Practical Settlement Bounds for Longest-Chain Consensus

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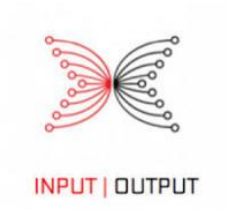
University of Connecticut
and IOG



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- transactions-carrying **blocks** appended in ever-growing **blocktree**
- blocks connected by hash links
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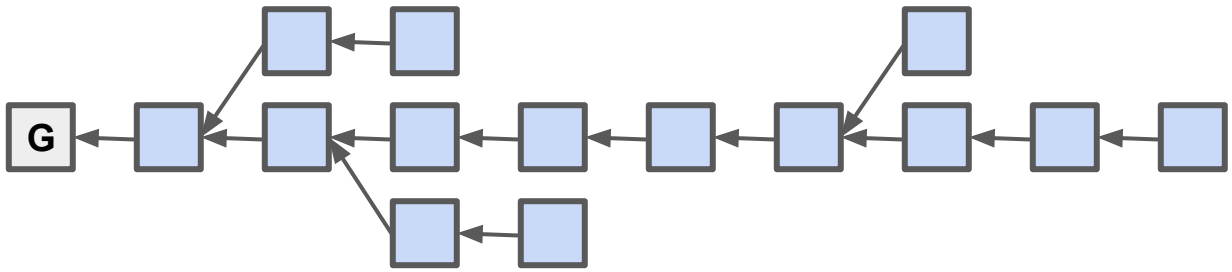

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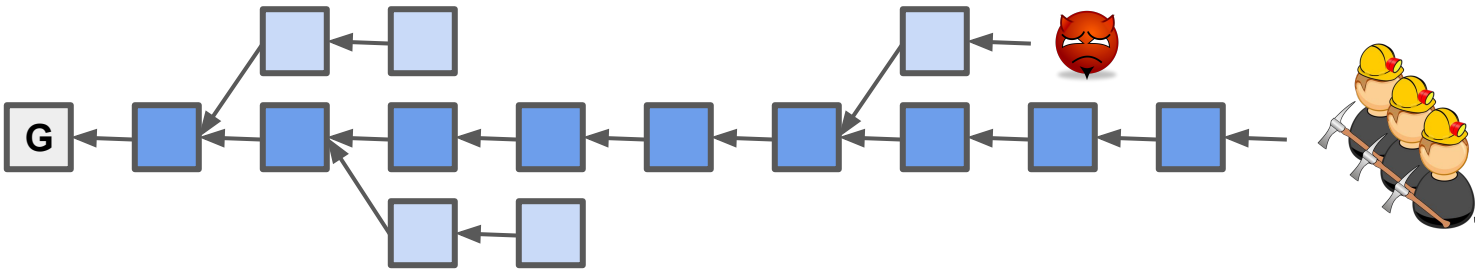

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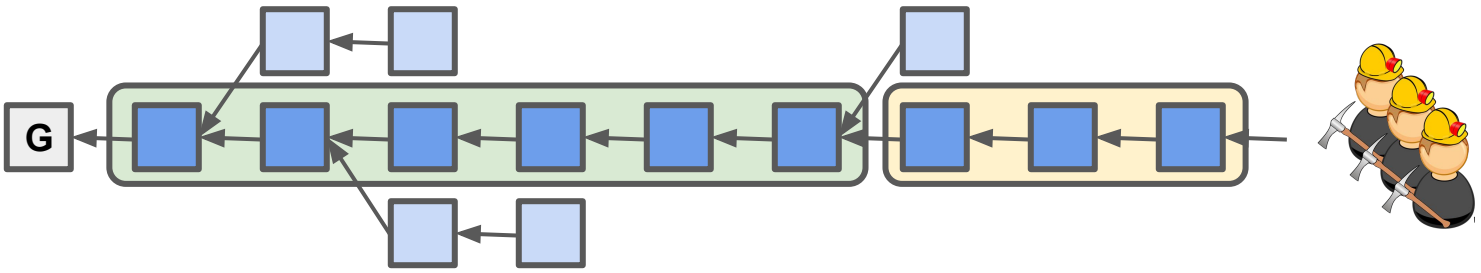
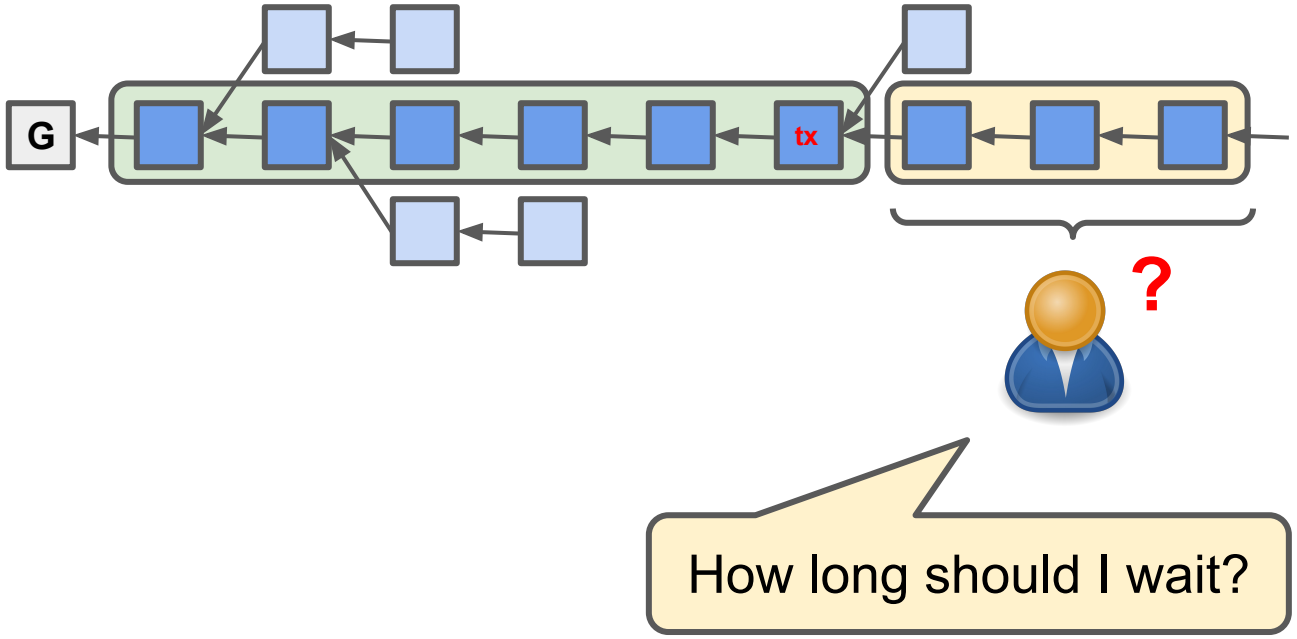
- **stable ledger state** : longest chain minus **unstable suffix**



Settlement is gradual and subjective!

How fast is longest-chain settlement?



Our Results

1. a rigorous method for obtaining settlement guarantees for longest-chain consensus:

Assuming
- some honest and adversarial power (hashing/stake)
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how many blocks guarantee settlement except with acceptable error?

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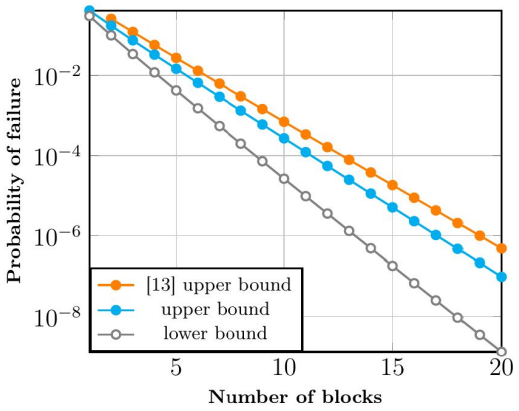
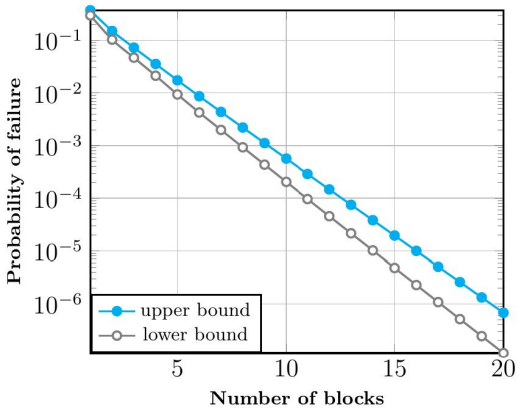
2. concrete numerical results of practical interest:

PoW (Ethereum)



PoS (Cardano)

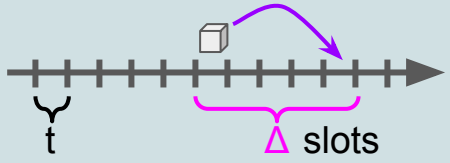


Our Model

Timeline and Message Delays:

Discrete model

- discrete slots of length t
- msgs delayed by: ≤ Δ slots



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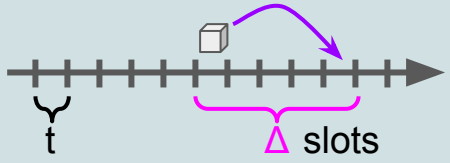

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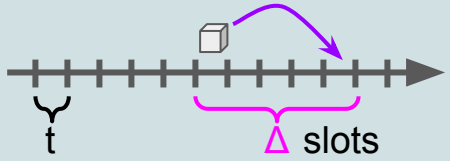

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Adversary: arbitrary strategy

- cannot break hash function or the lottery

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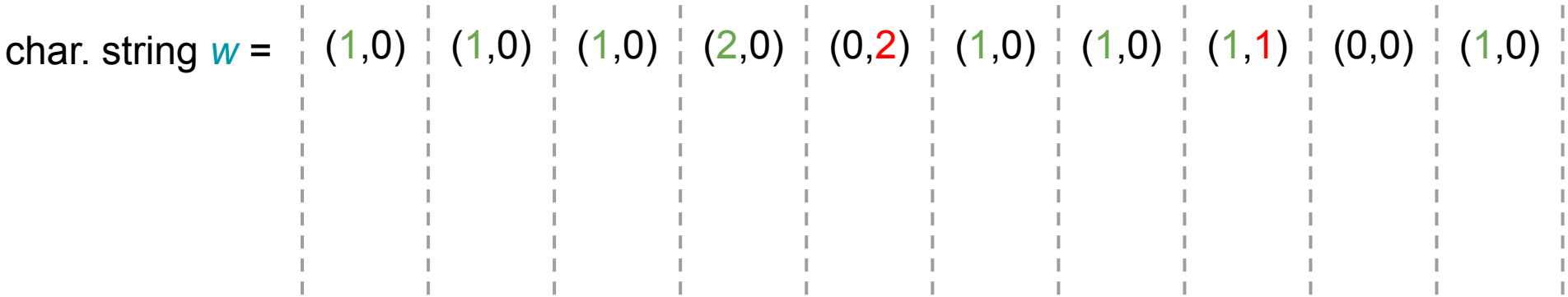
  - numbers of honest/adversarial lottery successes in each slot



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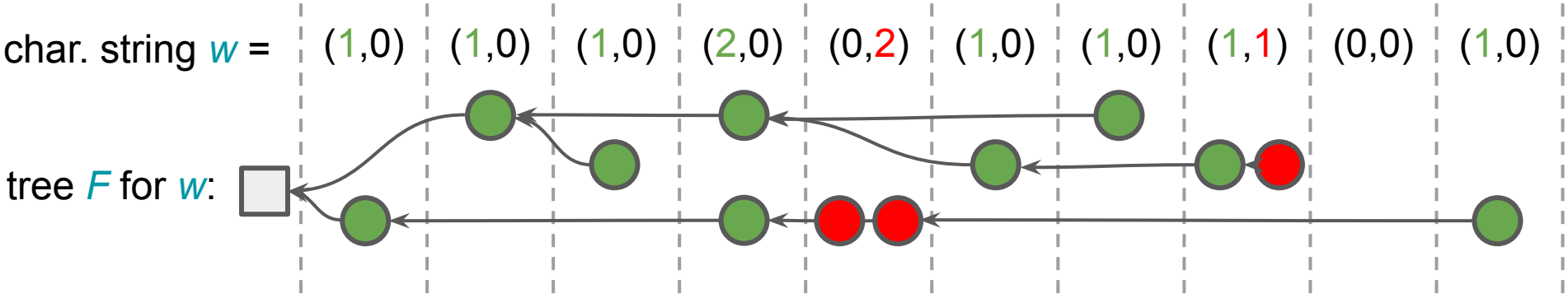

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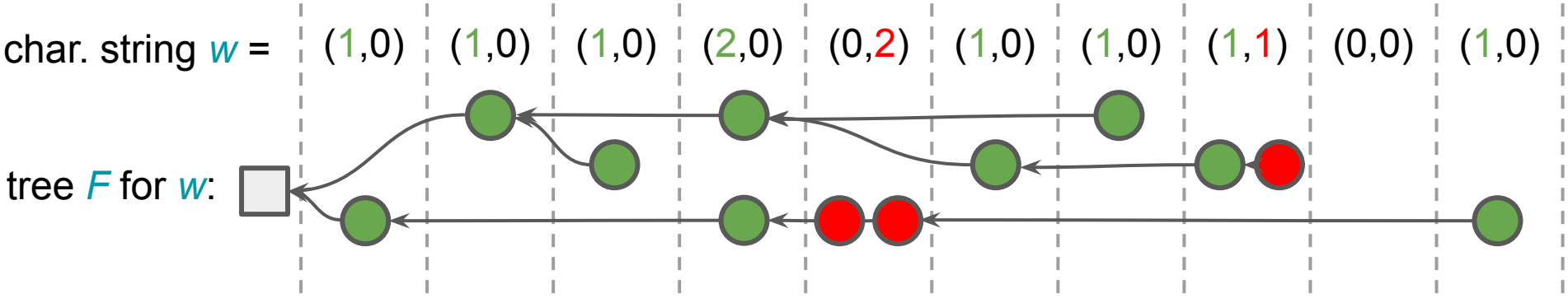

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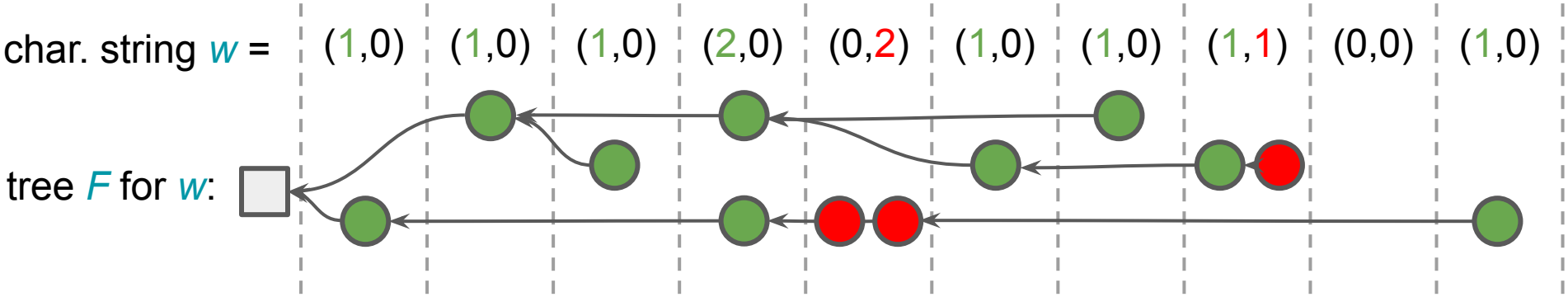
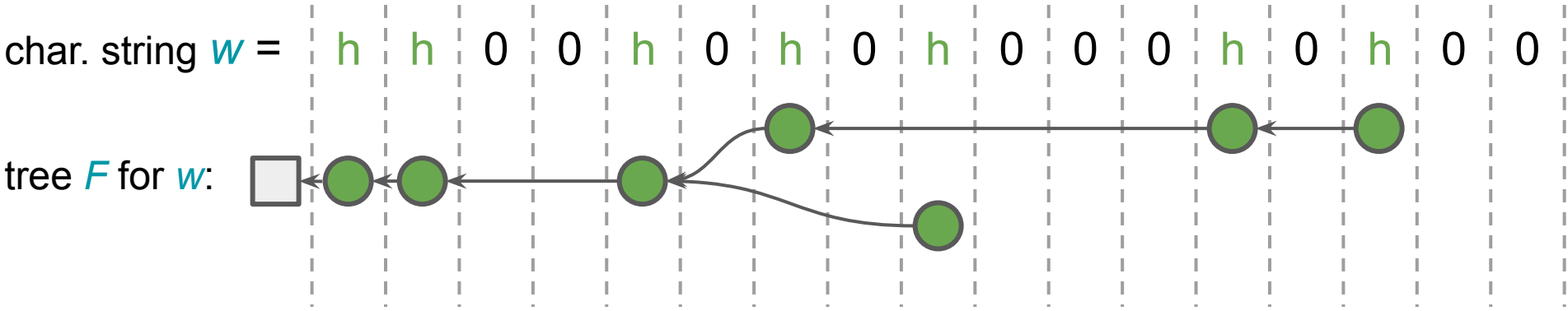


Does a string *w* admit a blocktree *F* with long diverging segments?

Characteristic Strings and Forks (Δ = 3)



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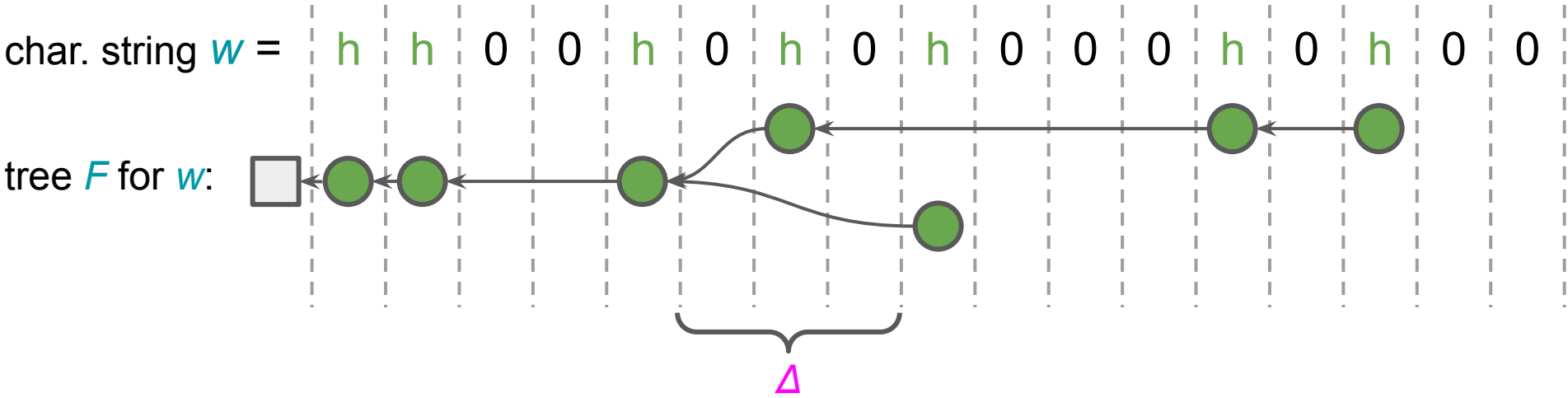

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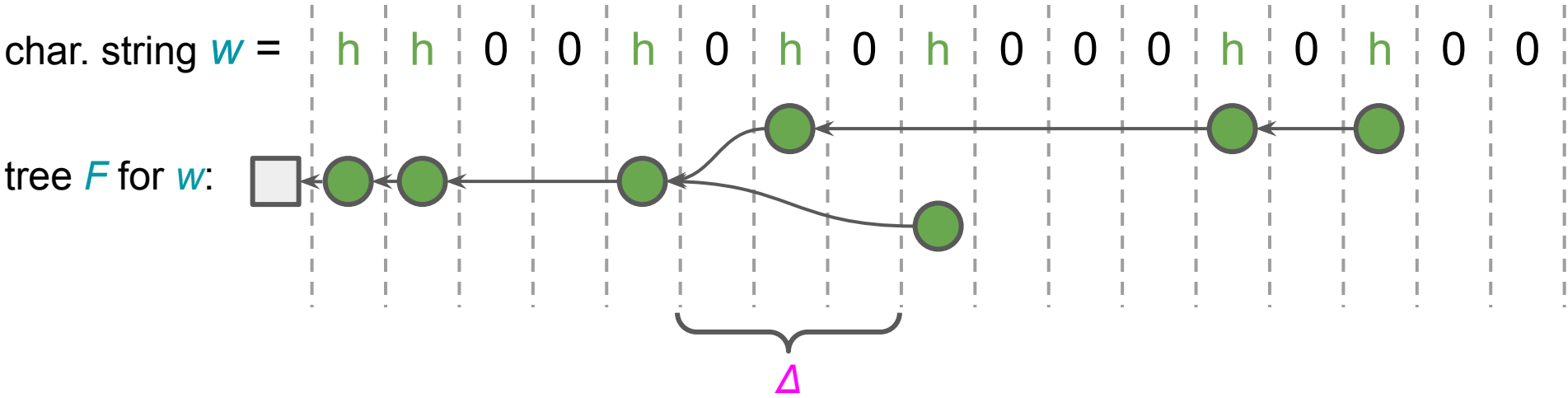

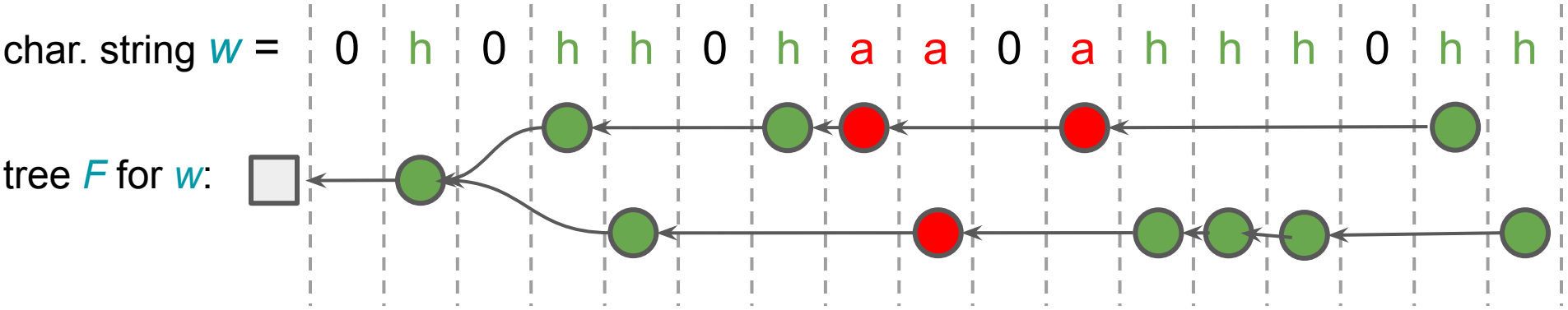

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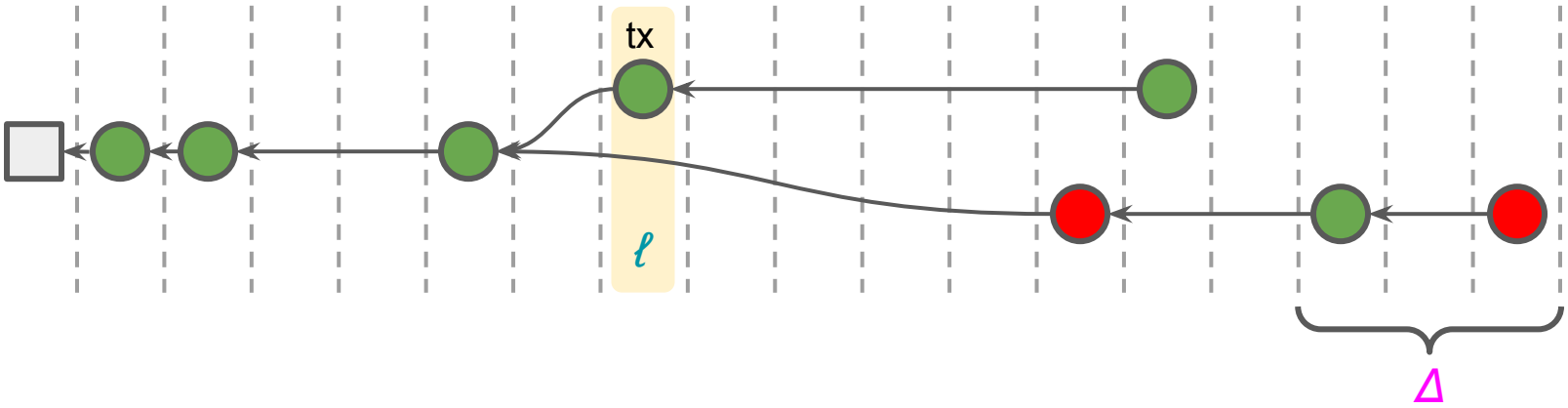

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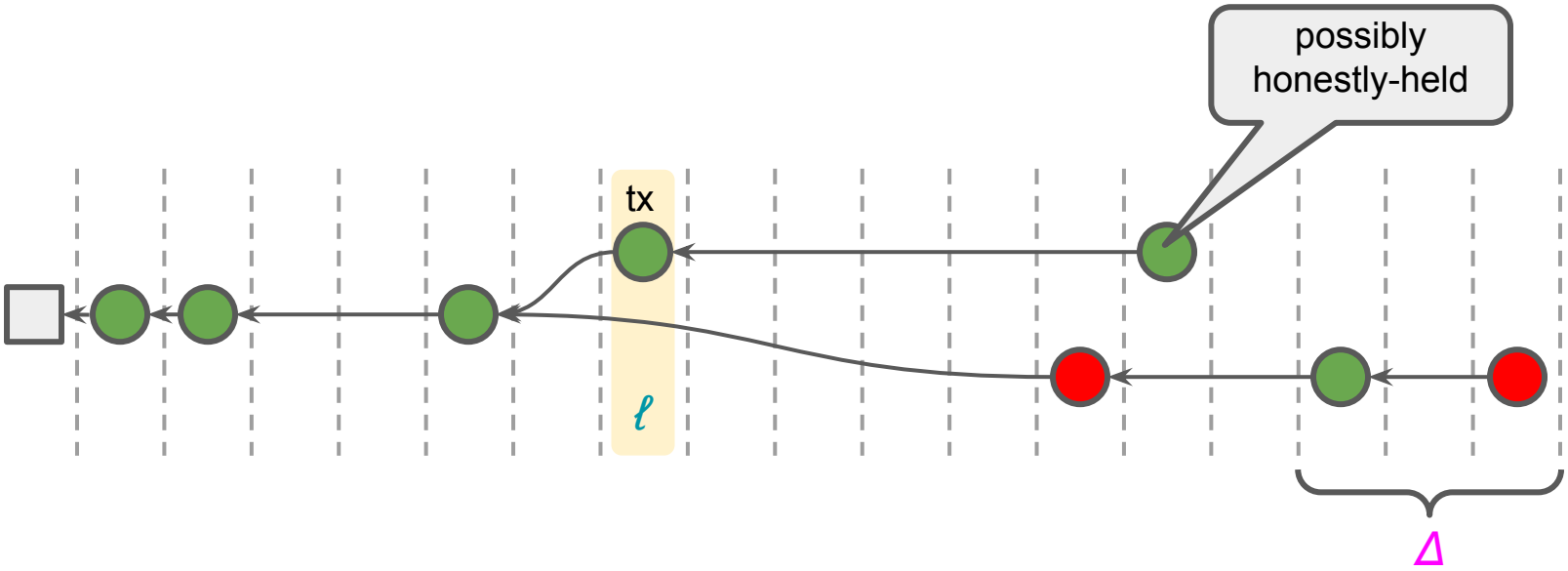

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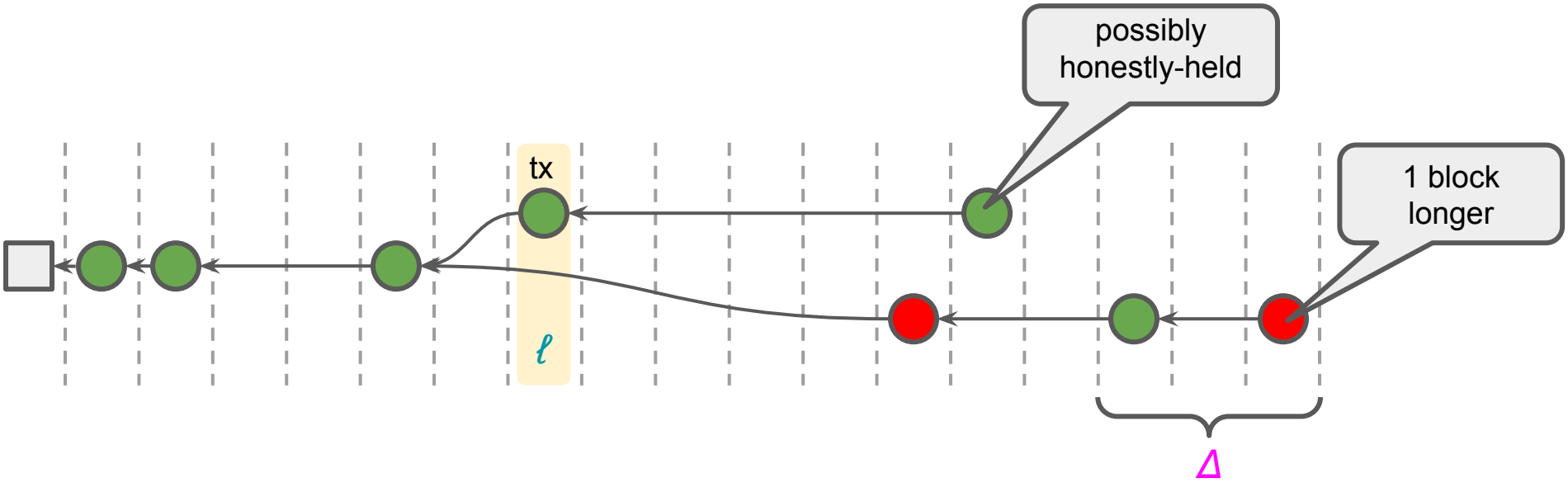

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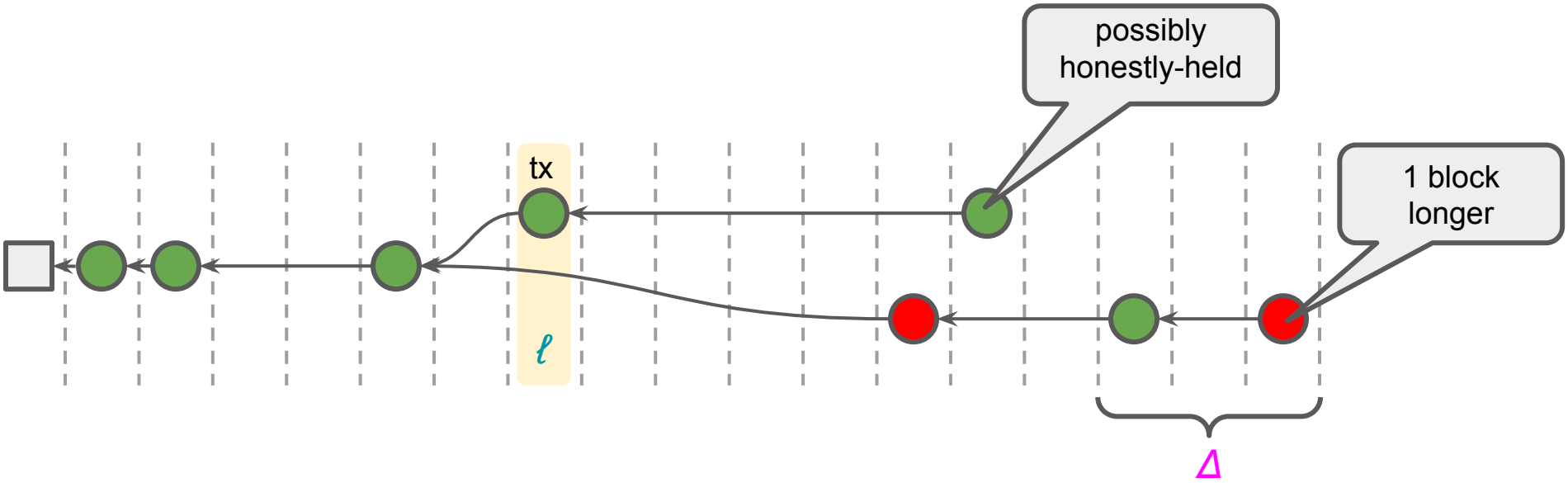
  - max. length advantage of a chain that differs from some "honest view" at ℓ



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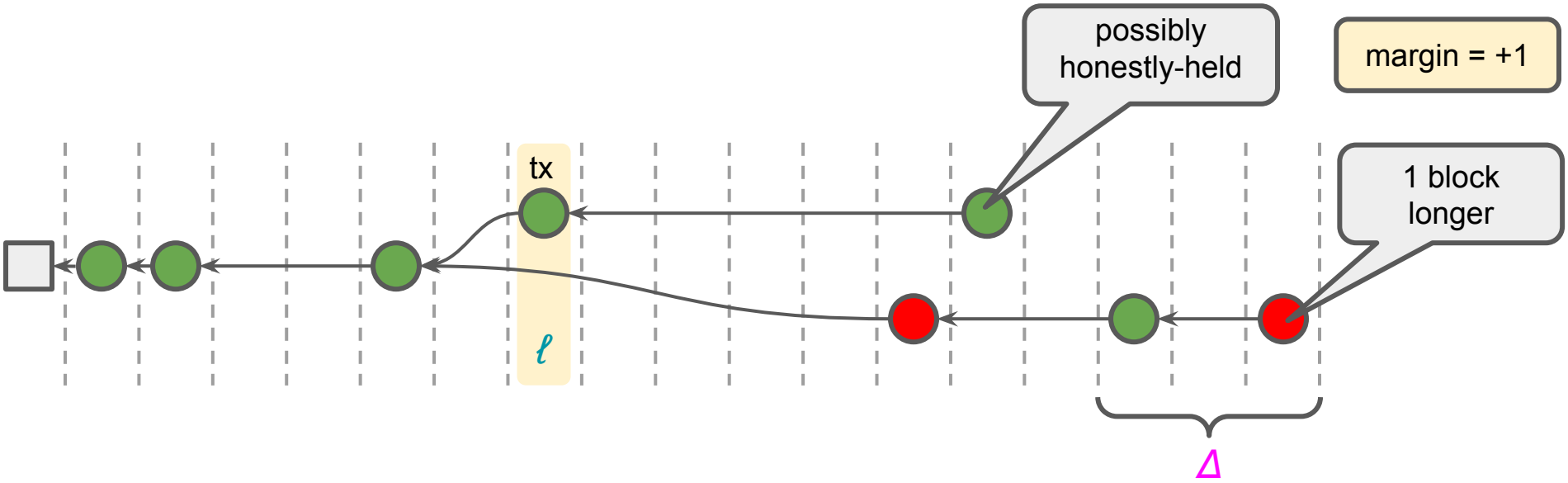

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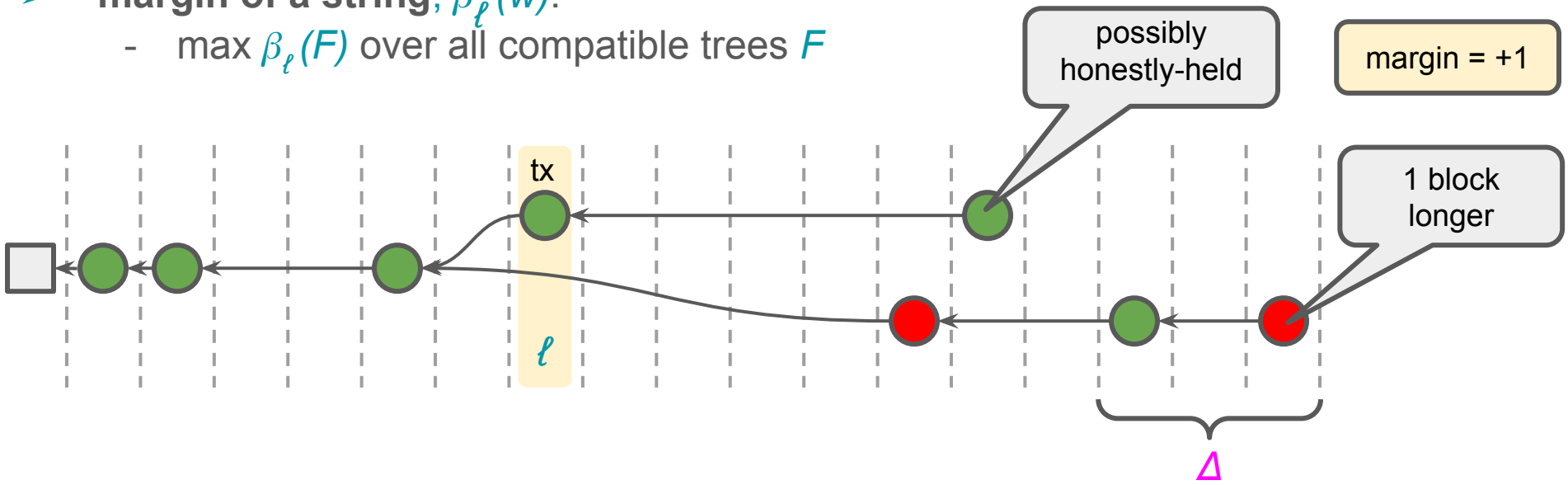

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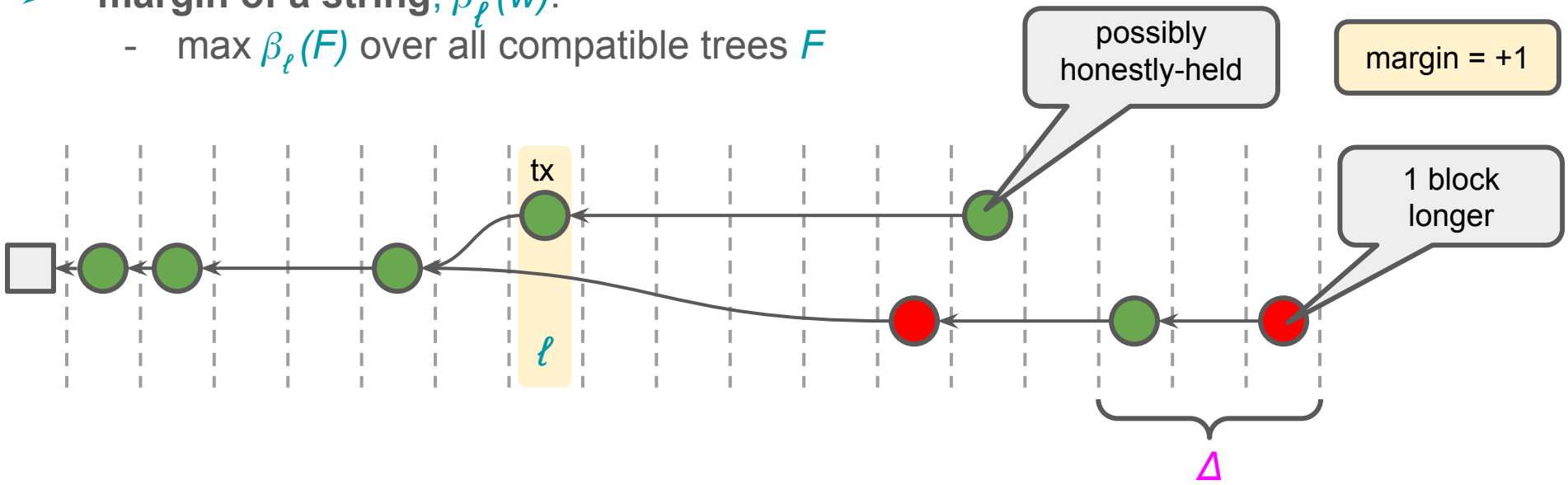

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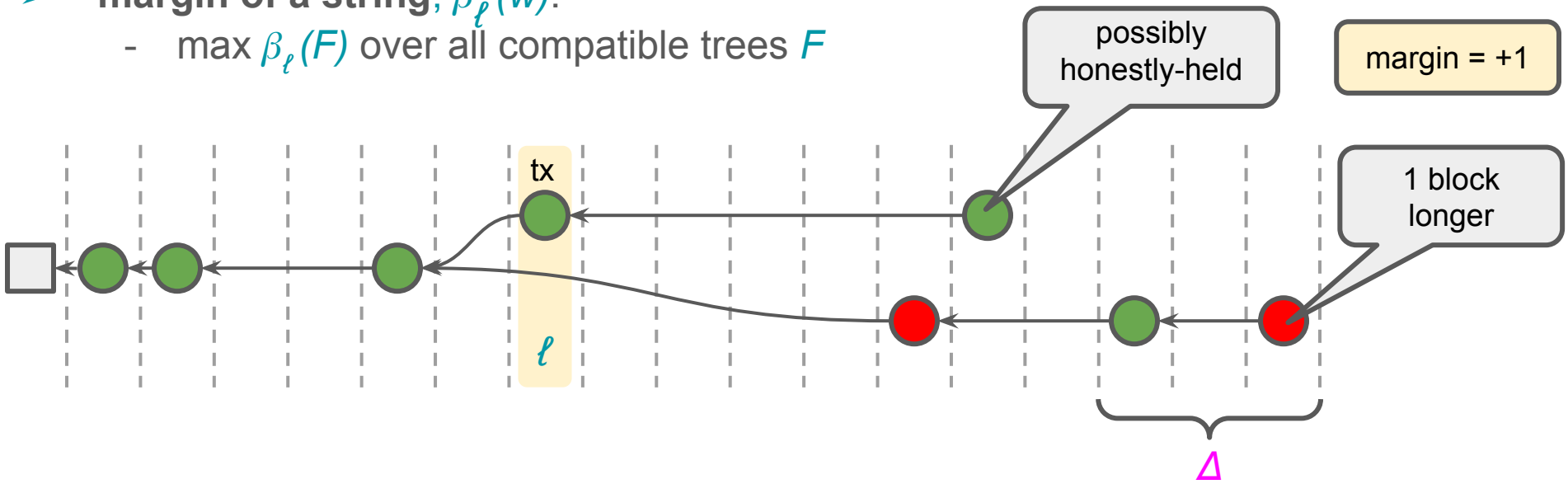

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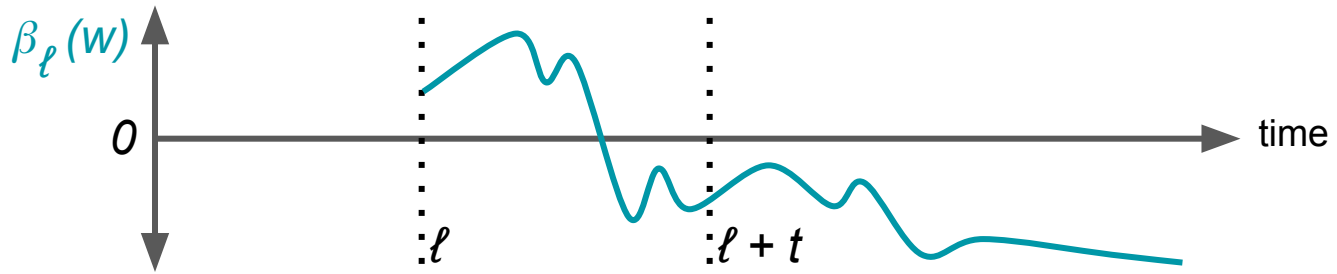

Detour: Longest-Chain Consistency Region

An easier but related question:

For which (r_h , r_a, Δ) do we get *any* eventual consistency?

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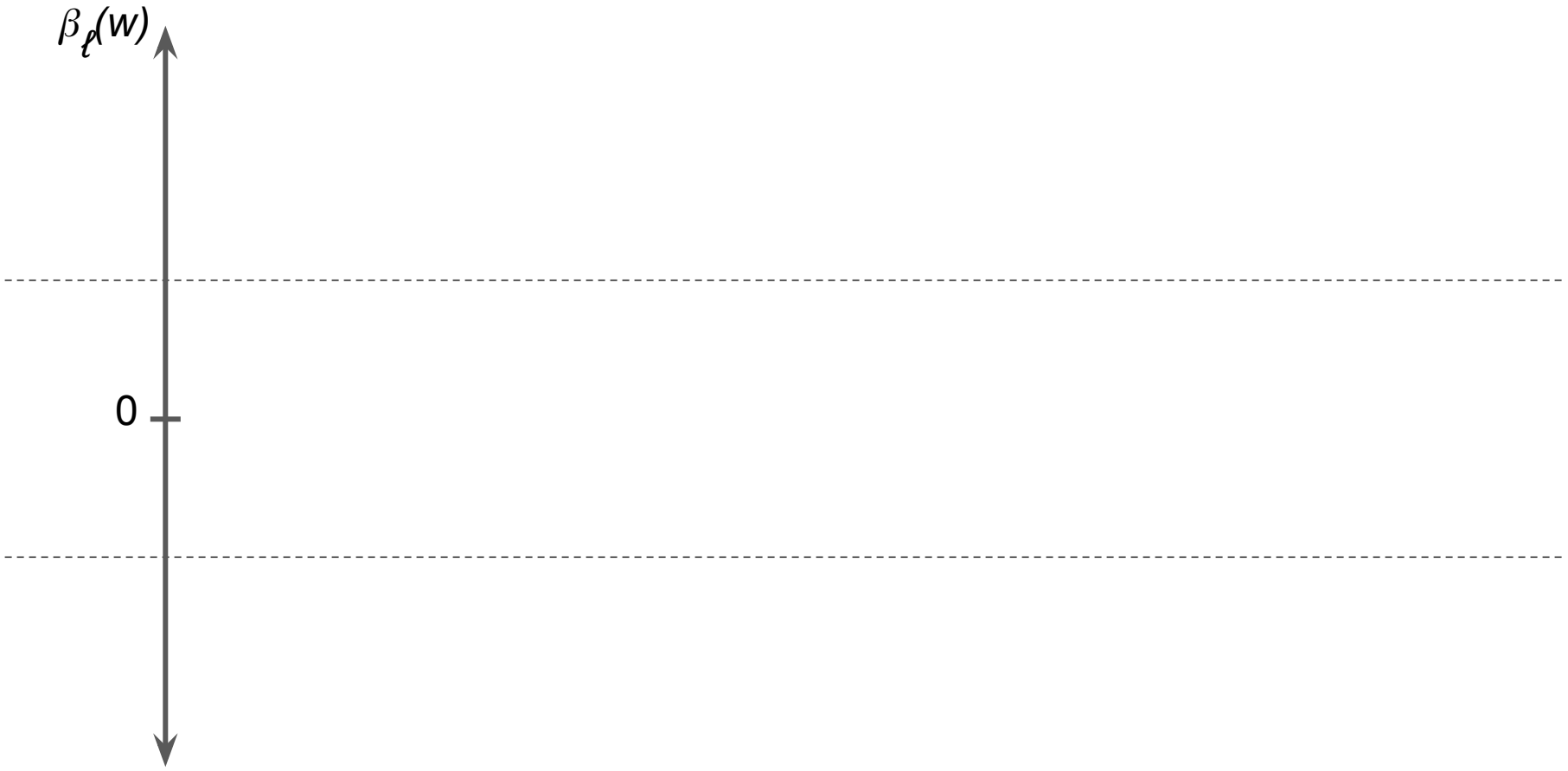
Fully answered in earlier work ([GKR,DKTTVWZ] @ CCS'20):

PoW/PoS longest-chain consensus is secure if $r_a < \frac{1}{\Delta + 1/r_h}$.

Intuition for the Consistency Region



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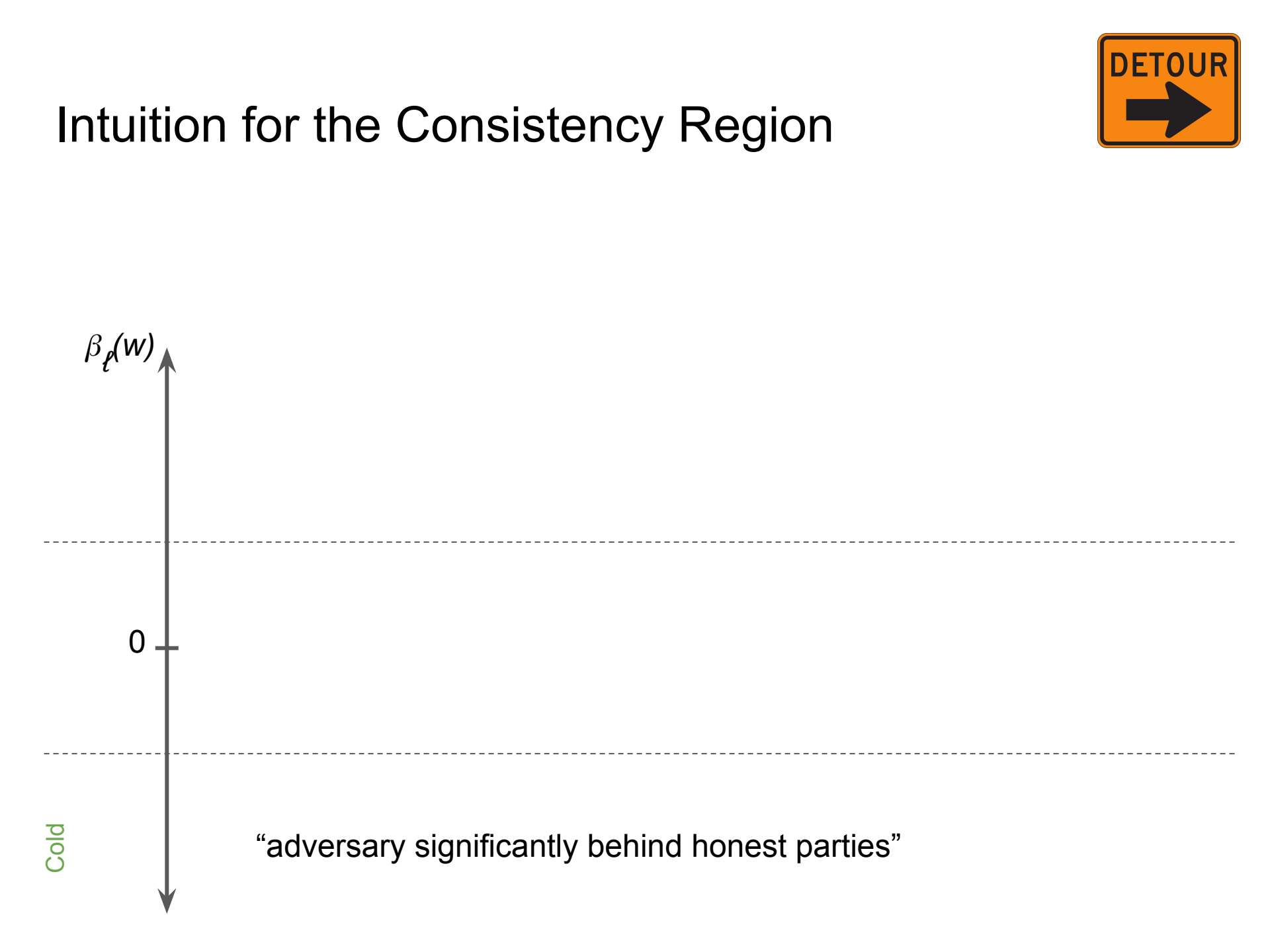

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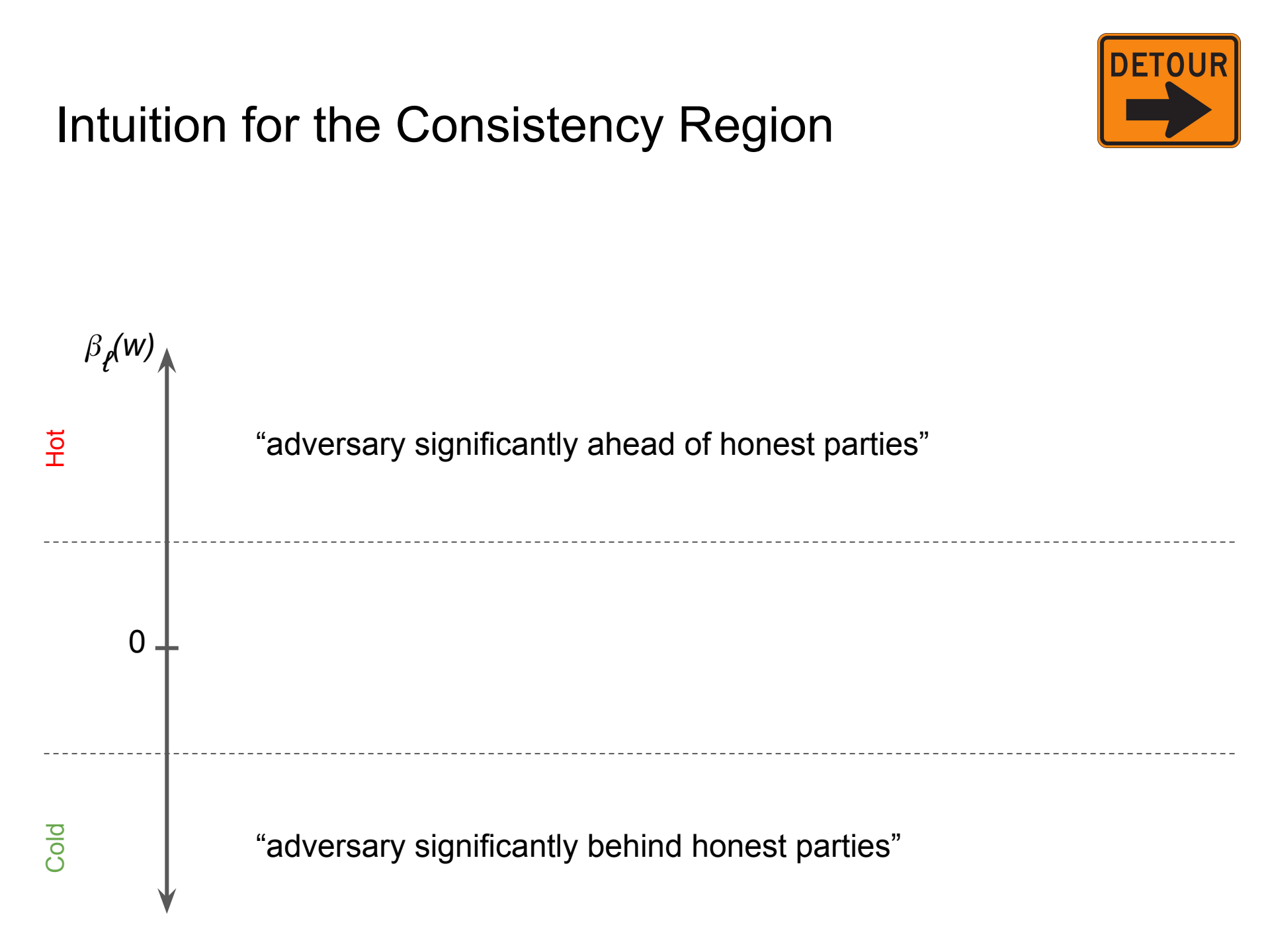

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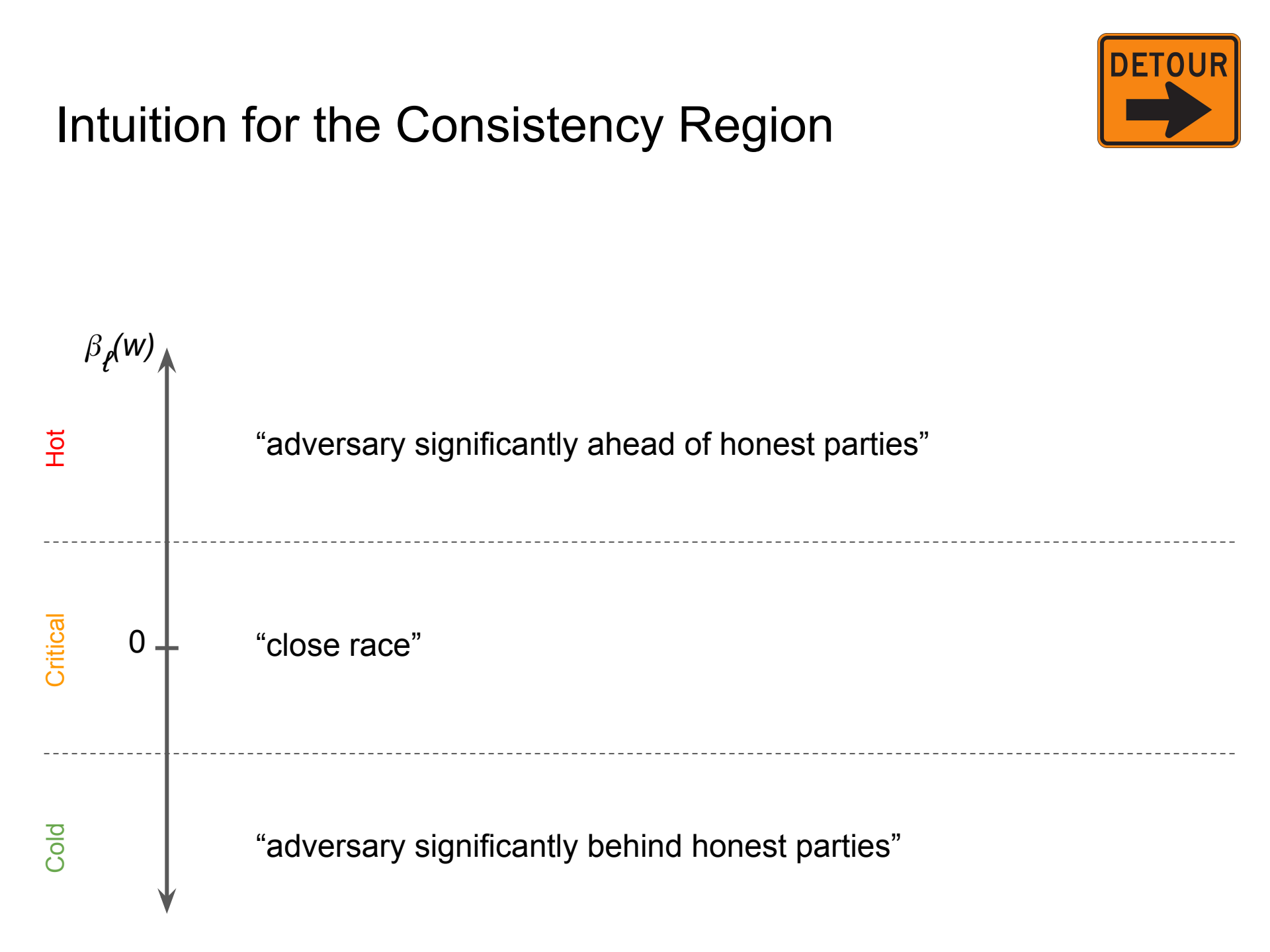

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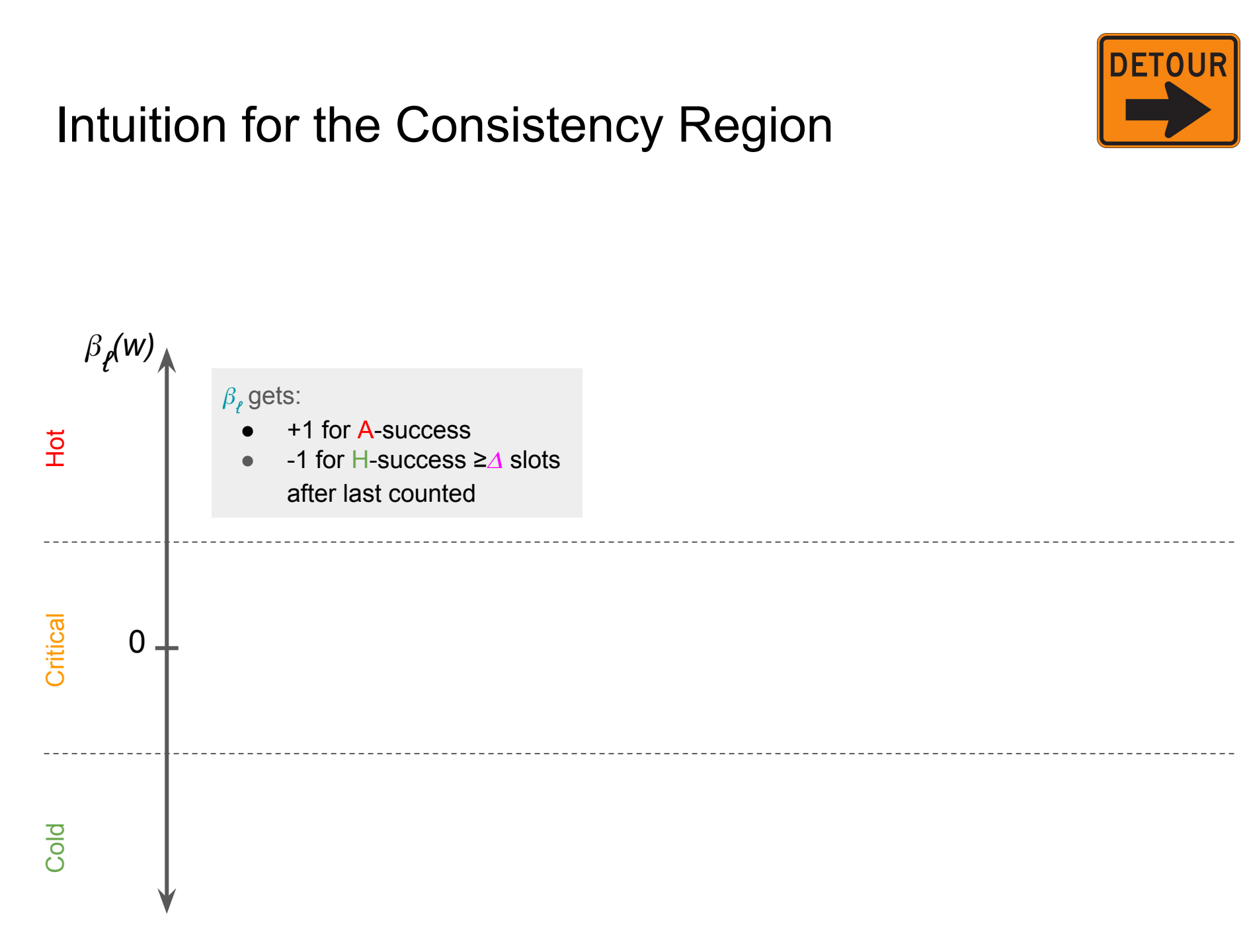

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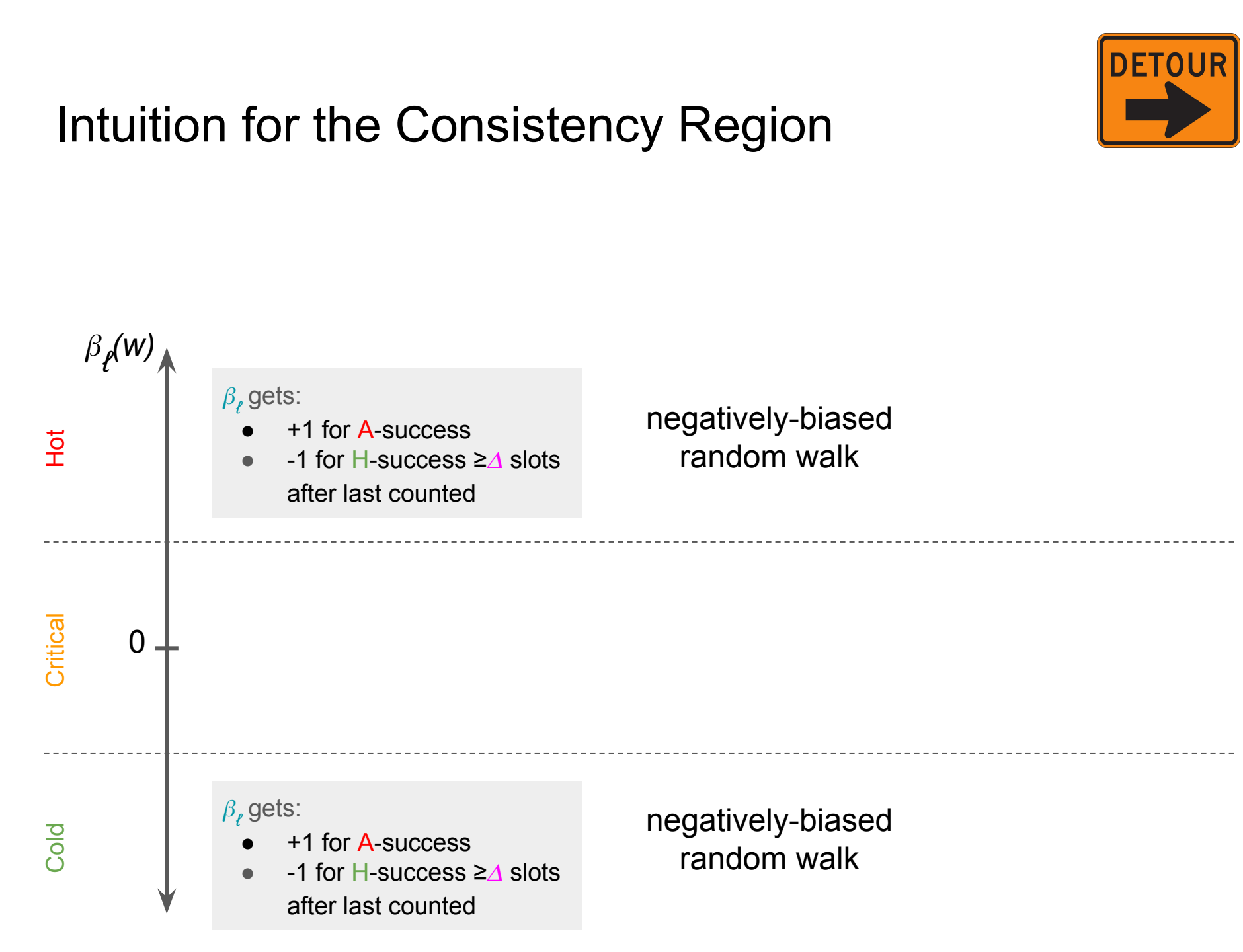

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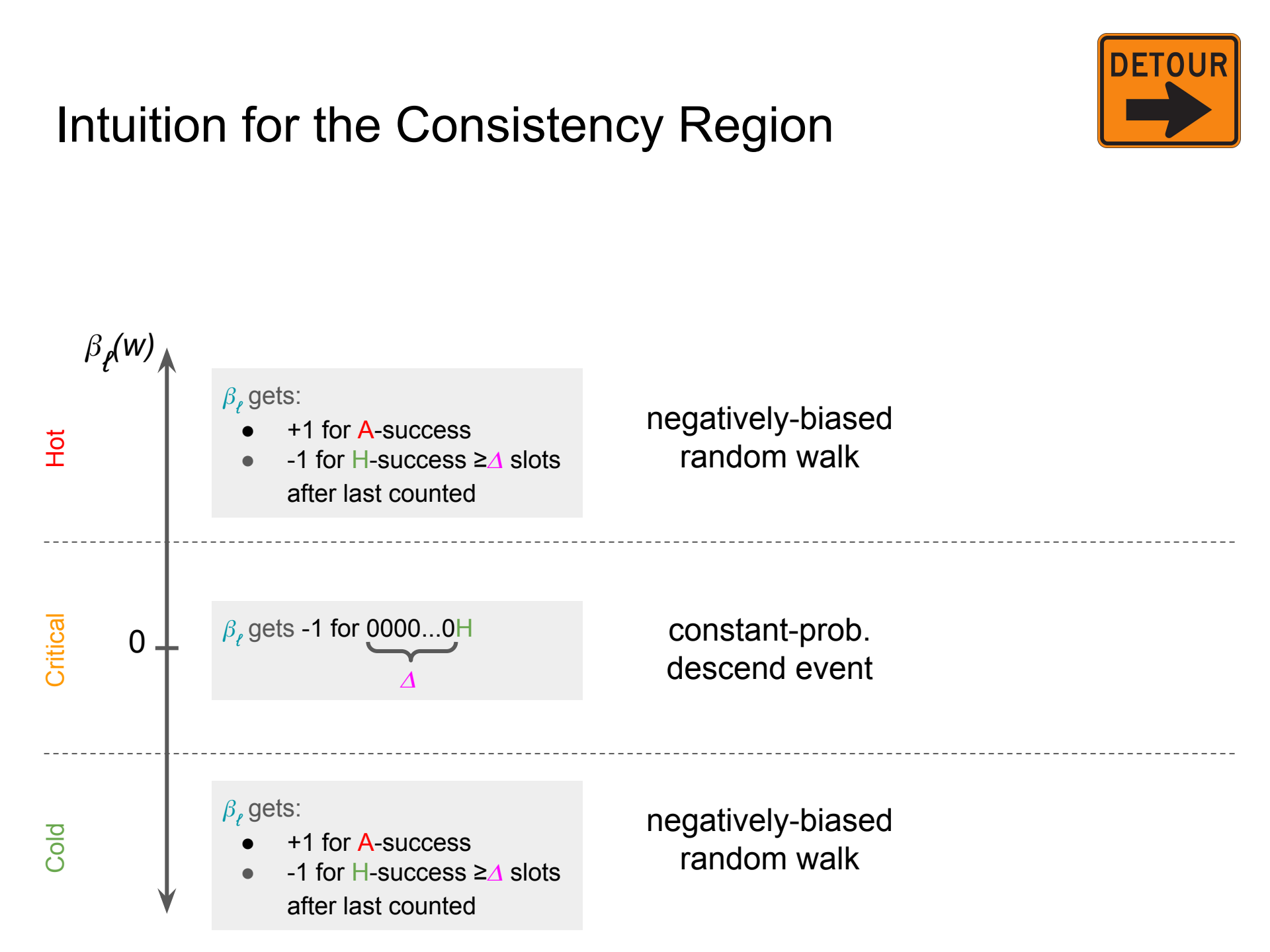

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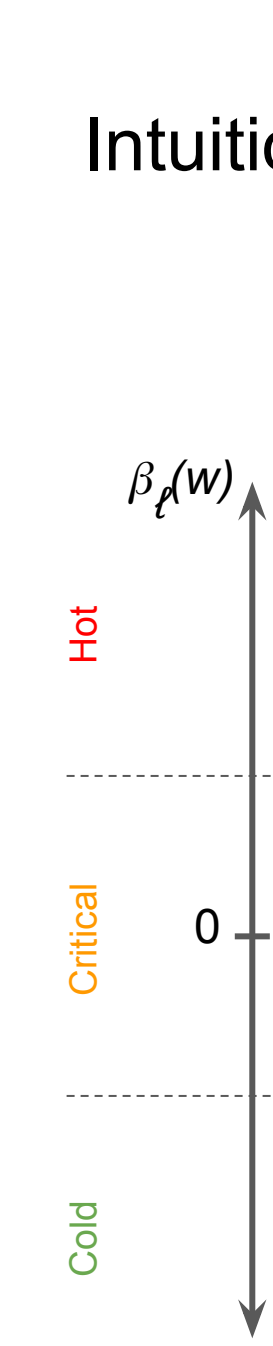

β_ℓ gets:
- +1 for A-success
- -1 for H-success ≥Δ slots after last counted

negatively-biased random walk

- descends with certainty
- leaves quickly

β_ℓ gets -1 for $\underbrace{0000\ldots0}_{\Delta}$H

constant-prob. descend event

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- stays forever with constant prob.
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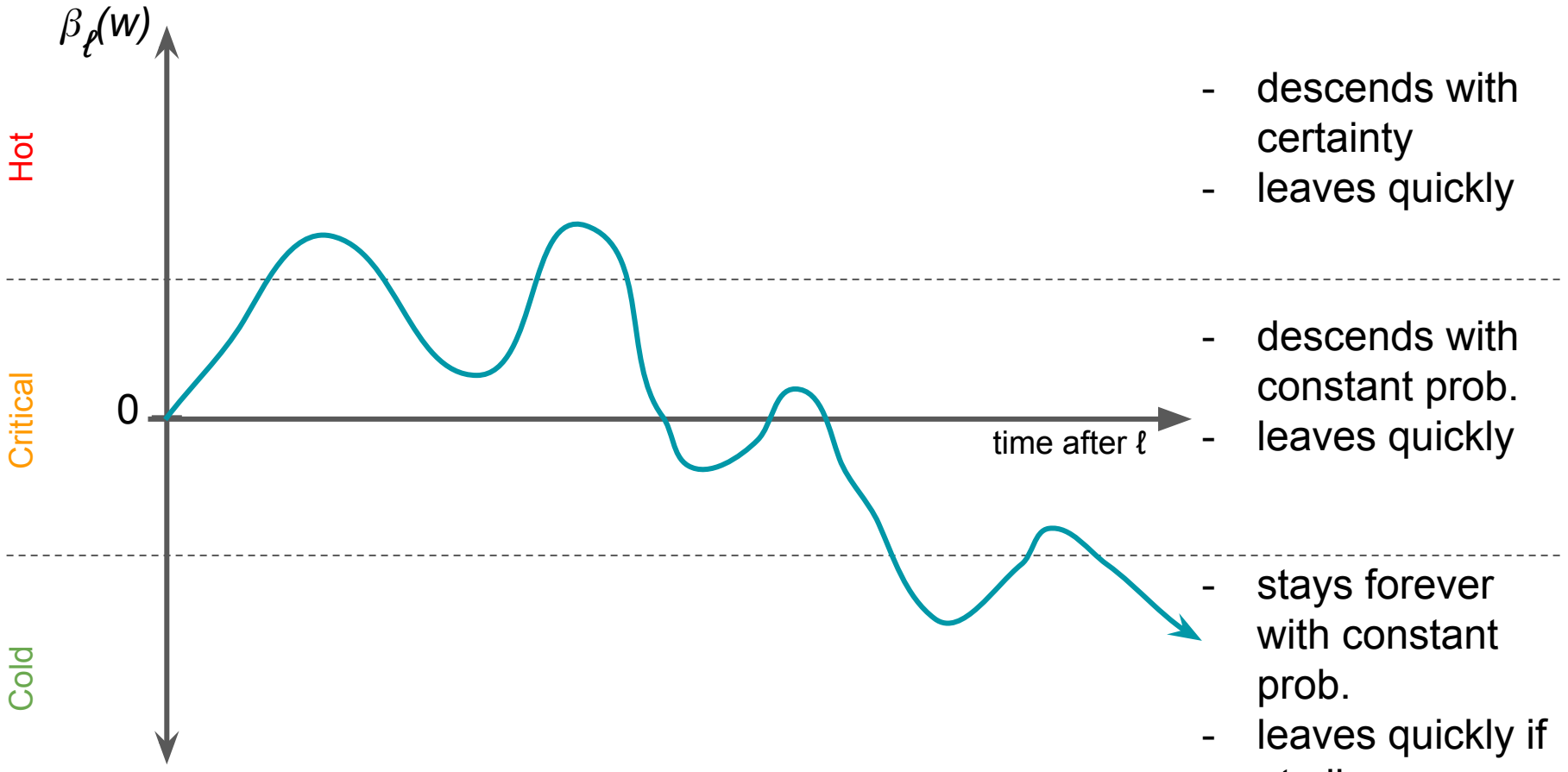
β_ℓ escapes to -∞ at a linear rate
(at time t is at position -Ω(t) except with probability exp(-Ω(t))



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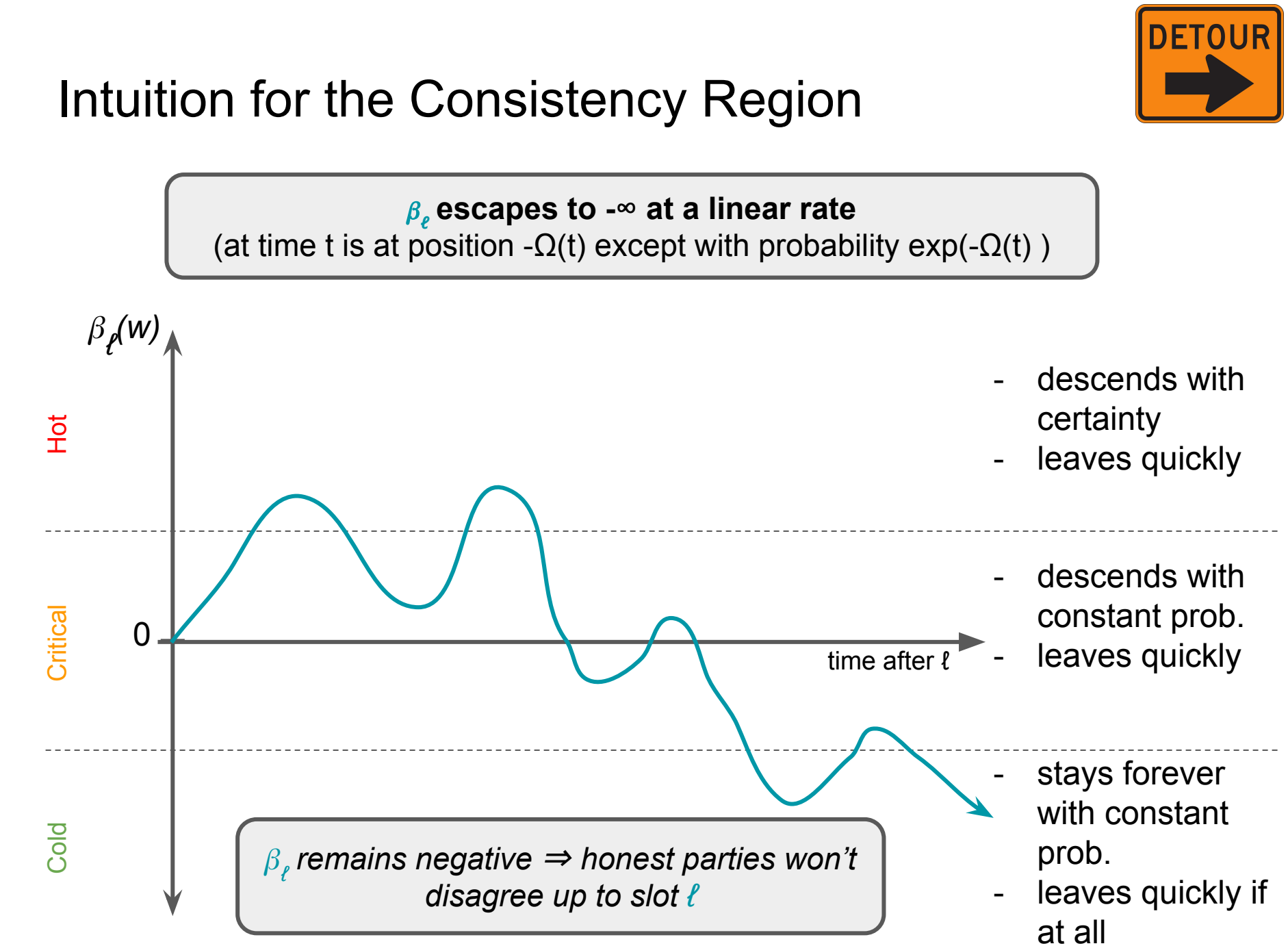

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$\beta_\ell(w)$

Hot

Critical

Cold

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time after ℓ

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β_ℓ remains negative ⇒ honest parties won't disagree up to slot ℓ

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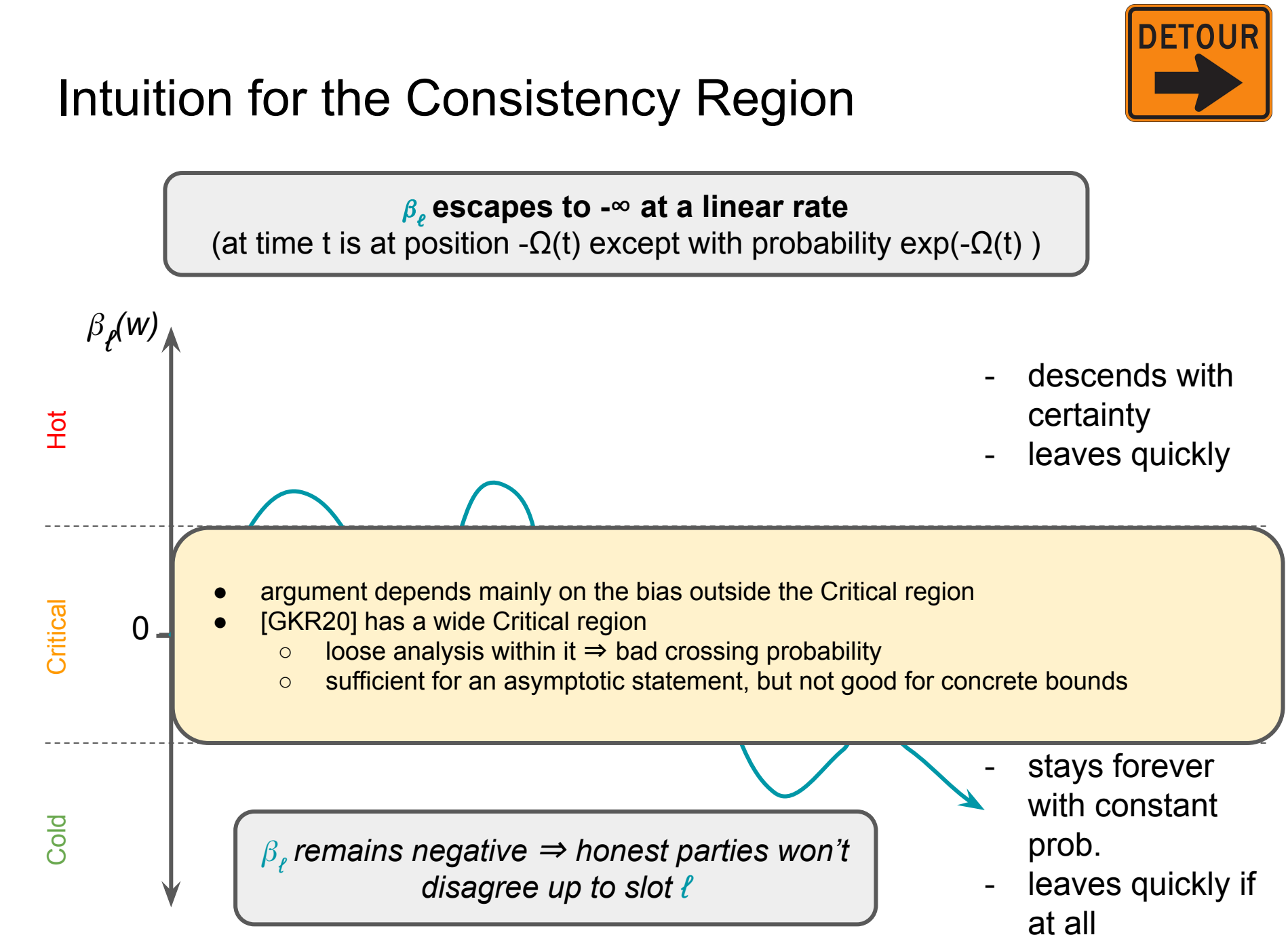

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- [GKR20] has a wide Critical region
  - loose analysis within it ⇒ bad crossing probability
  - sufficient for an asymptotic statement, but not good for concrete bounds

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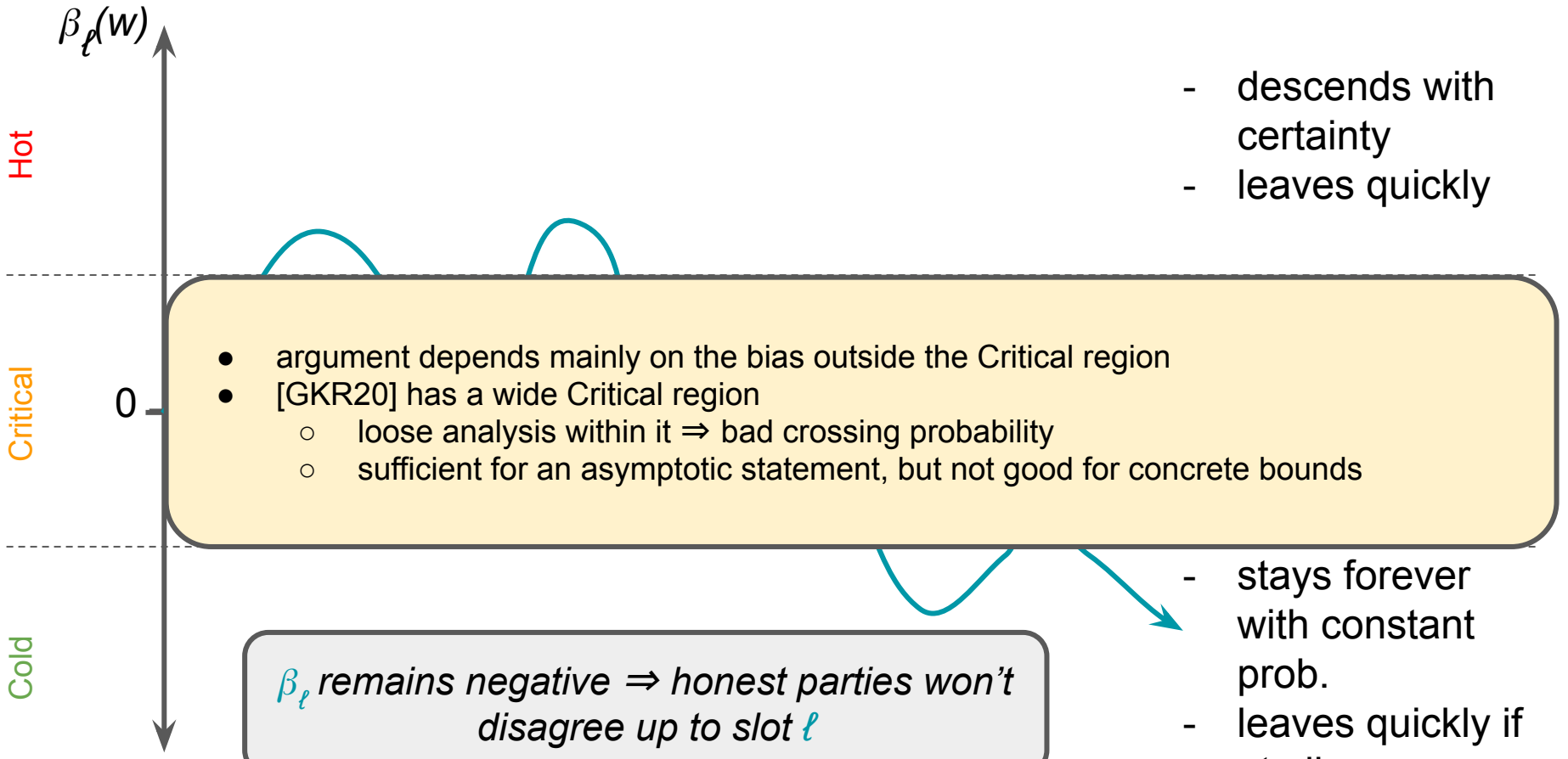

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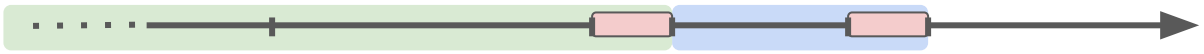

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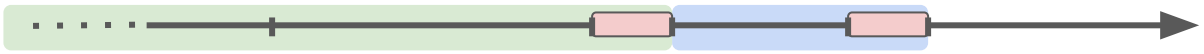
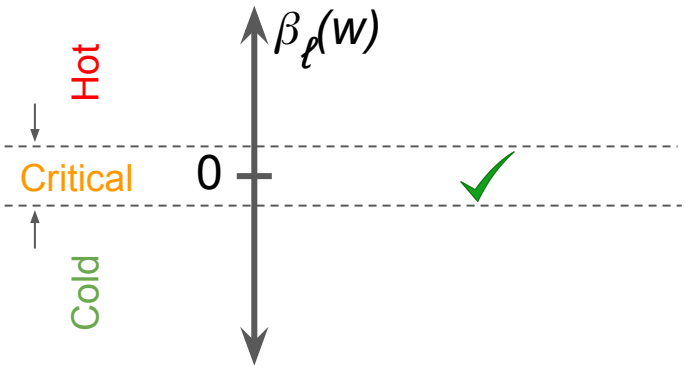
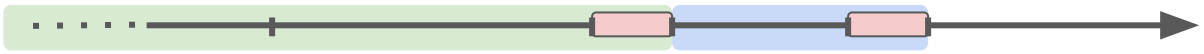


- allows for:
  - a narrow critical region
  - a practically tight analysis of margin while crossing it

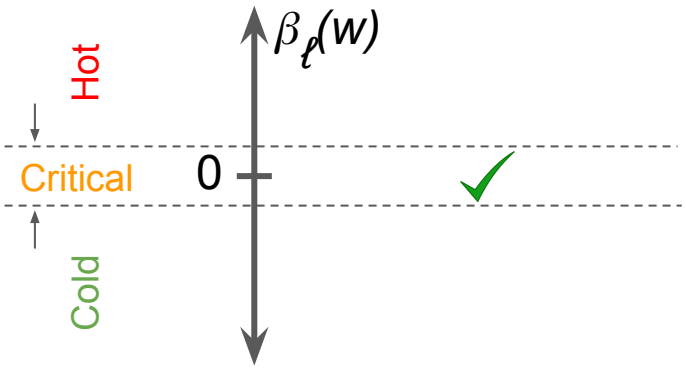


This Work: Technical Overview in a Nutshell

- a novel way to analyze the execution in larger chunks (“**phases**”)



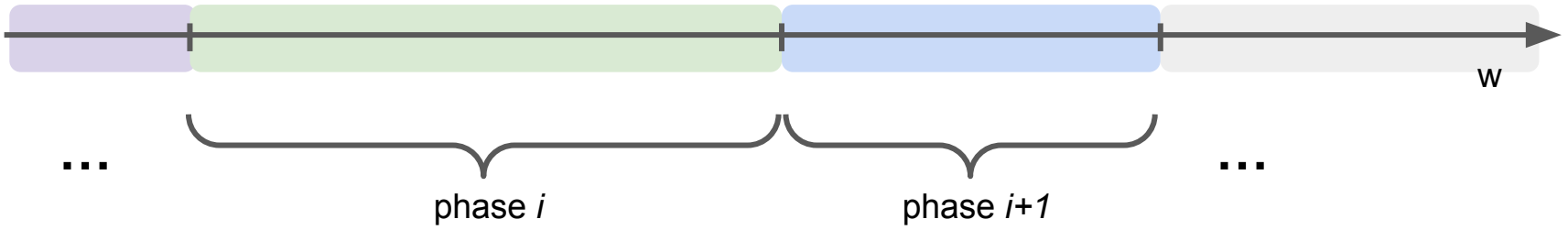
- allows for:
  - a narrow critical region
  - a practically tight analysis of margin while crossing it



- margin recurrences that can be simulated for practical settlement bounds

Splitting Execution into Phases

- consecutive, non-overlapping slot sequences
- **Goal:** honest party producing a block is aware of all honest blocks produced in all previous phases



Splitting Execution into Phases

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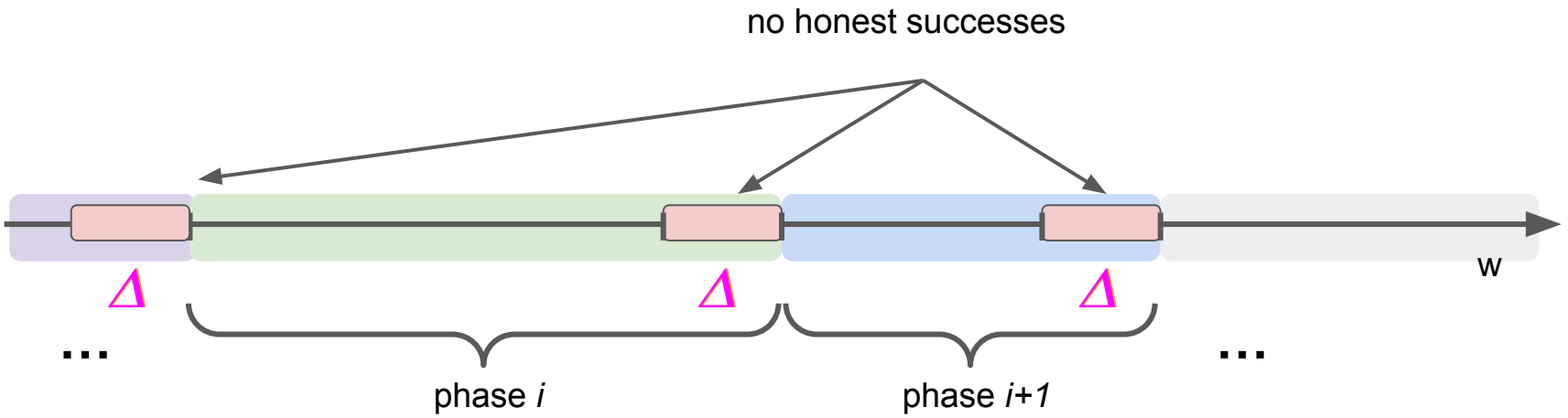
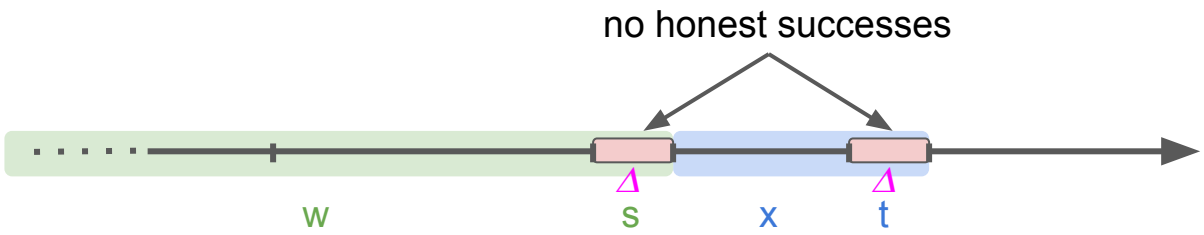
- **Goal:** honest party producing a block is aware of all honest blocks produced in all previous phases
- **Definition:** phase ends with Δ-long honest silence



Analysis Plan: Phase Recurrences



1. Devise recurrences upper-bounding β_ℓ(wsxt) based on
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$$\beta_\ell(\text{wsxt}) \le \beta_\ell(\text{ws}) + F(\text{xt})$$

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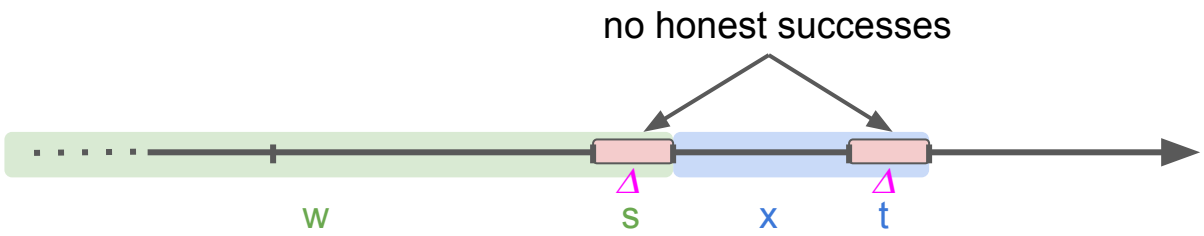

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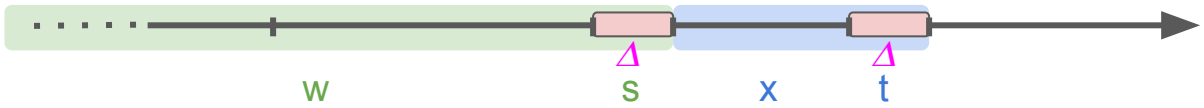
2. Iteratively upper-bound $\beta_\ell(.)$ throughout the full execution

PoW Phase Recurrences: Intuition

- “ideal” recurrence

$$\beta_\ell(\text{wsxt}) = \beta_\ell(\text{ws}) + \ldots - \ldots$$



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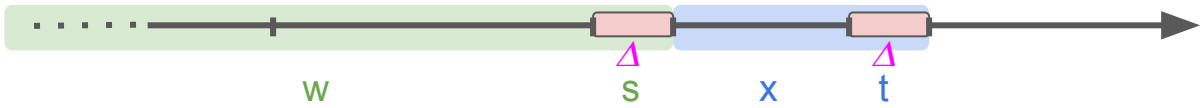

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$$\beta_\ell(\text{wsxt}) = \beta_\ell(\text{ws}) + \#_a(\text{xt}) - h_\Delta(\text{x})$$

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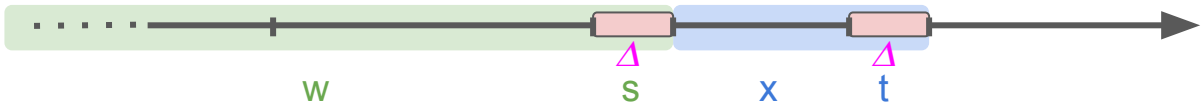
-1 for each “honest depth” increase



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- **Hot & cold regions:** "ideal" recurrence

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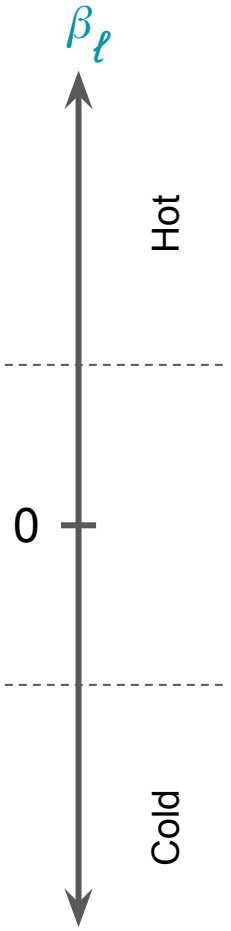

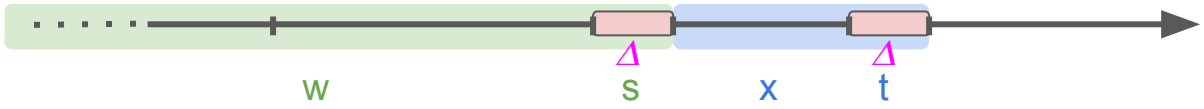

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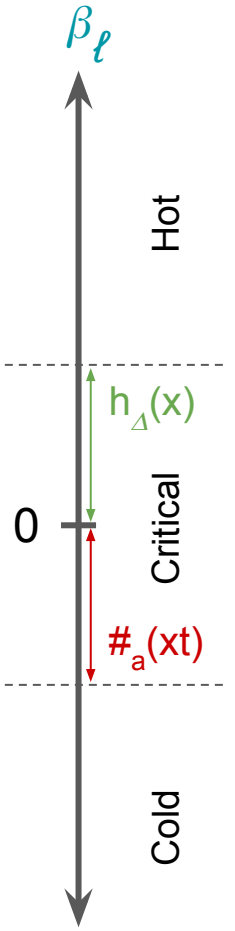

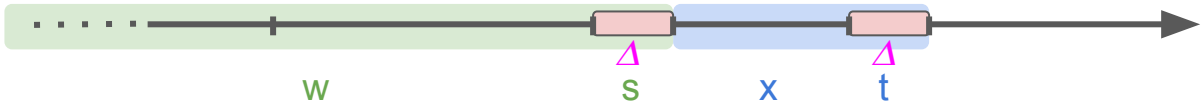

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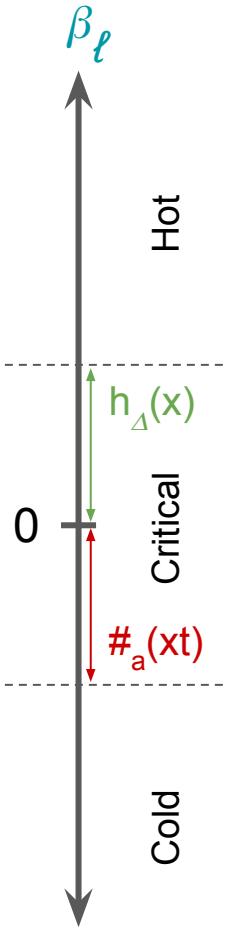

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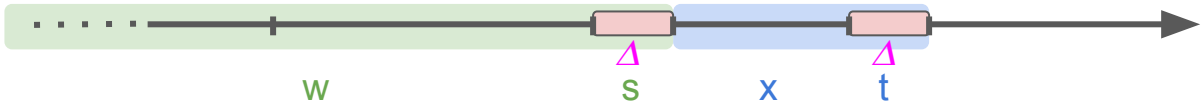

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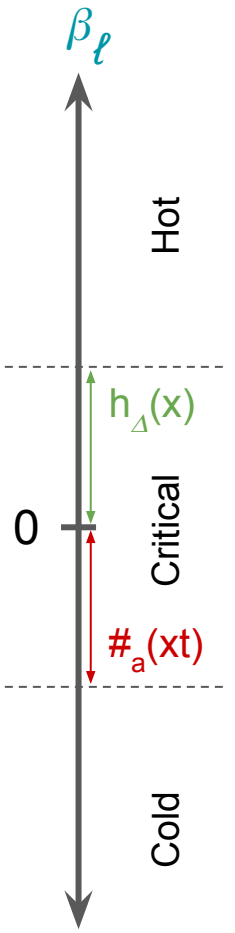

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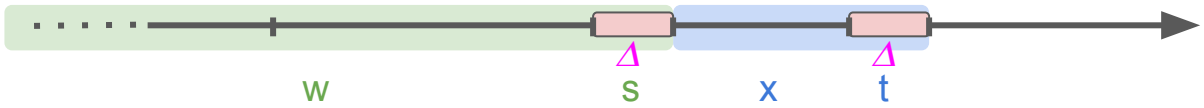

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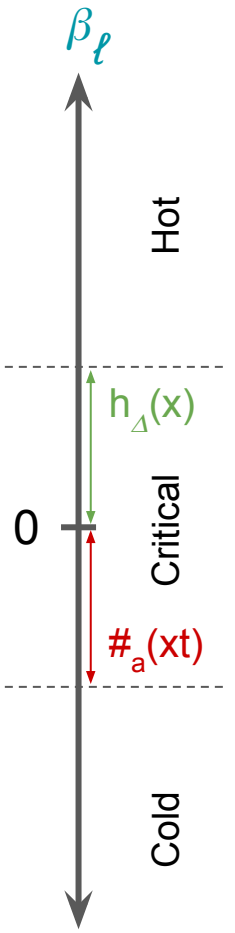

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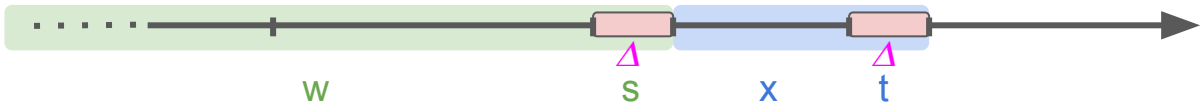

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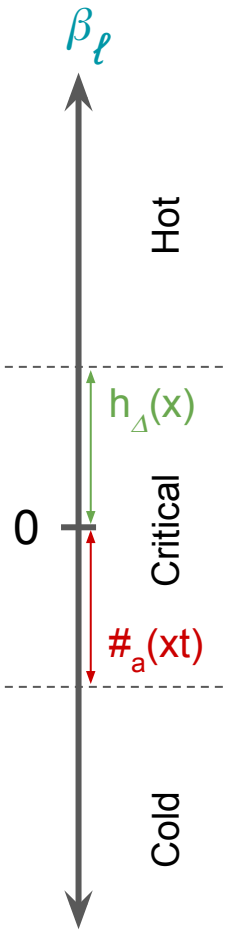

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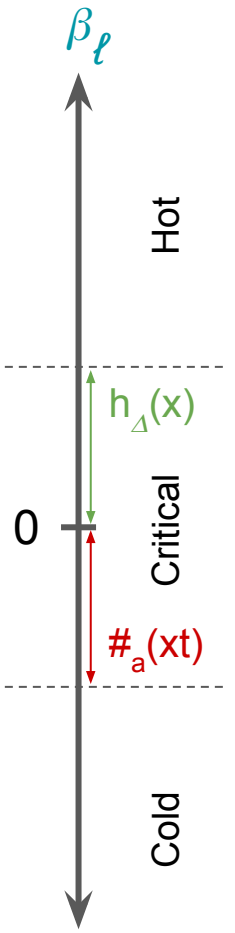

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Proofs: Tree surgery.

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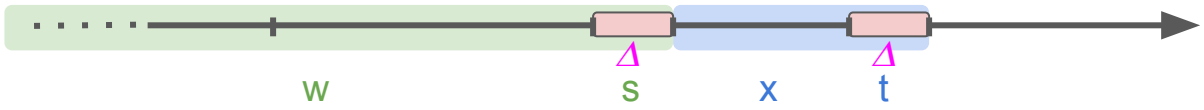
- the recurrence must compute these in tandem
  - determine both values for wsxt based on both values on ws

$$Q(wsxt) \leq Q(ws) + F(xt)$$



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- PoW recurrences give slightly faster settlement

Explicit Results

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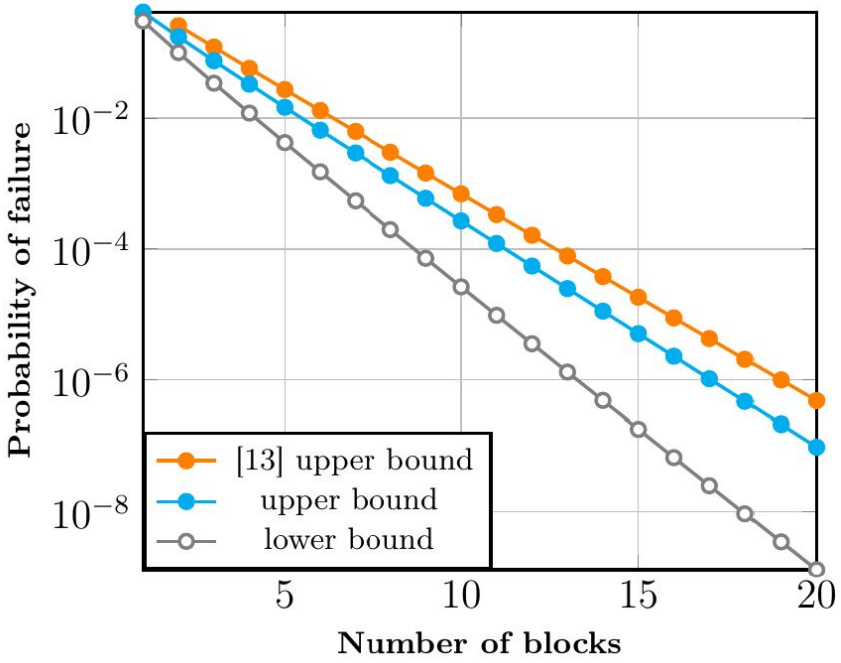
Ethereum (PoW)



- block time: 13 seconds
- Δ = 2 seconds
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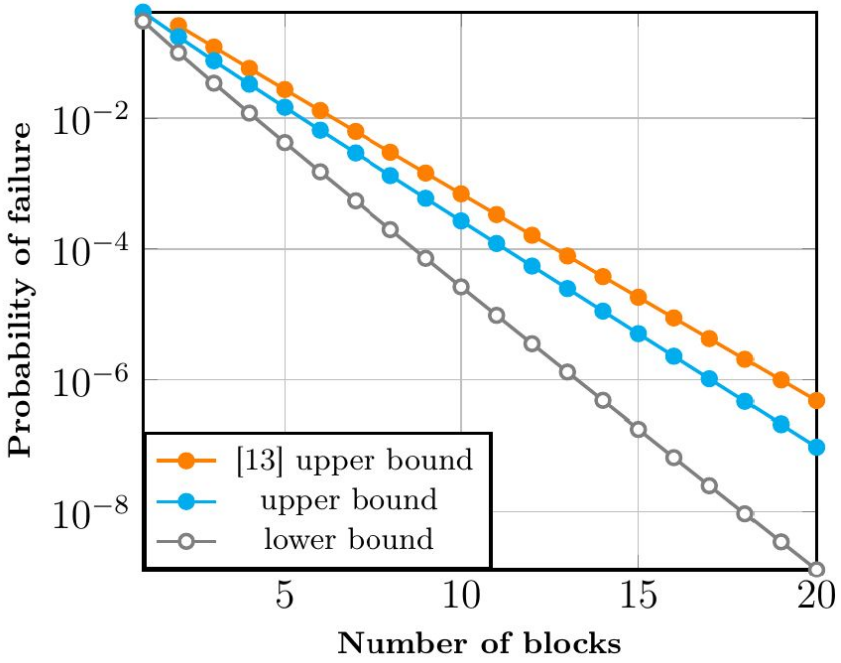

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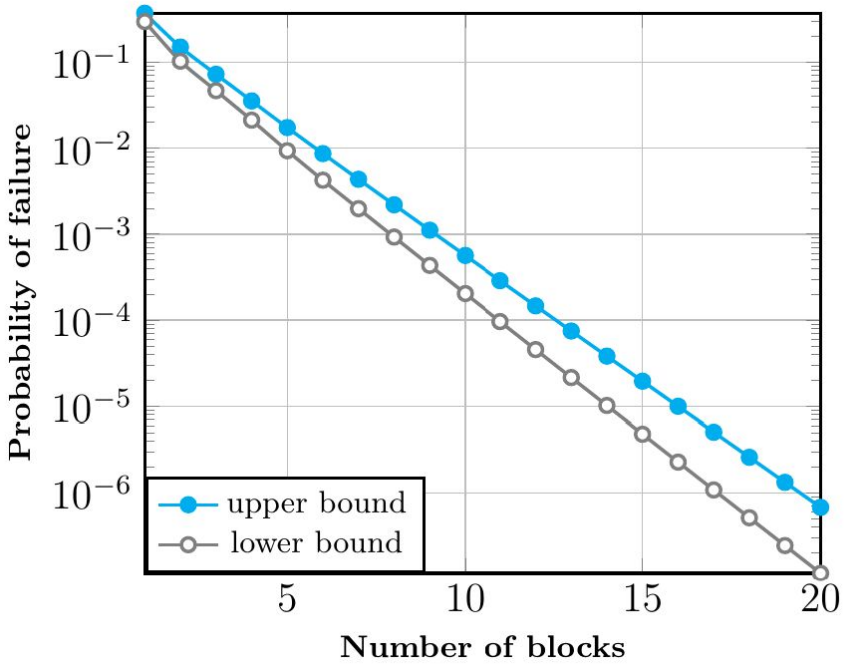
Cardano (PoS)



- block time: 20 seconds
- Δ = 2 seconds
- adversarial stake: 10%

Explicit Results

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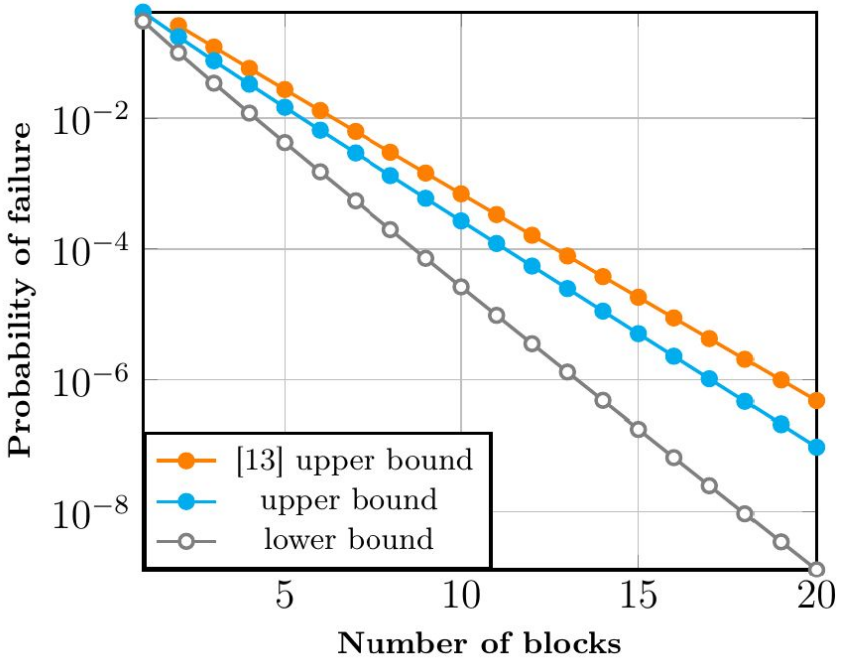

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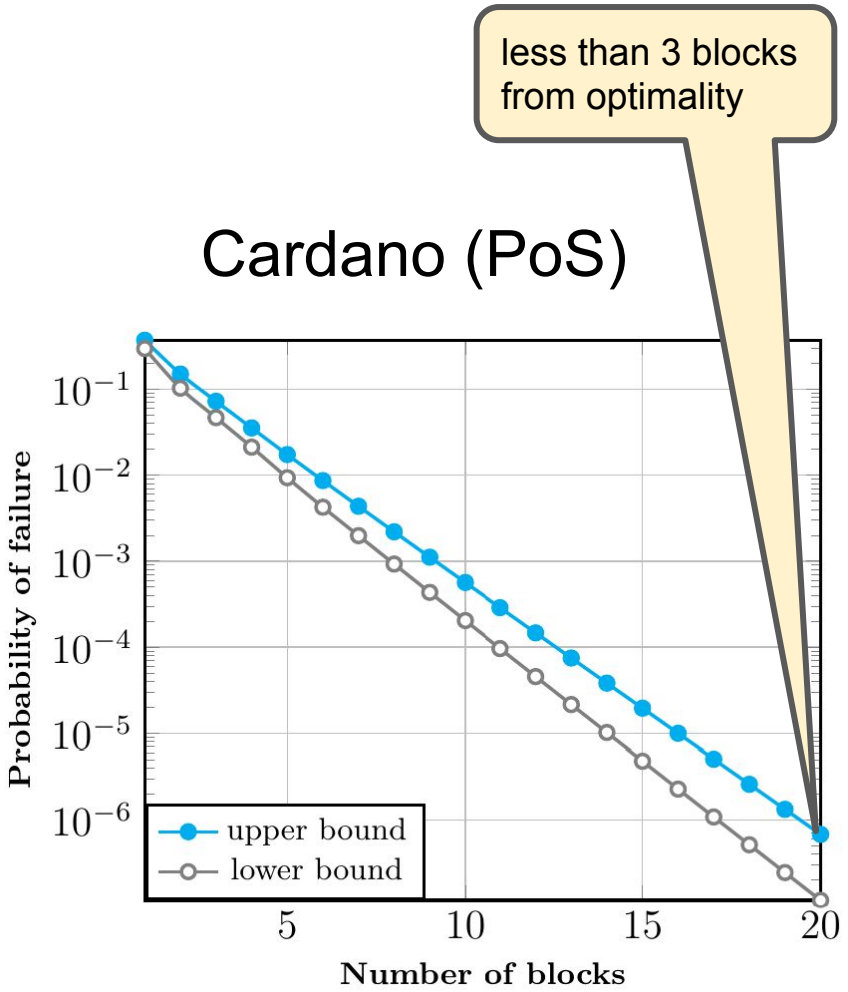

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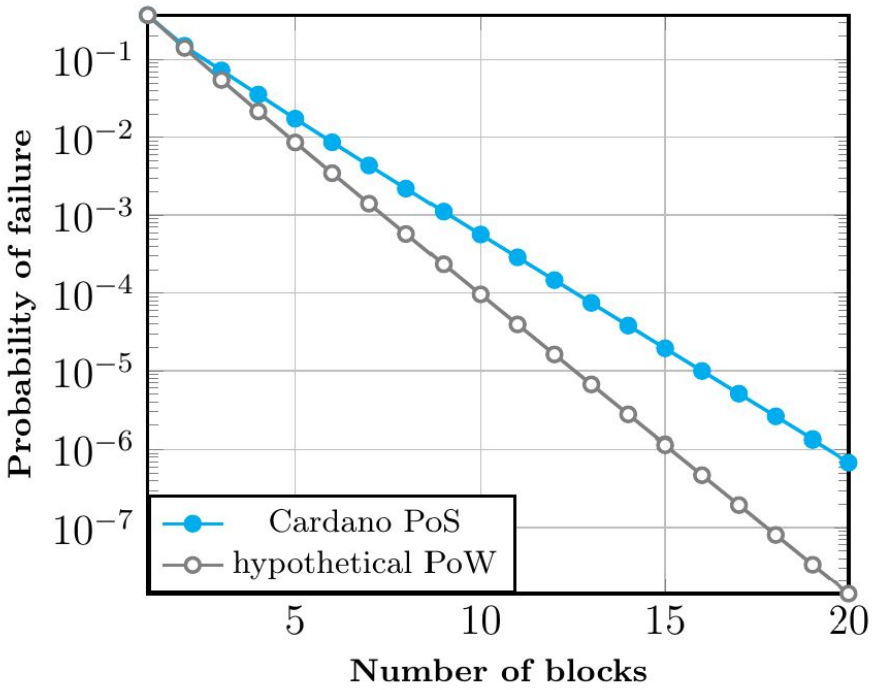
Explicit Results: Comparing PoW to PoS



- block time: 13 seconds
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Thank you for your attention!