Fast Blind Rotation for Bootstrapping FHEs

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Part I

Preliminaries

Fully Homomorphic Encryption

- Two-layer framework:
  1. Noise-based somewhat HE
  2. Homomorphically computes the decryption of SHE (Bootstrapping)

The modulo q operation is expensive!

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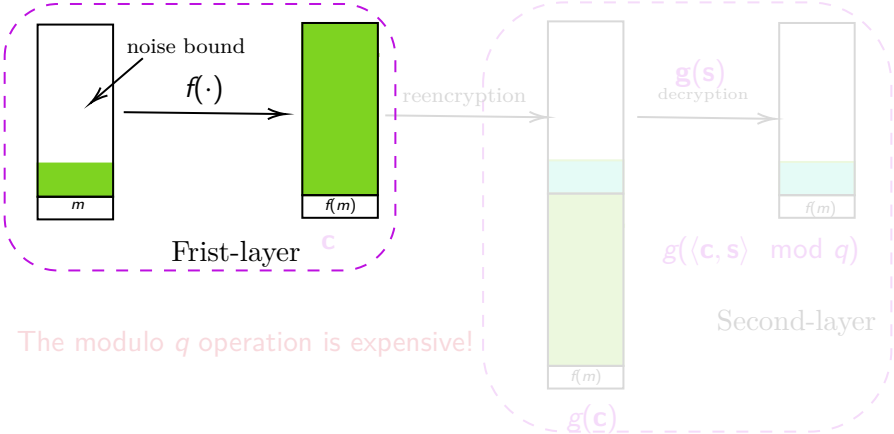

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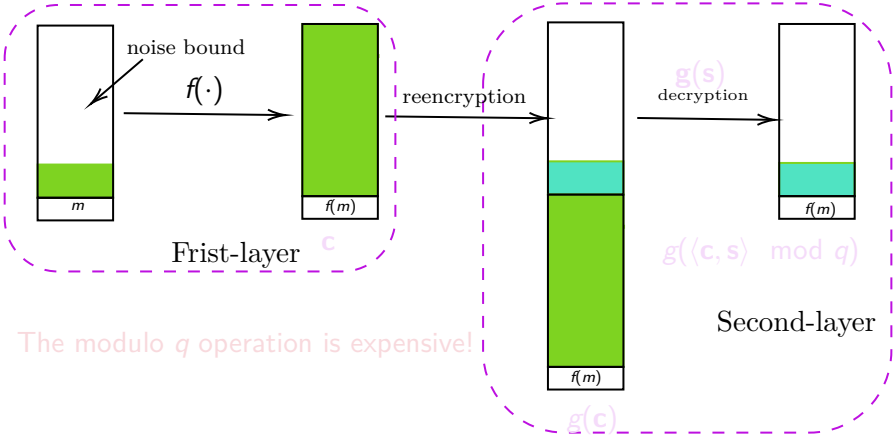

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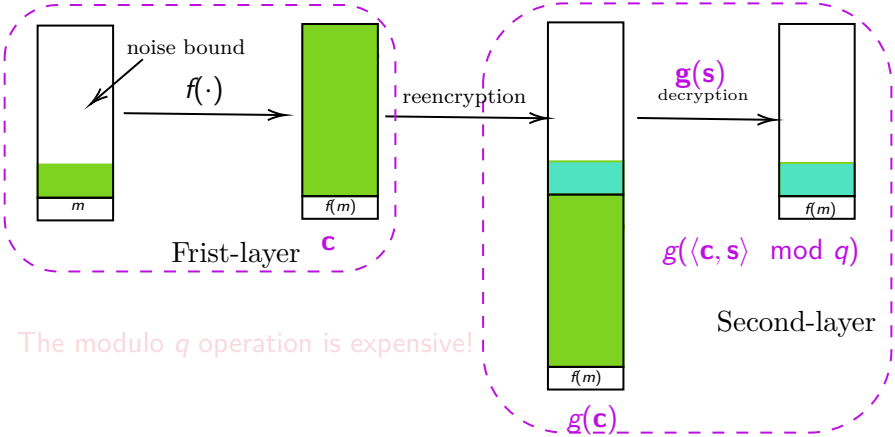

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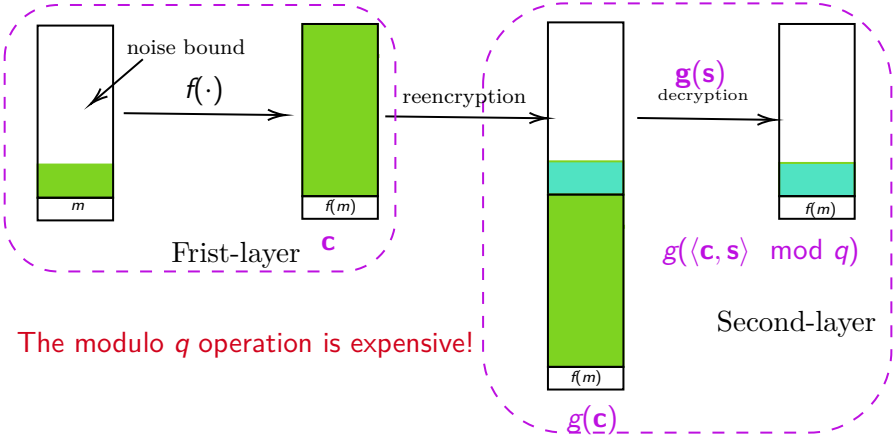

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Blind Rotation

- Homomorphically decrypt an LWE ciphertext on the exponent.

Definition (Blind Rotation(case $q = 2N$))

- Input:
  - LWE ciphertext $(\mathbf{a}, b = \sum_{i=0}^{n-1} a_i s_i - \text{noised}(m)) \in \mathbb{Z}_q^{n+1}$;
  - $r(X) \in R_Q = \mathbb{Z}_Q[X]/(X^N + 1)$; evaluation key **EVK**;
- Output: $g(r(X) \cdot X^{\text{noised}(m)})$

- The modulo q operation can be done for free in the exponent.

$$X^{\text{noised}(m)} = X^{\sum_{i=0}^{n-1} a_i s_i - b \mod q} = X^{-b} X^{\sum_{i=0}^{n-1} a_i s_i}$$

- The constant term of $r(X) \cdot X^{\text{noised}(m)}$ is exactly m.

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- RLWE-based blind rotation:
  - AP/FHEW [ASP14, DM15]
    - decompose $a_i = \sum_j a_{i,j} B^j$, all secret key distribution, large evaluation key
  - GINX/TFHE [GINX16, CGGI20]
    - decompose $s_i = \sum_{u \in U} s_{i,u} u$ for some public set U, limited secret key distribution, small evaluation key
  - Lee et al. [LMK+23]
    - ring automorphism, all secret key distribution, small evaluation key
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- Recommended key distributions in [ACC+21]: Uniform, Gaussian, and Ternary distribution.
- Final[BIP+22], NTRU-vum[Klu22] use binary or ternary secrets for performance consideration.
- Potential Problem: small secrets are subject to special attacks [Alb17, AGVW17, SC19, EJK20].
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Part II

Technical Contributions

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- We design a new NTRU-based GSW-like encryption
  - Faster external product
  - Faster key-switching and ring automorphism
- We propose a new blind rotation using NTRU and ring automorphism
  - performance asymptotically independent from the key distributions
  - all secret key distribution, small evaluation key
- We use our new blind rotation to bootstrap an LWE-based scheme
  - Faster bootstrapping time(53% faster than TFHE)
  - Smaller evaluation key (18MB)
- Extra improvement by using large key distributions (17%)

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Part III

NTRU-based GSW-like Scheme

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- Parameters (τ, Δ) depends on the encoding

$$(\tau, \Delta) = \begin{cases} \left(1, \left\lfloor \frac{Q}{t} \right\rceil\right), & \text{if noised}(m) = e + \left\lfloor \frac{q}{t} \right\rceil \cdot m; \\ (t, 1), & \text{if noised}(m) = t \cdot e + m; \\ (1, 1), & \text{if noised}(m) = e + m. \end{cases}$$

- scalar ciphertext: $\mathrm{NTRU}_{Q,f,\tau,\Delta}(u) := \tau \cdot g/f + \Delta \cdot u/f \in R_Q$
- vector ciphertext: $\mathrm{NTRU}'_{Q,f,\tau}(v) := (\tau \cdot g_0/f + B^0 \cdot v, \cdots, \tau \cdot g_{d-1}/f + B^{d-1} \cdot v) \in R_Q^d$

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- Basic operation
  - $\mathrm{BitDecom}_B(c) = (c_0, c_1, \ldots, c_{d-1}) \in R_B^d$ such that $c = \sum_{i=0}^{d-1} c_i \cdot B^i$
  - $$c \odot \mathbf{c}' = \sum_{i=0}^{d-1} c_i c_i' = \tau \cdot \left(\sum_{i=0}^{d-1} g_i c_i \right) / f' + \tau \cdot gv/f + \Delta \cdot uv/f$$
- External product
  - Let $f = f'$, $c \odot \mathbf{c}' = \tau \cdot \left(\sum_{i=0}^{d-1} g_i c_i + gv \right) / f + \Delta \cdot uv/f$
  - d multiplications on R_q
- Key Switching
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Part IV

New blind rotation

Basic idea

- Recall blind rotation: homomorphically decrypt the LWE ciphertext on the exponent

$$r(X)X^{\sum_{i=0}^{n-1} a_i s_i - b \mod q} = r(X)X^{-b}X^{\sum_{i=0}^{n-1} a_i s_i}$$

- Basic Construction:
  - Given a ciphertext $\mathrm{NTRU}_{f(X)}(X^{s_i})$, applying $X \to X^{a_i}$.
  - Perform once key-switching: convert the secret key $f(X^{a_i})$ to f(x).

$$\mathrm{NTRU}_{f(X)}(X^{s_i}) \xrightarrow{\text{EvalAuto}} \mathrm{NTRU}_{f(X^{a_i})}(X^{a_i s_i}) \xrightarrow{\text{KS}} \mathrm{NTRU}_{f(X)}(X^{a_i s_i})$$

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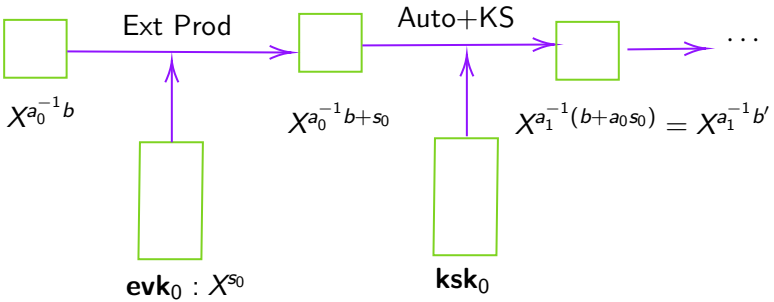

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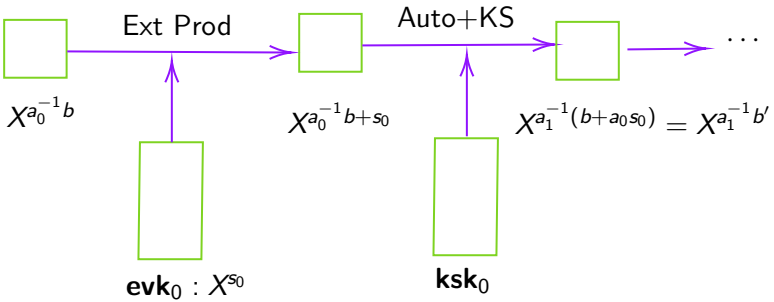

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Construction

- Problem I: Each $a_i \in \mathbb{Z}_q$ can be not coprime to 2N (probability 1/2)
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  - Define $S = \{2i+1 : 1 \leq i \leq q-1\} \subset \mathbb{Z}_{2N}$, $S \cup \{1\}$ is a multiplicative subgroup of $\mathbb{Z}_{2N}$
  - Easily check:$\forall w, \hat{w} \in S \cup \{1\}$ $w^{-1}, \hat{w}^{-1}, w\hat{w} \in S \cup \{1\}$
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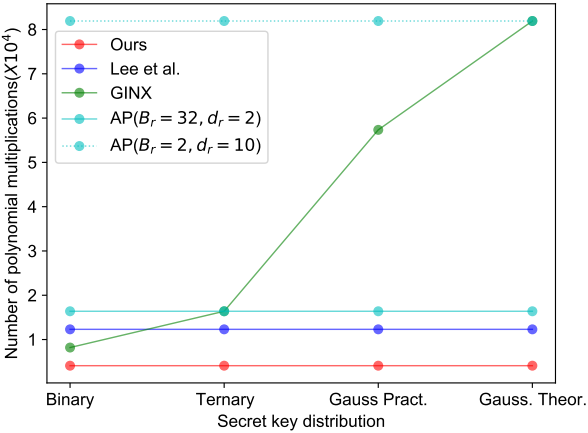
Ours
Lee et al.
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AP($B_r = 32, d_r = 2$)
AP($B_r = 2, d_r = 10$)
Number of polynomial multiplications($X10^4$)
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2
3
4
5
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7
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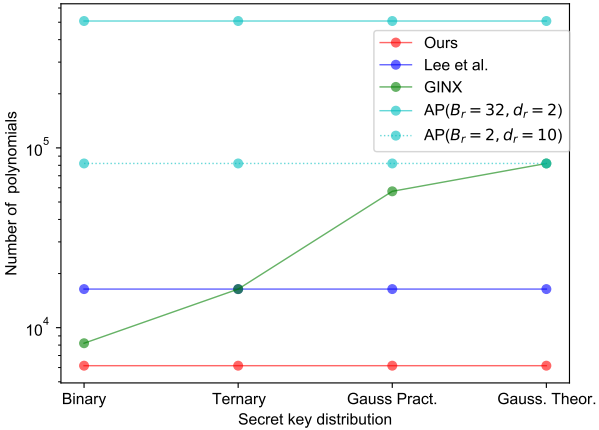
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Part V

Bootstrapping

Bootstrapping for LWE

- Overview of bootstrapping in RLWE/RGSW setting

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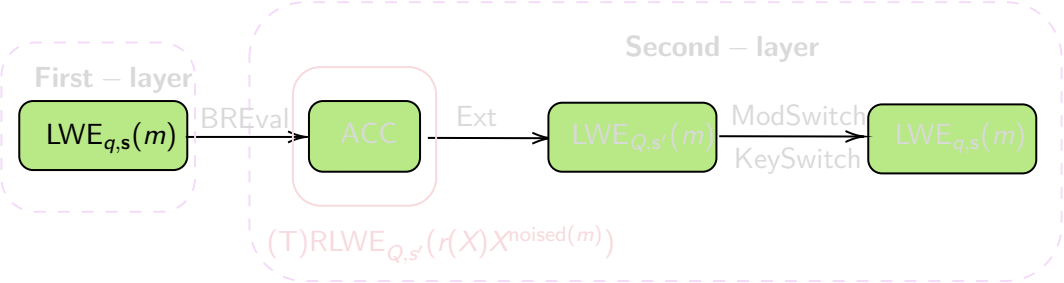

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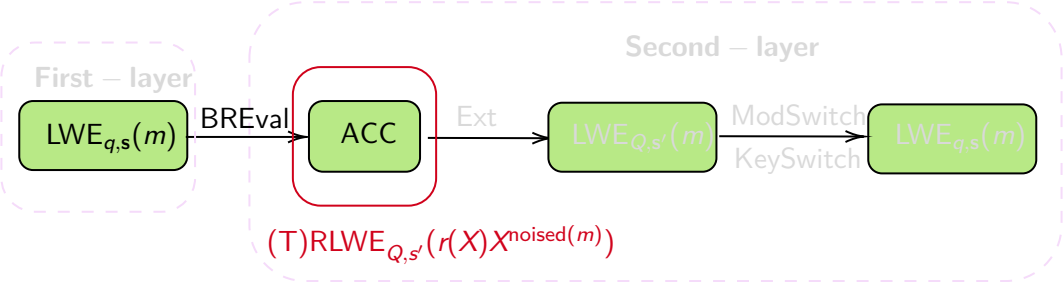

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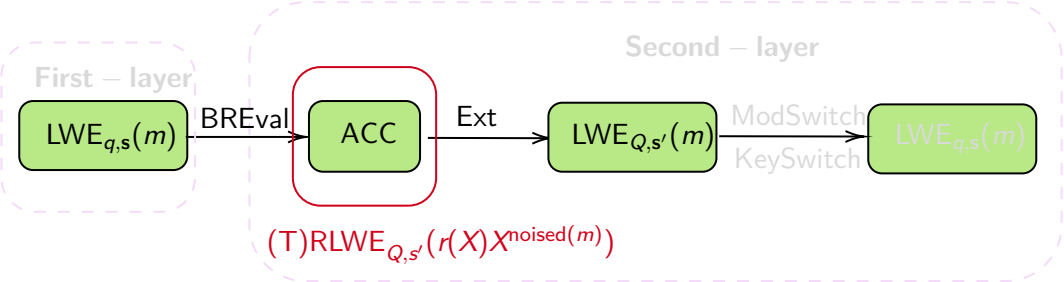

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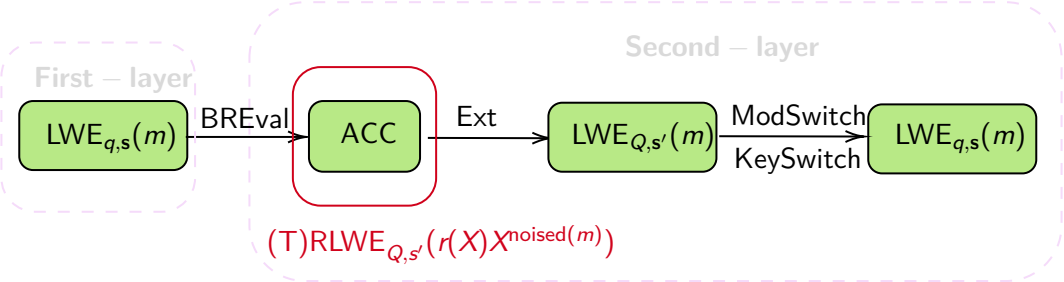

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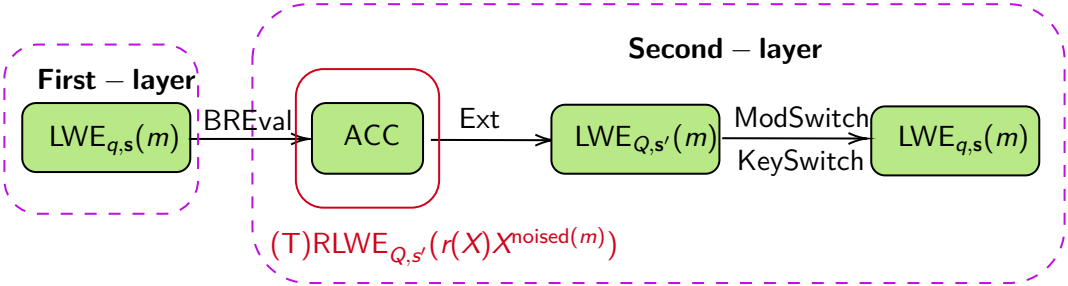

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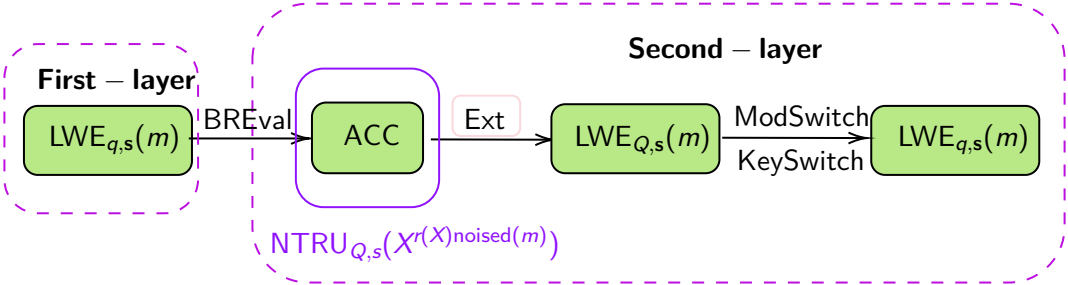

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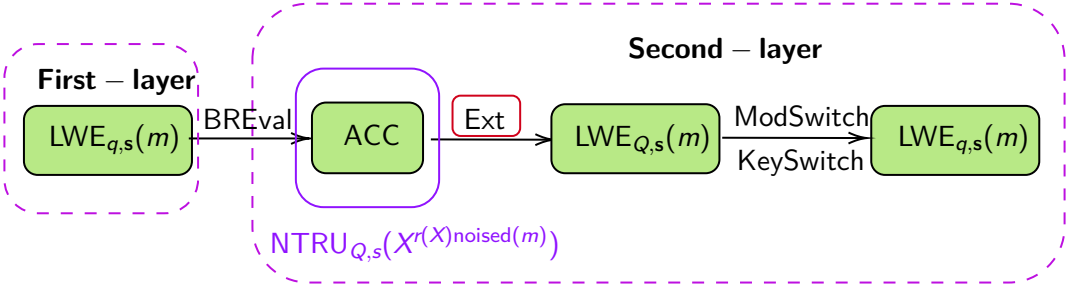

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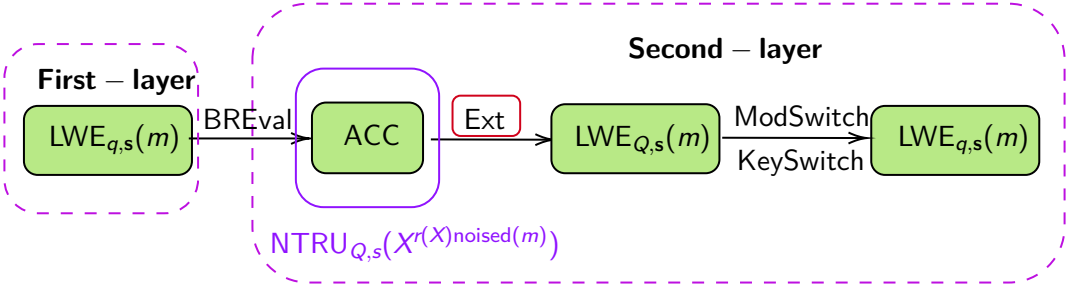

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Part VI

Experimental Results

Experimental Results

Table: Parameters for bootstrapping LWE-based first-layer ciphertexts.

Parameters	Key distrib.	n	q	N	Q	B	Q_{ks}	B_{ks}
STD128 [MP21]	Ternary	512	1024	1024	2^{27}	2^7	2^{14}	2^7
P128T	Ternary	512	1024	1024	$995329 \approx 2^{19.9}$	2^4	2^{14}	2^7
P128G	Gaussian	465	1024	1024	$995329 \approx 2^{19.9}$	2^4	2^{14}	2^7
STD192 [MP21]	Ternary	1024	1024	2048	2^{37}	2^{13}	2^{19}	28
P192T	Ternary	1024	1024	2048	$44421121 \approx 2^{25.4}$	2^9	2^{19}	28
P192G	Gaussian	870	1024	2048	$44421121 \approx 2^{25.4}$	2^9	2^{17}	28

Experimental Results

Table: Timings and key sizes for bootstrapping ($\lambda = 128$)

Algorithms	Parameters	Key distrib.	Timings (ms)	EVK (MB)	KSK (MB)	Boots. key (MB)
FHEW/AP	STD128 [MP21]	Ternary	359	1674	224	1898
TFHE/GINX	STD128 [MP21]	Ternary	234	54	224	278
Ours	P128T	Ternary	112	18.65	224	242.65
	P128G	Gaussian	100	17.90	203.44	221.34

Extra 10% improvement over P128T

Experimental Results

Table: Timings and key sizes for bootstrapping ($\lambda = 192$)

Algorithms	Parameters	Key distrib.	Timings (ms)	EVK (MB)	KSK (MB)	Boots. key (MB)
FHEW/AP	STD192 [MP21]	Ternary	1200	6682	532	7214
TFHE/GINX	STD192 [MP21]	Ternary	859	222	532	754
Ours	P192T	Ternary	320	38.10	532	570.10
	P192G	Gaussian	273	34.30	404.41	438.71

Extra 17% improvement over P192T

Conclusion

- An NTRU-based GSW-like scheme with faster external product and key switching
- A new blind rotation technique based on our scheme and ring automorphism
  - Arbitrary secret distribution, small evaluation key
- Faster bootstrapping than TFHE/FHEW
  - Improve parameter by using Gaussian distribution
  - Large secret distribution offers better security

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