On Perfect Linear Approximations and Differentials over Two-Round SPNs

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Christof Beierle, Patrick Felke, Gregor Leander, Patrick Neumann, Lukas Stennes

CASA
Cyber Security in the Age of Large-Scale Adversaries

Ruhr Universität Bochum RUB
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Horst Görtz Institut

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- Security of symmetric primitives based on resilience to existing attacks

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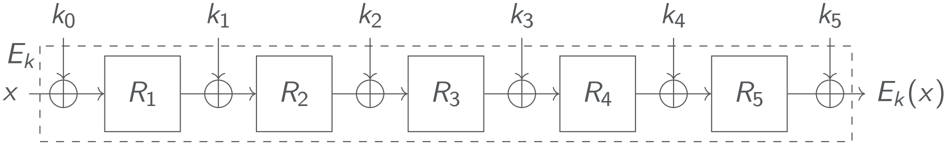
- Only possible if E_k has structure

For Round-Based Primitives



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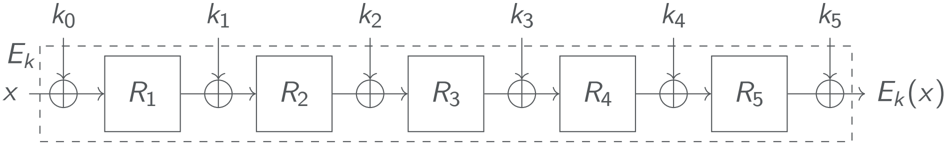

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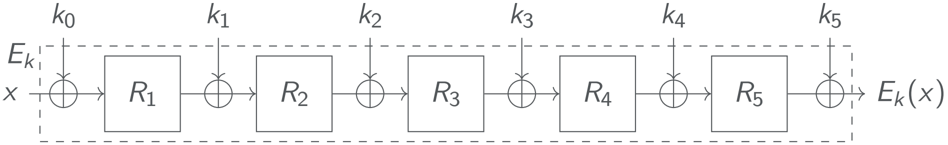

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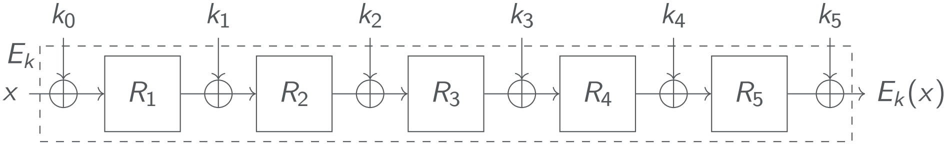

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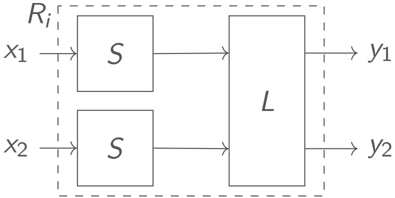
- Here: focus on SPNs



For Two Rounds

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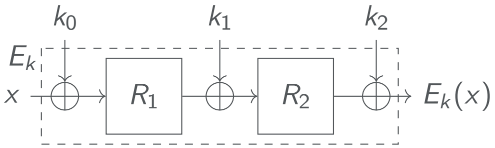

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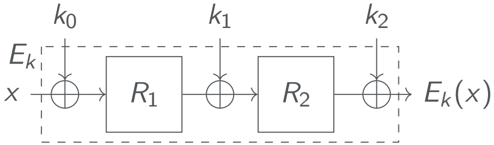
$$\mathrm{P}[\, \alpha \xrightarrow{E_k} \beta \,]$$



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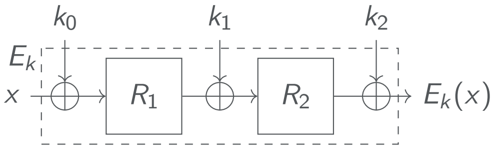

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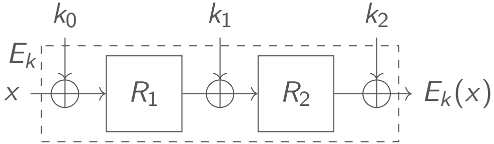

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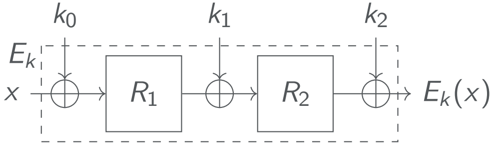

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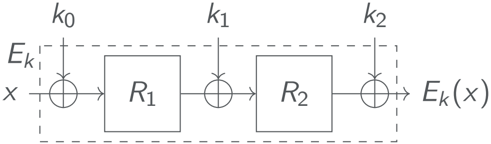

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- Gives only average $\mathrm{P}[\alpha \xrightarrow{E_k} \beta]$ (over the key)

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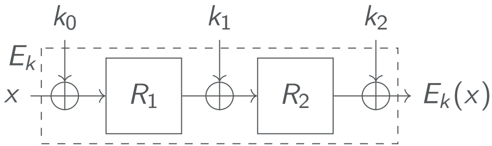

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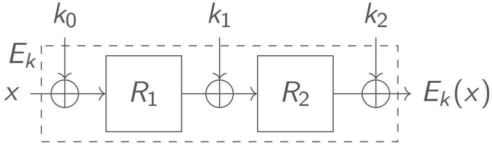

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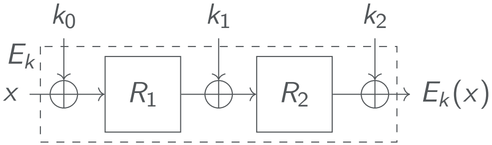
- Can we do better?

For Two Rounds and all keys



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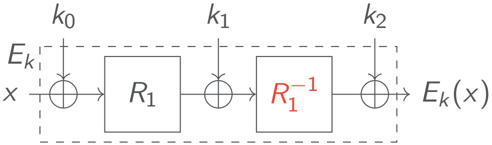

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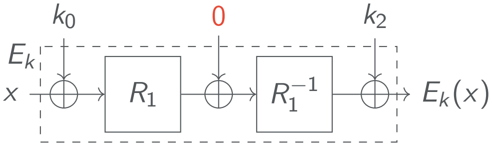

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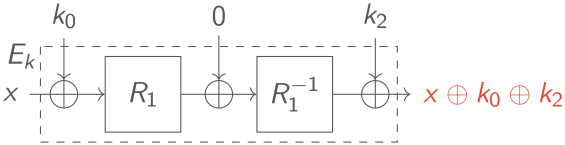

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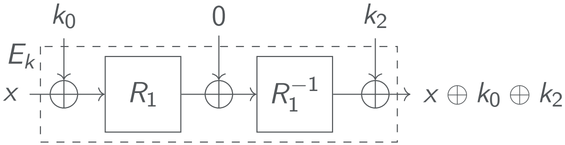

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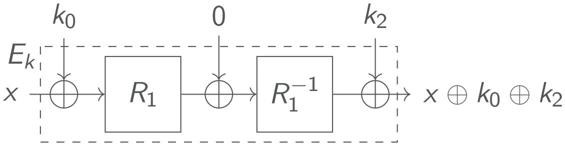

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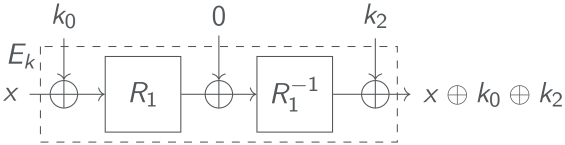

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As a First Step

Answer existence of k such that

- $\left|C[\gamma \xrightarrow{E_k} \zeta]\right| = 1$ (perfect linear approximation), or
- $P[\alpha \xrightarrow{E_k} \beta] = 1$ (perfect differential)

for two-round SPNs

Existence of Perfect Linear Approximations

- Perfect linear approximation: there exist $\gamma, \zeta \neq 0$ s.t.

$$\left|\mathrm{cor}(\gamma \xrightarrow{E_k} \zeta)\right| = 1 \iff \exists c : \quad \langle \gamma, x \rangle = \langle \zeta, E_k(x) \rangle \oplus c \quad \forall x$$

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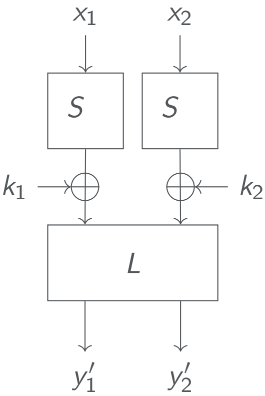
  - For two-round SPNs: can be (efficiently) answered

Existence of Perfect Linear Approximations over Two-Round SPNs



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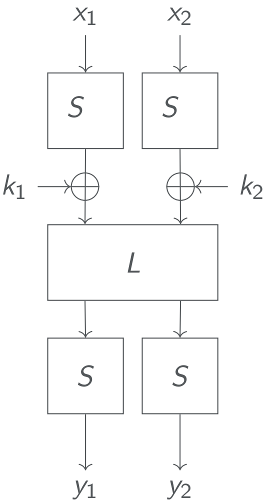

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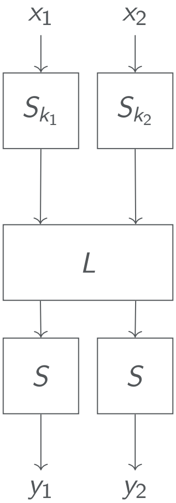

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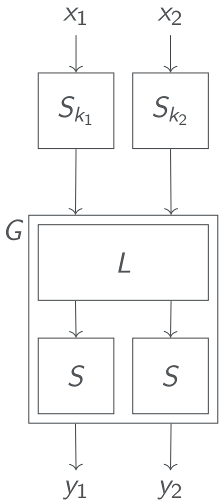

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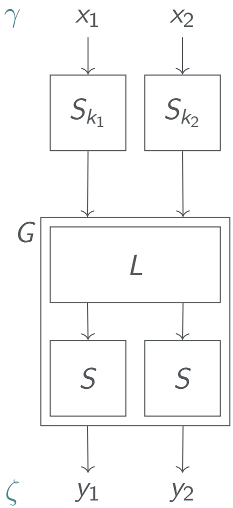

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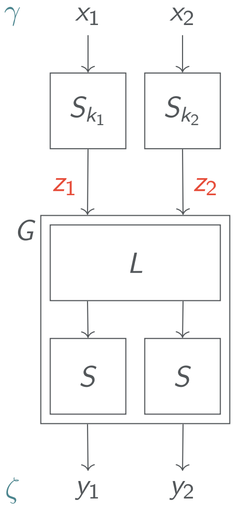

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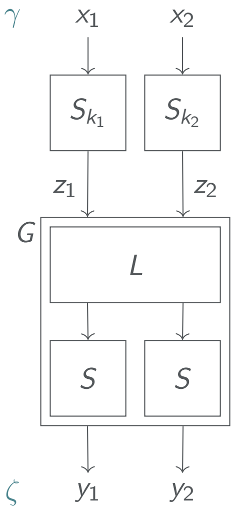

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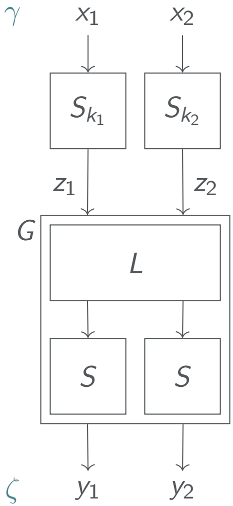

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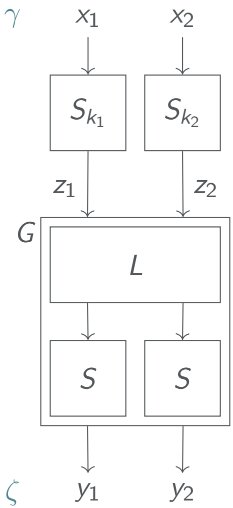

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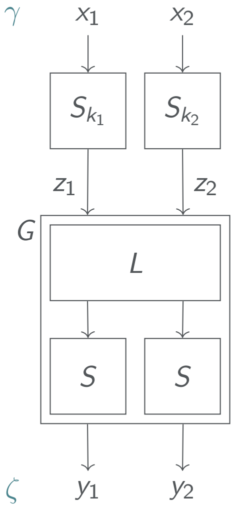

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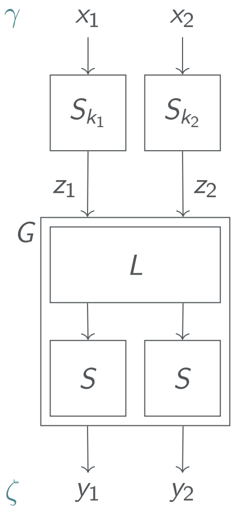

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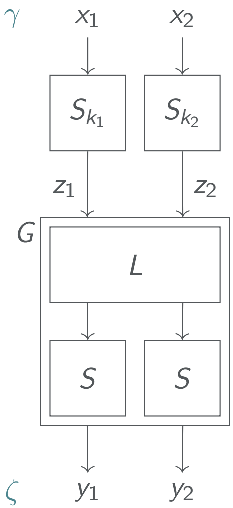

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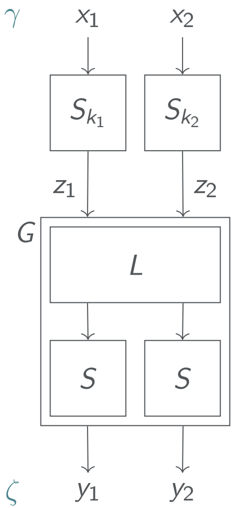
$$\underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1') \\ S_{k_2}^{-1}(z_2') \end{pmatrix} \right\rangle}_{\langle \zeta, G \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \rangle \oplus c} \oplus \underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1') \\ S_{k_2}^{-1}(0) \end{pmatrix} \right\rangle}_{\langle \zeta, G \begin{pmatrix} z_1' \\ 0 \end{pmatrix} \rangle \oplus c} = \underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(0) \\ S_{k_2}^{-1}(z_2') \end{pmatrix} \right\rangle}_{\langle \zeta, G \begin{pmatrix} 0 \\ z_2' \end{pmatrix} \rangle \oplus c} \oplus \underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(0) \\ S_{k_2}^{-1}(0) \end{pmatrix} \right\rangle}_{\langle \zeta, G \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rangle \oplus c}$$

$$\Longrightarrow \quad \left\langle \zeta, G \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \oplus G \begin{pmatrix} z_1' \\ 0 \end{pmatrix} \oplus G \begin{pmatrix} 0 \\ z_2' \end{pmatrix} \oplus G \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\rangle = 0$$

Existence of Perfect Linear Approximations over Two-Round SPNs

CASA
Cyber Security in the Age of Large-Scale Adversaries



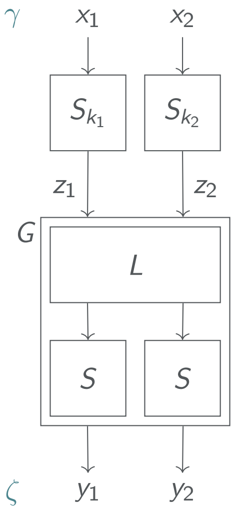
$$\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1) \\ S_{k_2}^{-1}(z_2) \end{pmatrix} \right\rangle = \left\langle \gamma, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\rangle = \left\langle \zeta, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle \oplus c = \left\langle \zeta, G \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \right\rangle \oplus c$$

$$\underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1') \\ S_{k_2}^{-1}(z_2') \end{pmatrix} \right\rangle}_{\left\langle \zeta, G \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \right\rangle \oplus c} \oplus \underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1') \\ S_{k_2}^{-1}(0) \end{pmatrix} \right\rangle}_{\left\langle \zeta, G \begin{pmatrix} z_1' \\ 0 \end{pmatrix} \right\rangle \oplus c} = \underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(0) \\ S_{k_2}^{-1}(z_2') \end{pmatrix} \right\rangle}_{\left\langle \zeta, G \begin{pmatrix} 0 \\ z_2' \end{pmatrix} \right\rangle \oplus c} \oplus \underbrace{\left\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(0) \\ S_{k_2}^{-1}(0) \end{pmatrix} \right\rangle}_{\left\langle \zeta, G \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\rangle \oplus c}$$

$$\Longrightarrow \quad \left\langle \zeta, G \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \oplus G \begin{pmatrix} z_1' \\ 0 \end{pmatrix} \oplus G \begin{pmatrix} 0 \\ z_2' \end{pmatrix} \oplus G \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\rangle = 0$$

For every z': linear equation in ζ

Existence of Perfect Linear Approximations over Two-Round SPNs



$$\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1) \\ S_{k_2}^{-1}(z_2) \end{pmatrix} \rangle = \langle \gamma, \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rangle = \langle \zeta, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \rangle \oplus c = \langle \zeta, G \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \rangle \oplus c$$

$$\underbrace{\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1') \\ S_{k_2}^{-1}(z_2') \end{pmatrix} \rangle}_{\langle \zeta, G \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \rangle \oplus c} \oplus \underbrace{\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(z_1') \\ S_{k_2}^{-1}(0) \end{pmatrix} \rangle}_{\langle \zeta, G \begin{pmatrix} z_1' \\ 0 \end{pmatrix} \rangle \oplus c} = \underbrace{\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(0) \\ S_{k_2}^{-1}(z_2') \end{pmatrix} \rangle}_{\langle \zeta, G \begin{pmatrix} 0 \\ z_2' \end{pmatrix} \rangle \oplus c} \oplus \underbrace{\langle \gamma, \begin{pmatrix} S_{k_1}^{-1}(0) \\ S_{k_2}^{-1}(0) \end{pmatrix} \rangle}_{\langle \zeta, G \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rangle \oplus c}$$

$$\Longrightarrow \quad \langle \zeta, G \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \oplus G \begin{pmatrix} z_1' \\ 0 \end{pmatrix} \oplus G \begin{pmatrix} 0 \\ z_2' \end{pmatrix} \oplus G \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rangle = 0$$

For every z': linear equation in ζ independent of key!

Existence of Perfect Linear Approximations over Two-Round SPNs

CASA
CYBER SECURITY IN THE AGE OF LARGE-SCALE ADVERSARIES

Cipher	Linear
	$r = 2$
Boomslang	
CRAFT	
MANTIS	
Midori64	
SKINNY-64	
SKINNY-128	
AES	✓
GIFT-64/128	✓
LED	✓
PRESENT	✓
PRINCE	✓
Streebog	✓
Ascon	✓
iSCREAM	✓
Keccak-100	✓
Kuznechik	✓
PRIDE	✓
RECTANGLE	✓

✓	Non-existence
✗	Existence
$\perp$	Abort
–	Not tested

Existence of Perfect Linear Approximations over Two-Round SPNs

Cipher	Linear $r = 2$
Boomslang	✗
CRAFT	
MANTIS	
Midori64	
SKINNY-64	
SKINNY-128	
AES	✓
GIFT-64/128	✓
LED	✓
PRESENT	✓
PRINCE	✓
Streebog	✓
Ascon	✓
iSCREAM	✓
Keccak-100	✓
Kuznechik	✓
PRIDE	✓
RECTANGLE	✓

✓	Non-existence
✗	Existence
$\perp$	Abort
–	Not tested

Existence of Perfect Linear Approximations over Two-Round SPNs

Cipher	Linear
	$r = 2$
Boomslang	✗
CRAFT	✗
MANTIS	
Midori64	
SKINNY-64	
SKINNY-128	
AES	✓
GIFT-64/128	✓
LED	✓
PRESENT	✓
PRINCE	✓
Streebog	✓
Ascon	✓
iSCREAM	✓
Keccak-100	✓
Kuznechik	✓
PRIDE	✓
RECTANGLE	✓

✓ Non-existence
✗ Existence
$\perp$ Abort
– Not tested

Existence of Perfect Linear Approximations over Two-Round SPNs

Cipher	Linear $r = 2$
Boomslang	✗
CRAFT	✗
MANTIS	✗
Midori64	✗
SKINNY-64	
SKINNY-128	
AES	✓
GIFT-64/128	✓
LED	✓
PRESENT	✓
PRINCE	✓
Streebog	✓
Ascon	✓
iSCREAM	✓
Keccak-100	✓
Kuznechik	✓
PRIDE	✓
RECTANGLE	✓

✓ Non-existence
✗ Existence
$\perp$ Abort
– Not tested

Existence of Perfect Linear Approximations over Two-Round SPNs

Cipher	Linear
	$r = 2$
Boomslang	✗
CRAFT	✗
MANTIS	✗
Midori64	✗
SKINNY-64	✗
SKINNY-128	✗
AES	✓
GIFT-64/128	✓
LED	✓
PRESENT	✓
PRINCE	✓
Streebog	✓
Ascon	✓
iSCREAM	✓
Keccak-100	✓
Kuznechik	✓
PRIDE	✓
RECTANGLE	✓

✓ Non-existence
✗ Existence
$\perp$ Abort
– Not tested

Existence of Perfect Linear Approximations over Two-Round SPNs

CASA
Cyber Security in the Age of Large-Scale Adversaries

Cipher	Linear $r = 2$	Linear $r = 3$
Boomslang	✗	✓
CRAFT	✗	✓
MANTIS	✗	✓
Midori64	✗	✓
SKINNY-64	✗	✓
SKINNY-128	✗	$\perp$
AES	✓	✓
GIFT-64/128	✓	✓
LED	✓	✓
PRESENT	✓	✓
PRINCE	✓	✓
Streebog	✓	✓
Ascon	✓	✓
iSCREAM	✓	$\perp$
Keccak-100	✓	✓
Kuznechik	✓	$\perp$
PRIDE	✓	✓
RECTANGLE	✓	✓

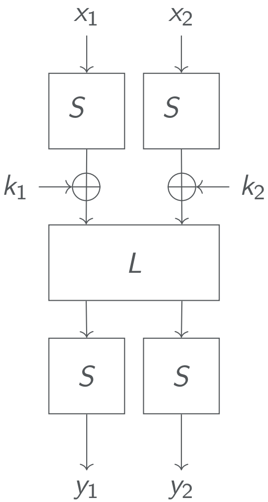
✓ Non-existence
✗ Existence
$\perp$ Abort
– Not tested

Existence of Perfect Linear Approximations over Two-Round SPNs

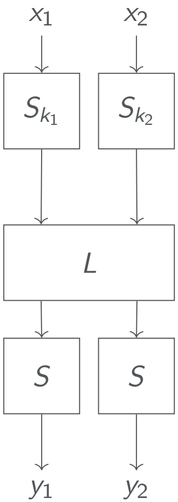
Cipher	Linear		
	$r = 2$	$r = 3$	$r = 4$
Boomslang	✗	✓	✗
CRAFT	✗	✓	✓
MANTIS	✗	✓	✗
Midori64	✗	✓	✗
SKINNY-64	✗	✓	✓
SKINNY-128	✗	$\perp$	$\perp$
AES	✓	✓	$\perp$
GIFT-64/128	✓	✓	✓
LED	✓	✓	✓
PRESENT	✓	✓	✓
PRINCE	✓	✓	✓
Streebog	✓	✓	$\perp$
Ascon	✓	✓	–
iSCREAM	✓	$\perp$	–
Keccak-100	✓	✓	–
Kuznechik	✓	$\perp$	–
PRIDE	✓	✓	–
RECTANGLE	✓	✓	–

✓ Non-existence
✗ Existence
$\perp$ Abort
– Not tested

Existence of Perfect Differentials over Two-Round SPNs



Existence of Perfect Differentials over Two-Round SPNs



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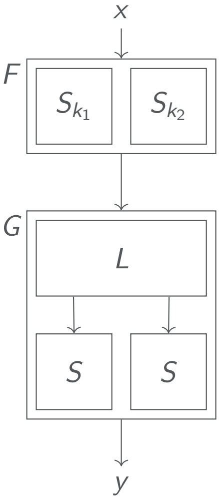

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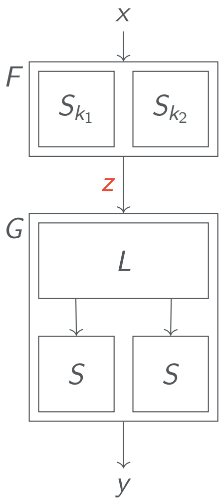

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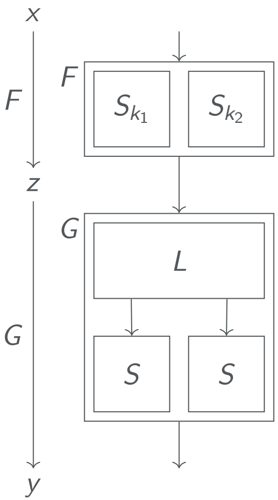

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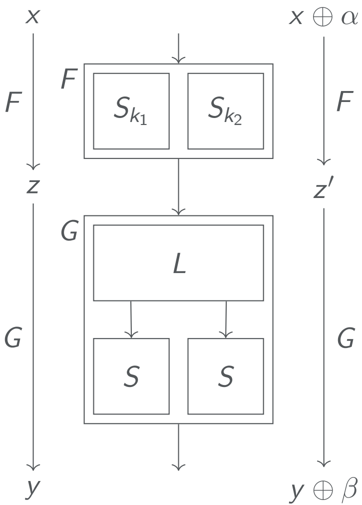

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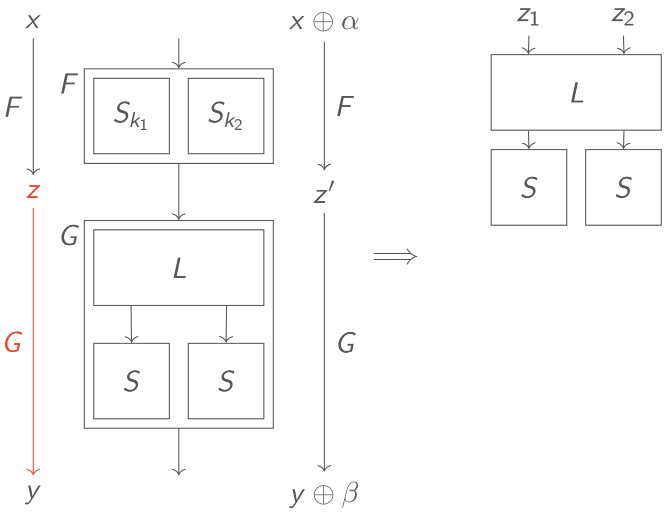

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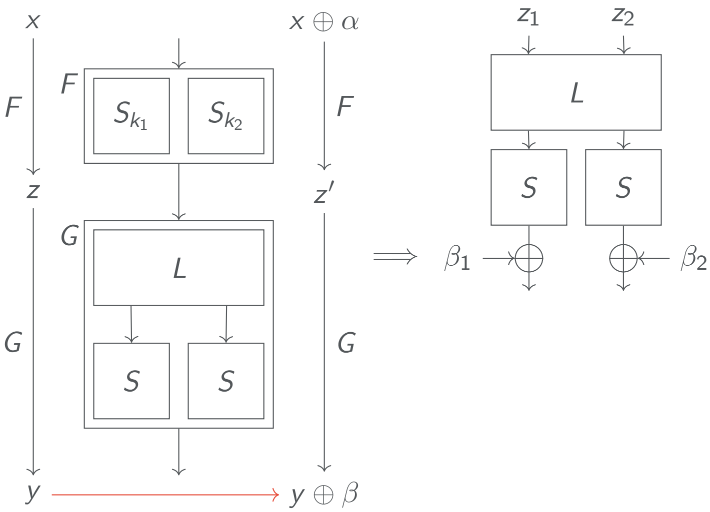

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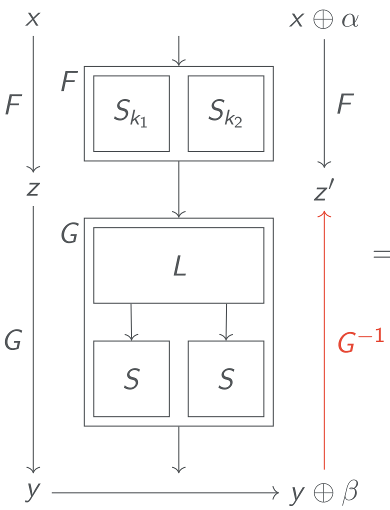
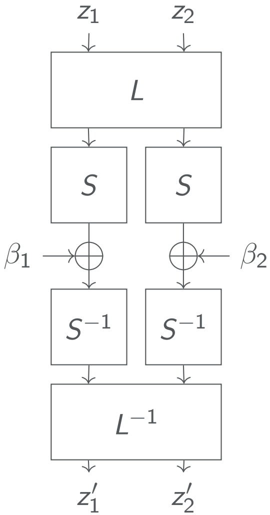


$\Longrightarrow$



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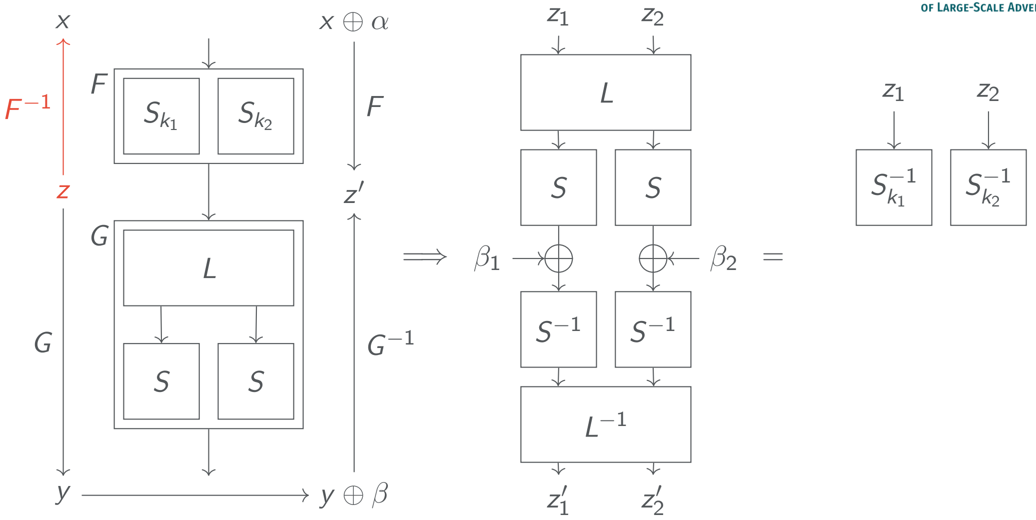

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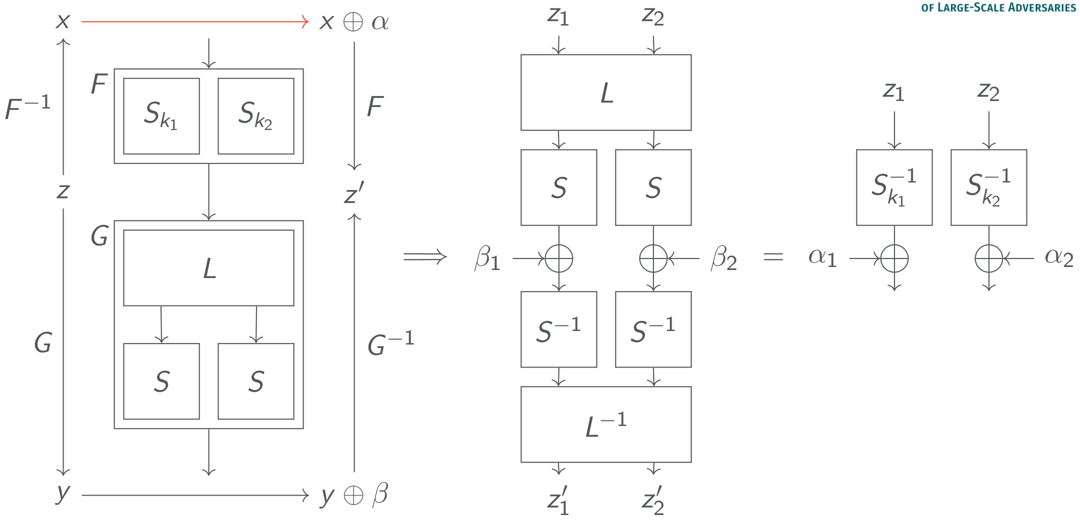

Existence of Perfect Differentials over Two-Round SPNs

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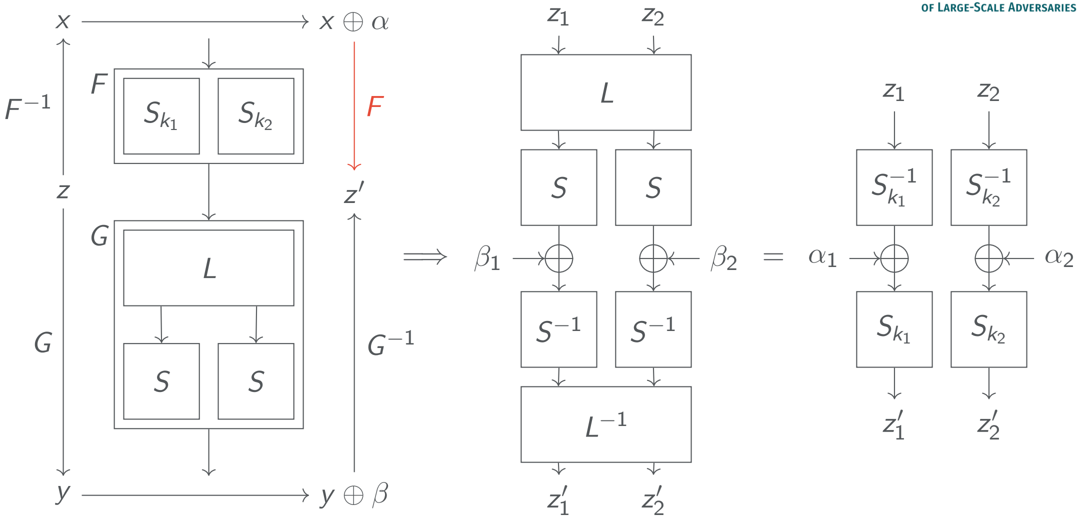
CYBER SECURITY IN THE AGE OF LARGE-SCALE ADVERSARIES



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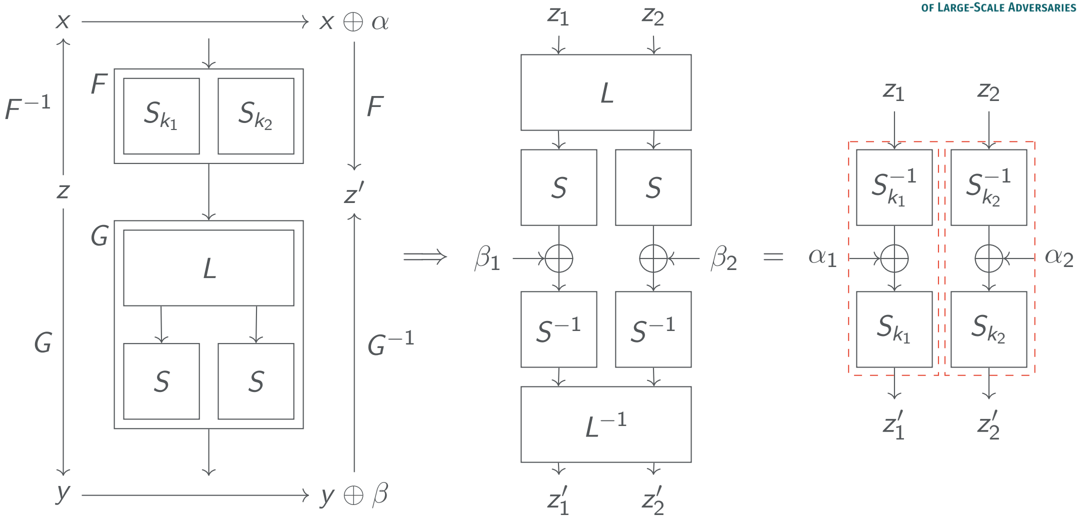

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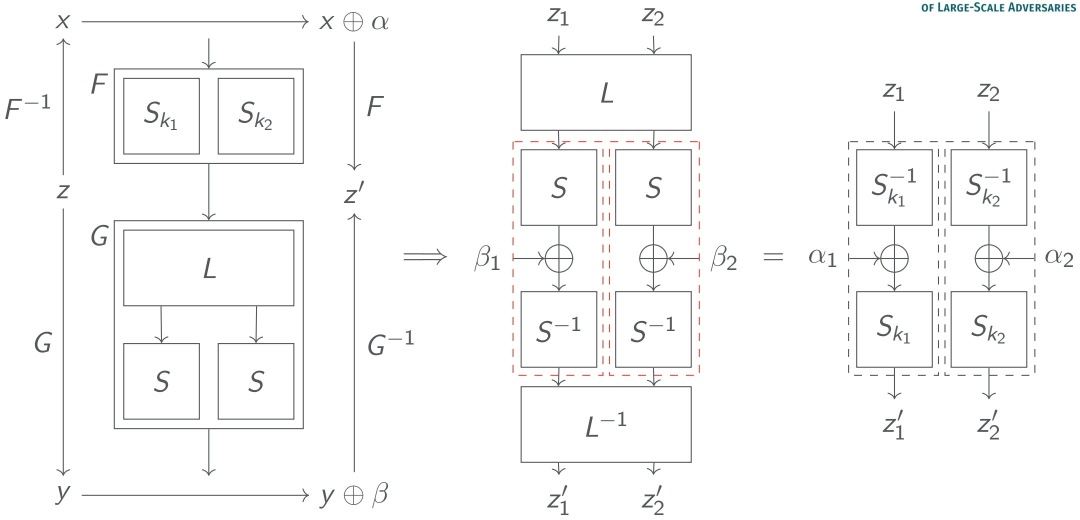

Existence of Perfect Differentials over Two-Round SPNs

Theorem 1 ([Lambin, Leander and N., EC'23], informal)

If an SPN-round-function has two essentially different decompositions then there exist a perfect linear approximation and a perfect differential over (at least) one of its s-boxes.

Existence of Perfect Differentials over Two-Round SPNs

CASA
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- Here: $\hat{S}\colon x \mapsto S^{-1}(S(x) \oplus \beta_i)$ are the s-boxes

Existence of Perfect Differentials over Two-Round SPNs

CASA
Cyber Security in the Age of Large-Scale Adversaries

Theorem 1 ([Lambin, Leander and N., EC'23], informal)

If an SPN-round-function has two essentially different decompositions then there exist a perfect linear approximation and a perfect differential over (at least) one of its s-boxes.

- Here: $\hat{S} : x \mapsto S^{-1}(S(x) \oplus \beta_i)$ are the s-boxes
- Perfect differential over $\hat{S}$ would imply

$$\exists \delta \neq 0, \delta' : \qquad S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta) \oplus \beta_i) \qquad = \delta' \qquad \forall x$$

Existence of Perfect Differentials over Two-Round SPNs

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$$\begin{aligned} \exists \delta \neq 0, \delta' : \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta) \oplus \beta_i) &= \delta' \quad & \forall x \\ \Longrightarrow \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta') \oplus \beta_i) &= \delta \quad & \forall x \end{aligned}$$

Existence of Perfect Differentials over Two-Round SPNs

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$$
\begin{aligned}
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\Longrightarrow \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta') \oplus \beta_i) &&= \delta && \forall x \\
\Longrightarrow \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta \oplus \delta') \oplus \beta_i) &&= \delta \oplus \delta' && \forall x
\end{aligned}
$$

Existence of Perfect Differentials over Two-Round SPNs

CYBER SECURITY IN THE AGE OF LARGE-SCALE ADVERSARIES

Theorem 1 ([Lambin, Leander and N., EC'23], informal)

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$$\begin{aligned}
\exists \delta \neq 0, \delta' : \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta) \oplus \beta_i) &&= \delta' && \forall x \\
\Longrightarrow \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta') \oplus \beta_i) &&= \delta && \forall x \\
\Longrightarrow \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta \oplus \delta') \oplus \beta_i) &&= \delta \oplus \delta' && \forall x
\end{aligned}$$

- I.e. S would have maximal boomerang uniformity [Boura and Canteaut, ToSC'18]

Existence of Perfect Differentials over Two-Round SPNs

CASA — Cyber Security in the Age of Large-Scale Adversaries

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If an SPN-round-function has two essentially different decompositions then there exist a perfect linear approximation and a perfect differential over (at least) one of its s-boxes.

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$$\begin{aligned}
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\implies \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta') \oplus \beta_i) && = \delta && \forall x \\
\implies \quad & S^{-1}(S(x) \oplus \beta_i) \oplus S^{-1}(S(x \oplus \delta \oplus \delta') \oplus \beta_i) && = \delta \oplus \delta' && \forall x
\end{aligned}$$

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Existence of Perfect Differentials over Two-Round SPNs

- Use theory from [Lambin, Leander and N., EC'23]

Existence of Perfect Differentials over Two-Round SPNs

- Use theory from [Lambin, Leander and N., EC'23]
- Exemplary implication

Corollary 2

If L has differential branch number of at least 3 and if S does not have

1. *maximal boomerang uniformity, or*
2. *linear structures*

then there cannot exist any perfect differential over two rounds.

Existence of Perfect Linear Approximations and Differentials over Two-Round SPNs

CASA
CYBER SECURITY IN THE AGE OF LARGE-SCALE ADVERSARIES

Cipher	Linear $r=2$	Linear $r=3$	Linear $r=4$	Differential $r=2$
Boomslang	✗	✓	✗	✓
CRAFT	✗	✓	✓	✓
MANTIS	✗	✓	✗	✓
Midori64	✗	✓	✗	✓
SKINNY-64	✗	✓	✓	✓
SKINNY-128	✗	$\perp$	$\perp$	✓
AES	✓	✓	$\perp$	✓
GIFT-64/128	✓	✓	✓	✓
LED	✓	✓	✓	✓
PRESENT	✓	✓	✓	✓
PRINCE	✓	✓	✓	✓
Streebog	✓	✓	$\perp$	✓
Ascon	✓	✓	–	✓
iSCREAM	✓	$\perp$	–	✓
Keccak-100	✓	✓	–	✓
Kuznechik	✓	$\perp$	–	✓
PRIDE	✓	✓	–	✓
RECTANGLE	✓	✓	–	✓

✓ Non-existence
✗ Existence
$\perp$ Abort
– Not tested

Existence of Perfect Linear Approximations and Differentials over Two-Round SPNs

CASA
Cyber Security in the Age of Large-Scale Adversaries

Cipher	Linear $r=2$	Linear $r=3$	Linear $r=4$	Differential $r=2$
Boomslang	✗	✓	✗	✓
CRAFT	✗	✓	✓	✓
MANTIS	✗	✓	✗	✓
Midori64	✗	✓	✗	✓
SKINNY-64	✗	✓	✓	✓
SKINNY-128	✗	⊥	⊥	✓
AES	✓	✓	⊥	✓
GIFT-64/128	✓	✓	✓	✓
LED	✓	✓	✓	✓
PRESENT	✓	✓	✓	✓
PRINCE	✓	✓	✓	✓
Streebog	✓	✓	⊥	✓
Ascon	✓	✓	–	✓
iSCREAM	✓	⊥	–	✓
Keccak-100	✓	✓	–	✓
Kuznechik	✓	⊥	–	✓
PRIDE	✓	✓	–	✓
RECTANGLE	✓	✓	–	✓

✓ Non-existence
✗ Existence
⊥ Abort
– Not tested

Thank you for your attention!