

# Correlation Intractability and SNARGs from Sub-exponential DDH



**Arka Rai Choudhuri**  
NTT Research



**Sanjam Garg**  
UC Berkeley and NTT  
Research



**Abhishek Jain**  
Johns Hopkins University  
and NTT Research



**Zhengzhong Jin**  
MIT



**Jiaheng Zhang**  
UC Berkeley

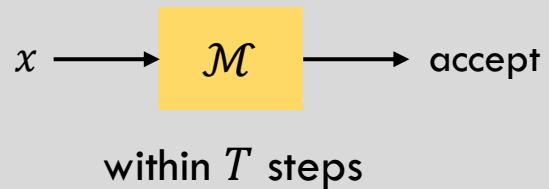
# Succinct Non-Interactive Arguments (SNARGs)



$\mathcal{M}, x$



$\mathcal{M}, x$



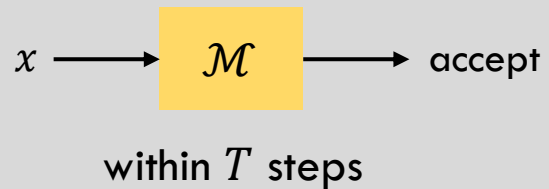
# Succinct Non-Interactive Arguments (SNARGs)



$\mathcal{M}, x$



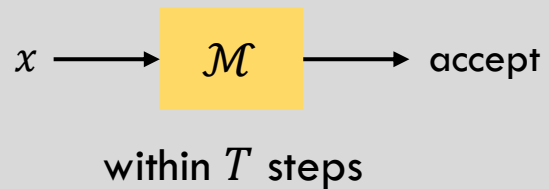
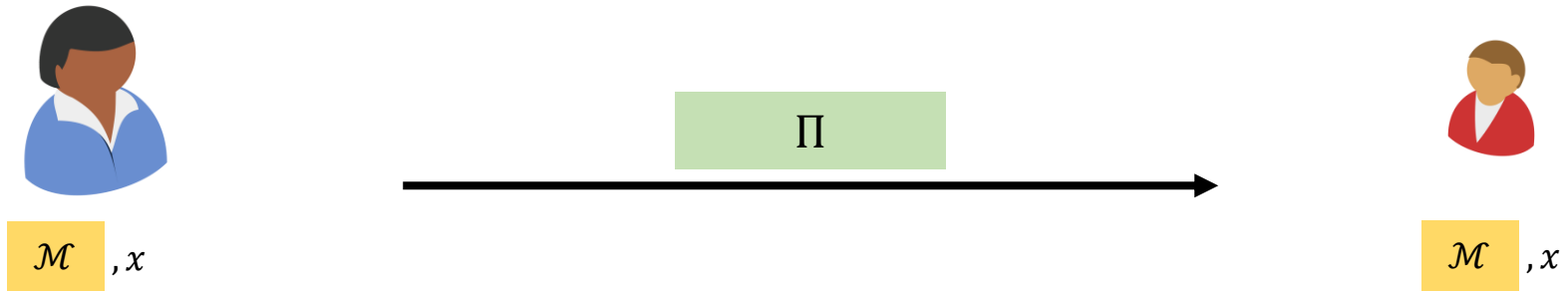
$\mathcal{M}, x$



wants to delegate computation to



# Succinct Non-Interactive Arguments (SNARGs)



# Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



$\mathcal{M}, x$

$\Pi$



$\mathcal{M}, x$

$x \longrightarrow \mathcal{M} \longrightarrow \text{accept}$

within  $T$  steps

# Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



$\mathcal{M}, x$

$\Pi$



$\mathcal{M}, x$

$\Pi$  is publicly verifiable

$x \longrightarrow \mathcal{M} \longrightarrow \text{accept}$

within  $T$  steps

# Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



$\mathcal{M}, x$

$\leftarrow \text{polylog}(T) \rightarrow$

$\Pi$



$\mathcal{M}, x$

Verifier **running time**:  
 $\text{polylog}(T)$

$\Pi$  is **publicly verifiable**

$x \rightarrow \mathcal{M} \rightarrow \text{accept}$

within  $T$  steps

# Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



$\mathcal{M}, x$

← polylog( $T$ ) →

$\Pi$



$\mathcal{M}, x$

Verifier running time:  
polylog( $T$ )

$\Pi$  is publicly verifiable

$x \rightarrow \mathcal{M} \rightarrow \text{accept}$

within  $T$  steps

No PPT  can produce accepting  $\Pi$  if

$x \rightarrow \mathcal{M} \rightarrow \text{accept}$

within  $T$  steps



# Succinct Non-Interactive Arguments (SNARGs)

Common Reference String (CRS)



$\mathcal{M}, x$

$\leftarrow \text{polylog}(T) \rightarrow$

$\Pi$



$\mathcal{M}, x$

Verifier running time:  
 $\text{polylog}(T)$

$\Pi$  is publicly verifiable

$x \rightarrow \mathcal{M} \rightarrow \text{accept}$

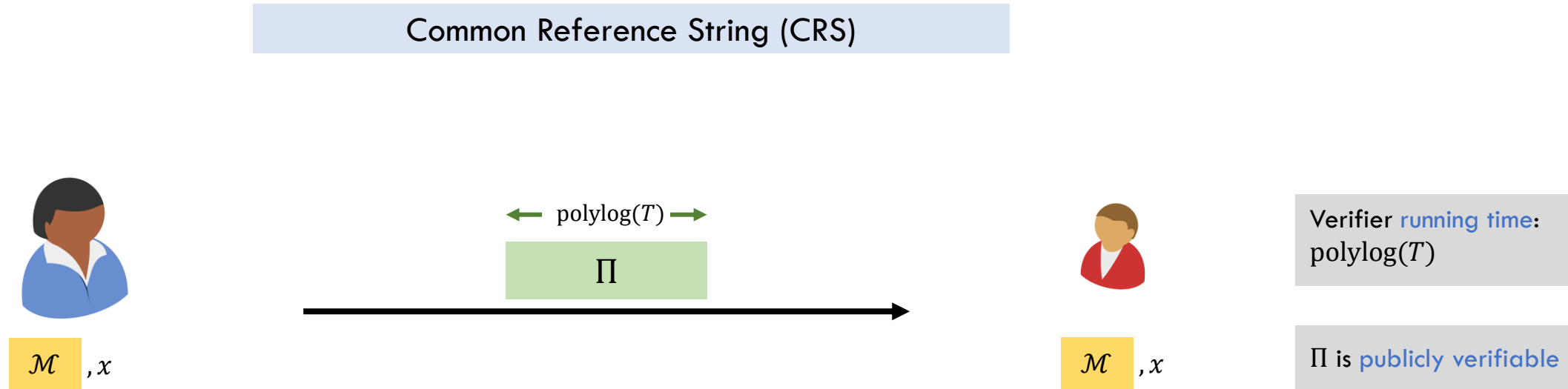
within  $T$  steps

No PPT  can produce accepting  $x, \Pi$  if

$x \rightarrow \mathcal{M} \rightarrow \text{accept}$

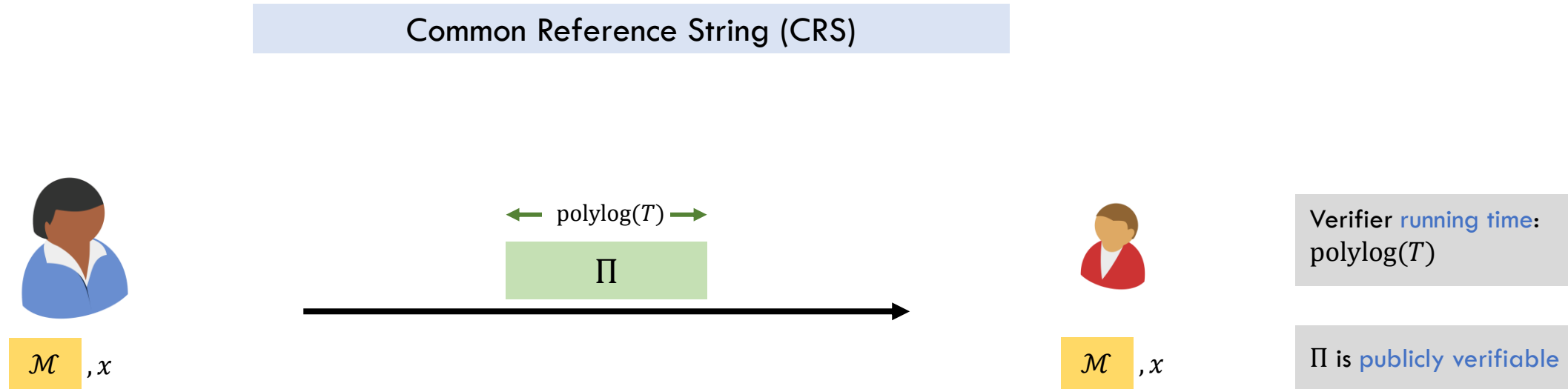
within  $T$  steps

# Succinct Non-Interactive Arguments (SNARGs)



What kind of computation can we hope to **delegate** based on **standard assumptions**?

# Succinct Non-Interactive Arguments (SNARGs)



What kind of computation can we hope to **delegate** based on **standard assumptions**?

Nondeterministic polynomial-time computation (NP)? **Unlikely!** [Gentry-Wichs'11]

# SNARGs for Batch NP (or BARGs)

CRS



$C, x_1, \dots, x_k$

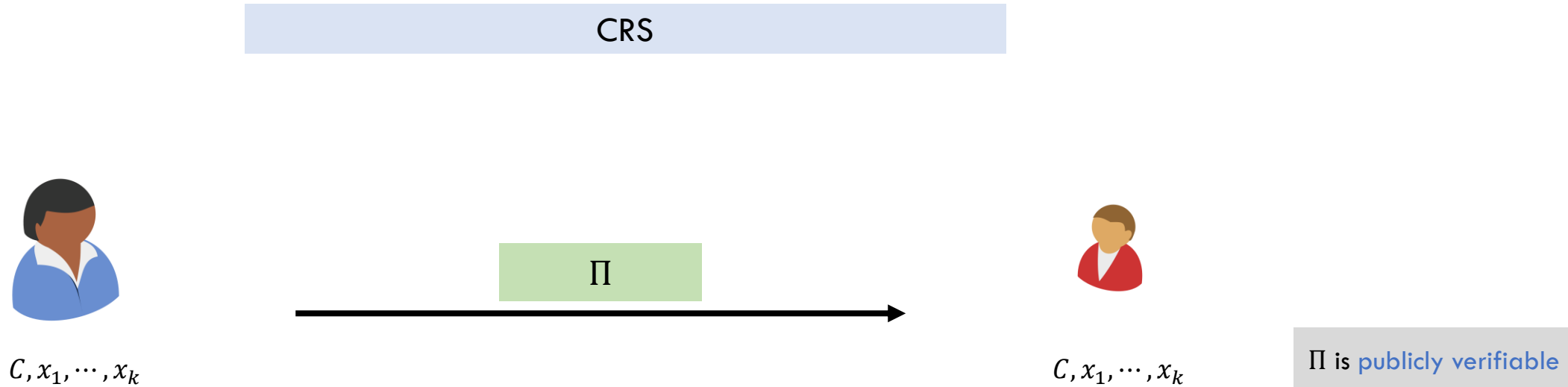


$C, x_1, \dots, x_k$

$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in \text{SAT}$

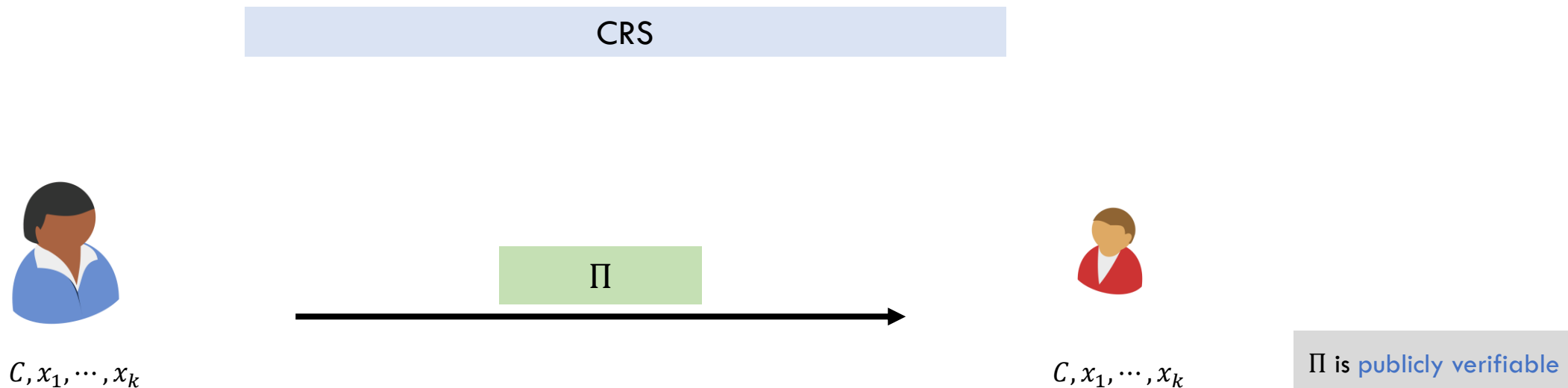
# SNARGs for Batch NP (or BARGs)



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

# SNARGs for Batch NP (or BARGs)



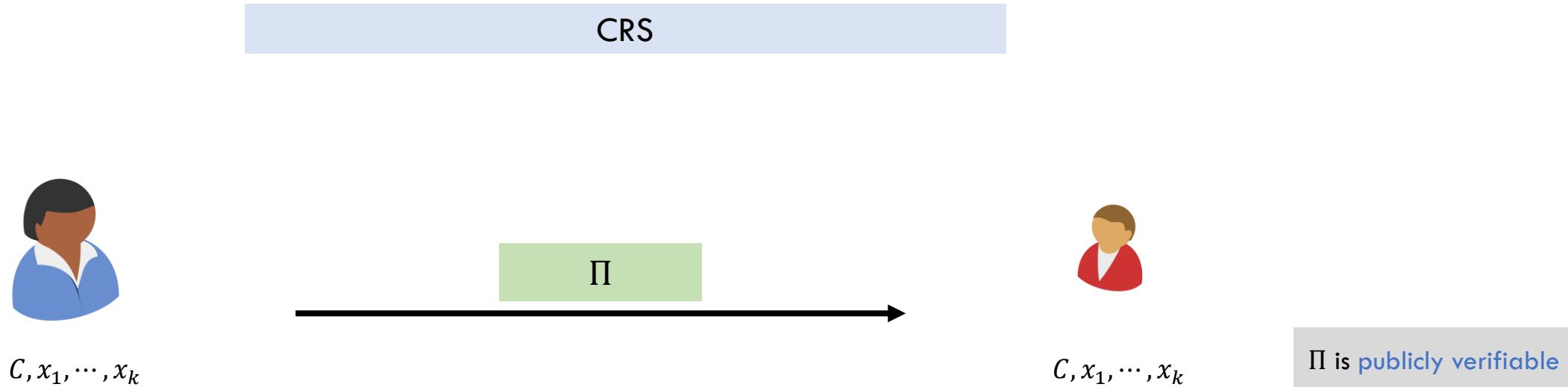
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

No PPT  can produce **accepting**  $\Pi$  if

$$\exists i^* \in [k], (C, x_{i^*}) \notin \text{SAT}$$

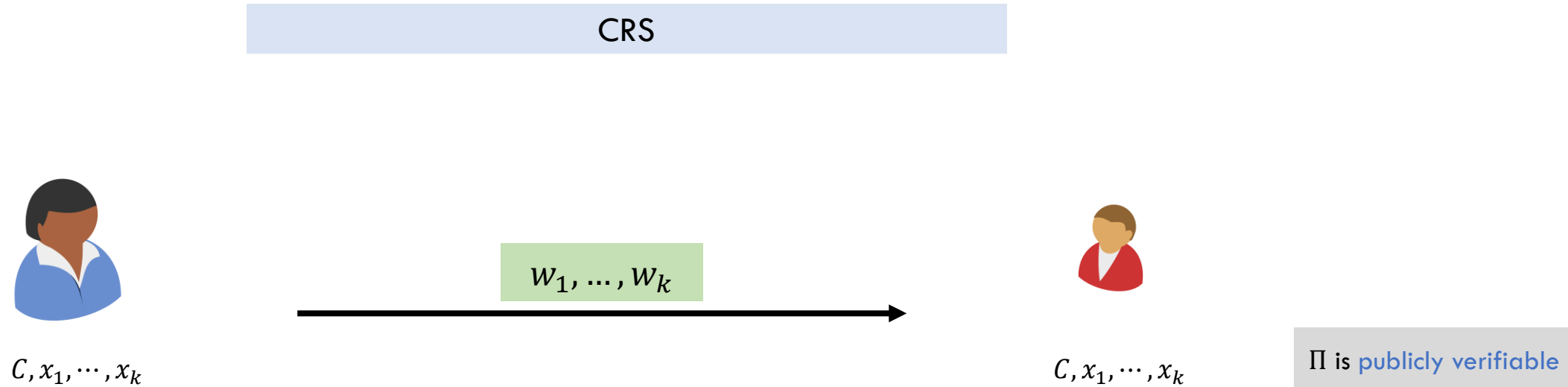
# SNARGs for Batch NP (or BARGs)



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

# SNARGs for Batch NP (or BARGs)

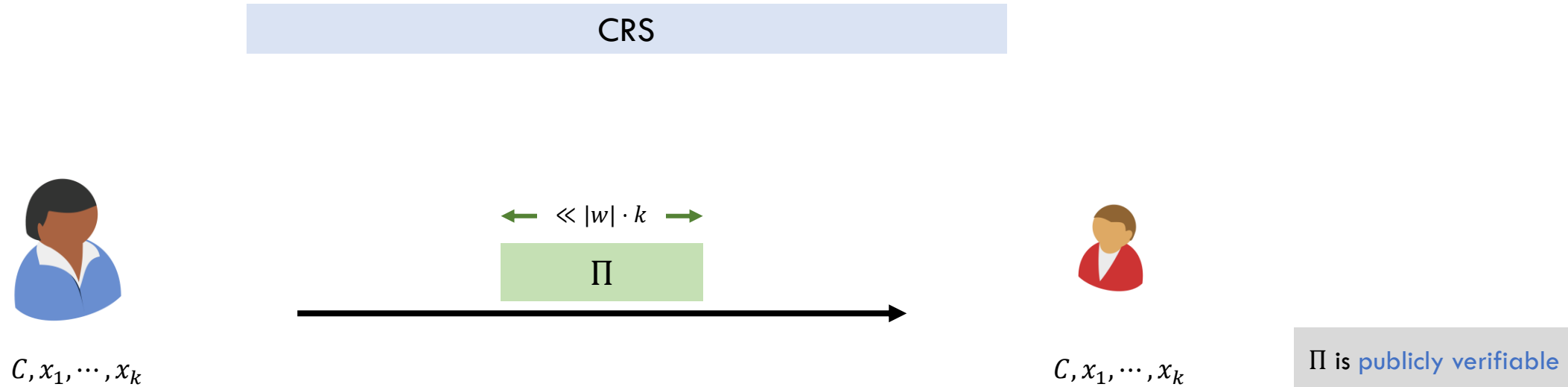


$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$



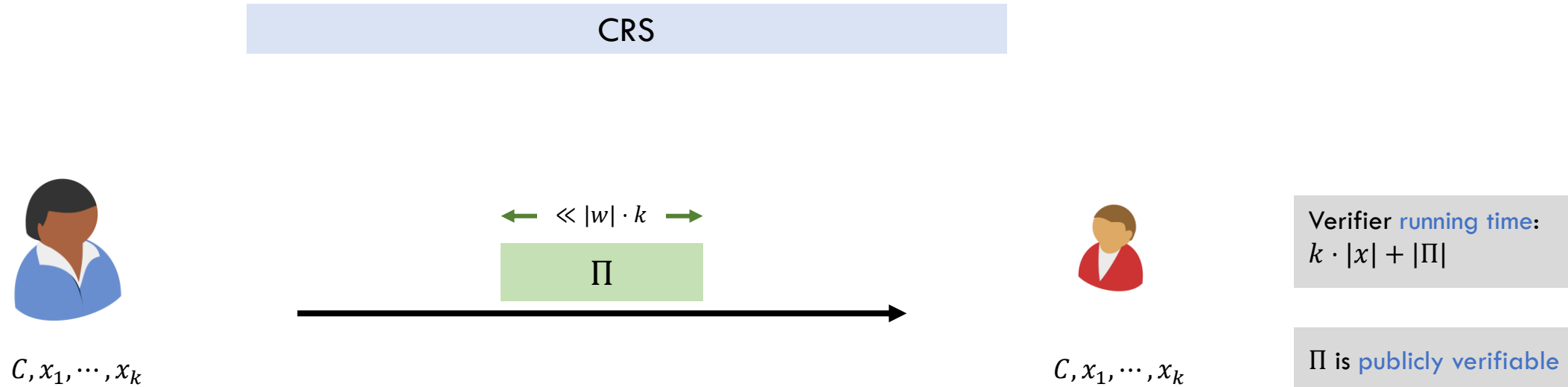
# SNARGs for Batch NP (or BARGs)



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

# SNARGs for Batch NP (or BARGs)



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

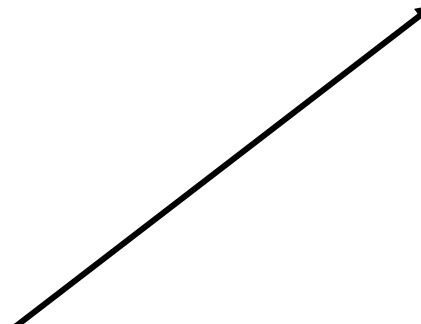
# Usefulness of BARGs



BARGs

# Usefulness of BARGs

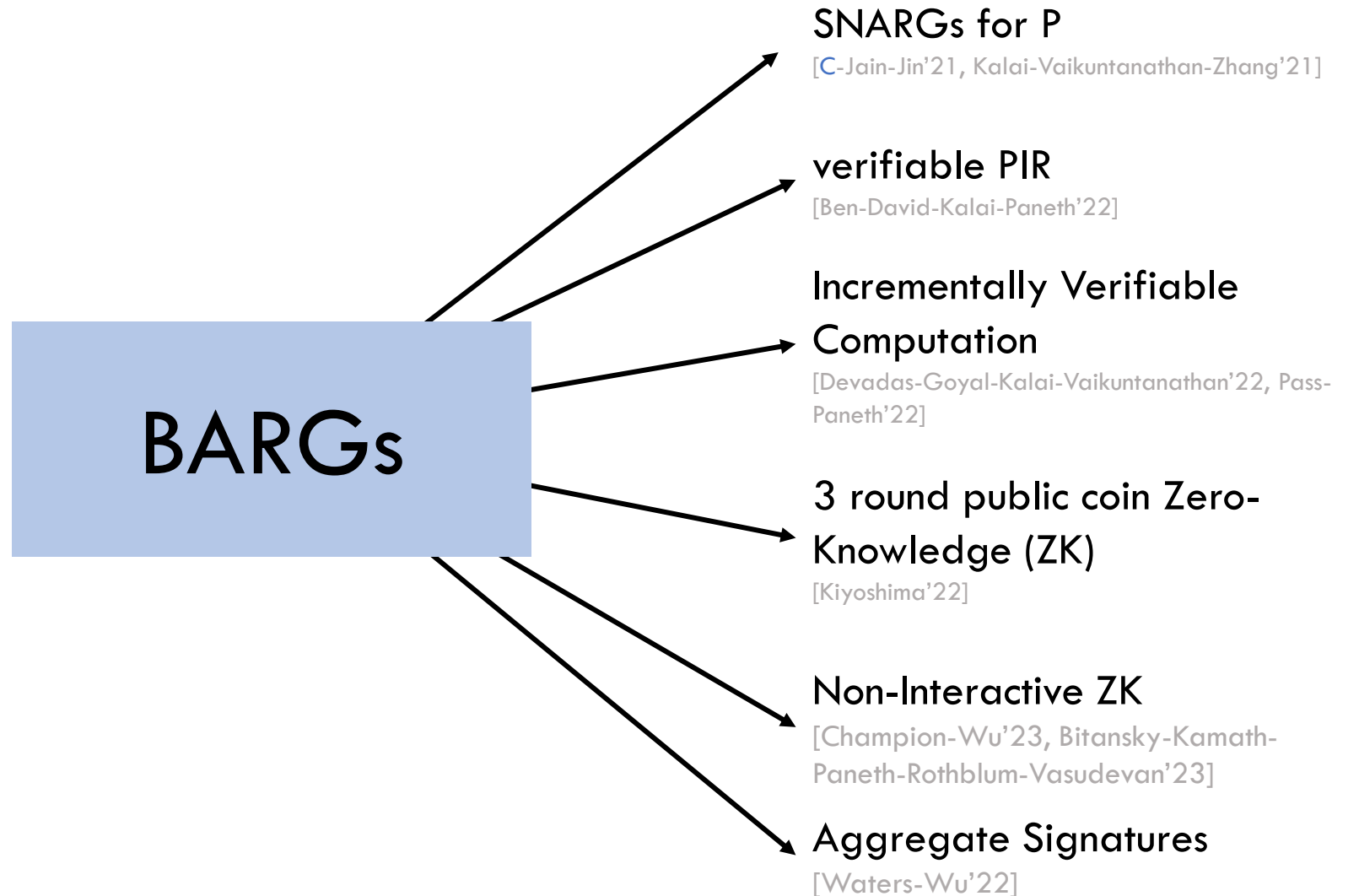
BARGs



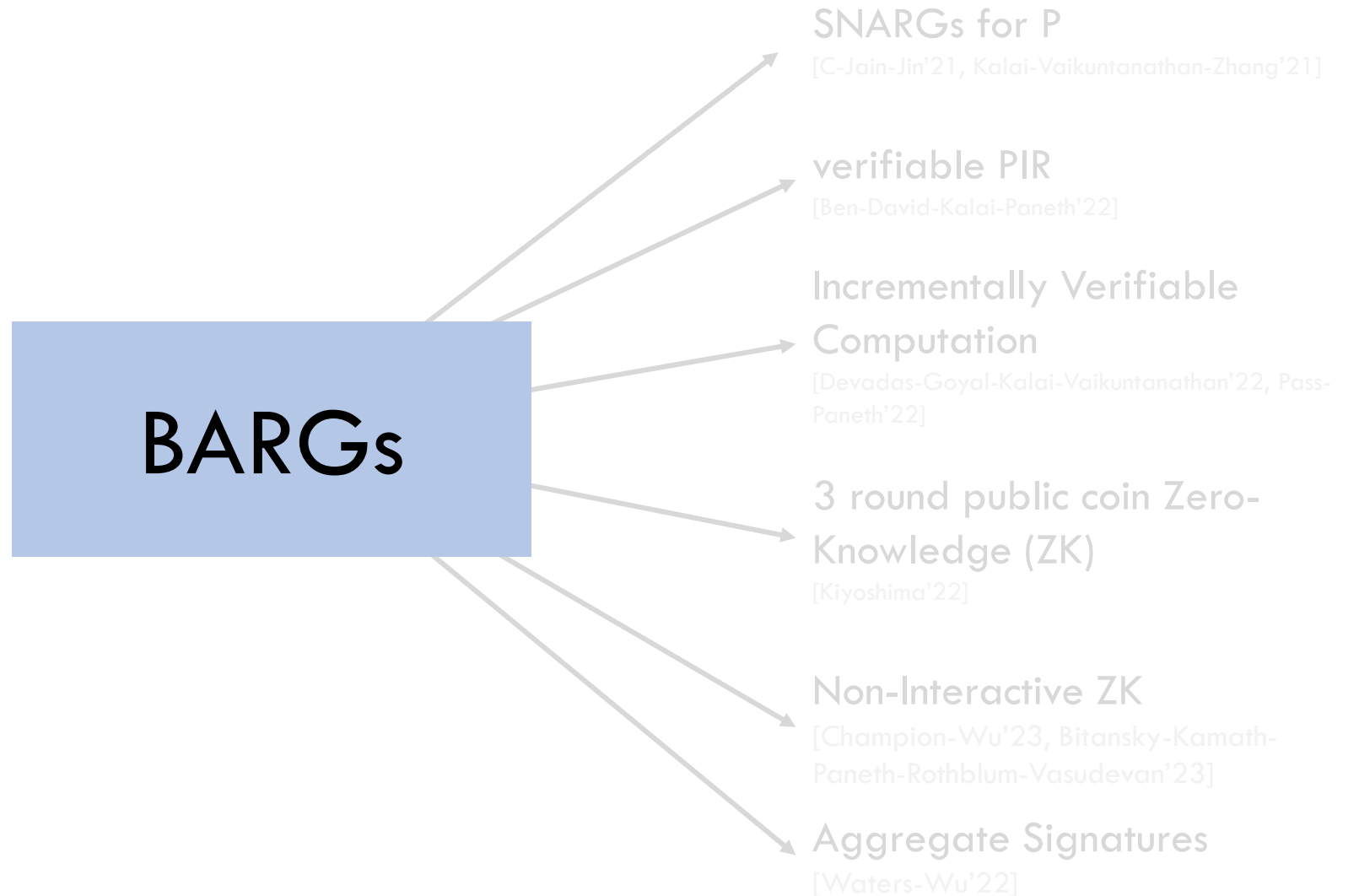
SNARGs for P

[C-Jain-Jin'21, Kalai-Vaikuntanathan-Zhang'21]

# Usefulness of BARGs



# Construction of BARGs



# Construction of BARGs

LWE

[C-Jain-Jin'21]

DLIN

[Waters-Wu'22]

Sub-exp DDH

+ QR

[C-Jain-Jin'21 $\alpha$ , Hulett-Jawale-Khurana-Srinivasan'22]

BARGs

SNARGs for P

[C-Jain-Jin'21, Kalai-Vaikuntanathan-Zhang'21]

verifiable PIR

[Ben-David-Kalai-Paneth'22]

Incrementally Verifiable

Computation

[Devadas-Goyal-Kalai-Vaikuntanathan'22, Pass-Paneth'22]

3 round public coin Zero-Knowledge (ZK)

[Kiyoshima'22]

Non-Interactive ZK

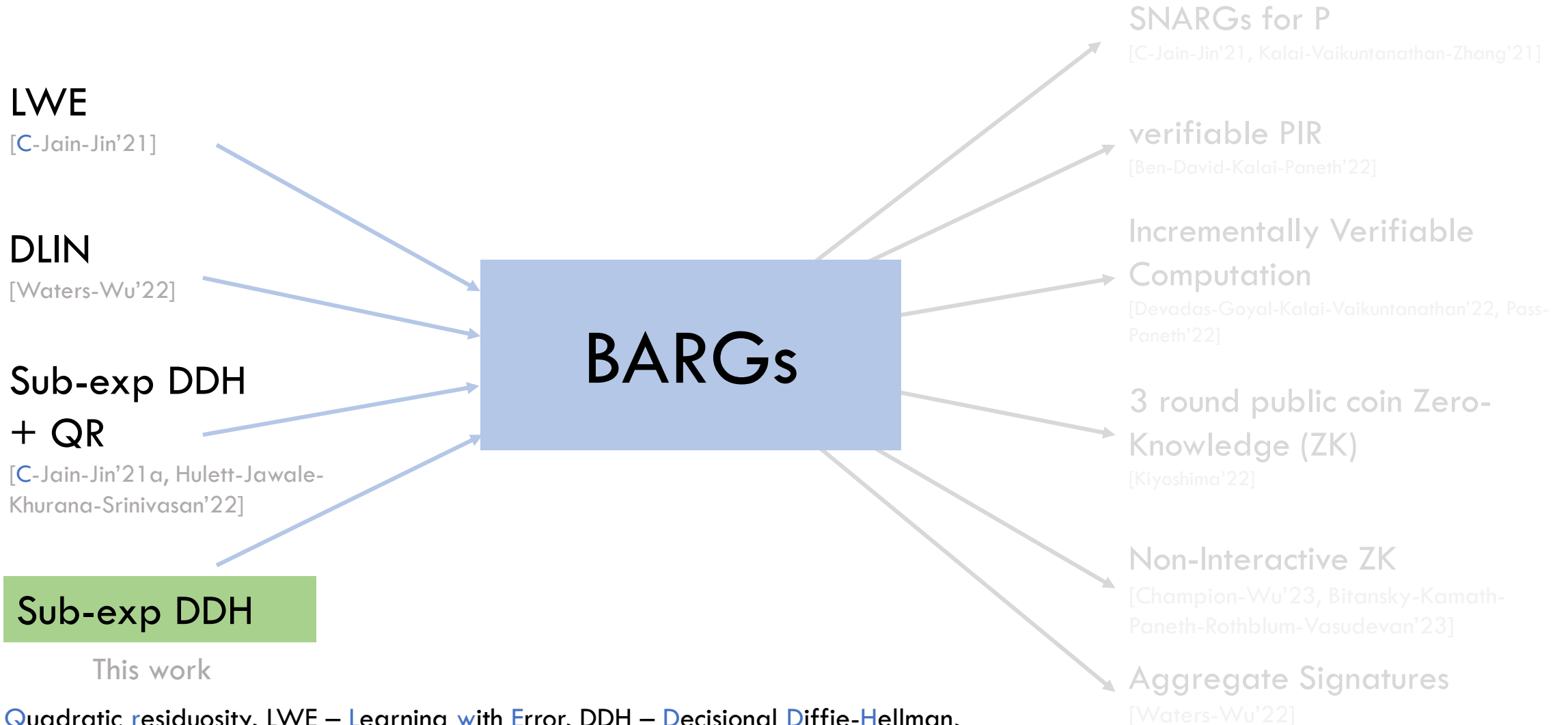
[Champion-Wu'23, Bitansky-Kamath-Paneth-Rothblum-Vasudevan'23]

Aggregate Signatures

[Waters-Wu'22]

QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman, DLIN – Decisional Linear Assumption over Bilinear Maps.

# Construction of BARGs



QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman, DLIN – Decisional Linear Assumption over Bilinear Maps.



# Our Results

## Theorem 1

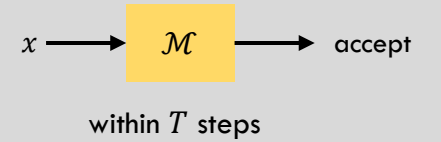
Assuming **sub-exponential hardness of DDH**, there exists **SNARGs** for **batch NP** where

$$|\Pi| = \text{poly}(\log k, |C|)$$

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

# Our Results



## Theorem 2

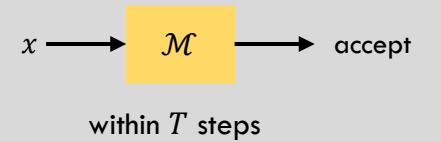
Assuming **sub-exponential hardness of DDH**, there exists **SNARGs for P** where

$$|\text{CRS}|, |\Pi|, |\text{👤}| = \text{polylog}(T)$$

# Our Results

Recent concurrent work [Kalai-Lombardi-Vaikuntanathan'23]:

SNARGs for **bounded depth circuits** assuming **sub-exponential hardness of DDH**.



## Theorem 2

Assuming **sub-exponential hardness of DDH**, there exists **SNARGs for P** where

$$|\text{CRS}|, |\Pi|, |\text{👤}| = \text{polylog}(T)$$

# Our Results

## Theorem 1

Assuming **sub-exponential hardness of DDH**, there exists SNARGs for batch NP where

$$|\Pi| = \text{poly}(\log k, |C|)$$

## Theorem 2

Assuming **sub-exponential hardness of DDH**, there exists SNARGs for P where

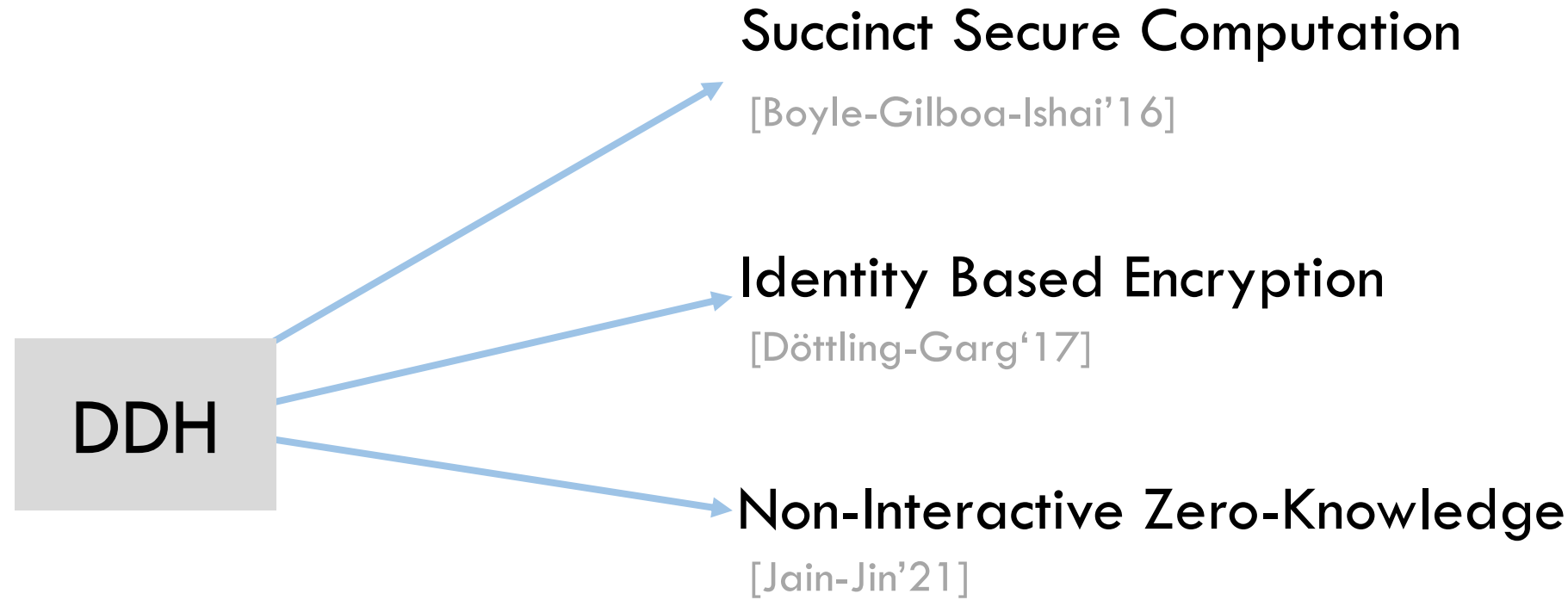
$$|\text{CRS}|, |\Pi|, |\text{decommit}| = \text{polylog}(T)$$

# Meta View: Advanced Primitives from DDH

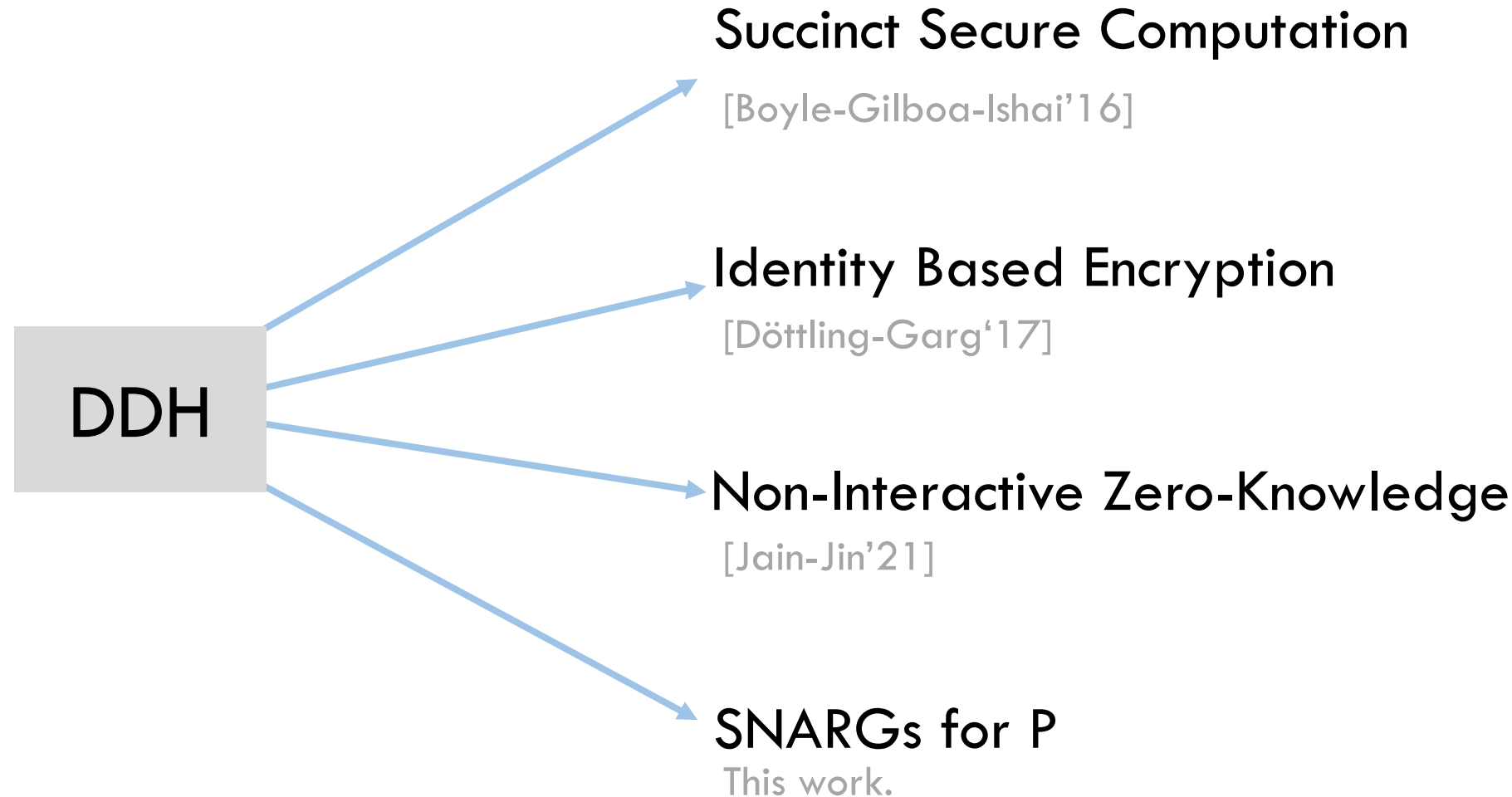


DDH

# Meta View: Advanced Primitives from DDH



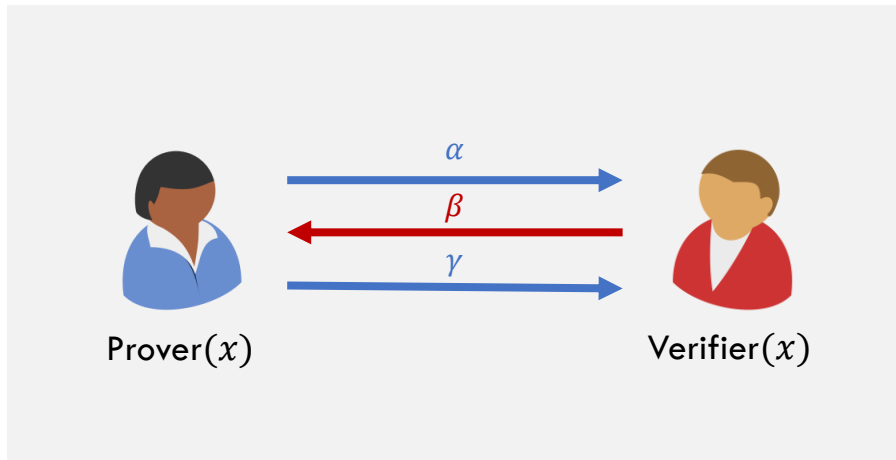
# Meta View: Advanced Primitives from DDH



# Tools and Techniques

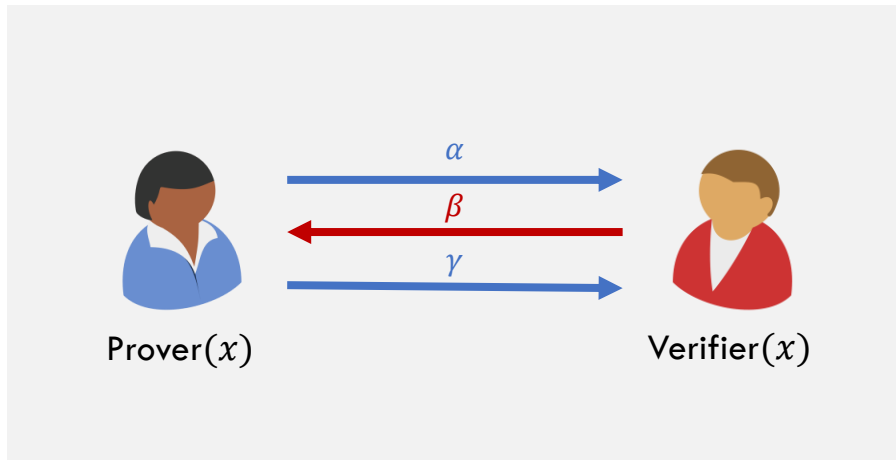


# Fiat-Shamir (FS) Methodology: Recipe for Success



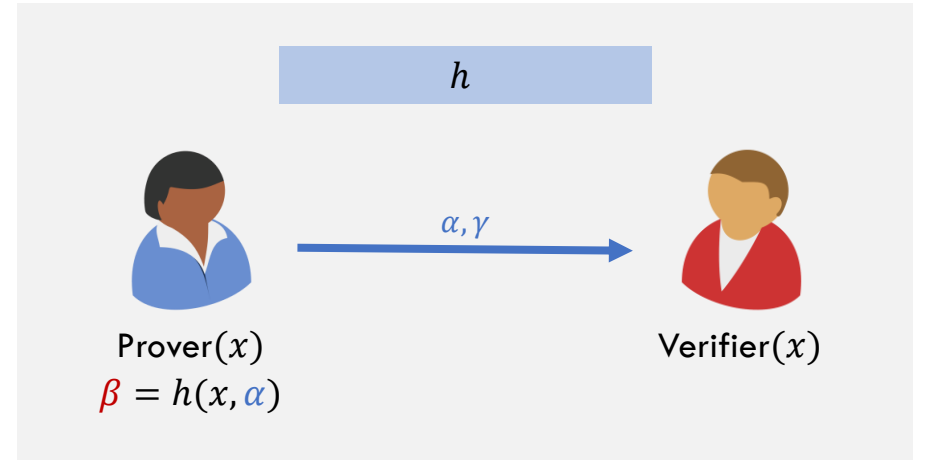
$\beta$  is a random string

# Fiat-Shamir (FS) Methodology

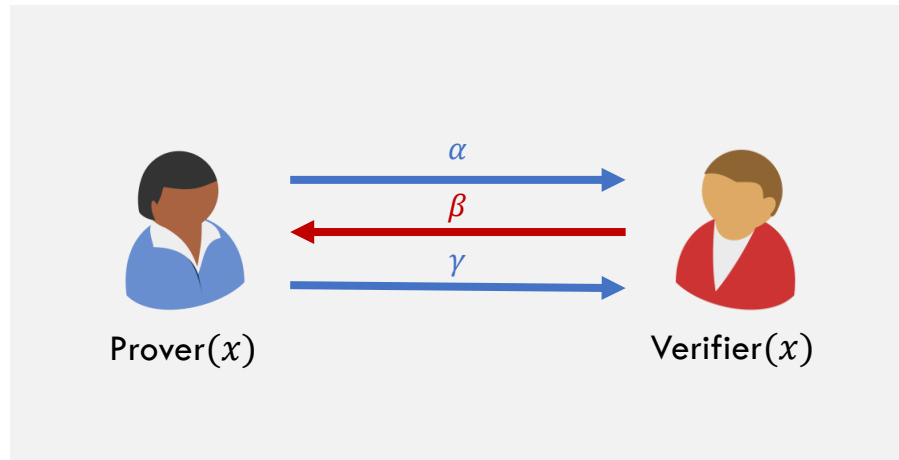


$\beta$  is a random string

→  
[Fiat-Shamir'86]

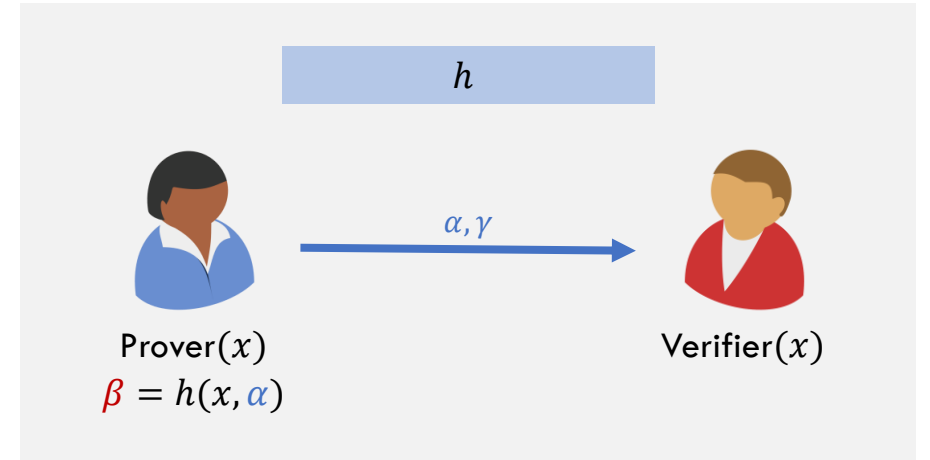


# Fiat-Shamir (FS) Methodology



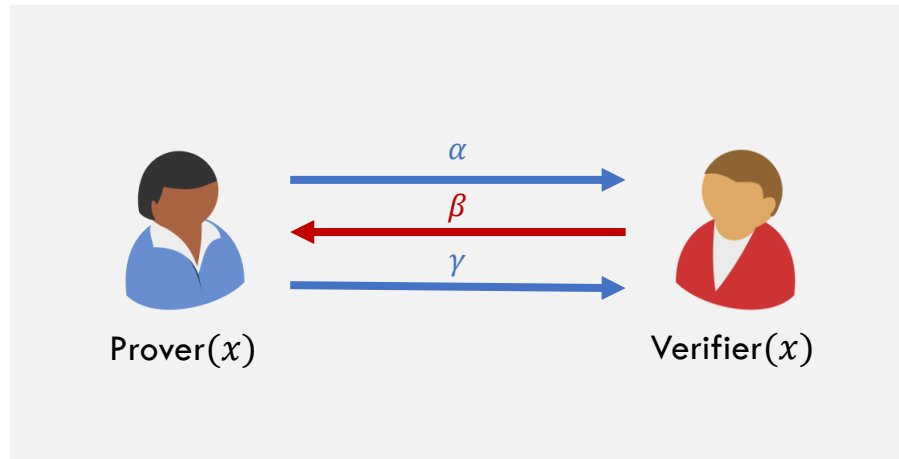
$\beta$  is a random string

→  
[Fiat-Shamir'86]



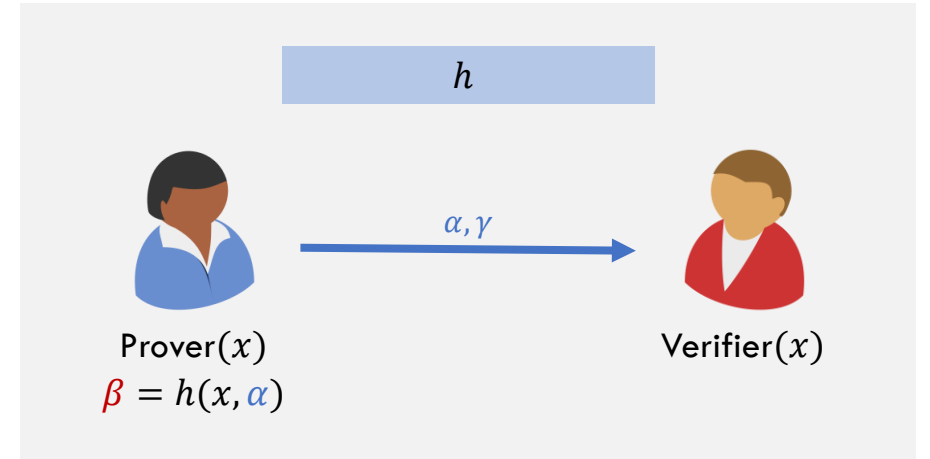
$\forall x \notin \mathcal{L}$   
 $\text{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$

# Fiat-Shamir (FS) Methodology



$\beta$  is a random string

→  
[Fiat-Shamir'86]

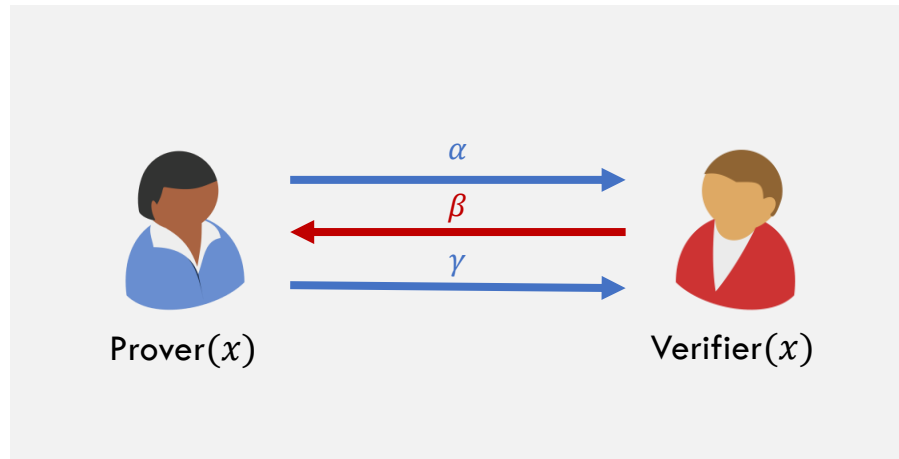


$$\forall x \notin \mathcal{L}$$
$$\text{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$

If  $x \notin \mathcal{L}$ , no PPT  can find  $\alpha$  such that

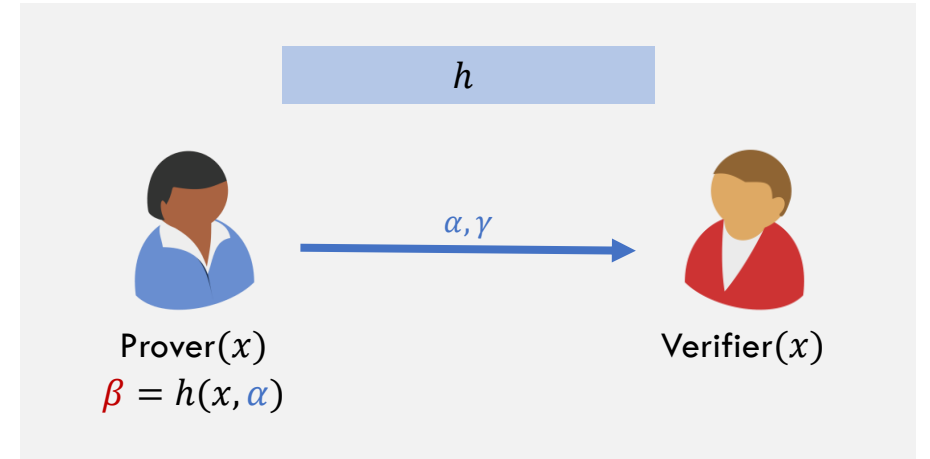
$$h(x, \alpha) \in \text{BAD}_{x,\alpha}$$

# Correlation Intractability [Canetti-Goldreich-Halevi'98]



$\beta$  is a random string

→ [Fiat-Shamir'86]



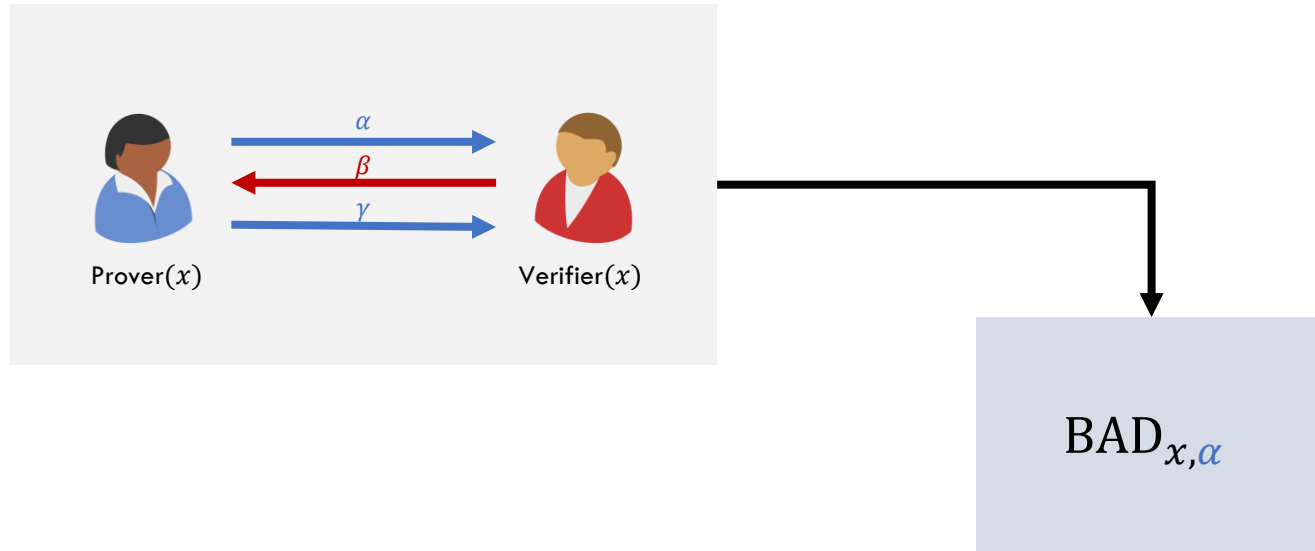
$\forall x \notin \mathcal{L}$   
 $\text{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$

If  $x \notin \mathcal{L}$ , no PPT can find  $\alpha$  such that

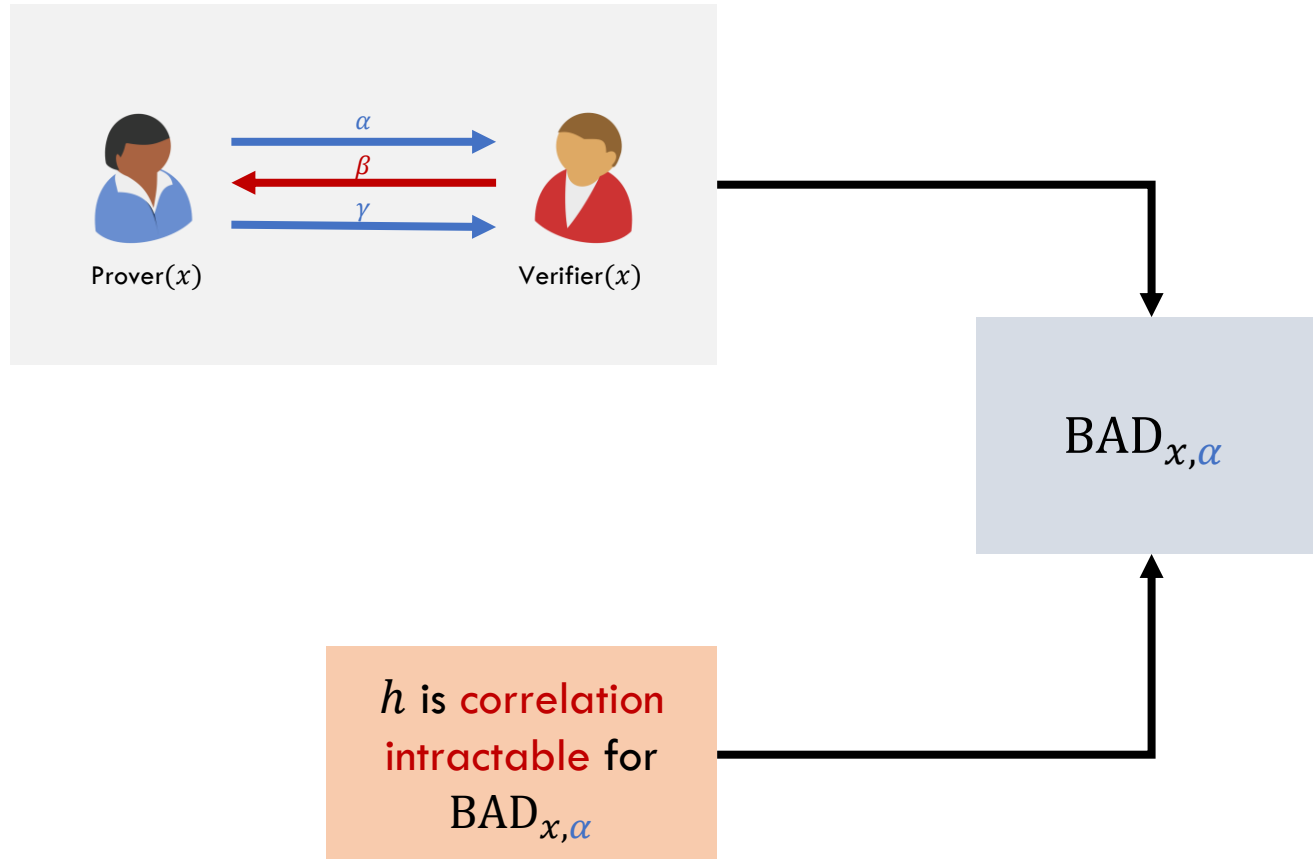
$h(x, \alpha) \in \text{BAD}_{x,\alpha}$

$h$  is **correlation intractable (CI)** for  $\text{BAD}_{x,\alpha}$

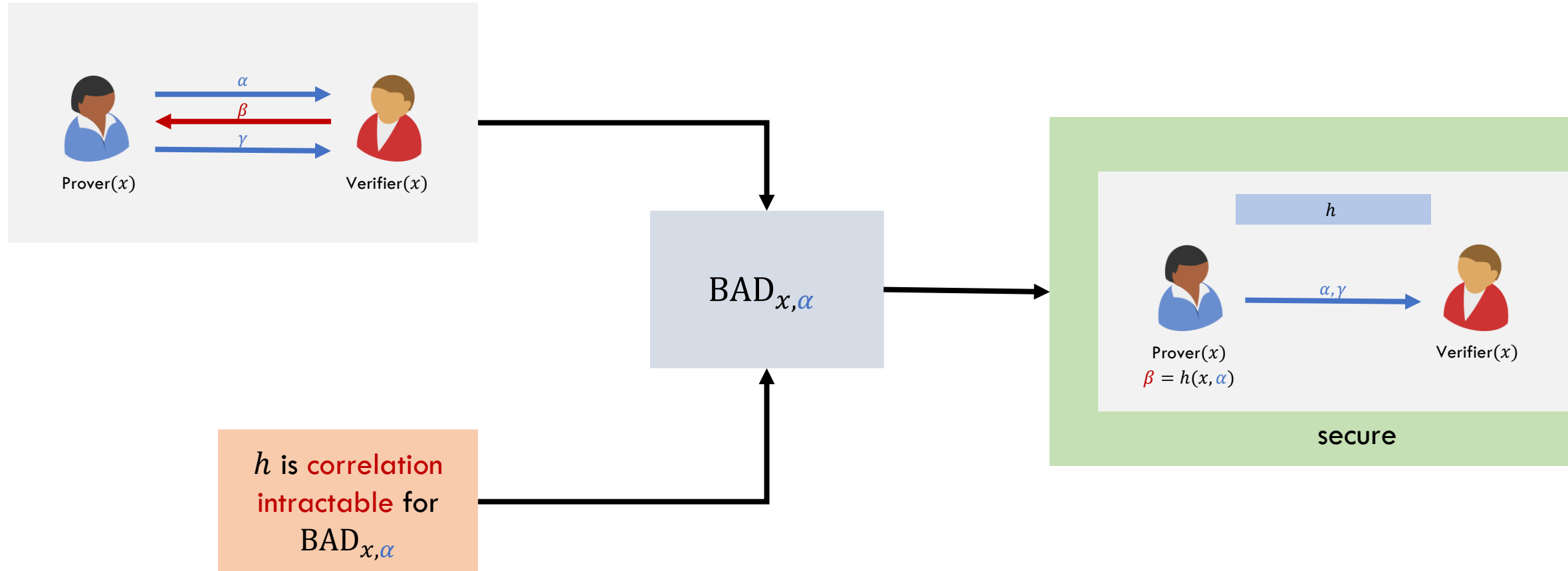
# Instantiating the FS Transform



# Instantiating the FS Transform

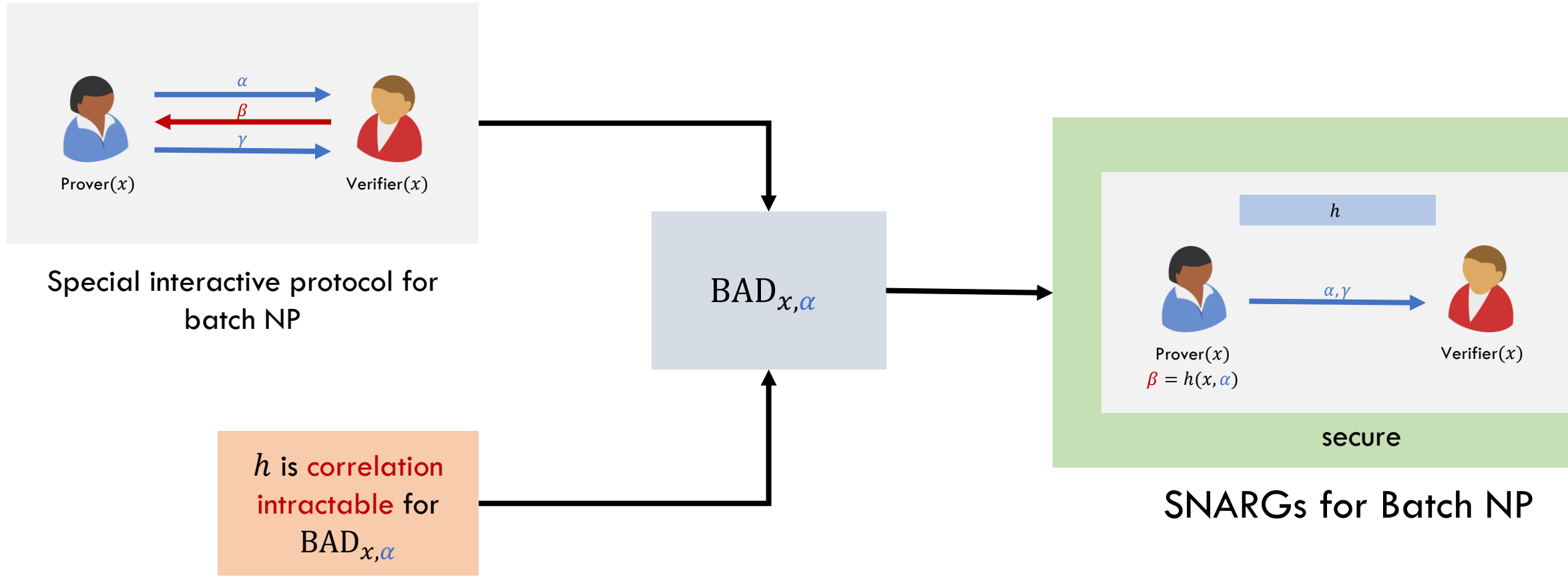


# Instantiating the FS Transform

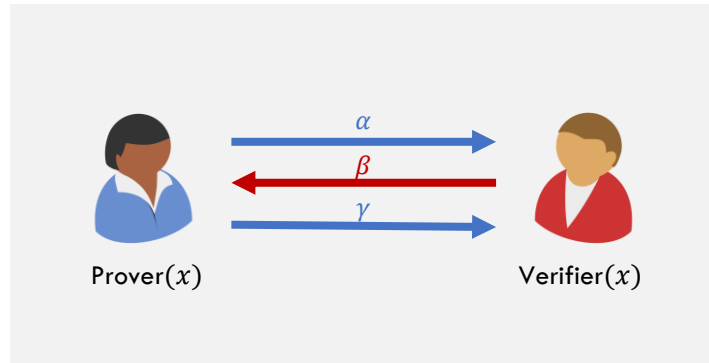




# [C-Jain-Jin'21] Methodology



# [C-Jain-Jin'21] Methodology



Special interactive protocol for batch NP

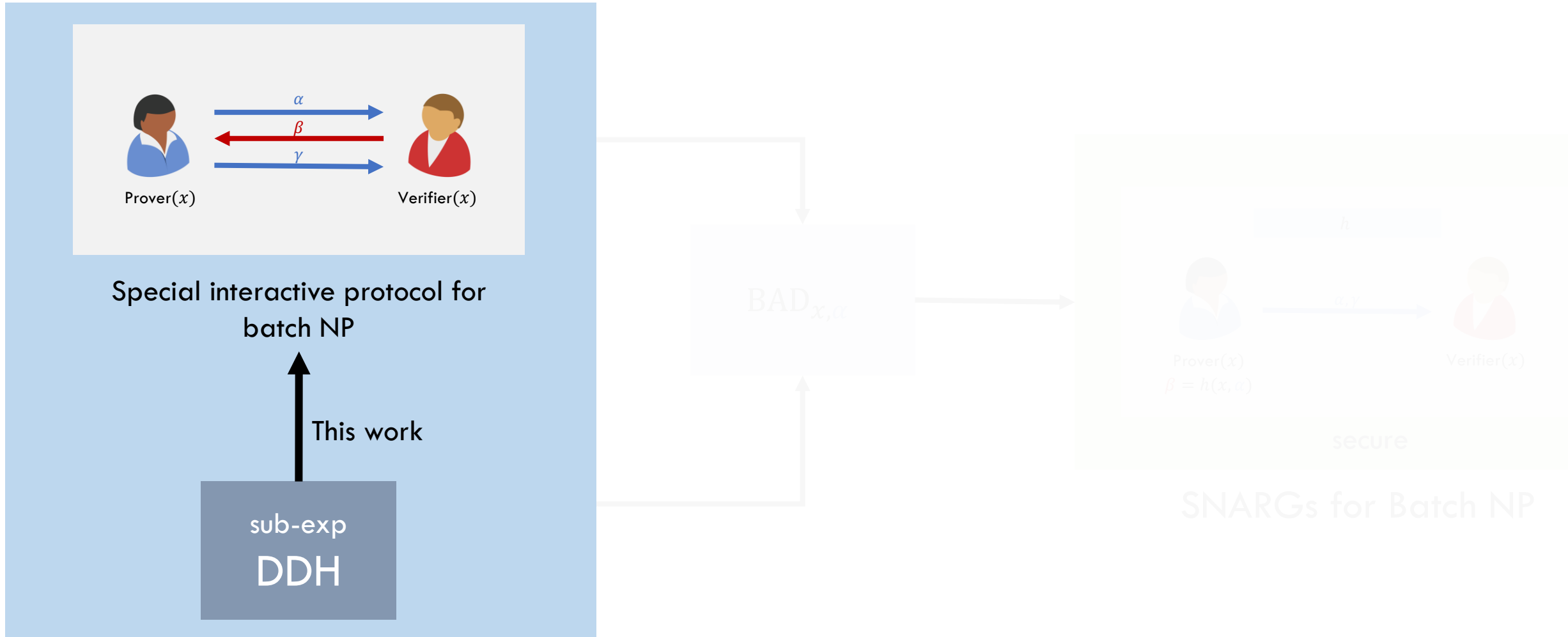
$h$  is correlation intractable for  $BAD_{x,\alpha}$

$BAD_{x,\alpha}$



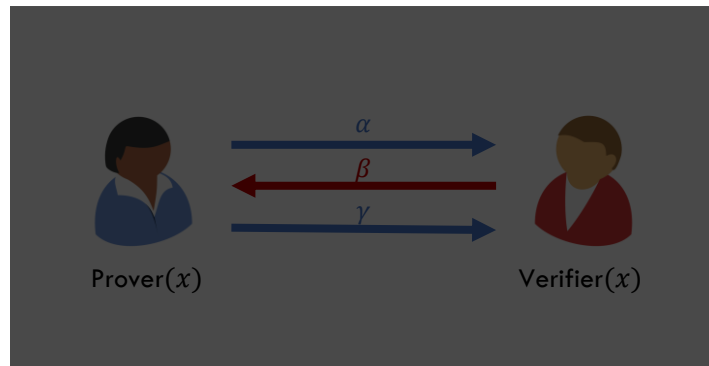
SNARGs for Batch NP

# [C-Jain-Jin'21] Methodology



see paper for details

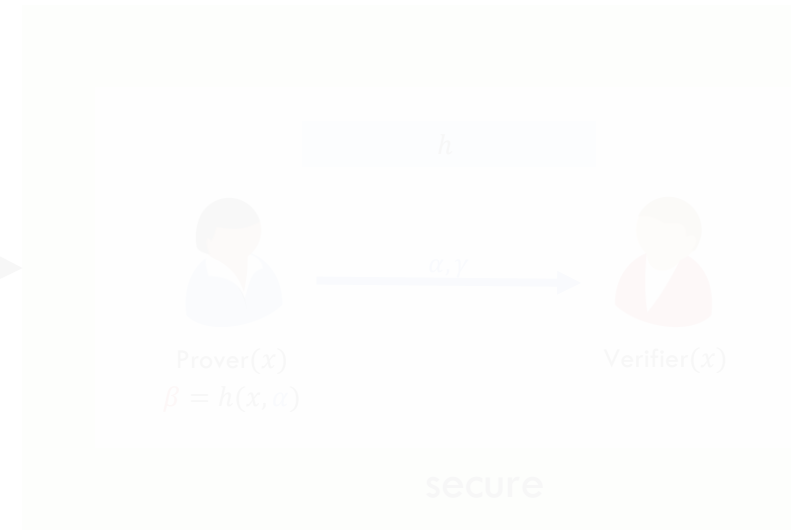
# [C-Jain-Jin'21] Methodology



**Magic Box**  
Special interactive protocol for  
batch NP

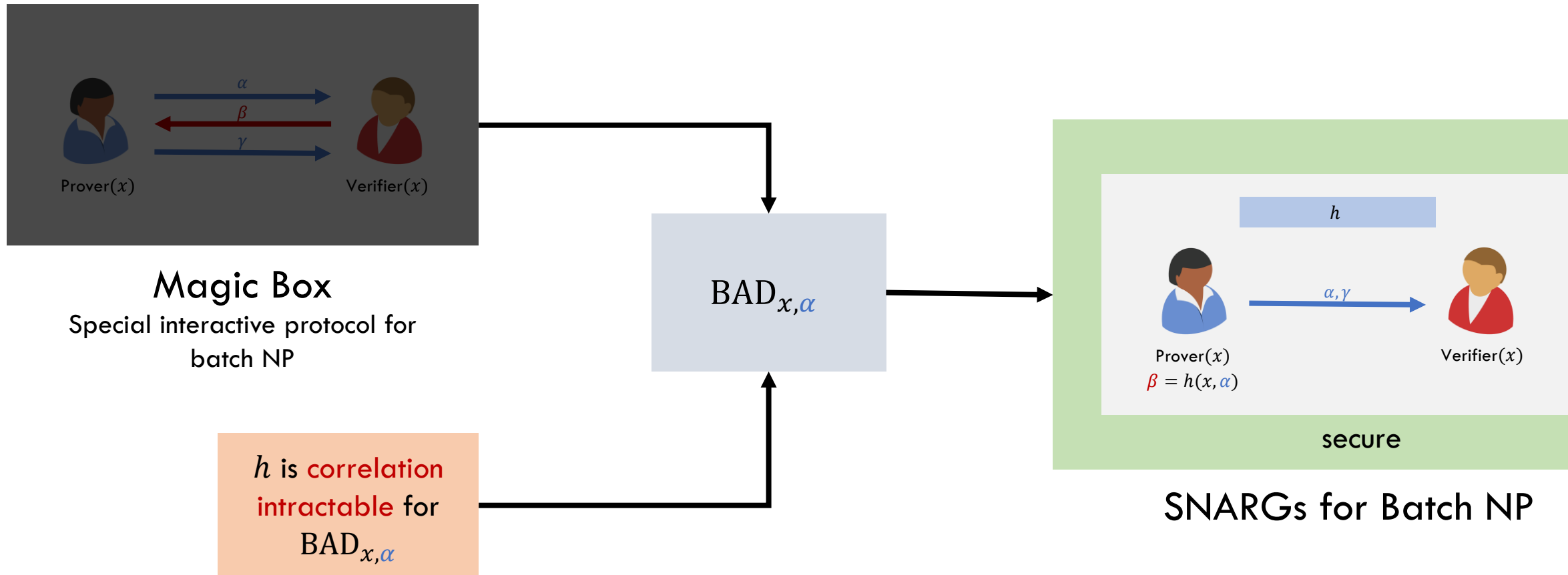
*$h$  is correlation  
intractable for  
 $BAD_{x,\alpha}$*

$BAD_{x,\alpha}$

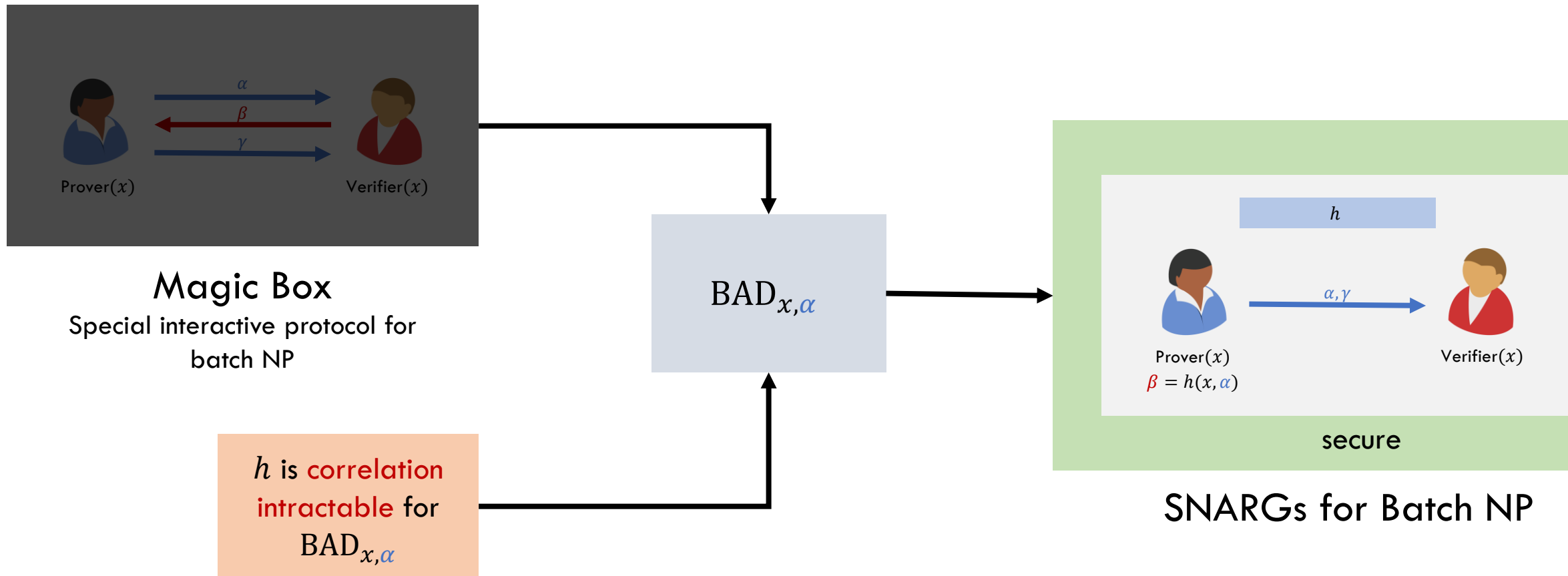


SNARGs for Batch NP

# [C-Jain-Jin'21] Methodology



# [C-Jain-Jin'21] Methodology



What properties does BAD $_{x,\alpha}$  have?

# Properties of $\text{BAD}_{x,\alpha}$

$\text{BAD}_{x,\alpha}$  is product verifiable.

$\forall x \notin \mathcal{L}$

$\text{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$

# Properties of $\text{BAD}_{x,\alpha}$

$$\text{BAD}_{x,\alpha} = \text{BAD}_{x,\alpha}^{(1)} \times \text{BAD}_{x,\alpha}^{(2)} \times \text{BAD}_{x,\alpha}^{(3)} \times \text{BAD}_{x,\alpha}^{(4)}$$

$\text{BAD}_{x,\alpha}$  is **product verifiable**.

$\forall x \notin \mathcal{L}$

$$\text{BAD}_{x,\alpha}^{(j)} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$



# Properties of $\text{BAD}_{x,\alpha}$

$$\text{BAD}_{x,\alpha} = \text{BAD}_{x,\alpha}^{(1)} \times \text{BAD}_{x,\alpha}^{(2)} \times \text{BAD}_{x,\alpha}^{(3)} \times \text{BAD}_{x,\alpha}^{(4)}$$

$\text{BAD}_{x,\alpha}$  is **product verifiable**.

$\forall x \notin \mathcal{L}$

$$\text{BAD}_{x,\alpha}^{(j)} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$

Exponentially many bad challenges even when  $\beta$  sampled from polynomial size challenge space.

# Properties of $\text{BAD}_{x,\alpha}$

$$\text{BAD}_{x,\alpha} = \text{BAD}_{x,\alpha}^{(1)} \times \text{BAD}_{x,\alpha}^{(2)} \times \text{BAD}_{x,\alpha}^{(3)} \times \text{BAD}_{x,\alpha}^{(4)}$$

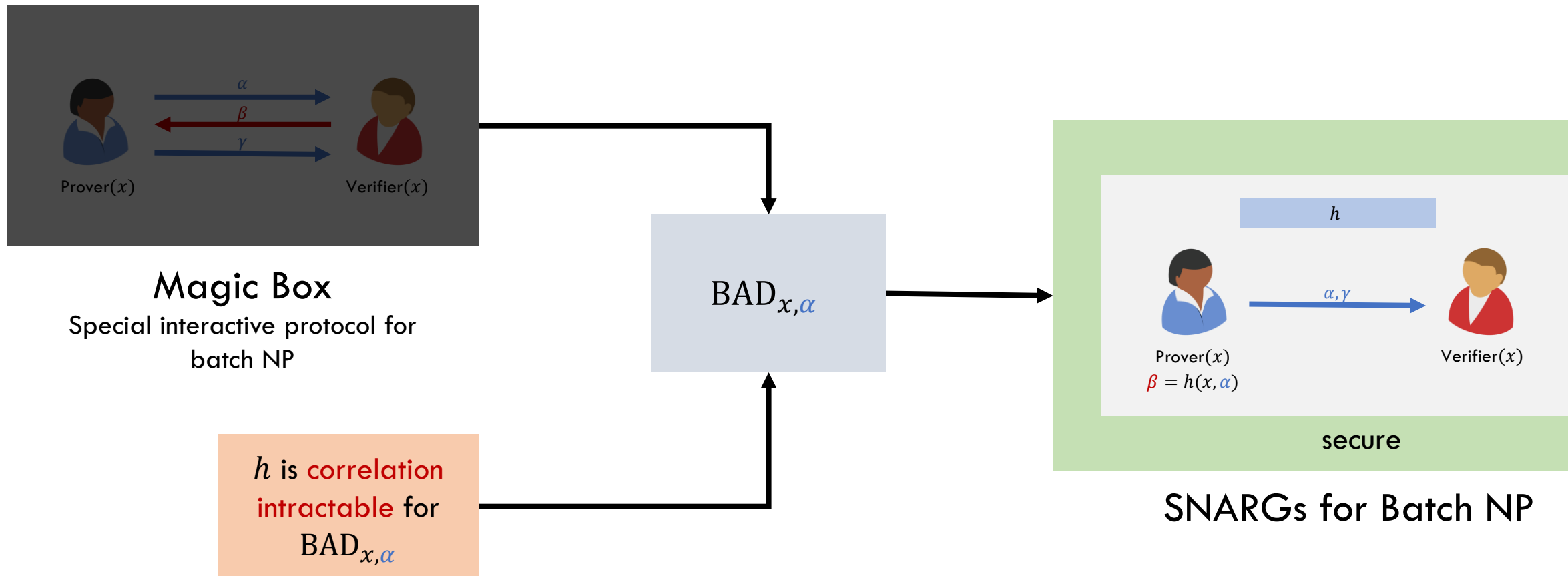
$\text{BAD}_{x,\alpha}$  is **product verifiable**.

Each  $\text{BAD}_{x,\alpha}^{(i)}$  is **efficiently verifiable**

$\forall x \notin \mathcal{L}$

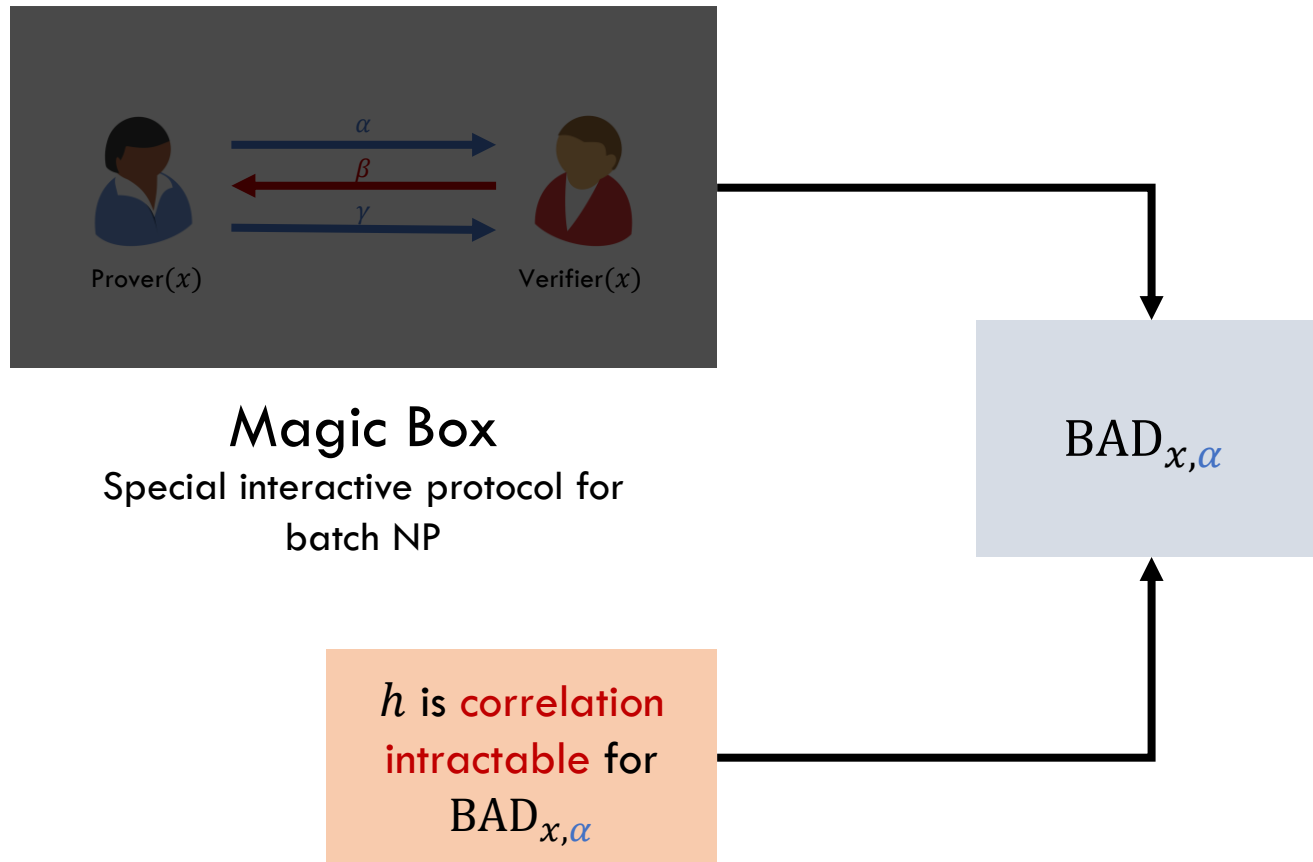
$$\text{BAD}_{x,\alpha}^{(j)} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$

# [C-Jain-Jin'21] Methodology



What properties does  $BAD_{x,\alpha}$  have?

# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

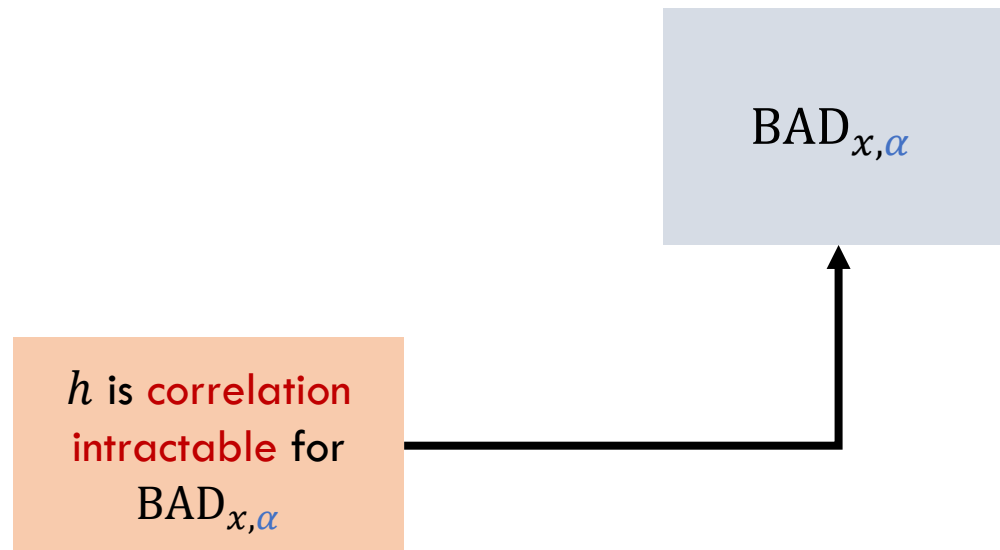
1 Bad challenges are a product set

2 Challenge space is of polynomial size

3 Bad challenges are product verifiable in  $TC^0$

$TC^0$  - Constant depth polynomial-size threshold circuits

# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

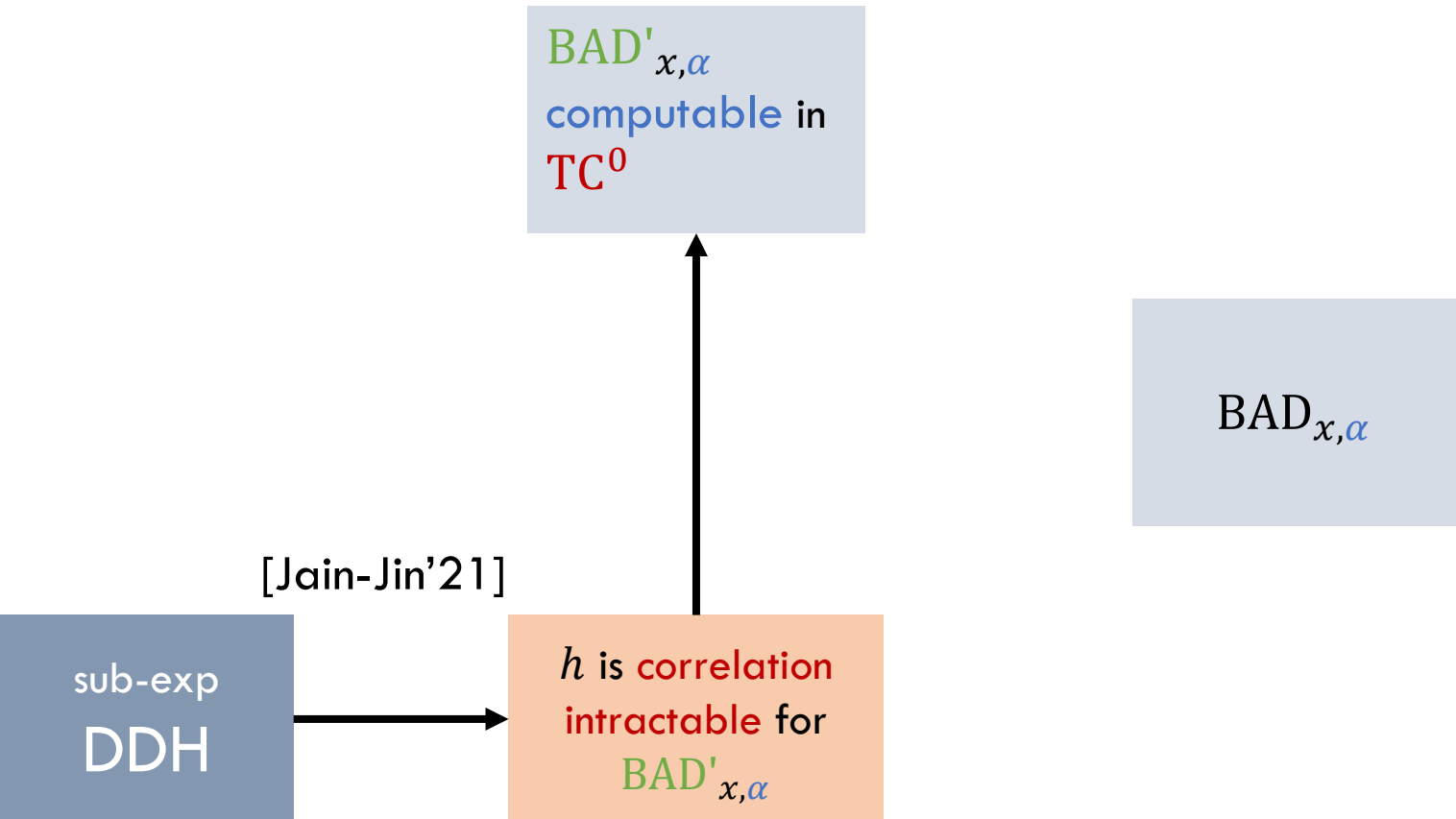
1 Bad challenges are a product set

2 Challenge space is of polynomial size

3 Bad challenges are product verifiable in  $TC^0$

$TC^0$  - Constant depth polynomial-size threshold circuits

# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

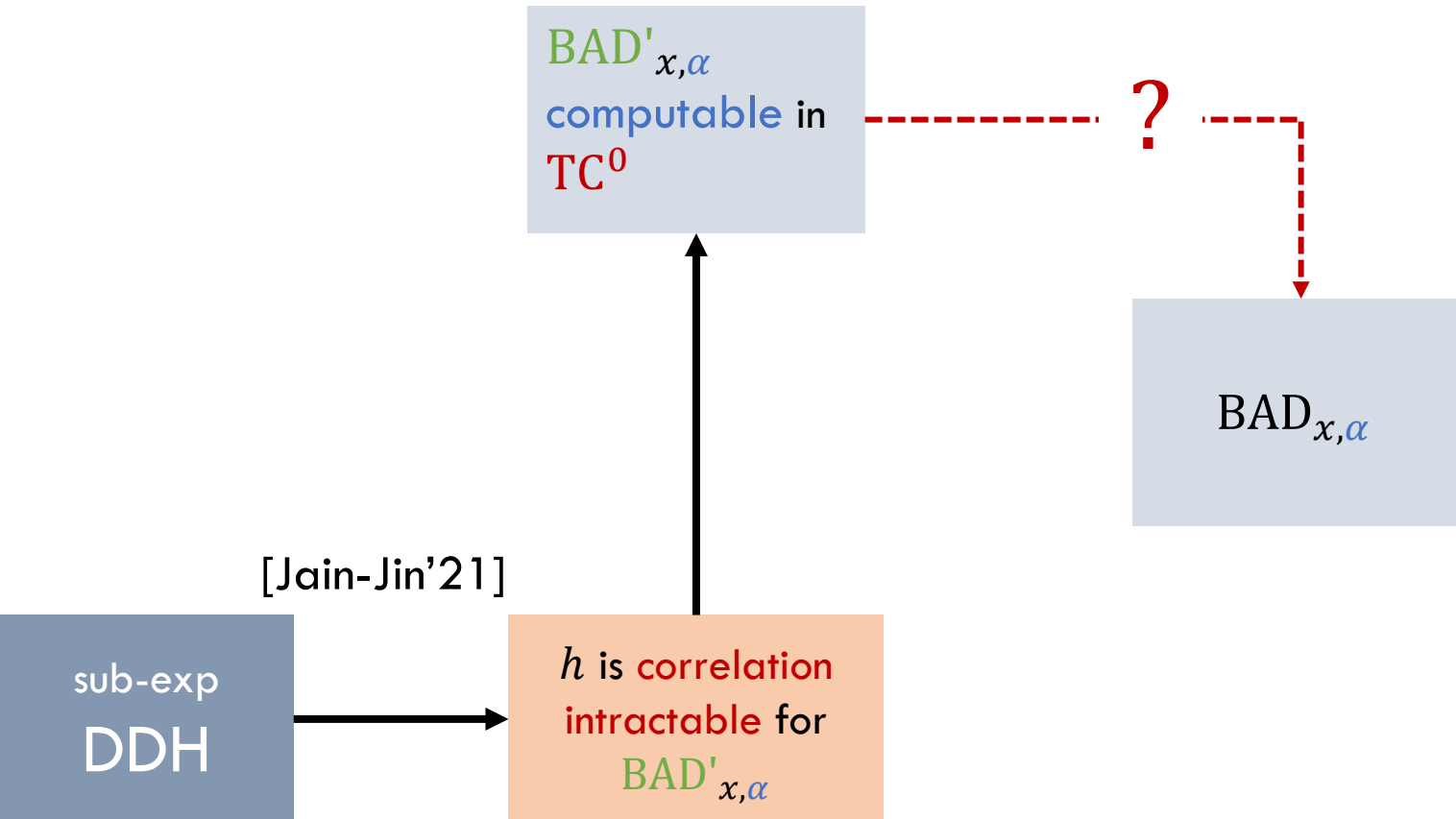
1 Bad challenges are a product set

2 Challenge space is of polynomial size

3 Bad challenges are product verifiable in  $TC^0$

$TC^0$  - Constant depth polynomial-size threshold circuits

# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

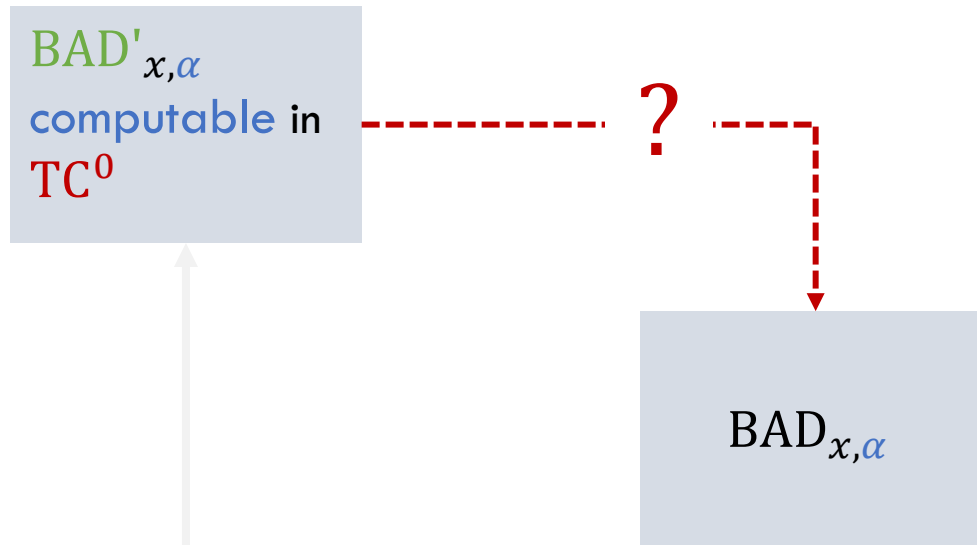
1 Bad challenges are a product set

2 Challenge space is of polynomial size

3 Bad challenges are product verifiable in  $TC^0$

$TC^0$  - Constant depth polynomial-size threshold circuits

# [C-Jain-Jin'21] Methodology



[Jain-Jin'21]

Difficulty [Holmgren-Lombardi-Rothblum'21]:  
 $BAD_{x,\alpha}$  has **exponentially many bad challenges**.

## $BAD_{x,\alpha}$ properties

1 Bad challenges are a product set

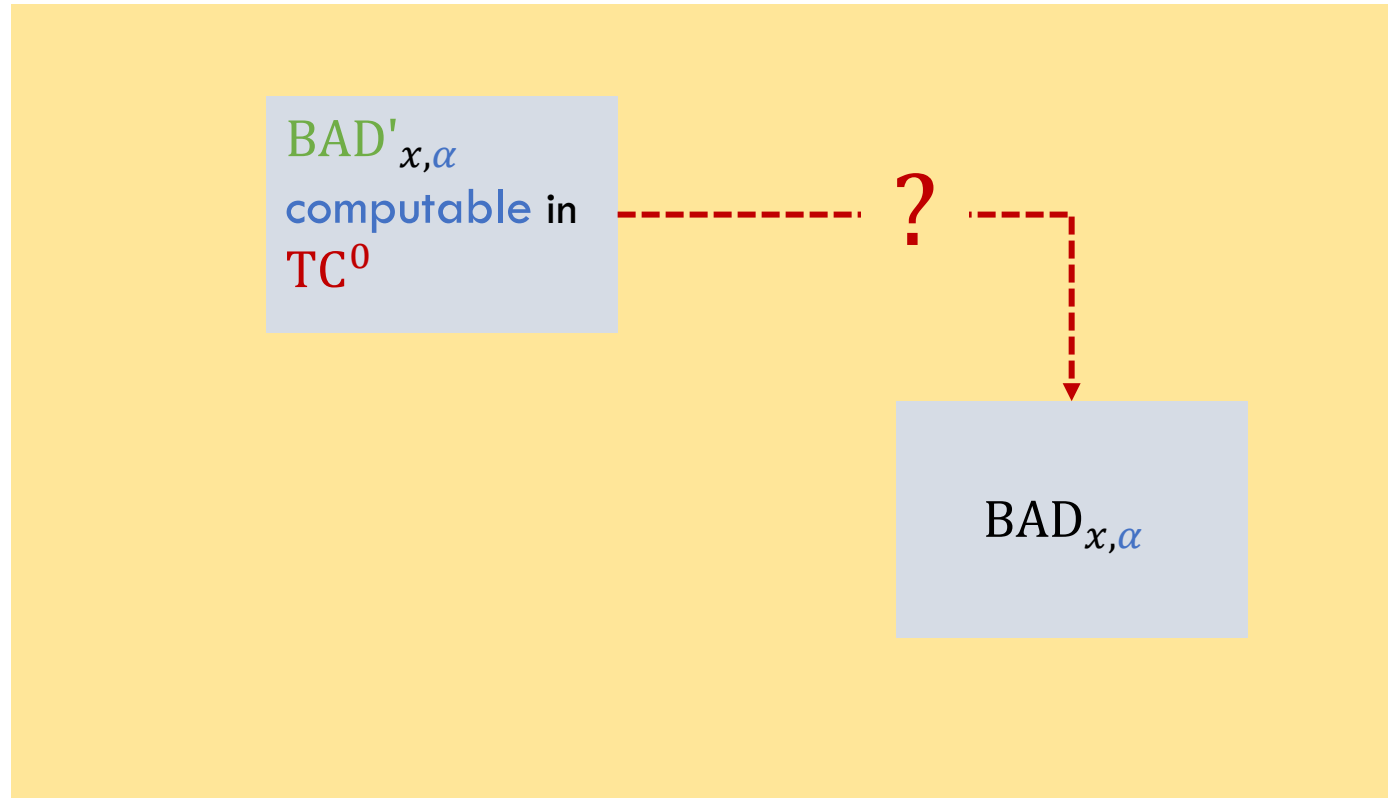
2 Challenge space is of polynomial size

3 Bad challenges are product verifiable in  $TC^0$

$TC^0$  - Constant depth polynomial-size threshold circuits



# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

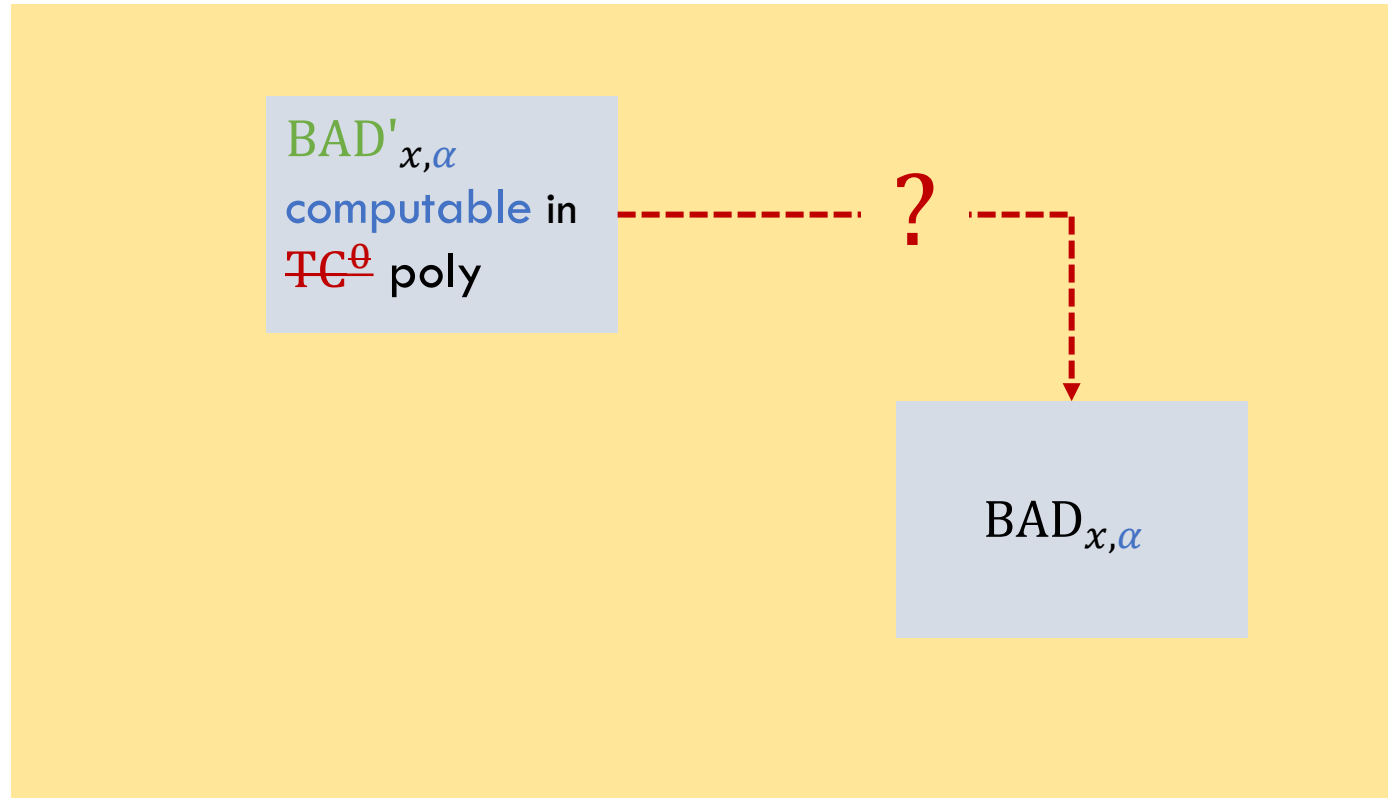
1 Bad challenges are a product set

2 Challenge space is of polynomial size

3 Bad challenges are product verifiable in  $TC^0$

$TC^0$  - Constant depth polynomial-size threshold circuits

# [C-Jain-Jin'21] Methodology

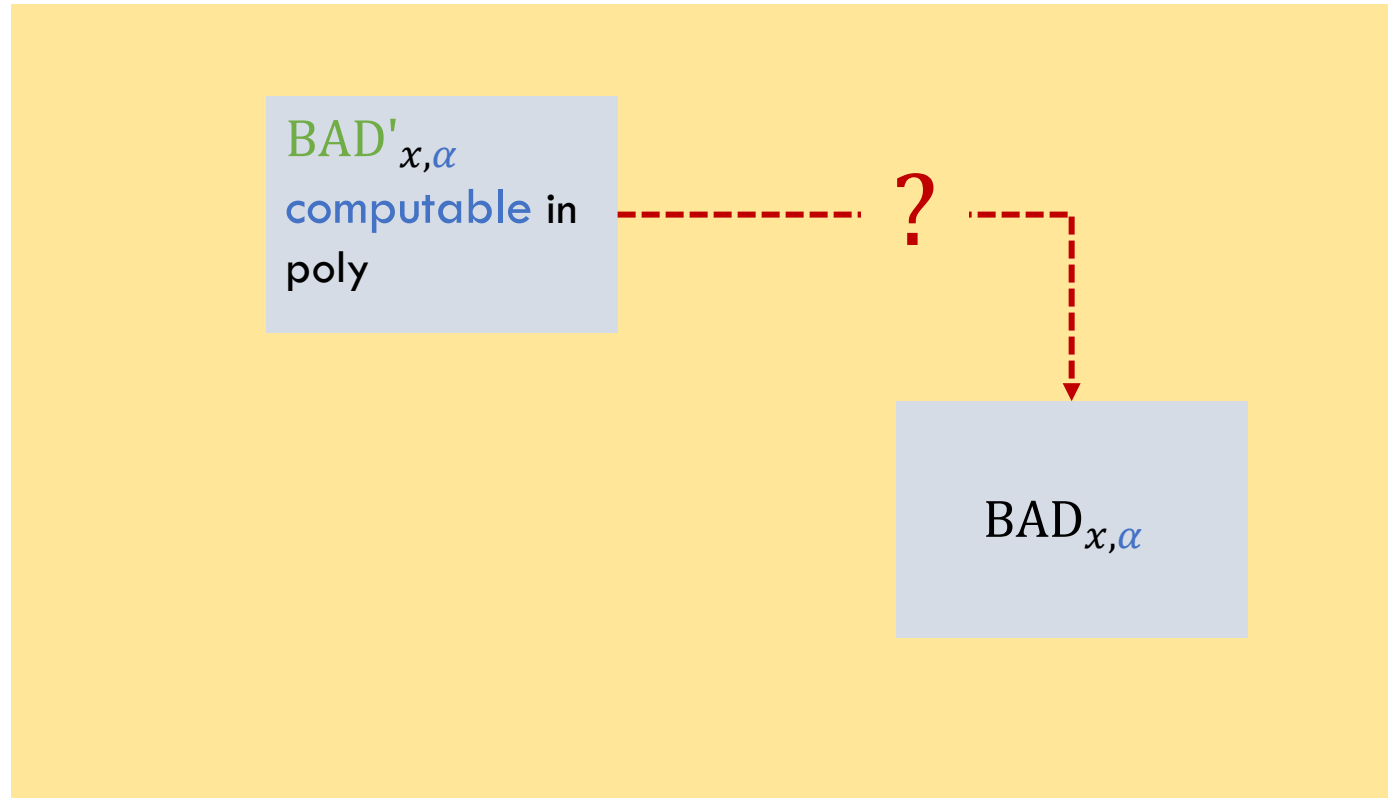


## $BAD_{x,\alpha}$ properties

- 1 Bad challenges are a product set
- 2 Challenge space is of polynomial size
- 3 Bad challenges are product verifiable in  $TC^\theta$  poly

$TC^\theta$  - Constant depth polynomial-size threshold circuits

# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

- 1 Bad challenges are a product set
- 2 Challenge space is of polynomial size
- 3 Bad challenges are product verifiable in poly

# Easy Case: Verifiable Unique Bad Challenge

BAD <sub>$x, \alpha$</sub> <sup>(1)</sup>

# Easy Case: Verifiable Unique Bad Challenge

$\text{BAD}_{x,\alpha}^{(1)}$

Compute Bad Challenge  
for  $\beta \in \text{ChallengeSpace}$

| if  $\beta \in \text{BAD}_{x,\alpha}^{(1)}$   
| | return  $\beta$

$\text{ChallengeSpace}$  polynomial size +  $\text{BAD}_{x,\alpha}^{(1)}$  efficiently verifiable  $\Rightarrow$   $\text{BAD}_{x,\alpha}^{(1)}$  efficiently computable.

# Easy Case: Verifiable Unique Bad Challenge

$$\text{BAD}_{x,\alpha} = \text{BAD}_{x,\alpha}^{(1)} \times \text{BAD}_{x,\alpha}^{(2)} \times \cdots \times \text{BAD}_{x,\alpha}^{(d)}$$

# Easy Case: Verifiable Unique Bad Challenge

$$\text{BAD}_{x,\alpha} = \text{BAD}_{x,\alpha}^{(1)} \times \text{BAD}_{x,\alpha}^{(2)} \times \cdots \times \text{BAD}_{x,\alpha}^{(d)}$$

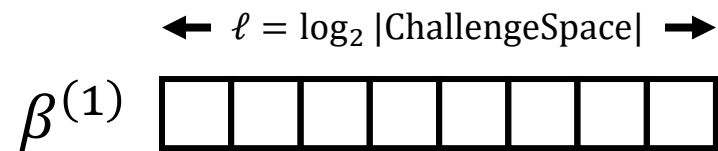
## Compute Bad Challenge

```
for  $i \in [d]$ 
  | for  $\beta^{(i)} \in \text{ChallengeSpace}$ 
  | | if  $\beta^{(i)} \in \text{BAD}_{x,\alpha}^{(i)}$ 
  | | | store  $\beta^{(i)}$ 
return  $(\beta^{(1)}, \dots, \beta^{(d)})$ 
```

poly repetitions + ChallengeSpace polynomial size +  $\text{BAD}_{x,\alpha}^{(i)}$  efficiently verifiable  $\Rightarrow \text{BAD}_{x,\alpha}$  efficiently computable.

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition



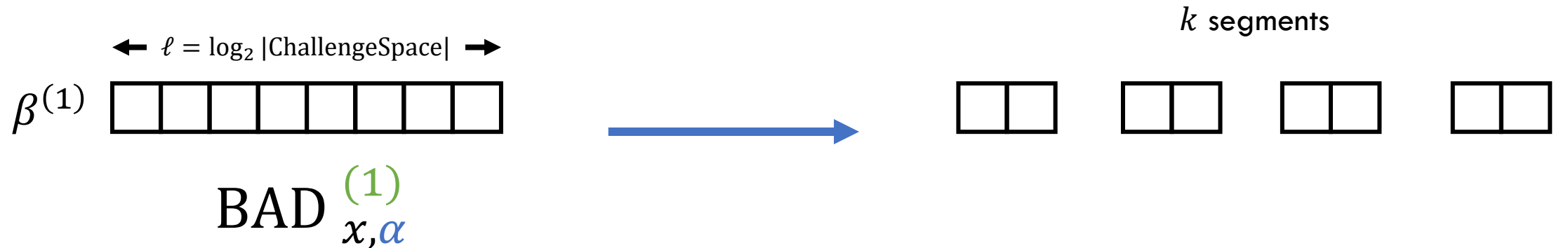
BAD  $^{(1)}$   
 $x, \alpha$

No restriction on number of bad challenges



# Reducing to Verifiable Unique Bad Challenge

No parallel repetition

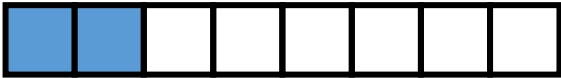
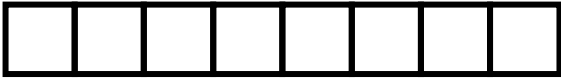


Each segment has  $\ell/k$  bits

# Sampling Challenges via Segments



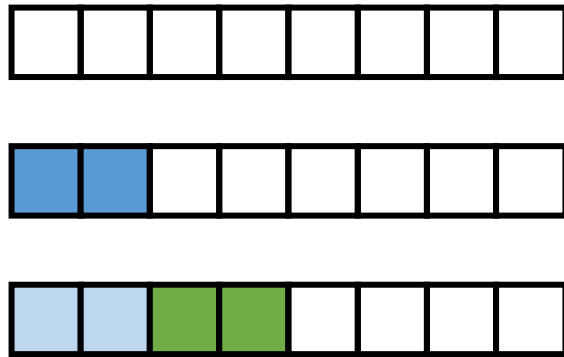
# Sampling Challenges via Segments



# Sampling Challenges via Segments



# Sampling Challenges via Segments

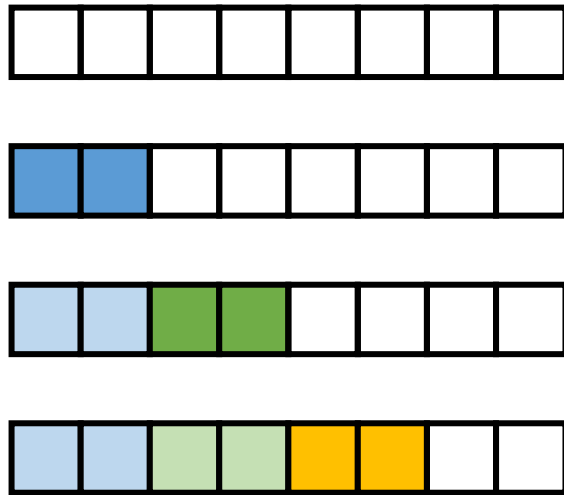


$$\text{[Blue Box]} = h(x, \alpha)$$

$$\text{[Green Box]} = h(x, \alpha, \text{[Light Blue Box]})$$

$h$  is correlation intractable for **efficiently verifiable unique bad challenge** relations.

# Sampling Challenges via Segments



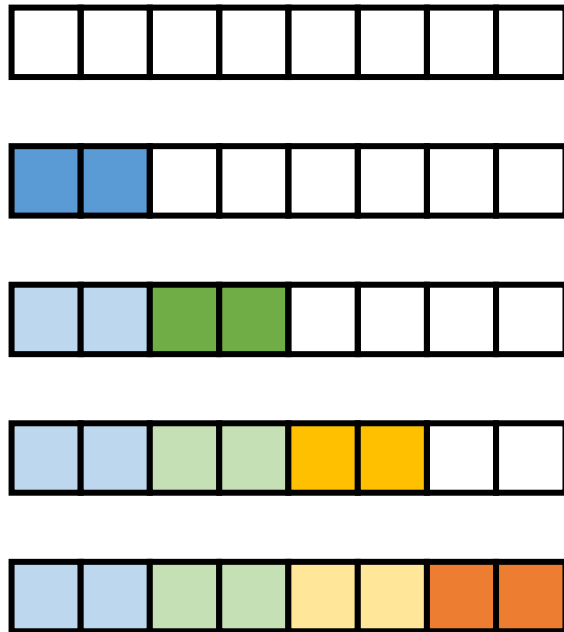
$$\text{[Blue Box]} = h(x, \alpha)$$

$$\text{[Green Box]} = h(x, \alpha, \text{[Light Blue Box]})$$

$$\text{[Yellow Box]} = h(x, \alpha, \text{[Light Blue Box] [Light Green Box]})$$

$h$  is correlation intractable for **efficiently verifiable unique bad challenge** relations.

# Sampling Challenges via Segments



$$\boxed{\text{blue}} \boxed{\text{blue}} = h(x, \alpha)$$

$$\boxed{\text{green}} \boxed{\text{green}} = h(x, \alpha, \boxed{\text{light blue}} \boxed{\text{light blue}})$$

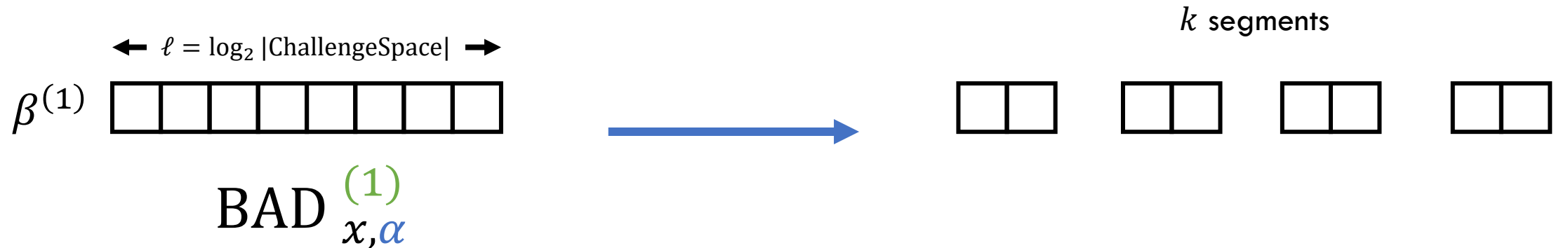
$$\boxed{\text{yellow}} \boxed{\text{yellow}} = h(x, \alpha, \boxed{\text{light blue}} \boxed{\text{light blue}} \boxed{\text{light green}} \boxed{\text{light green}})$$

$$\boxed{\text{orange}} \boxed{\text{orange}} = h(x, \alpha, \boxed{\text{light blue}} \boxed{\text{light blue}} \boxed{\text{light green}} \boxed{\text{light green}} \boxed{\text{yellow}} \boxed{\text{yellow}})$$

$h$  is correlation intractable for **efficiently verifiable unique bad challenge** relations.

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition

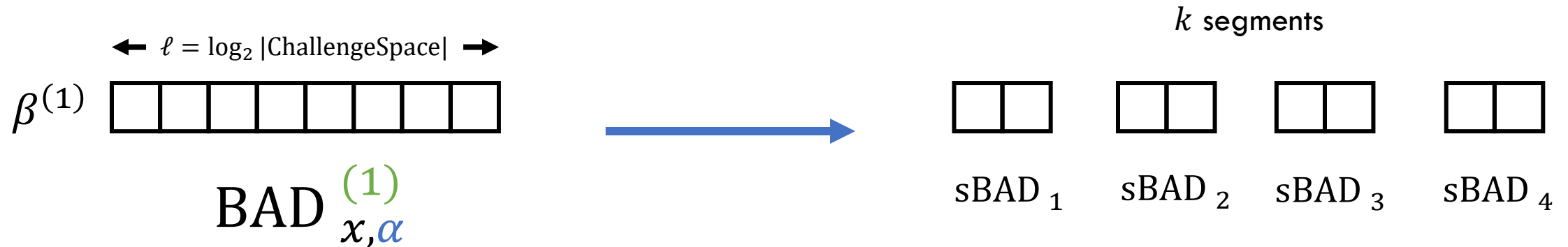


Each segment has  $\ell/k$  bits



# Reducing to Verifiable Unique Bad Challenge

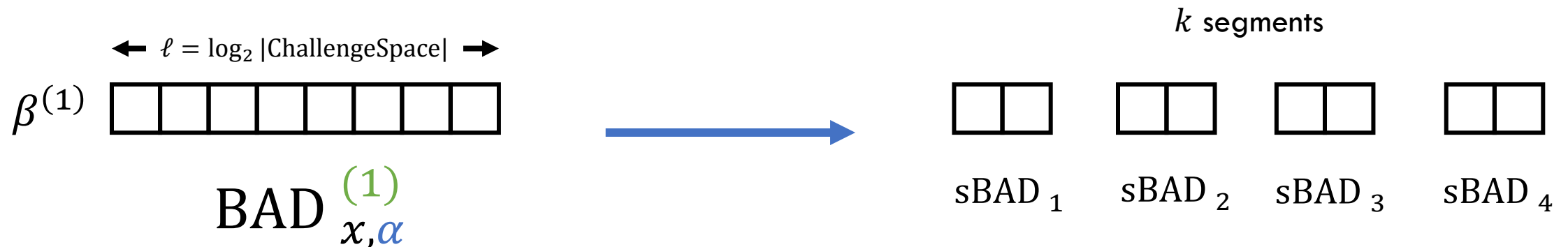
No parallel repetition



Each segment has  $\ell/k$  bits

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition



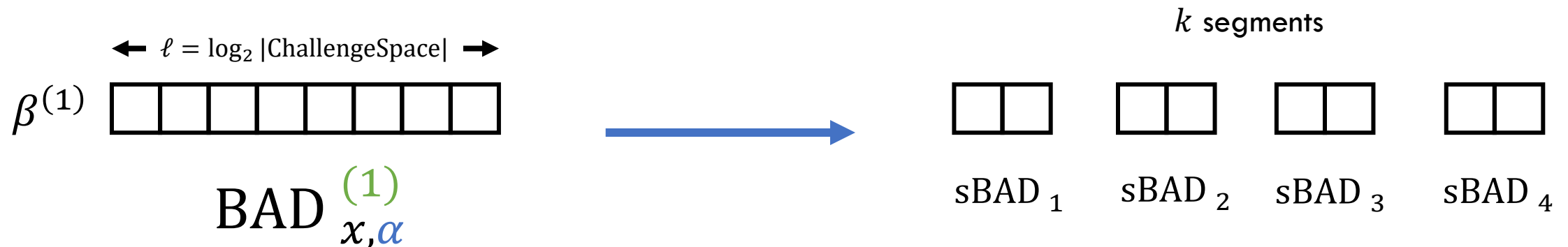
Requirements:

1. Each  $\text{sBAD}_j$  must be **efficiently verifiable unique bad challenge** relations.

Each segment has  $\ell/k$  bits

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition

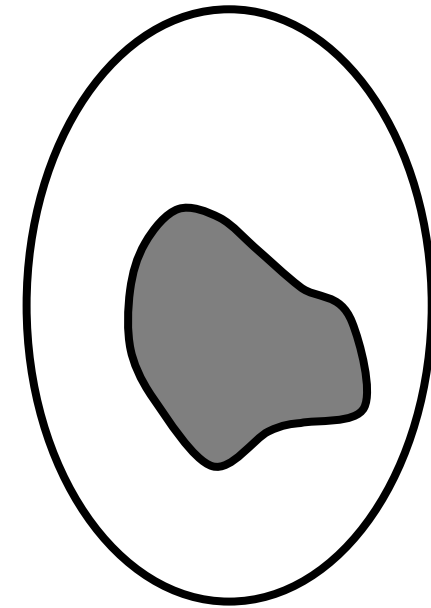


Requirements:

1. Each  $\text{sBAD}_j$  must be **efficiently verifiable unique bad challenge** relations.
2. If a challenge is bad, then there **must exist a bad segment**.

Each segment has  $\ell/k$  bits

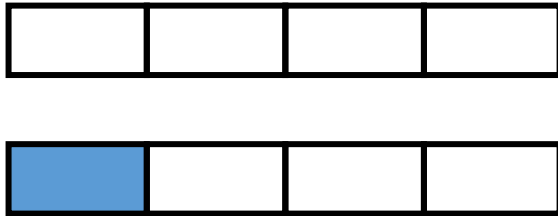
# Defining Bad Segments



Challenge space

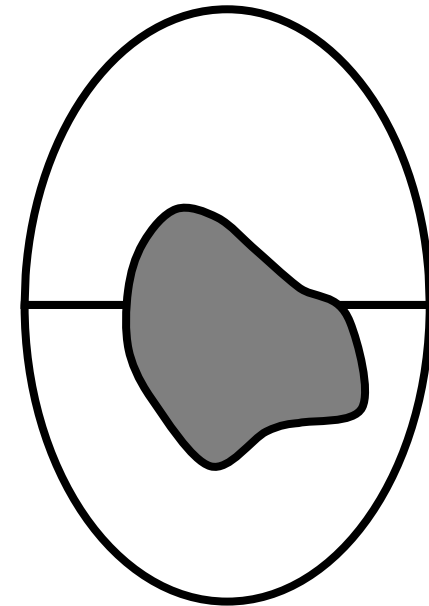
 All bad challenges for  $\text{BAD}_{x,\alpha}^{(1)}$

# Defining Bad Segments



Challenges with prefix 0

Challenges with prefix 1



 = 0

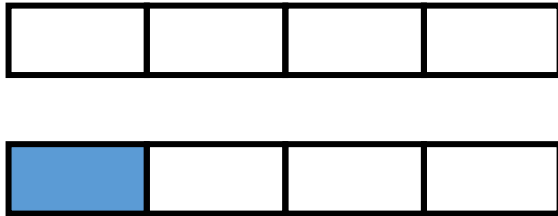
 = 1

$sBAD_1$

 is bad if

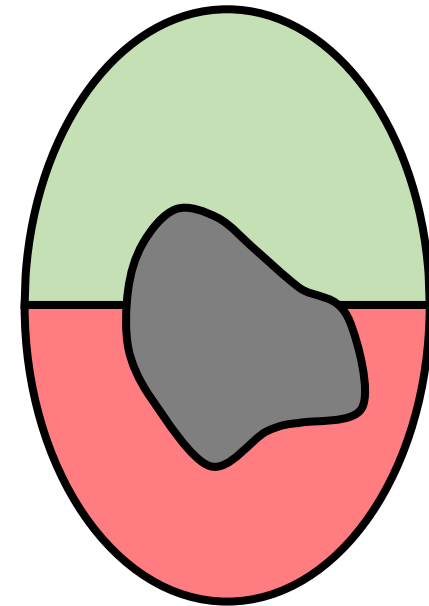
$\#bad\ challenges\ with\ prefix\  > \#bad\ challenges/2$

# Defining Bad Segments



Challenges with prefix 0

Challenges with prefix 1



 = 0

 = 1

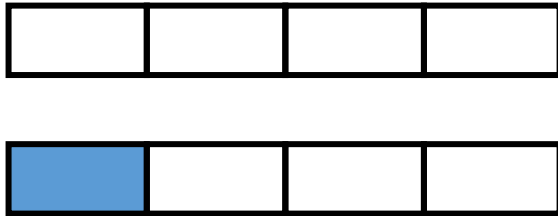
Bad Segment

$sBAD_1$

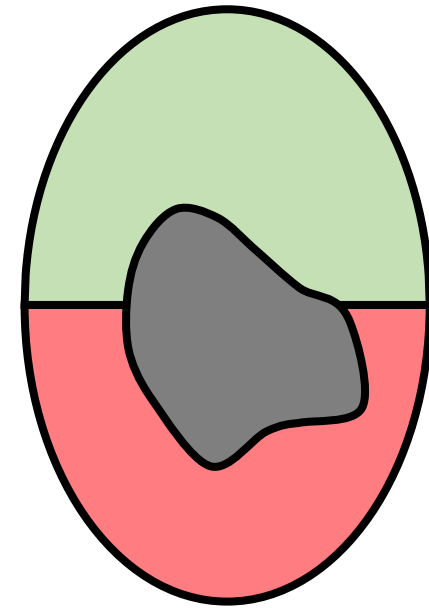
 is bad if

$\#bad\ challenges\ with\ prefix\ \langle \text{blue box} \rangle > \#bad\ challenges / 2$

# Defining Bad Segments



Challenges with prefix 0




 = 0

Challenges with prefix 1

 = 1

Bad Segment

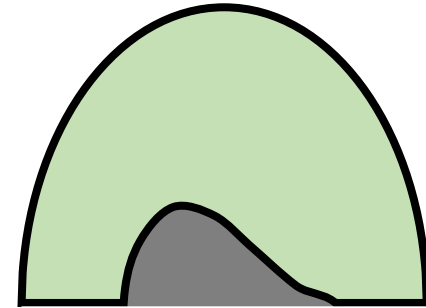
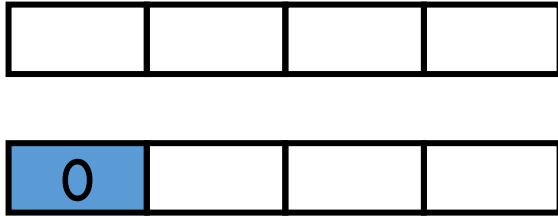
1. By pigeonhole principle, unique bad 
2. ChallengeSpace polynomial size +  $BAD_{x,\alpha}^{(1)}$  efficiently verifiable  $\Rightarrow$   $sBAD_1$  efficiently verifiable

$sBAD_1$

 is bad if

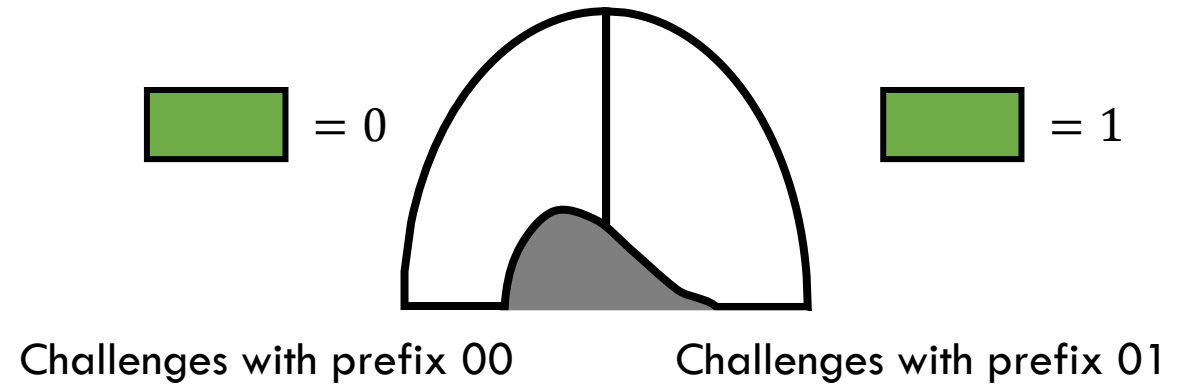
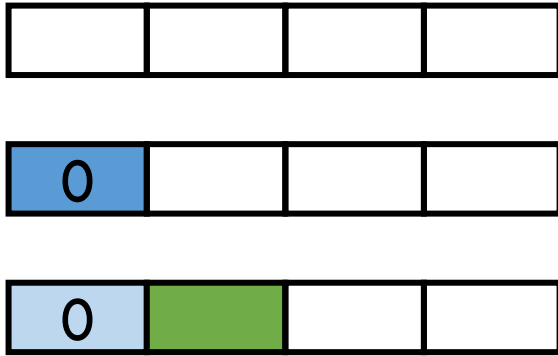
$\# \text{bad challenges with prefix } \langle \text{blue box} \rangle > \# \text{bad challenges} / 2$

# Defining Bad Segments

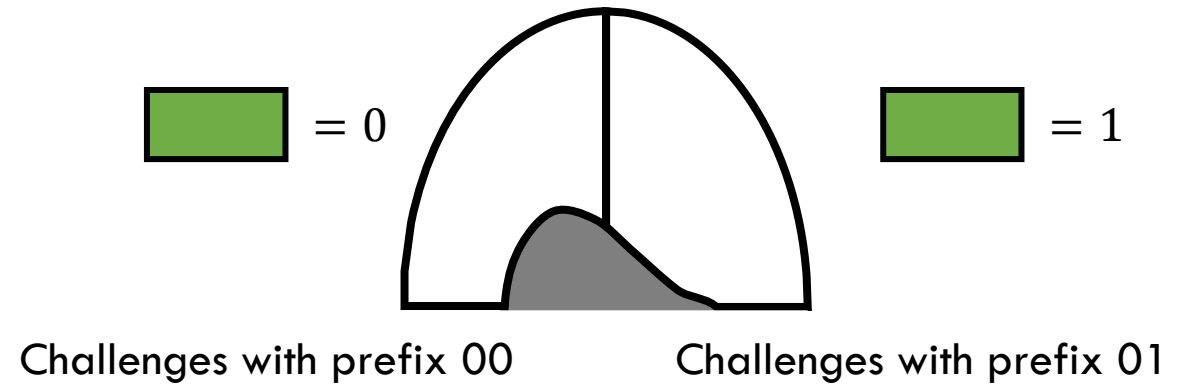
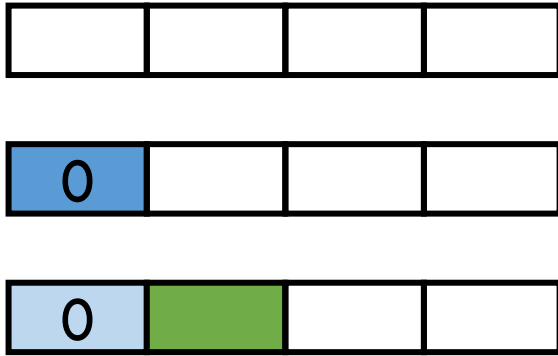






# Defining Bad Segments



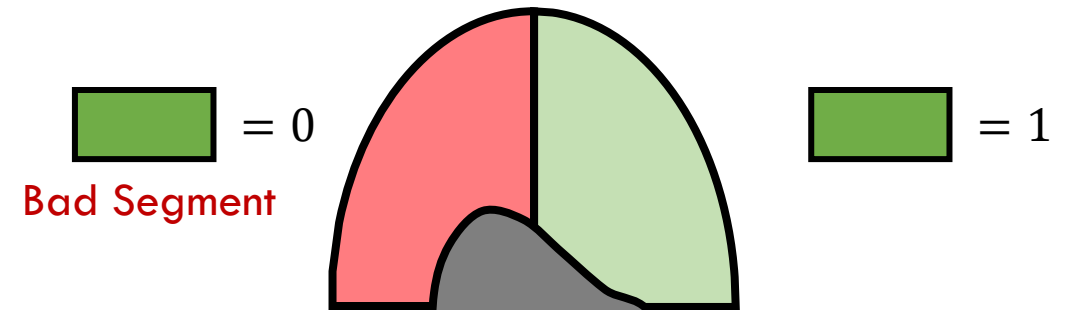
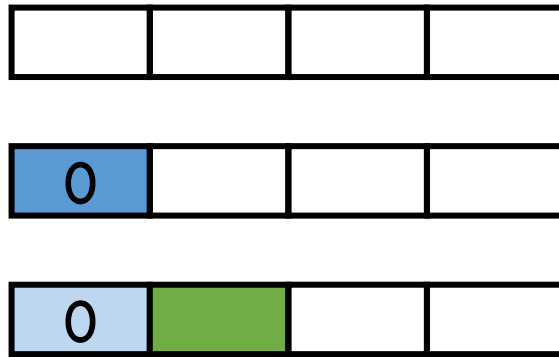
# Defining Bad Segments




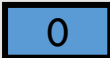



$sBAD_2$

 is bad given  if  
 $\# \text{bad challenges with prefix } \begin{matrix} \text{blue box with 0} \\ \text{green box} \end{matrix} > (\# \text{bad challenges with prefix } \begin{matrix} \text{blue box with 0} \end{matrix}) / 2$

# Defining Bad Segments

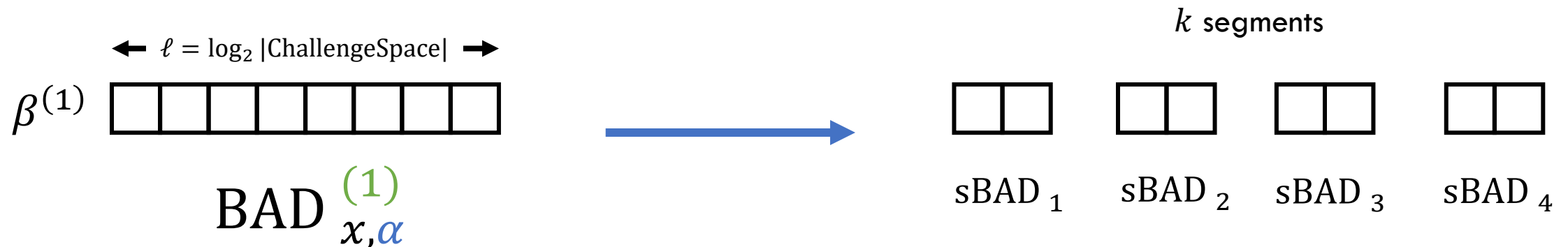


sBAD<sub>2</sub>

 is bad given  if  
#bad challenges with prefix   
> (#bad challenges with prefix  )/2

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition



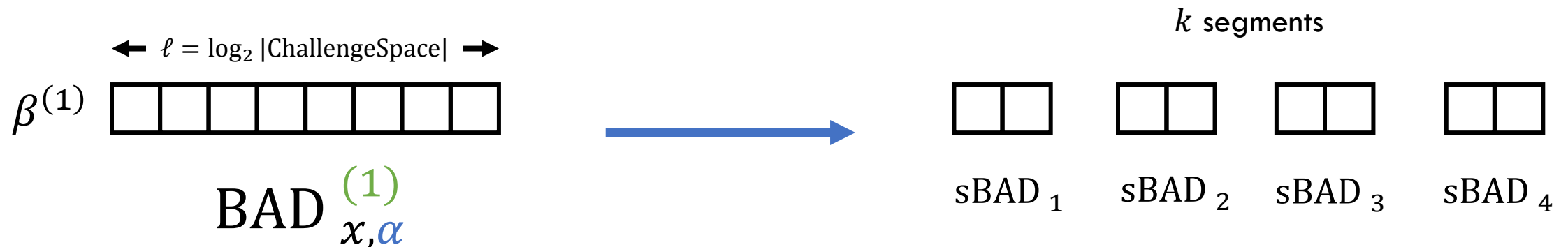
Requirements:

1. Each  $\text{sBAD}_j$  must be **efficiently verifiable unique bad challenge** relations.
2. If a challenge is bad, then there **must exist a bad segment**.

Each segment has  $\ell/k$  bits

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition



## Requirements:

1. Each  $\text{sBAD}_j$  must be efficiently verifiable unique bad challenge relations.
2. If a challenge is bad, then there must exist a bad segment.

Each segment has  $\ell/k$  bits

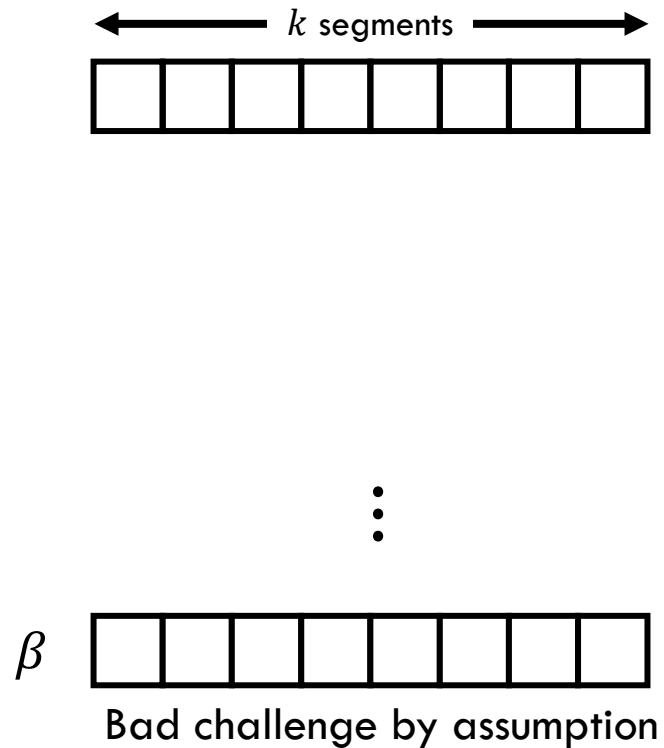
# Existence of a bad segment

$\beta$ 

--	--	--	--	--	--	--	--

  
Bad challenge by assumption

# Existence of a bad segment



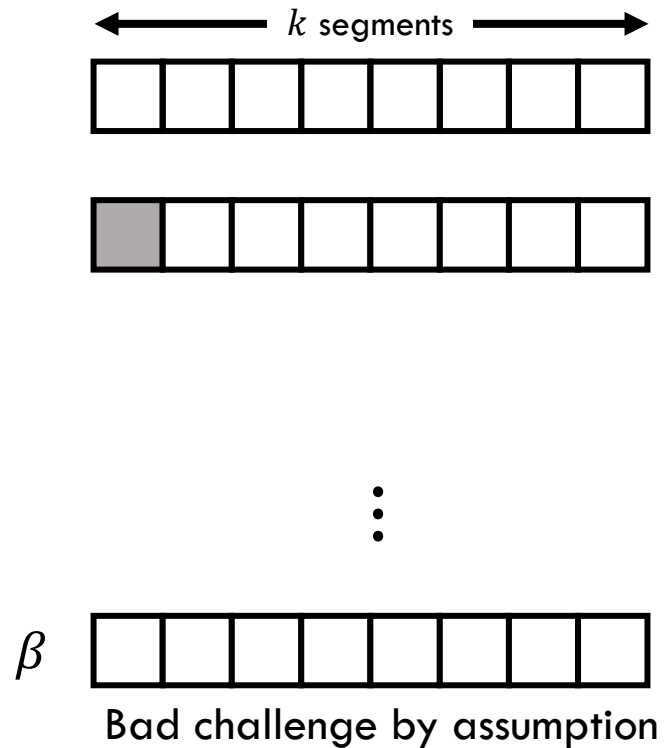
#bad challenges remaining

$T$

$T = \# \text{bad challenges } \text{BAD}_{x,\alpha}^{(1)}$

$k$  such that  $2^k > T$

# Existence of a bad segment



#bad challenges remaining

$T$

$< T/2$

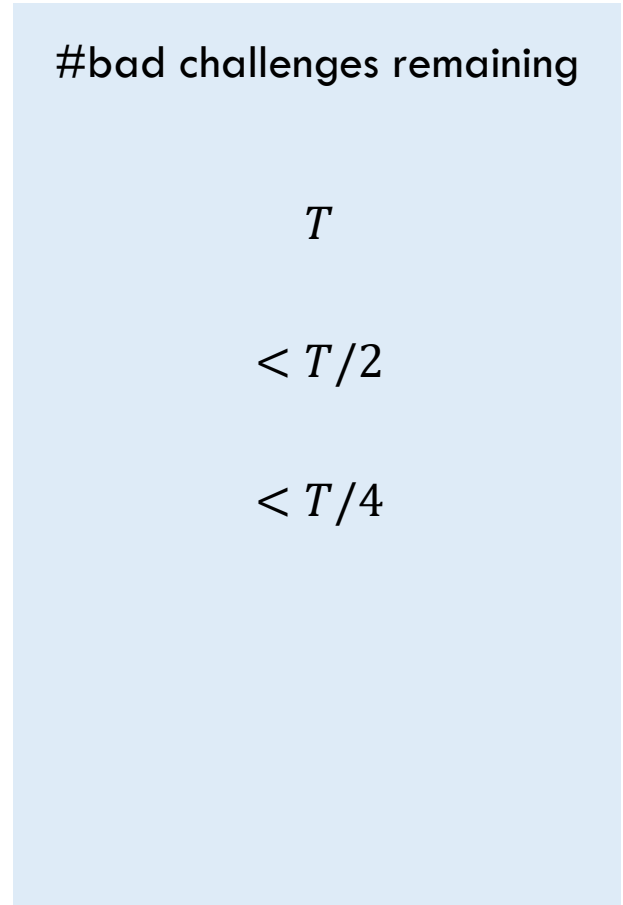
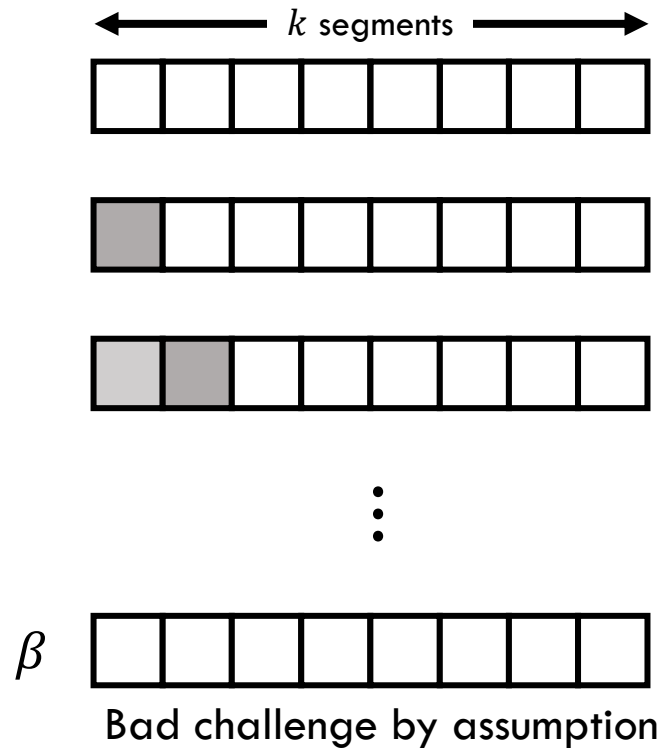
$T = \#\text{bad challenges } \text{BAD}_{x,\alpha}^{(1)}$

$k$  such that  $2^k > T$

If each segment is good



# Existence of a bad segment

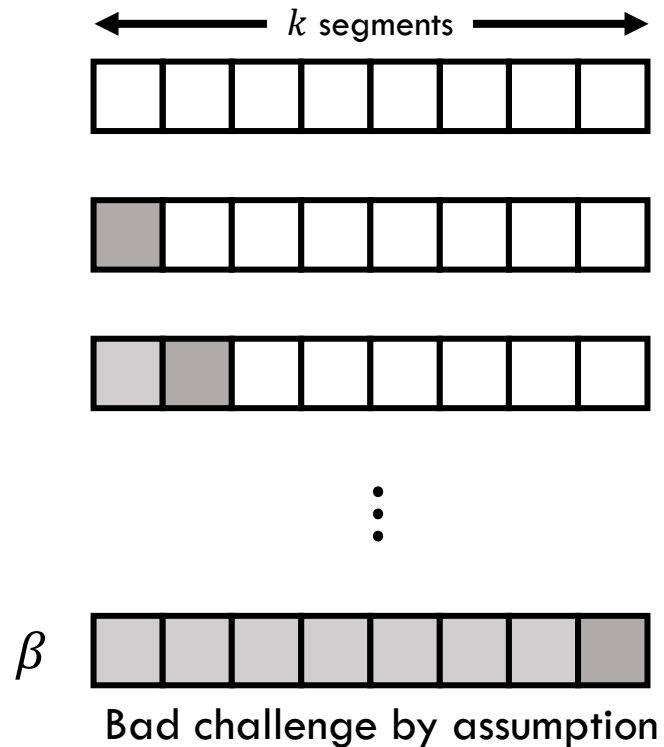


$$T = \#\text{bad challenges } \text{BAD}_{x,\alpha}^{(1)}$$

$$k \text{ such that } 2^k > T$$

If each segment is good

# Existence of a bad segment



#bad challenges remaining

$T$

$< T/2$

$< T/4$

$\vdots$

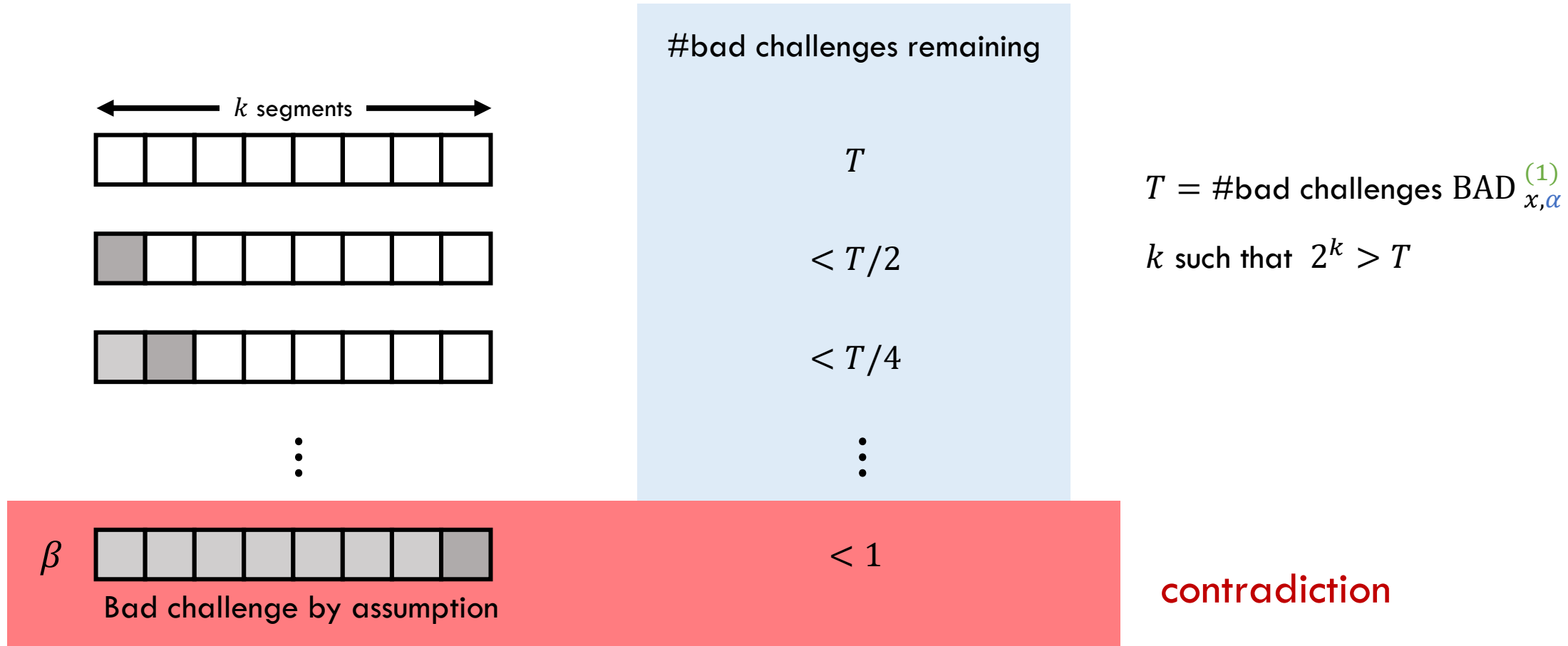
$< 1$

$T = \#\text{bad challenges } \text{BAD}_{x,\alpha}^{(1)}$

$k$  such that  $2^k > T$

If each segment is good

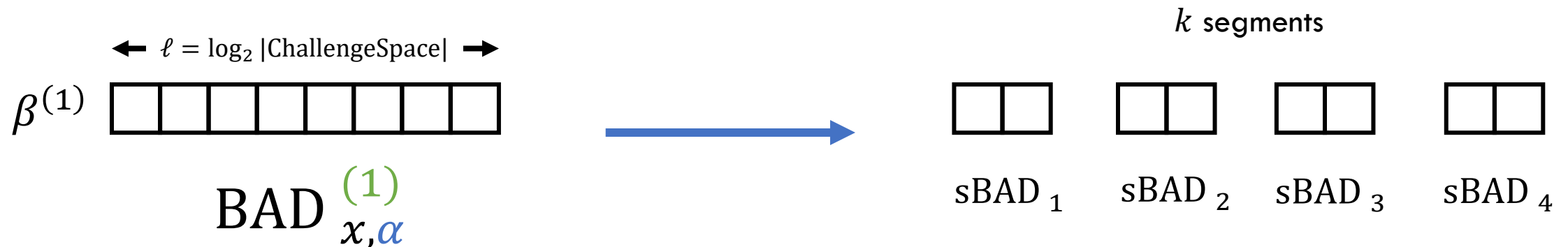
# Existence of a bad segment



If each segment is good

# Reducing to Verifiable Unique Bad Challenge

No parallel repetition

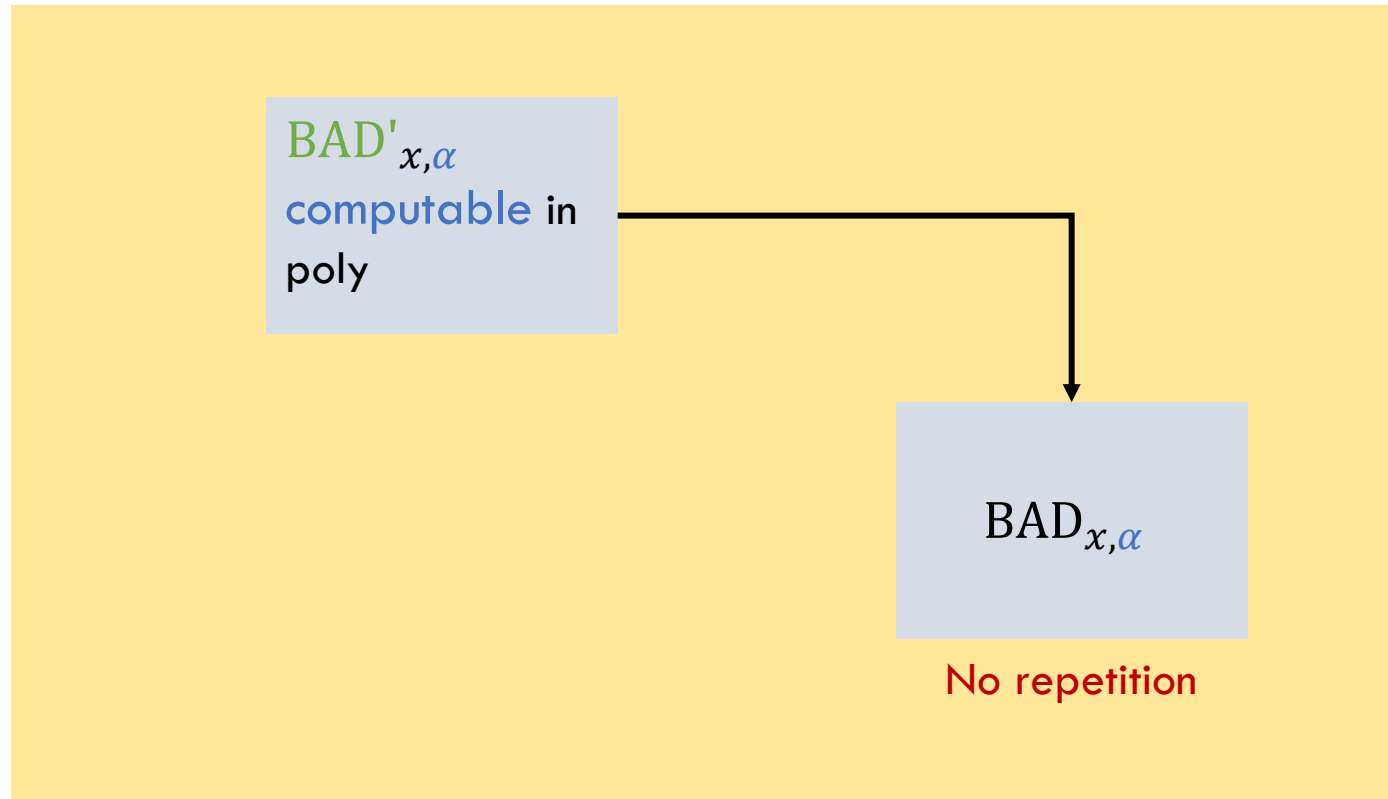


Requirements:

1. Each  $\text{sBAD}_j$  must be **efficiently verifiable unique bad challenge** relations.
2. If a challenge is bad, then there **must exist a bad segment**.

Each segment has  $\ell/k$  bits

# [C-Jain-Jin'21] Methodology



## $BAD_{x,\alpha}$ properties

- 1 Bad challenges are a product set
- 2 Challenge space is of polynomial size
- 3 Bad challenges are product verifiable in poly

# Concluding Remarks

See paper for:

1. Extension to parallel repetition.
2. Choice of parameters for size of segments, number of repetitions.
3. New somewhere extractable hash scheme necessary for “Magic box”.

# Recap: Our Results

## Theorem 1

Assuming **sub-exponential hardness of DDH**, there exists SNARGs for batch NP where

$$|\Pi| = \text{poly}(\log k, |C|)$$

## Theorem 2

Assuming **sub-exponential hardness of DDH**, there exists SNARGs for P where

$$|\text{CRS}|, |\Pi|, |\text{👤}| = \text{polylog}(T)$$

# Thank you. Questions?

Arka Rai Choudhuri

[arkarai.choudhuri@ntt-research.com](mailto:arkarai.choudhuri@ntt-research.com)

[ia.cr/2022/1486](https://ia.cr/2022/1486)