

# Tighter Security for Generic Authenticated Key Exchange in the QROM



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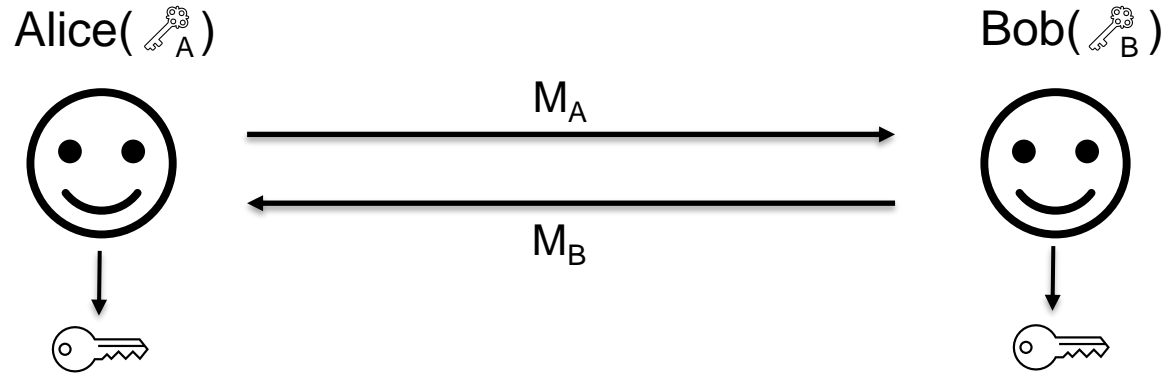
U N I K A S S E L  
V E R S I T Ä T



# Authenticated Key Exchange

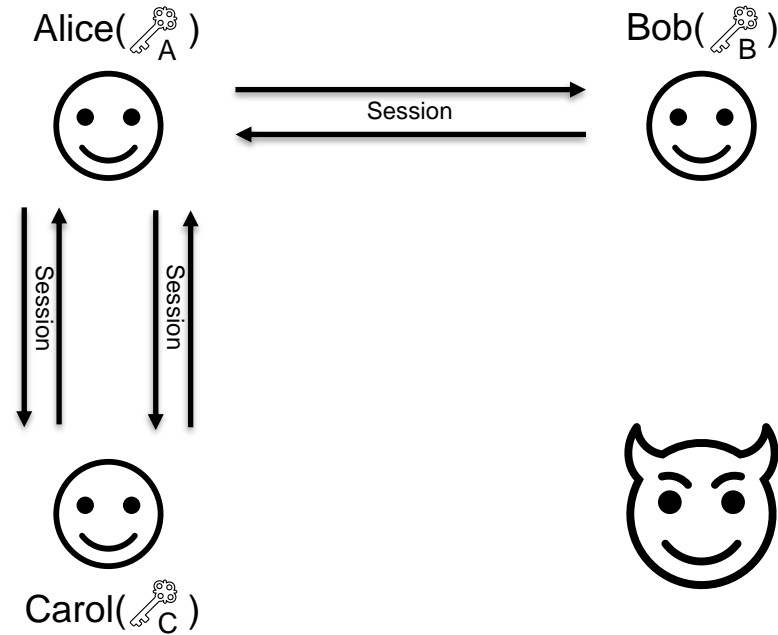


# Two-message AKE



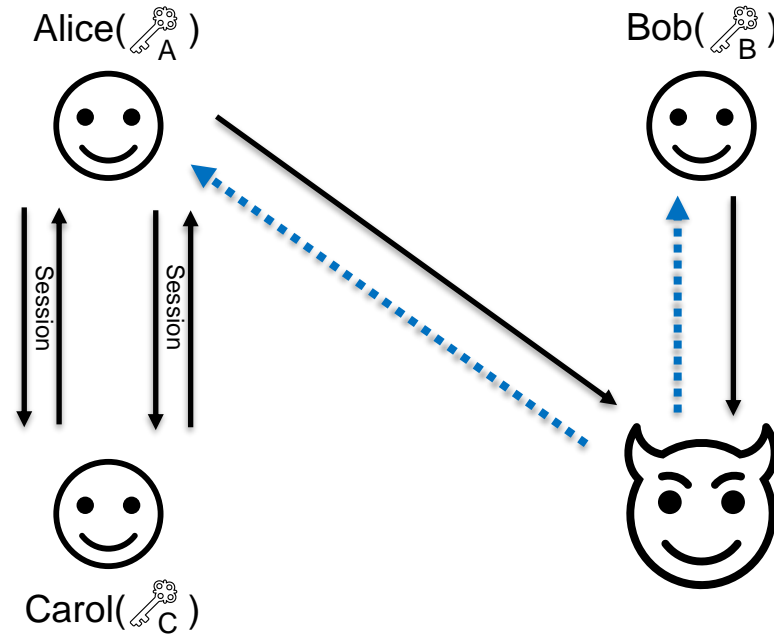
# Security of AKE

- Multi-user and Multi-session Settings



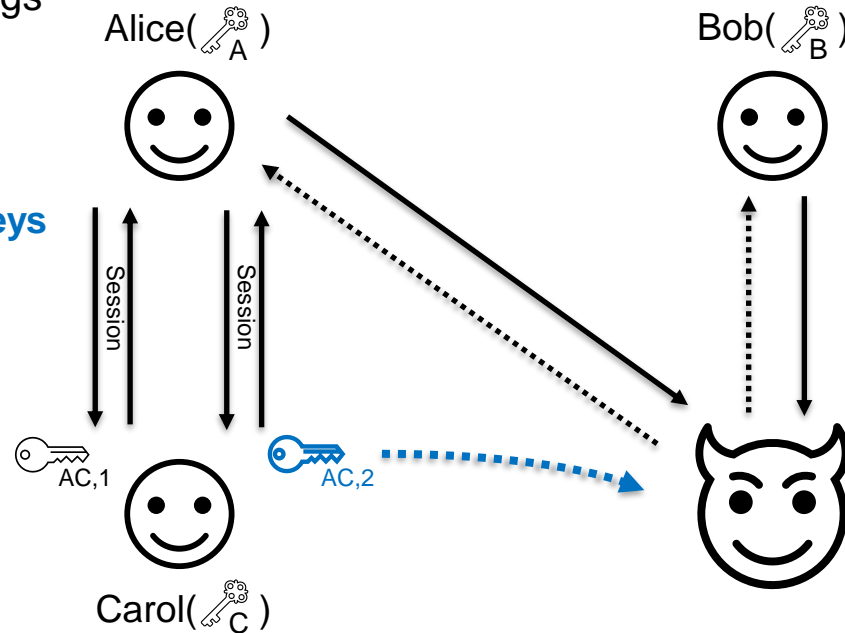
# Security of AKE

- Multi-user and Multi-session Settings
- Adversary Capabilities
  - **Control the network**



# Security of AKE

- Multi-user and Multi-session Settings
- Adversary Capabilities
  - Control the network
  - **Reveal established session keys**

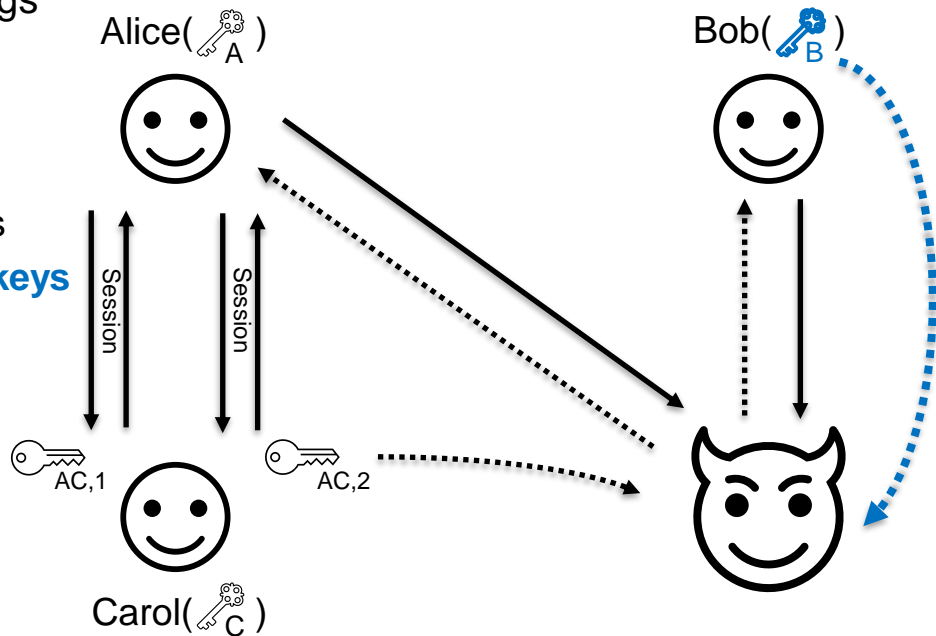


# Security of AKE

- Multi-user and Multi-session Settings

- Adversary Capabilities

- Control the network
- Reveal established session keys
- **Adaptively corrupt long-term keys**



# Security of AKE

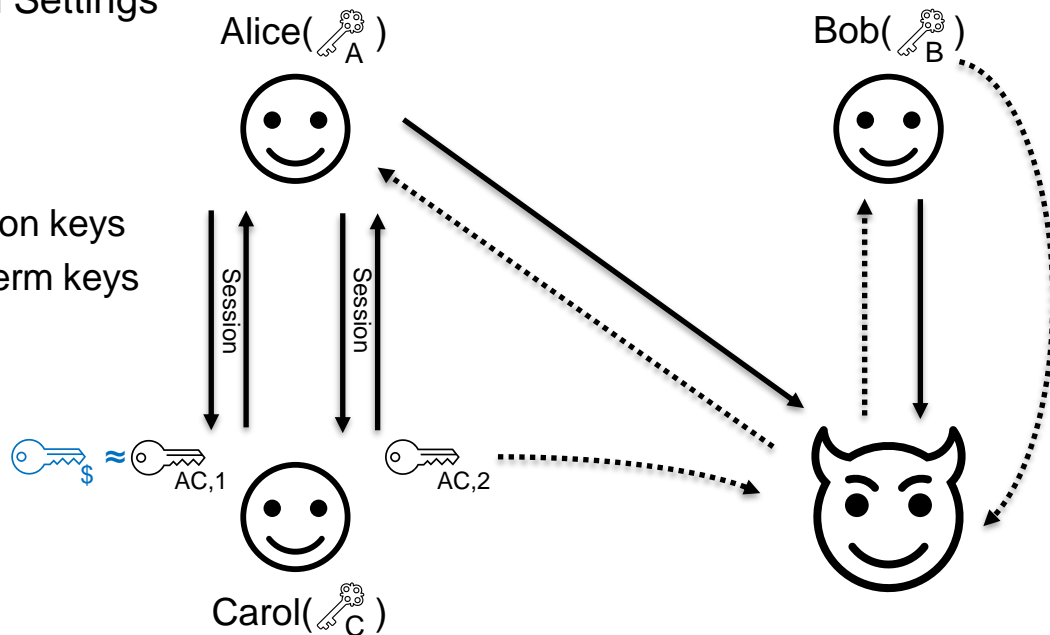
- Multi-user and Multi-session Settings

- Adversary Capabilities

- Control the network
- Reveal established session keys
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- Security Goals

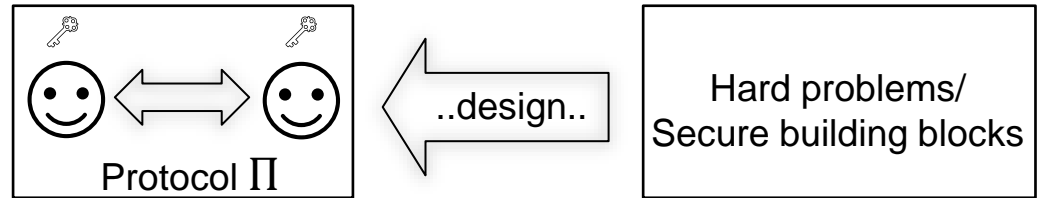
- Key Indistinguishability
- Authentication





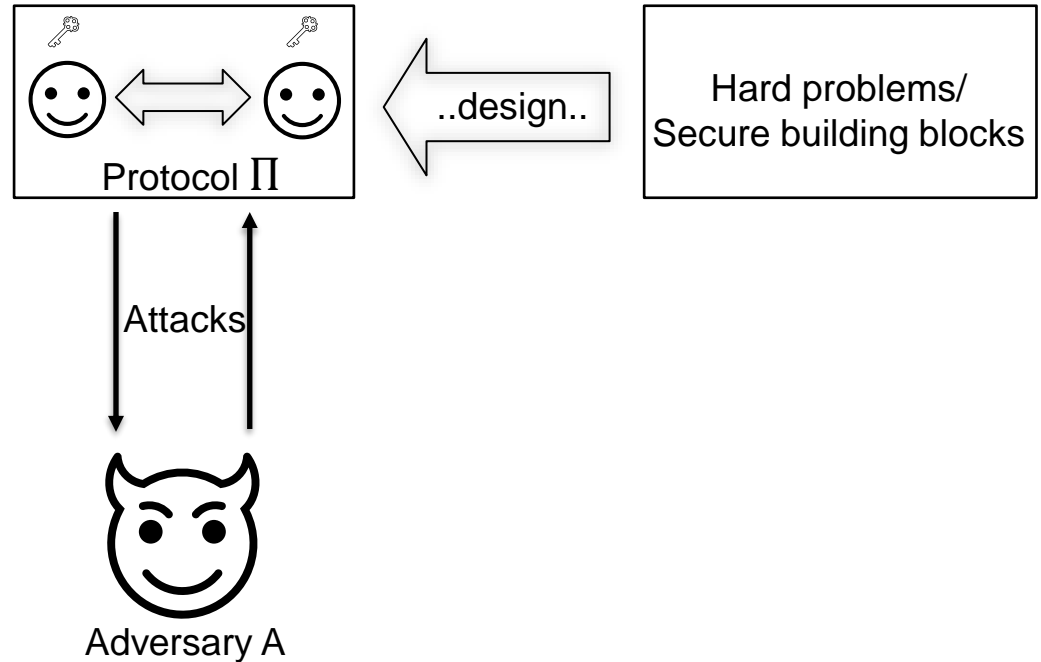
# Tightness of Security Reduction

- Security Proof via Reduction



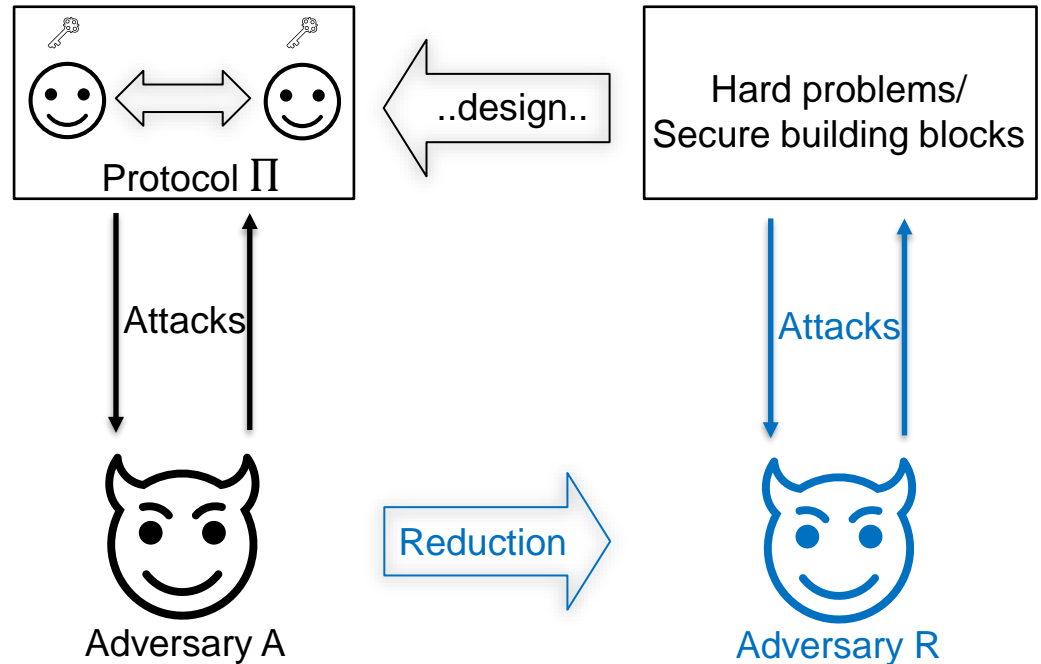
# Tightness of Security Reduction

- Security Proof via Reduction
  - A breaks  $\Pi$



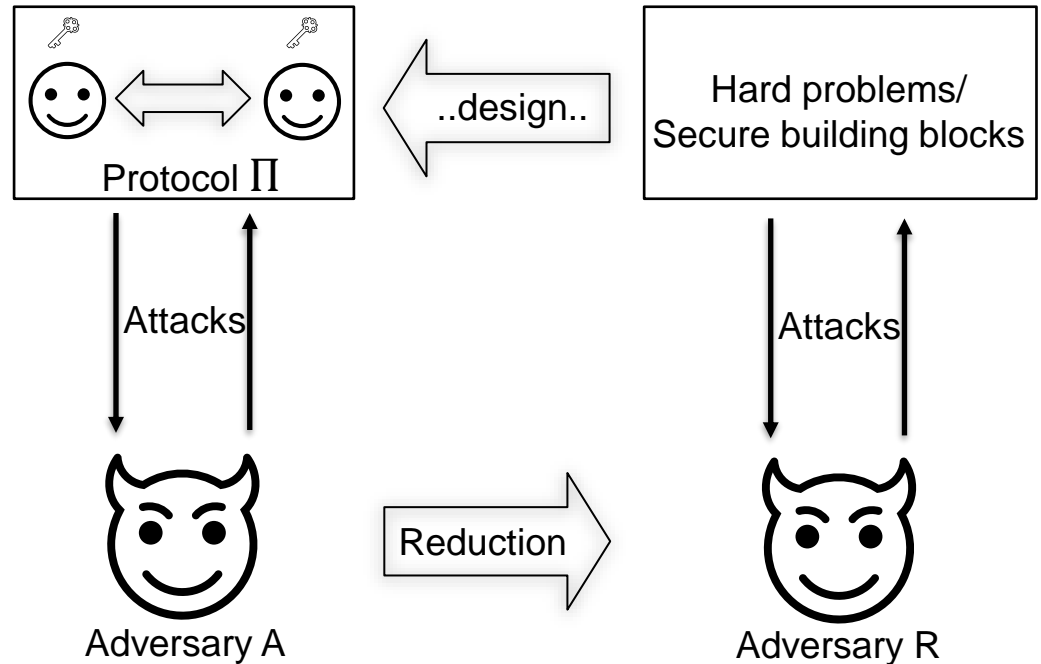
# Tightness of Security Reduction

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 $\Rightarrow R$  solves problems



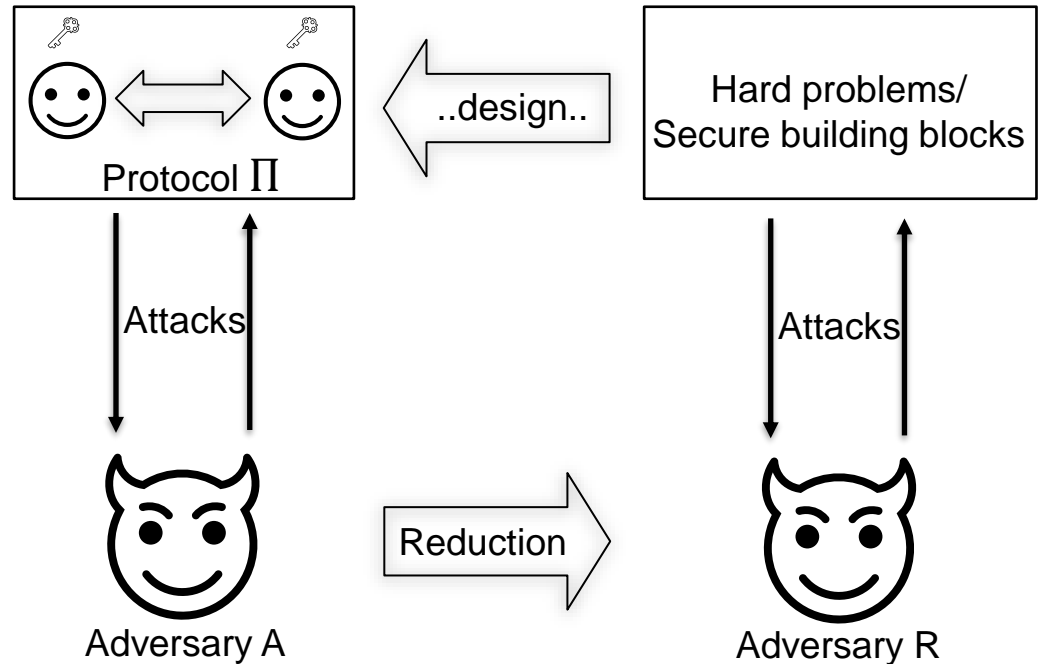
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  - $\text{Adv}(R) \leq L \cdot \text{Adv}(A)$
  - $L$ : Security loss



# Tightness of Security Reduction

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  - $\text{Adv}(R) \leq L \cdot \text{Adv}(A)$
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  - $L$  smaller  $\Rightarrow$  tighter



# Tightness of Security Reduction

- Security Proof via Reduction

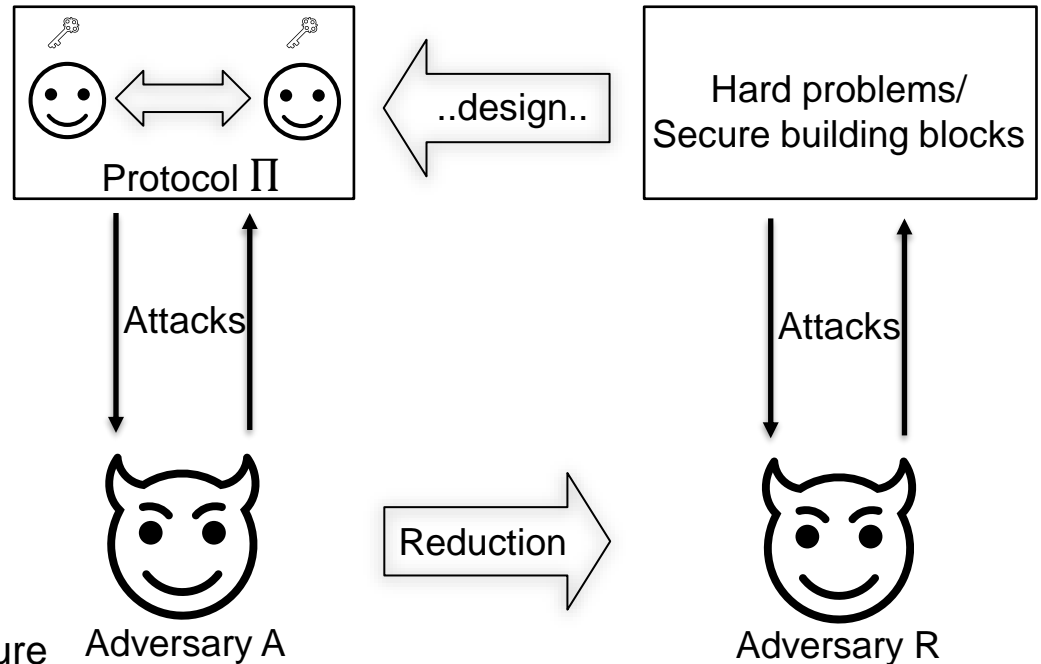
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- Tightness of Reduction

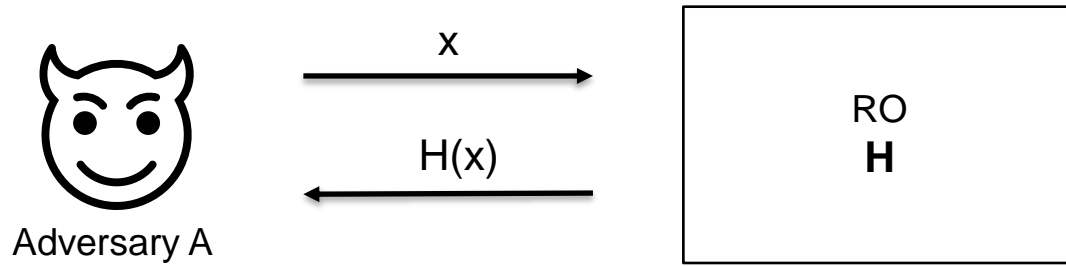
- $\text{Adv}(R) \leq L \cdot \text{Adv}(A)$
- $L$ : Security loss
- $L$  smaller  $\Rightarrow$  tighter

- Relevance: Parameter selection

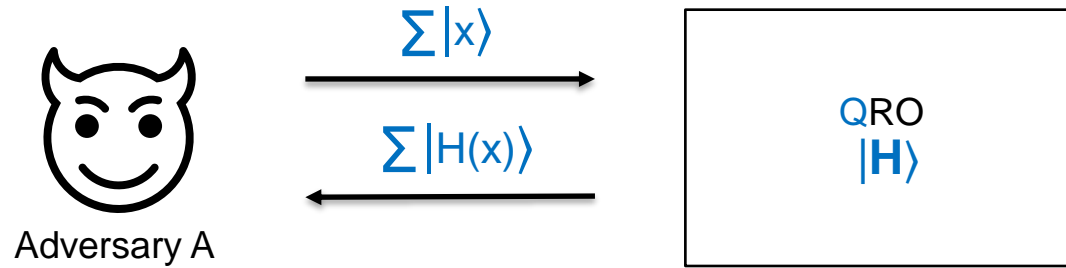
- $L$  is large  $\Rightarrow$  inefficient or insecure



# Quantum Random Oracle Model



# Quantum Random Oracle Model





# AKE in the (Q)ROM

Scheme	Construction	Assumption	Security Loss	Model
JKRS21	KEM	DDH	$\Theta(1)$	ROM
PWZ23	KEM	LWE	$\Theta(\lambda)$	ROM
HKSU20	PKE/KEM	LWE	$\Theta(N \cdot S \cdot \sqrt{\epsilon^{-1}})$	QROM
XAYLJ20	2KEM	Isogeny	$\Theta(N \cdot S \cdot \sqrt{\epsilon^{-1}})$	QROM

$\lambda$ : Security parameter

$N$ : Number of user;

$S$ : Number of session;

$\sqrt{\epsilon^{-1}}$ : Square-root security loss;

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<b>Our Goal</b>	KEM	Post-Quantum	Tight, or tighter?	QROM

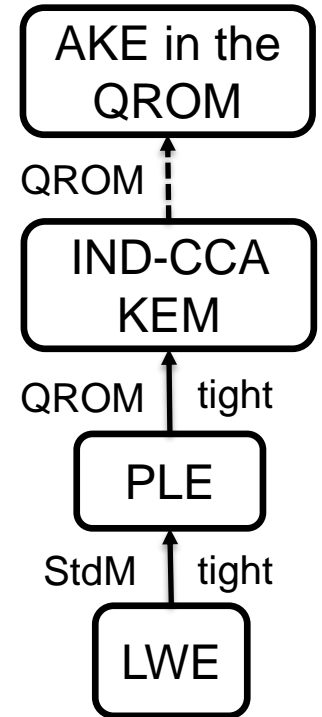
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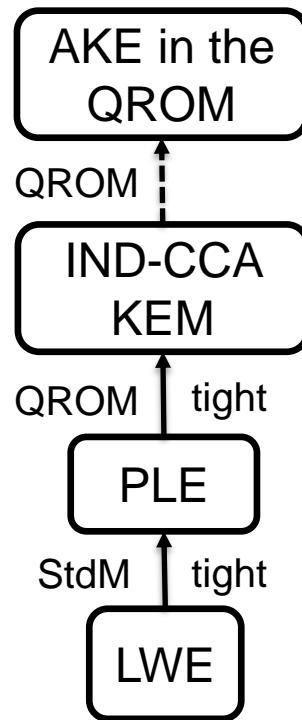
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# Our Contributions



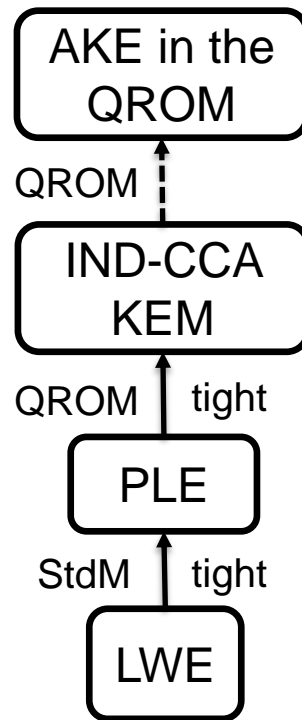
# Our Contributions

- LWE-based AKE with **Tighter** Security in the QROM
  - **Session-tight** and **without square-root loss**
  - Via multi-user-challenge(MUC)-IND-CCA secure KEM



# Our Contributions

- LWE-based AKE with **Tighter** Security in the QROM
  - **Session-tight** and **without square-root loss**
  - Via multi-user-challenge(MUC)-IND-CCA secure KEM
- Parameter-lossy Encryption (PLE)
  - Used to construct **tightly MUC-IND-CCA secure KEM**
  - (Almost-)Tight construction from LWE

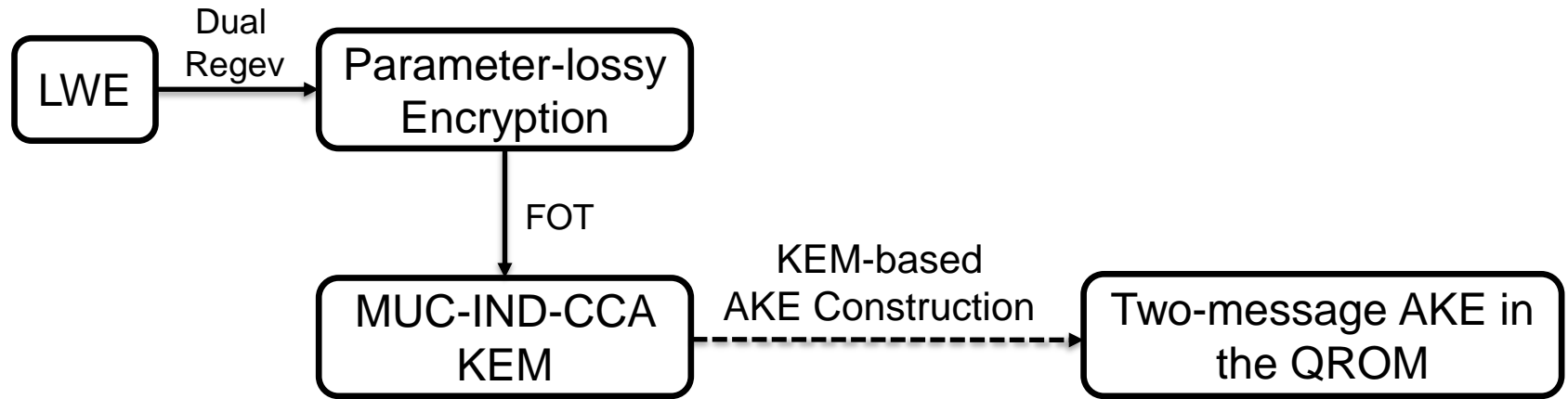


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**Session-tight** and **without square-root loss**

# Technical Outline

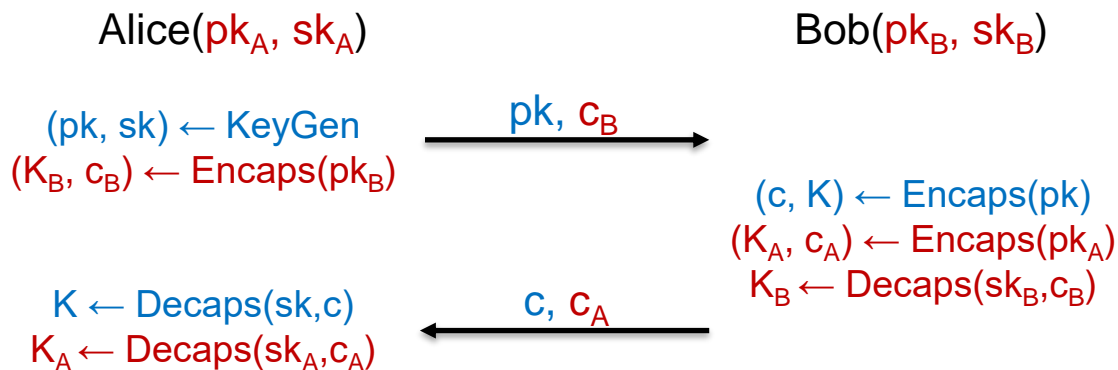


→ : (almost-)tightly

- - - - -> : non-tightly

# AKE from KEM

- Construction [JKRS21, PWZ23]: **Static KEM** + **Ephemeral KEM**

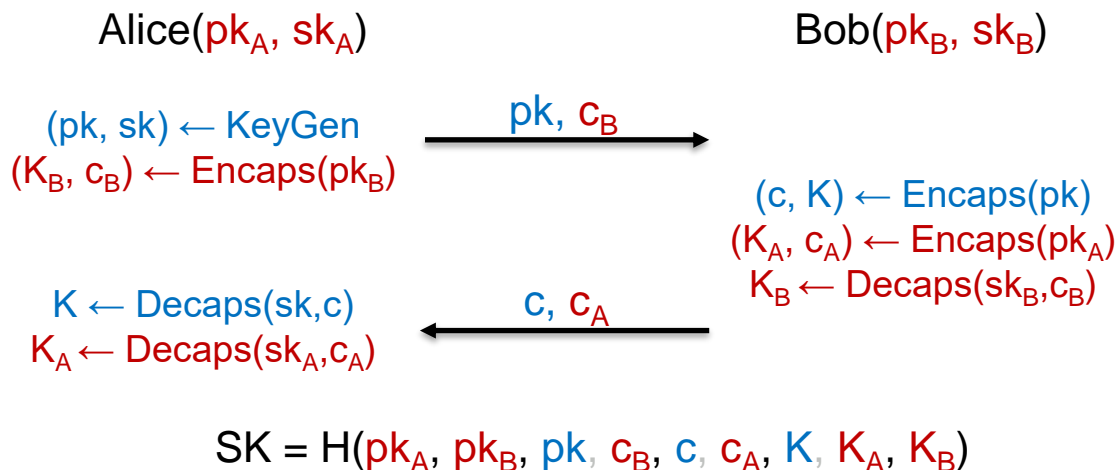


$$SK = H(pk_A, pk_B, pk, c_B, c, c_A, K, K_A, K_B)$$



# AKE from KEM

- Construction [JKRS21, PWZ23]: **Static KEM** + **Ephemeral KEM**



- Ephemeral KEM** should have **MUC-IND-CCA security**
- Static KEM** should have **MUC-IND-CCA security with strong corruptions** [JKRS21, PWZ23]

# AKE from KEM

- Construction [JKRS21, PWZ23]: **Static KEM** + **Ephemeral KEM** (in the QROM)
- To have tight security in the QROM:
  - **Ephemeral KEM** should have **MUC-IND-CCA security in the QROM**
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# AKE from KEM

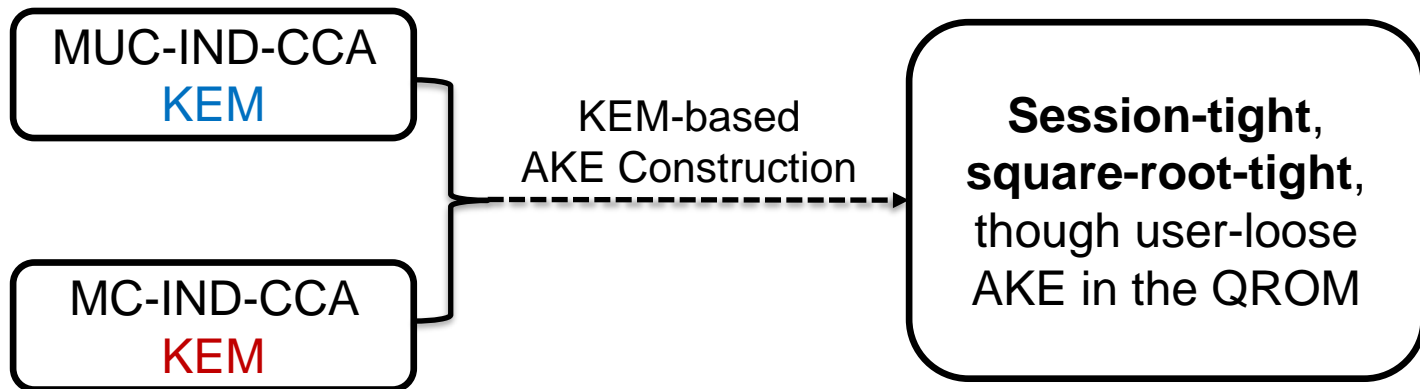
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- Both are unknown how to construct from **LWE**...
  - Cannot use the re-randomization technique
  - Unknown how to construct such **Static KEM** from **LWE** in the QROM

# AKE from KEM

- Construction [JKRS21, PWZ23]: **Static KEM** + **Ephemeral KEM** (in the QROM)
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- Both are unknown how to construct from **LWE**...
  - Cannot use the re-randomization technique
  - Unknown how to construct such **Static KEM** from **LWE** in the QROM
- If we have **MC-IND-CCA** **Static KEM** + **MUC-IND-CCA** **Ephemeral KEM**...

# AKE from KEM, tighter

- *MC-IND-CCA* **Static KEM** + *MUC-IND-CCA* **Ephemeral KEM**



# MUC-IND-CCA KEM & Parameter-lossy Encryption

MUC-IND-CCA  
KEM

MC-IND-CCA  
KEM

- Unknown how to construct from LWE (not “re-randomizable”)

# MUC-IND-CCA KEM & Parameter-lossy Encryption

MUC-IND-CCA  
KEM

MC-IND-CCA  
KEM

Parameter-lossy  
Encryption

- Unknown how to construct from LWE (not “re-randomizable”)
- Parameter-lossy Encryption
  - A multi-user version of lossy encryption [BHY09]
  - Lossy public keys & **lossy parameter**

# MUC-IND-CCA KEM & Parameter-lossy Encryption

- Lossy Encryption

1. Key indistinguishability:

$$\text{real } pk \approx_c \text{ lossy } lpk$$

2. Lossiness: (Informally) Ciphertexts have statistical indistinguishability under lossy key  $lpk...$

3.  $\Rightarrow$  MC-IND-CPA PKE



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- Parameter-lossy Encryption

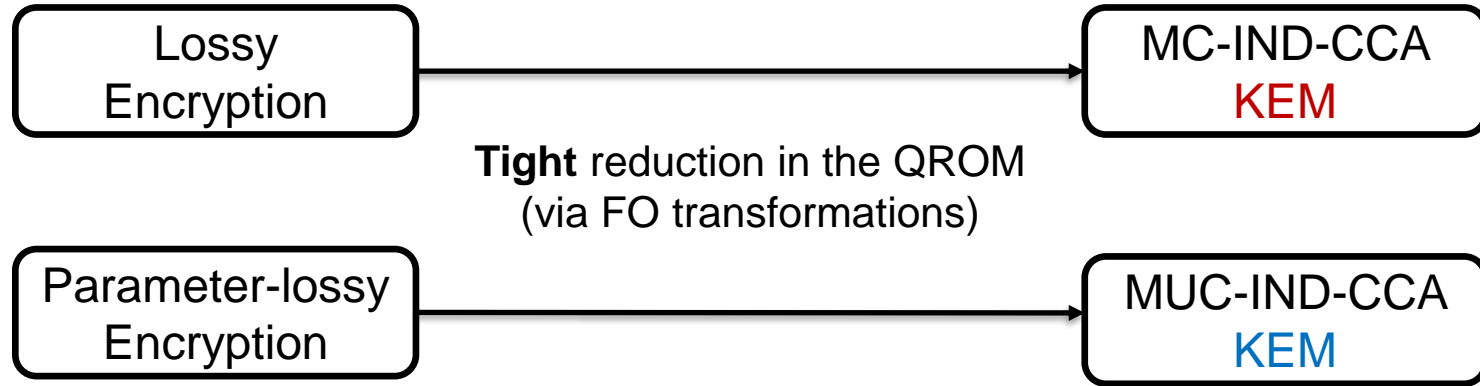
1. Parameter-key indistinguishability:

$$\text{real } (\mathbf{par}, pk_1, \dots, pk_\mu) \approx_c \text{ lossy } (\mathbf{lpar}, lpk_1, \dots, lpk_\mu)$$

2. Lossiness: Statistical indistinguishability under lossy parameter  $\mathbf{lpar}$  and lossy key  $lpk$ ...

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# MUC-IND-CCA KEM & Parameter-lossy Encryption



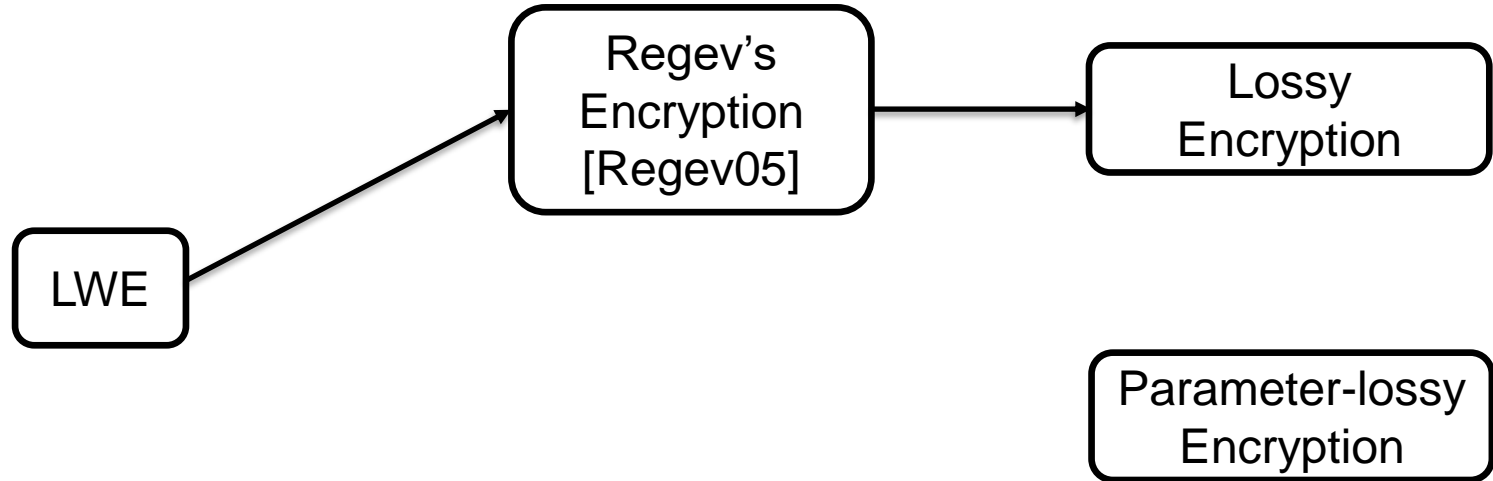
# Parameter-lossy Encryption from LWE

LWE

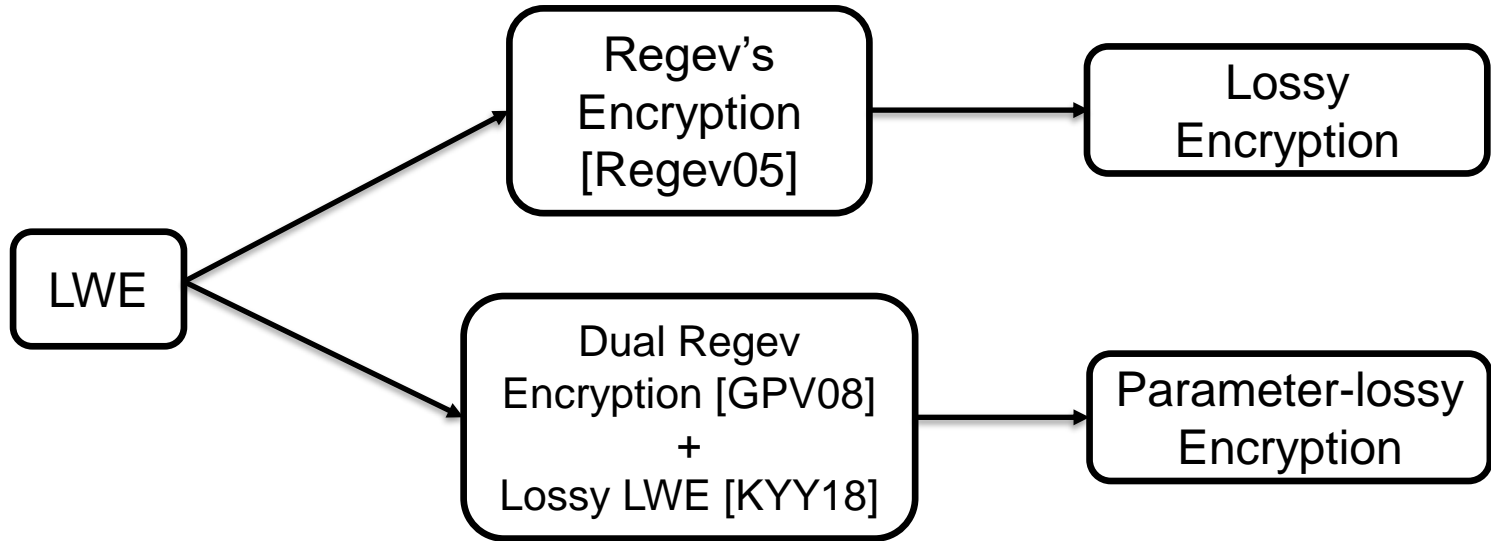
Lossy  
Encryption

Parameter-lossy  
Encryption

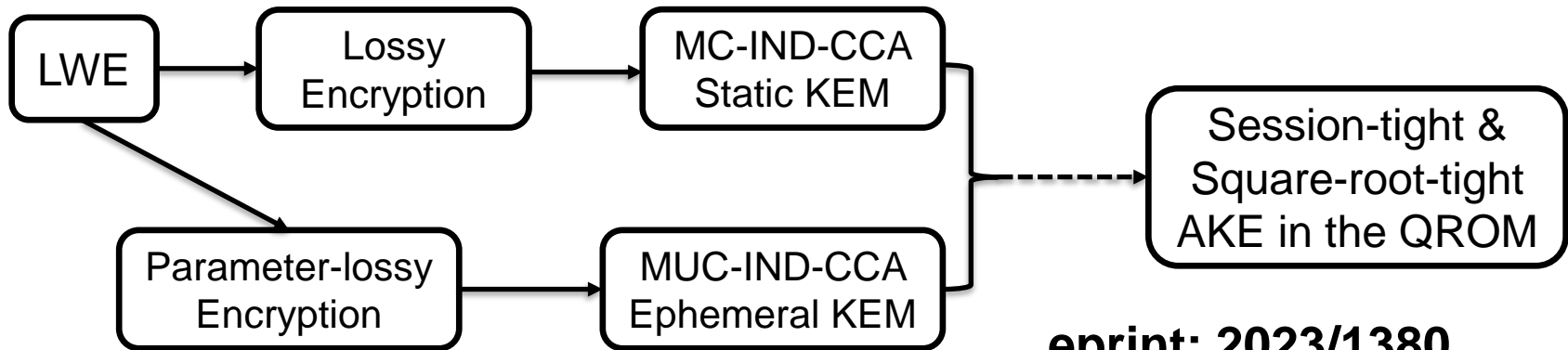
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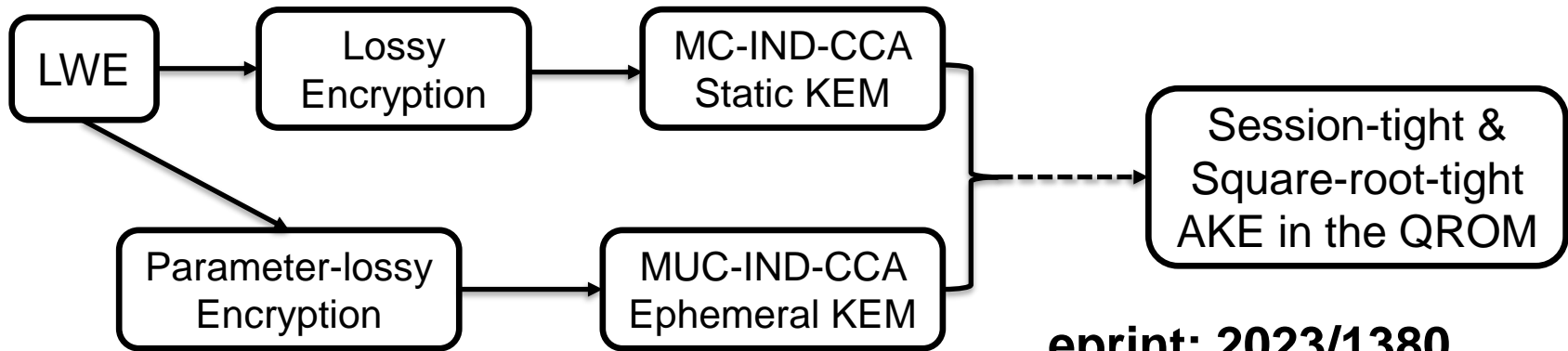
# Summary and Open Problems



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# Summary and Open Problems



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**Also user-tight in the QROM?**

# References

- KYY18 Shuichi Katsumata, Shota Yamada, Takashi Yamakawa: *Tighter security proofs for GPV-IBE in the quantum random oracle model*. ASIACRYPT 2018
- XAYLJ20 Haiyang Xue, Man Ho Au, Rupeng Yang, Bei Liang, Haodong Jiang: *Compact authenticated key exchange in the quantum random oracle model*. eprint 2018/1282
- HKSU20 Kathrin Hövelmanns, Eike Kiltz, Sven Schäge, Dominique Unruh: *Generic authenticated key exchange in the quantum random oracle model*. PKC 2020
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- PWZ23 Jiaxin Pan, Benedikt Wagner, Runzhi Zeng: *Lattice-based authenticated key exchange with tight security*. CRYPTO 2023