TreePIR: Sublinear Time Polylog Bandwidth Private Information Retrieval from DDH

Arthur Lazzaretti

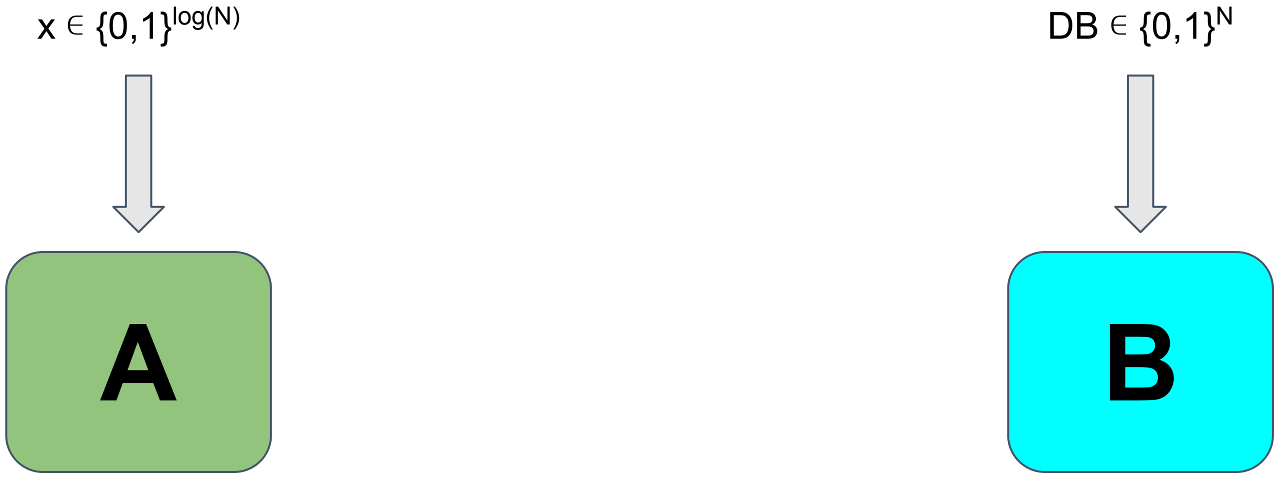
joint work with Charalampos Papamanthou

Private Information Retrieval [CGKM ‘95, KO ‘97,....]

A

B

Private Information Retrieval [CGKM ‘95, KO ‘97,....]



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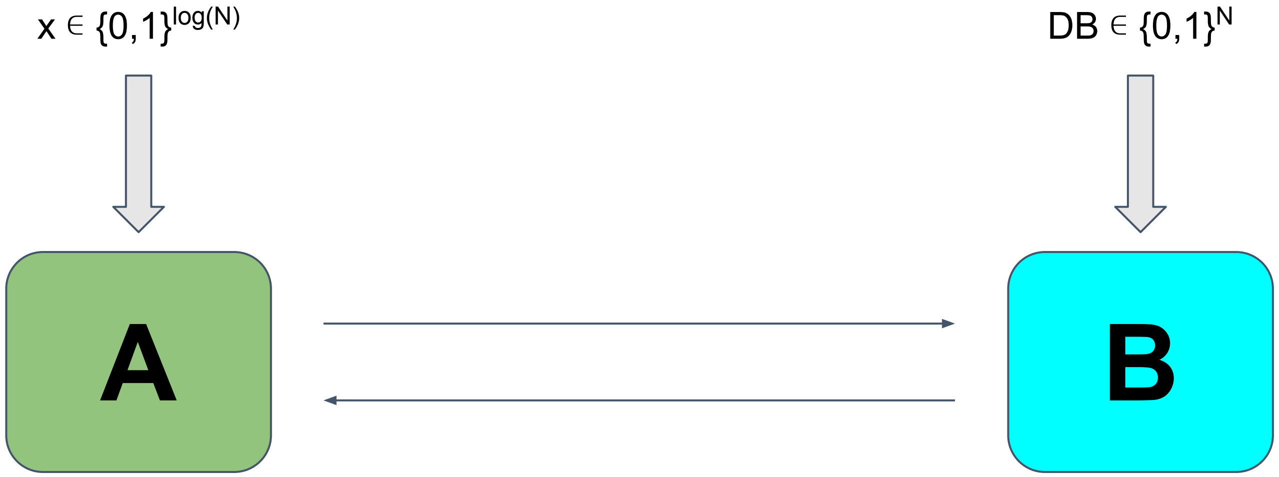

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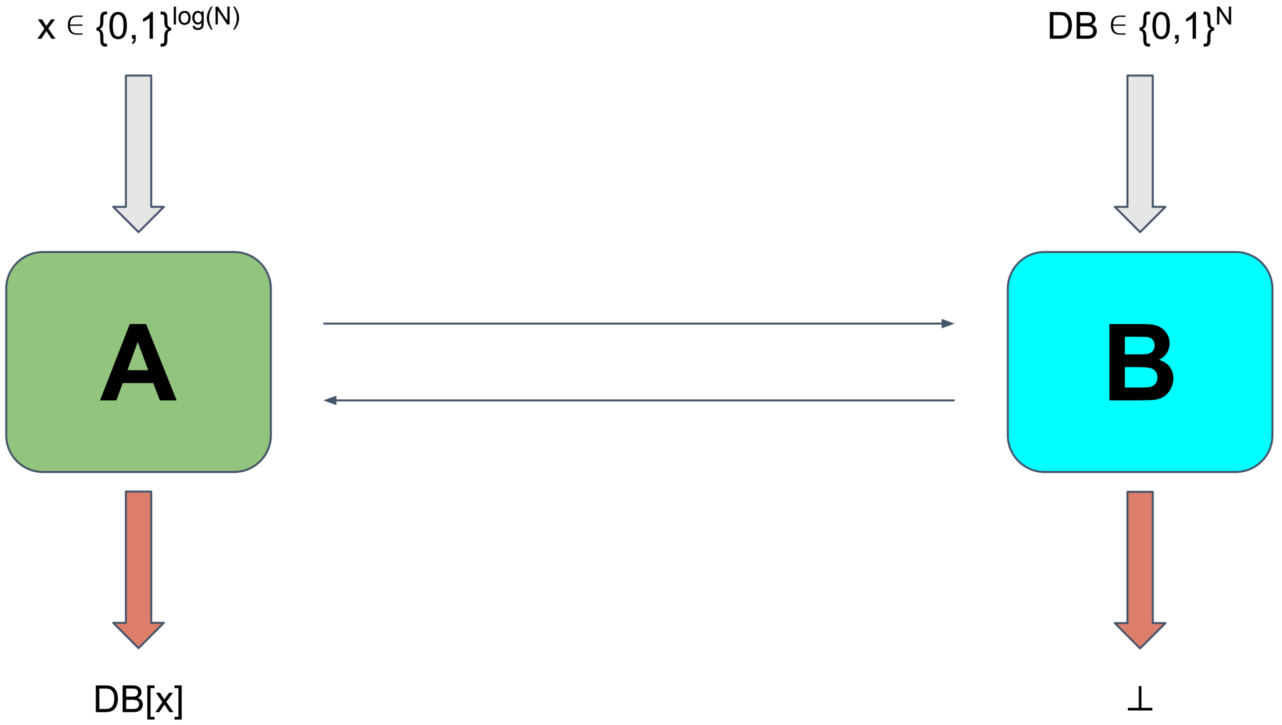

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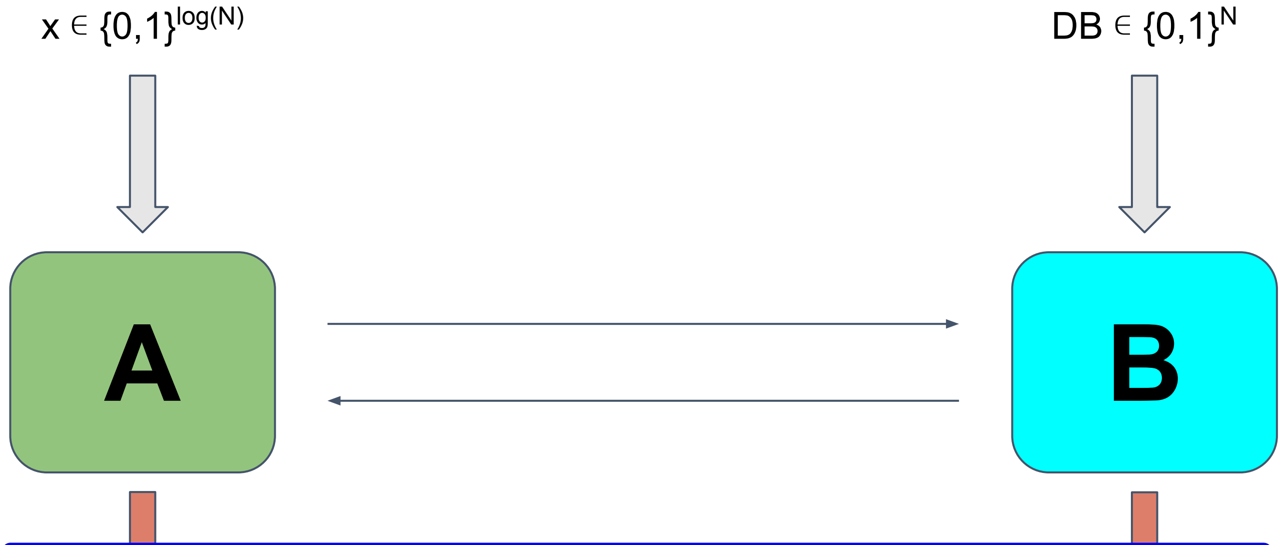
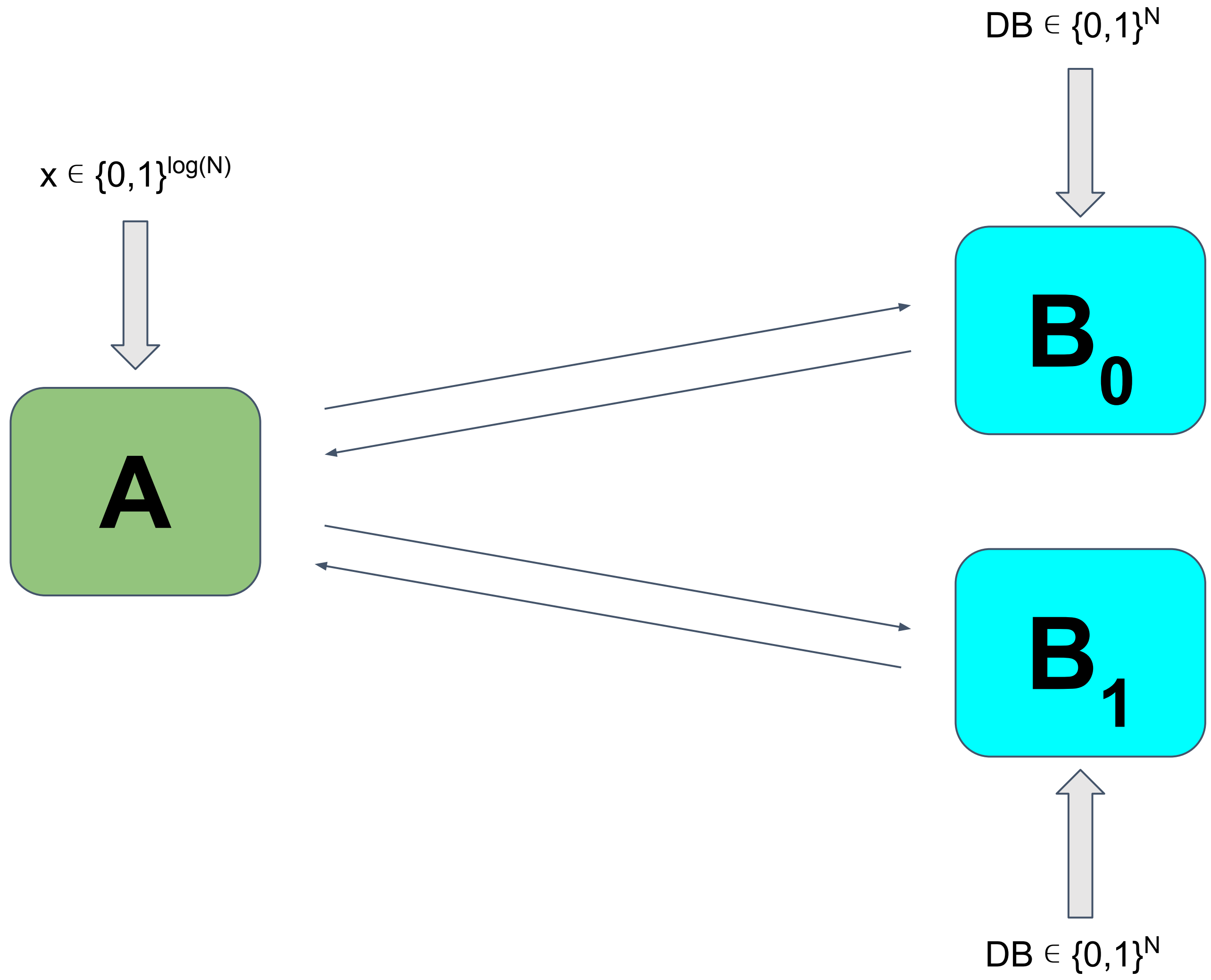


Privacy:
There exists a PPT simulator A* with no knowledge of x that can act as A and B cannot tell the difference

Two-Server Private Information Retrieval [CGKM ‘95, KO ‘97,....]



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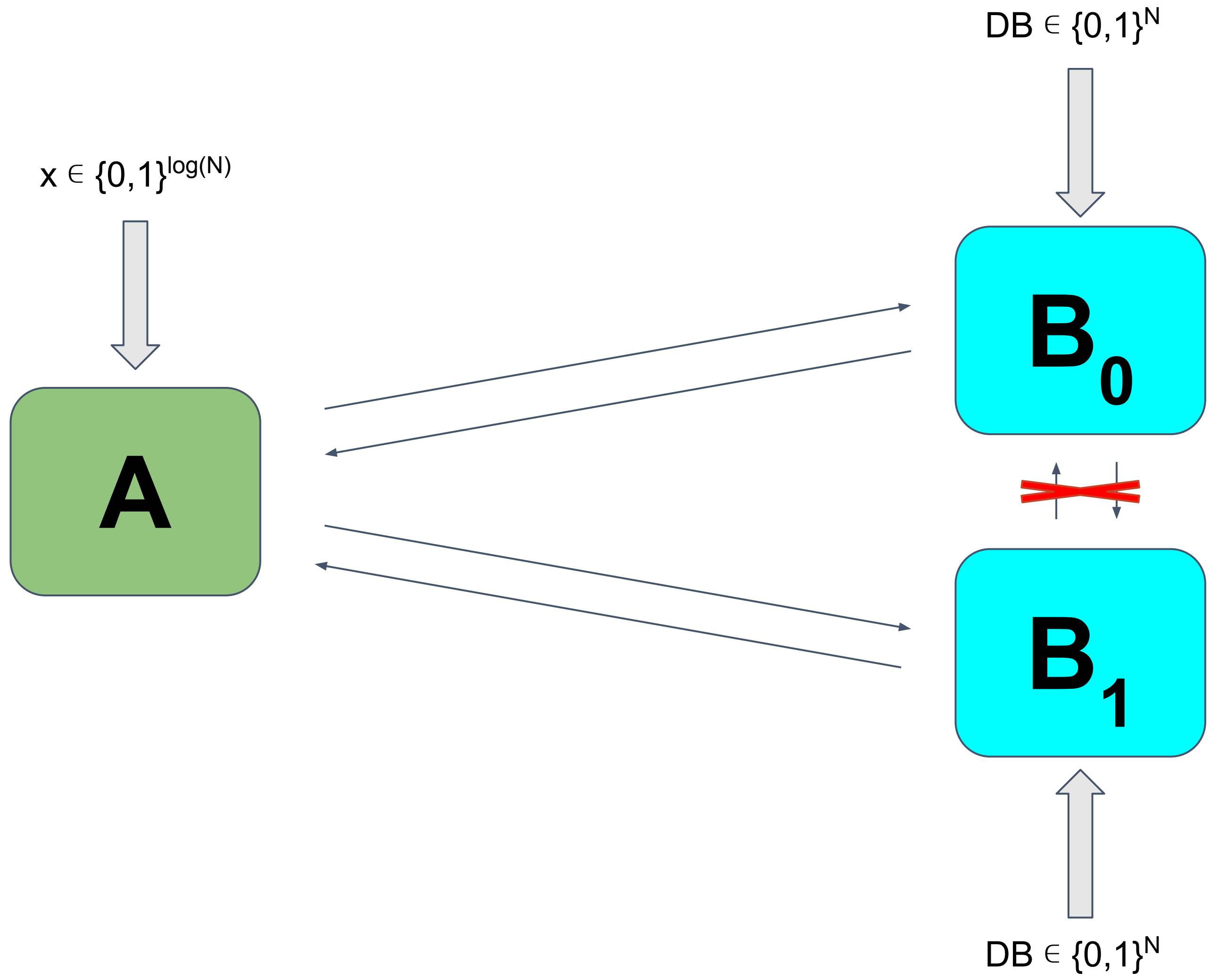

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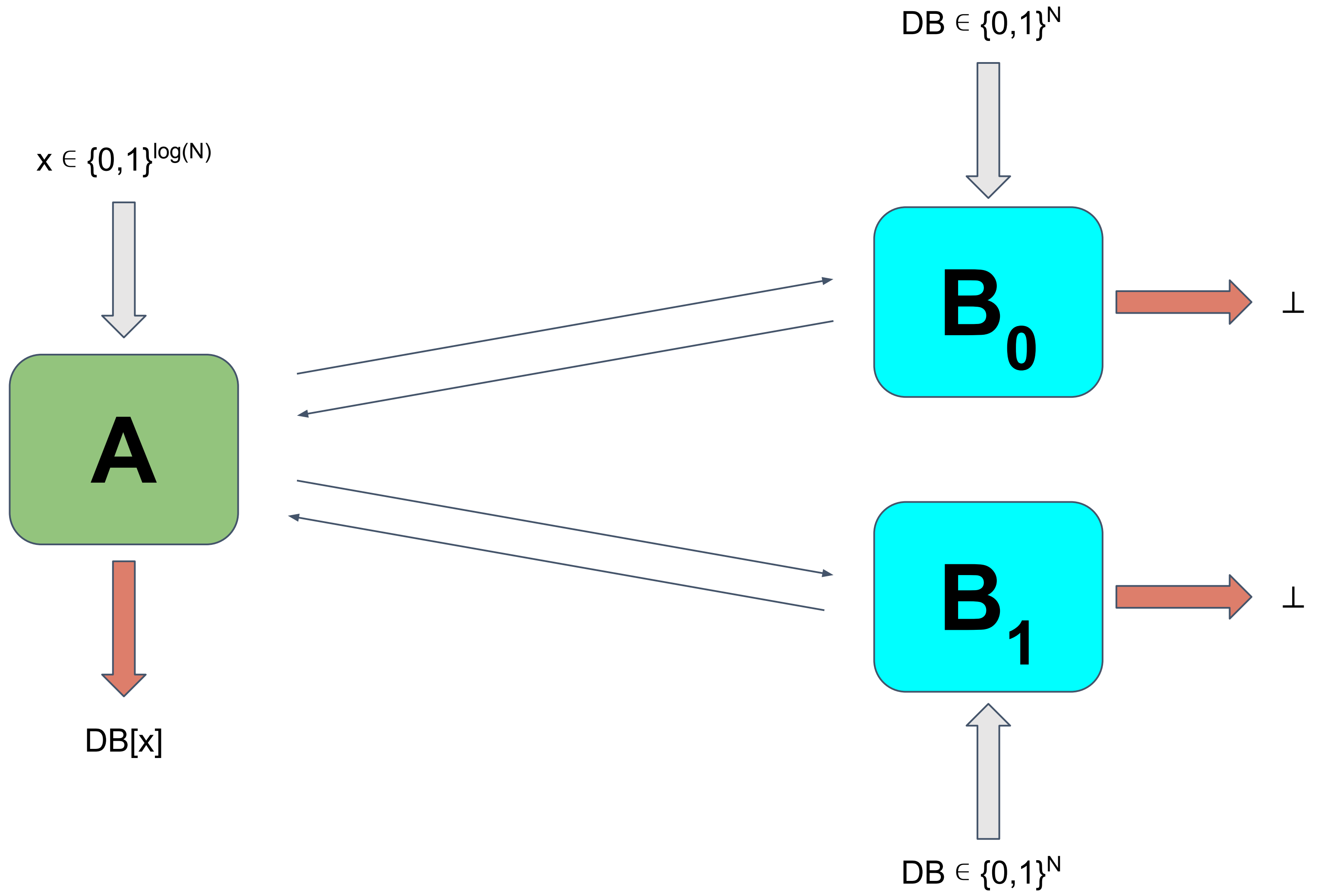
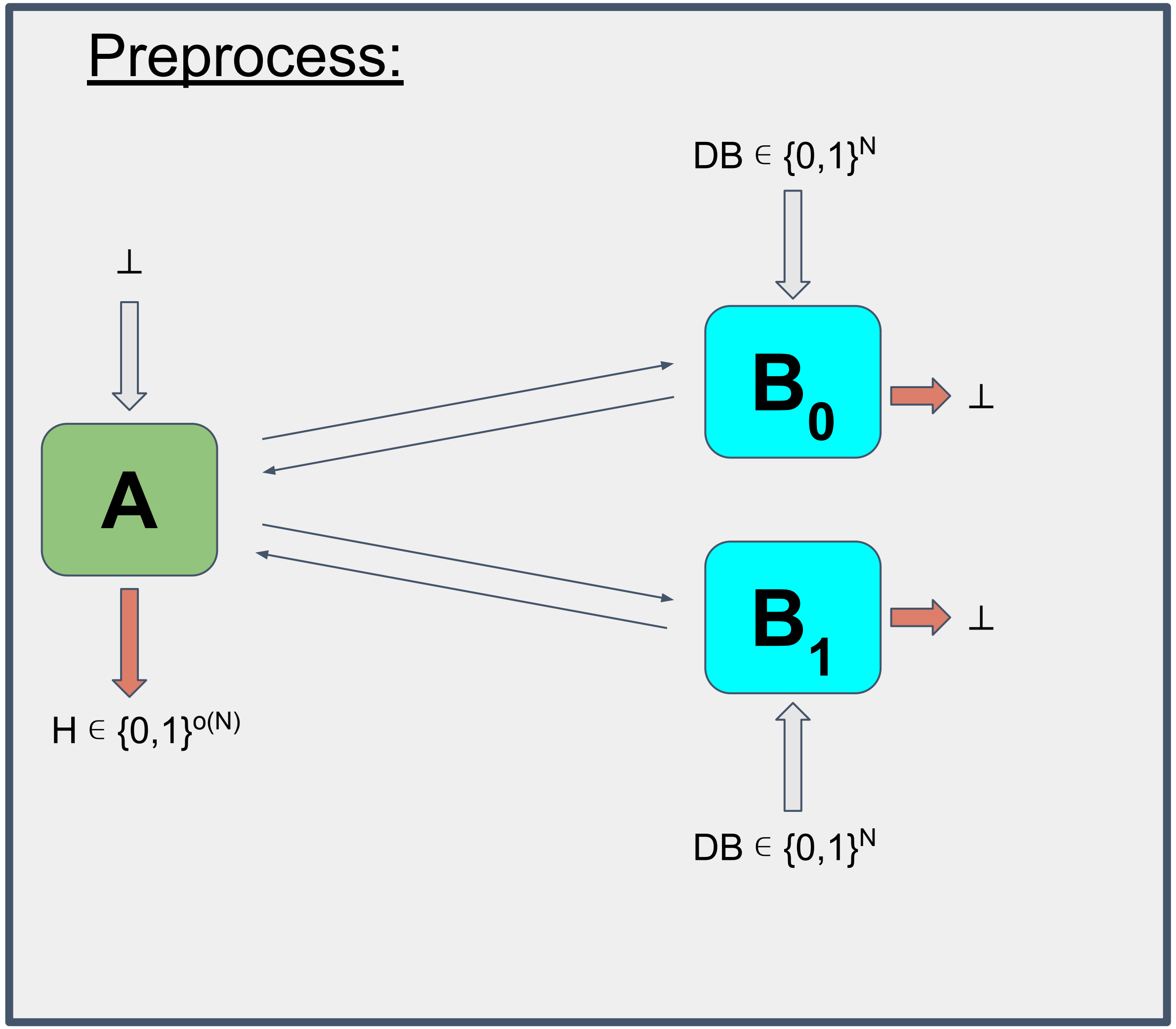


Two-Server PIR, Client Preprocessing [BIM ‘04, …, CK ‘20, ...]



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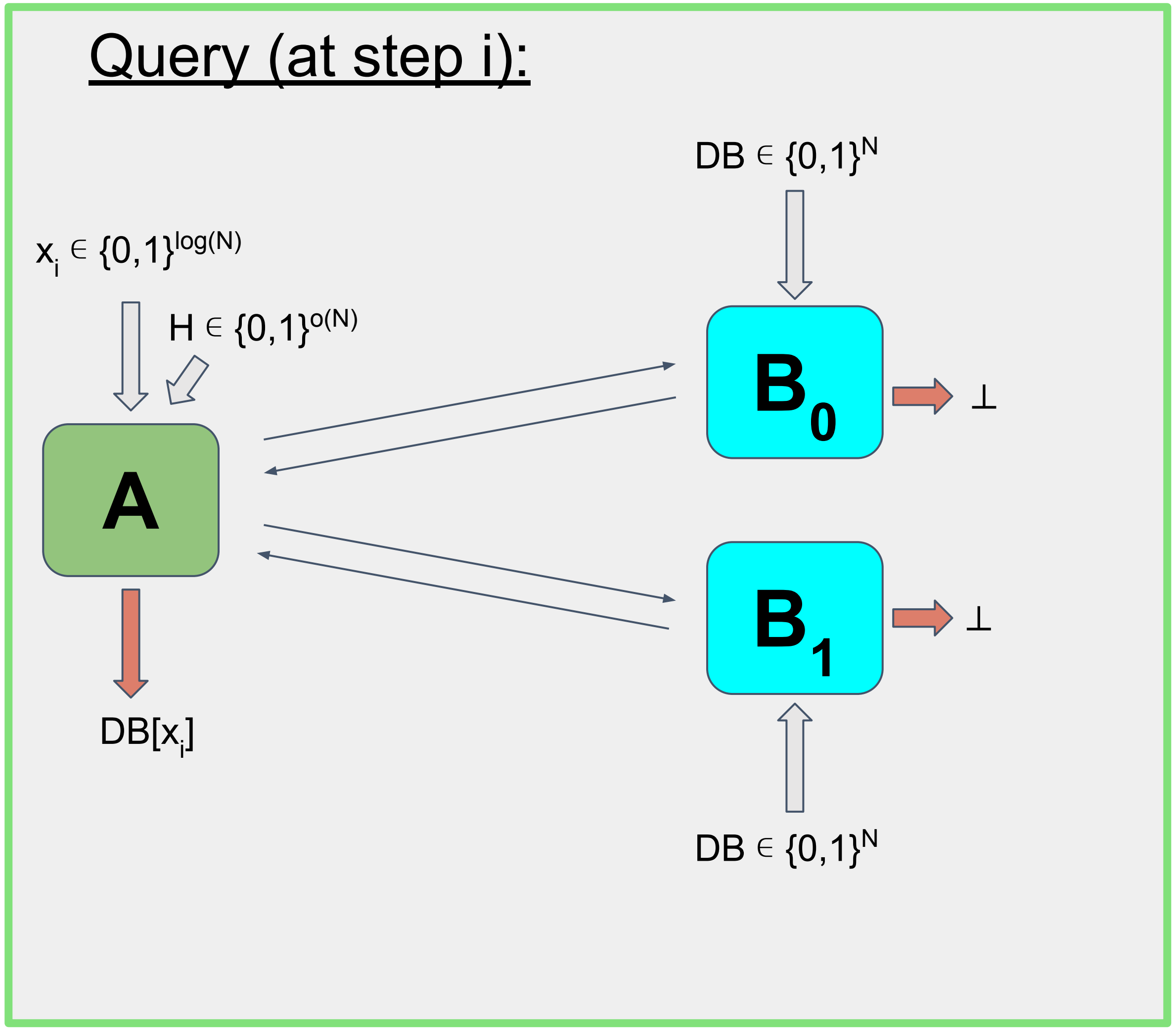

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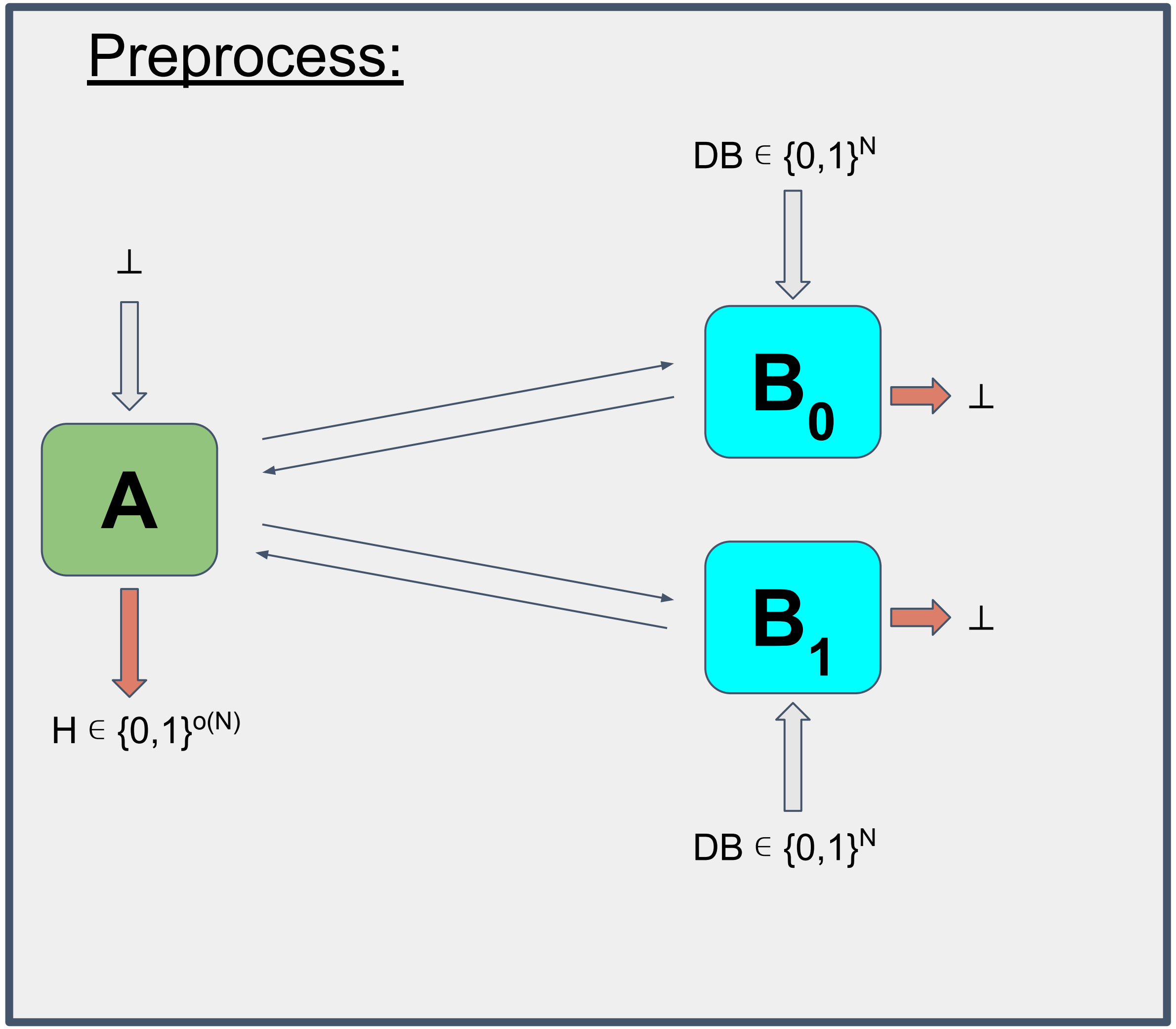

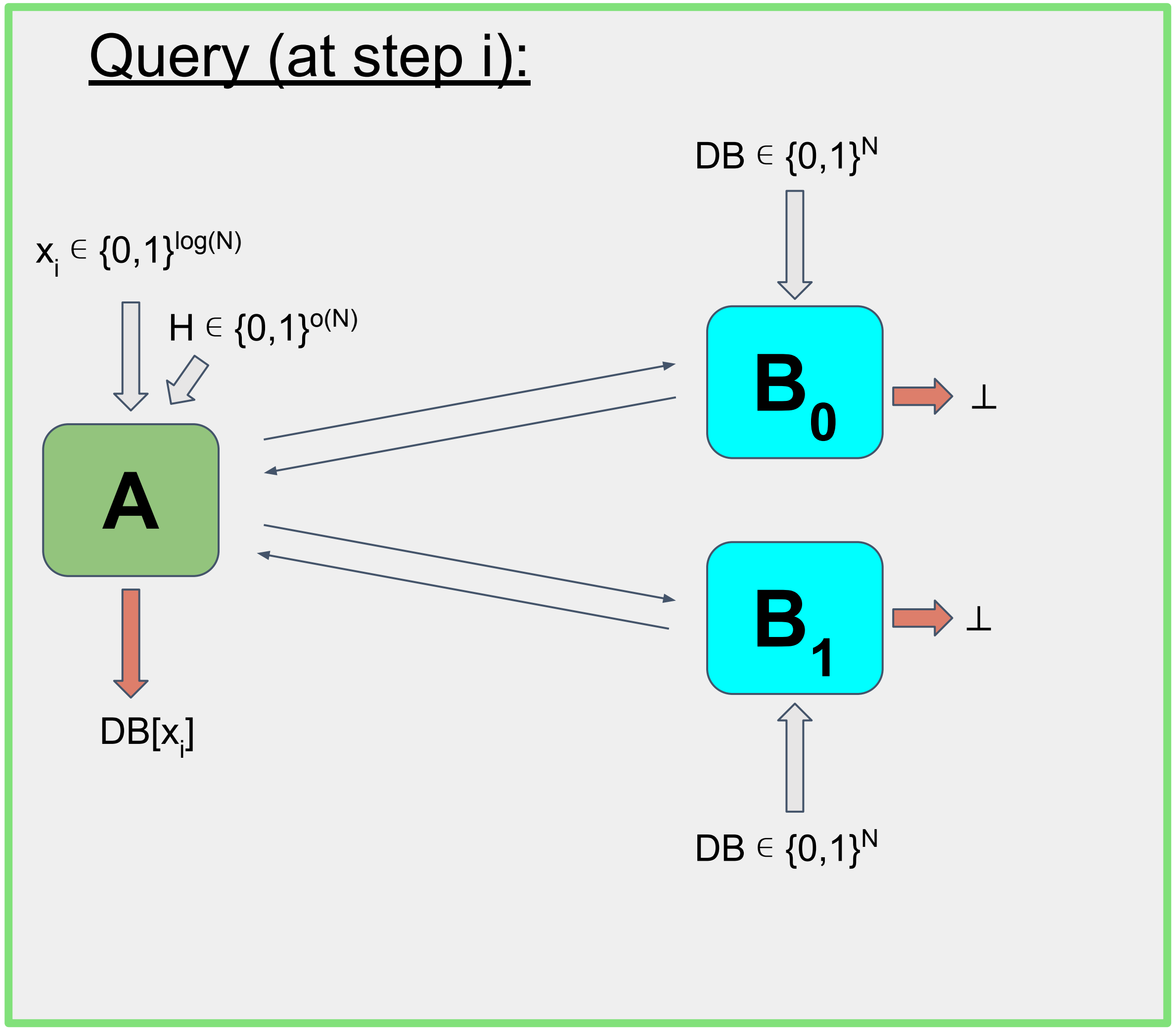

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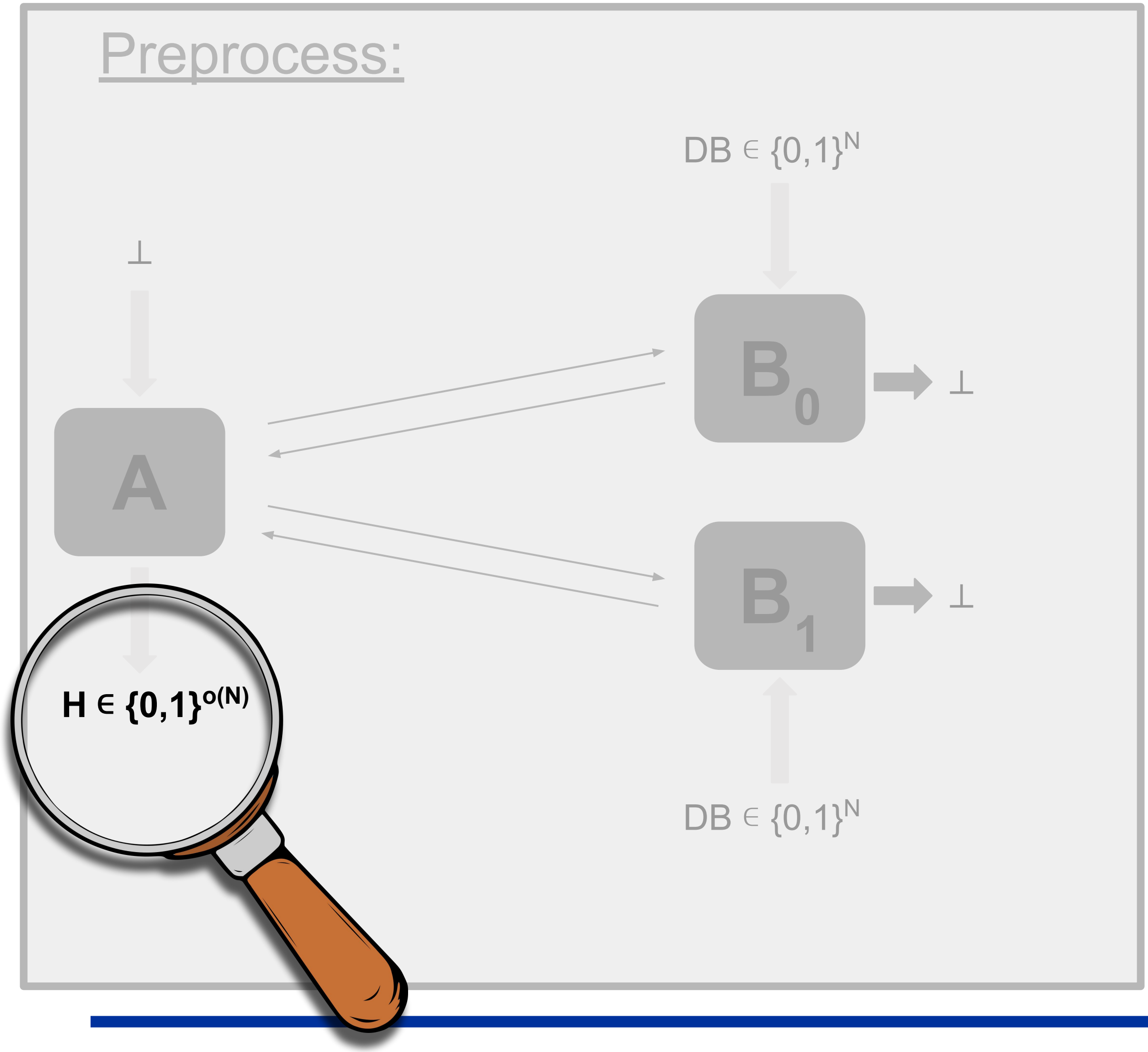
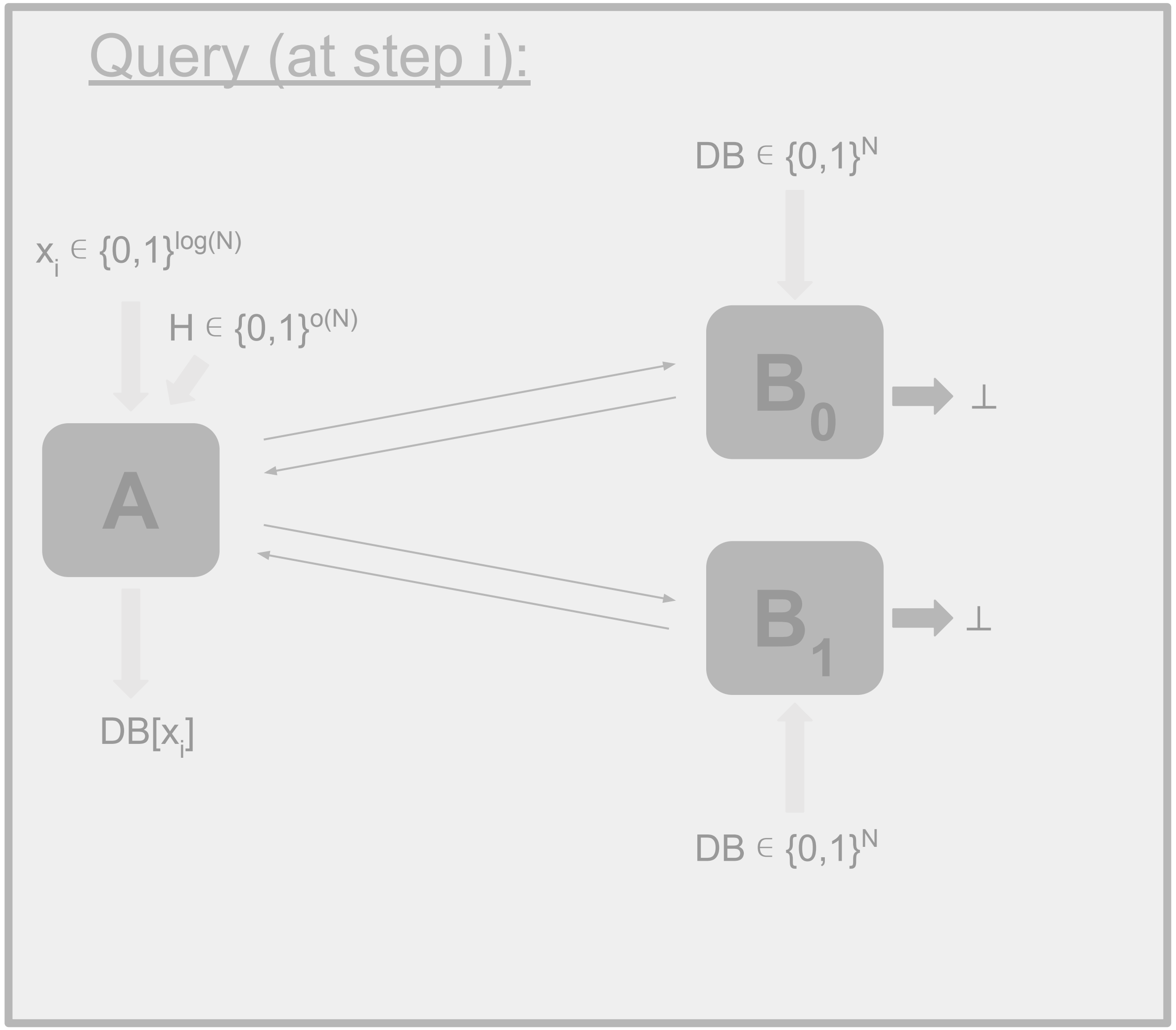
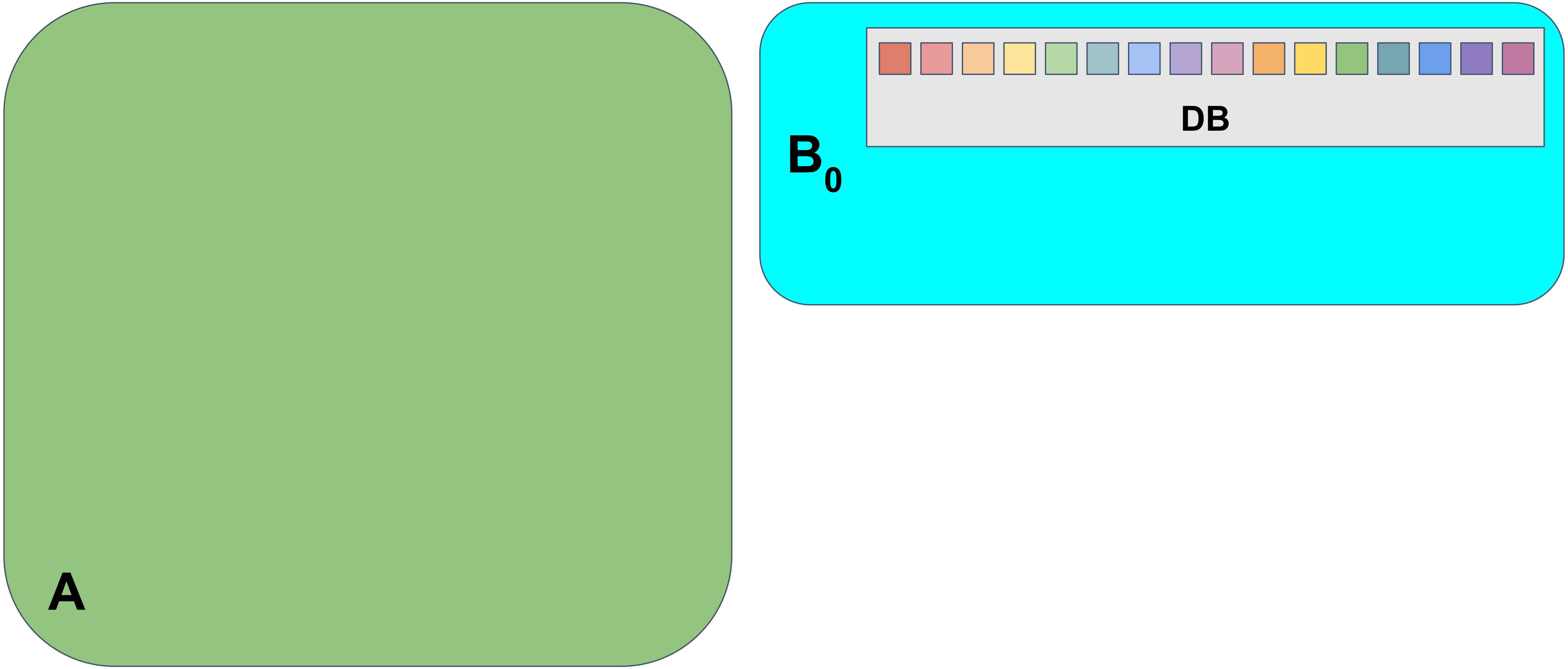




The Client’s Hint [CK ‘20, KC ‘21, …]



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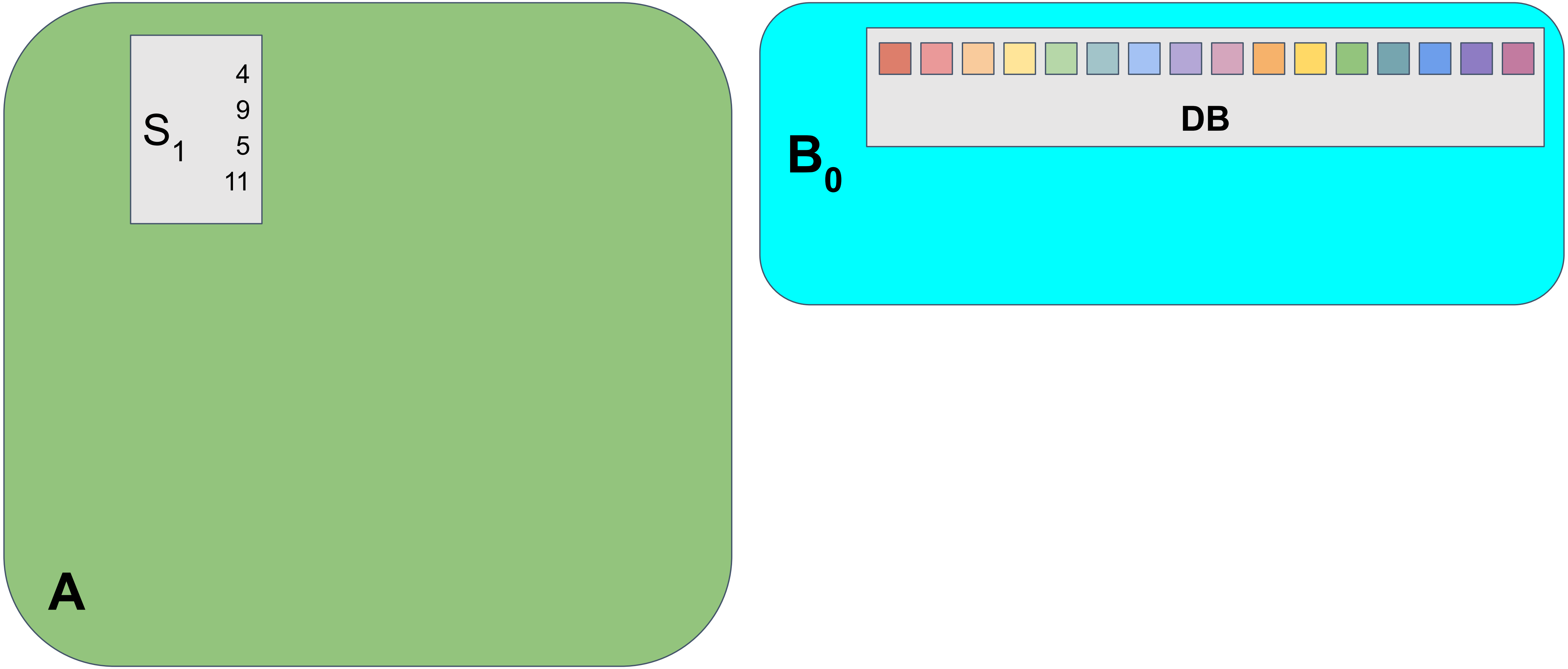

S_1

4

9

5

11

A

B_0

DB

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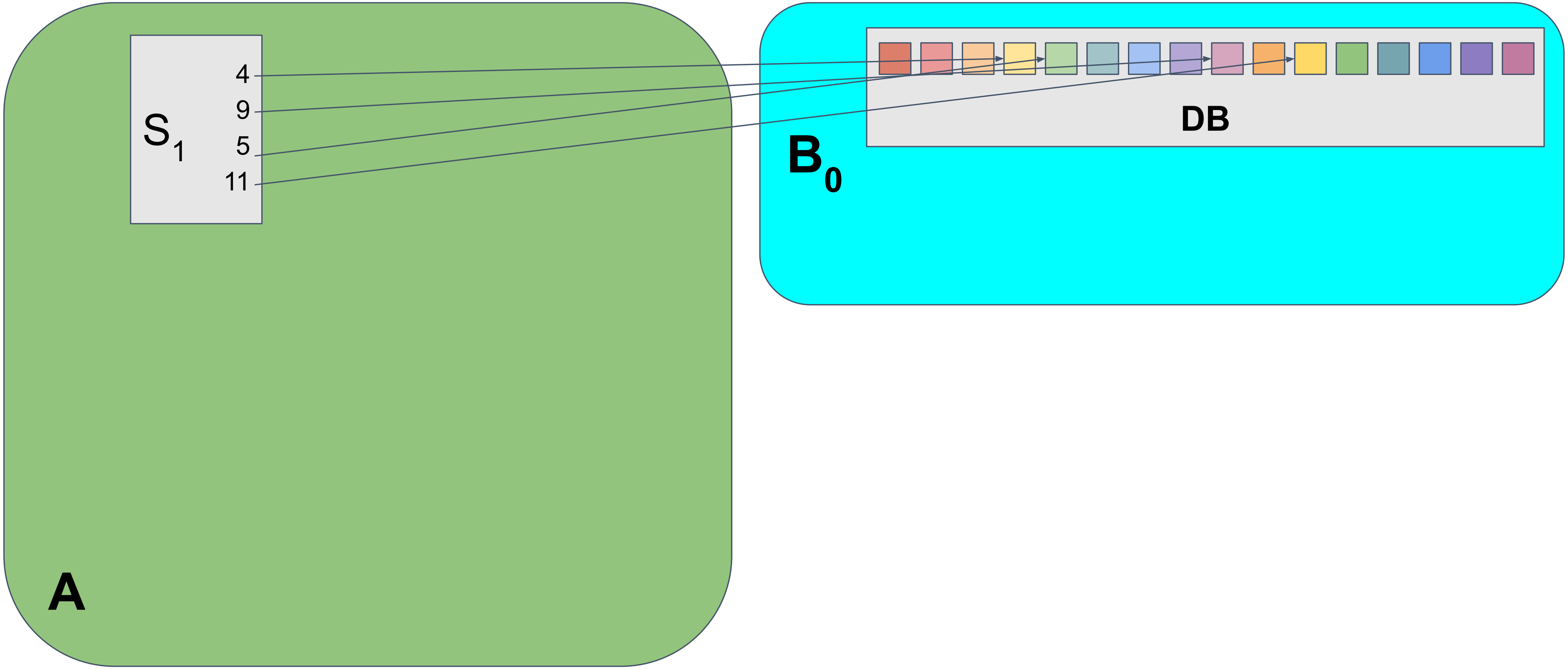

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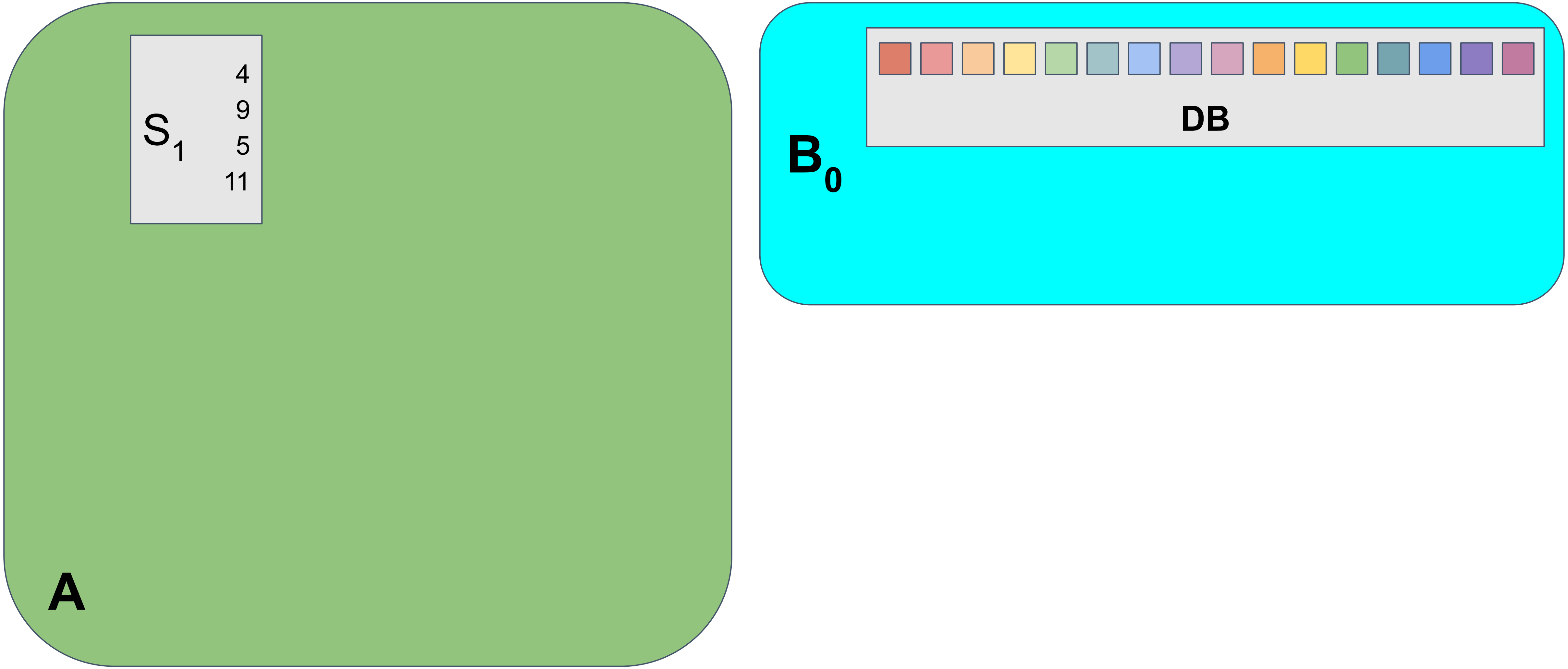

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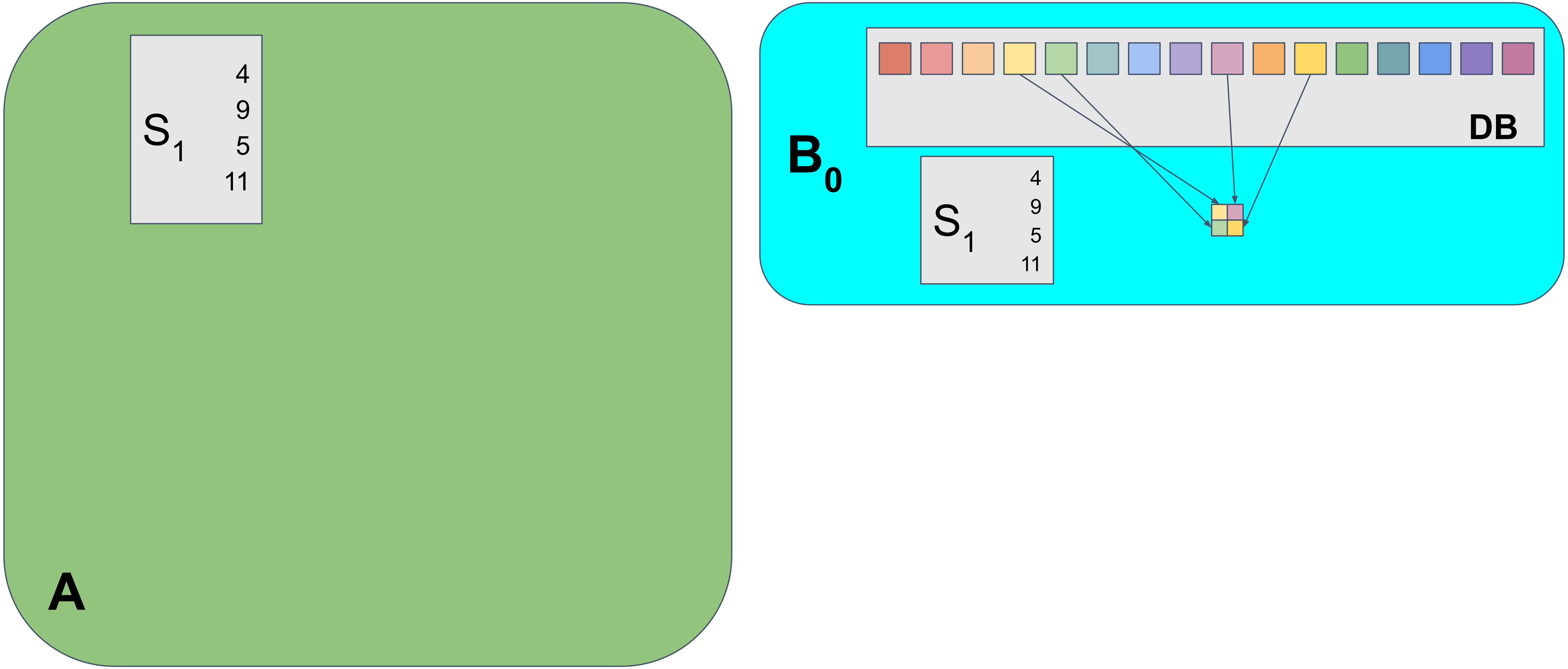

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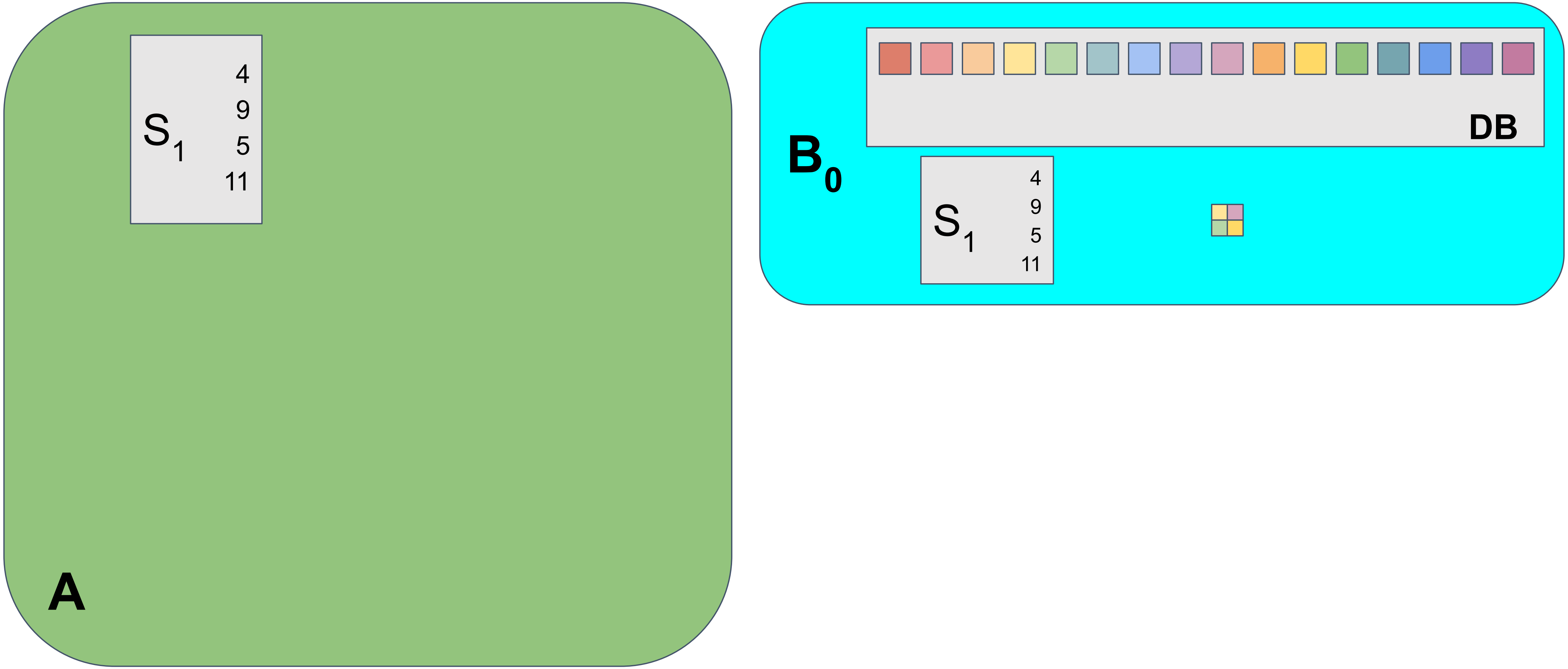

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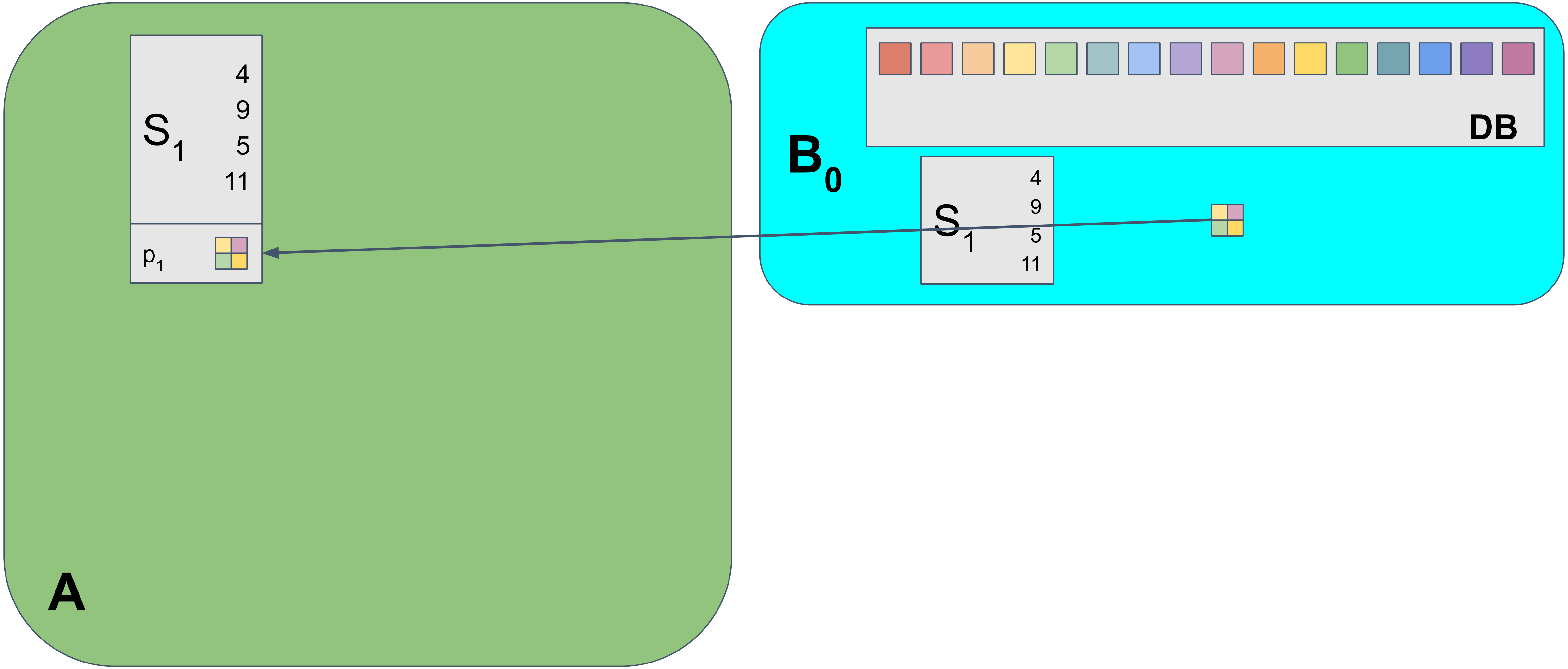

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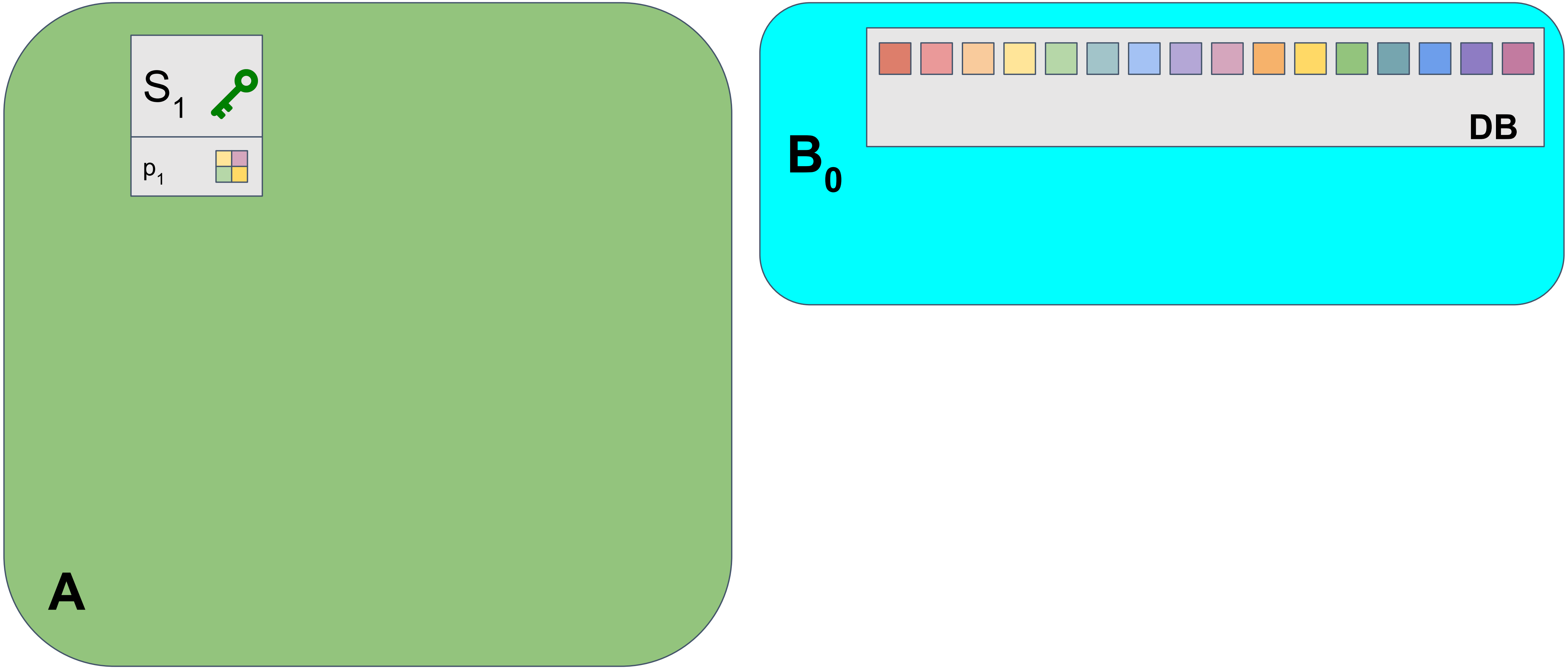

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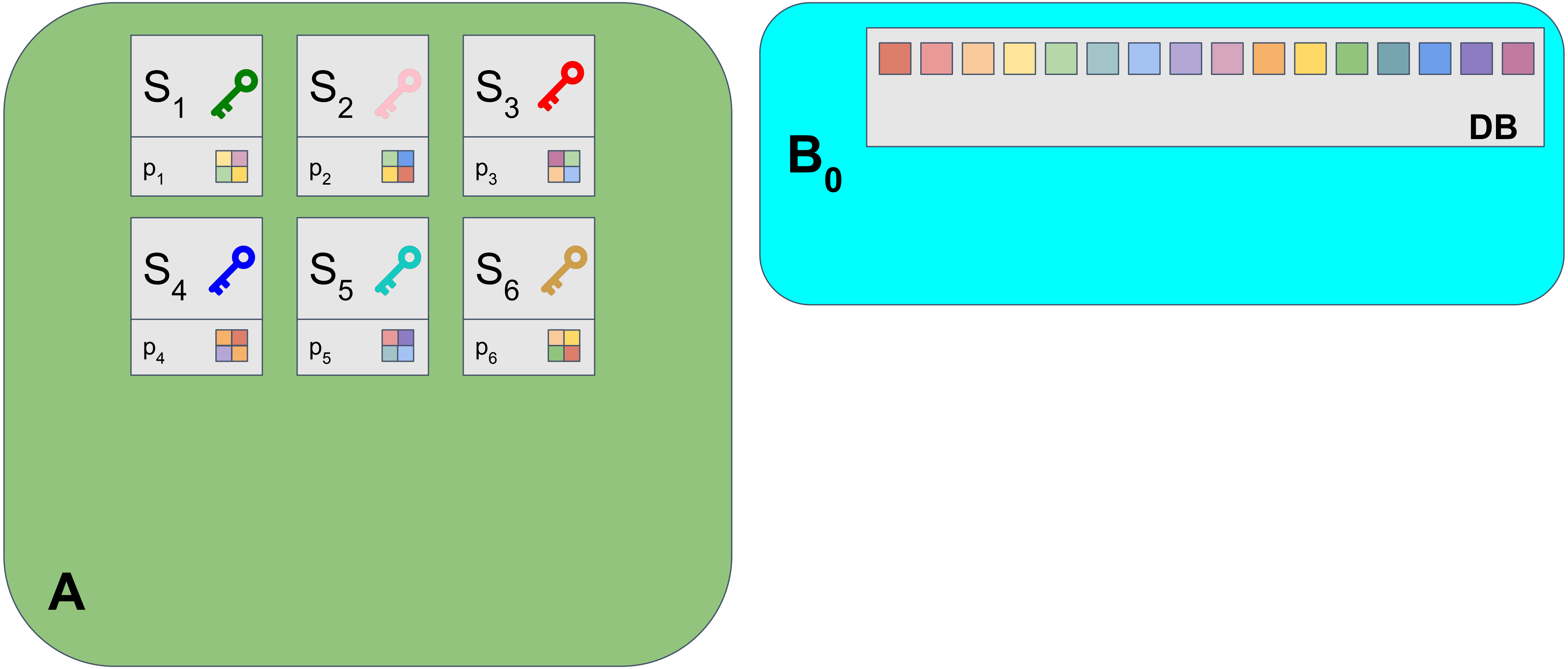

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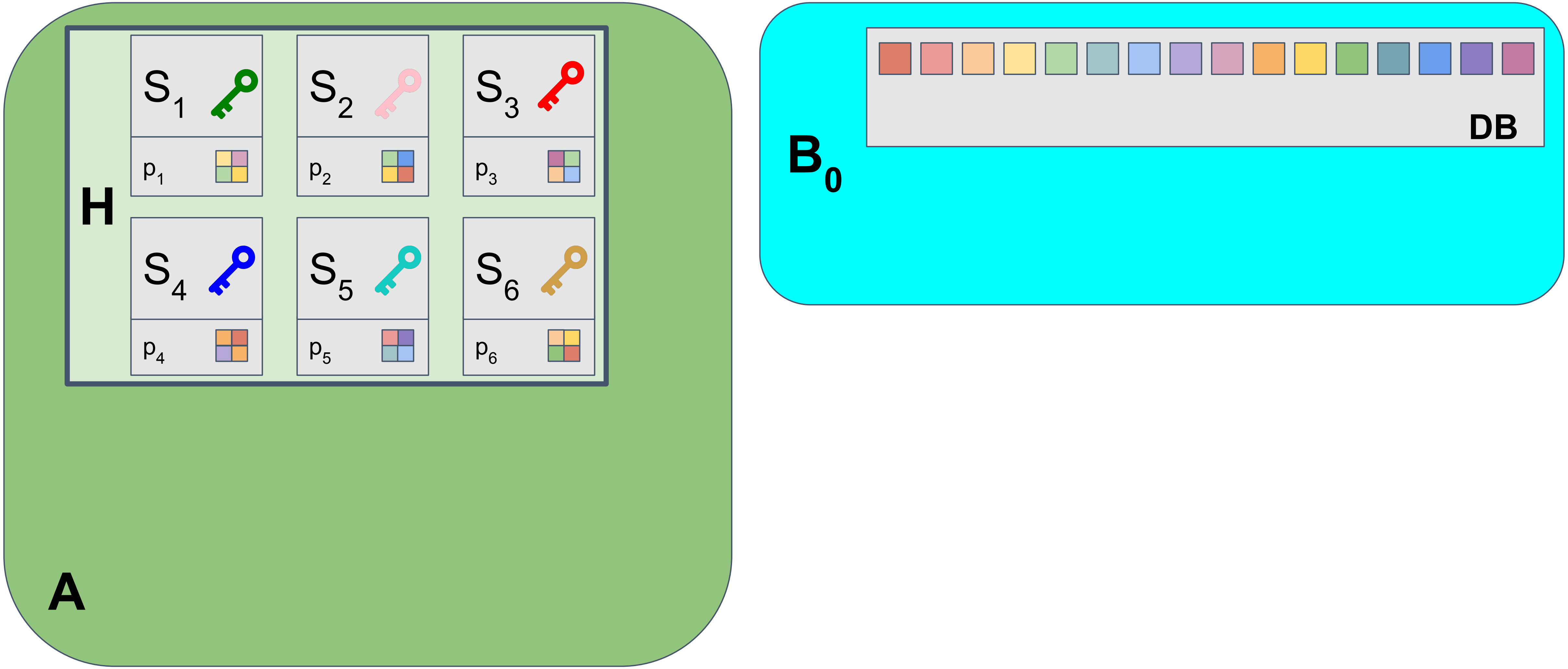

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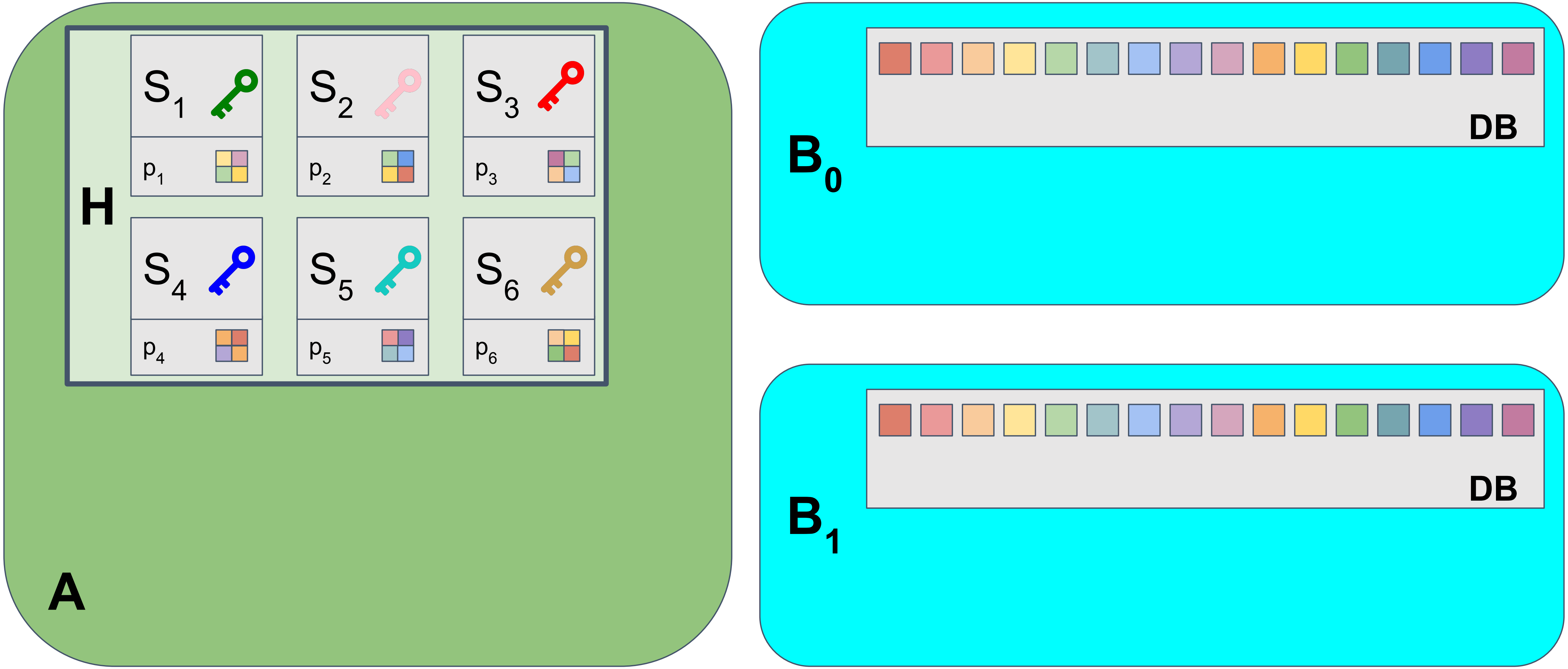

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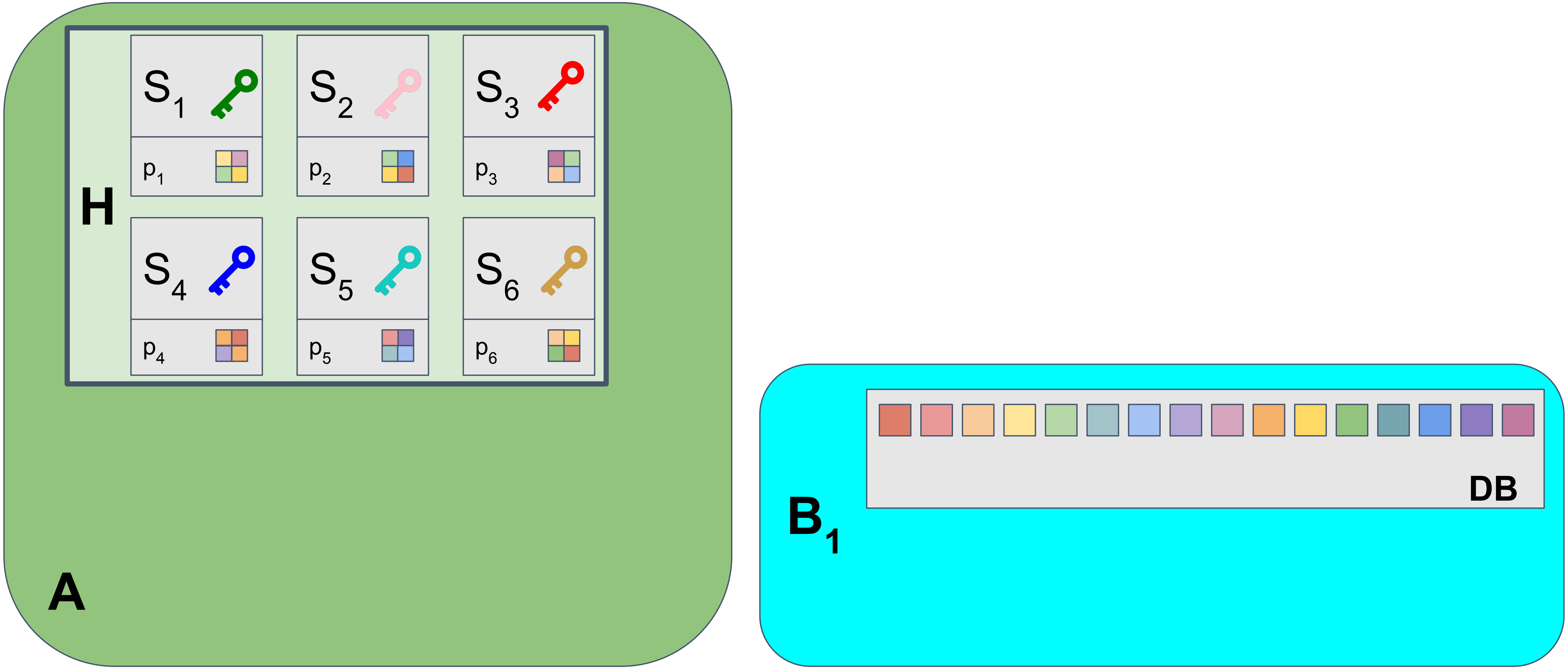
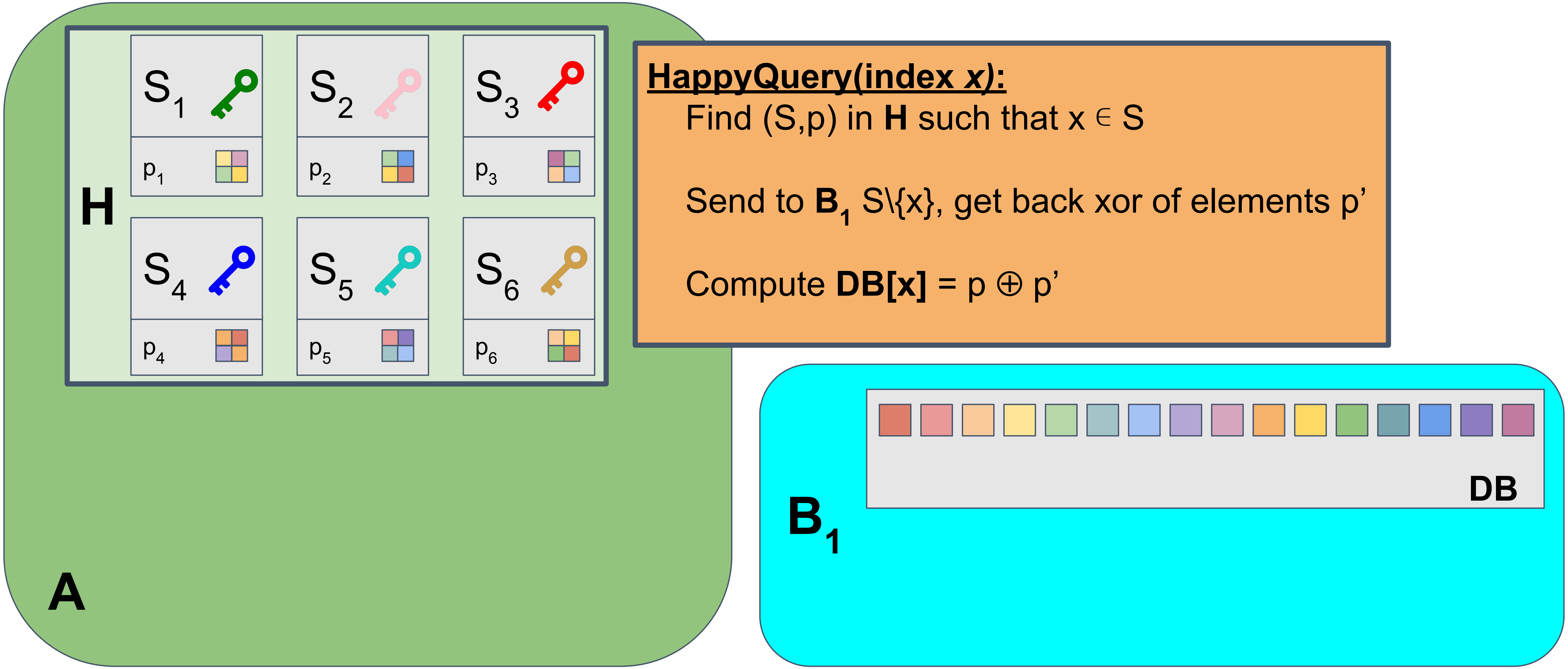


Query Outline



```
HappyQuery(index x):
    Find (S,p) in H such that $x \in S$

    Send to B$_1$ S\{x}, get back xor of elements p'

    Compute DB[x] = $p \oplus p'$
```

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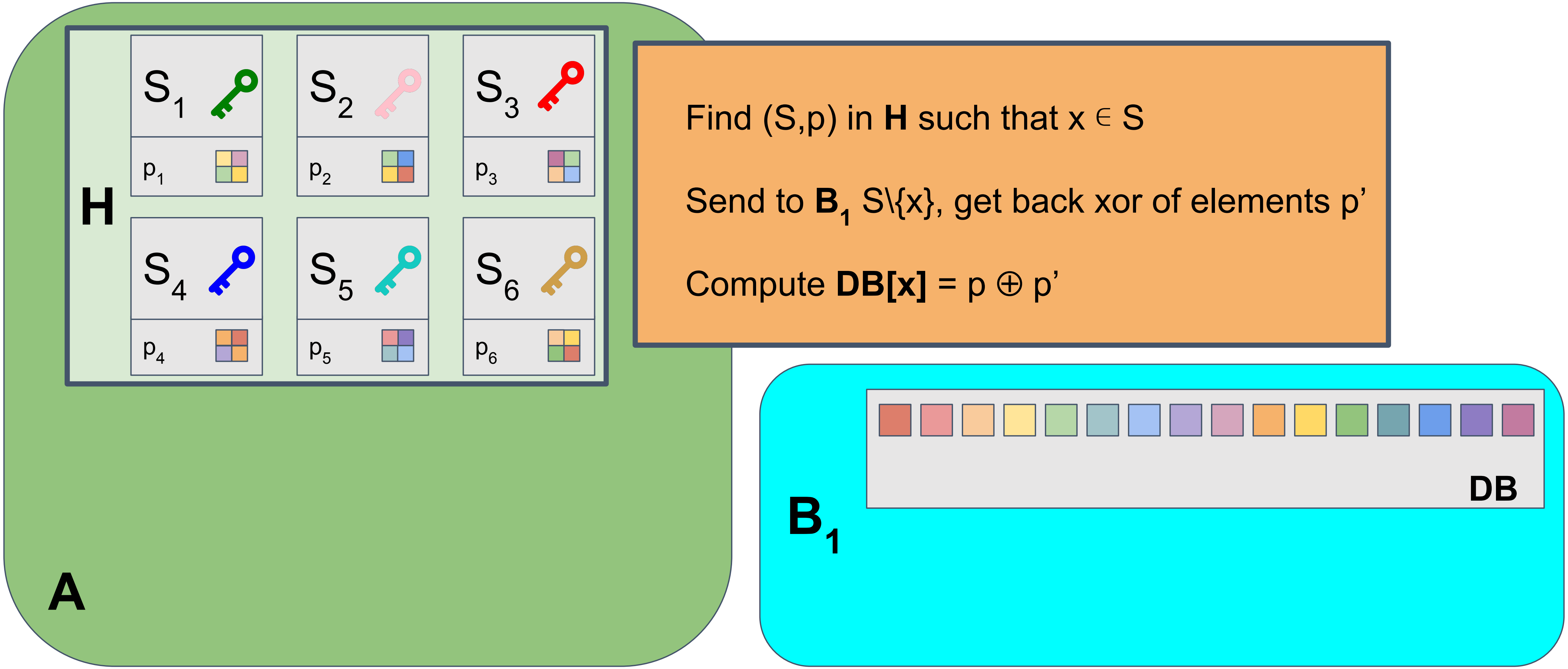

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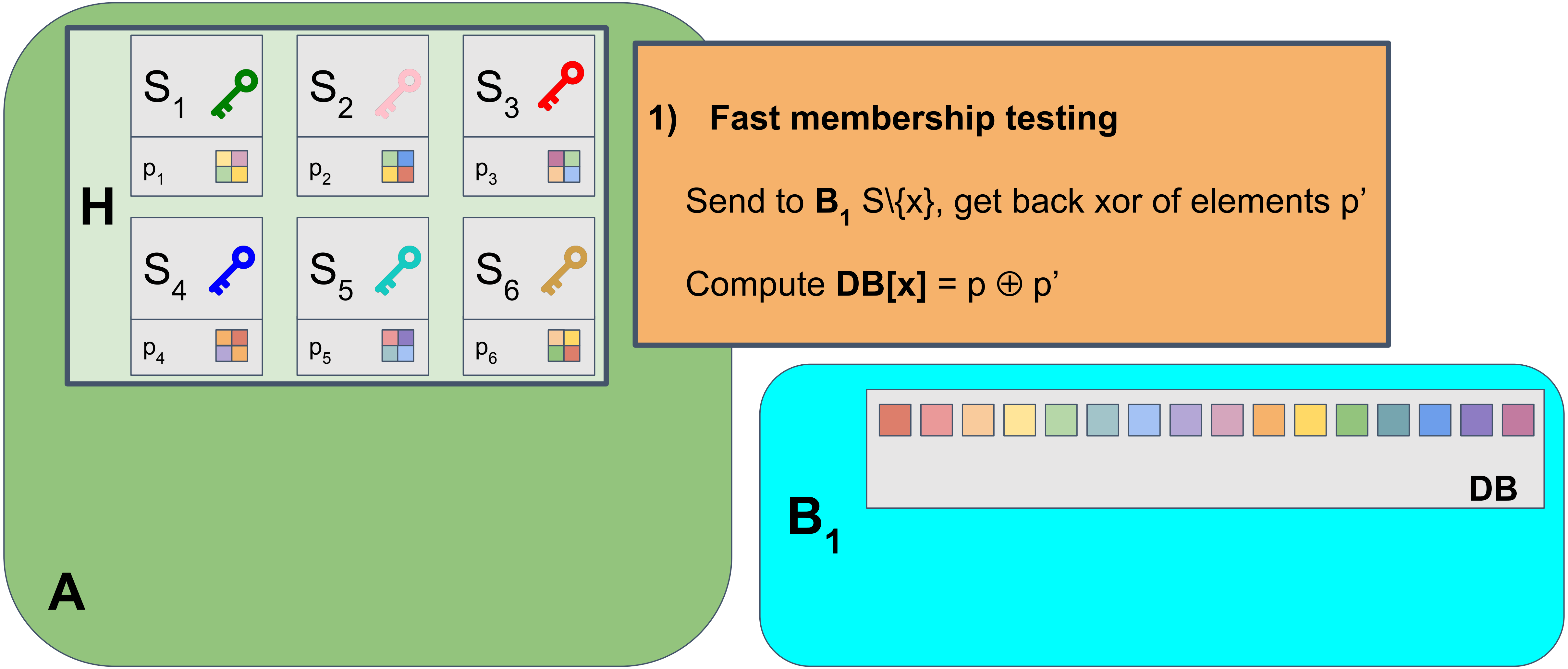
Query Requirements



1) **Fast membership testing**

Send to **B_1** $S\setminus\{x\}$, get back xor of elements p'

Compute **DB[x]** = $p \oplus p'$

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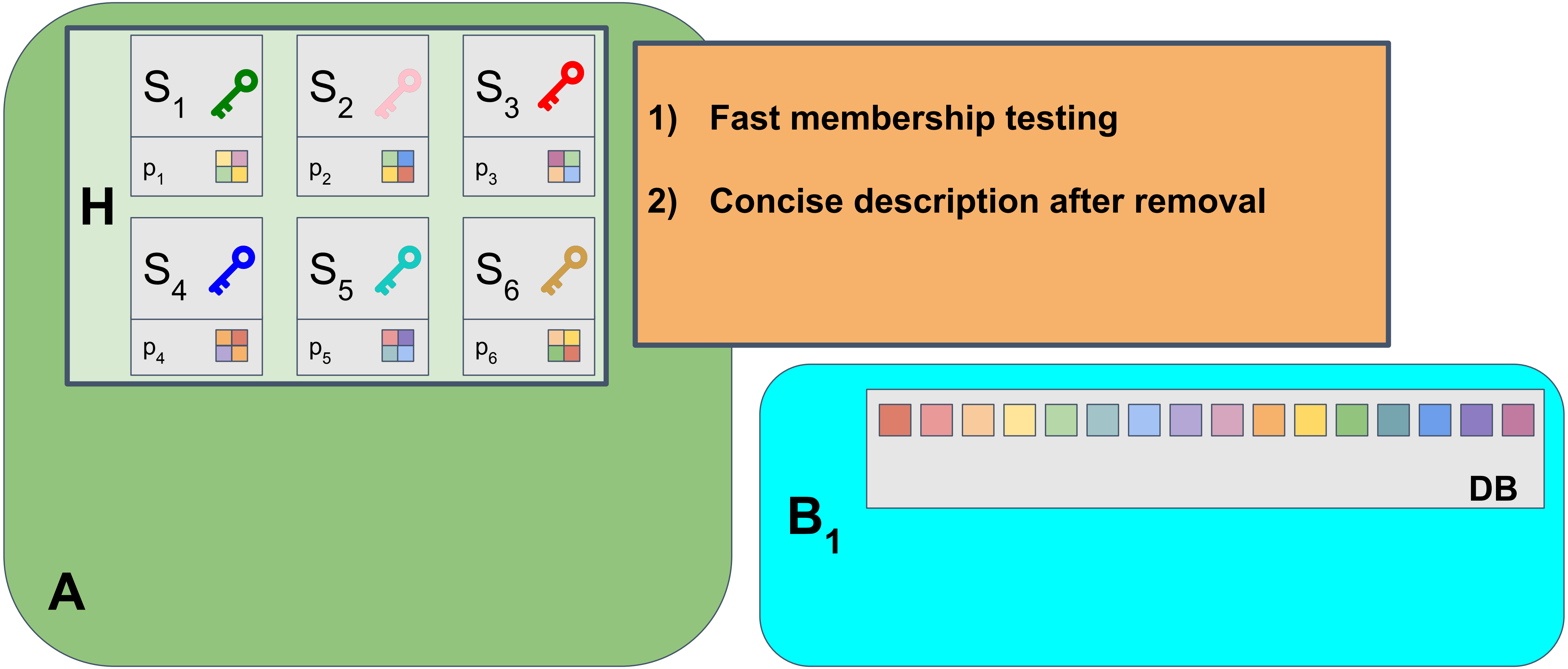

1) **Fast membership testing**

2) **Concise description after removal**

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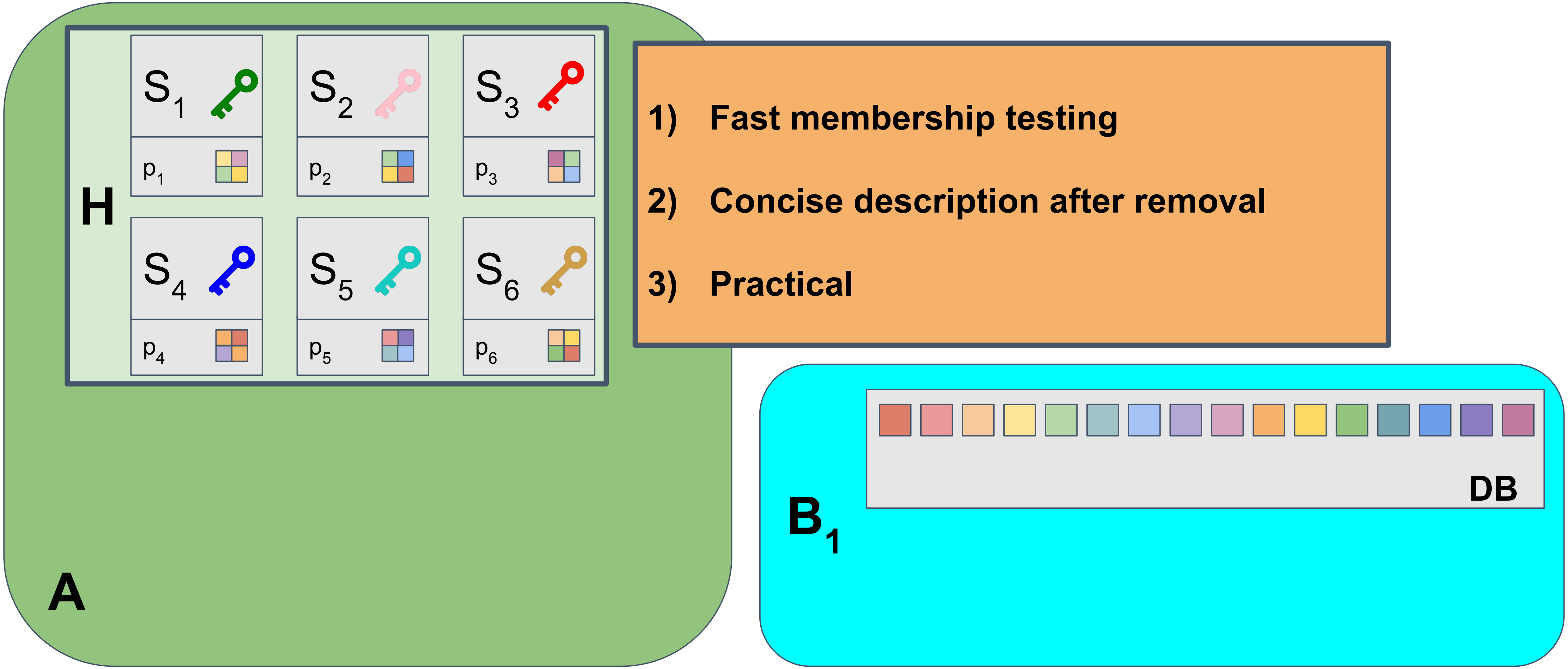
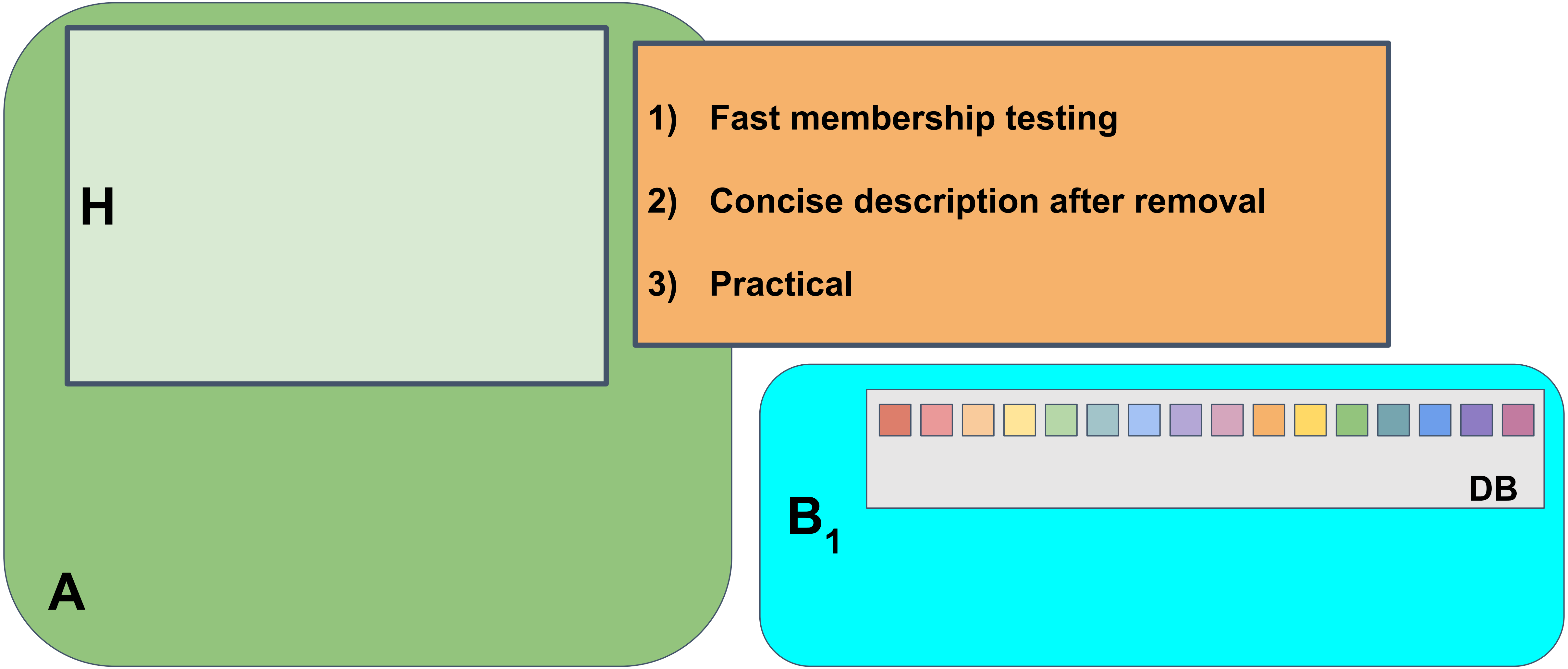


1) **Fast membership testing**
2) **Concise description after removal**
3) **Practical**

Modifying Client's Hint



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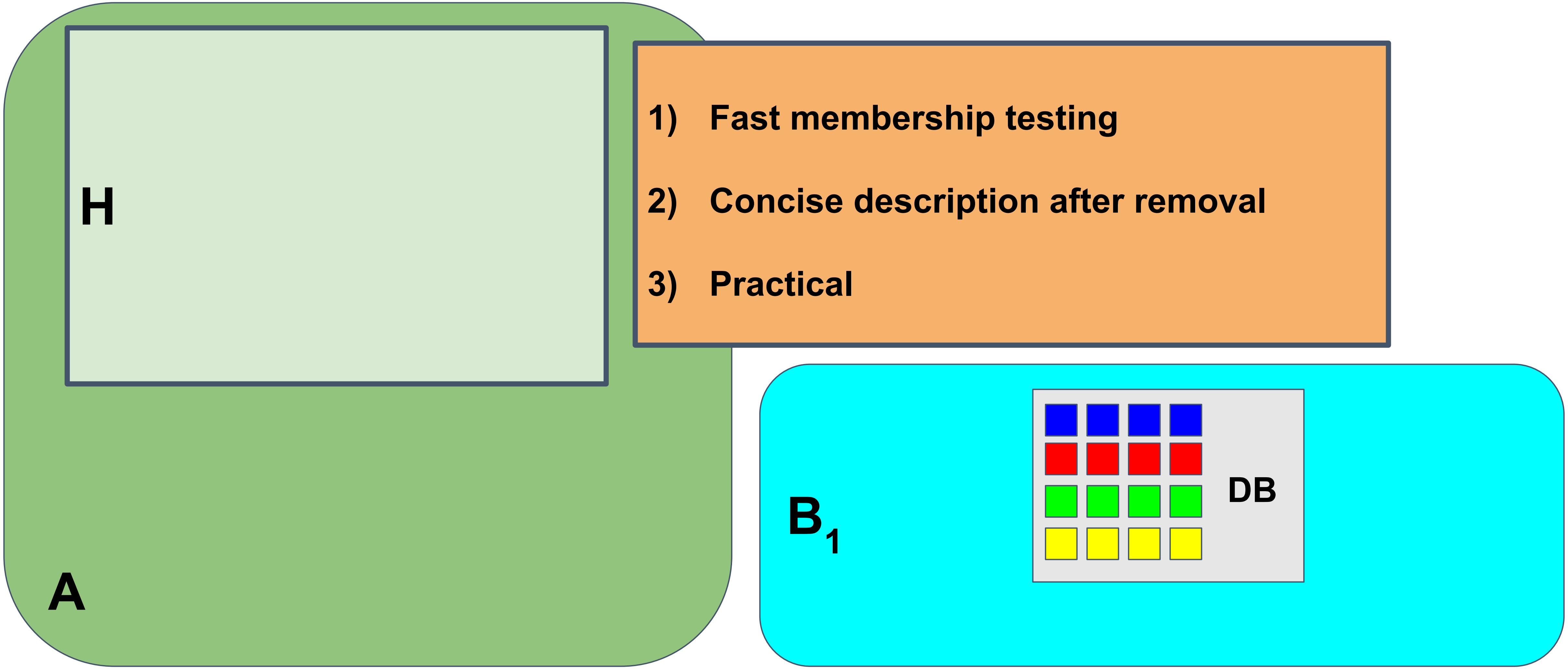

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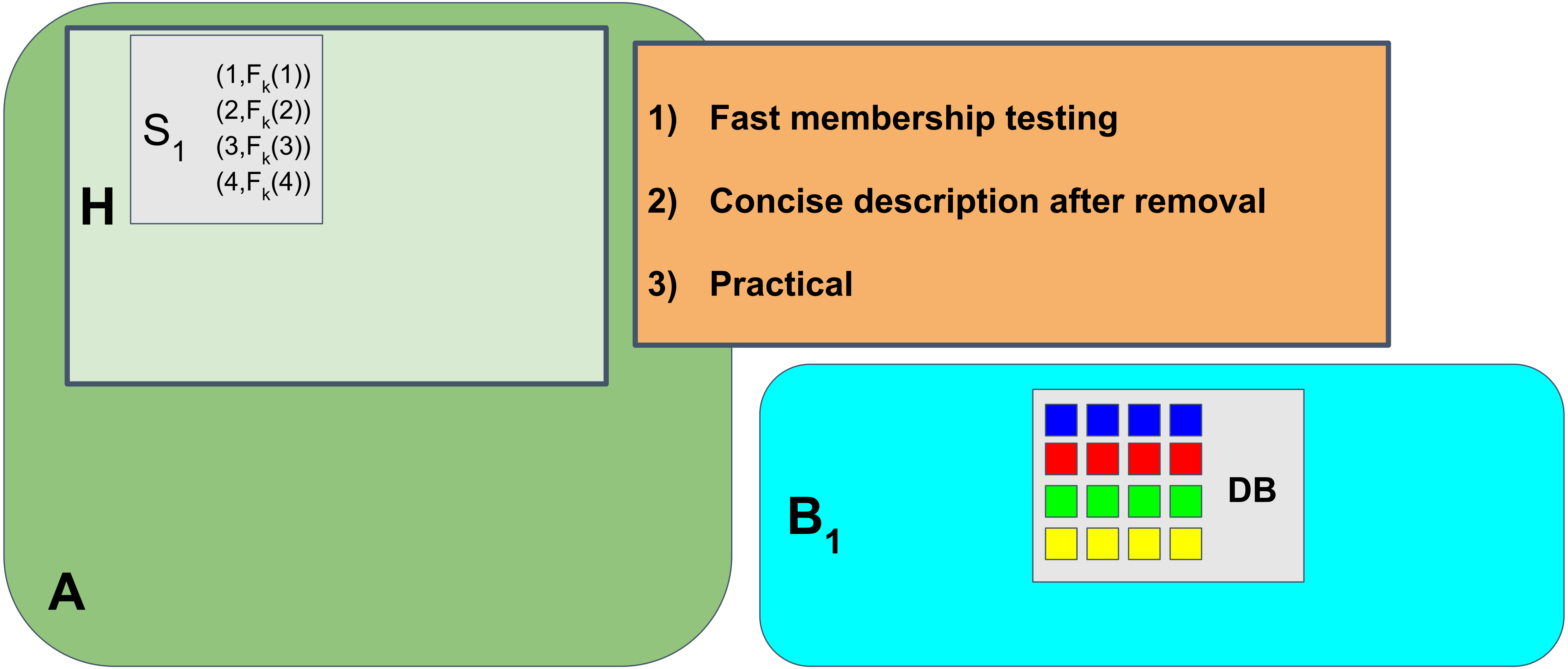

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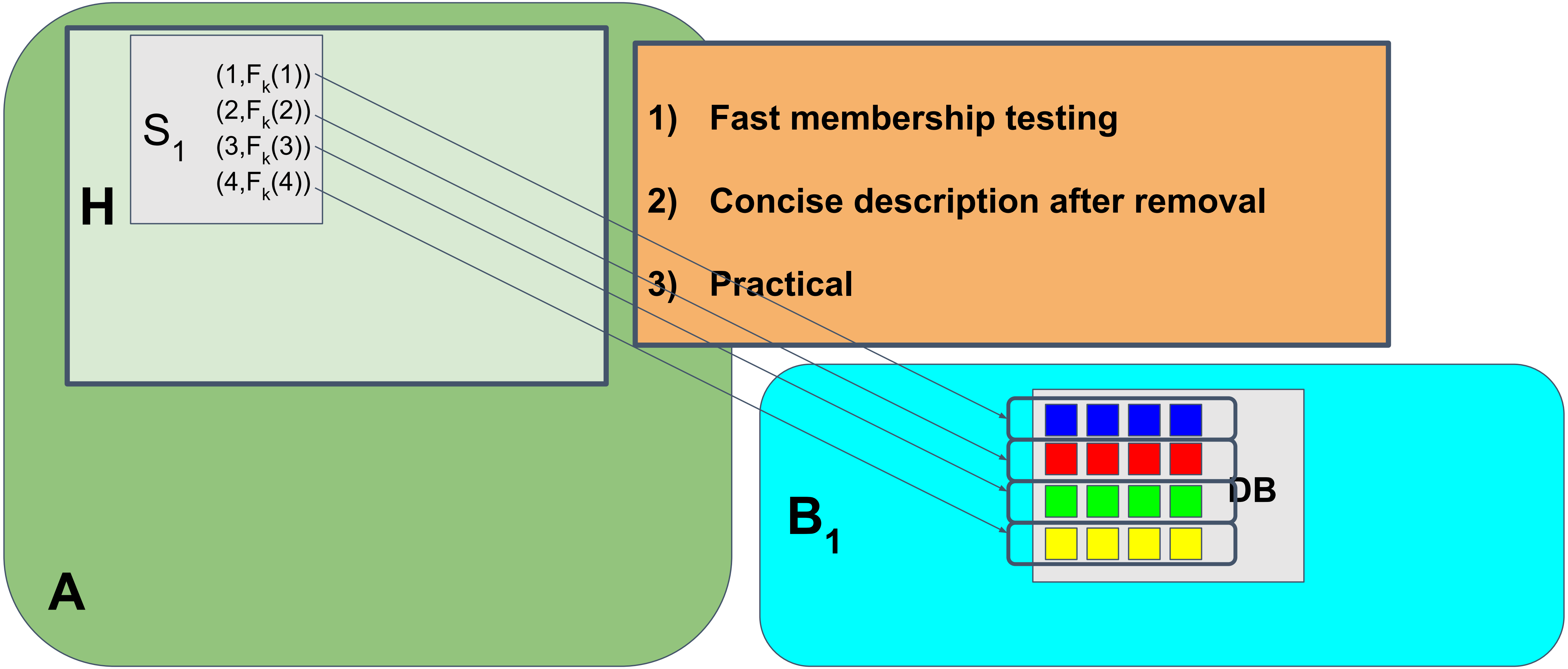

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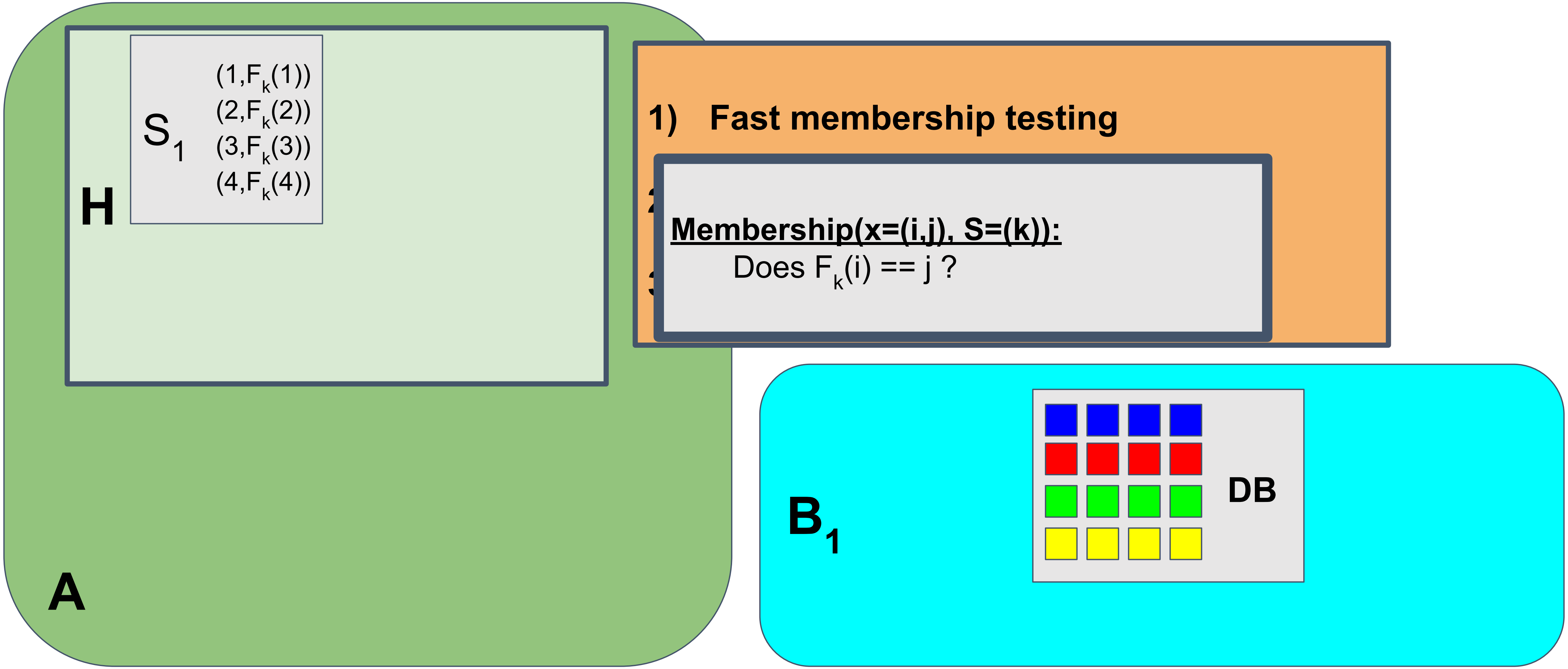
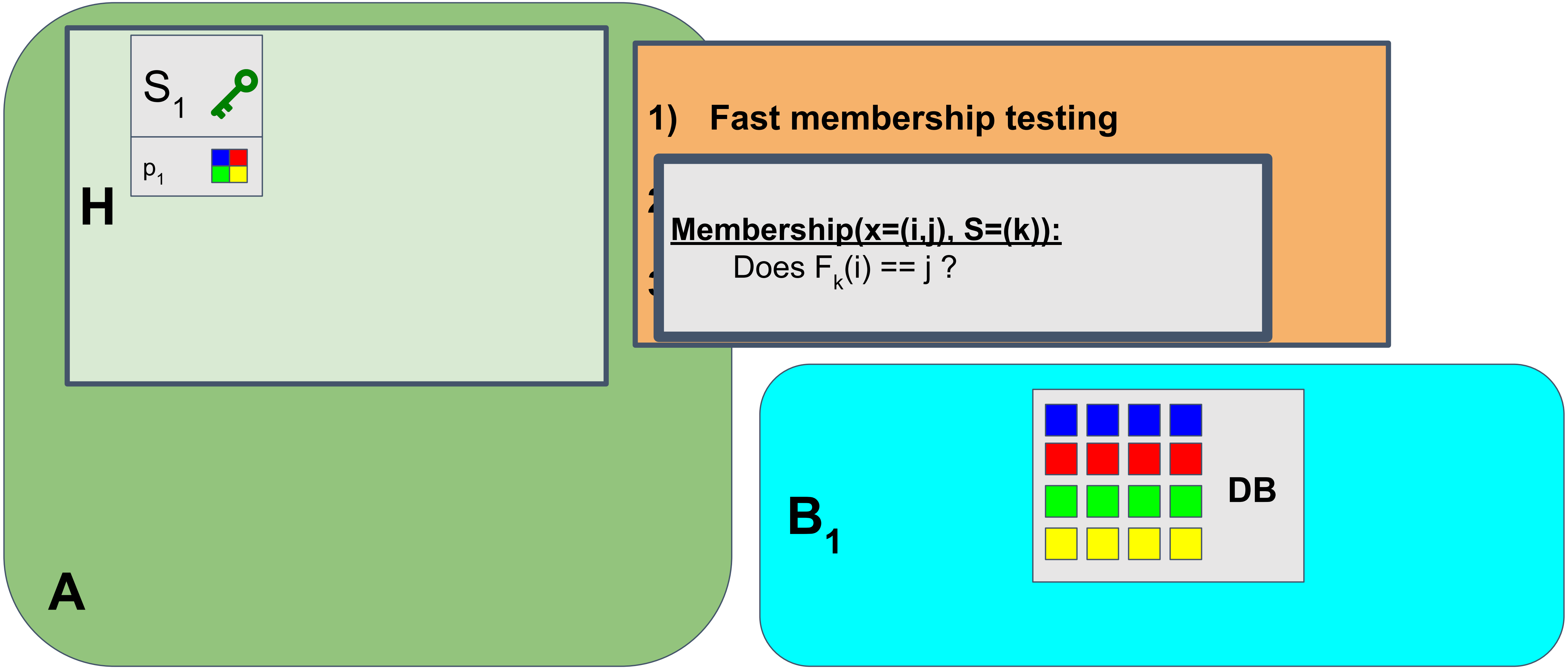


1) **Fast membership testing**

<u>Membership(x=(i,j), S=(k)):</u>

Does $F_k(i) == j$?

Modifying Client’s Hint



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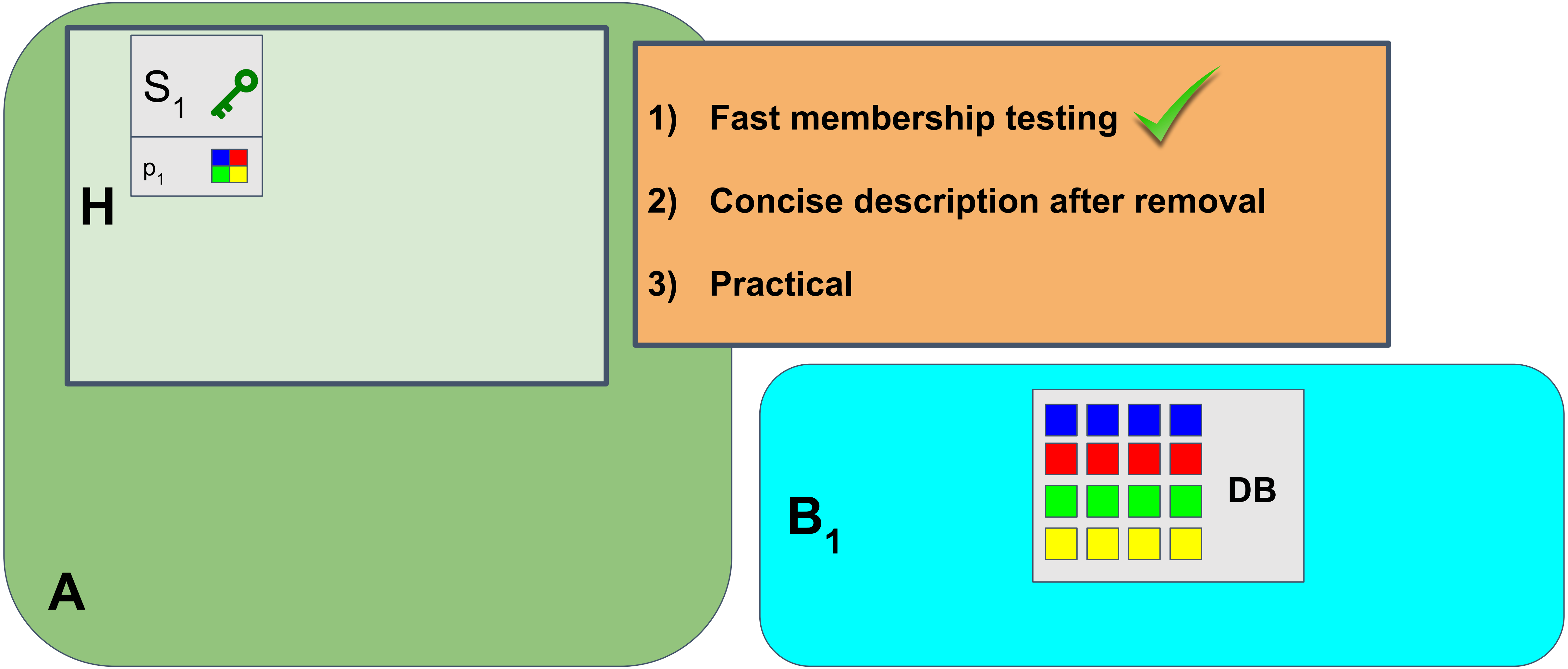

1) **Fast membership testing** ✓

2) **Concise description after removal**

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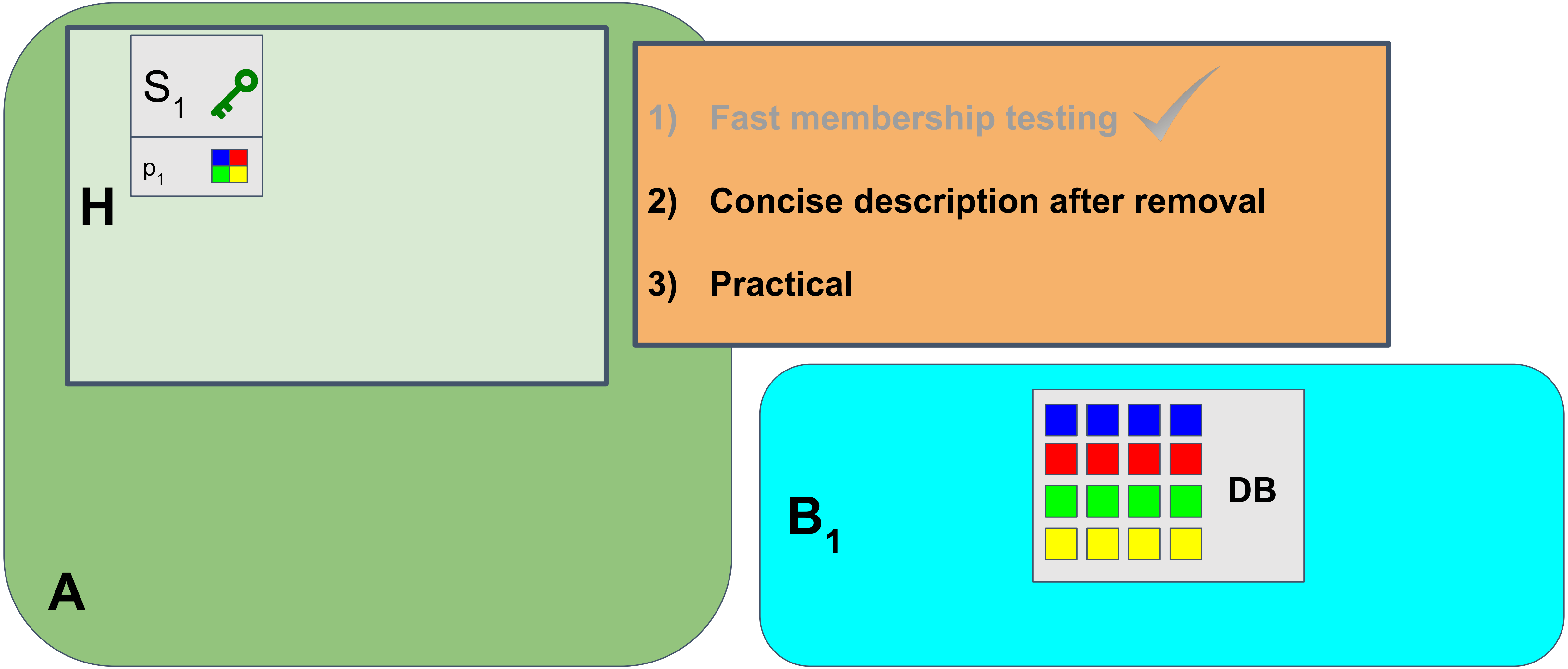
Modifying Client's Hint



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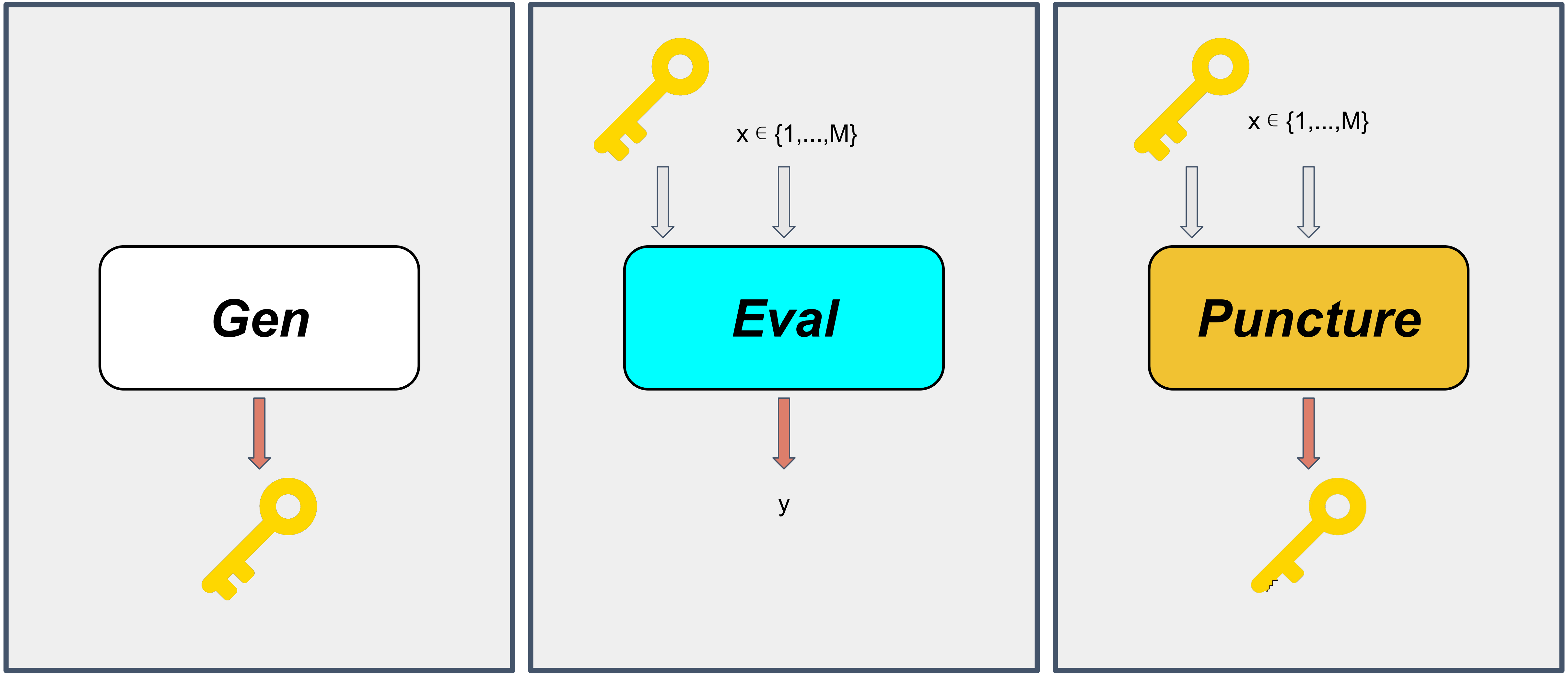
Tool: Puncturable Pseudorandom Function [GGM ‘84, BW ‘13, KPTZ ‘13, BGI ‘14, …]



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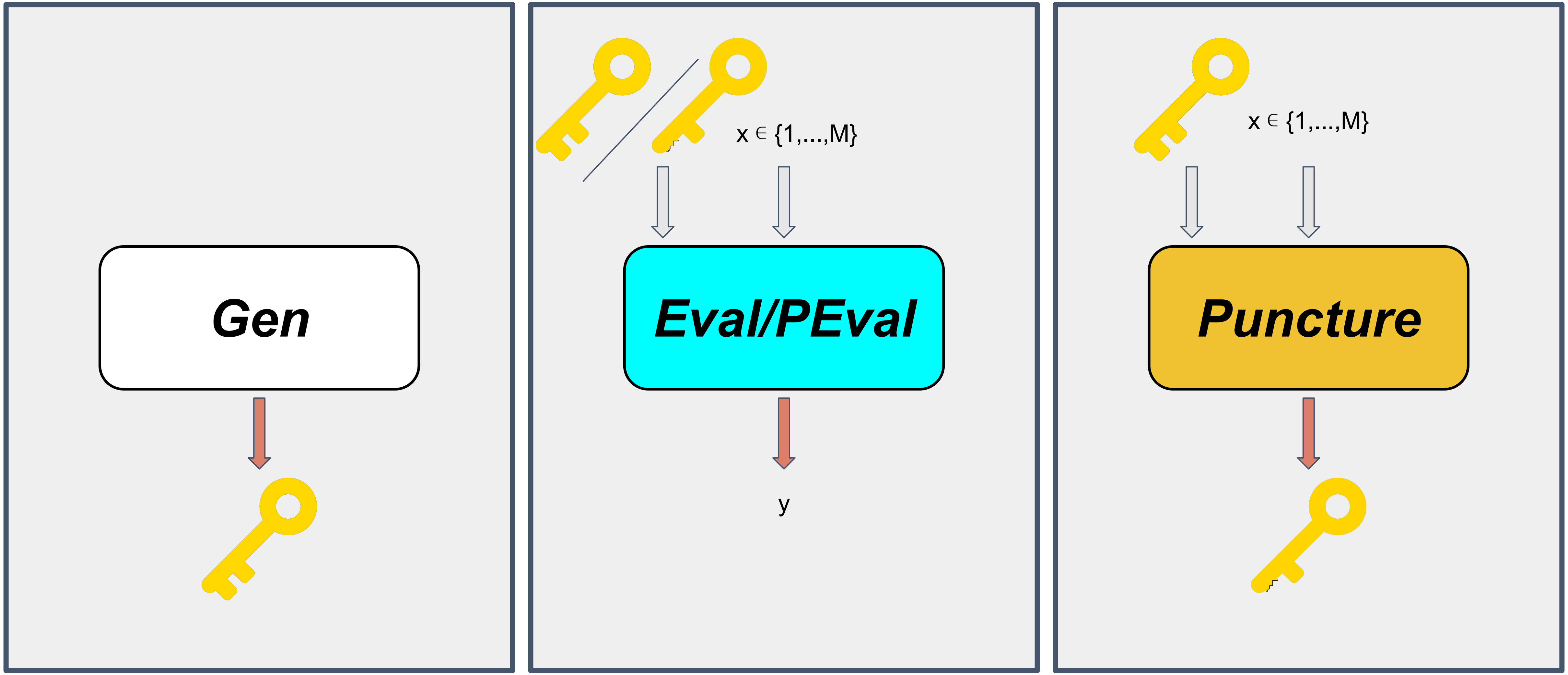

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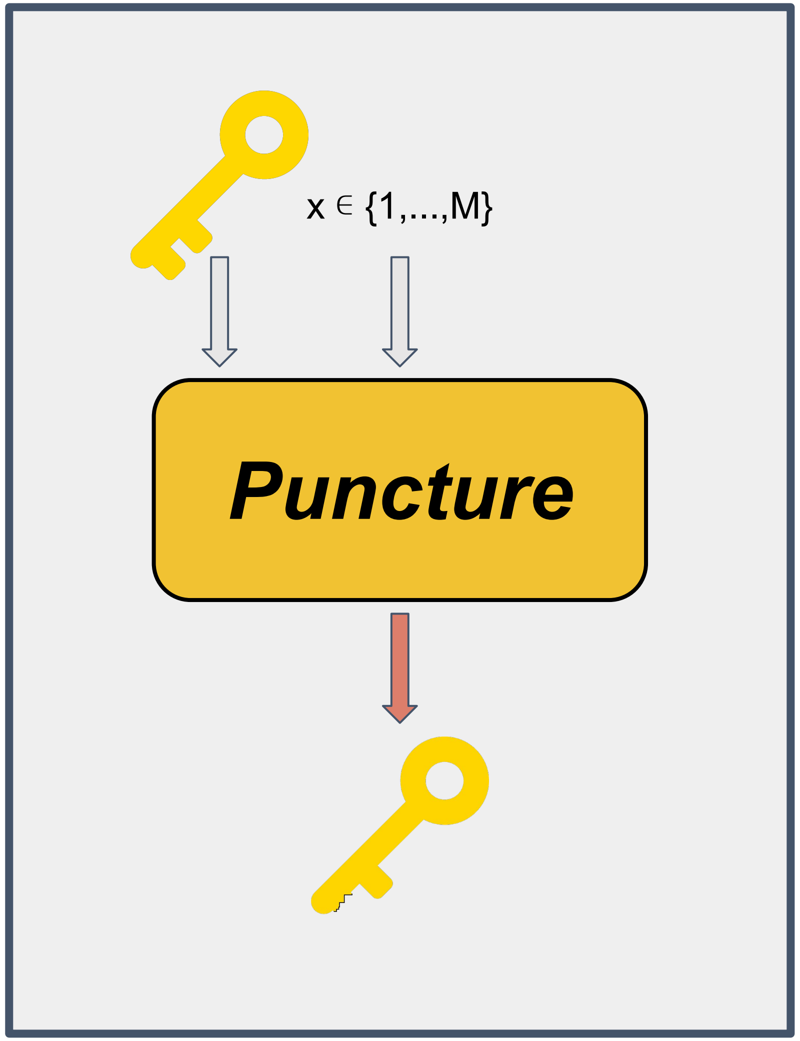
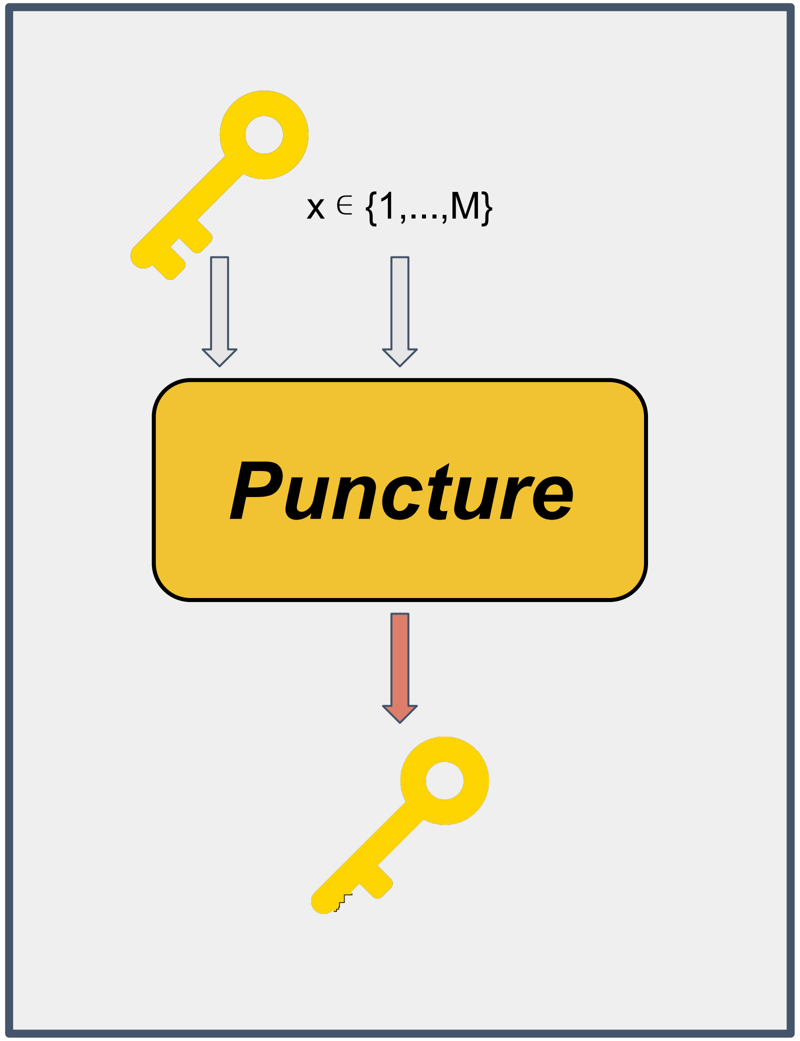


Correctness:

For any input x’ ≠ x, punctured key evaluates to same output as original key

Puncture on Puncturable PRF [BW ‘13, KPTZ ‘13, BGI ‘14, …]



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Security:

New key contains no information about evaluation at punctured point x

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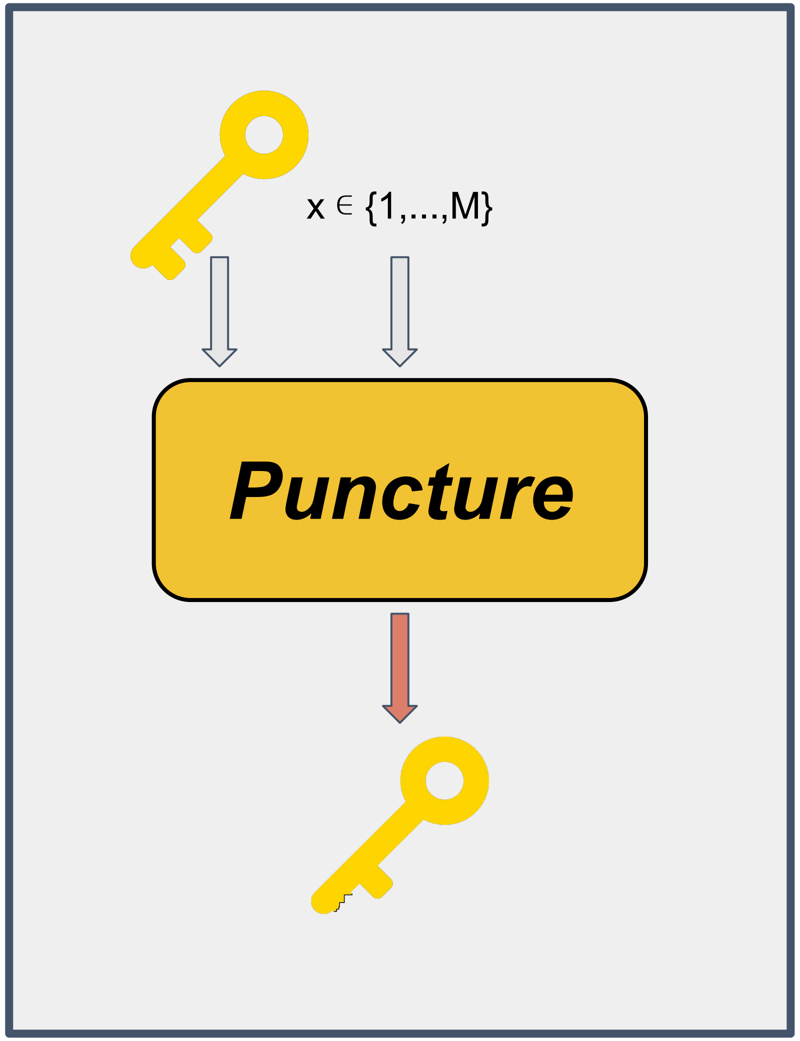

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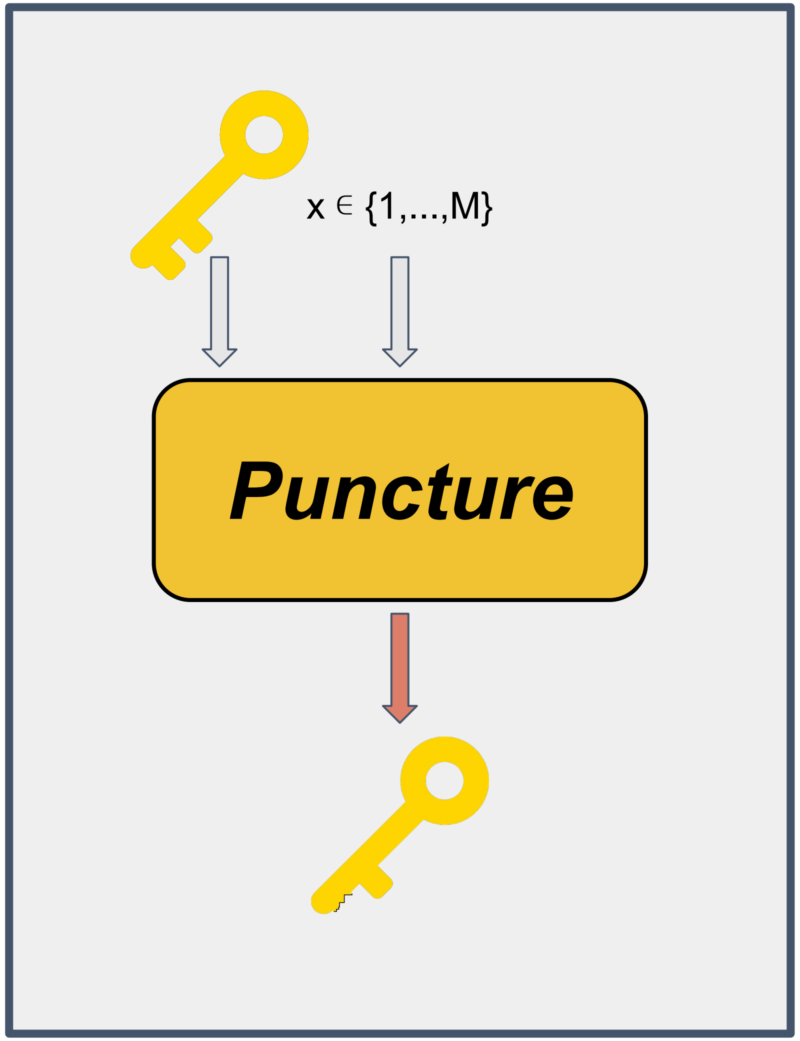
Recall: if x = (i,j) and sets are made of tuples $(i,F_k(i))$, if we send to $\mathbf{B_0}$ k’ ← Puncture(k,i), it hides $F_k(i)$ but *does not hide i*

Puncture on Privately Puncturable PRF [BLW ‘15, BKM ‘17, CC ‘17, ...]



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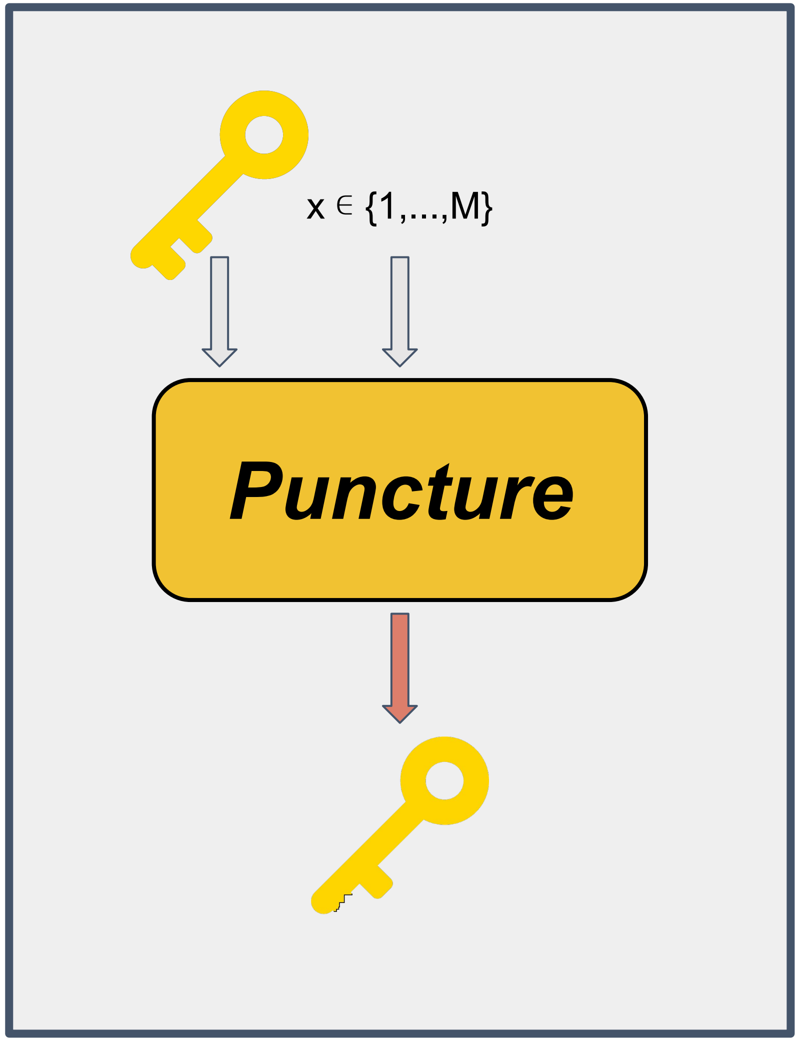

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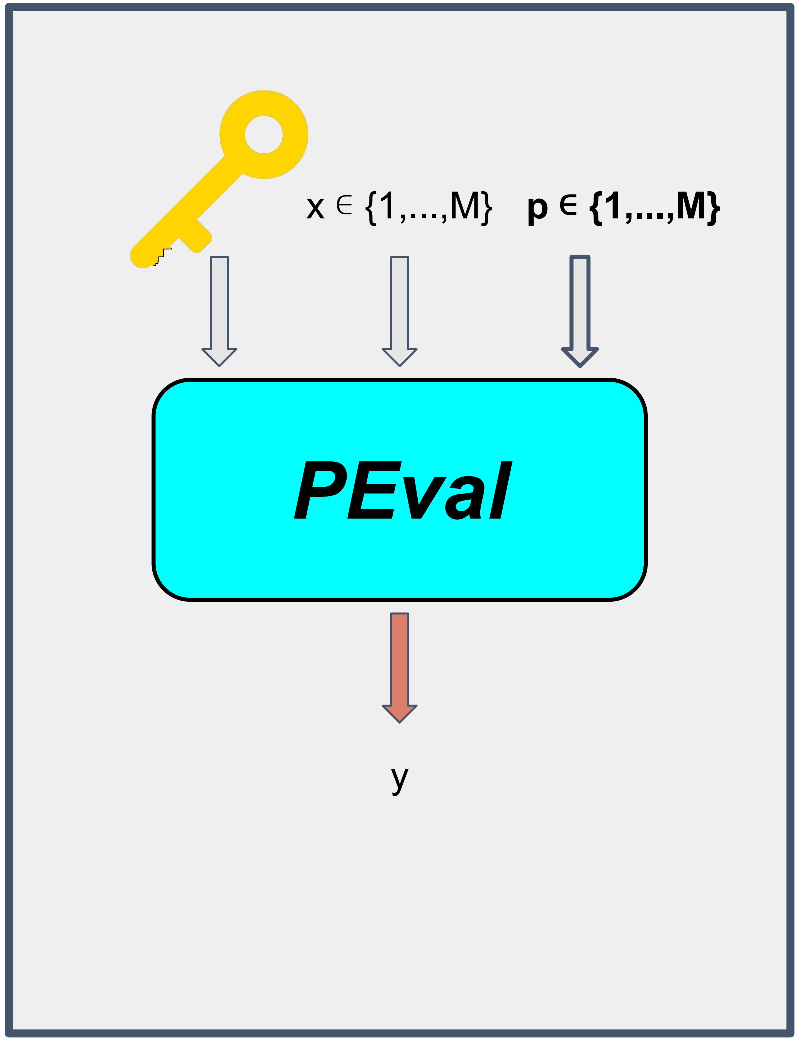
[SACM ‘21]: First construction of sublinear time, polylog bandwidth PIR, using ppPRFs

Weak Privately Puncturable PRF



Weak Correctness:

For any input x' ≠ x, punctured key evaluates to same output as original key for a correct guess of the punctured point x

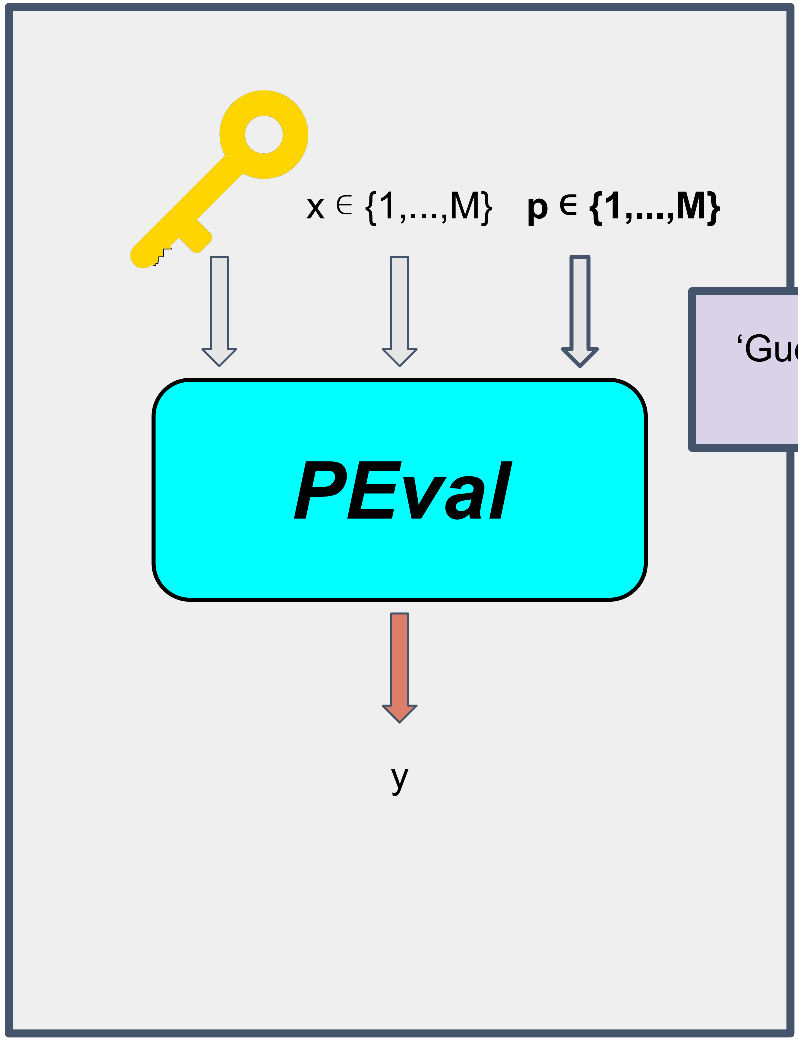
Security:

New key contains no information about evaluation at punctured point x

Privacy:

New key contains no information about punctured point x

Weak Privately Puncturable PRF



Weak Correctness:

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‘Guess’ of point that was punctured

ntains no information about evaluation at punctured point x

Privacy:

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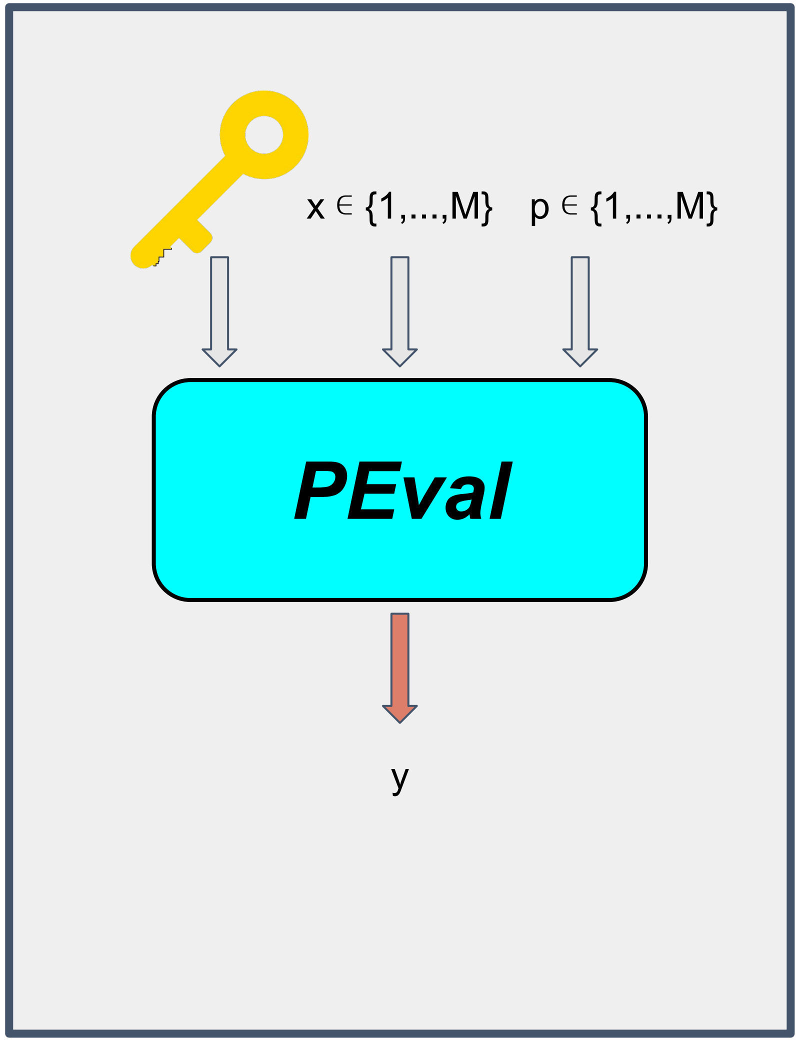

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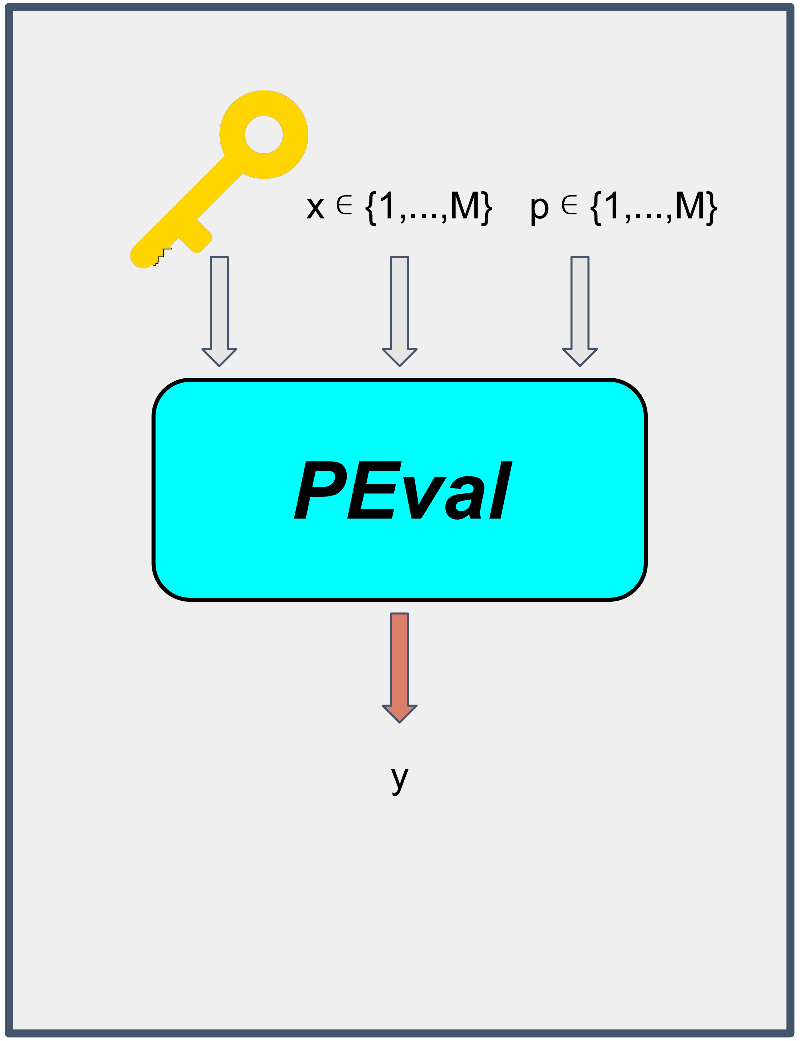
Efficient Full Evaluation:

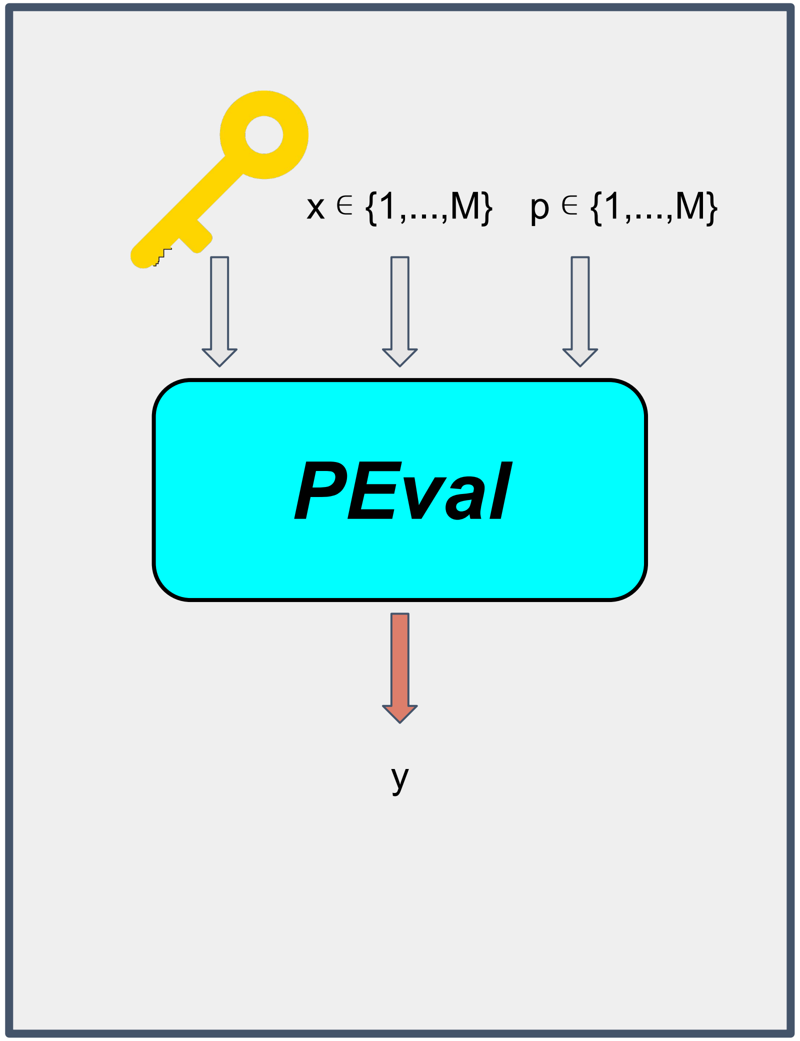
Weak Privately Puncturable PRF



Efficient Full Evaluation (Simplified):

Can compute a function over evaluations of entire domain in time quasi-linear in domain size.

Weak Privately Puncturable PRF



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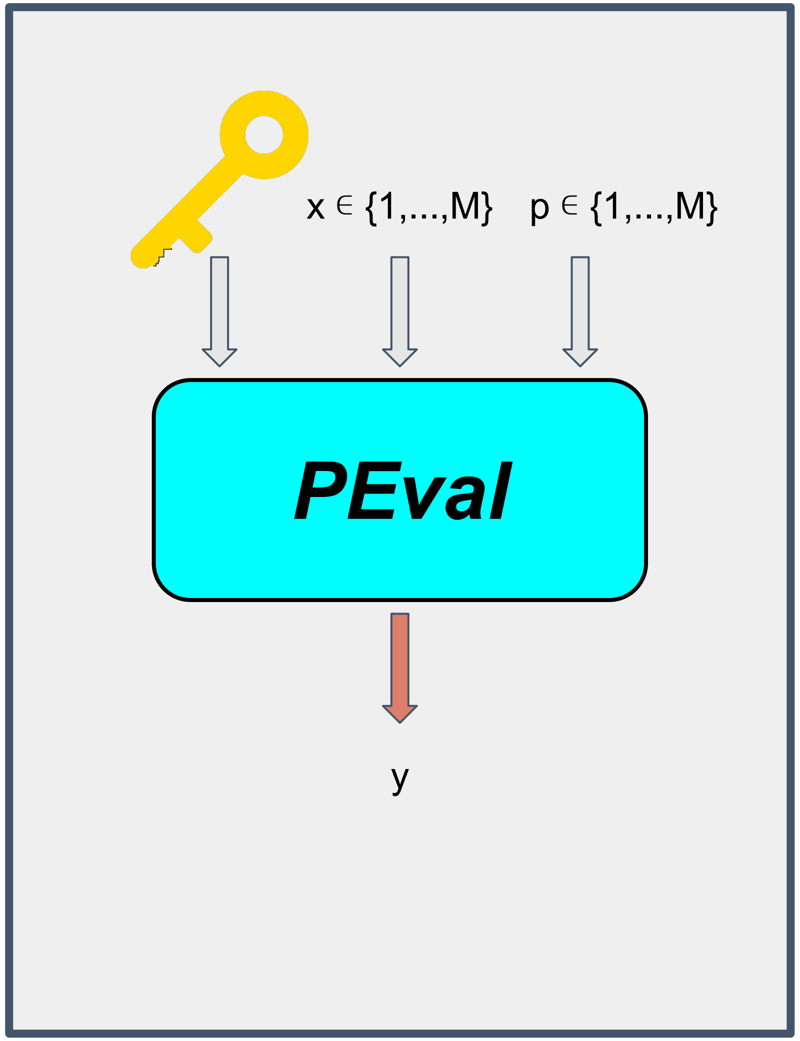
Can compute a function over evaluations of entire domain in time quasi-linear in domain size.

XOR:

Output array of length M where the i-th element is:

$$\bigoplus_{x \in \{1,...M\}} \mathrm{PEval}(k', x, i)$$

Weak Privately Puncturable PRF



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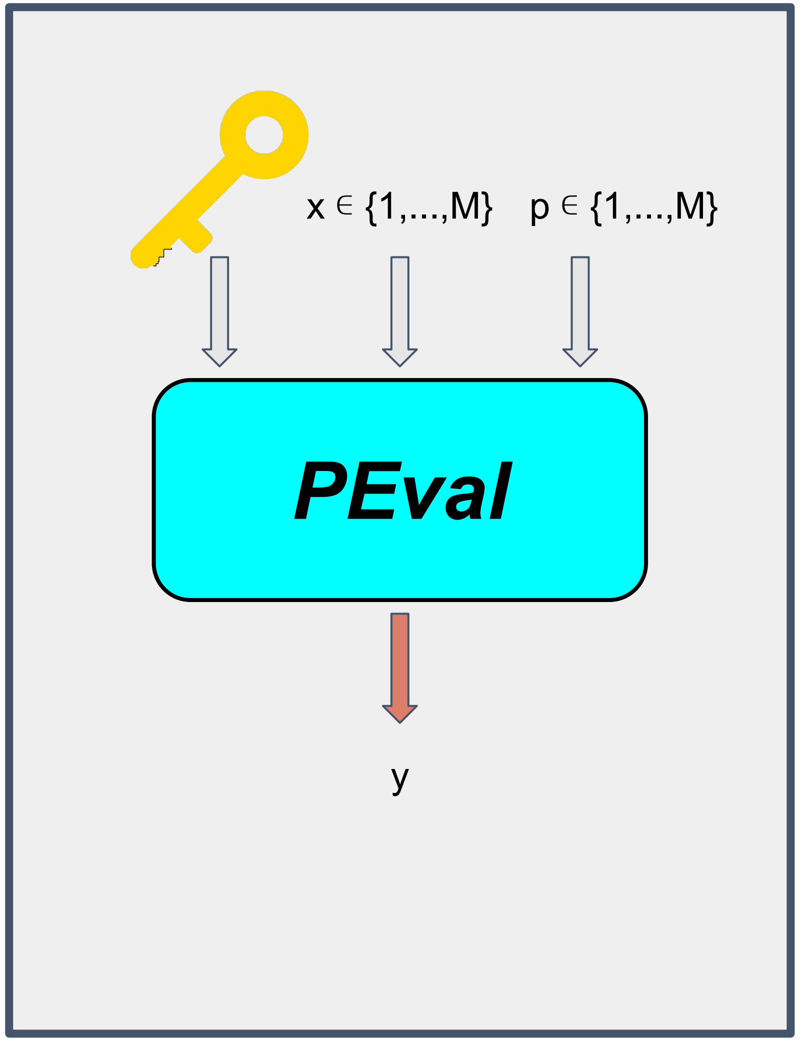

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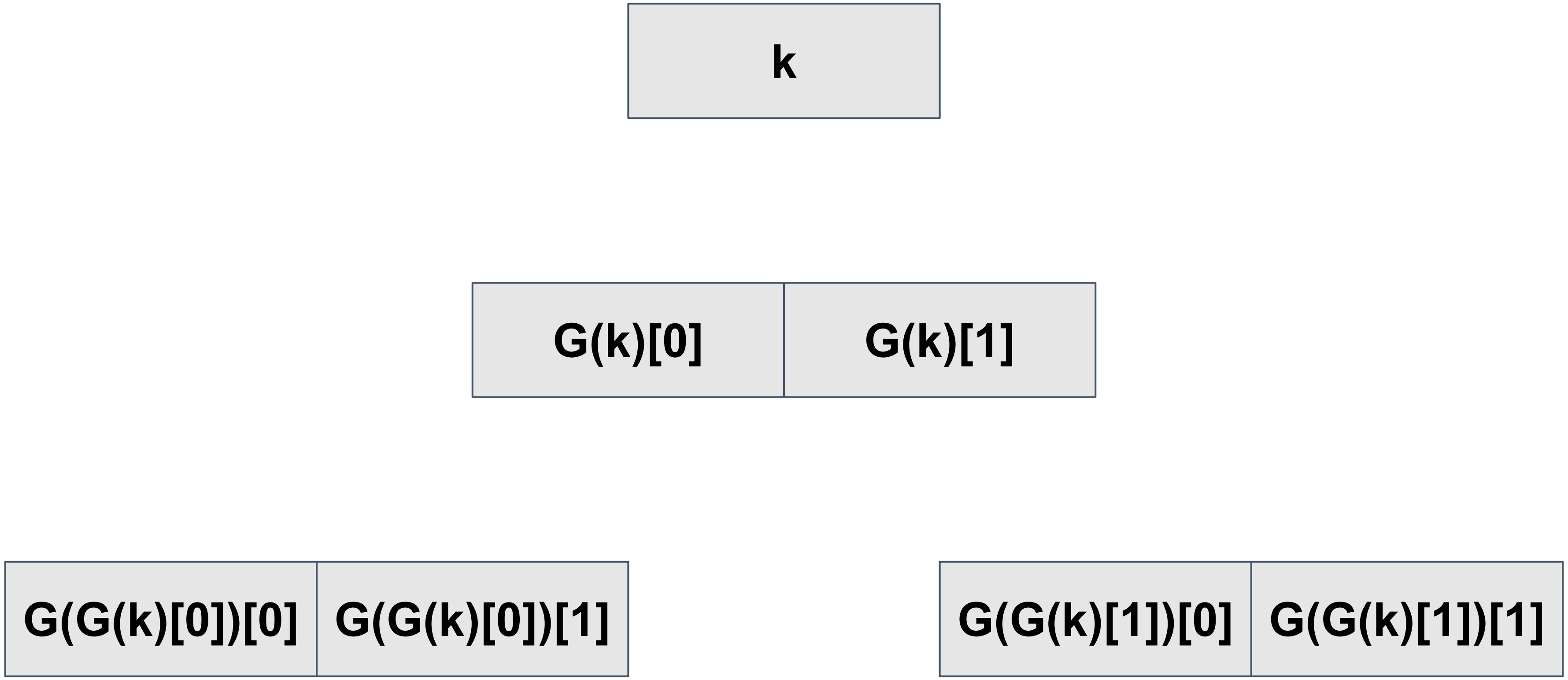
By Efficient Full Evaluation, we can output this array in quasi-linear time in M.

GGM PRF



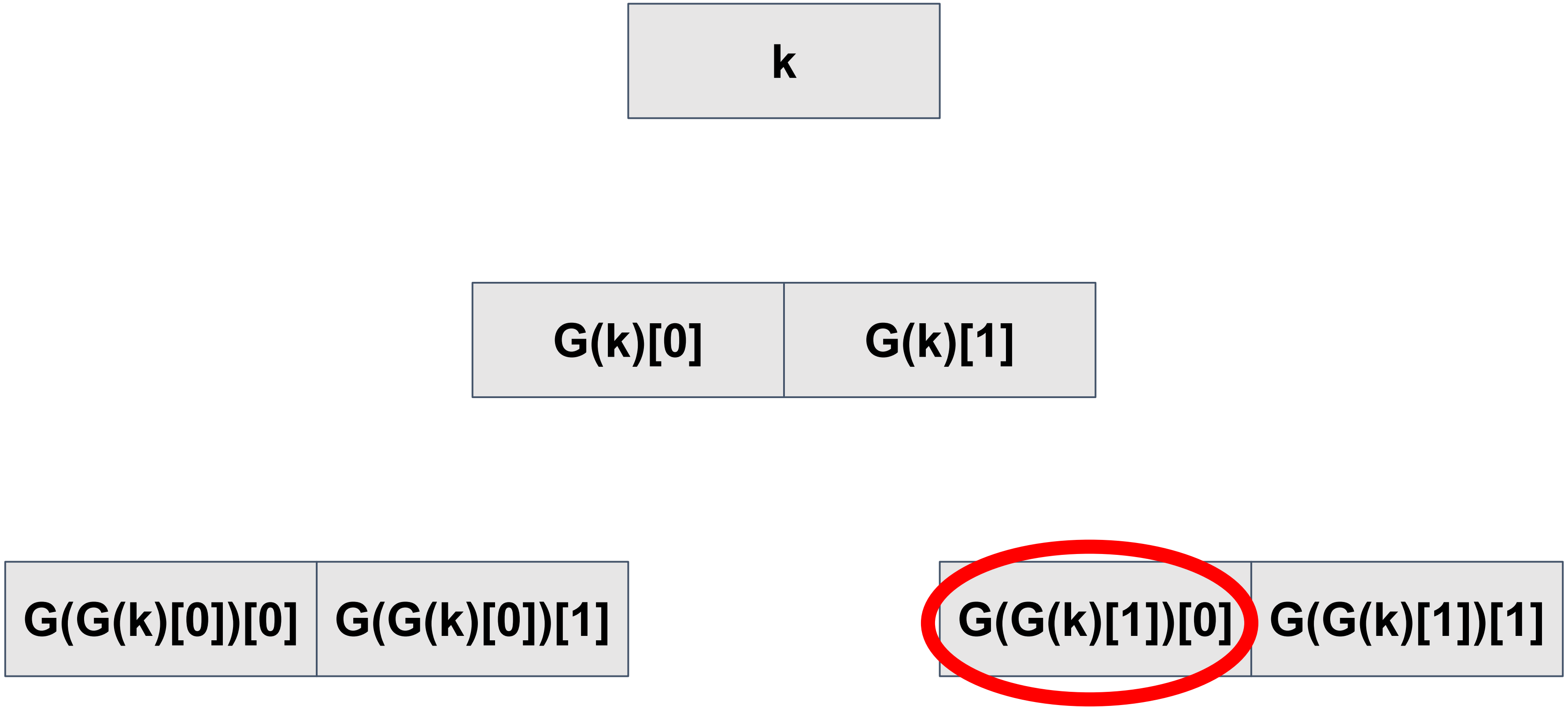
k

G(k)[0] G(k)[1]

G(G(k)[0])[0] G(G(k)[0])[1]

G(G(k)[1])[0] G(G(k)[1])[1]

GGM PRF



k

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G(k)[1]

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G(G(k)[0])[1]

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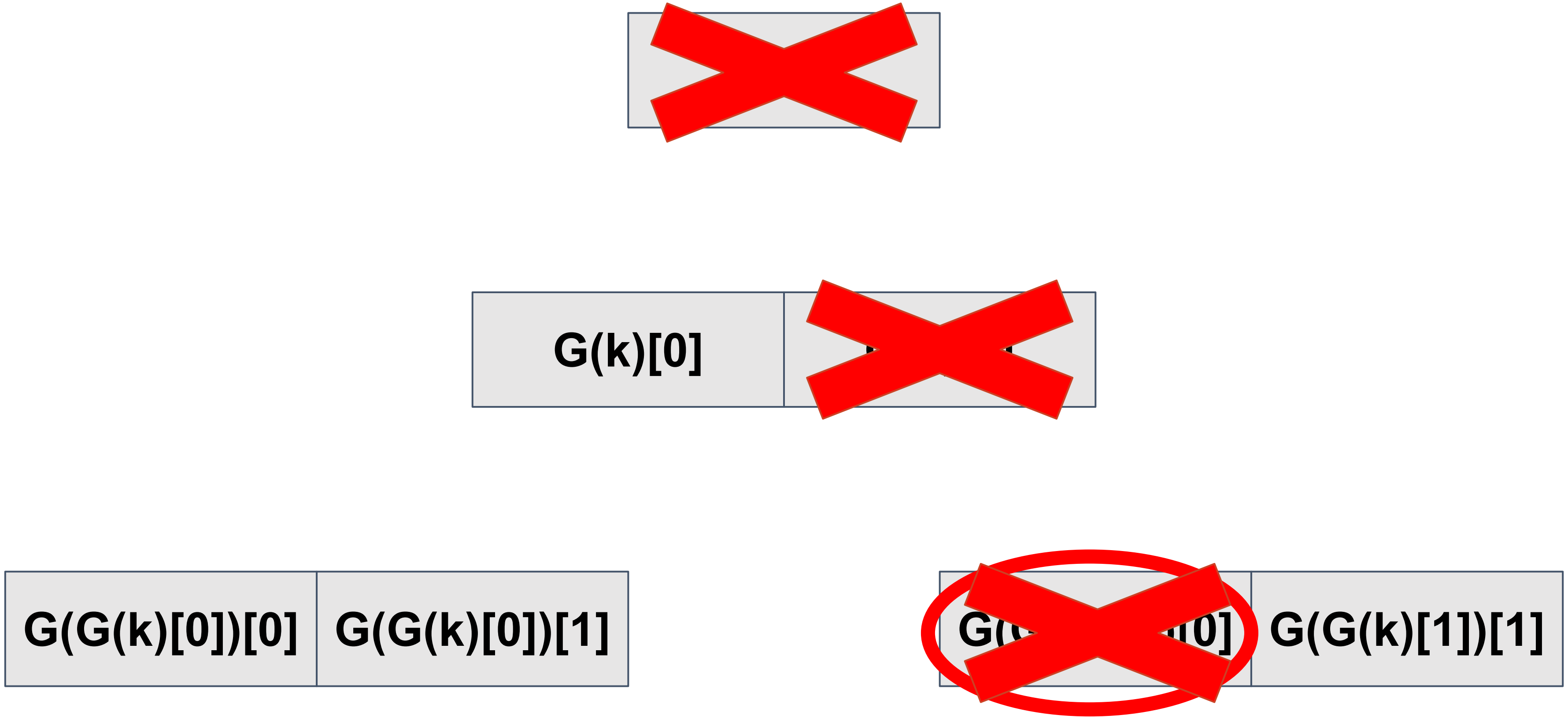
G(G(k)[1])[1]

GGM PRF



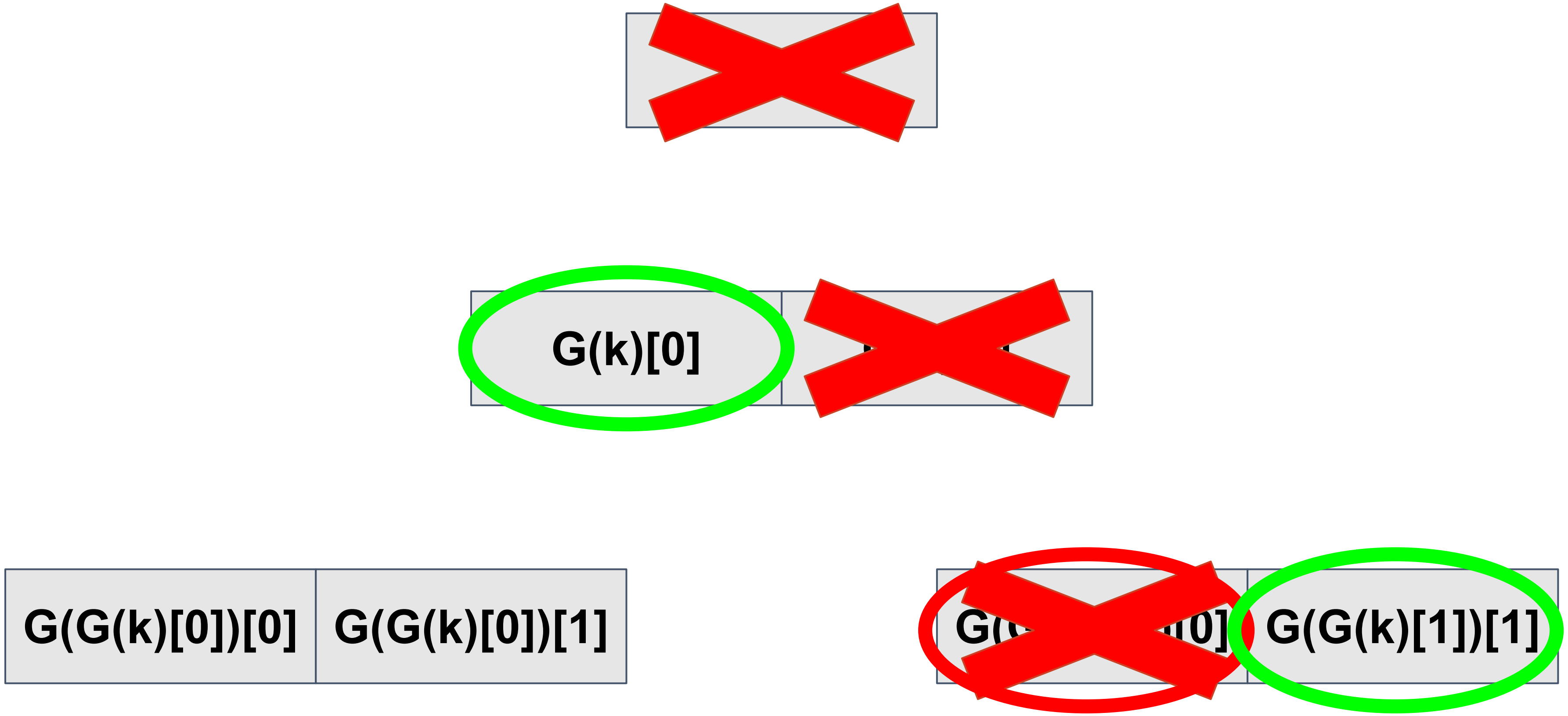
G(k)[0]

G(G(k)[0])[0]

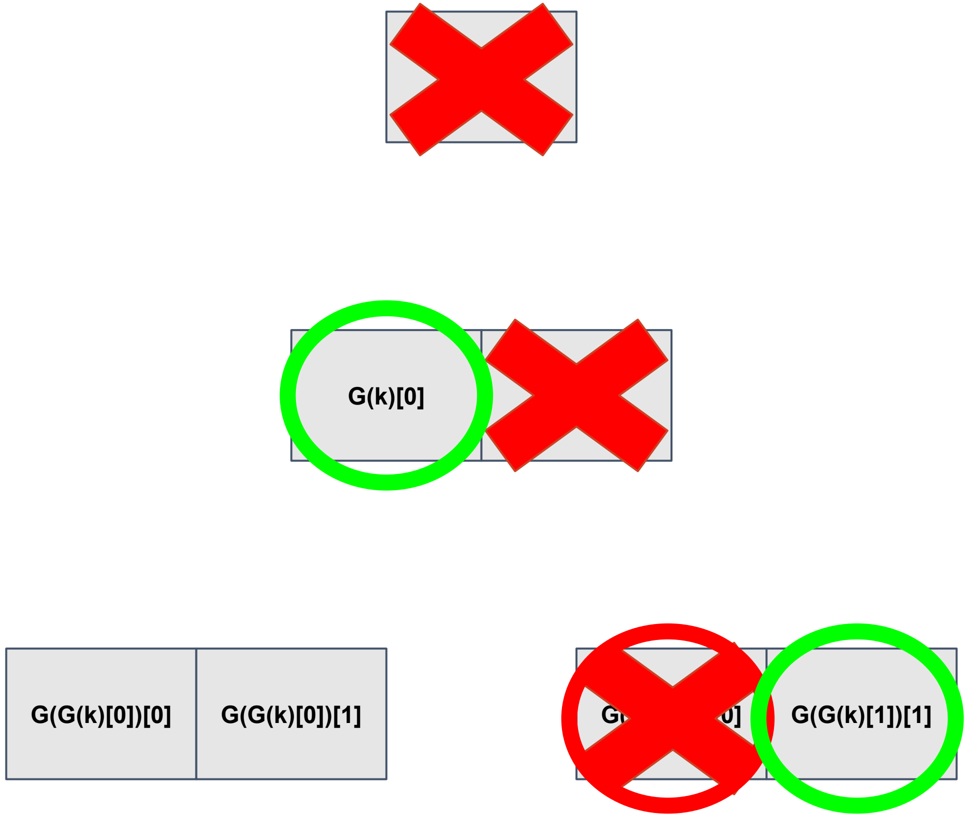
G(G(k)[0])[1]

G(G(k)[1])[1]

GGM PRF

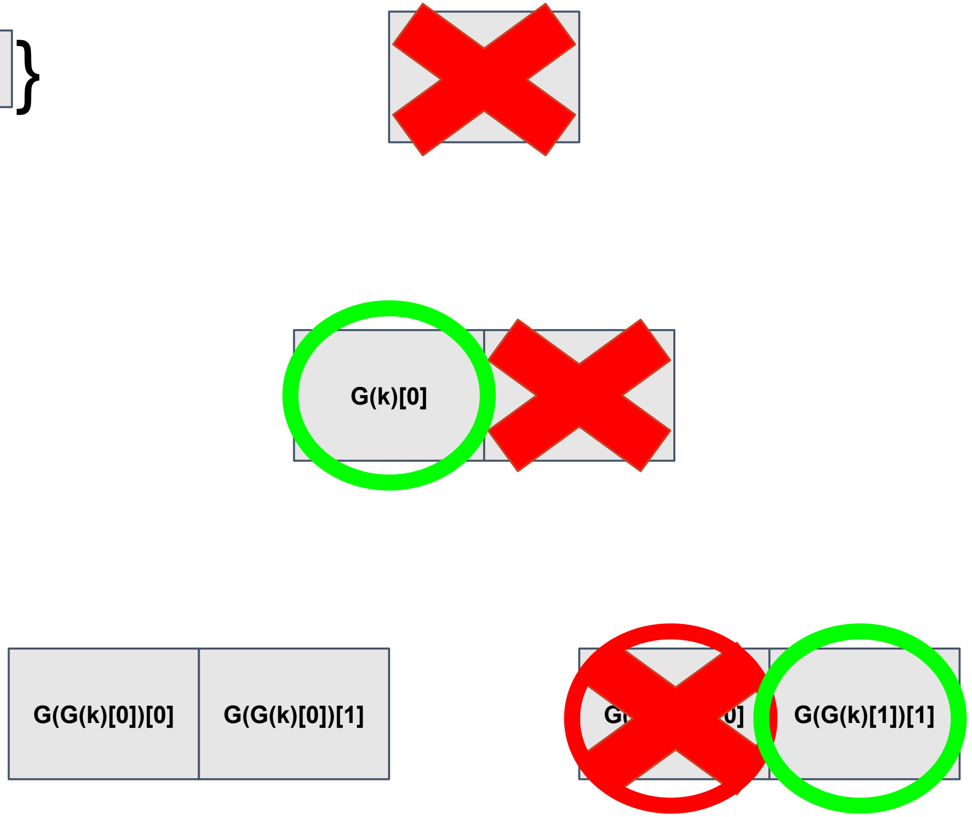


GGM PRF



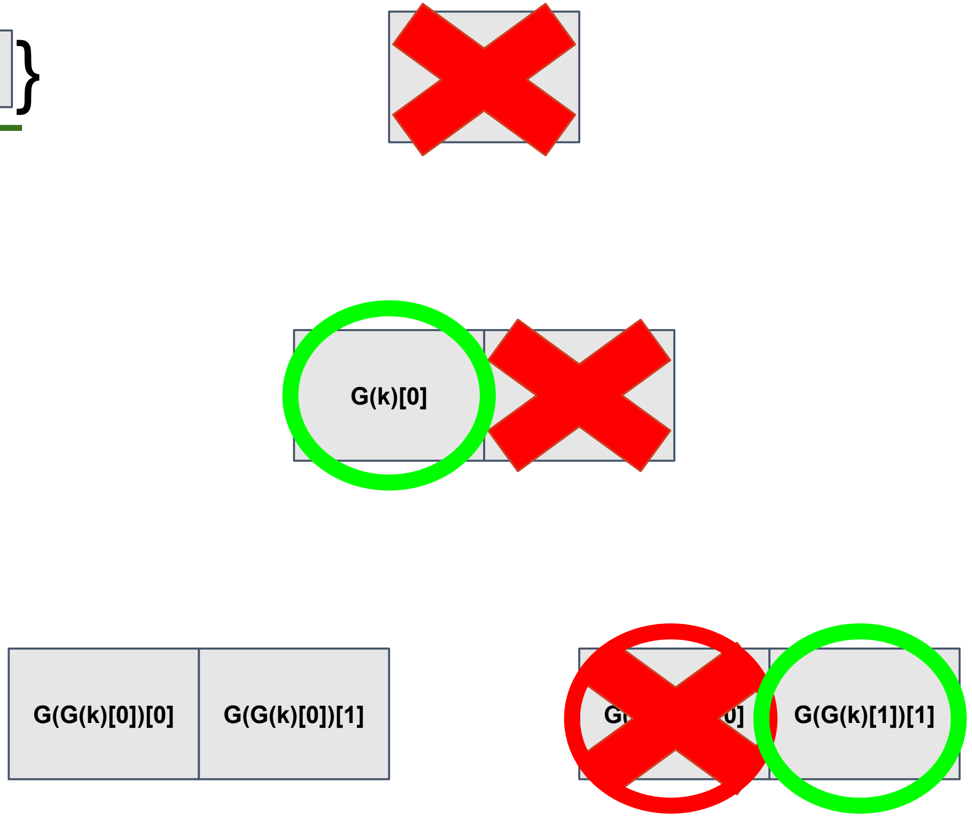
GGM PRF

Punctured Key: { 3, G(k)[0], G(G(k)[1])[1] }



GGM PRF

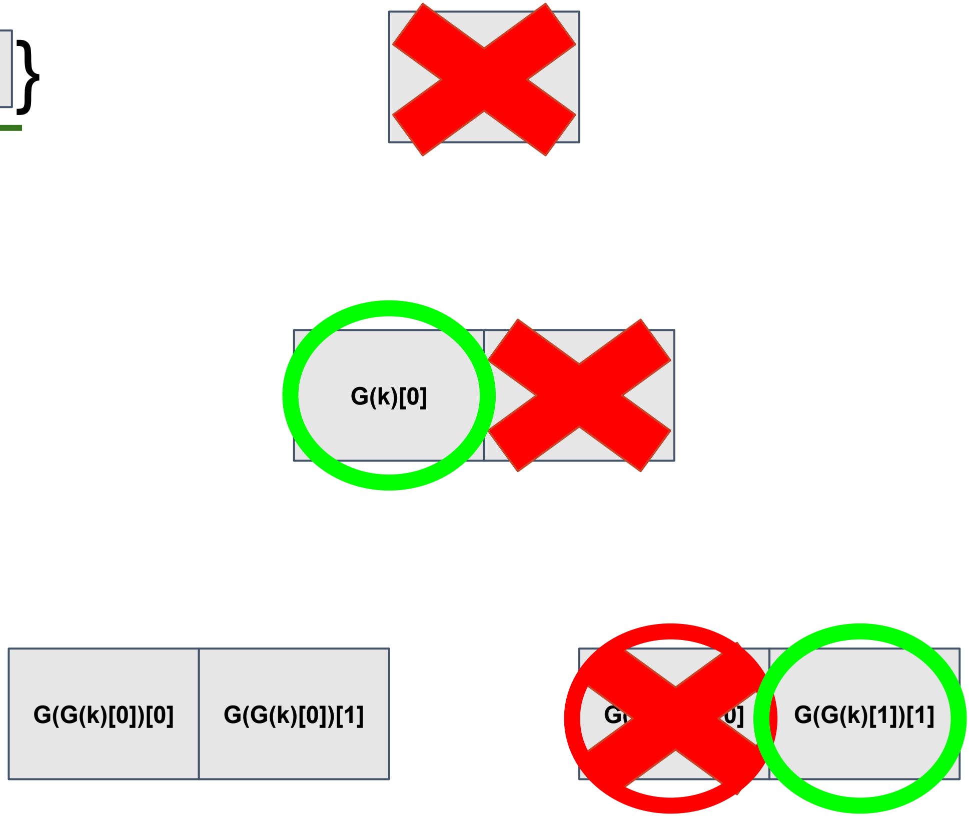
Punctured Key: $\{ \underbrace{3,}_{\text{index punctured}} \underbrace{\boxed{G(k)[0]}, \boxed{G(G(k)[1])[1]}}_{\text{Left-to-right ordering of nodes in adjacent path}} \}$



Weak Privately Puncturable PRF

Privately
Punctured Key: { ~~8~~, G(k)[0], G(G(k)[1])[1] }

Left-to-right ordering of nodes in adjacent path



Weak ppPRF

Privately

Punctured Key: $\{\cancel{3}, \overbrace{\boxed{G(k)[0]}, \boxed{G(G(k)[1])[1]}}^{\text{Left-to-right ordering of nodes in adjacent path}}\}$

Recipient can guess punctured index:

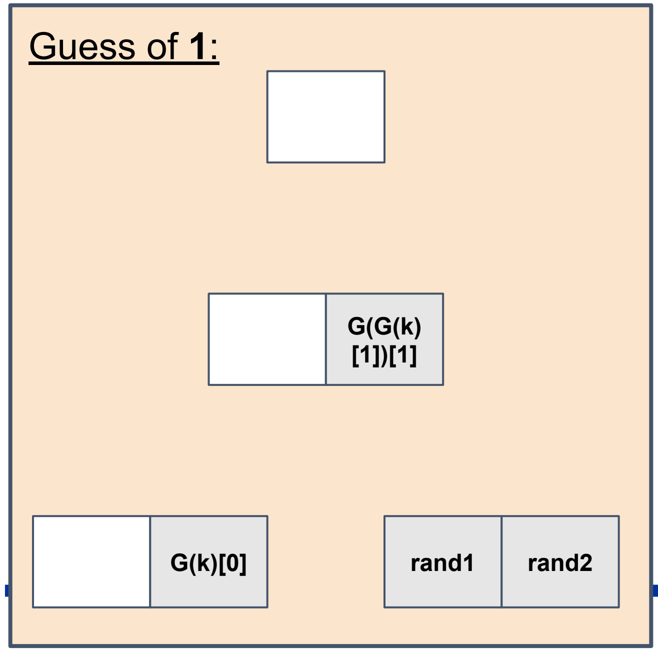
Weak ppPRF

Left-to-right ordering of nodes in adjacent path

Privately

Punctured Key: { ~~3~~, G(k)[0], G(G(k)[1])[1] }

Recipient can guess punctured index:

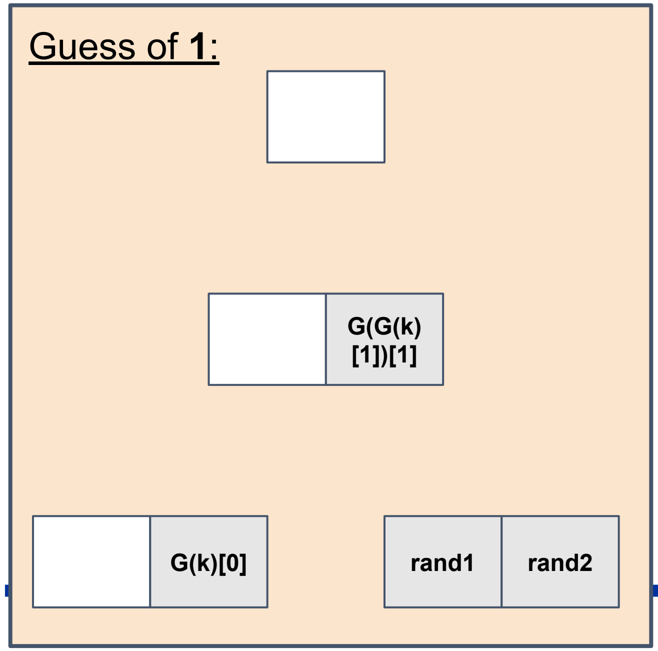


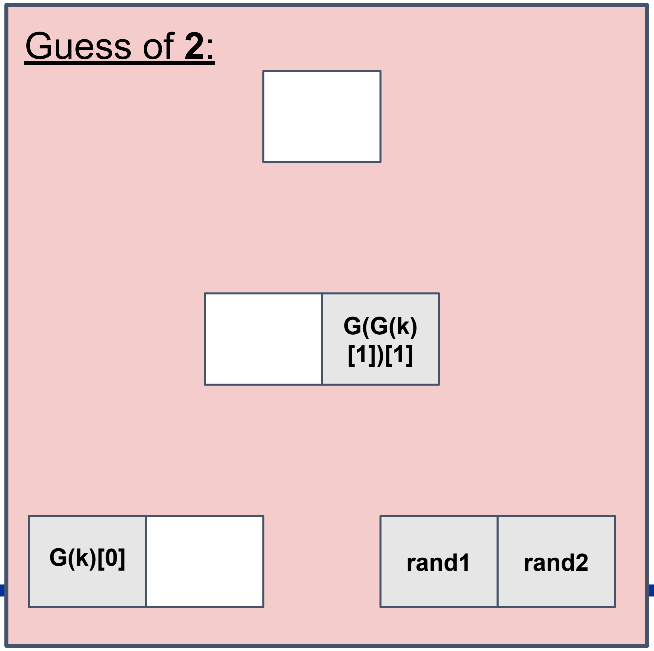
Weak ppPRF

Left-to-right ordering of nodes in adjacent path

Privately
Punctured Key: { ~~3~~, G(k)[0], G(G(k)[1])[1] }

Recipient can guess punctured index:





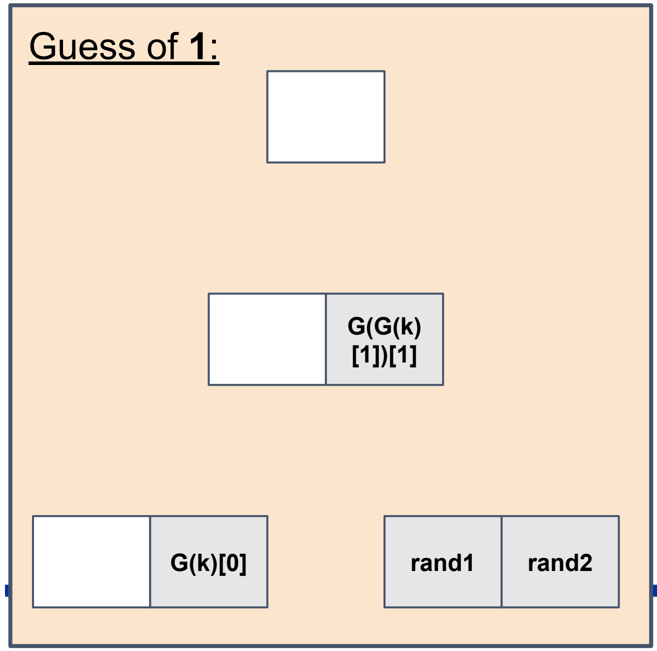
Weak ppPRF

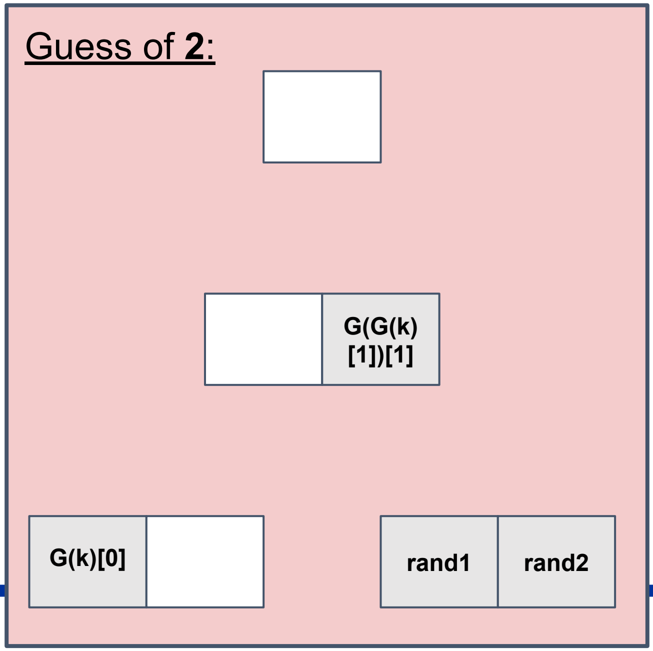
Left-to-right ordering of nodes in adjacent path

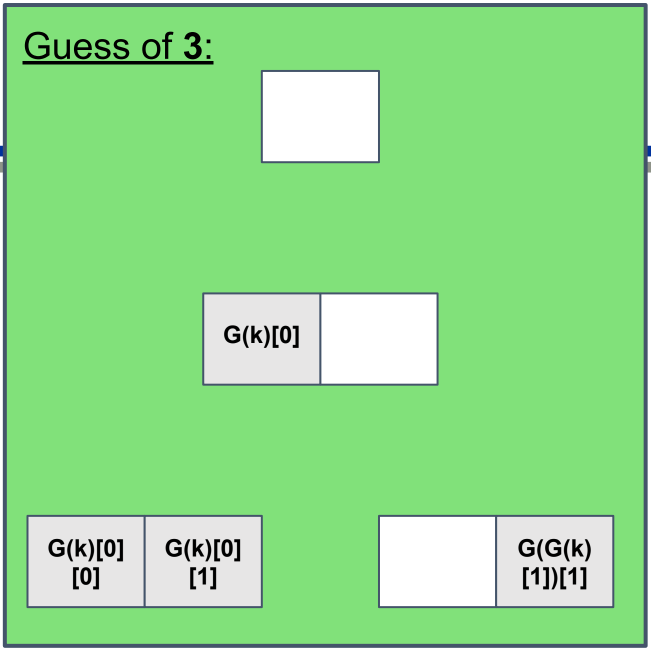
Privately

Punctured Key: { ~~3~~, G(k)[0] , G(G(k)[1])[1] }

Recipient can guess punctured index:


Guess of **1**:


Guess of **2**:

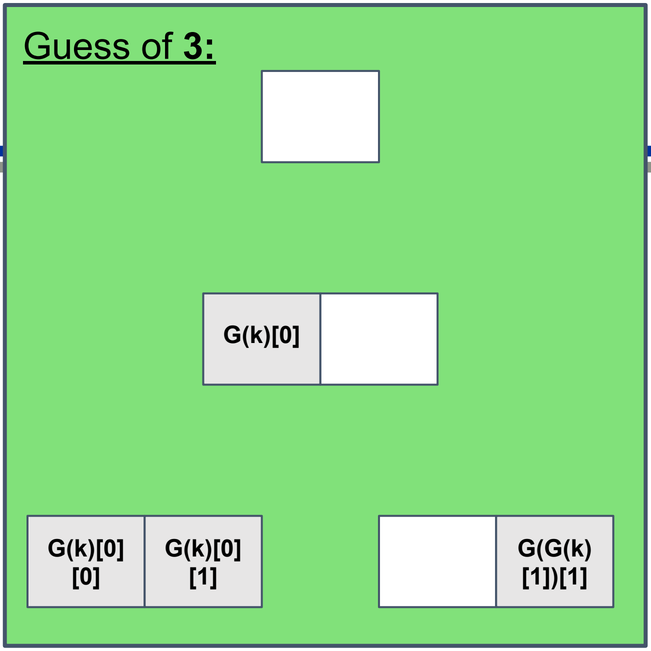

Guess of **3**:

Weak ppPRF

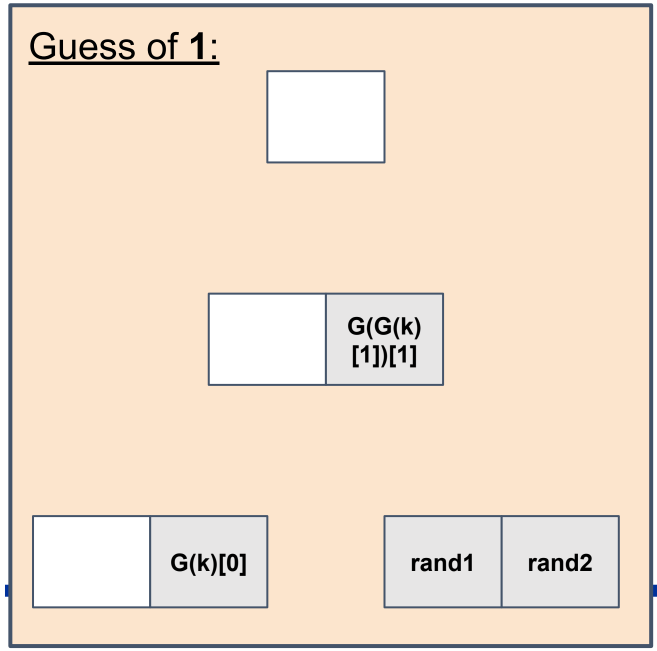
Left-to-right ordering of nodes in adjacent path

Privately Punctured Key: { ~~3~~, G(k)[0], G(G(k)[1])[1] }

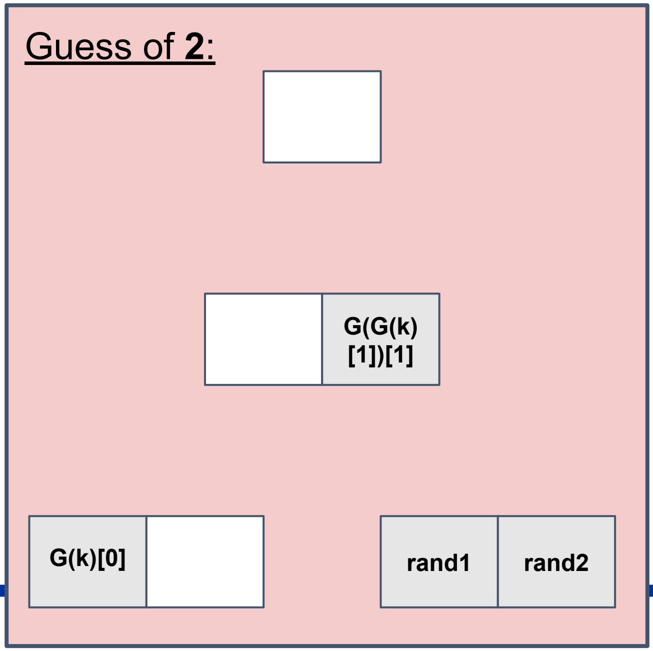
Recipient can guess punctured index:



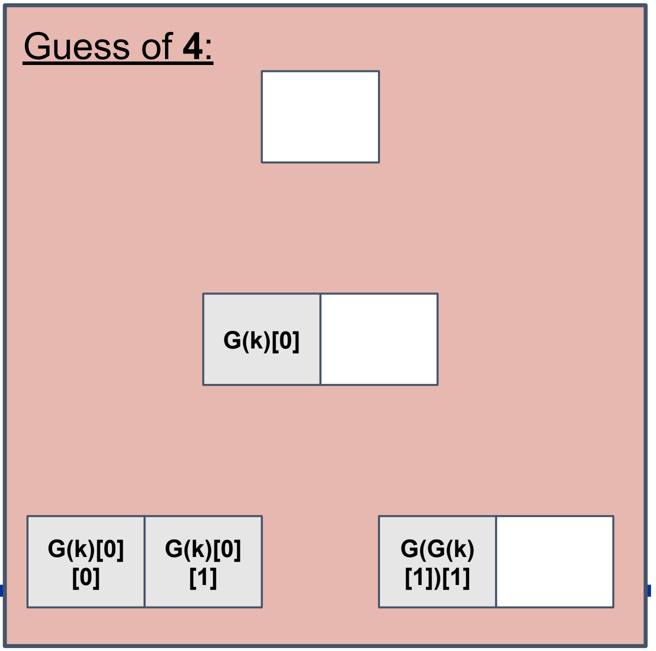
Guess of **3:**



Guess of **1**:

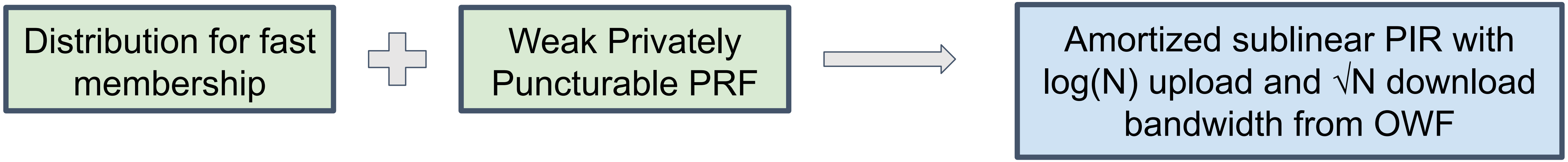


Guess of **2**:



Guess of **4**:

TreePIR

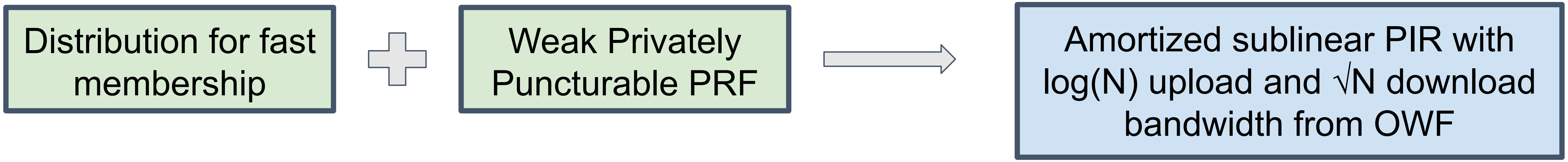


Distribution for fast membership

Weak Privately Puncturable PRF

Amortized sublinear PIR with log(N) upload and √N download bandwidth from OWF

TreePIR



Distribution for fast membership

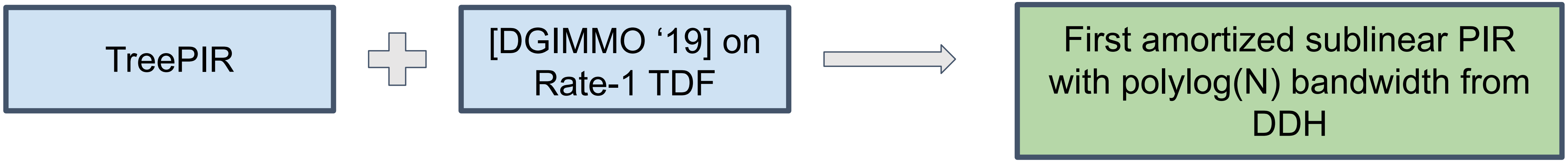
Weak Privately Puncturable PRF

Amortized sublinear PIR with log(N) upload and $\sqrt{N}$ download bandwidth from OWF

Database of 2^{32} bit entries

PIR Scheme	Client storage	Amortized query time	Online Bandwidth
TreePIR	1MB	3.5s	16.6KB
Checklist [KC21]	8GB	12.5s	0.5KB

TreePIR plus recursion

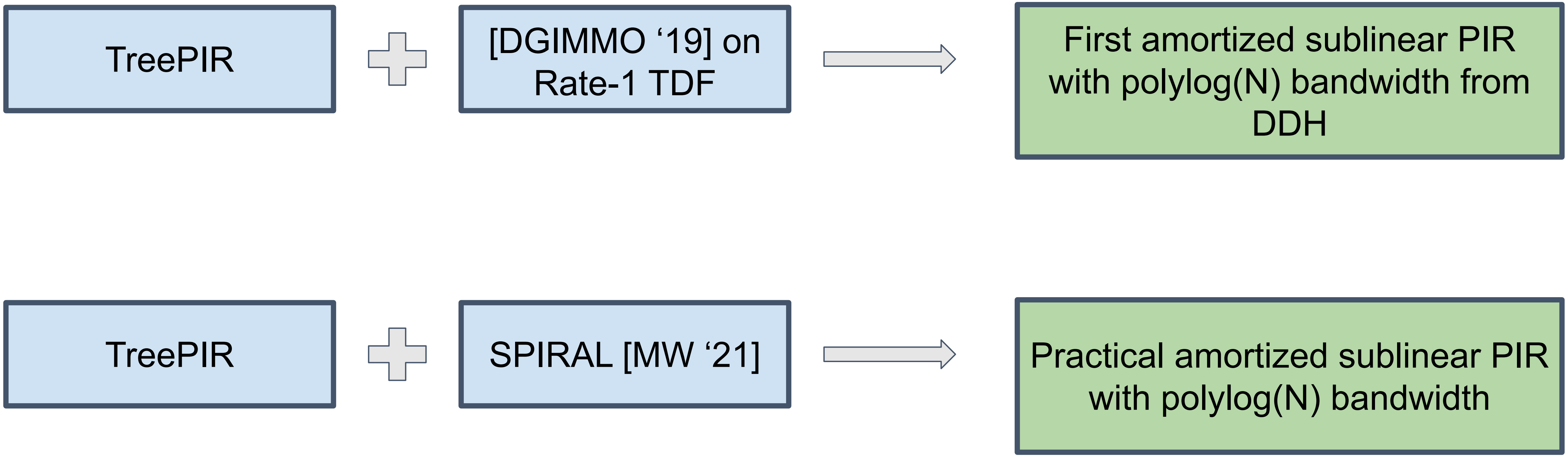


TreePIR

[DGIMMO ‘19] on Rate-1 TDF

First amortized sublinear PIR with polylog(N) bandwidth from DDH

TreePIR plus recursion



TreePIR + [DGIMMO '19] on Rate-1 TDF → First amortized sublinear PIR with polylog(N) bandwidth from DDH

TreePIR + SPIRAL [MW '21] → Practical amortized sublinear PIR with polylog(N) bandwidth

New works building on TreePIR

- Piano: Extremely Simple, Single-Server PIR with Sublinear Server Computation [ZLTS '23] (to appear IEEE S&P '24)
- Simple and Practical Amortized Sublinear Private Information Retrieval [MIR '23] (ePrint)

Sources for icons:

https://icon-library.com/icon/key-icon-png-7.html.html

https://www.onlygfx.com/magnifying-glass-clipart-png-transparent/

https://www.freepnglogos.com/images/tick-33835.html