

Combined Fault and Leakage Resilience: Composability, Constructions and Compiler

Sebastian Berndt Thomas Eisenbarth Sebastian Faust Marc Gourjon
Maximilian Ortl Okan Seker

Fault and Side-Channel Attacks

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Classical setting: Black-box model



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E.g. plain-text/cipher-text
(M, C)



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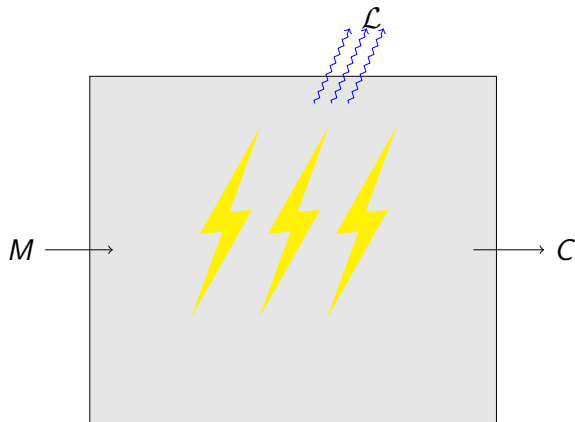
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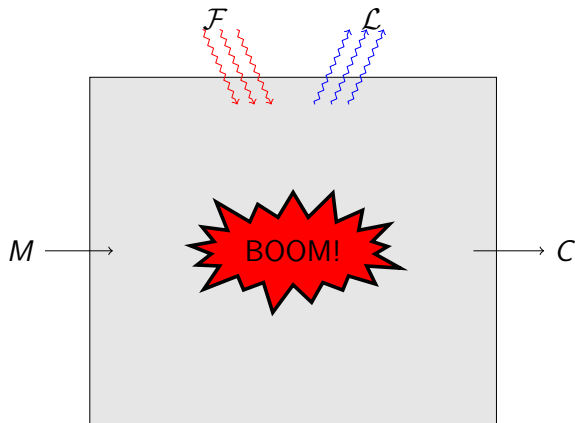
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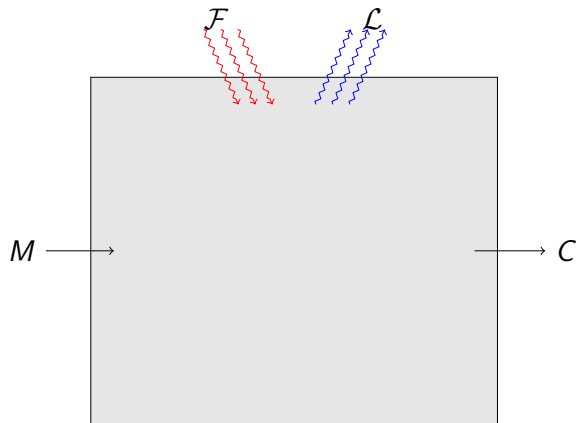
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- ▶ Side-Channel Attack
E.g. Power consumption
- ▶ Fault Attacks
E.g. Electromagnetic Pulses

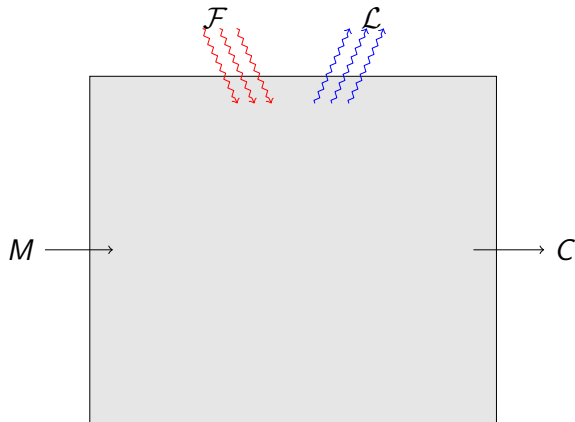


Security Model



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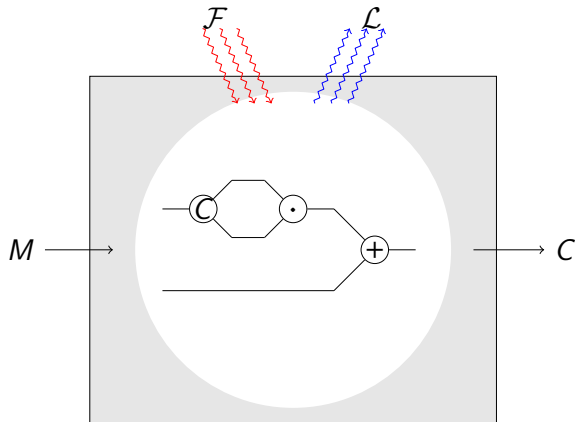
Computational model



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► Arithmetic circuit

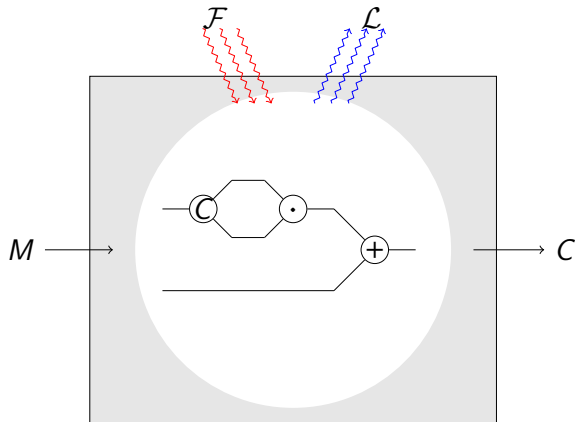


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Adversarial model



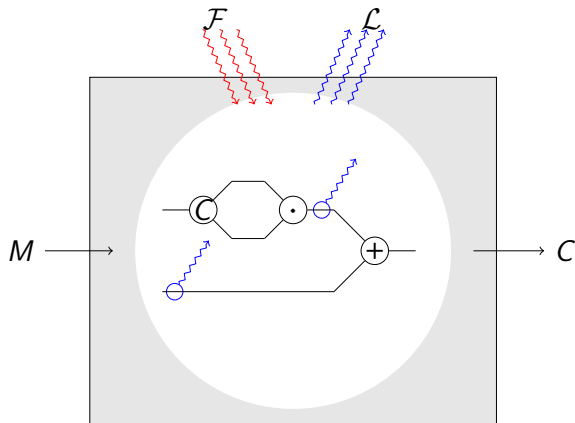
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 d arbitrary wires can be probed



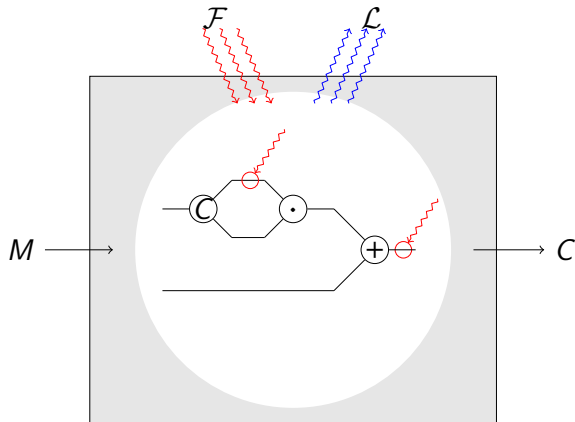
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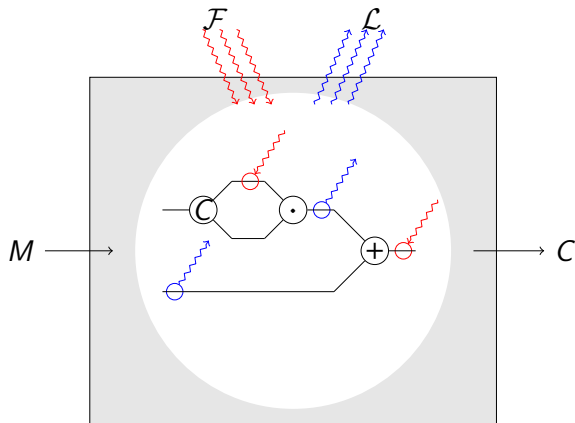
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- ▶ Combined model



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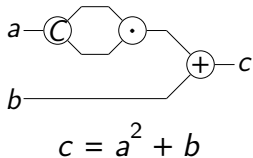
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$O(d + e) \leq O(d \cdot e) \Rightarrow$ This work uses Shamir Secret Sharing

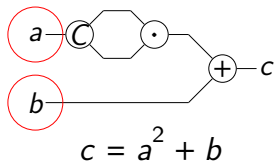
Countermeasure: Compute on Encodings

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- ▶ Encode the inputs a and b



Compiler

→

a_0

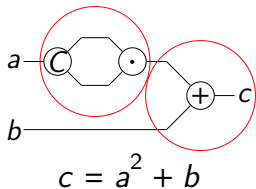
a_1

b_0

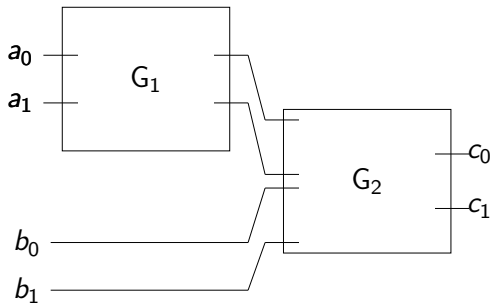
b_1

Countermeasure: Compute on Encodings

- ▶ Encode the inputs a and b
- ▶ Only compute on encodings

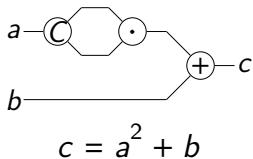


Compiler
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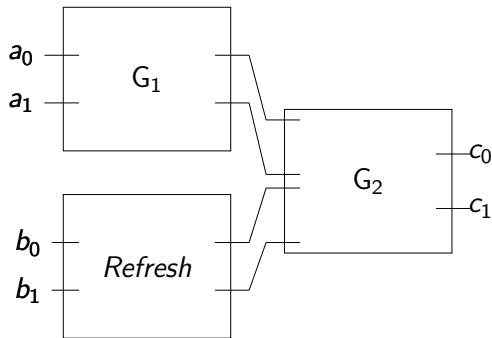


Countermeasure: Compute on Encodings

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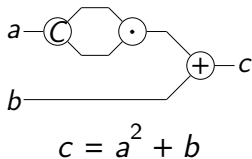


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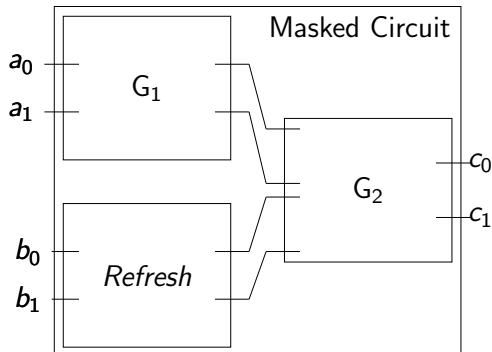


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[SFRES18]

[DN19]

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Improved Security: This work even allows $d/2$ probes in each gadget.

Paper Overview

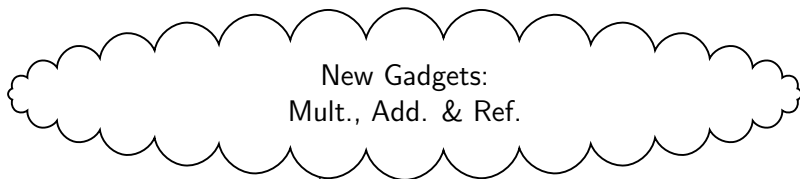
Paper Overview



New Gadgets:
Mult., Add. & Ref.

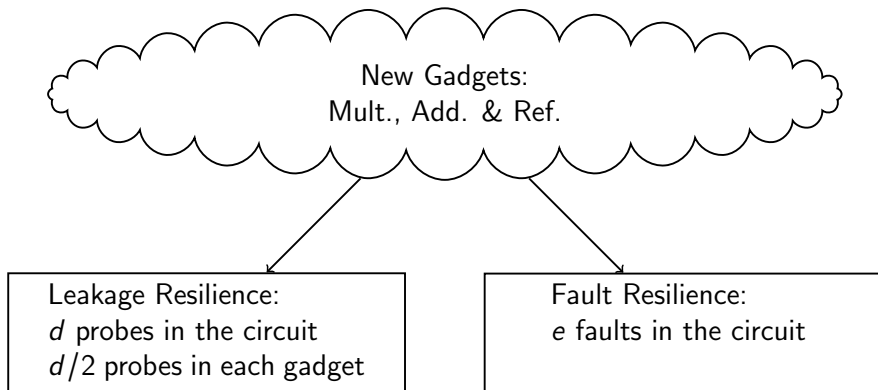
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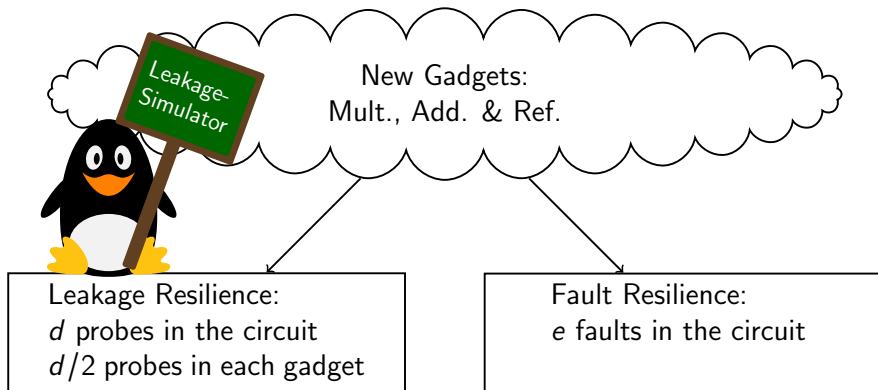


Leakage Resilience:
 d probes in the circuit
 $d/2$ probes in each gadget

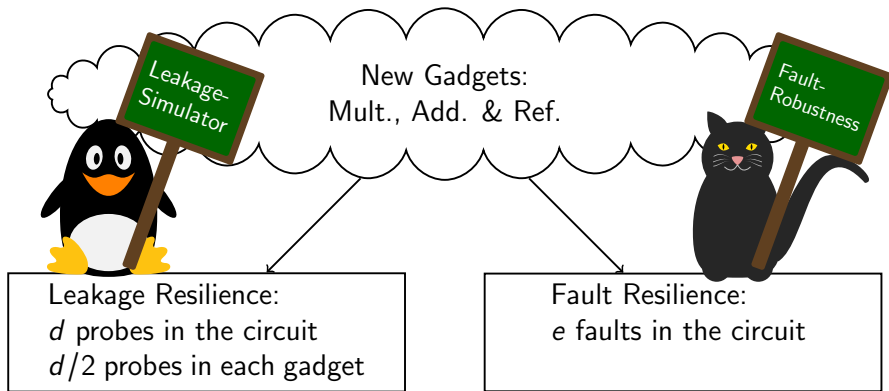
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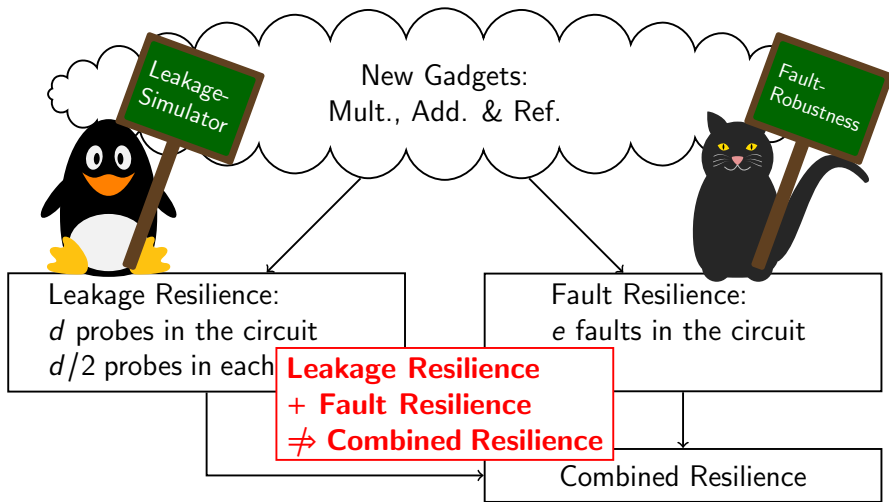
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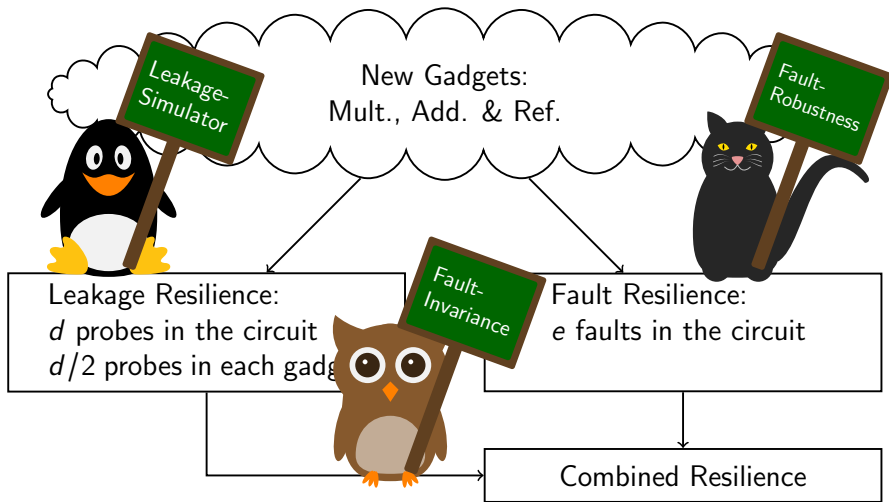
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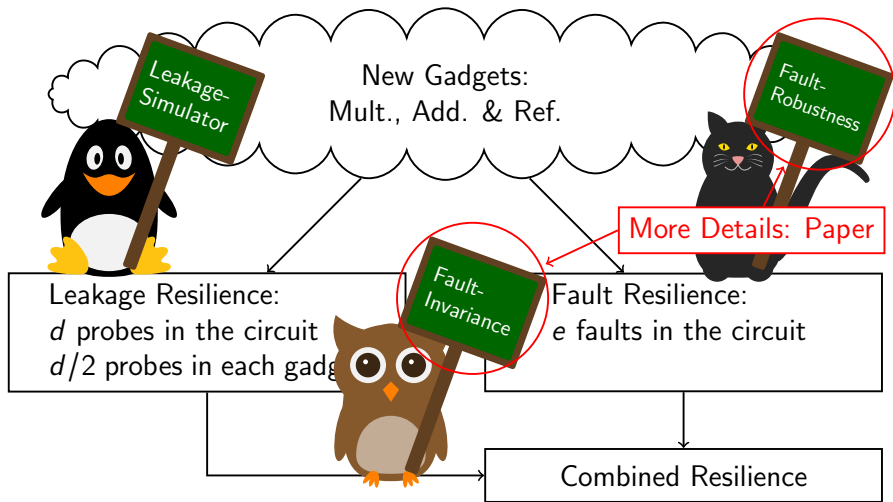
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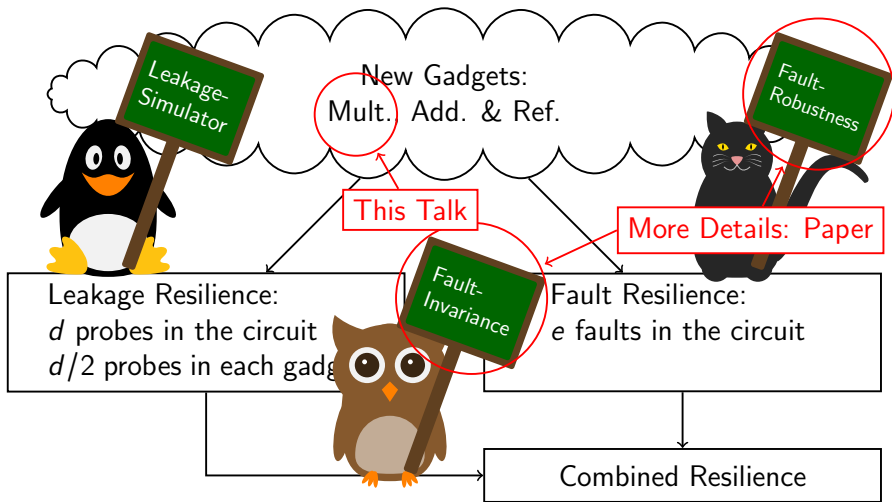
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$r_1 \dots r_d \leftarrow \$ \mathbb{F}$

for $j = 0, \dots, n - 1$:

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return $[[s]]_d^n := (s_0, \dots, s_{n-1})$

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- ▶ Leakage Resilience: Any set of d shares s_j is uniformly, independently distributed.
- ▶ Fault Resilience: If only e shares of $d + e + 1$ shares are faulted the polynomial has a degree $> d$!

Fixed SotA Compiler for Affine Circuits

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▶ Compute $c_i \leftarrow a_i + b_i$

▶ Result $\llbracket c \rrbracket_d^n = \llbracket a + b \rrbracket_d^n$

$$\left. \begin{array}{l} \text{Addition of } \llbracket a \rrbracket_d^n \text{ and } \llbracket b \rrbracket_d^n \\ \text{▶ Compute } c_i \leftarrow a_i + b_i \\ \text{▶ Result } \llbracket c \rrbracket_d^n = \llbracket a + b \rrbracket_d^n \end{array} \right\} \left\{ \begin{array}{l} (\sum_{i=1}^d r_i x^i + a) + (\sum_{i=1}^d r_i x^i + b) \\ = \sum_{i=1}^d (r_i + r_i') x^i + (a + b) \end{array} \right.$$

Refresh of $\llbracket a \rrbracket_d^n$

▶ Generate $\llbracket b \rrbracket_d^n \leftarrow \text{Enc}(0)$

▶ Compute $\llbracket c \rrbracket_d^n = \llbracket a + b \rrbracket_d^n$

$$\left. \begin{array}{l} \text{Refresh of } \llbracket a \rrbracket_d^n \\ \text{▶ Generate } \llbracket b \rrbracket_d^n \leftarrow \text{Enc}(0) \\ \text{▶ Compute } \llbracket c \rrbracket_d^n = \llbracket a + b \rrbracket_d^n \end{array} \right\} \left\{ \begin{array}{l} \text{Probing Attack: Our Paper} \\ \text{Fix: } d \text{ Refreshes in a row} \end{array} \right.$$

Fixed SotA Compiler with $n = 2d + e + 1$

Fixed SotA Compiler with $n = 2d + e + 1$

Multiplication of $\llbracket a \rrbracket_d^n$ and $\llbracket b \rrbracket_d^n$

Fixed SotA Compiler with $n = 2d + e + 1$

Multiplication of $\llbracket a \rrbracket_d^n$ and $\llbracket b \rrbracket_d^n$

- Compute $c_i \leftarrow a_i \cdot b_i$

Fixed SotA Compiler with $n = 2d + e + 1$

Multiplication of $\llbracket a \rrbracket_d^n$ and $\llbracket b \rrbracket_d^n$

- ▶ Compute $c_i \leftarrow a_i \cdot b_i$
- ▶ Result $\llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n$

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Fixed SotA Compiler with $n = 2d + e + 1$

Multiplication of $\llbracket a \rrbracket_d^n$ and $\llbracket b \rrbracket_d^n$

- ▶ Compute $c_i \leftarrow a_i \cdot b_i$
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$$\left. \begin{array}{l} \text{Multiplication of } \llbracket a \rrbracket_d^n \text{ and } \llbracket b \rrbracket_d^n \\ \text{▶ Compute } c_i \leftarrow a_i \cdot b_i \\ \text{▶ Result } \llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n \\ \text{▶ Reduce } \llbracket c \rrbracket_{2d}^n \text{ down to } \llbracket c \rrbracket_d^n \end{array} \right\} \left\{ \begin{array}{l} (\sum_{i=1}^d r_i x^i + a) \cdot (\sum_{i=1}^d r_i x^i + b) \\ = \sum_{i=1}^{2d} (r_i'') x^i + (a \cdot b) \end{array} \right.$$

Fixed SotA Compiler with $n = 2d + e + 1$

Multiplication of $\llbracket a \rrbracket_d^n$ and $\llbracket b \rrbracket_d^n$

- ▶ Compute $c_i \leftarrow a_i \cdot b_i$
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- ▶ Reduce $\llbracket c \rrbracket_{2d}^n$ down to $\llbracket c \rrbracket_d^n$
- ▶ **Problem: Requires $n = 2d + e + 1$**

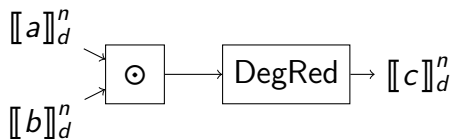
$$\left. \begin{array}{l} \text{Multiplication of } \llbracket a \rrbracket_d^n \text{ and } \llbracket b \rrbracket_d^n \\ \text{▶ Compute } c_i \leftarrow a_i \cdot b_i \\ \text{▶ Result } \llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n \\ \text{▶ Reduce } \llbracket c \rrbracket_{2d}^n \text{ down to } \llbracket c \rrbracket_d^n \\ \text{▶ Problem: Requires } n = 2d + e + 1 \end{array} \right\} \left\{ \begin{array}{l} (\sum_{i=1}^d r_i x^i + a) \cdot (\sum_{i=1}^d r_i x^i + b) \\ = \sum_{i=1}^{2d} (r_i'') x^i + (a \cdot b) \end{array} \right.$$

Compiler with $n = d + e + 1$

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Instead of

- ▶ Compute $\llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n$
- ▶ Reduce $\llbracket c \rrbracket_{2d}^n$ down to $\llbracket c \rrbracket_d^n$



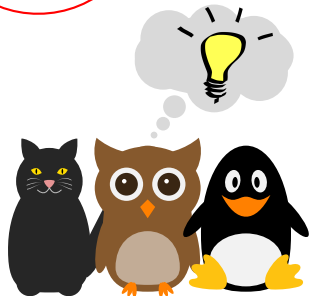
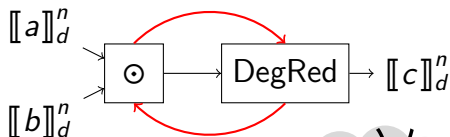
Compiler with $n = d + e + 1$

Instead of

- ▶ Compute $\llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n$
- ▶ Reduce $\llbracket c \rrbracket_{2d}^n$ down to $\llbracket c \rrbracket_d^n$

Reverse the steps

- ▶ Reduce $\llbracket a \rrbracket_d^n, \llbracket b \rrbracket_d^n$ down to $\llbracket a \rrbracket_{d/2}^n, \llbracket b \rrbracket_{d/2}^n$
- ▶ Compute $\llbracket c \rrbracket_d^n = \llbracket a \cdot b \rrbracket_d^n$



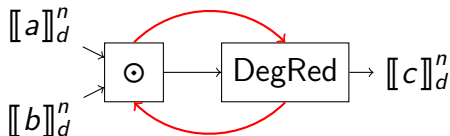
Compiler with $n = d + e + 1$

Instead of

- ▶ Compute $\llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n$
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Reverse the steps

- ▶ Reduce $\llbracket a \rrbracket_d^n, \llbracket b \rrbracket_d^n$ down to $\llbracket a \rrbracket_{d/2}^n, \llbracket b \rrbracket_{d/2}^n$
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- ▶ Problem d probes in $\llbracket a \rrbracket_{d/2}^n$ reveal a



Compiler with $n = d + e + 1$

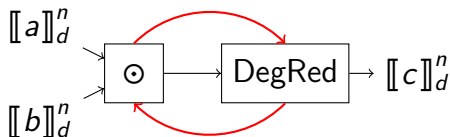
Instead of

- ▶ Compute $\llbracket c \rrbracket_{2d}^n = \llbracket a \cdot b \rrbracket_{2d}^n$
- ▶ Reduce $\llbracket c \rrbracket_{2d}^n$ down to $\llbracket c \rrbracket_d^n$

Reverse the steps

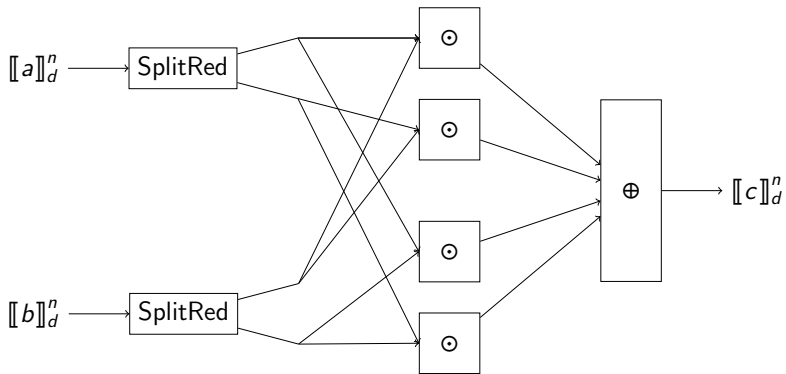
- ▶ Reduce $\llbracket a \rrbracket_d^n, \llbracket b \rrbracket_d^n$ down to $\llbracket a \rrbracket_{d/2}^n, \llbracket b \rrbracket_{d/2}^n$
- ▶ Compute $\llbracket c \rrbracket_d^n = \llbracket a \cdot b \rrbracket_d^n$
- ▶ Problem d probes in $\llbracket a \rrbracket_{d/2}^n$ reveal a

Solution: SplitRed splits $\llbracket a \rrbracket_d^n$ into $\llbracket a' \rrbracket_d^n$ and $\llbracket a'' \rrbracket_d^n$ with $\llbracket a' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n = \llbracket a \rrbracket_{d/2}^n$

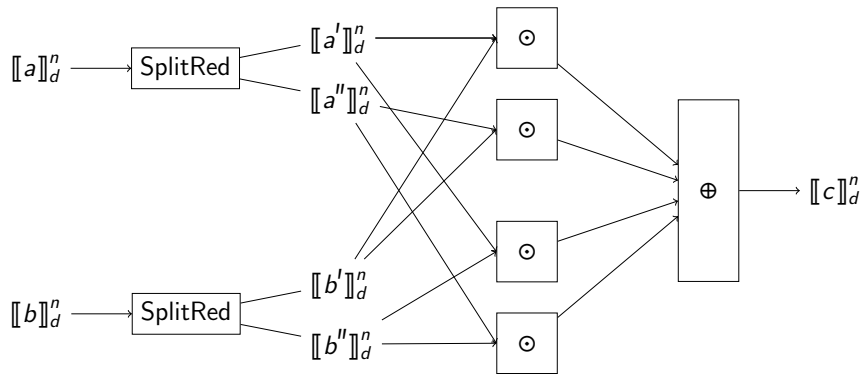


The Multiplication

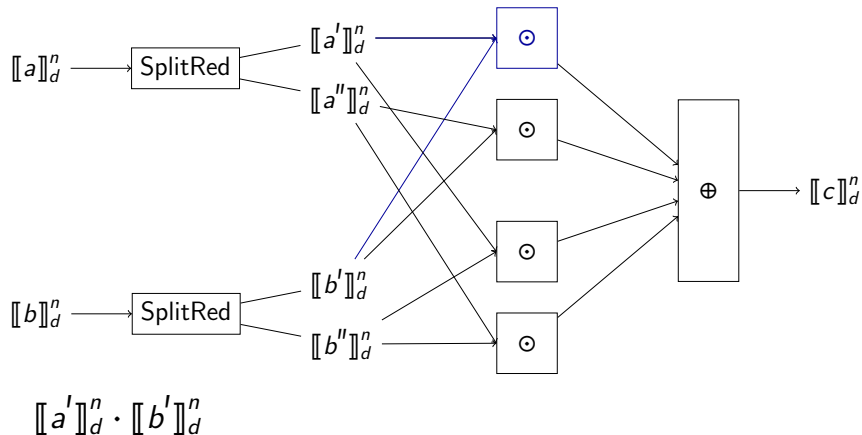
The Multiplication



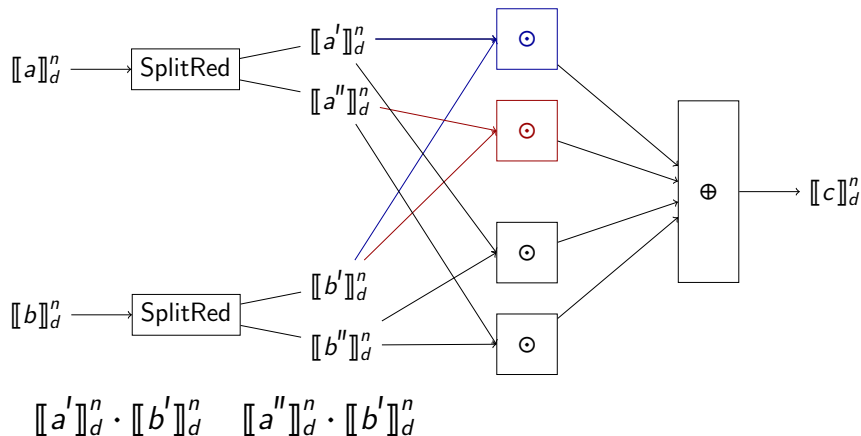
The Multiplication



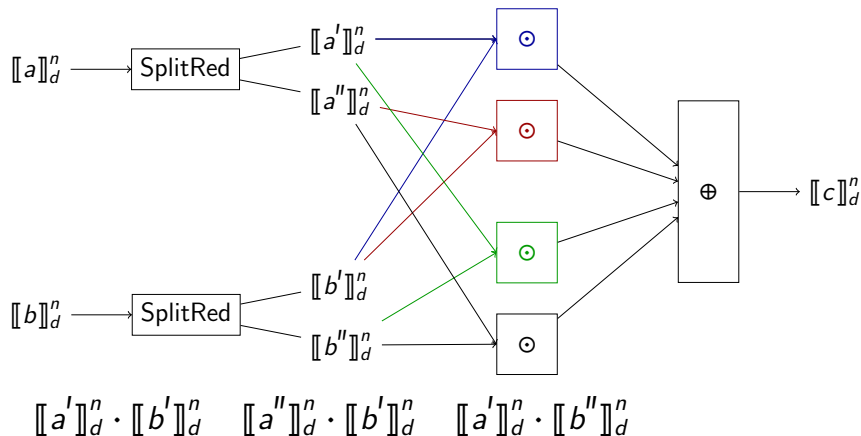
The Multiplication



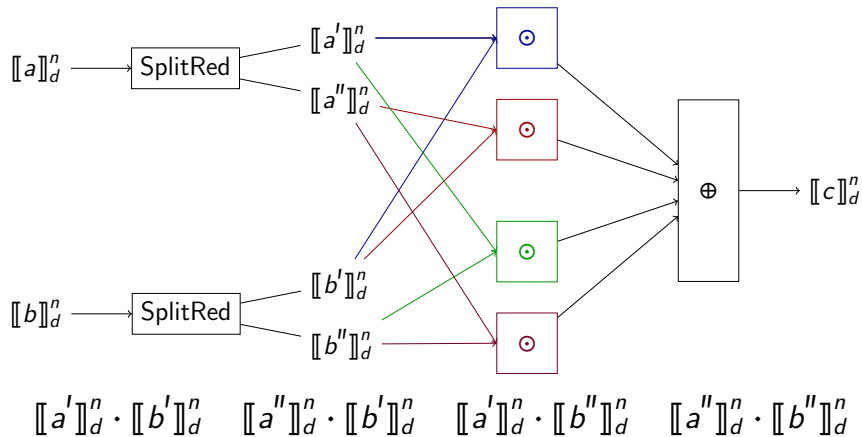
The Multiplication



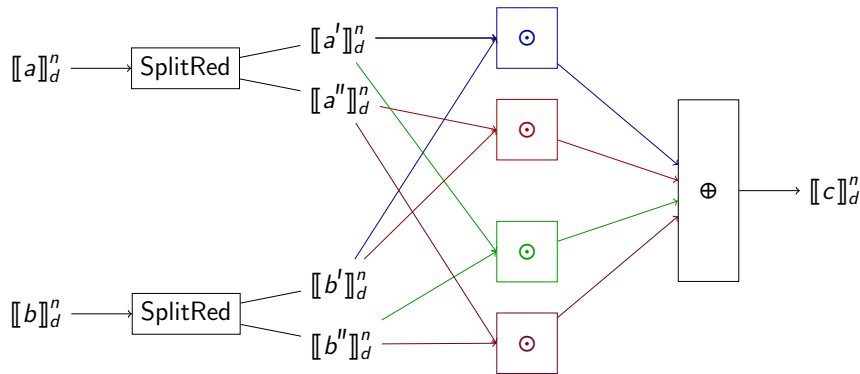
The Multiplication



The Multiplication

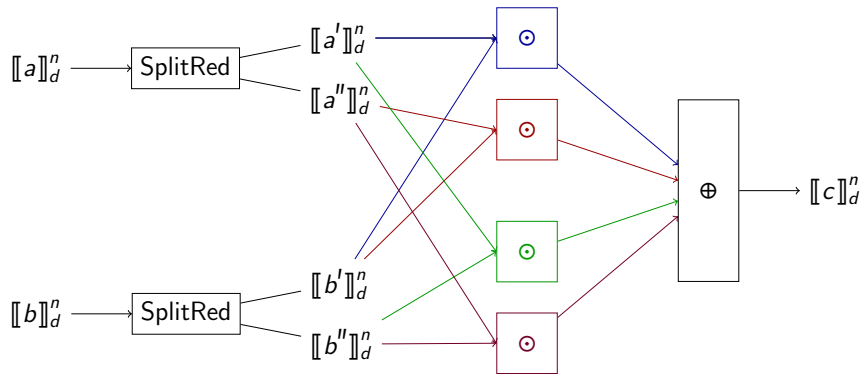


The Multiplication



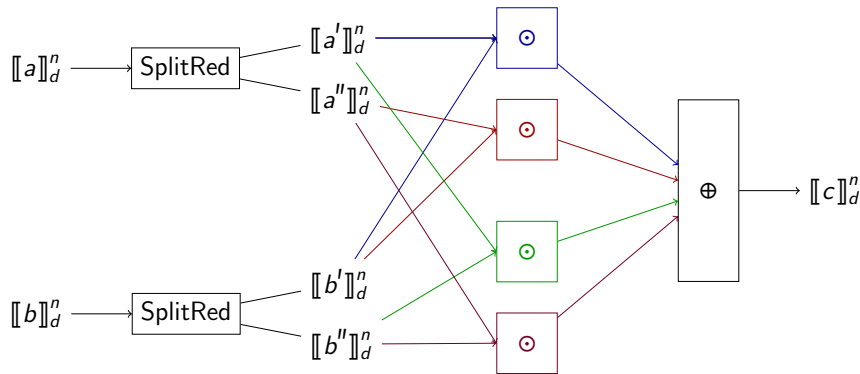
$$\llbracket c \rrbracket_d^n = \llbracket a' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n$$

The Multiplication



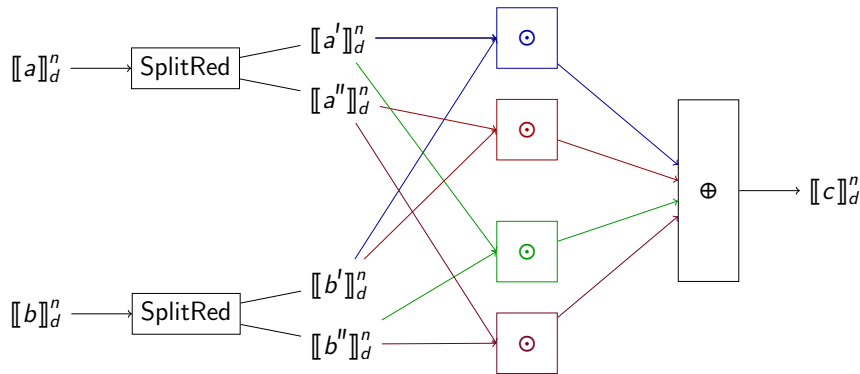
$$\begin{aligned}\llbracket c \rrbracket_d^n &= \llbracket a' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n \\ &= (\llbracket a' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n) \cdot (\llbracket b' \rrbracket_d^n + \llbracket b'' \rrbracket_d^n)\end{aligned}$$

The Multiplication



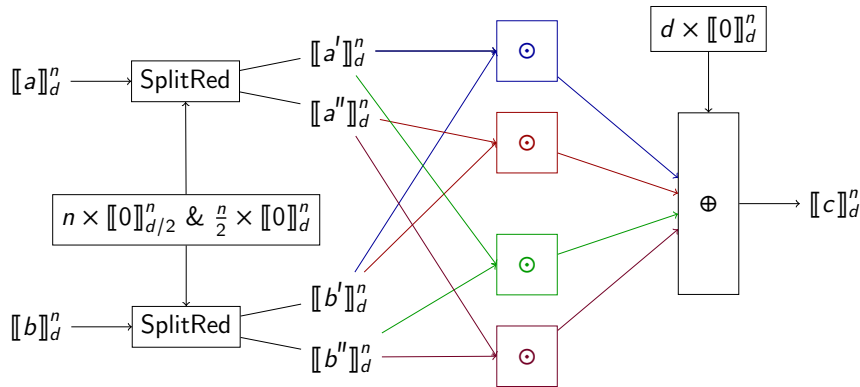
$$\begin{aligned}\llbracket c \rrbracket_d^n &= \llbracket a' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n \\ &= (\llbracket a' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n) \cdot (\llbracket b' \rrbracket_d^n + \llbracket b'' \rrbracket_d^n) = \llbracket a \rrbracket_{d/2}^n \cdot \llbracket b \rrbracket_{d/2}^n\end{aligned}$$

The Multiplication



$$\begin{aligned}
 \llbracket c \rrbracket_d^n &= \llbracket a' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n \\
 &= (\llbracket a' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n) \cdot (\llbracket b' \rrbracket_d^n + \llbracket b'' \rrbracket_d^n) = \llbracket a \rrbracket_{d/2}^n \cdot \llbracket b \rrbracket_{d/2}^n = \llbracket a \cdot b \rrbracket_d^n
 \end{aligned}$$

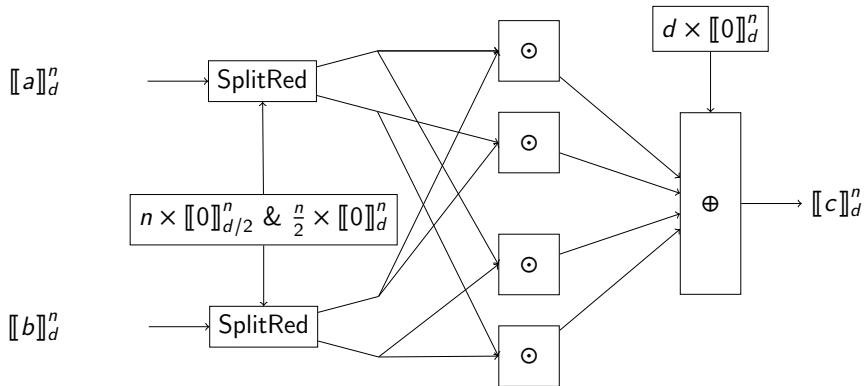
The Multiplication



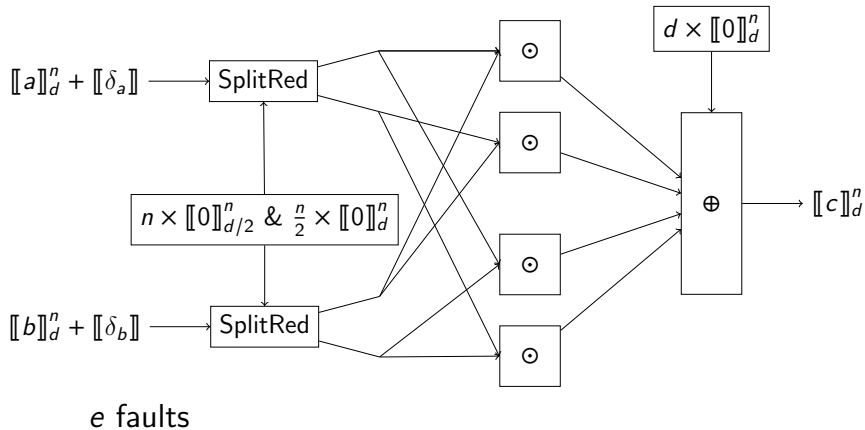
$$\begin{aligned}
 \llbracket c \rrbracket_d^n &= \llbracket a' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b' \rrbracket_d^n + \llbracket a' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n \cdot \llbracket b'' \rrbracket_d^n \\
 &= (\llbracket a' \rrbracket_d^n + \llbracket a'' \rrbracket_d^n) \cdot (\llbracket b' \rrbracket_d^n + \llbracket b'' \rrbracket_d^n) = \llbracket a \rrbracket_{d/2}^n \cdot \llbracket b \rrbracket_{d/2}^n = \llbracket a \cdot b \rrbracket_d^n
 \end{aligned}$$

Security: Fault-Robustness

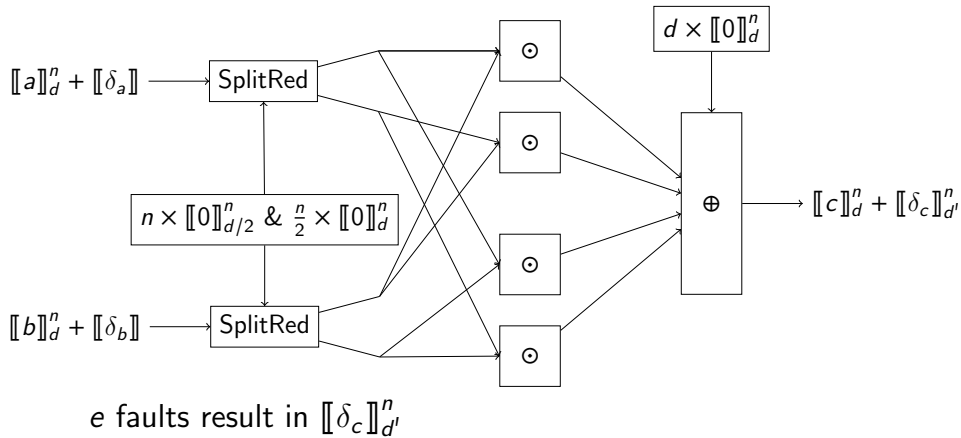
Security: Fault-Robustness



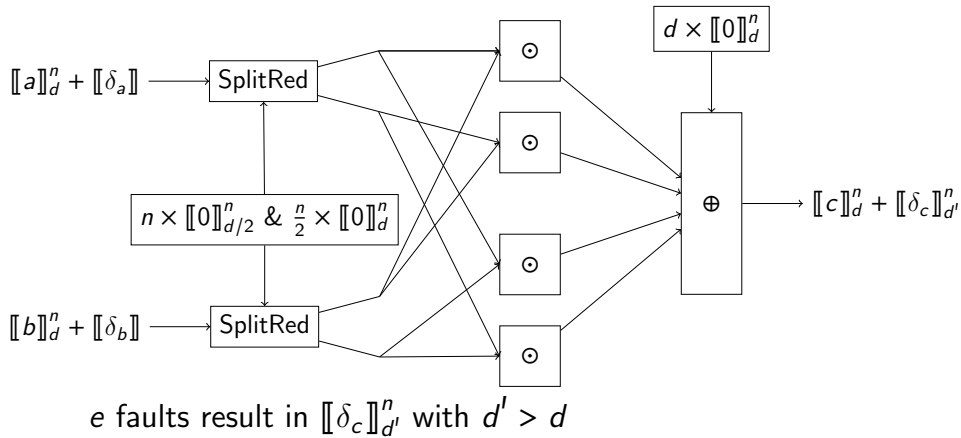
Security: Fault-Robustness



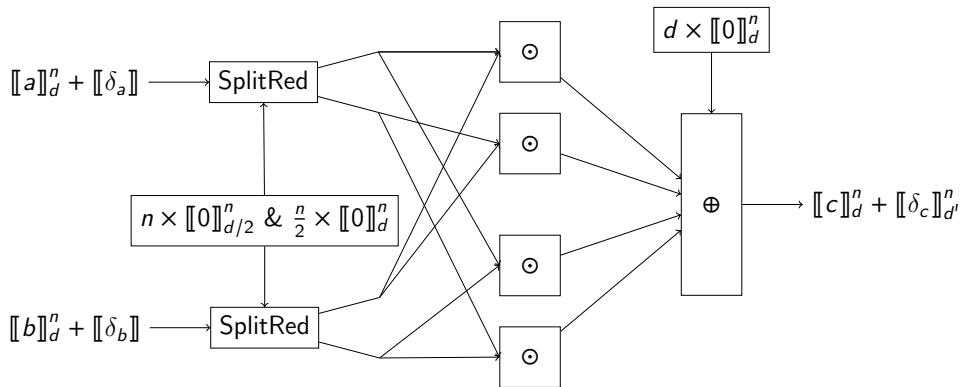
Security: Fault-Robustness



Security: Fault-Robustness



Security: Fault-Robustness



e faults result in $\llbracket \delta_c \rrbracket_{d'}^n$ with $d' > d$ or $\llbracket \delta_c \rrbracket_{d'}^n = (0, \dots, 0)$

Security: Combined Resilience

Security: Combined Resilience

(a) e-Fault Robustness

Security: Combined Resilience

- (a) ϵ -Fault Robustness
- (b) d -Probing Security (or $d/2$ probes in each gadget)

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 - ▶ Each intermediate value can be written as a function $f(x_0, x_1, \dots)$ with input values (x_0, x_1, \dots)

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Security: Combined Resilience

(a) ϵ -Fault Robustness

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(c) Fault Invariance

- ▶ Each intermediate value can be written as a function $f(x_0, x_1, \dots)$ with input values (x_0, x_1, \dots)
- ▶ Let f' be the corresponding value in the faulted gadget
- ▶ There are functions χ, χ_0, χ_1 s.t. $f' = \chi(f(\chi_0(x_0), \chi_1(x_1), \dots))$

Security: Combined Resilience

(a) ϵ -Fault Robustness

(b) d -Probing Security (or $d/2$ probes in each gadget)

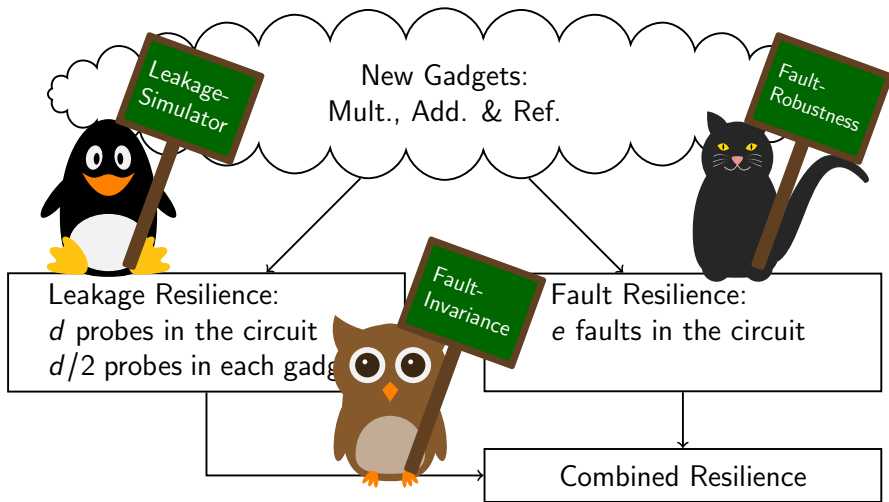
(c) Fault Invariance

- ▶ Each intermediate value can be written as a function $f(x_0, x_1, \dots)$ with input values (x_0, x_1, \dots)
- ▶ Let f' be the corresponding value in the faulted gadget
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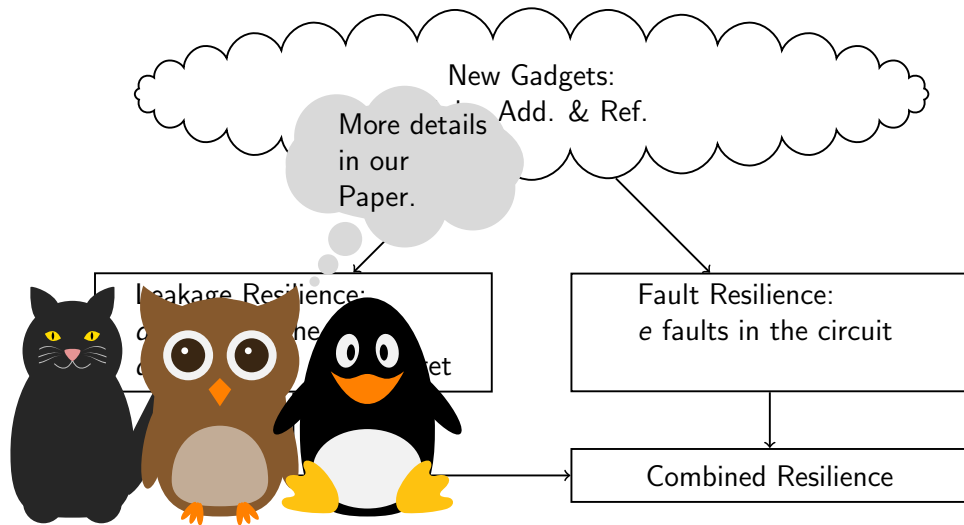
(a)+(b)+(c)= Combined Resilience

Conclusion

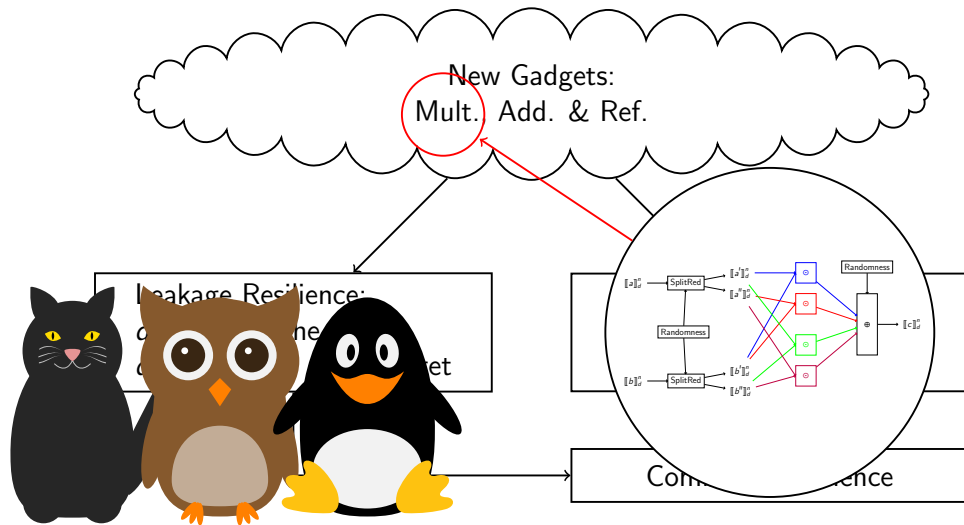
Conclusion



Conclusion



Conclusion



Conclusion

