Merged Talk:

Fixing and Mechanizing the Security Proof of Fiat-Shamir with Aborts and Dilithium

Manuel Barbosa Gilles Barthe Christian Doczkal Jelle Don
Serge Fehr Benjamin Grégoire Yu-Hsuan Huang Andreas
Hülsing Yi Lee Xiaodi Wu

A Detailed Analysis of Fiat-Shamir with Aborts

Julien Devevey Pouria Fallahpour Alain Passelègue Damien Stehlè

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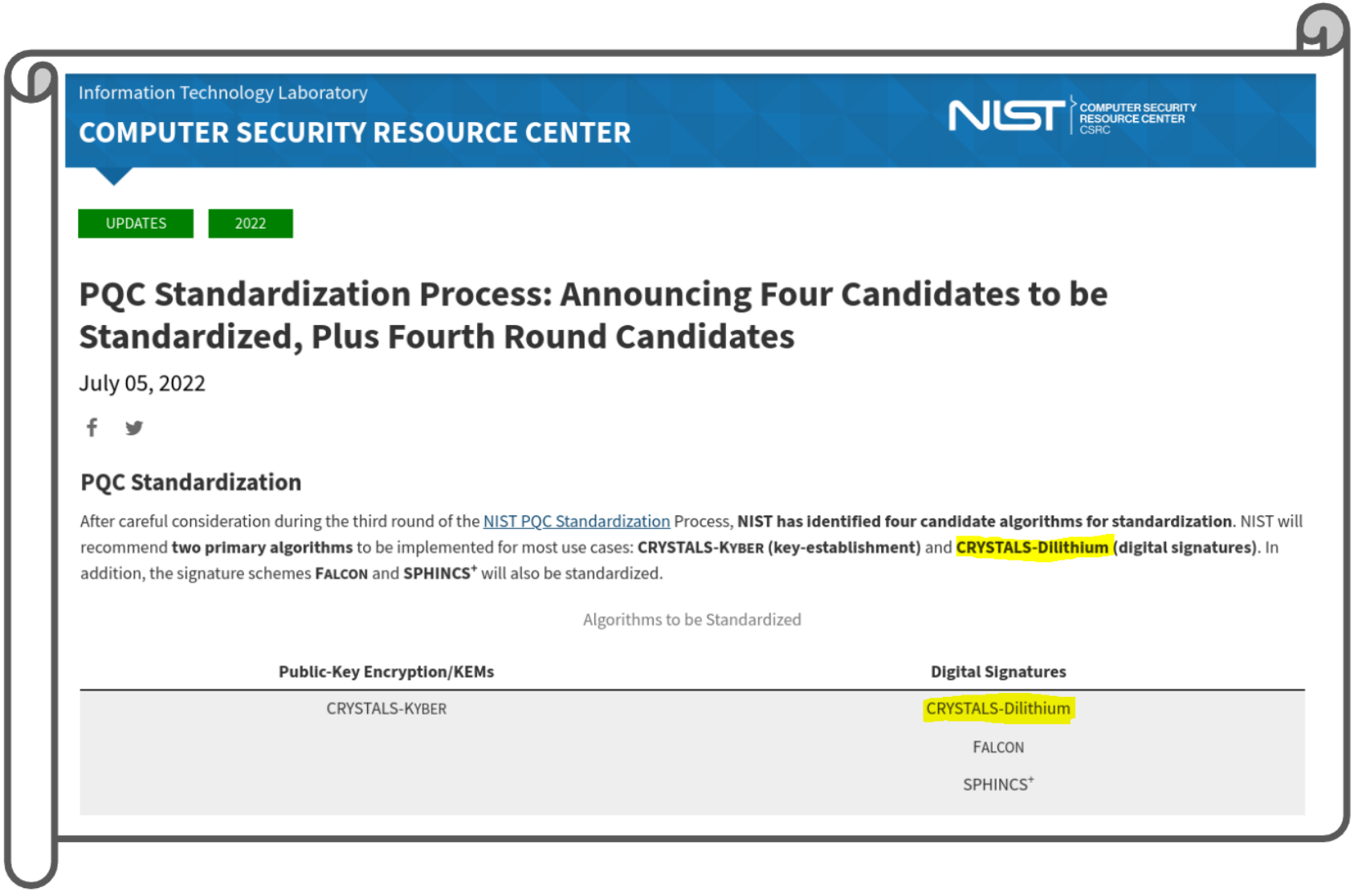
Story Time: Dilithium



Information Technology Laboratory

COMPUTER SECURITY RESOURCE CENTER

NIST COMPUTER SECURITY RESOURCE CENTER CSRC

UPDATES 2022

PQC Standardization Process: Announcing Four Candidates to be Standardized, Plus Fourth Round Candidates

July 05, 2022

PQC Standardization

After careful consideration during the third round of the NIST PQC Standardization Process, **NIST has identified four candidate algorithms for standardization**. NIST will recommend **two primary algorithms** to be implemented for most use cases: **CRYSTALS-KYBER (key-establishment)** and **CRYSTALS-Dilithium (digital signatures)**. In addition, the signature schemes **FALCON** and **SPHINCS+** will also be standardized.

Algorithms to be Standardized

Public-Key Encryption/KEMs	Digital Signatures
CRYSTALS-KYBER	CRYSTALS-Dilithium
	FALCON
	SPHINCS+

Figure: Annoucement of NIST PQC Winners

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Dilithium: a signature scheme based on FSwA

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History of FSwA:

- adapted from [FS86]

Amos Fiat (left) and Adi Shamir (right)

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History of FSwA:

- adapted from [FS86]
- origins: [Lyu09, Lyu12]

Fiat-Shamir With Aborts: Applications to Lattice and Factoring-Based Signatures

Vadim Lyubashevsky *

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- quantum analysis: [KLS18]

A Concrete Treatment of Fiat-Shamir Signatures in the Quantum Random-Oracle Model

Eike Kiltz [1] Vadim Lyubashevsky [2] Christian Schaffner [3]

February 20, 2018

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[3] QuSoft and ILLC, University of Amsterdam
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Plot-twist: gap in all previous security proofs of FSwA!

Invalidates claimed proven security of:

- Dilithium
- all other FSwA schemes (e.g. SeaSign[DFG19])

Our Work: Three-fold Contributions

1. **Identify a gap** in the proof of FSwA:
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3. **Formal verification** via Easycrypt, classically:
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Impact:

- We **fully restored** the security of Dilithium.
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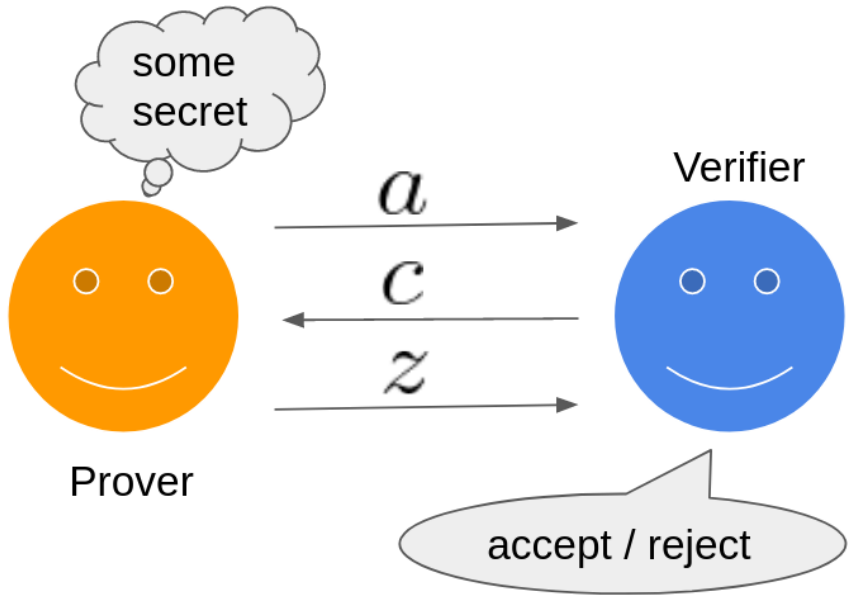
Concurrent work [Devevey, Fallahpour, Passelégue, Stehlé].

Outline

- Sigma Protocols
- Fiat-Shamir (with Aborts) Paradigm
- Fixing the Flaw
- Mechanizing Proofs

Sigma Protocol

A 3-round protocol where:
Prover convinces Verifier that he knows some secret.



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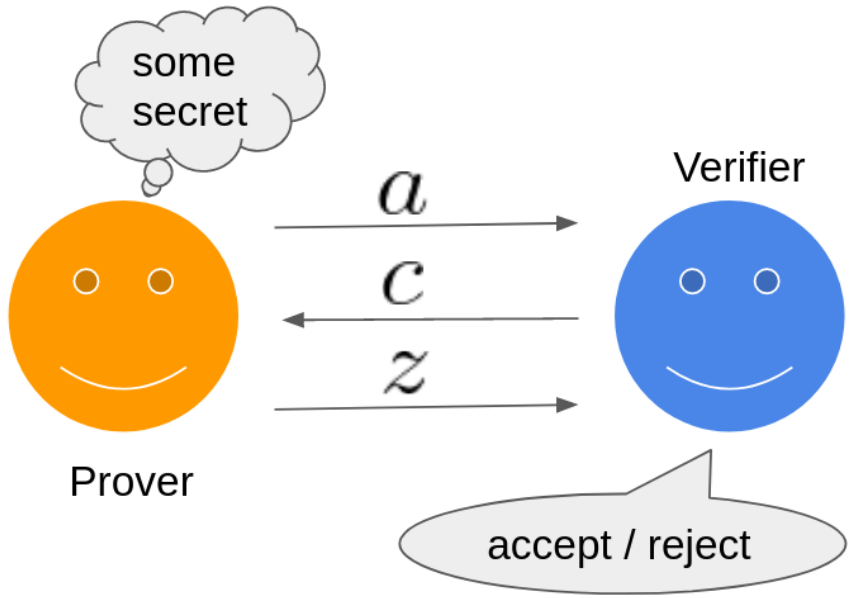

Knowledge soundness:

"Verifier can be convinced only if the Prover knows the secret."

Honest-Verifier Zero-Knowledge (HVZK)[1]:

"Verifier learns nothing about the secret from the protocol."

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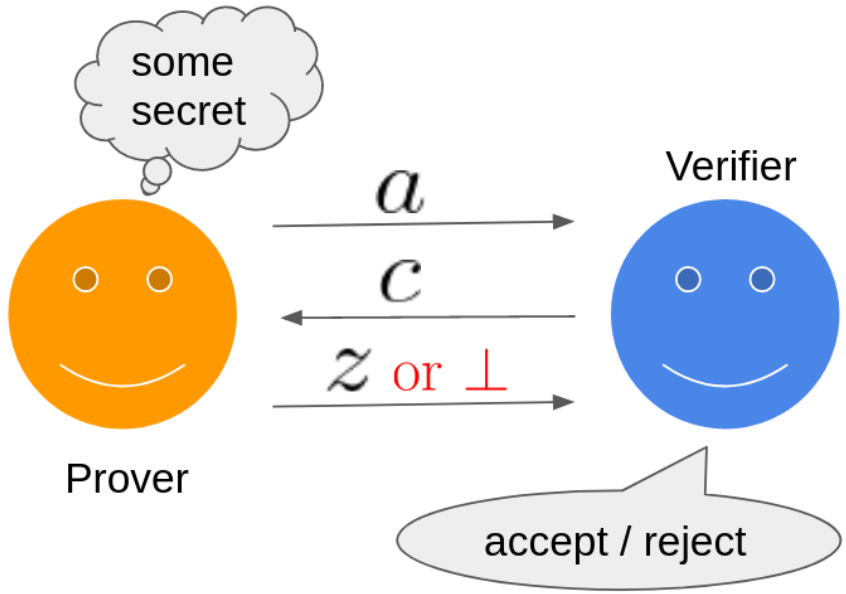
Aborting Sigma Protocol

In **aborting** Σ-protocols, Prover may abort with some probability.

Relevant in the case of lattices or isogenies,



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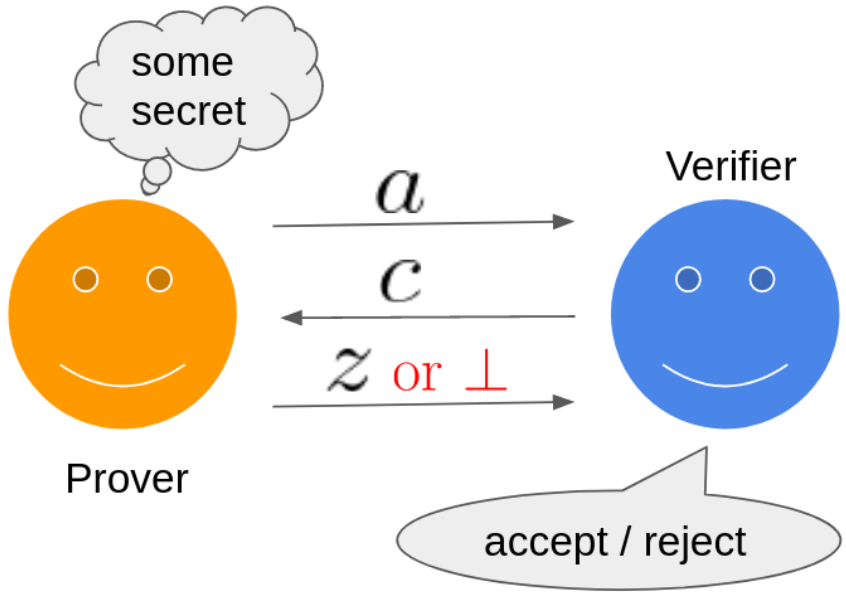

- Protocol is **repeated until** $z \neq \perp$ to convince Verifier
- Typically satisfies weaker version of HVZK (acHVZK)

acHVZK: the transcript **conditioned on** $z \neq \perp$, can be simulated

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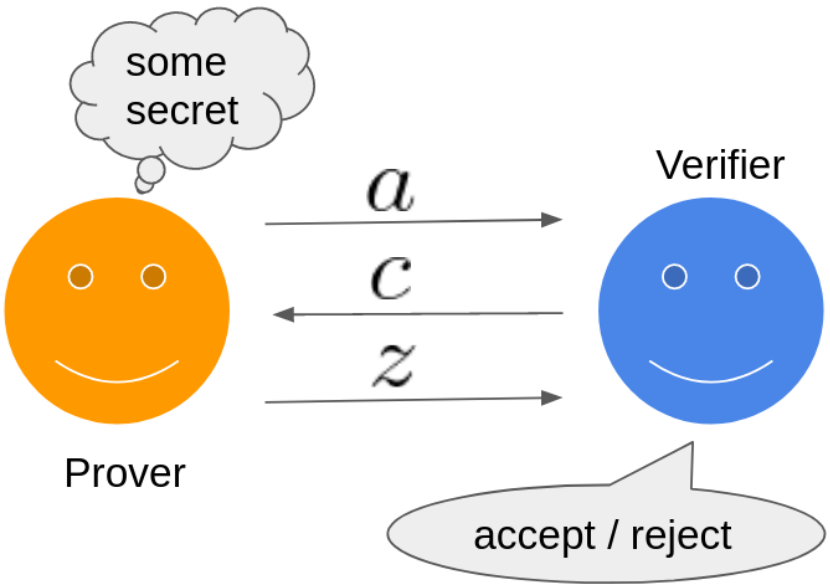
Fiat-Shamir Paradigm

a "recipe" for designing signature schemes / NIZK

Ingredient: a Sigma protocol Σ



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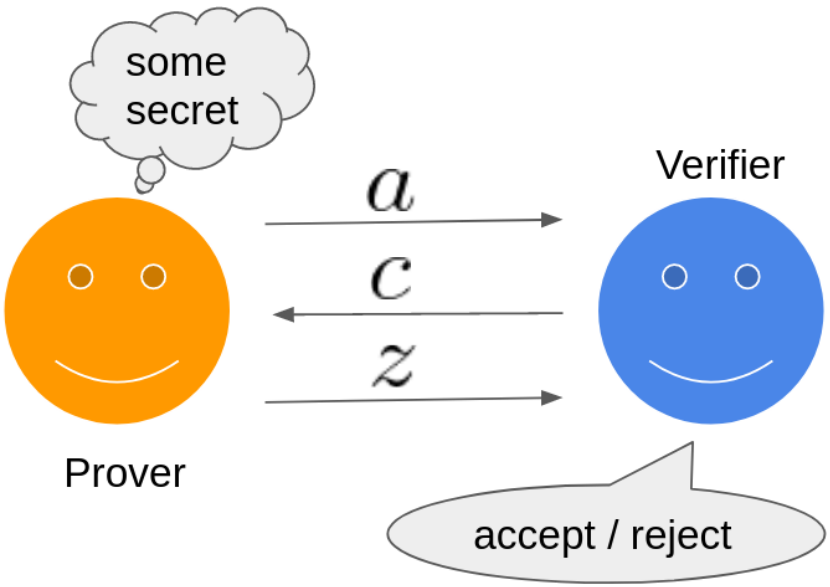
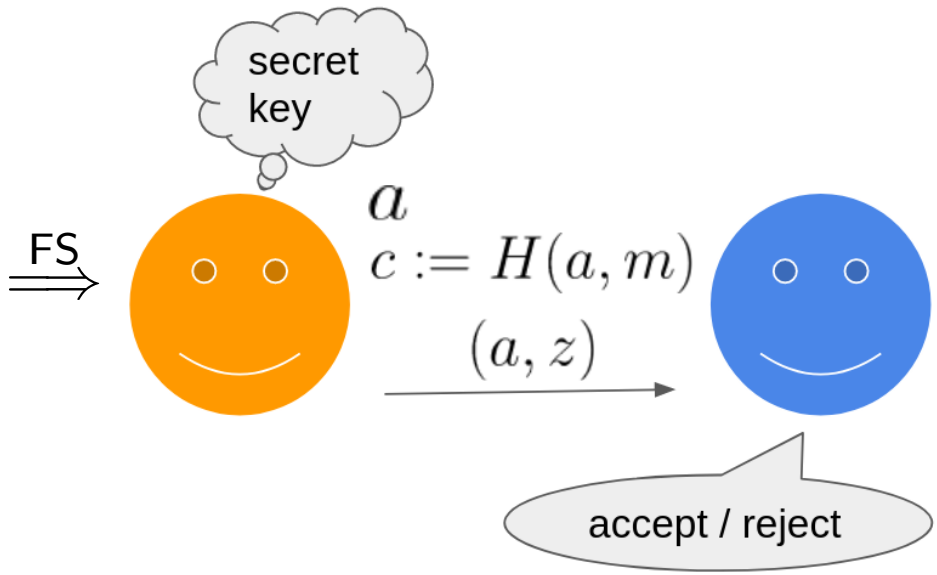


Result: a signature FS[Σ]



Fiat-Shamir with Aborts Paradigm

a "recipe" for designing signature schemes / NIZK

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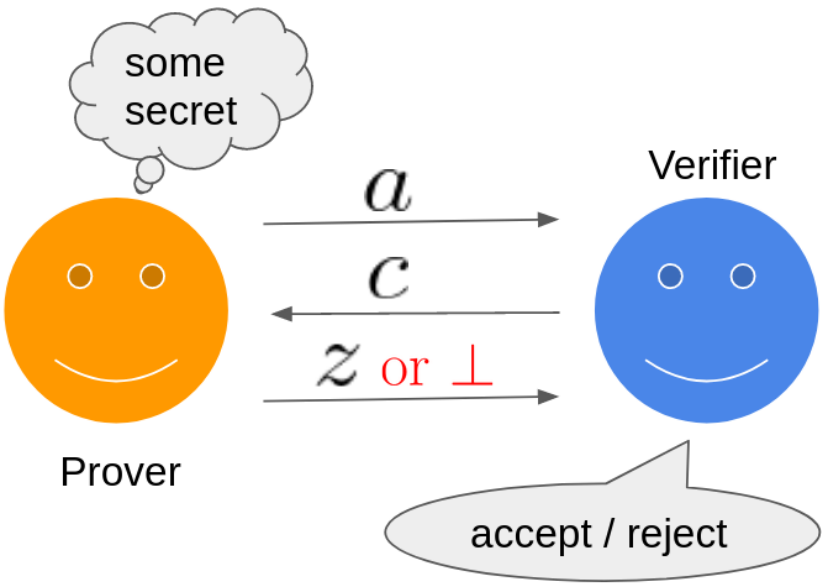
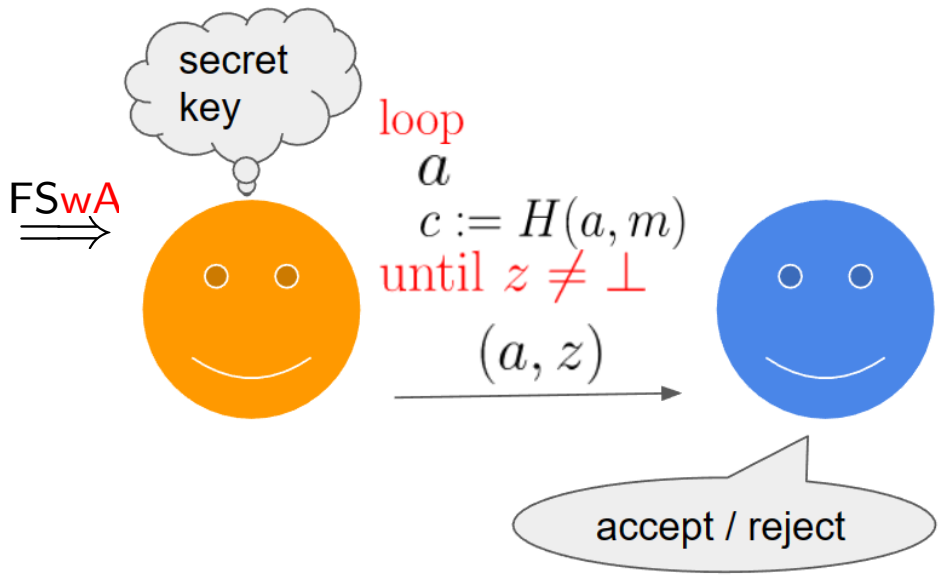


Result: a signature FSwA[Σ]



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- (weak) HVZK $\overset{!}{\Rightarrow}$ CMA-to-NMA reduction, i.e. seeing valid signatures does not help forging

"$\overset{!}{\Rightarrow}$" holds for Fiat-Shamir,
but here lies the catch for FSwA

Security of Fiat-Shamir (with Aborts)

Reduction: a forger could've simulated signatures by himself

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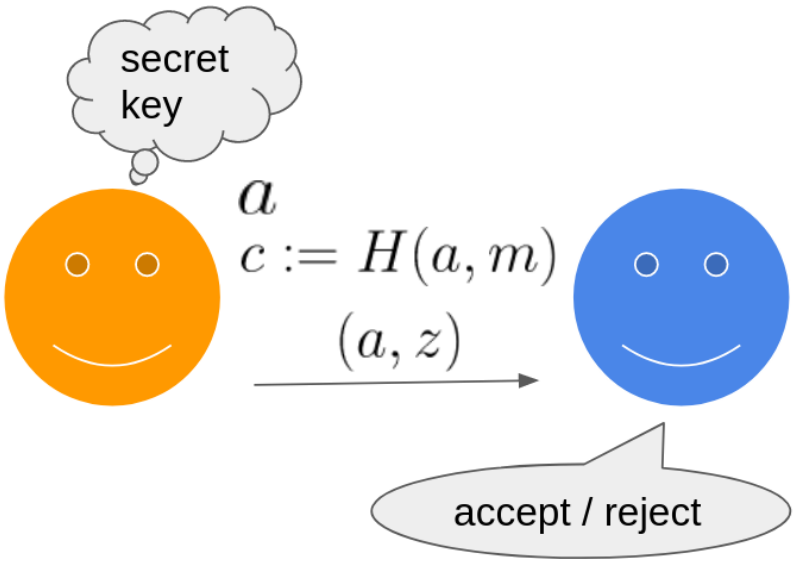

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$H(a, m) := c$

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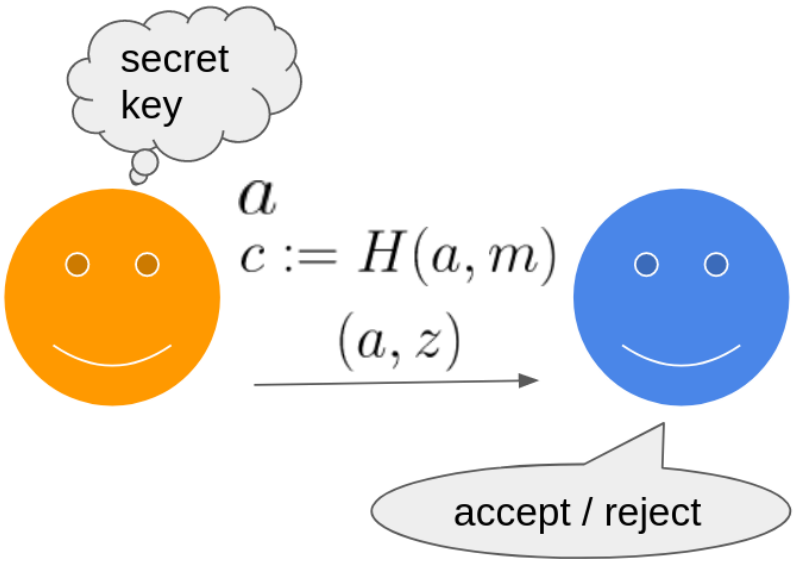

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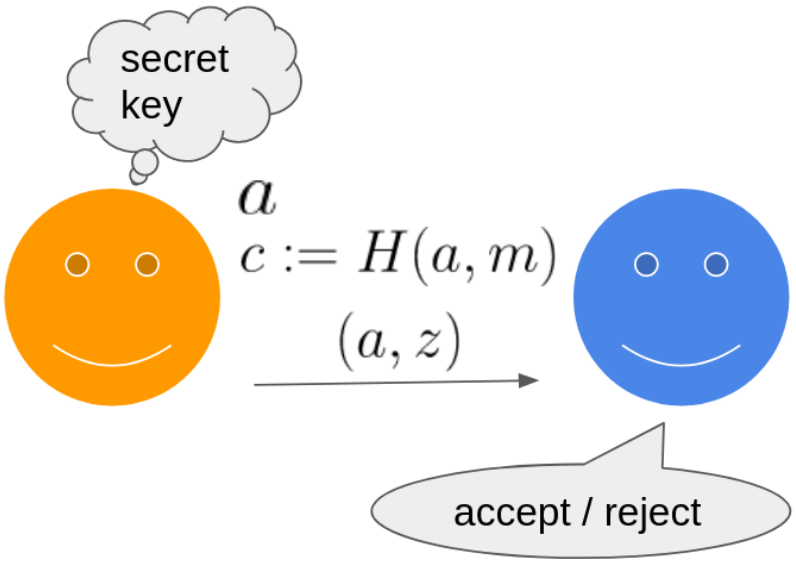

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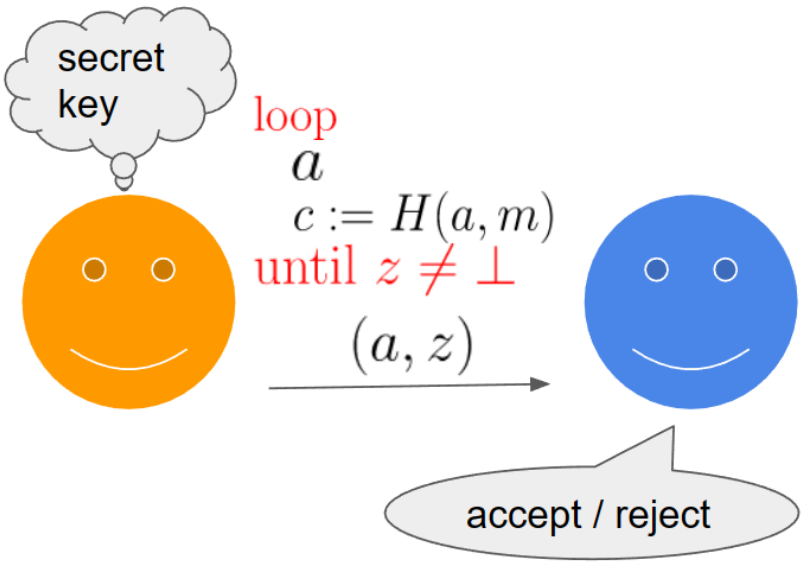

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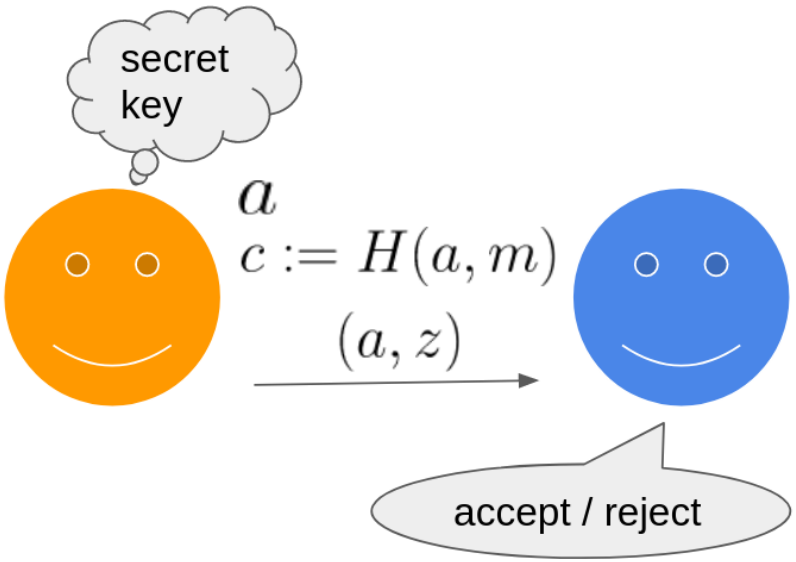

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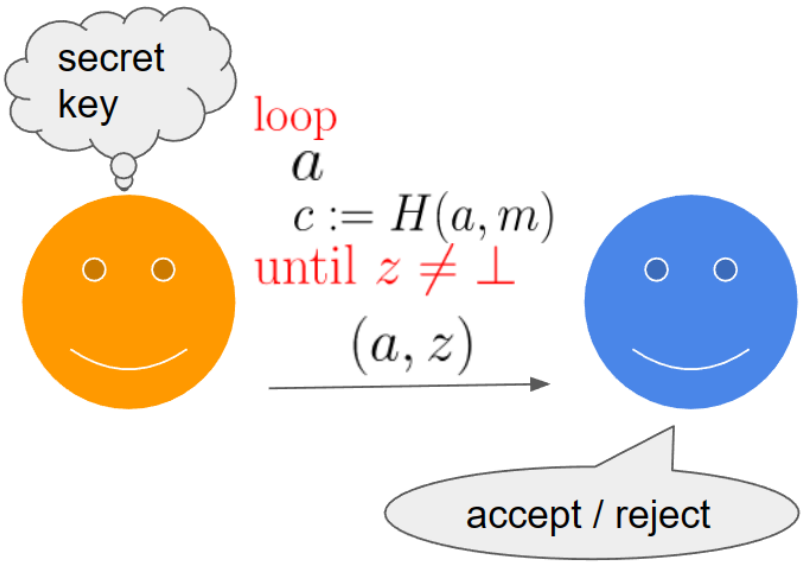

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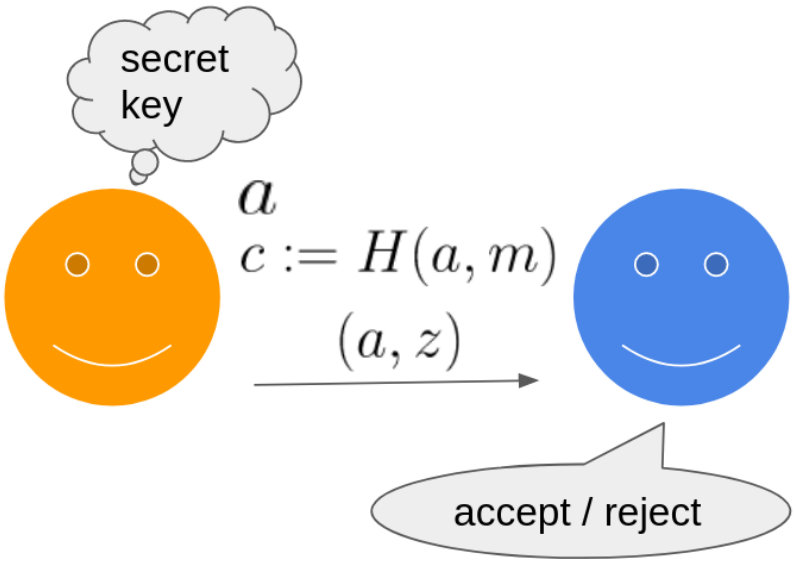

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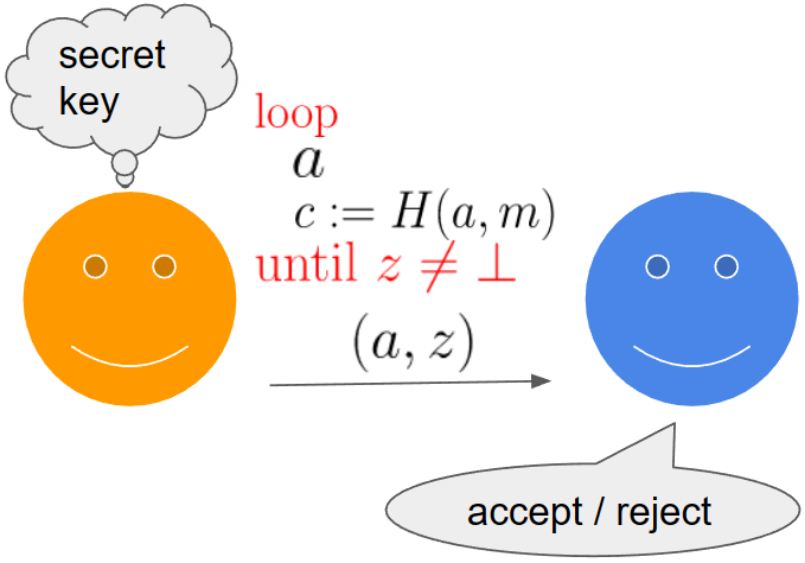

simulated by $\Longrightarrow$ $(a, c, z) \leftarrow$ acHVZK-Sim, $H(a, m) := c$

c is not random and independent of a

H is biased, ruining previous proofs!

Outline

Sigma Protocol

Fiat-Shamir (with Aborts) Paradigm

- Fixing the Flaw
- Mechanizing Proofs

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No easy patch. Had to redo the proof from scratch!

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Generically, obtain worse security loss.

Fixing the Flaw

Assuming perfect (weak) HVZK, for CMA-to-NMA reduction:

- [KLS18]: quantum and classical loss $\leq \epsilon := \max_{a^\circ} \Pr[a = a^\circ]$
- Ours:

$$\text{quantum loss} \leq O\left(\sqrt{q_H^2 q_S \epsilon} + \sqrt{q_S^3 \epsilon}\right)$$

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Restored via better control over ϵ, partially computer-aided

- [KLS18]: $\epsilon \lesssim 2^{-255}$
- Ours, for NIST3 parameters: $\epsilon \lesssim 2^{-844}$

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Role of Formal Verification Efforts

- Machine-checked proofs using EasyCrypt
- Discovery of the [KLS18] flaw

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    - Underlying aborting sigma-protocol is acHVZK
    - NMA to lattice assumptions
    - Commitment-recovery optimization
- $+$ several thousand lines of library extensions
- Novelty: Expected number of iterations, infinite hybrids

Our Mechanized Proof: Future Directions

- Extend results to optimized implementation
- Incorporate side-channel resistance

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Call for action: Re-examine your own FSwA signatures!

Thank you for listening!

Eprint: ia.cr/2023/246

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