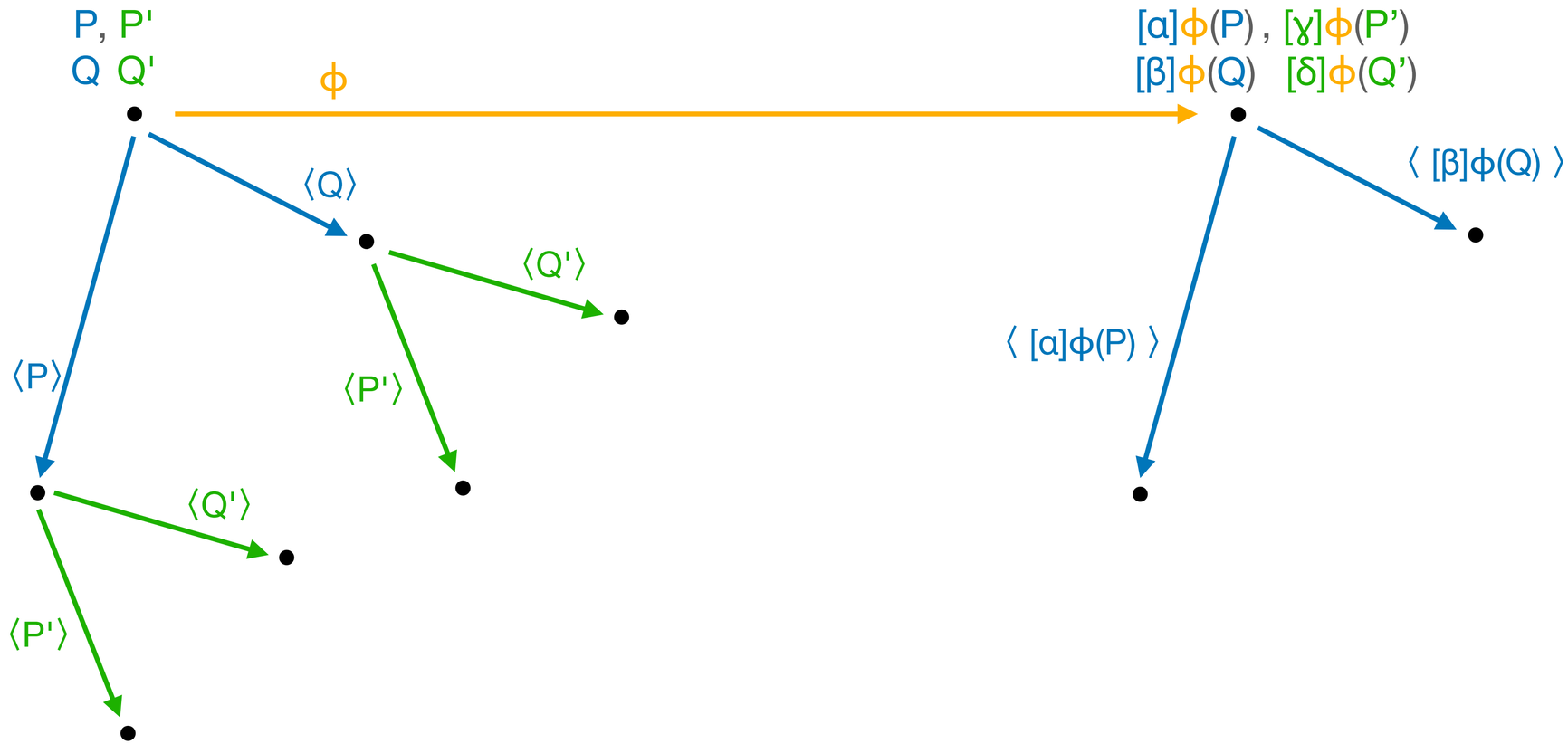

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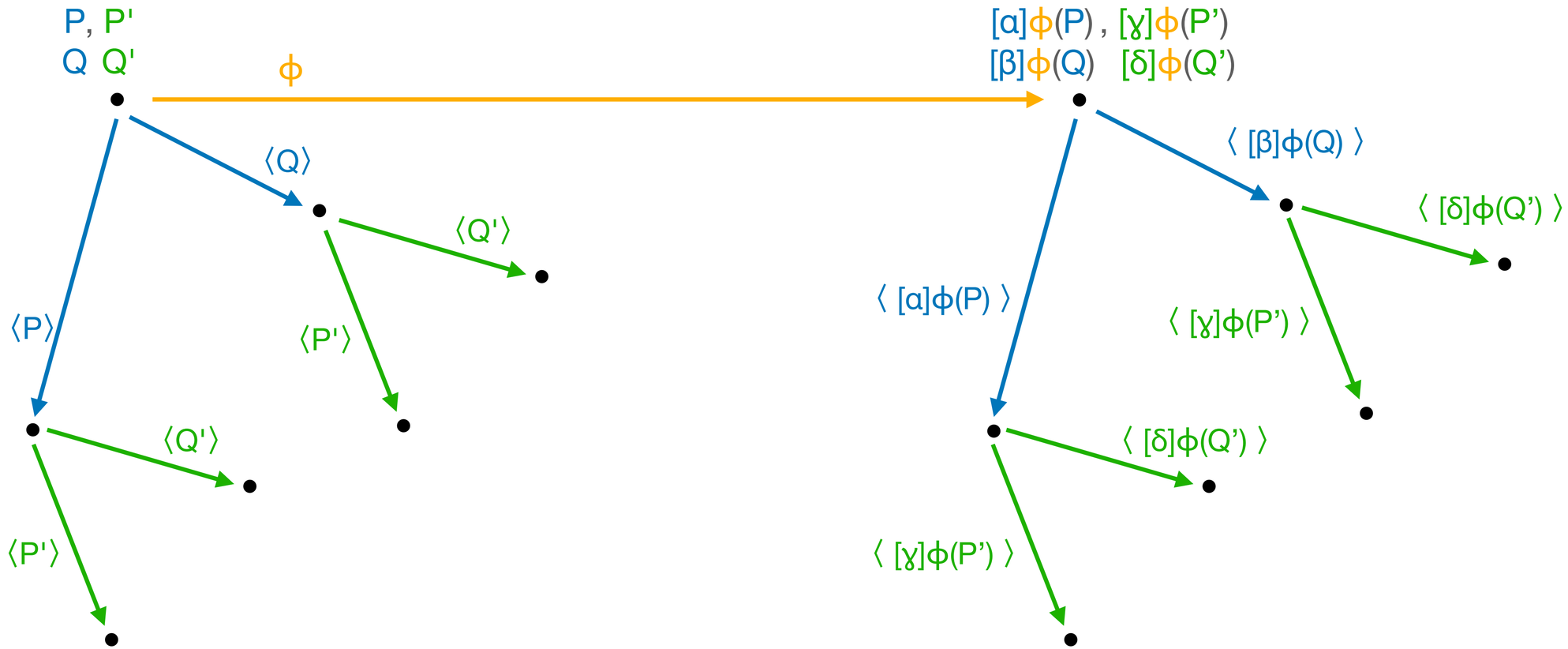

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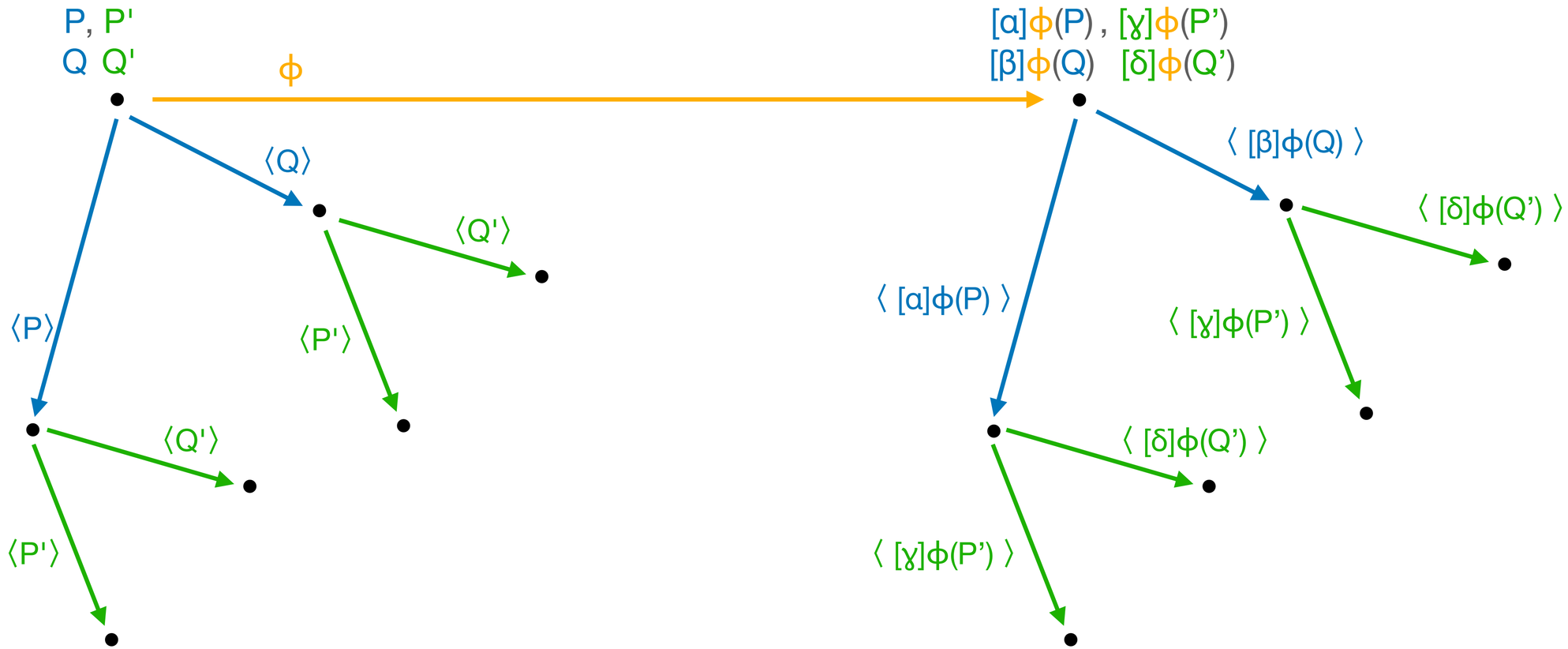

$e([\alpha]P, [\beta]Q) = e(P, Q)^{\alpha\beta}$

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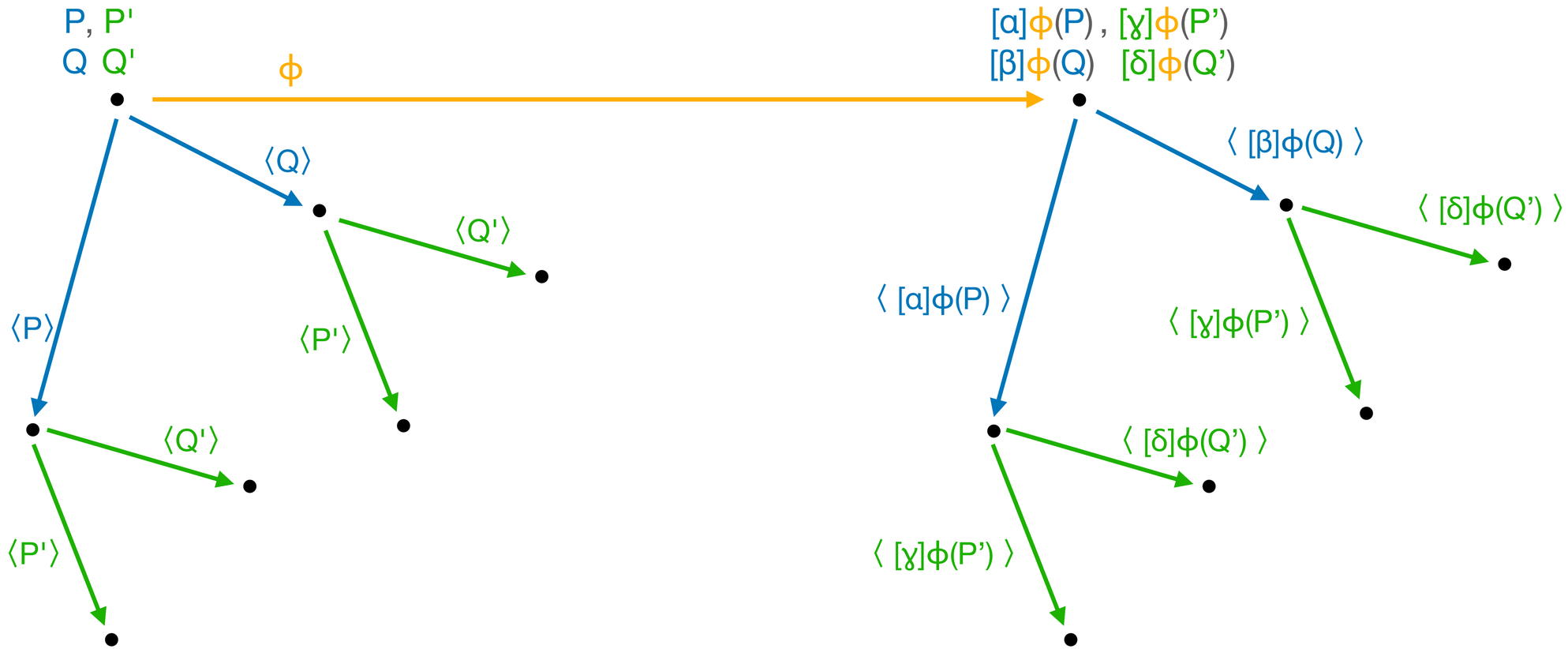
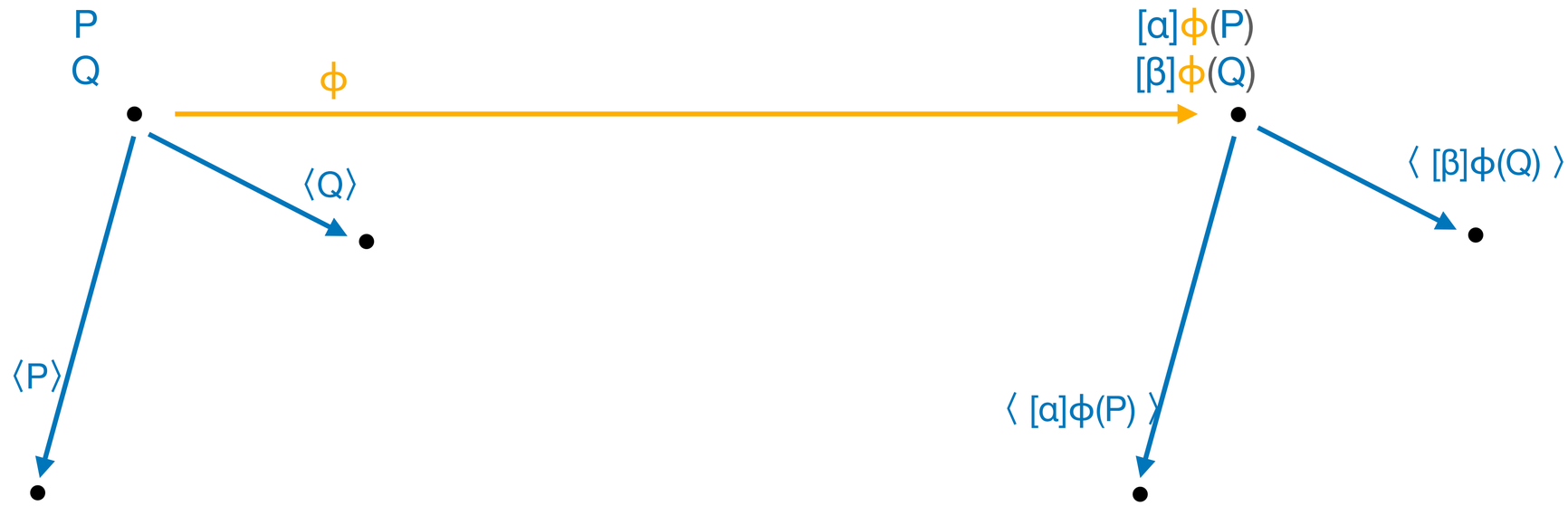


$e([\alpha]P, [\beta]Q) = e(P, Q)^{\alpha\beta}$ ⇒ $p \approx 2^{2400}$

Even better?



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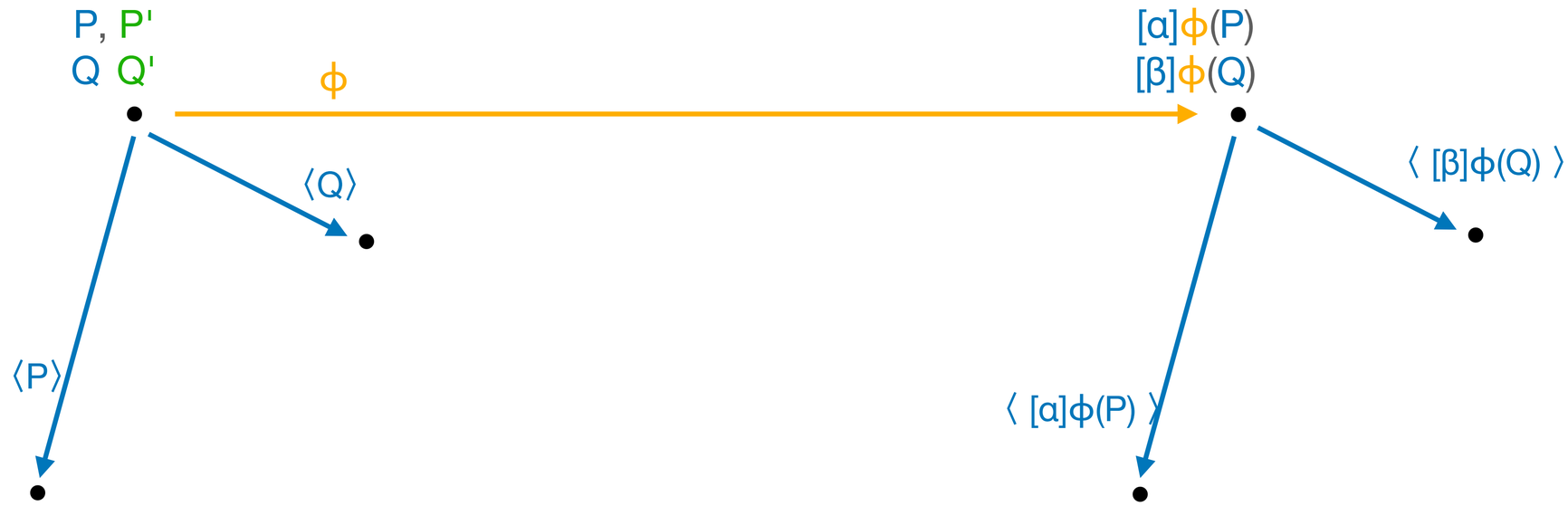

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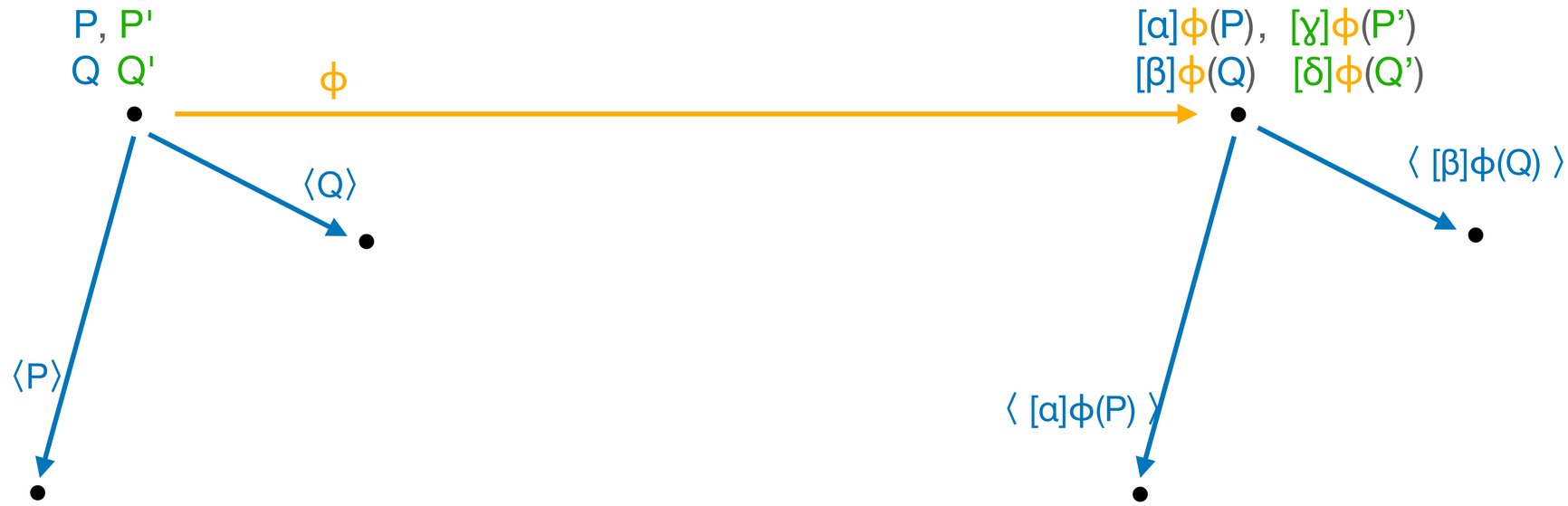

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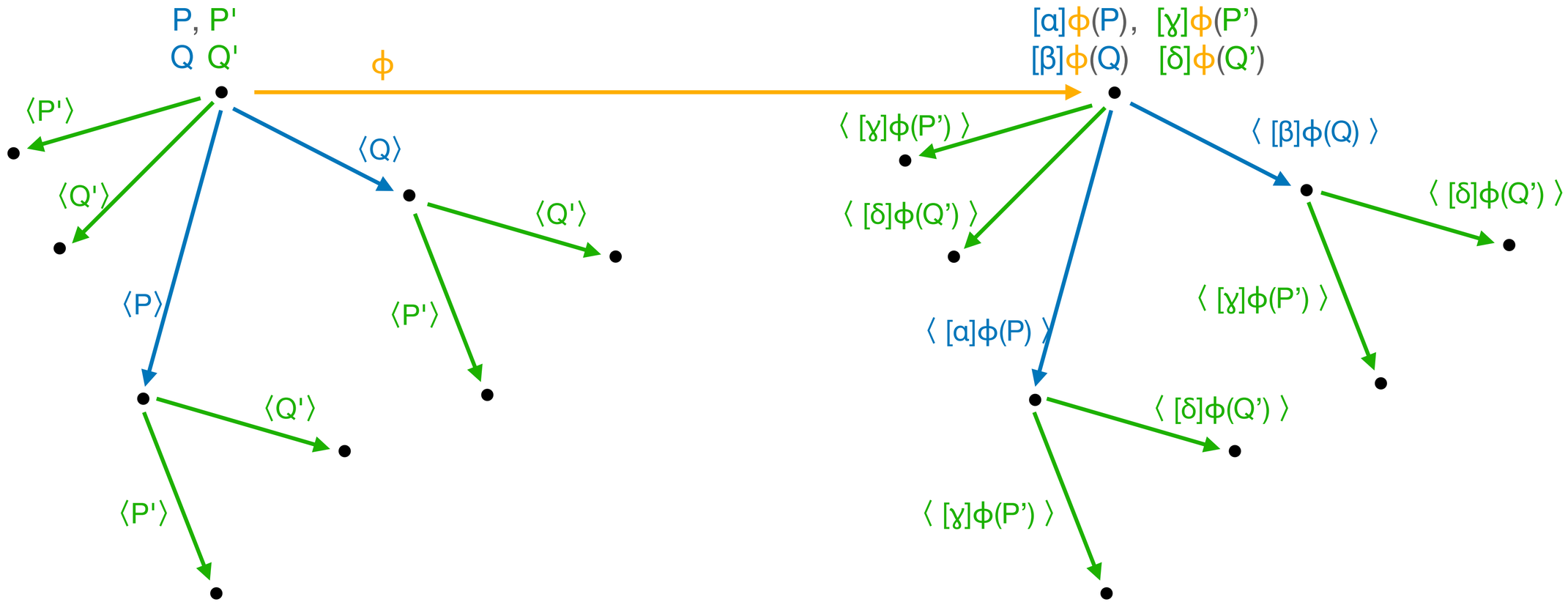

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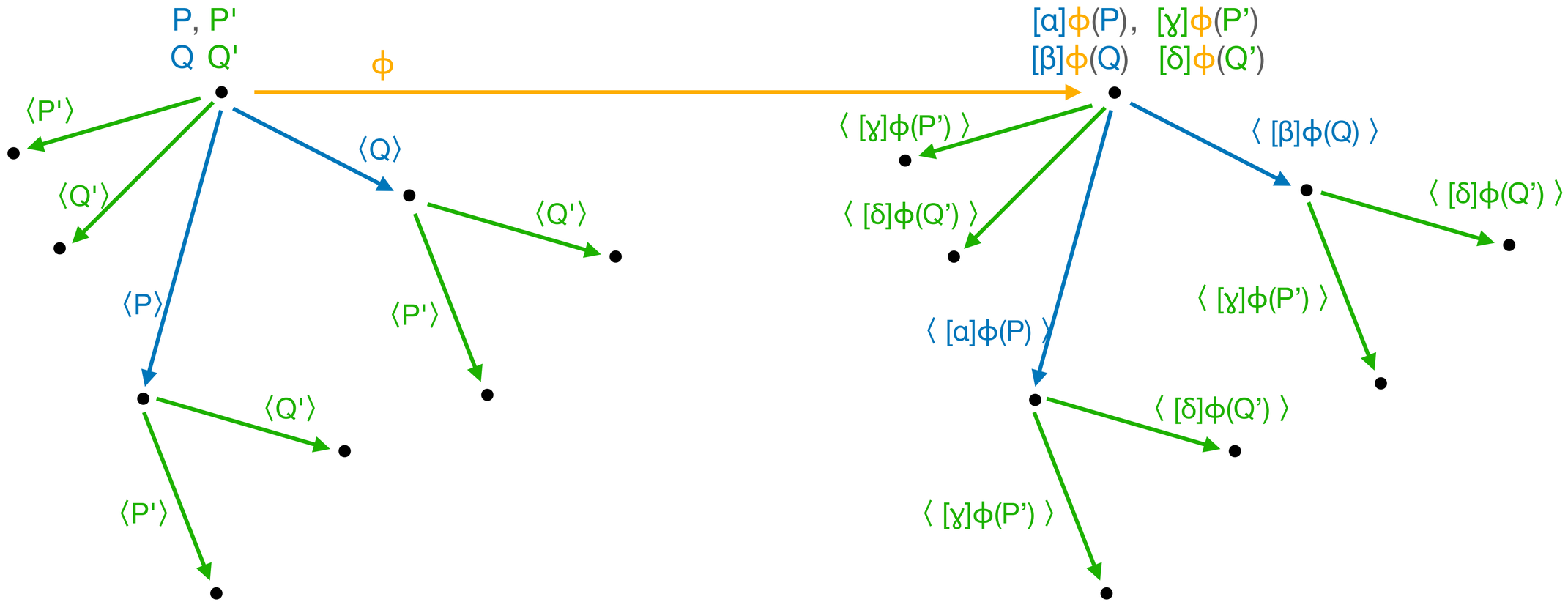

$\Rightarrow \quad p \approx 2^{1500}$

New key exchanges: binSIDH and terSIDH

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-

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$i \in \{1, 2, \ldots, t\}$

P_i, Q_i

P_i, Q_i

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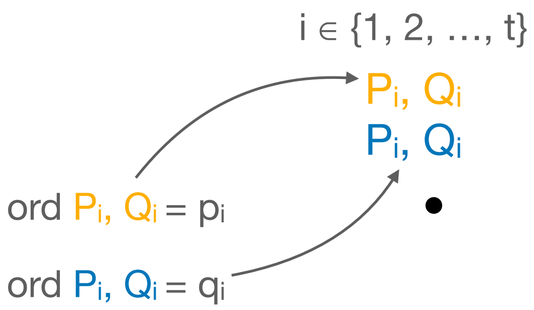

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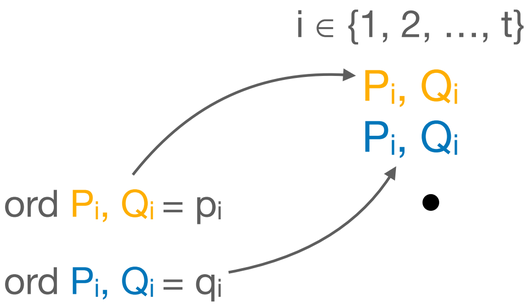

$$\ker \phi = \left\langle \frac{P_1}{Q_1}, \frac{P_2}{Q_2}, \frac{P_3}{Q_3}, \dots, \frac{P_t}{Q_t} \right\rangle$$

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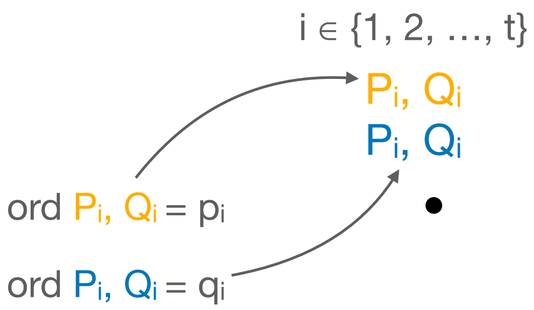

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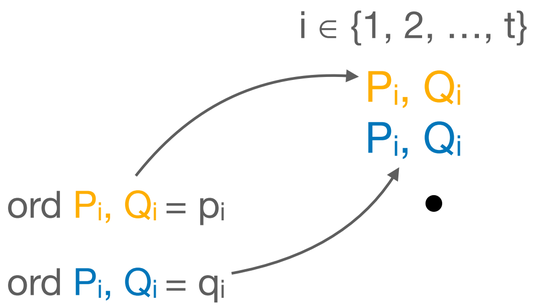

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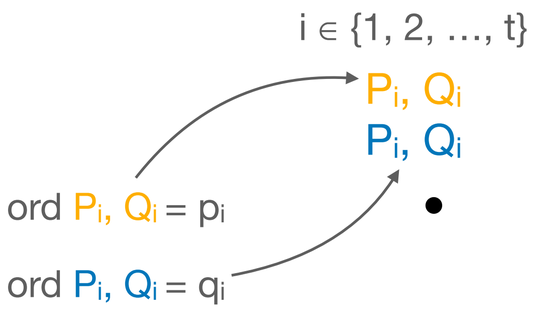

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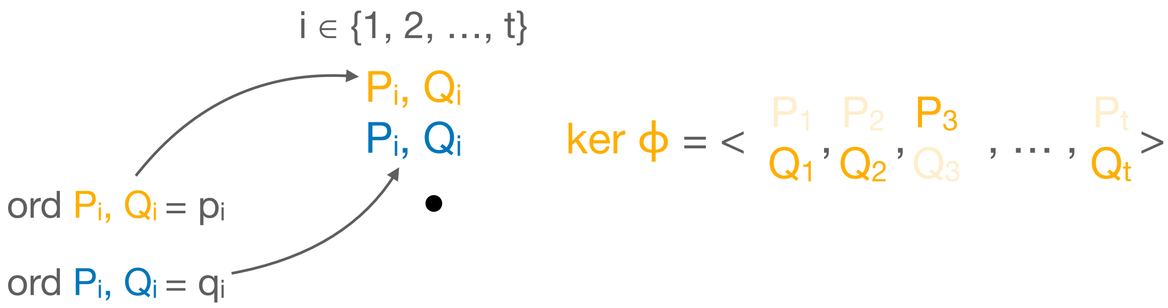

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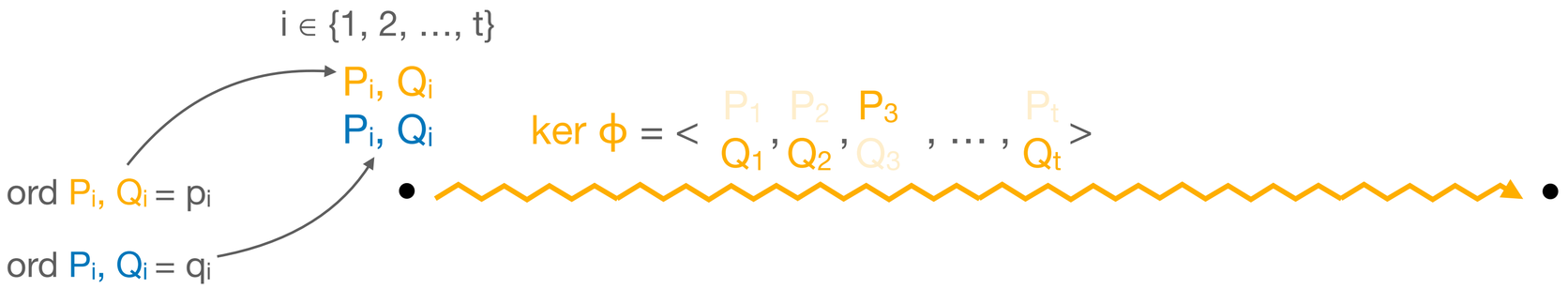

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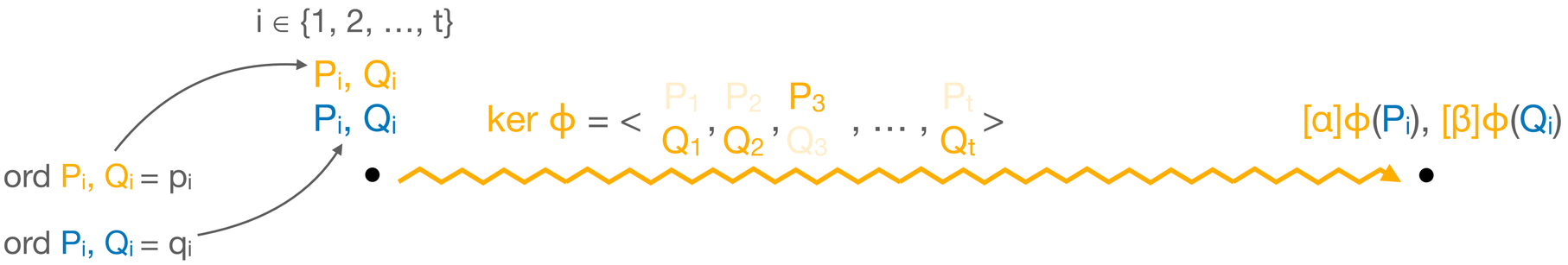

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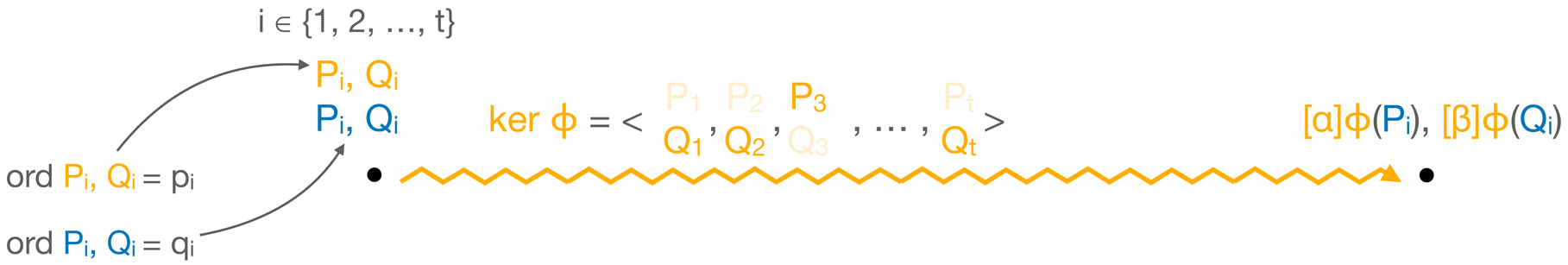

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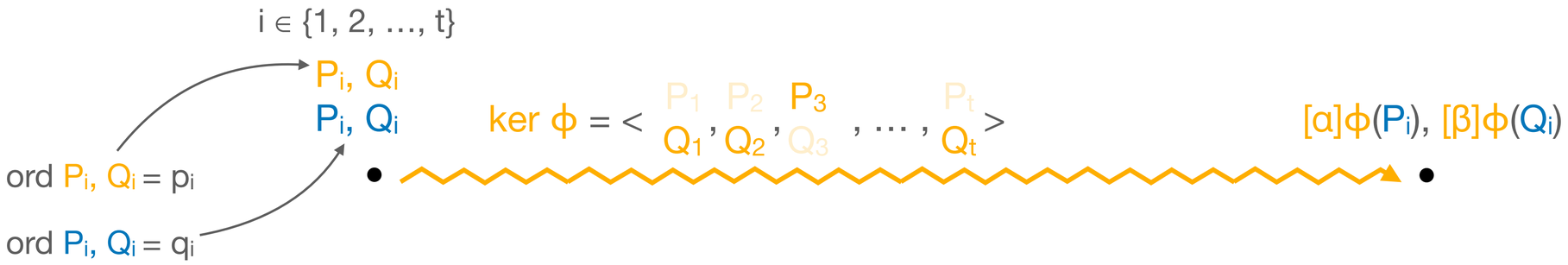

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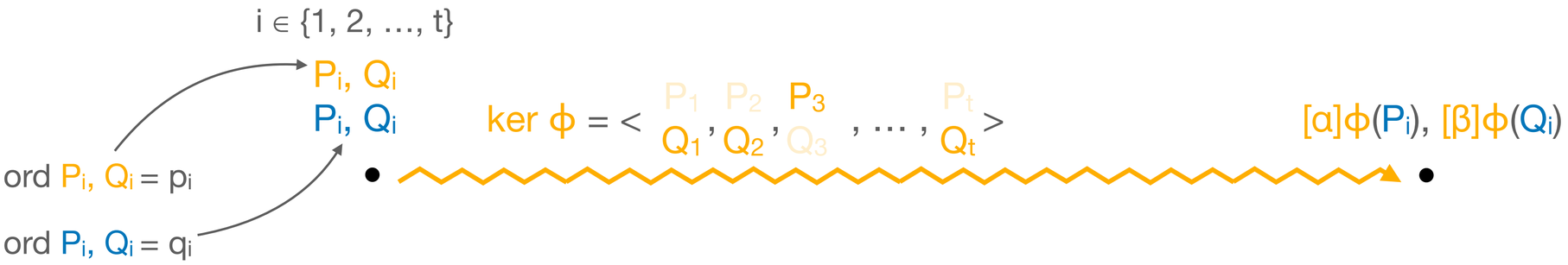

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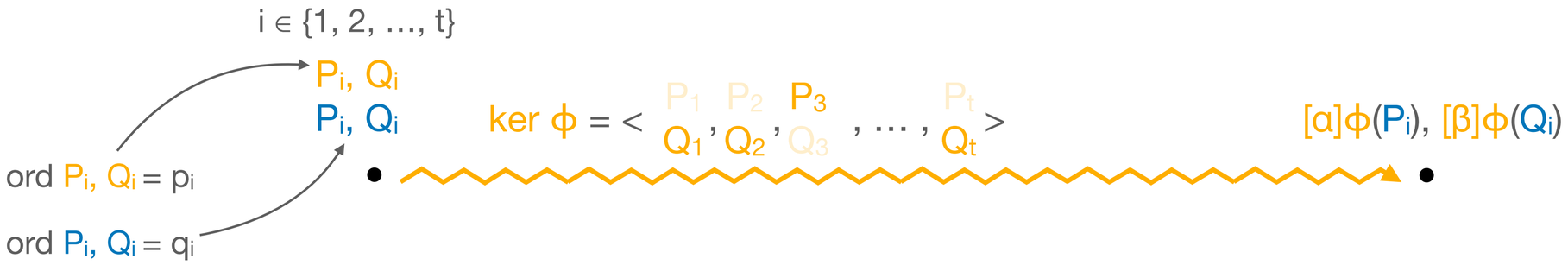

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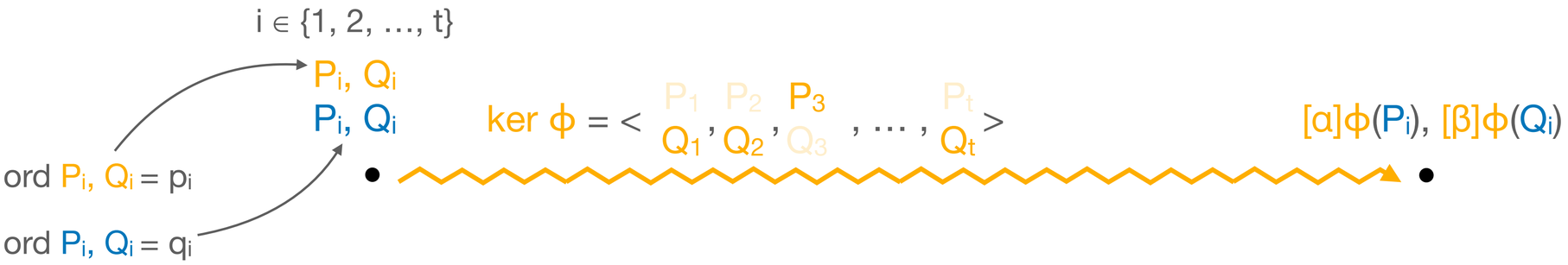

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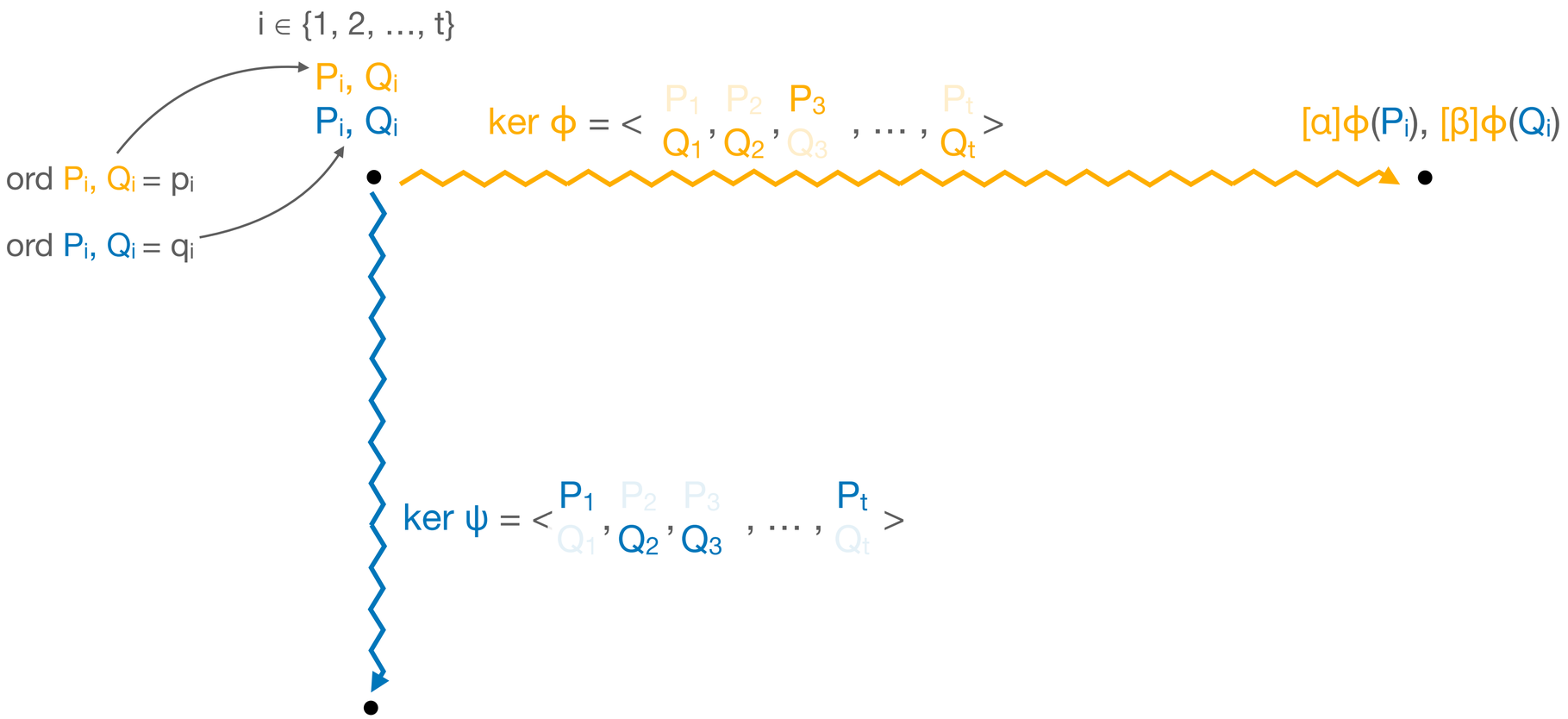

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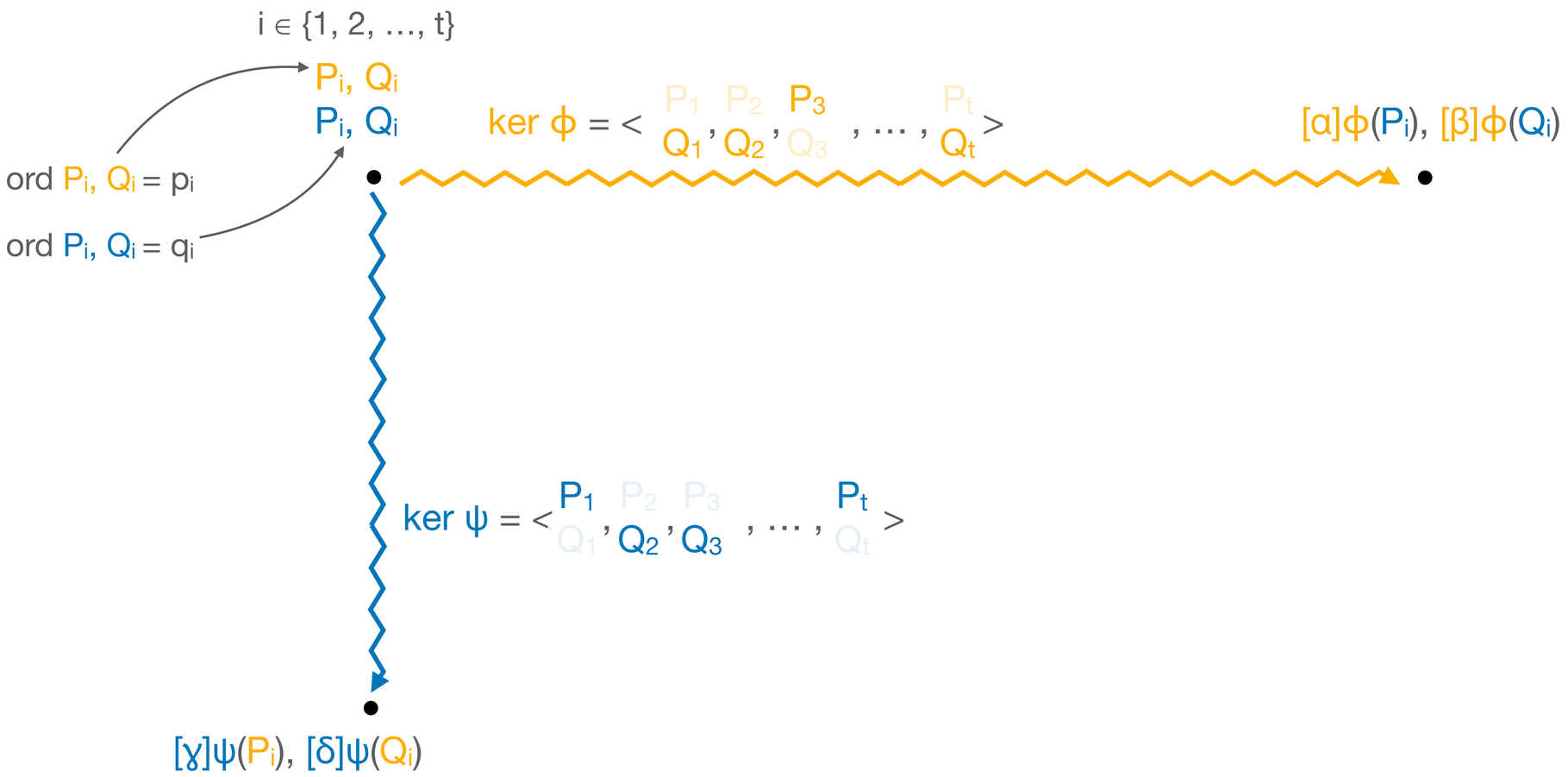

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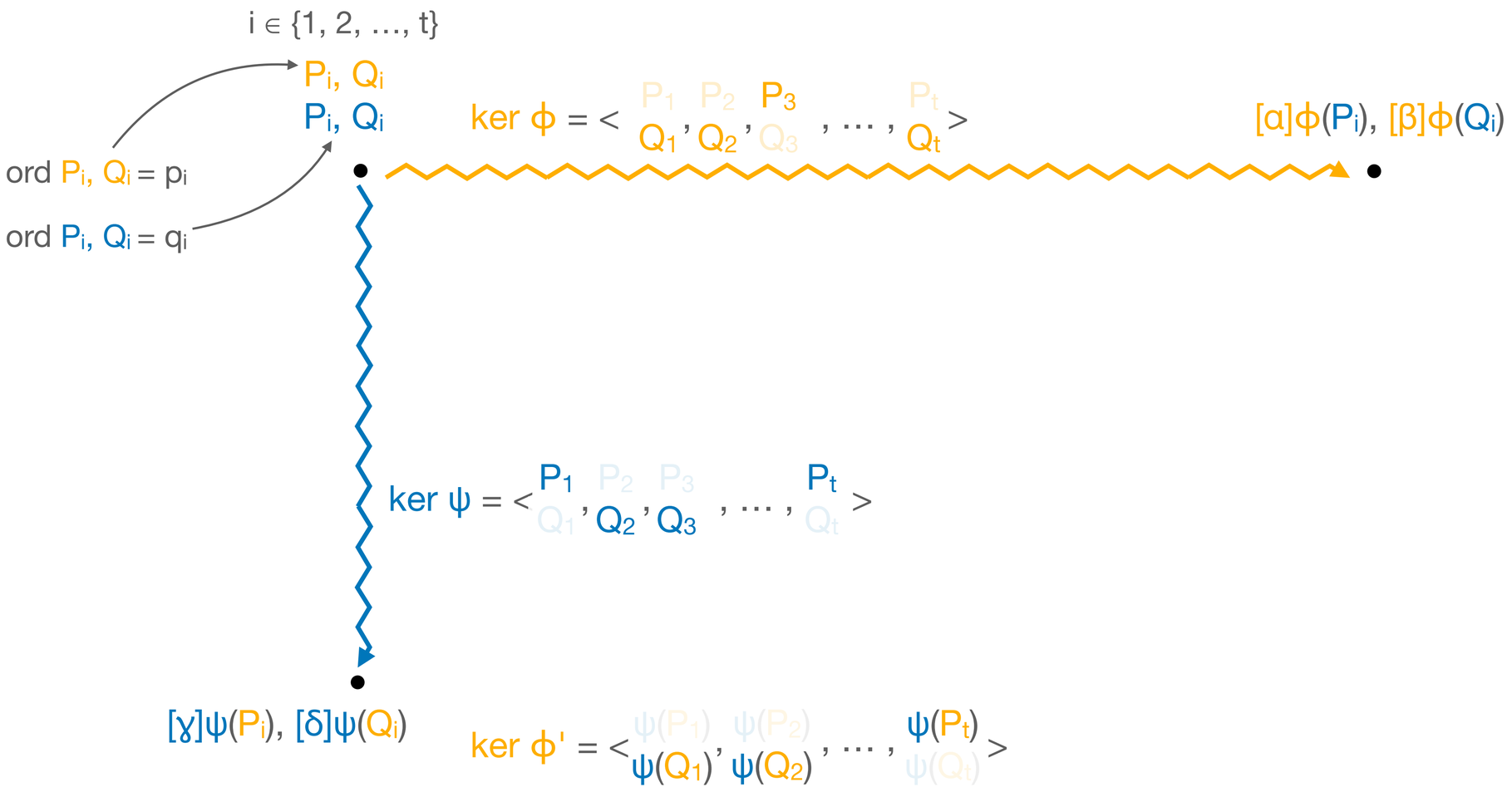

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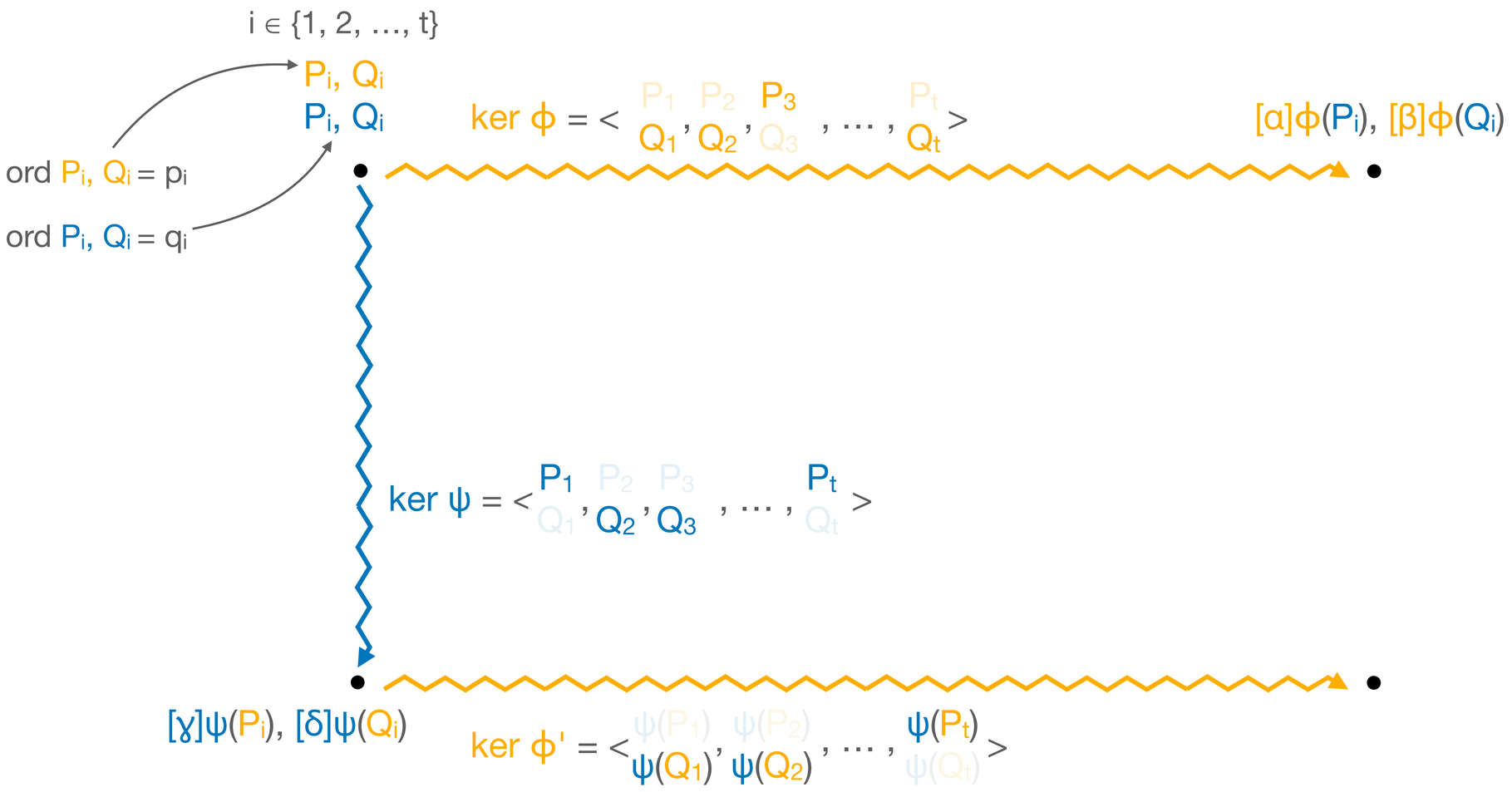

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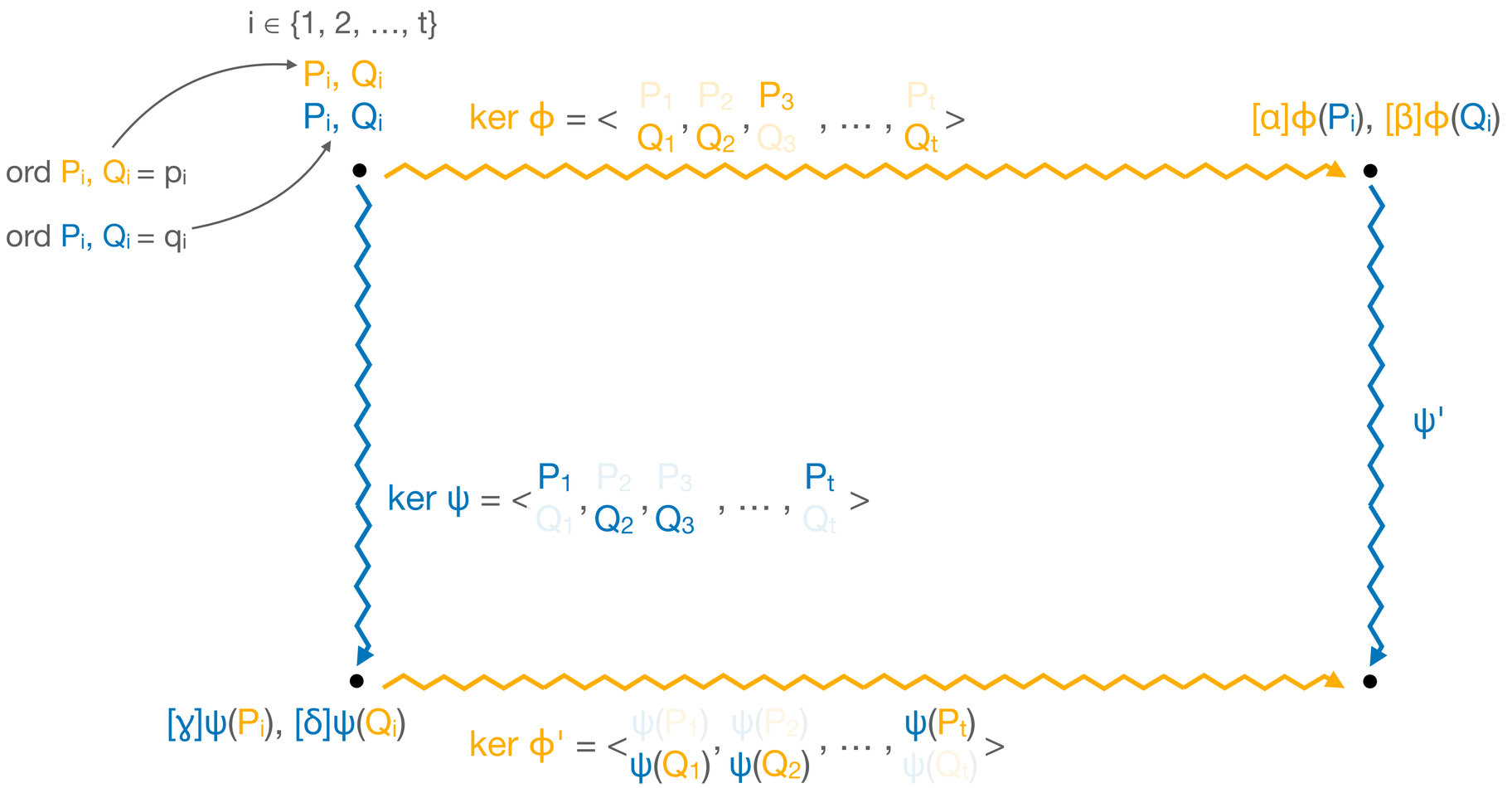

Is it secure?

Compute ϕ, given:

- **E_0, E_1,**
- **P, Q,**
- **[α]ϕ(P), [β]ϕ(Q),**
- **deg ϕ**

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only 2 subgroups!

A hybrid construction

What if Alice computes bin/ter-SIDH isogenies and Bob computes SIDH-like isogenies?

1. bin/terSIDH isogenies are quite long
2. We can safely reveal the exact torsion images of some small points (with current params, up to $2^{2\lambda}$ is safe)
3. Other party can compute SIDH-like isogenies

Results

			Timings (s)			
	λ	$\log p$	KeyGen_A	KeyGen_B	$\mathsf{SharedKey}_A$	$\mathsf{SharedKey}_B$
$\mathsf{binSIDH}^{\mathsf{hyb}}$	128	2004	0.23	14.33	0.22	10.66
	192	3126	0.62	56.77	0.61	42.85
	256	4267	1.41	157.58	1.34	117.07
$\mathsf{terSIDH}^{\mathsf{hyb}}$	**128**	**1532**	**0.16**	**3.21**	**0.16**	**1.96**
	192	2373	0.47	13.44	0.44	10.01
	256	3216	0.94	34.66	0.90	23.57

Conclusion

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3. Faster and more compact than previous constructions, with encouraging results for terSIDH-hyb