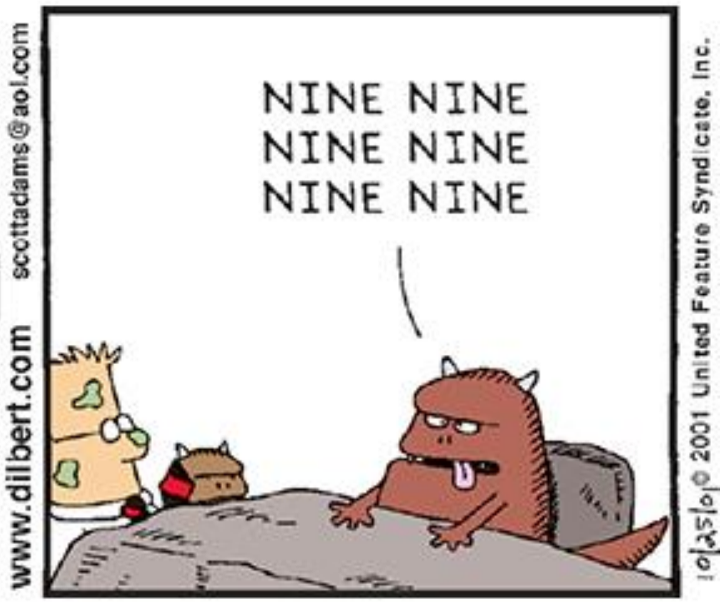
Extractors

Low Entropy Requirements Colliding with Non-Malleability

Divesh Aggarwal Eldon Chung Maciej Obremski

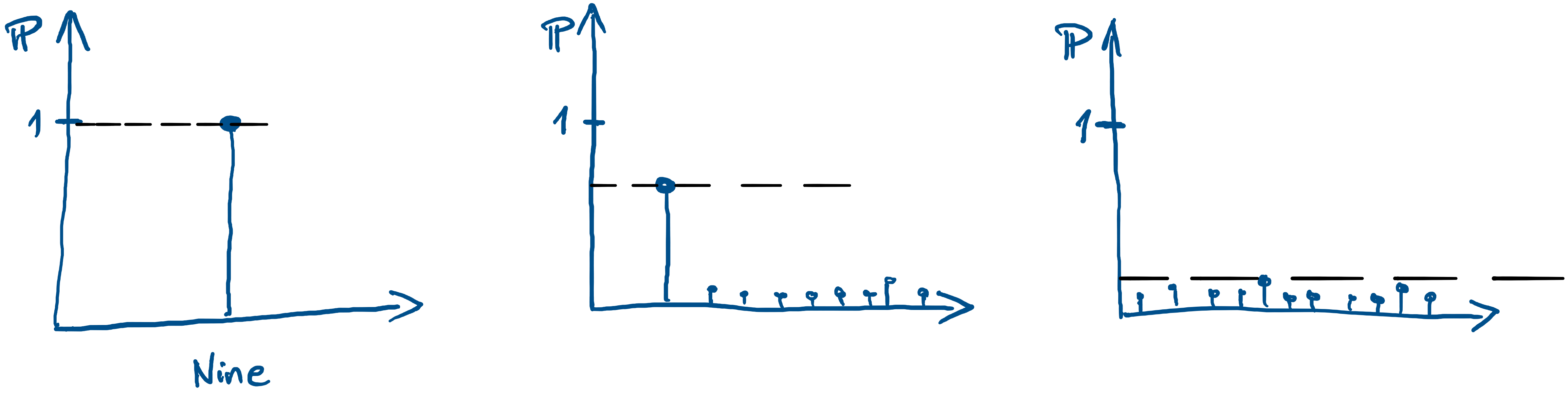
National University of Singapore

Randomness



All processes are random
but some processes are
more random than others.

Min-Entropy

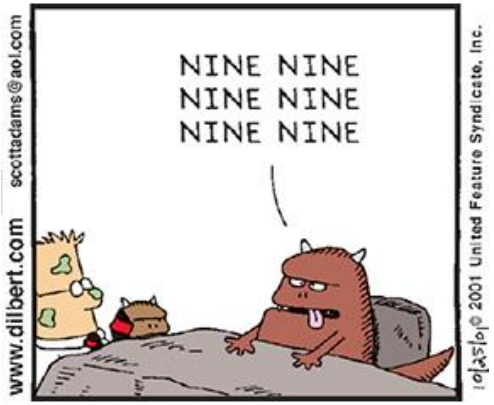


$$H_\infty(X) = -\log \max_x \mathbb{P}(X=x)$$

$X \sim \text{uniform}\{0,1\}^n$

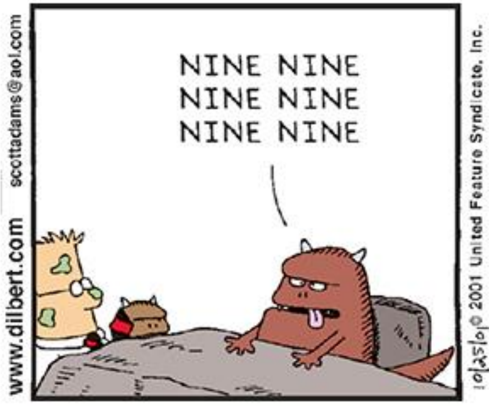
$H_\infty(X) = n$ ← this is maximum possible.

Extractors



$\xrightarrow{?}$

Extractors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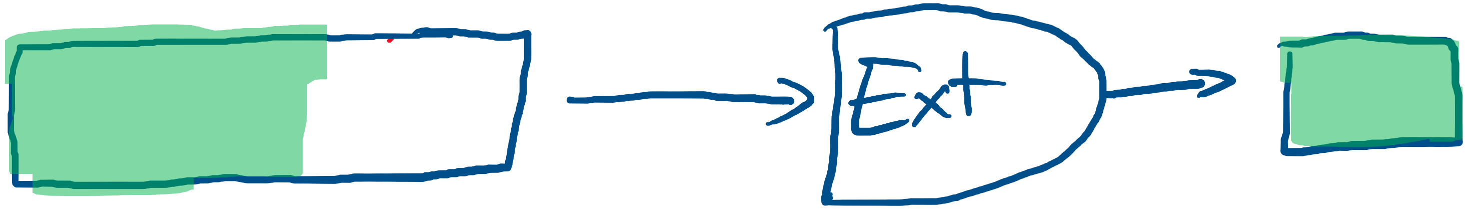
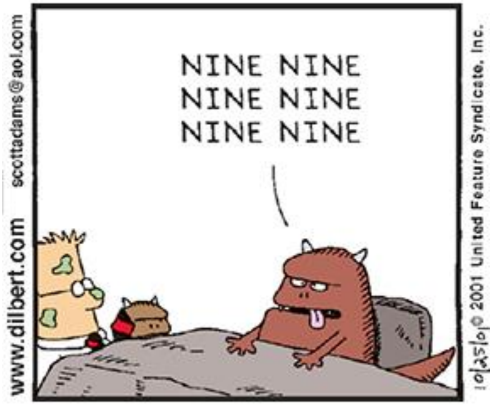


Nope

Extractors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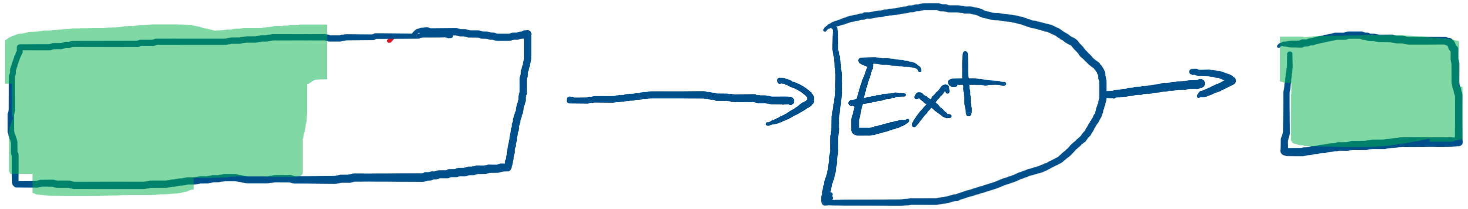

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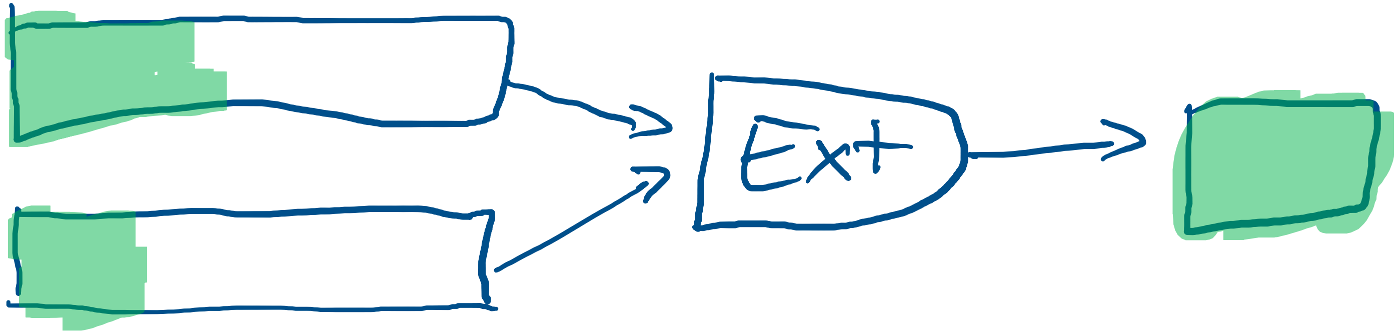
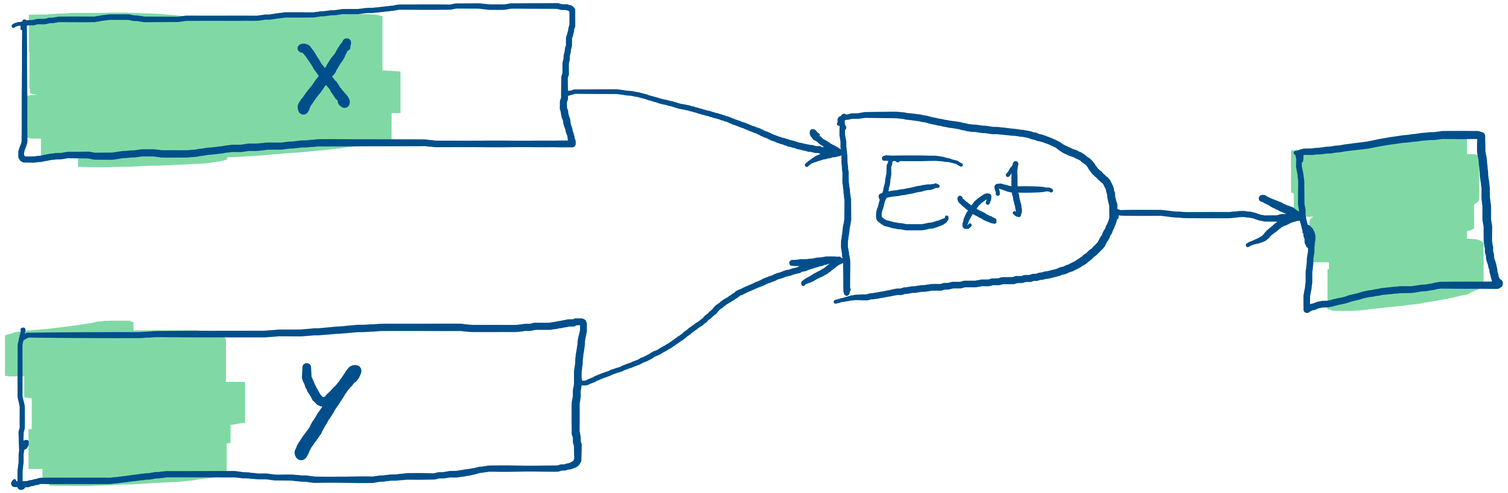


Yey

Non-Malleability



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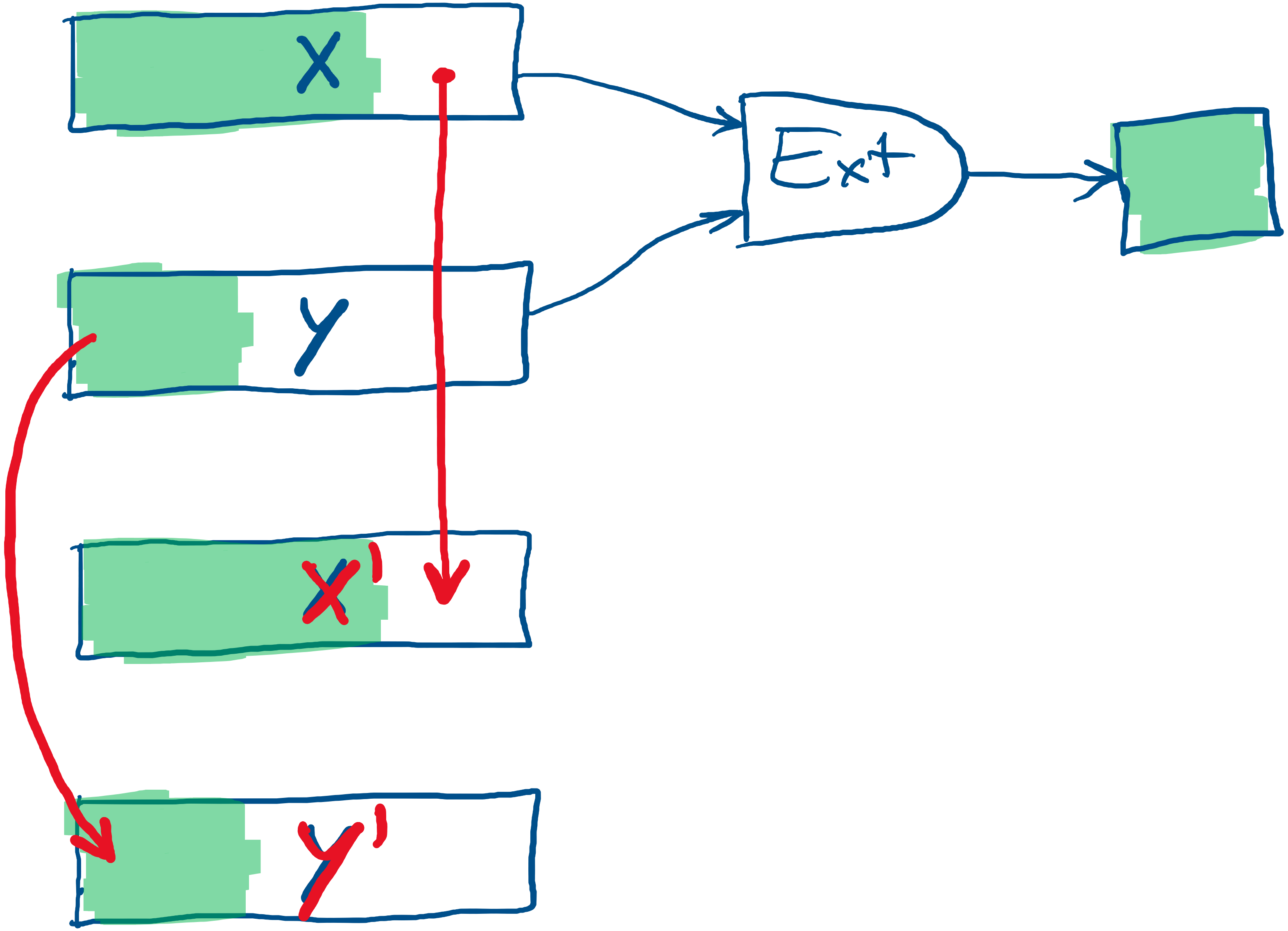

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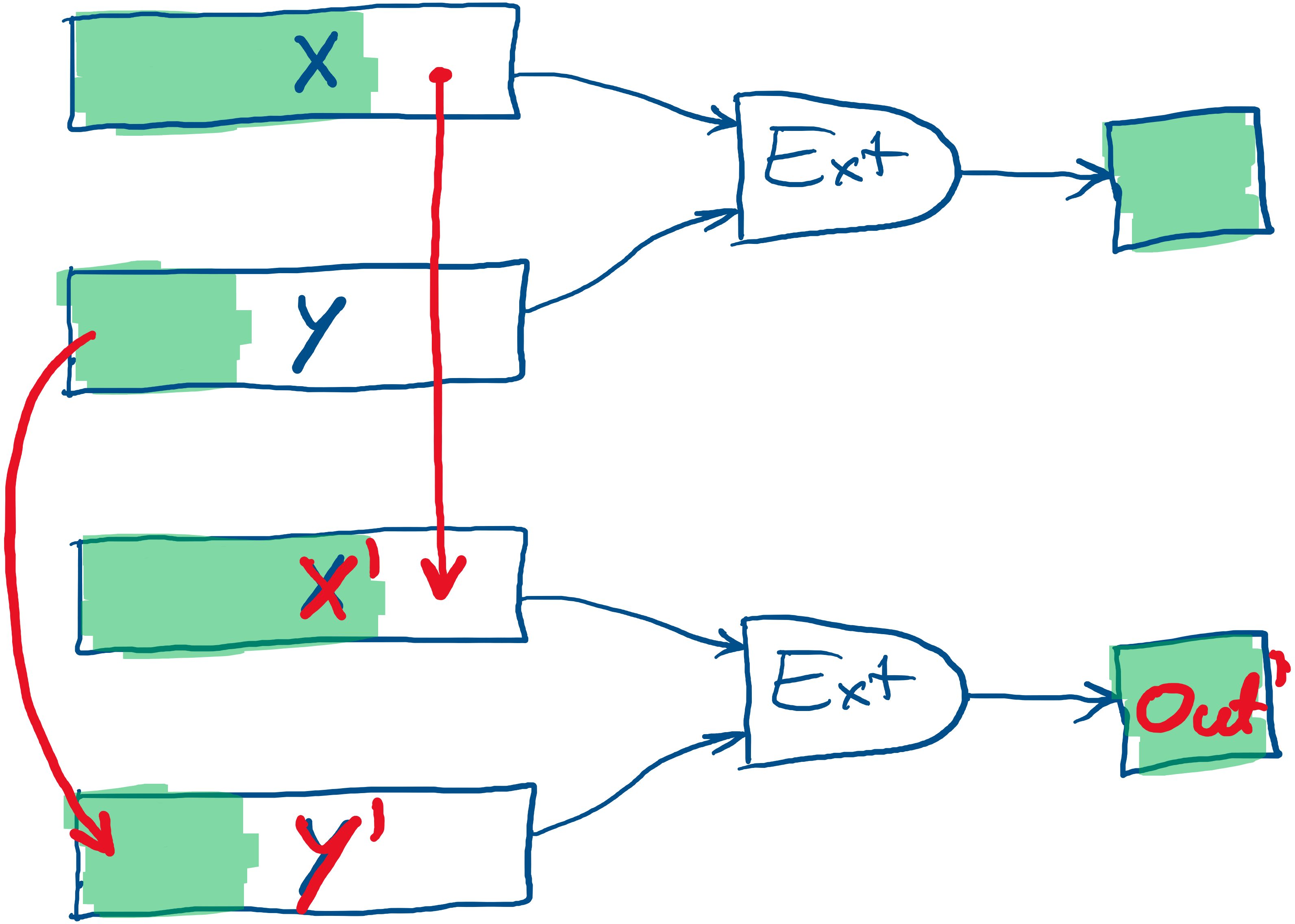

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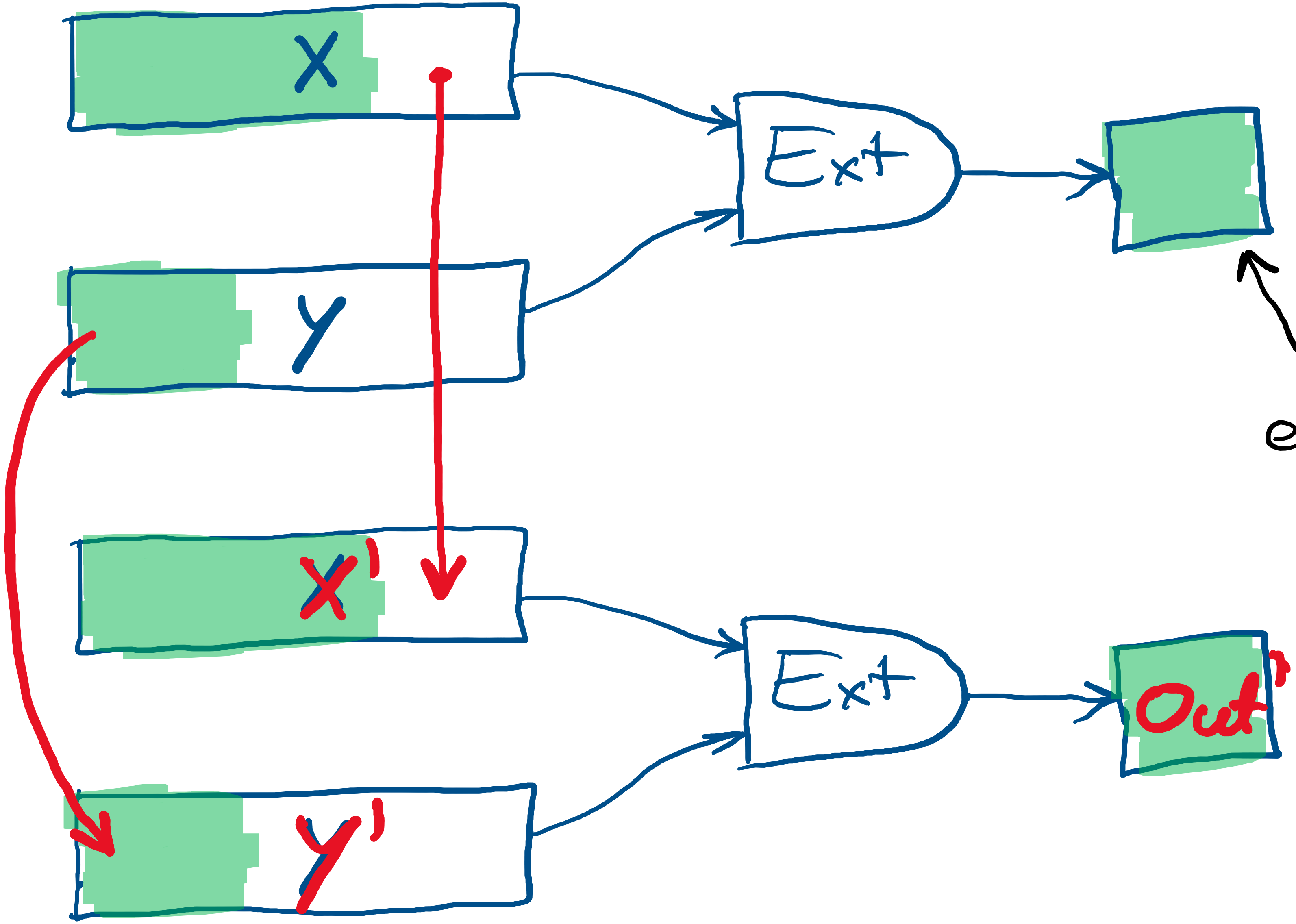
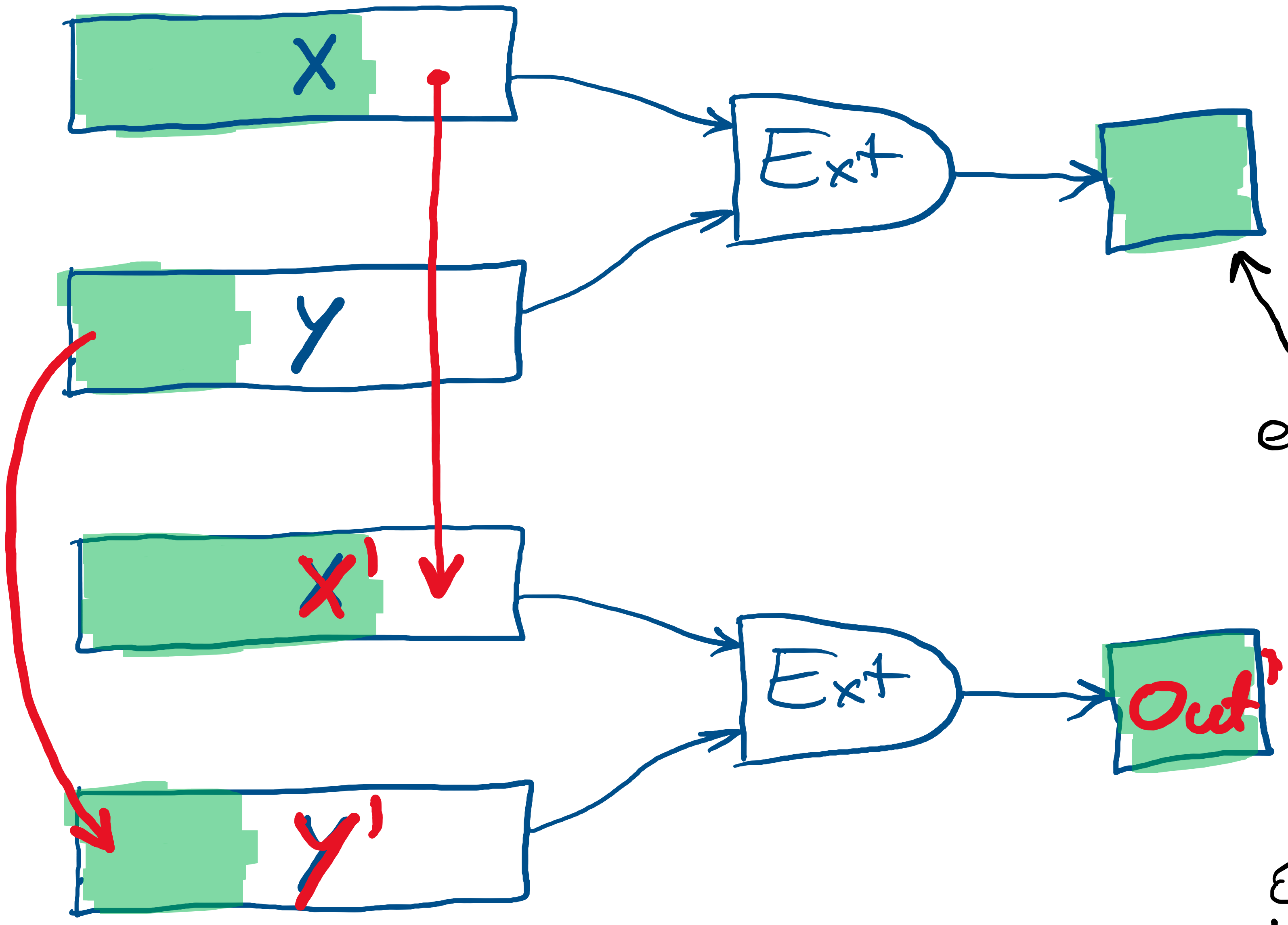


Remains uniform even given Out'

Non-Malleability



Remains uniform even given Out'

Even if X and X' are correlated, and Y, Y' are correlated Out and Out' are Independent.

What we Know (negligible error)

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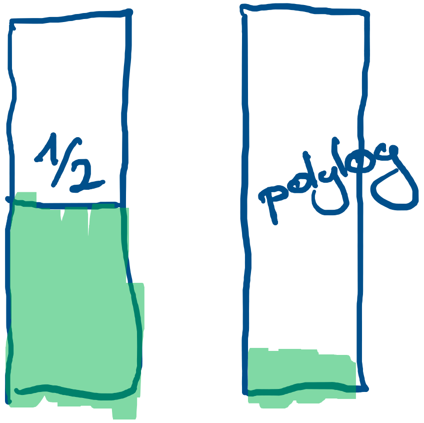
Raz



Not

NM

What we know (negligible error)

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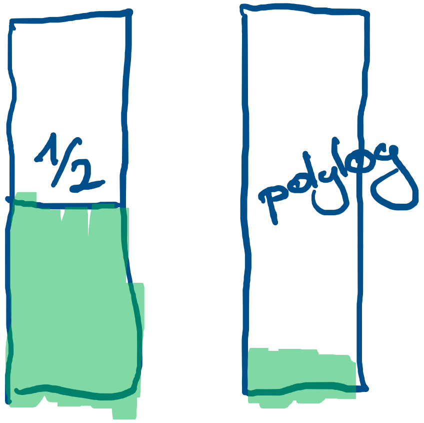
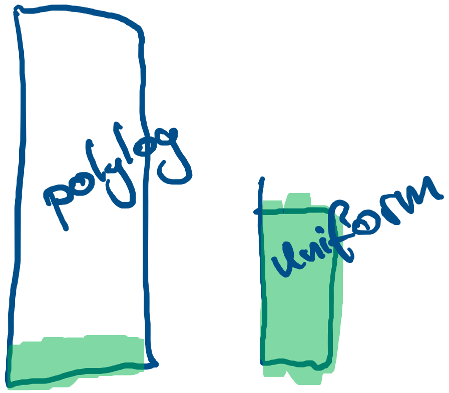


Not NM

Seeded nmExt



NM with respect to seed

What we Know (negligible error)

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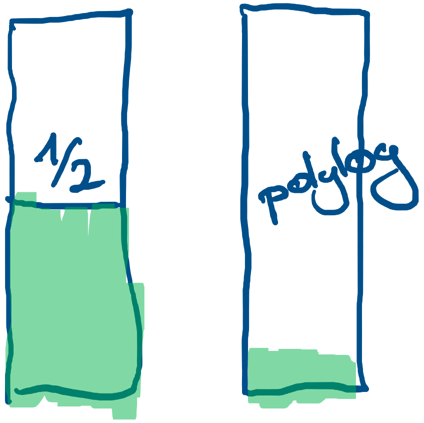

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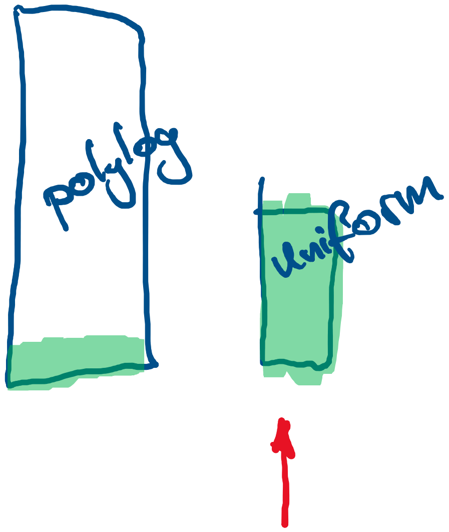
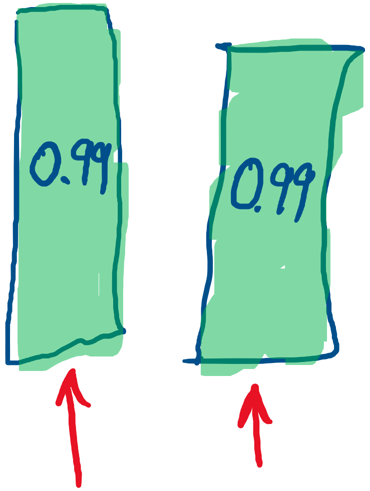


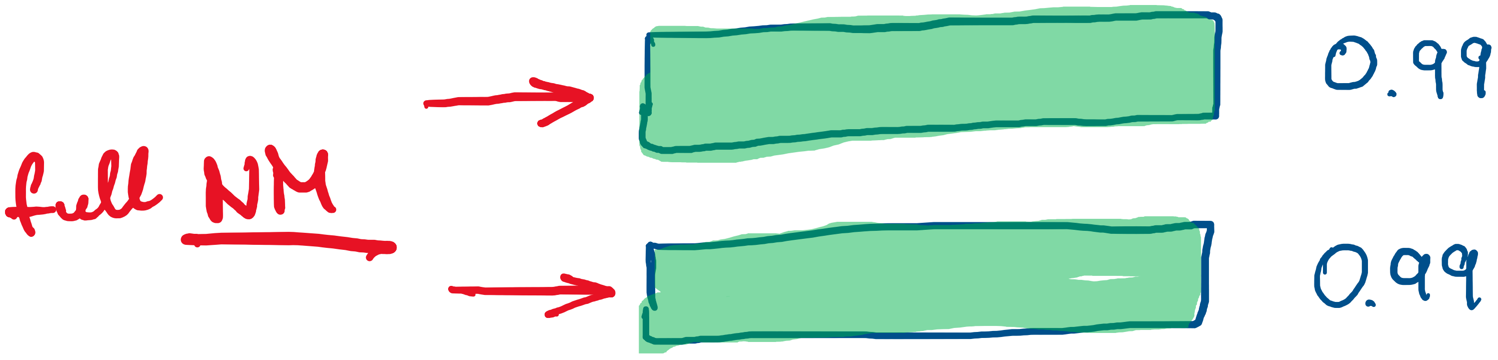
NM with respect to seed

2 nmExt

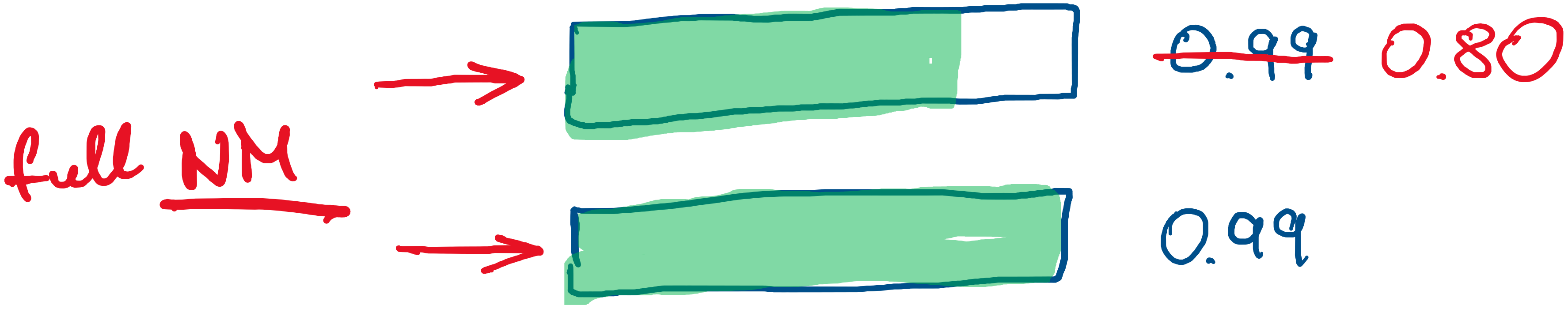


full NM

What we know NOW



What we Know NOW



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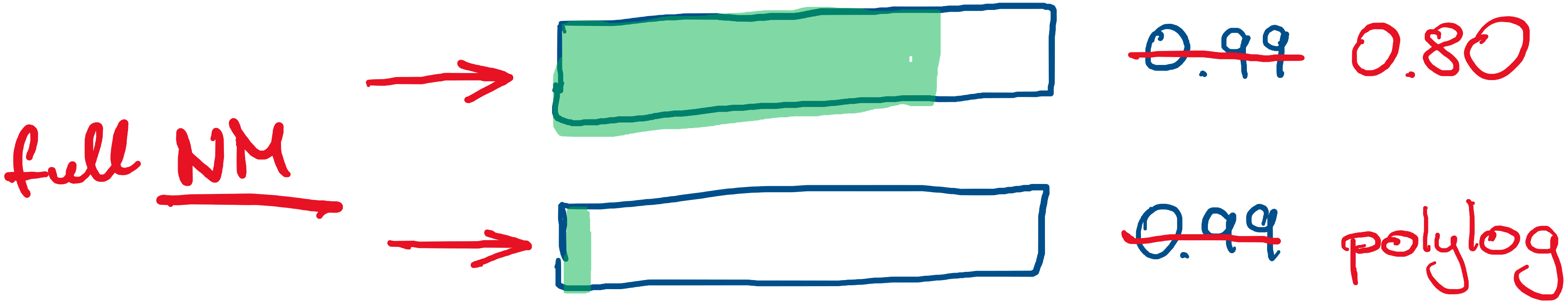
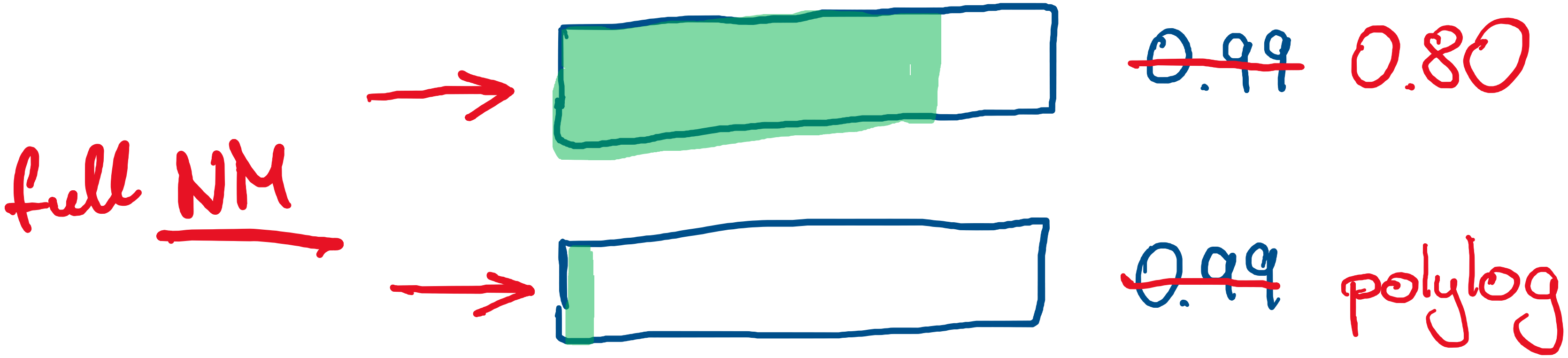


Best of all Worlds



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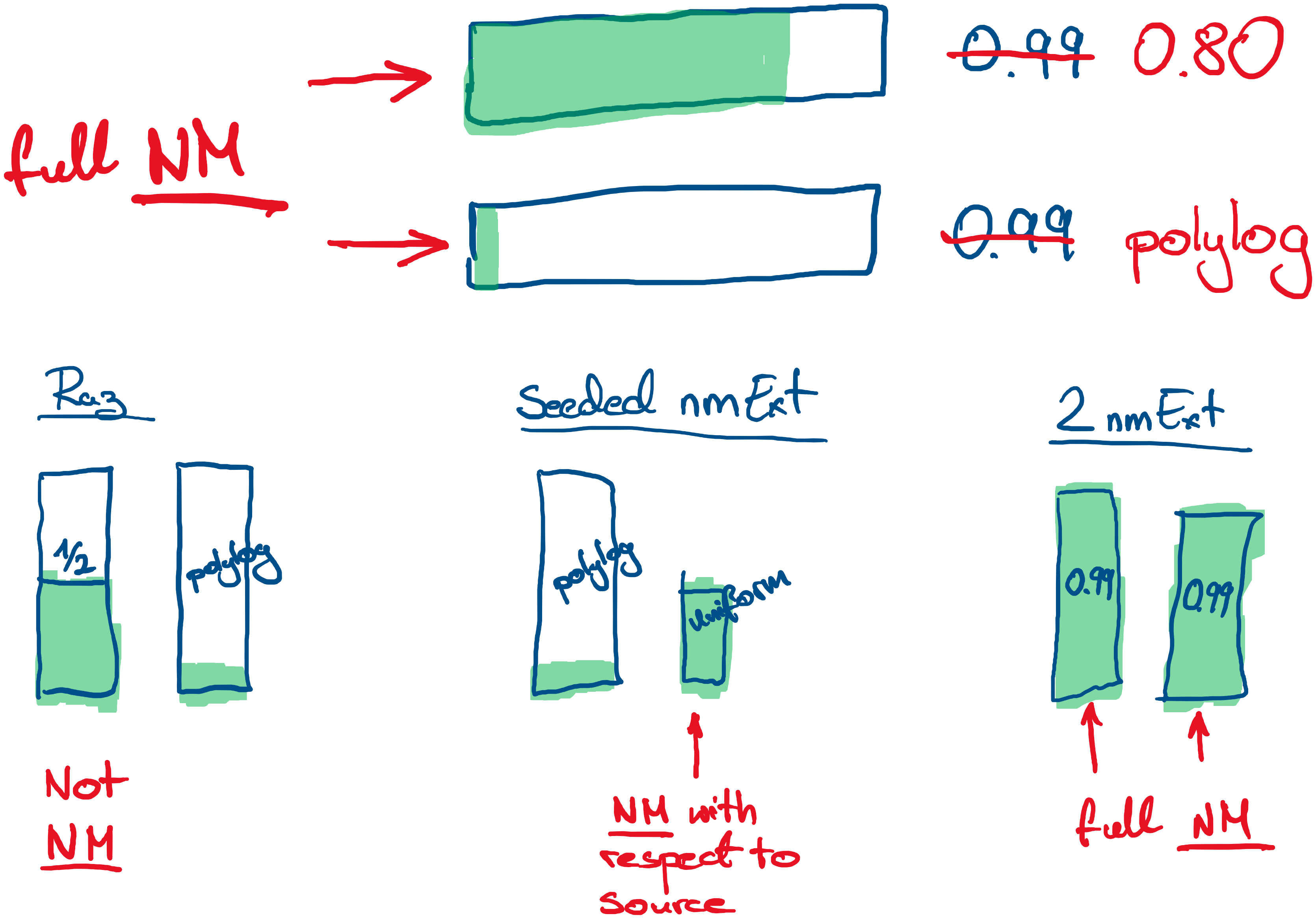

A bit more history

Goyal, Srinivasan, Zhu '21 considered following

$$2\mathrm{NMExt}(\boxed{X}, \boxed{Y_1 \| Y_2}) =$$

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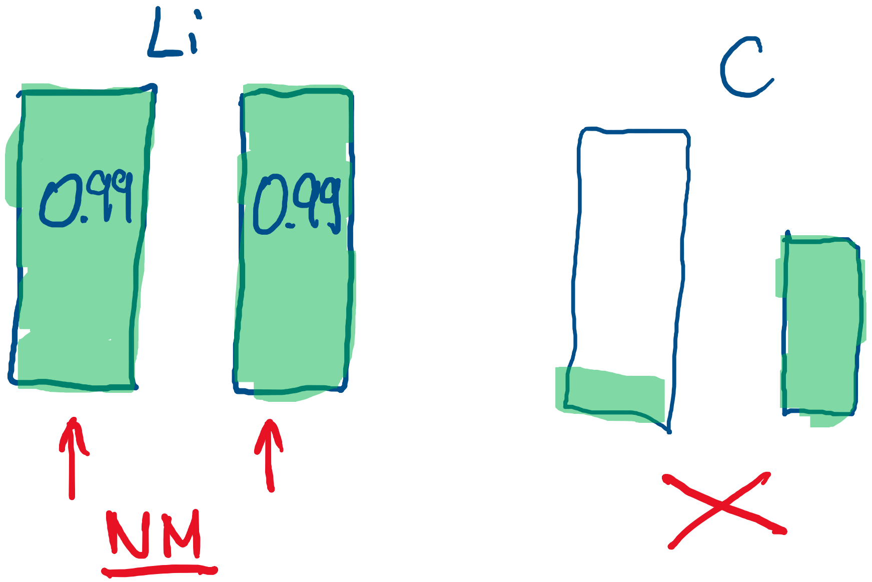

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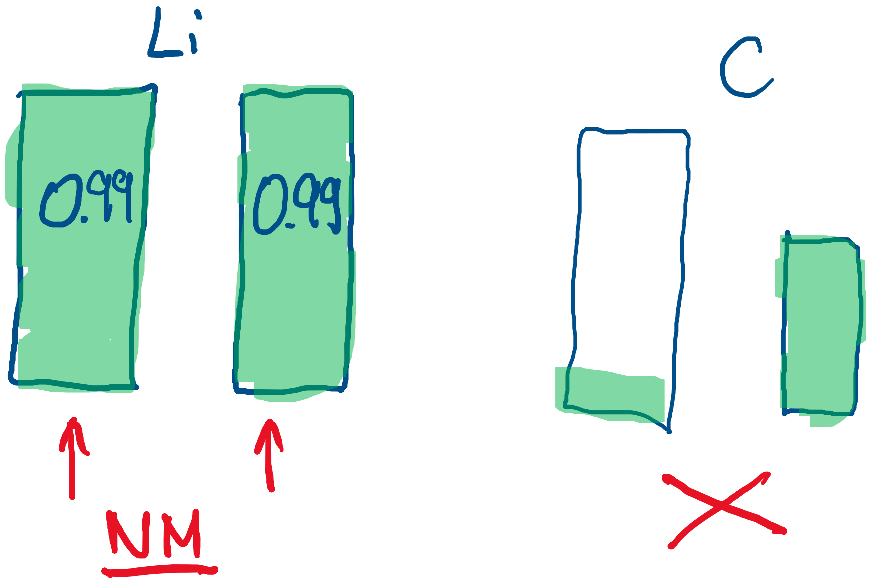

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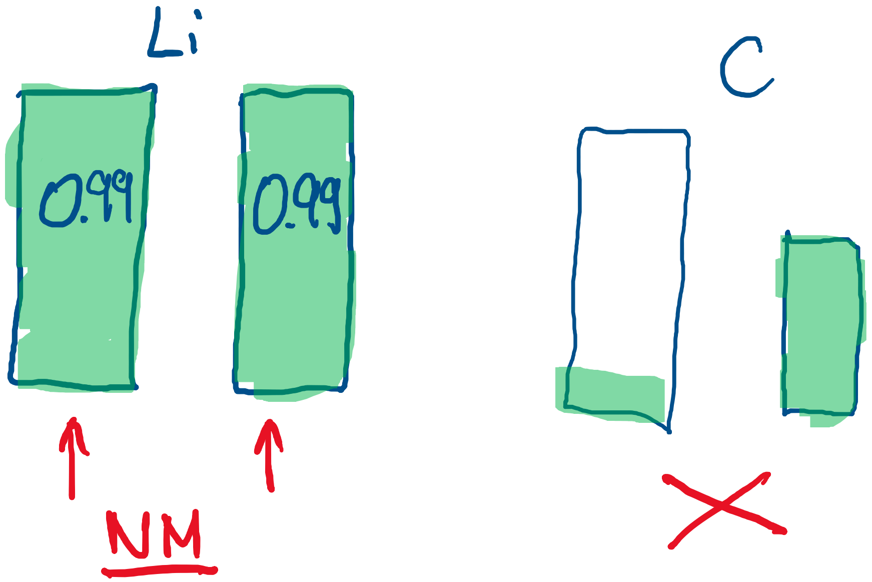

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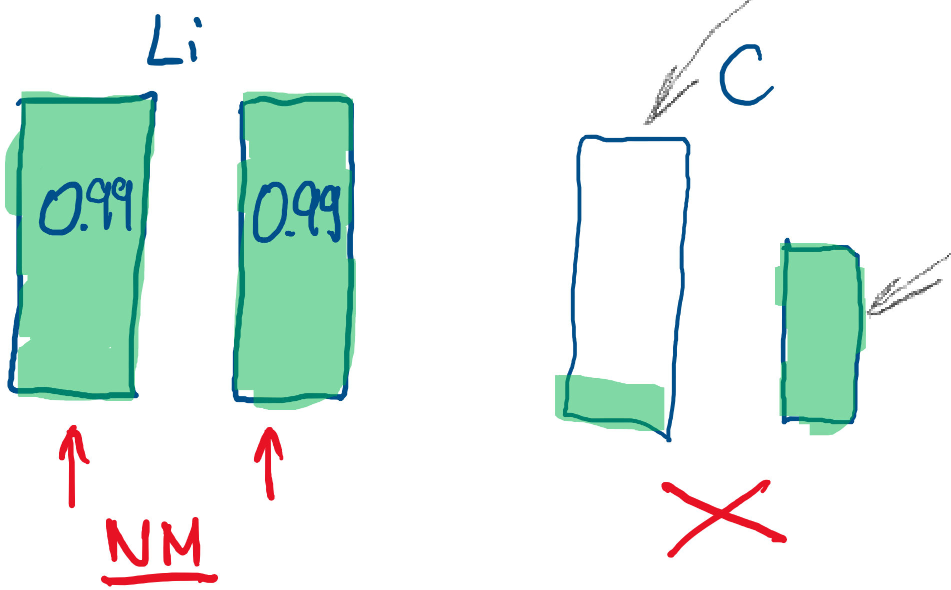

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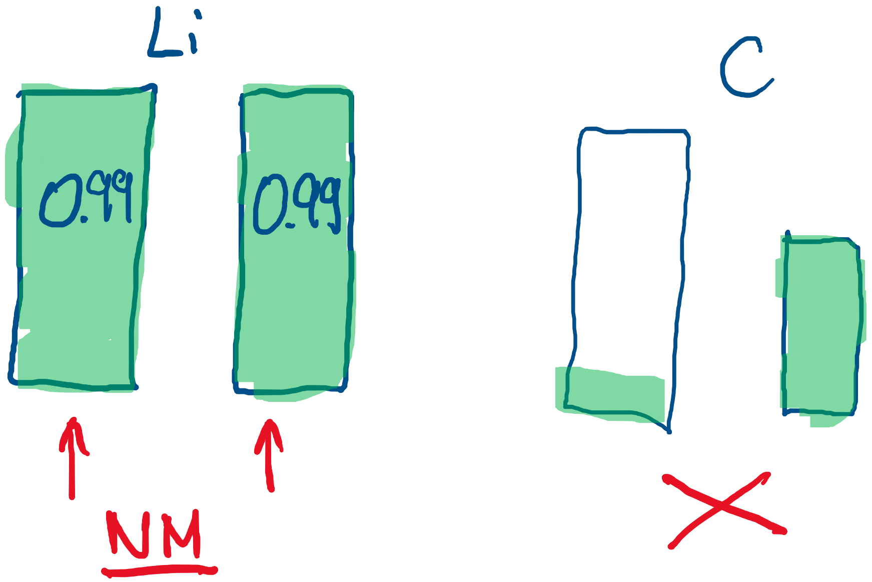

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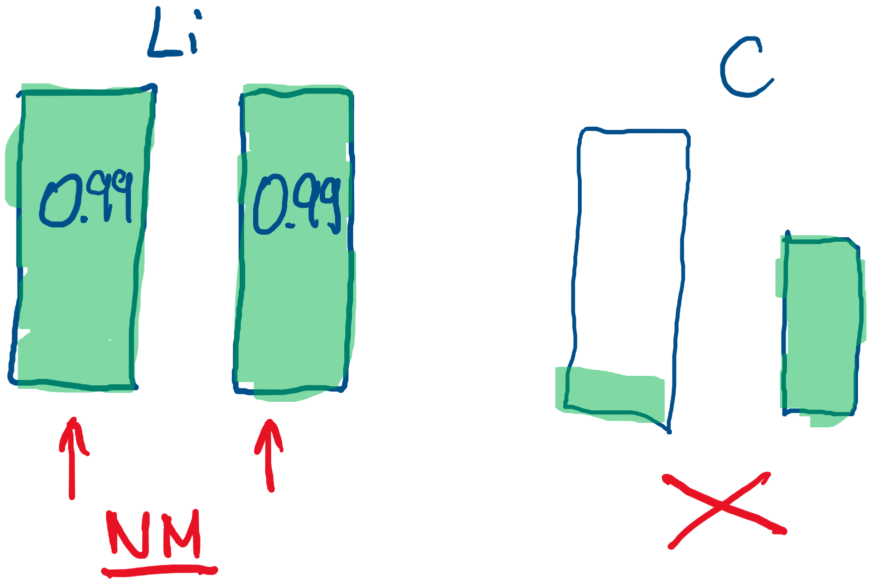

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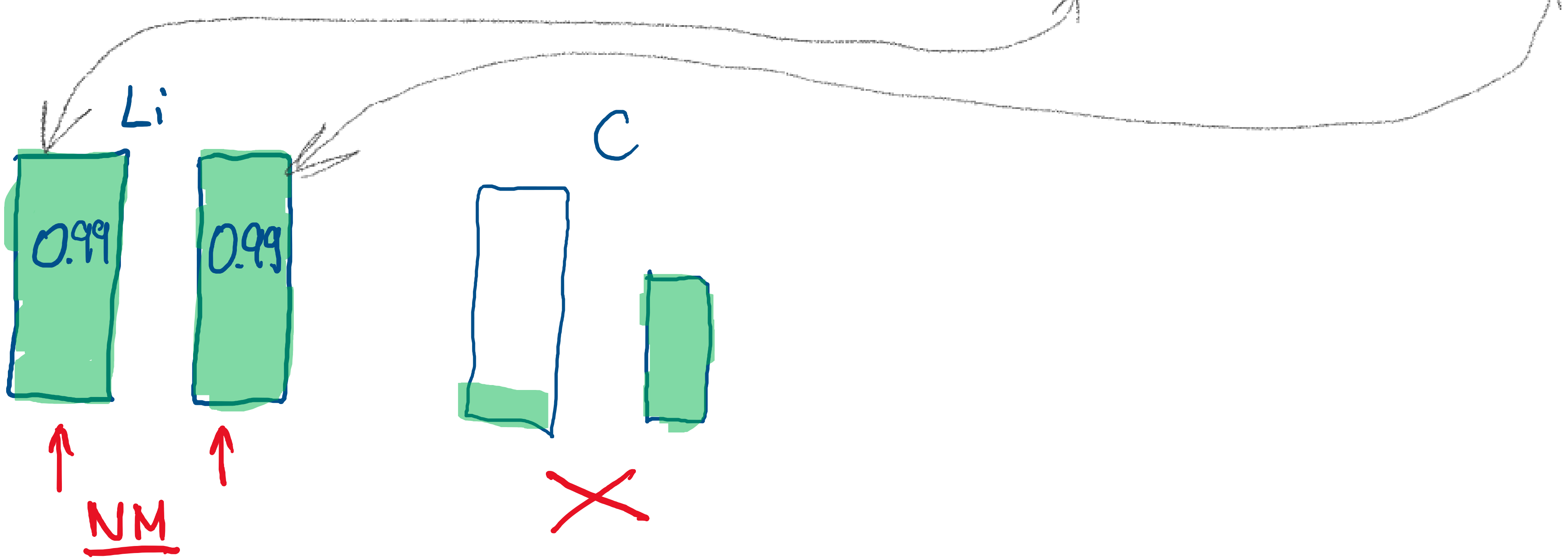

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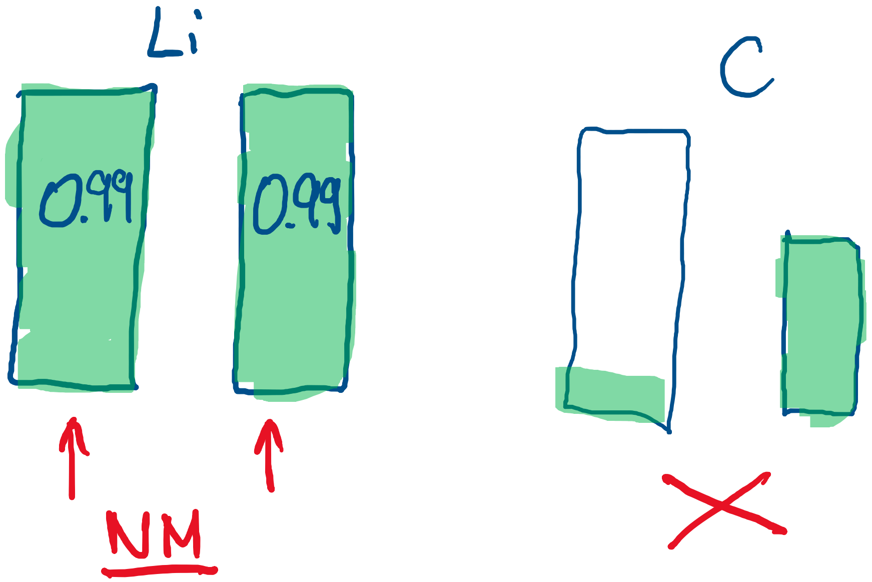

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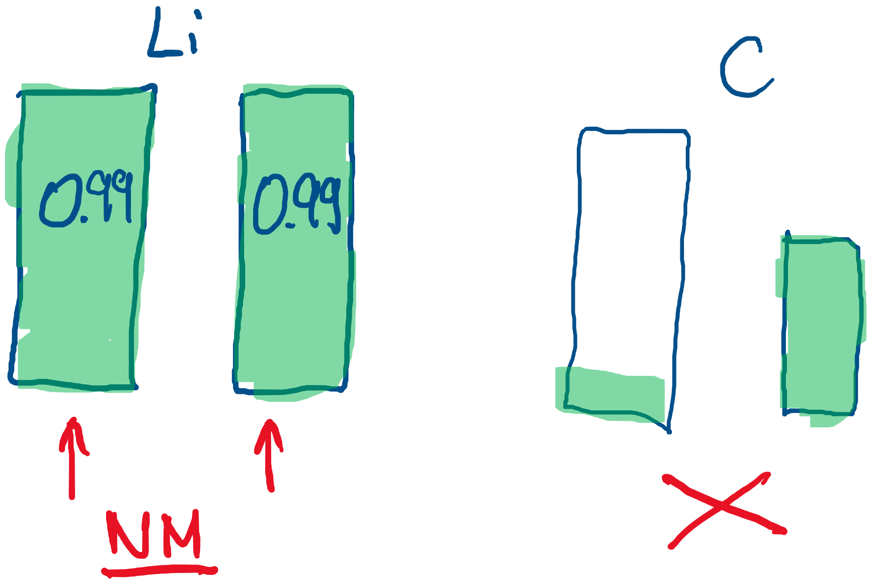

Problem

if $Y_1' \neq Y_1$

$X' \neq X$

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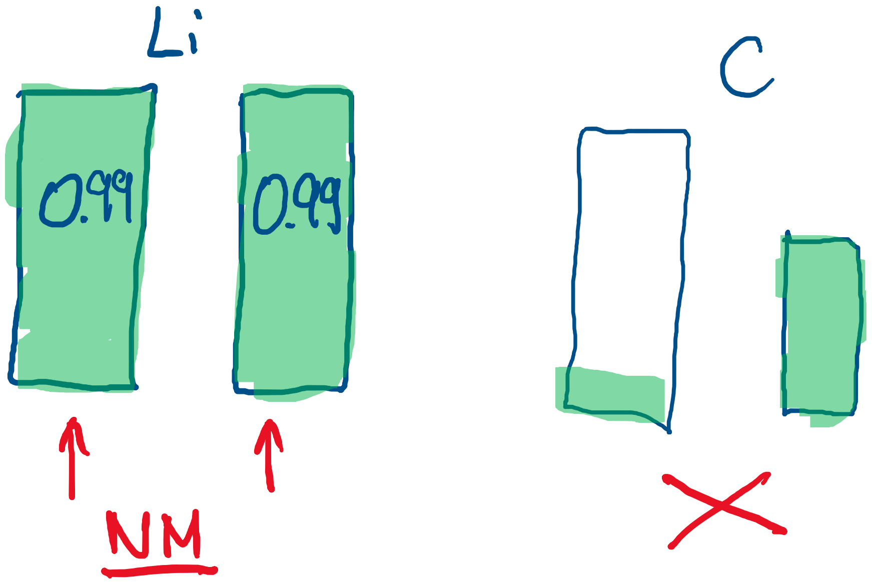

Problem

if $Y_1' \neq Y_1$ but $C(X', Y_1') = C(X, Y_1)$

$X' \neq X$ then

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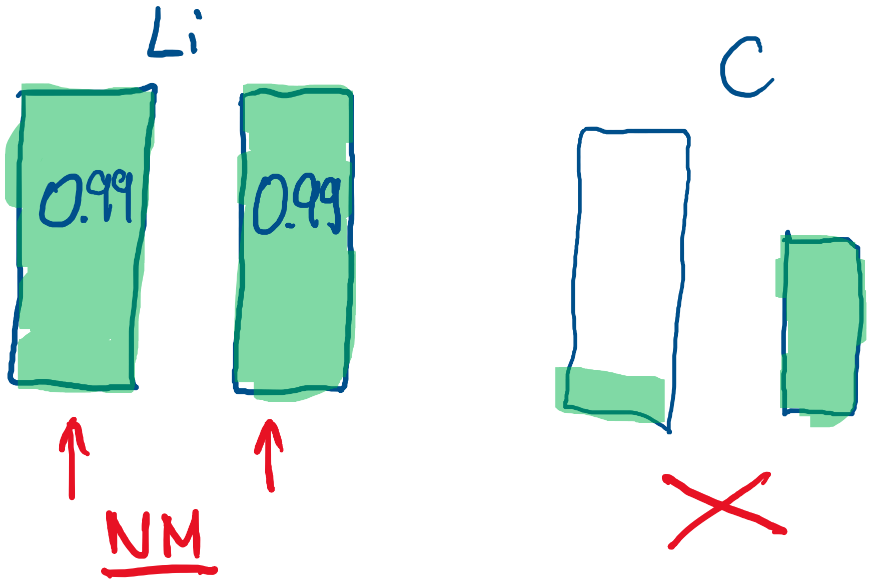

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$X' \neq X$ then

$Out' = Out.$

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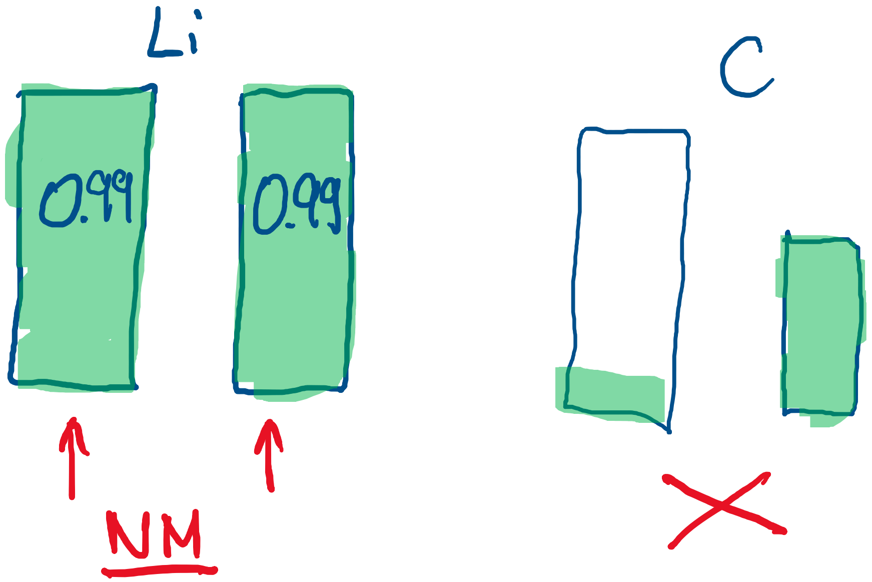
also has to be uniform since Li doesn't tollerate low entropy.

$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{Li}\left(C\left(\boxed{X}, \boxed{Y_1}\right); \boxed{Y_2}\right)$$



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then

$\mathrm{Out}' = \mathrm{Out}.$

GSZ transform with a twist(s)

Original

$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{Li}\left(C\left(\boxed{X}, \boxed{Y_1}\right); \boxed{Y_2} \right)$$

GSZ transform with a twist(s)

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Twist

$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = E\left(\qquad\qquad\qquad \right)$$

GSZ transform with a twist(s)

Original

$$\mathrm{2NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{Li}\left(C\left(\boxed{X}, \boxed{Y_1}\right); \boxed{Y_2}\right)$$

Twist

$$\mathrm{2NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = E\left(C\left(\boxed{X}, \boxed{Y_1}\right), \qquad \right)$$

GSZ transform with a twist(s)

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$$2\text{NMExt}(\boxed{X}, \boxed{Y_1 \| Y_2}) = E\left(C(\boxed{X}, \boxed{Y_1}), \boxed{Y_1 \| Y_2}\right)$$

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Twist

Problem $C(X', Y_1') = C(X, Y_1)$

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$$2NMExt(\boxed{X}, \boxed{Y_1 \| Y_2}) = E(C(\boxed{X}, \boxed{Y_1}), \boxed{Y_1 \| Y_2})$$

if $Y_1' \neq Y_1$ E will see it!

GSZ transform with a twist(s)

Original

$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{Li}\left(C\left(\boxed{X}, \boxed{Y_1}\right); \boxed{Y_2}\right)$$

Twist

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Problem $C(X', Y_1) = C(X, Y_1)$

GS2 transform with a twist(s)

Original

$$2NMExt(\boxed{X}, \boxed{Y_1 \| Y_2}) = Li(C(\boxed{X}, \boxed{Y_1}); \boxed{Y_2})$$

Twist

Problem $C(X', Y_1') = C(X, Y_1)$

$$2NMExt(\boxed{X}, \boxed{Y_1 \| Y_2}) = E(C(\boxed{X}, \boxed{Y_1}), \boxed{Y_1 \| Y_2})$$

Problem

$$C(X', Y_1) = C(X, Y_1)$$

All we need is ~~love~~ Collision Resistance

Leftover Hash Lemma

- if $\mathbb{P}_{Y \leftarrow \$}\big(C(x_0, Y) = C(x_1, Y) \big)$ is tiny for all $x_0 \neq x_1$

  then C is a good Extractor!

Leftover Hash Lemma

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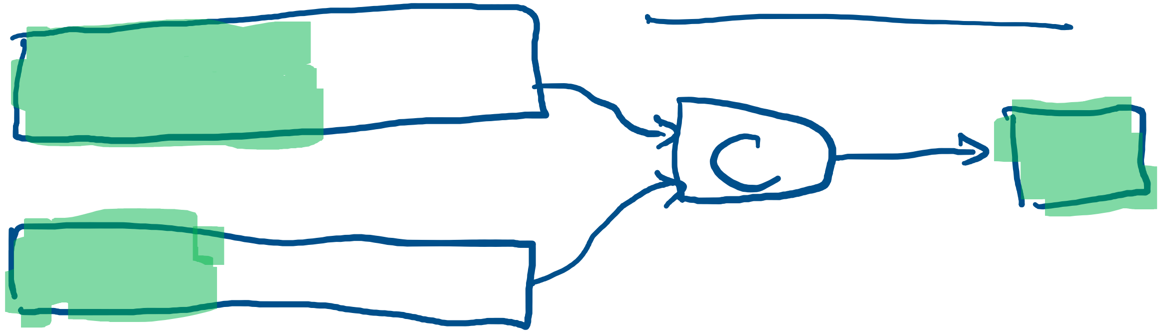
  then C is a good Extractor!

- Infact in [OS'18] we showed inverse is almost true too.

Collision Resilient Extractors.

Extraction



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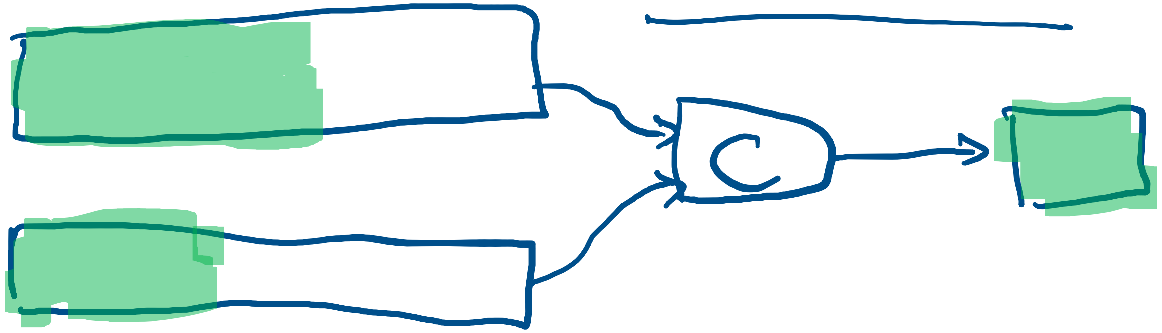


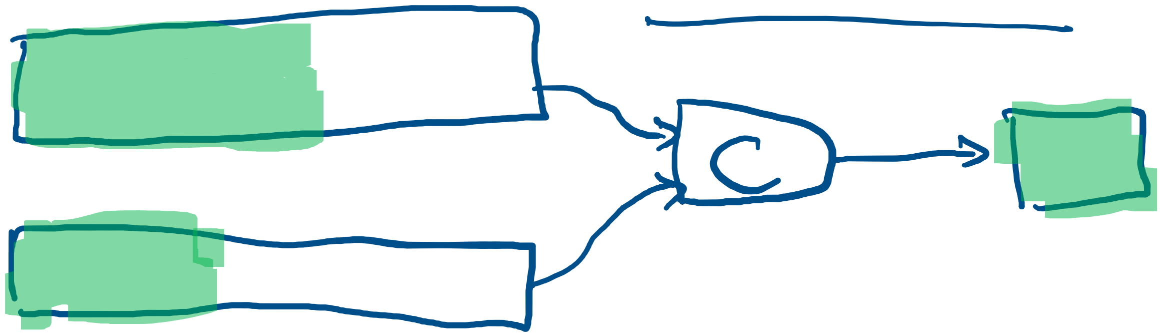
CR

$$\mathbb{P}_{Y \leftarrow \$}\left(C(X, Y) = C(X', Y)\right) \leq \text{tiny}$$

X and X'
are arbitrarly
correlated

Collision Resilient Extractors.

Extraction



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X

makes it even easier.

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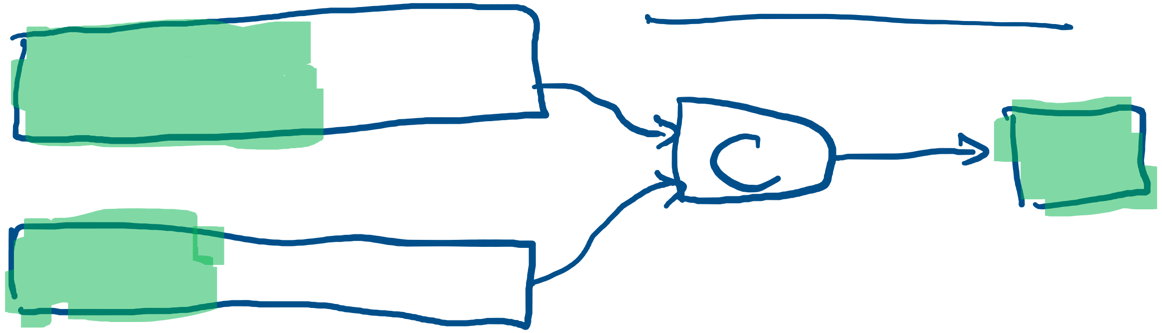

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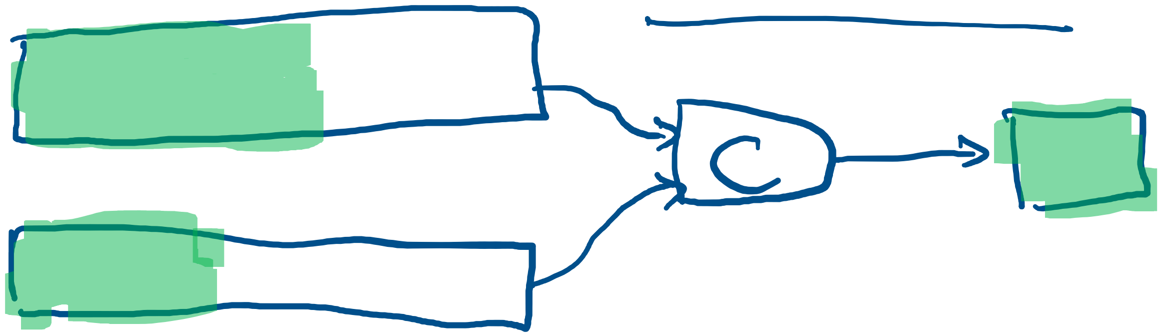
What is CR:

Collision Resilient Extractors.

Extraction



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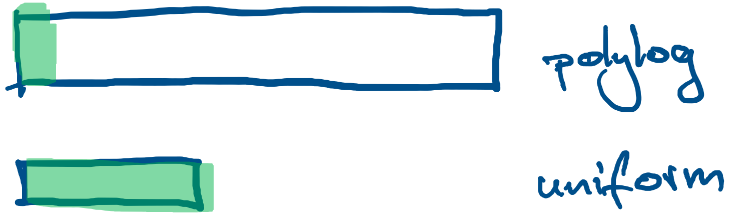
What is CR:

- Seeded Ext (can be compiled)



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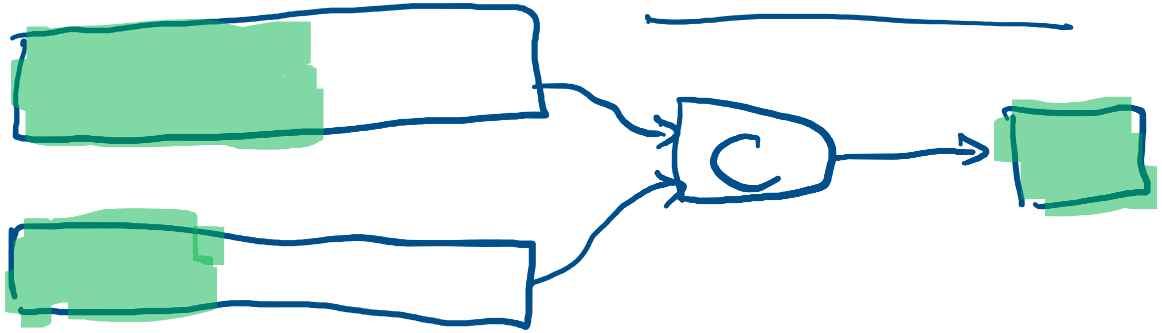

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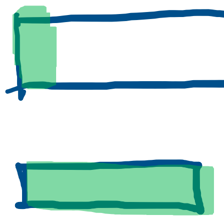
What is CR:

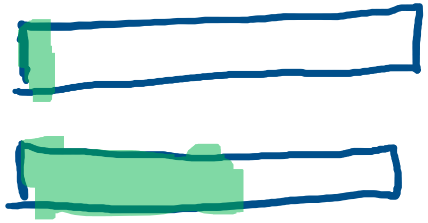
- Seeded Ext (can be compiled)



polylog

uniform

- Raz



polylog

$\frac{1}{2}$

Construction

$$\underline{\mathrm{FNMExt}}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \underline{\mathrm{Li}}\left(\underline{\mathrm{Tre}}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

Construction

$$\underline{\mathrm{FNMExt}}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \underline{\mathrm{Li}}\left(\underline{\mathrm{Tre}}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

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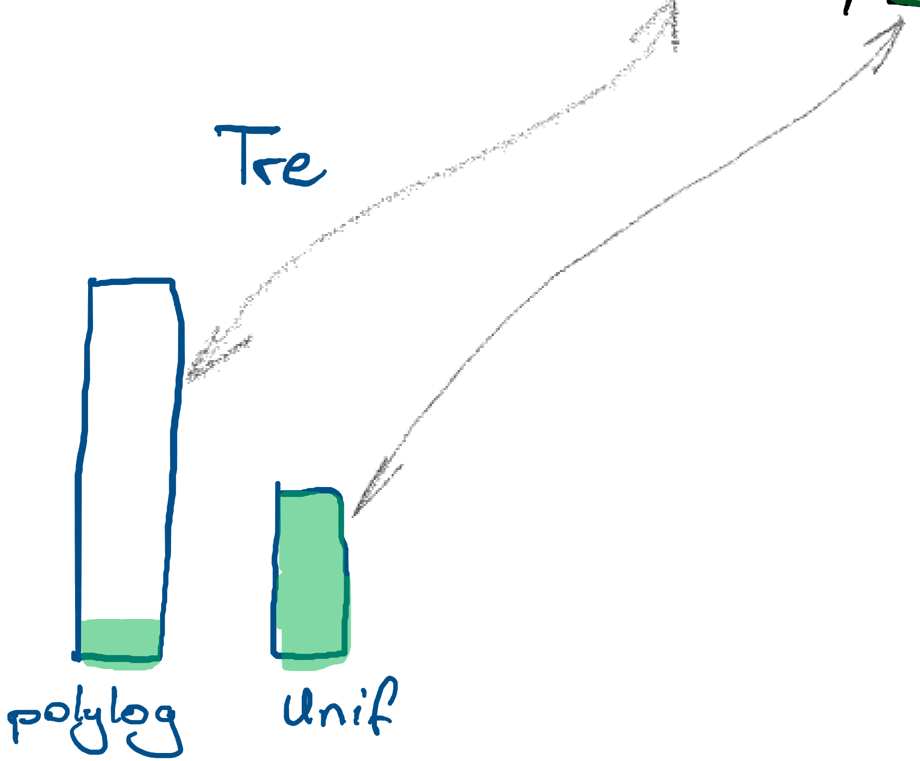
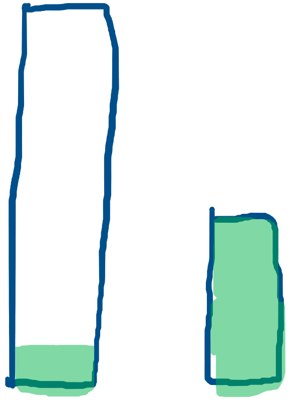


Construction

$$\underline{\mathrm{FNMExt}}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \underline{\mathrm{Li}}\left(\underline{\mathrm{Tre}}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

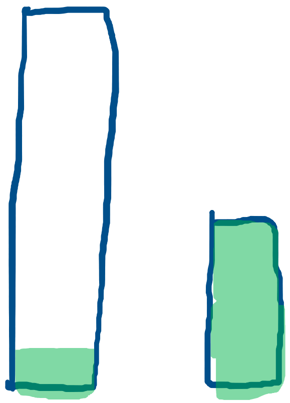
Tre



Construction

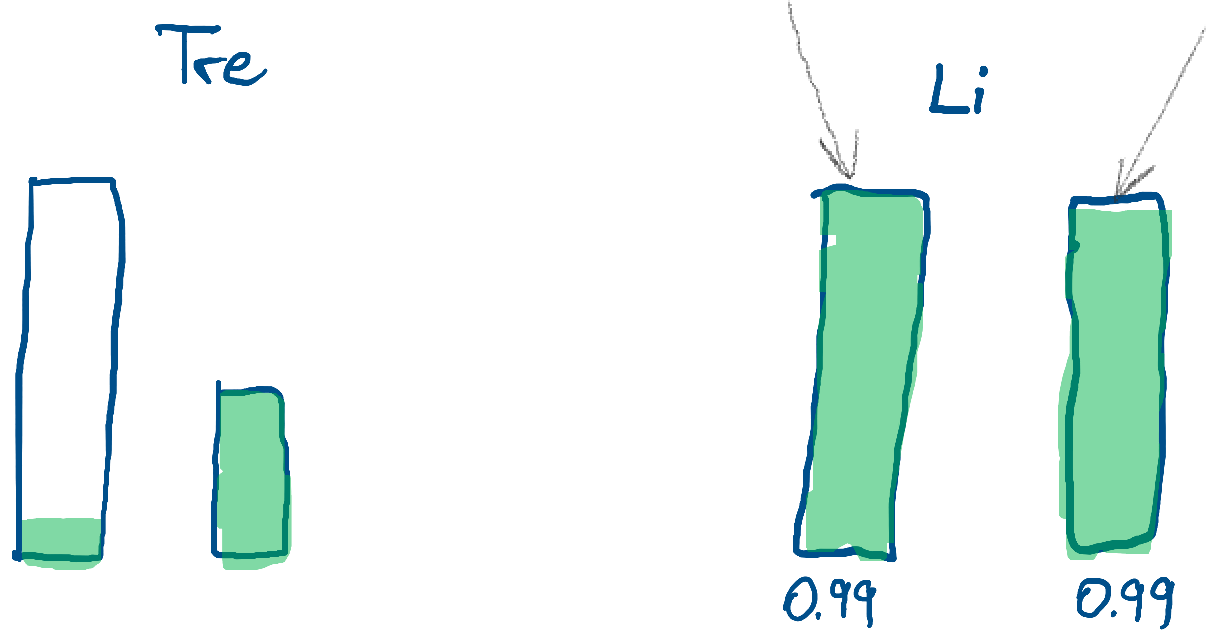
$$\underline{\mathrm{FNMExt}}\left(\boxed{x}, \boxed{Y_1 \| Y_2}\right) = \underline{\mathrm{Li}}\left(\underline{\mathrm{Tre}}\left(\boxed{x}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

Tre



Construction

$$\underline{\mathrm{FNMExt}}\left(\boxed{x}, \boxed{Y_1 \| Y_2}\right) = \underline{\mathrm{Li}}\left(\underline{\mathrm{Tre}}\left(\boxed{x}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

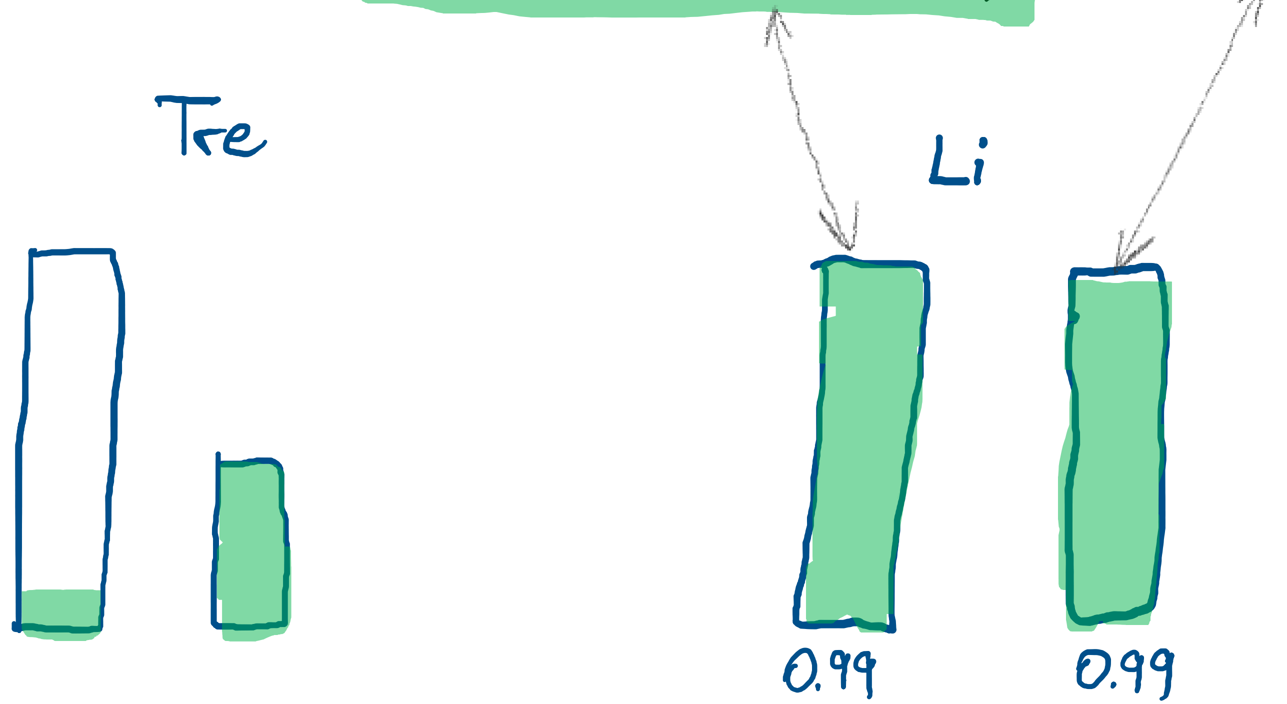


Construction

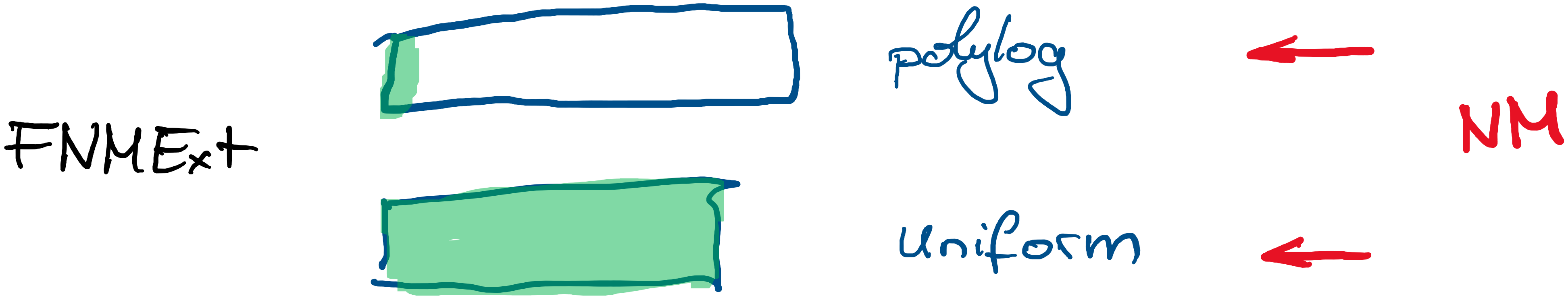
$$\underline{\mathrm{FNMExt}}(\boxed{X}, \boxed{Y_1 \| Y_2}) = \underline{\mathrm{Li}}(\underline{\mathrm{Tre}}(\boxed{X}, \boxed{Y_1}), \boxed{Y_1 \| Y_2})$$

gives NonMalleability

gives low entropy of X.



Construction



Construction

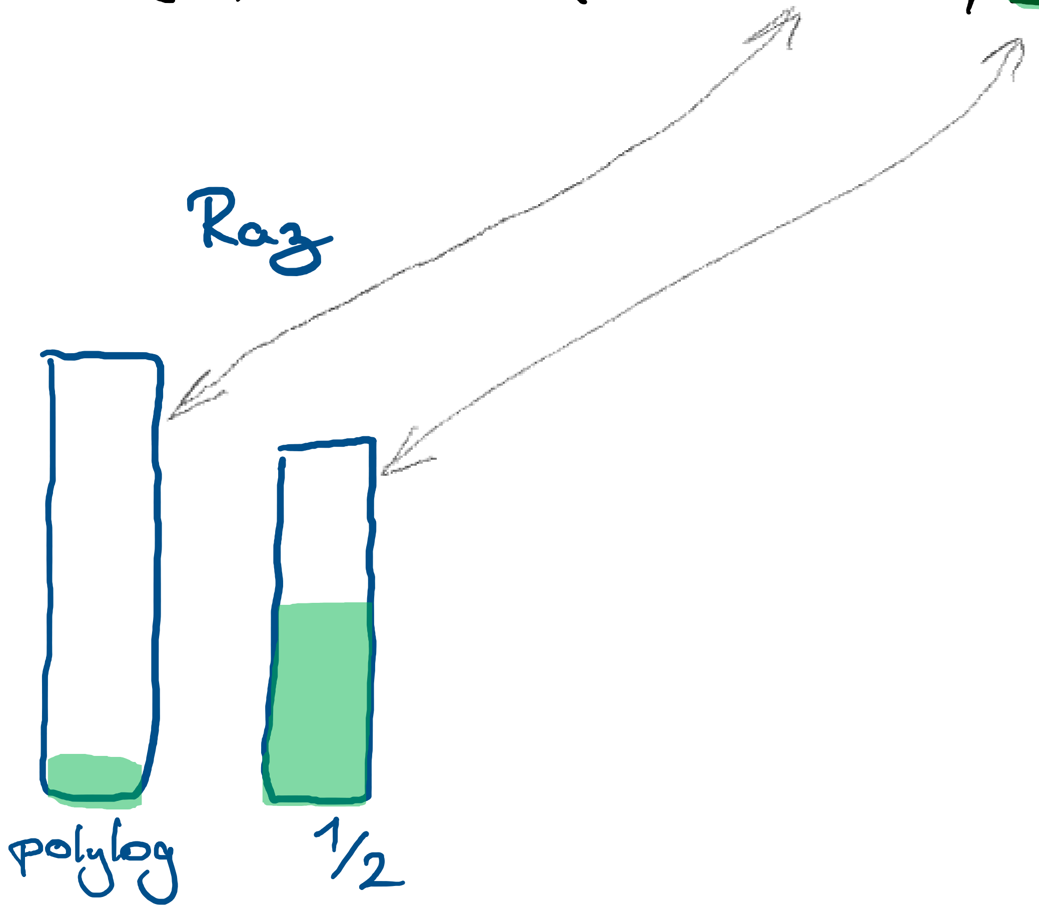
$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{FNMExt}\left(\mathrm{Raz}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

Construction

$$\mathrm{2NMExt}\left(\boxed{X},\ \boxed{Y_1 \| Y_2}\right) = \mathrm{FNMExt}\left(\mathrm{Raz}\left(\boxed{X},\ \boxed{Y_1}\right),\ \boxed{Y_1 \| Y_2}\right)$$

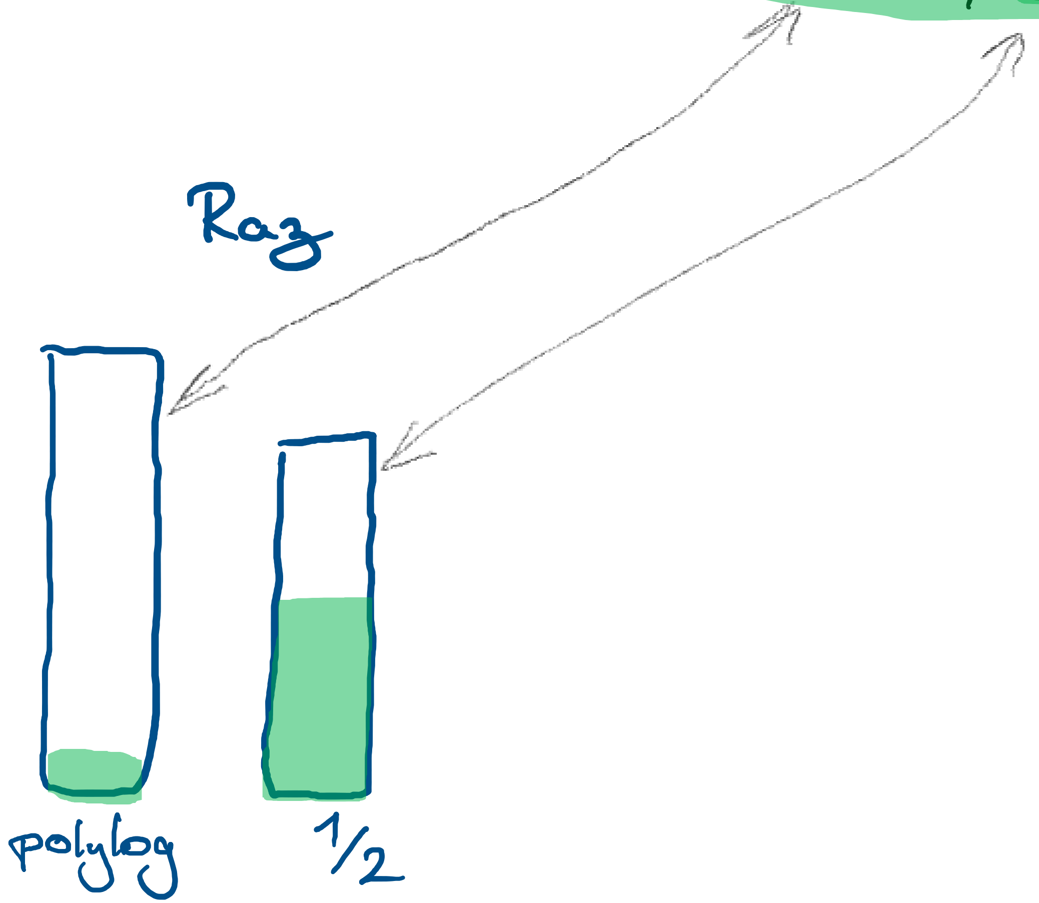
Construction

$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{FNMExt}\left(\mathrm{Raz}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$



Construction

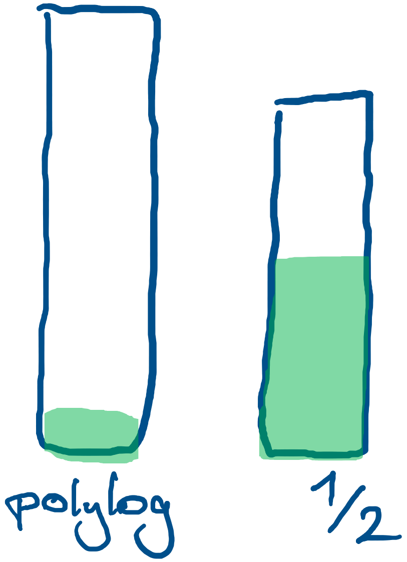
$$2\mathrm{NMExt}\left(\boxed{\;X\;},\ \boxed{Y_1 \| Y_2}\right) = \mathrm{FNMExt}\left(\mathrm{Raz}\left(\boxed{\;X\;},\ \boxed{Y_1\;}\right),\ \boxed{Y_1 \| Y_2}\right)$$



Construction

$$2\mathrm{NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{FNMExt}\left(\mathrm{Raz}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

Raz

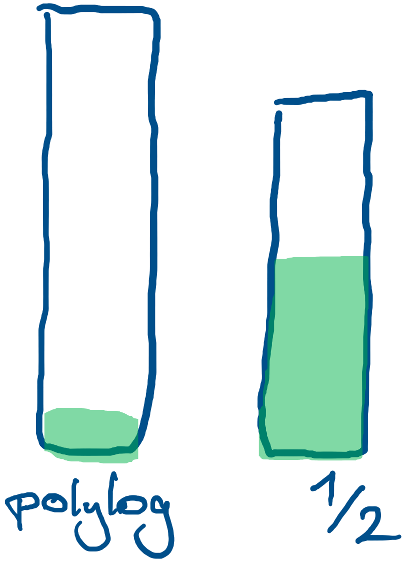


Construction

$$2\text{NMExt}(\boxed{X}, \boxed{Y_1 \| Y_2}) = \text{FNMExt}(\text{Raz}(\boxed{X}, \boxed{Y_1}), \boxed{Y_1 \| Y_2})$$

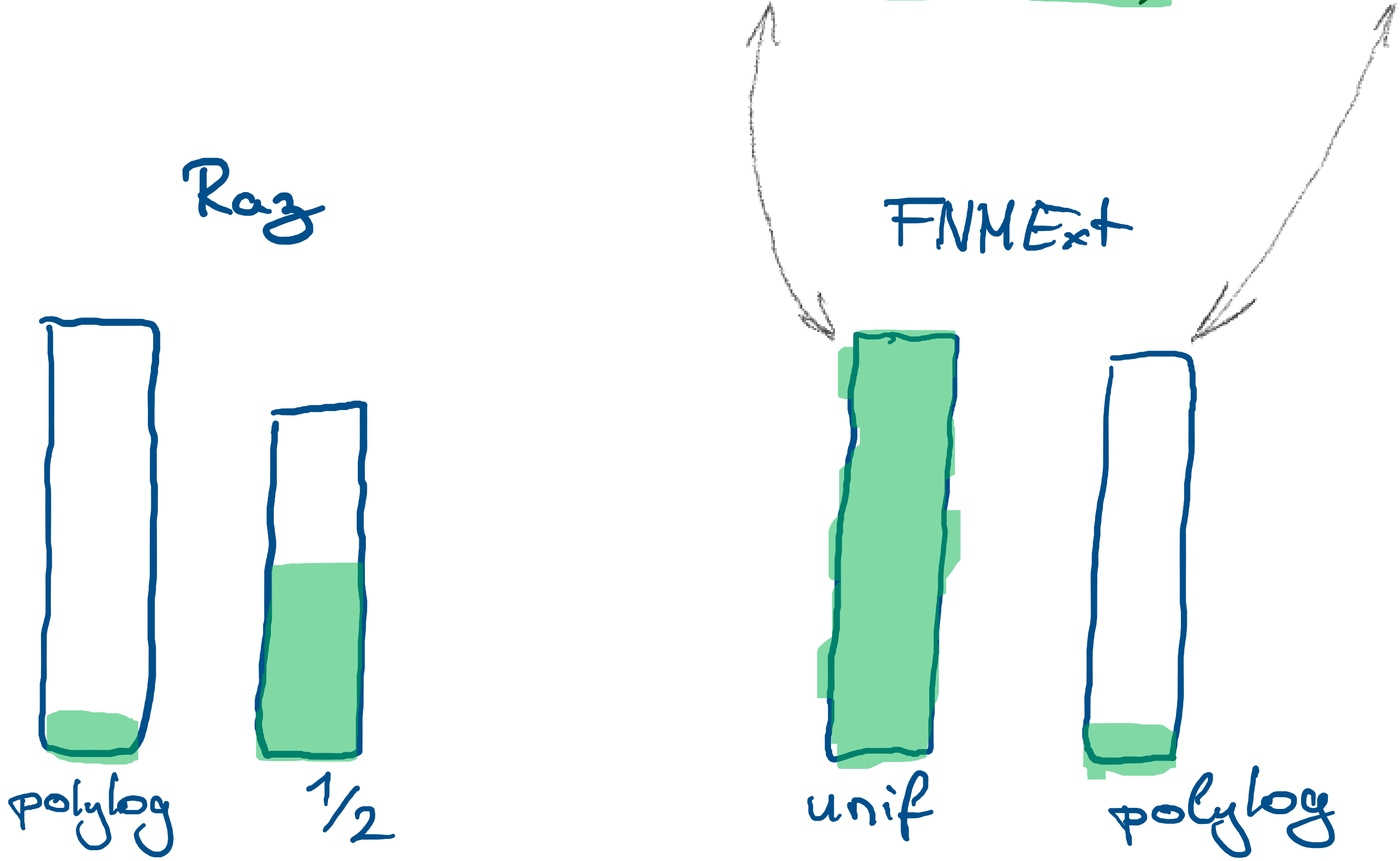
Entropy in Y_1 has been „used up" so there is little left.

Raz



Construction

$$\mathrm{2NMExt}\left(\boxed{X}, \boxed{Y_1 \| Y_2}\right) = \mathrm{FNMExt}\left(\mathrm{Raz}\left(\boxed{X}, \boxed{Y_1}\right), \boxed{Y_1 \| Y_2}\right)$$

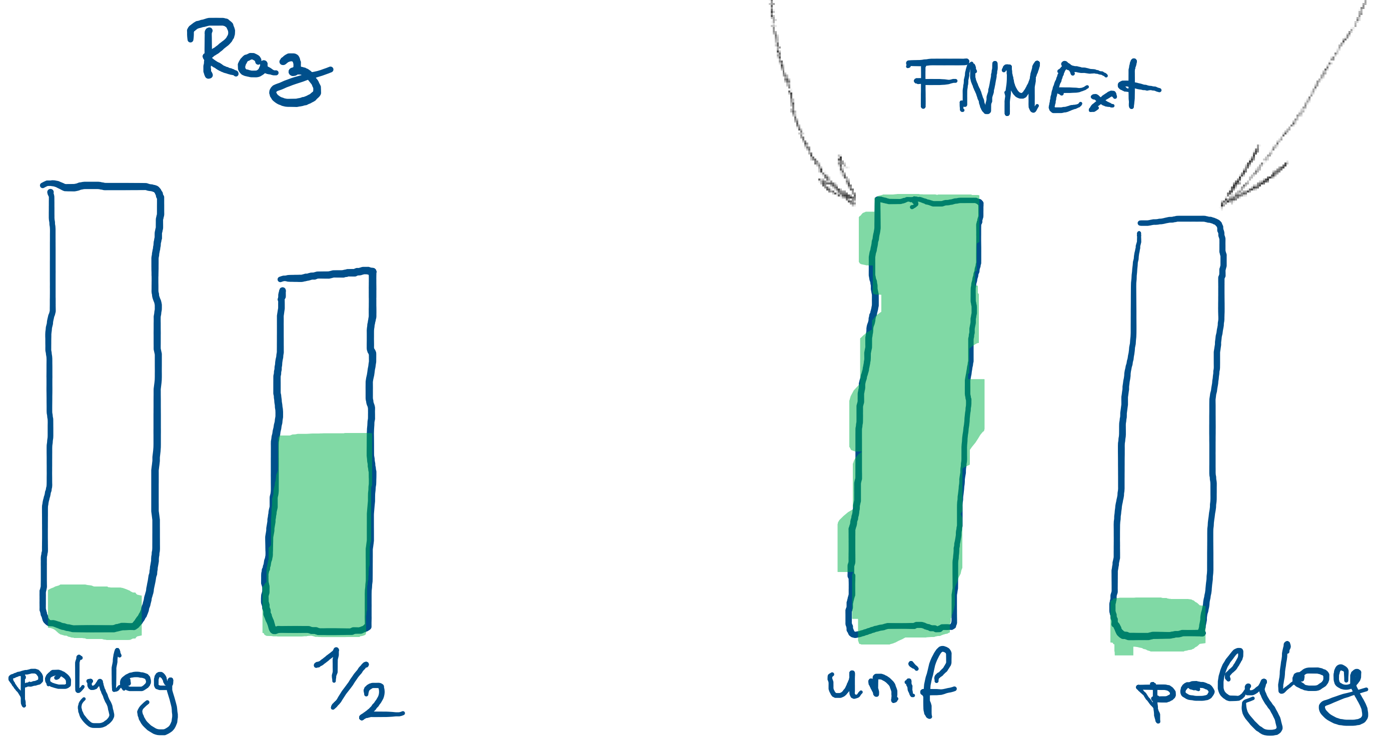


Construction

gives NonMalleability

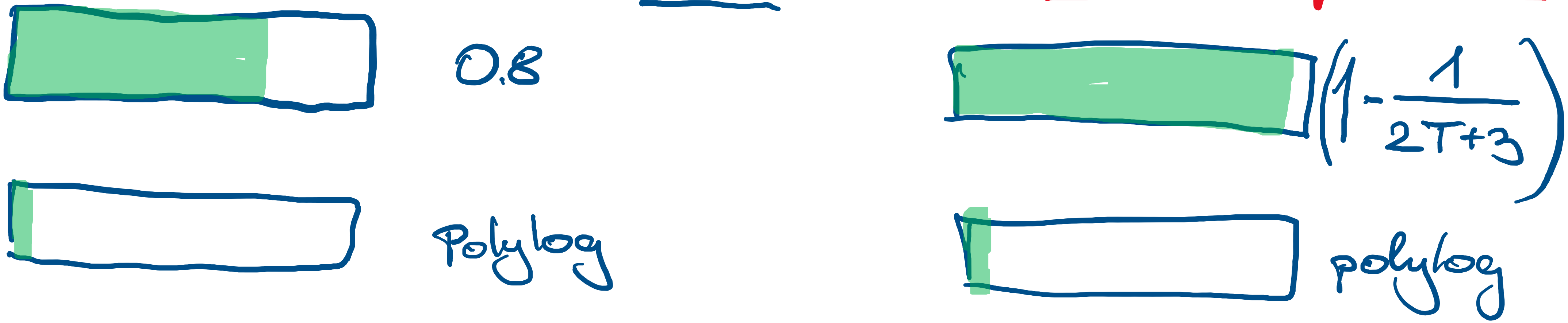
allows for lower entropies

$$\mathrm{2NMExt}(\boxed{X}, \boxed{Y_1 \| Y_2}) = \mathrm{FNMExt}(\mathrm{Raz}(\boxed{X}, \boxed{Y_1}), \boxed{Y_1 \| Y_2})$$



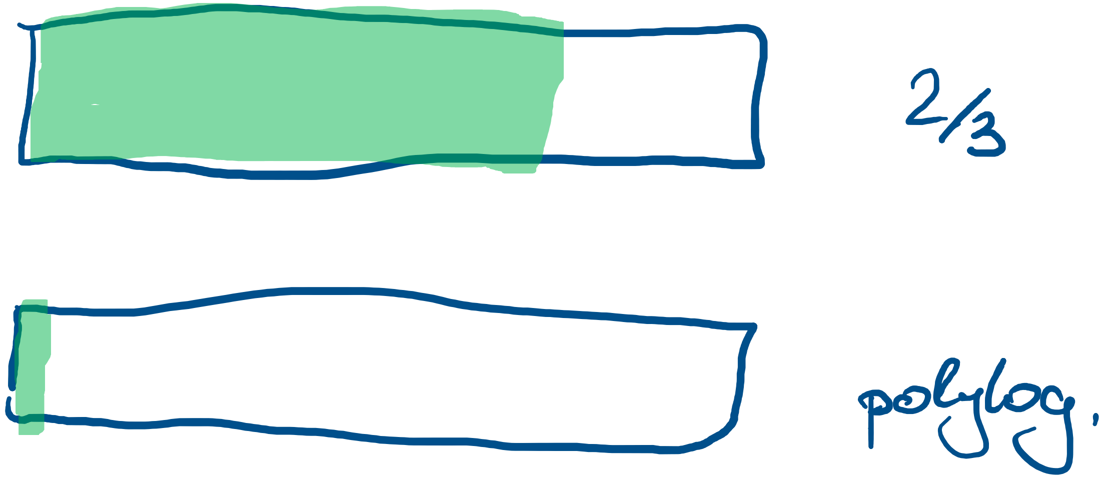
Final Result

Full NM — And — T-Multitamper NM



Subsequent Work

Li :



single tampering.

Why Care?

Why Care ?

2NMExt are cool tools:

NMC

Privacy Amplification

Network Extraction

Leak Resi
Tamper Resi

NM Secret Sharing

Why Care?

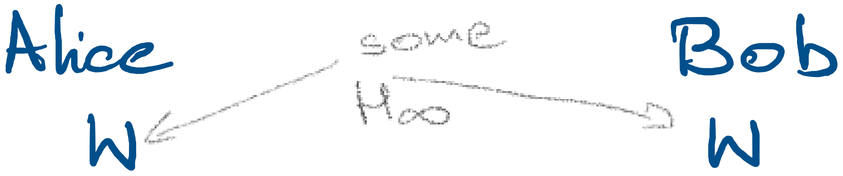
2NMExt are cool tools:

Privacy Amplification

Why Care ?

2NMExt are cool tools:

Privacy Amplification



Alice

Bob

W

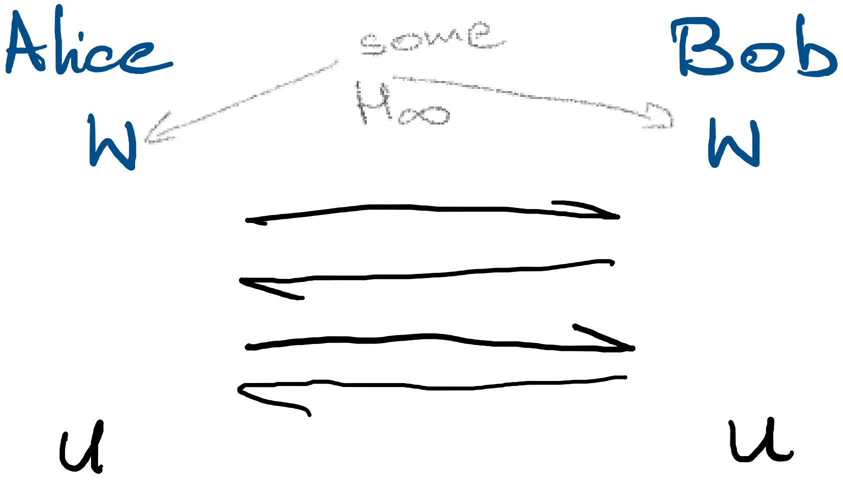
some H_∞

W

Why Care?

2NMExt are cool tools:

Privacy Amplification



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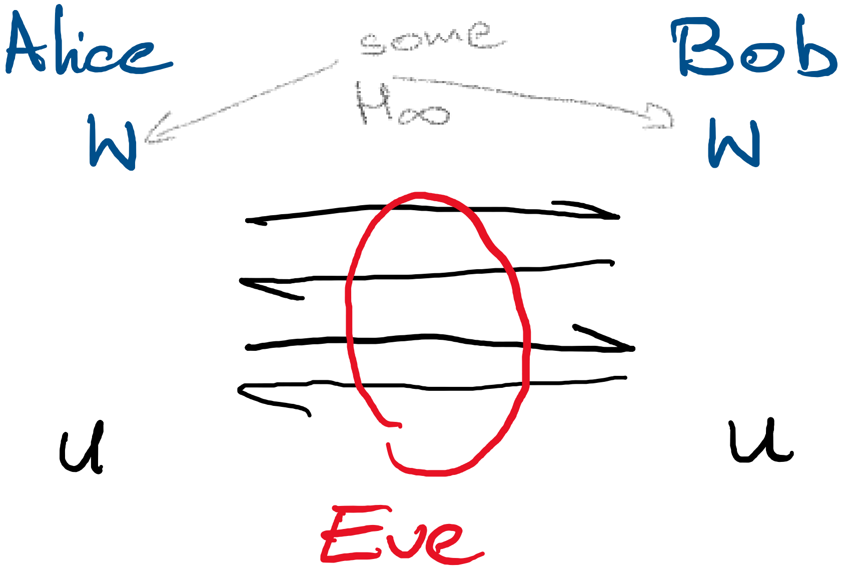
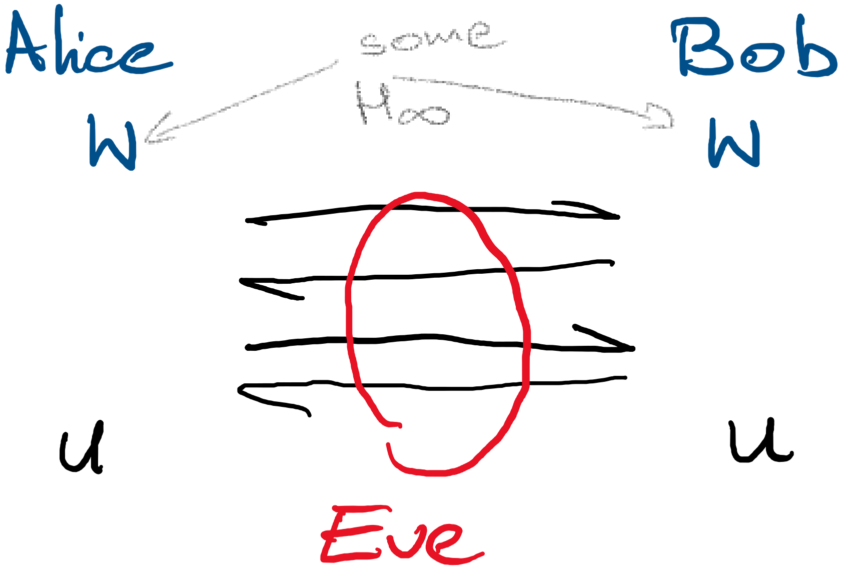


Why Care?

2NMExt are cool tools:

Privacy Amplification

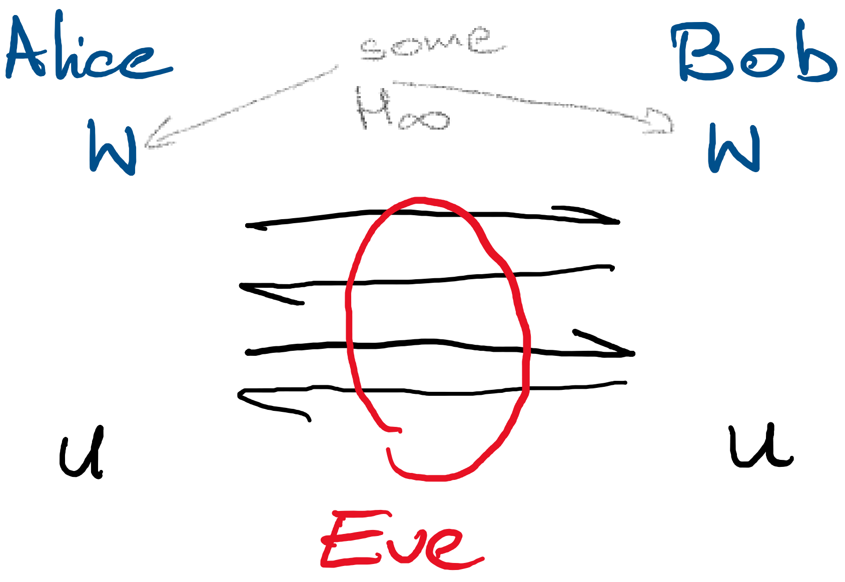


Seeded nmExt

Why Care ?

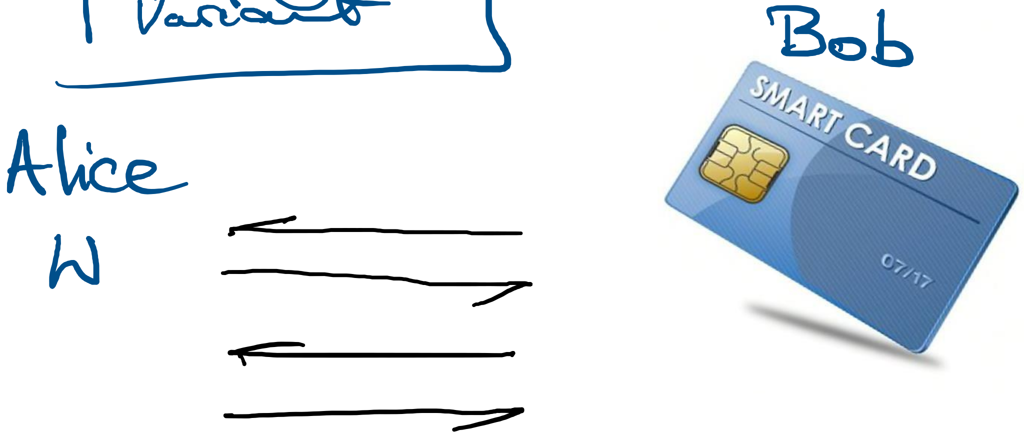
2NMExt are cool tools:

Privacy Amplification



Seeded nmExt

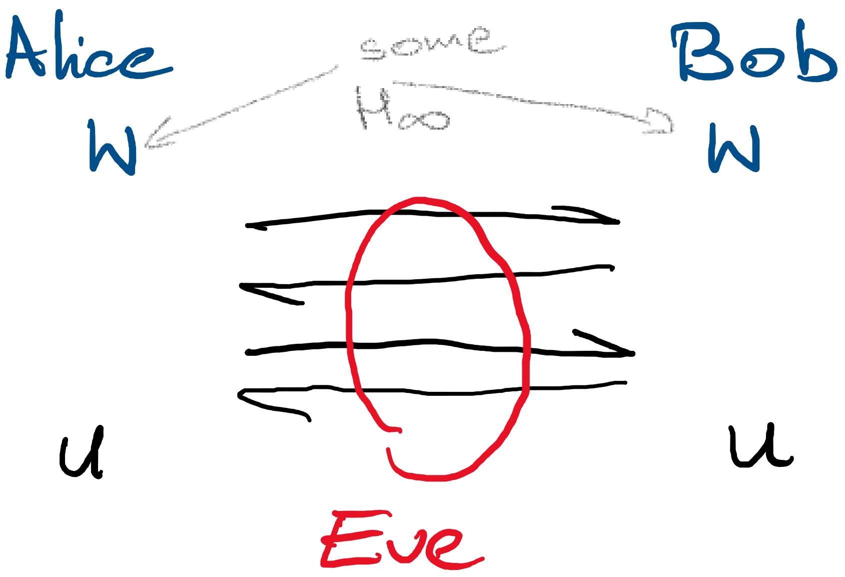
Stronger Variant



Why Care?

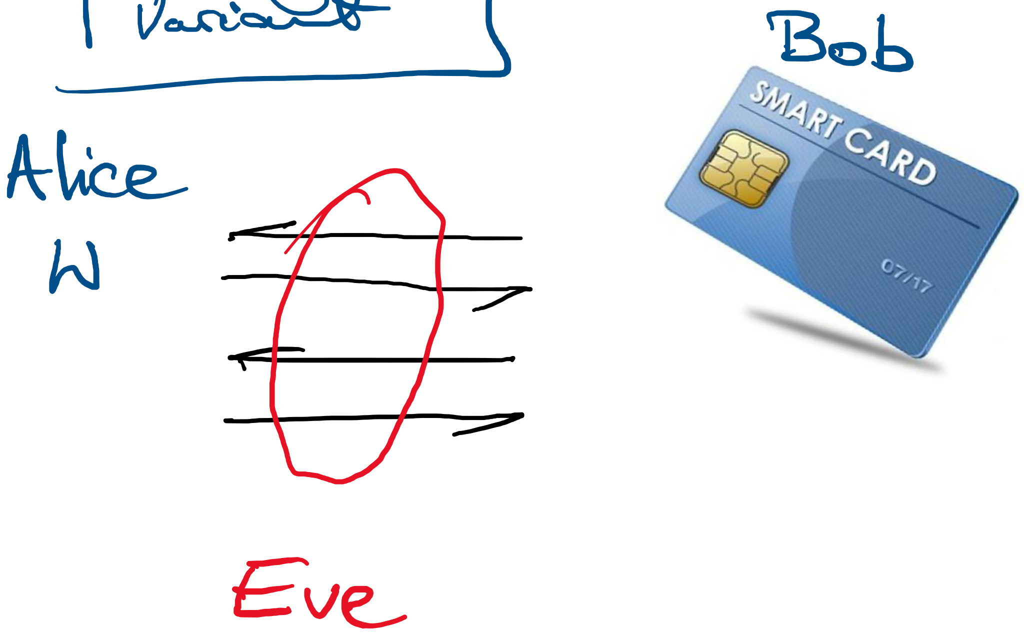
2NMExt are cool tools:

Privacy Amplification



Seeded nmExt

Stronger Variant



Why Care?

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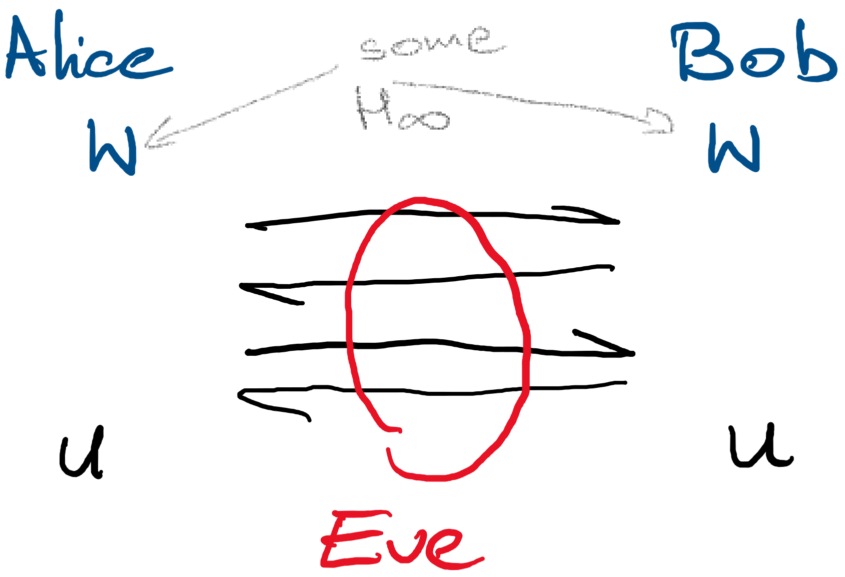
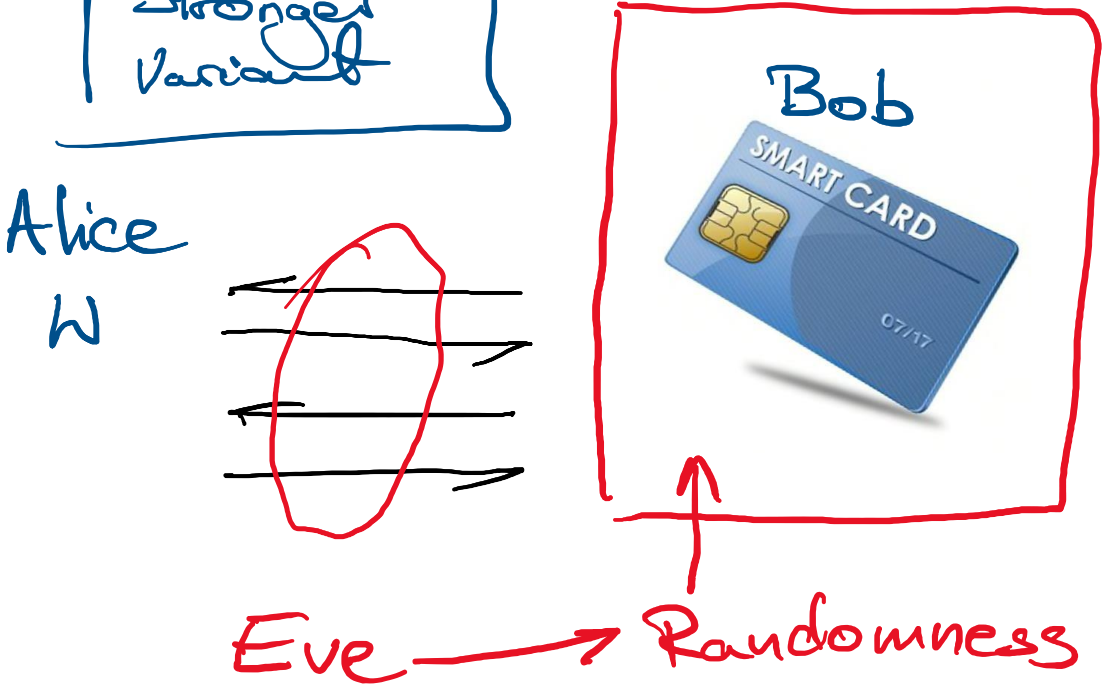
Privacy Amplification



Seeded nmExt

Stronger Variant



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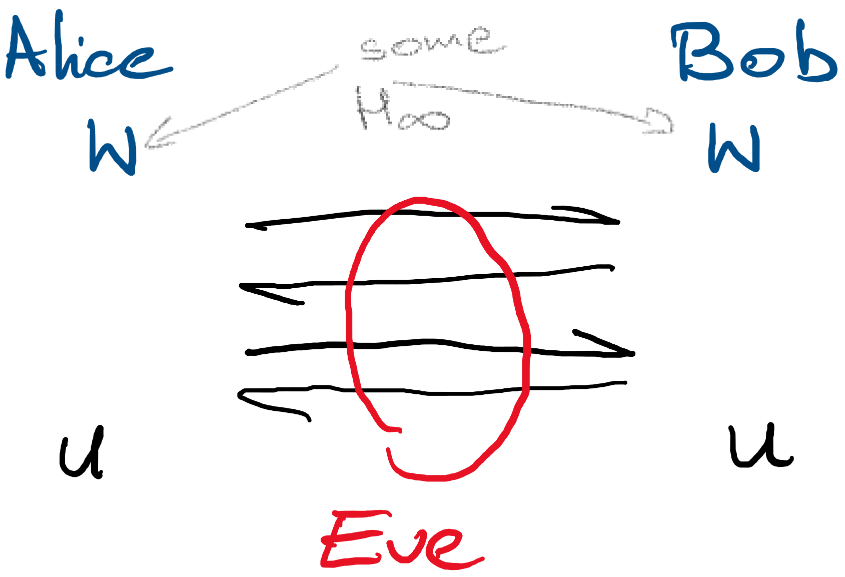

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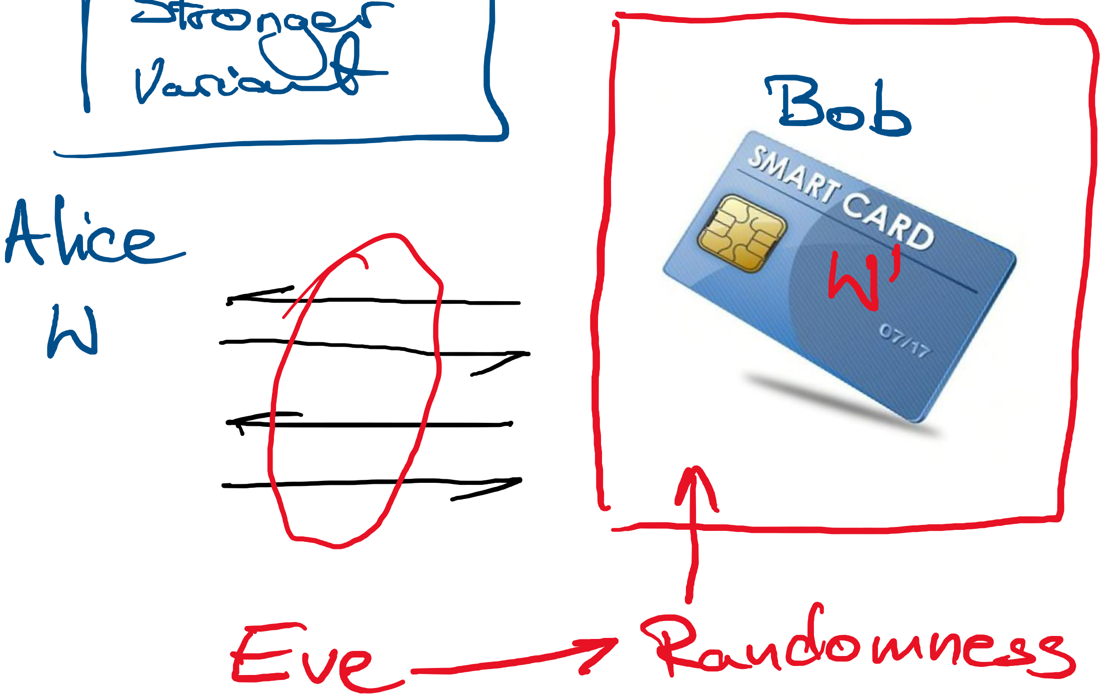
Stronger Variant



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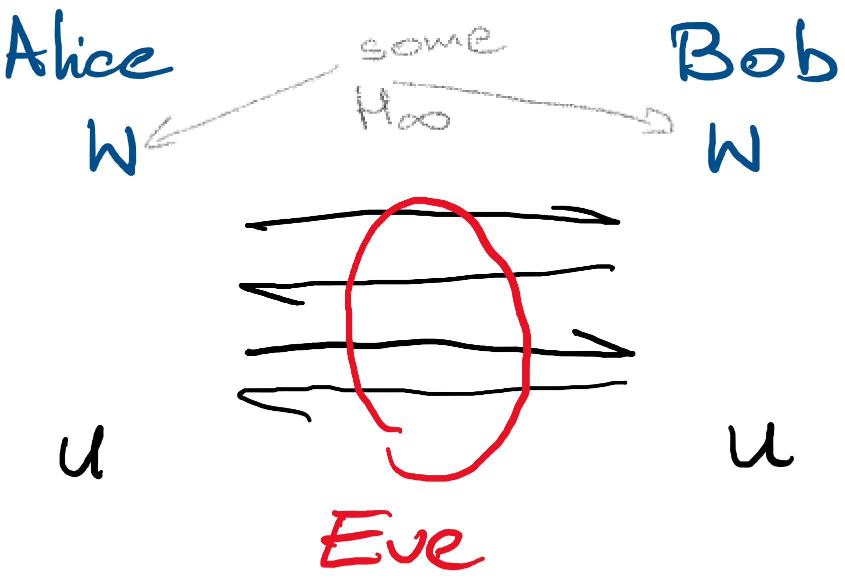

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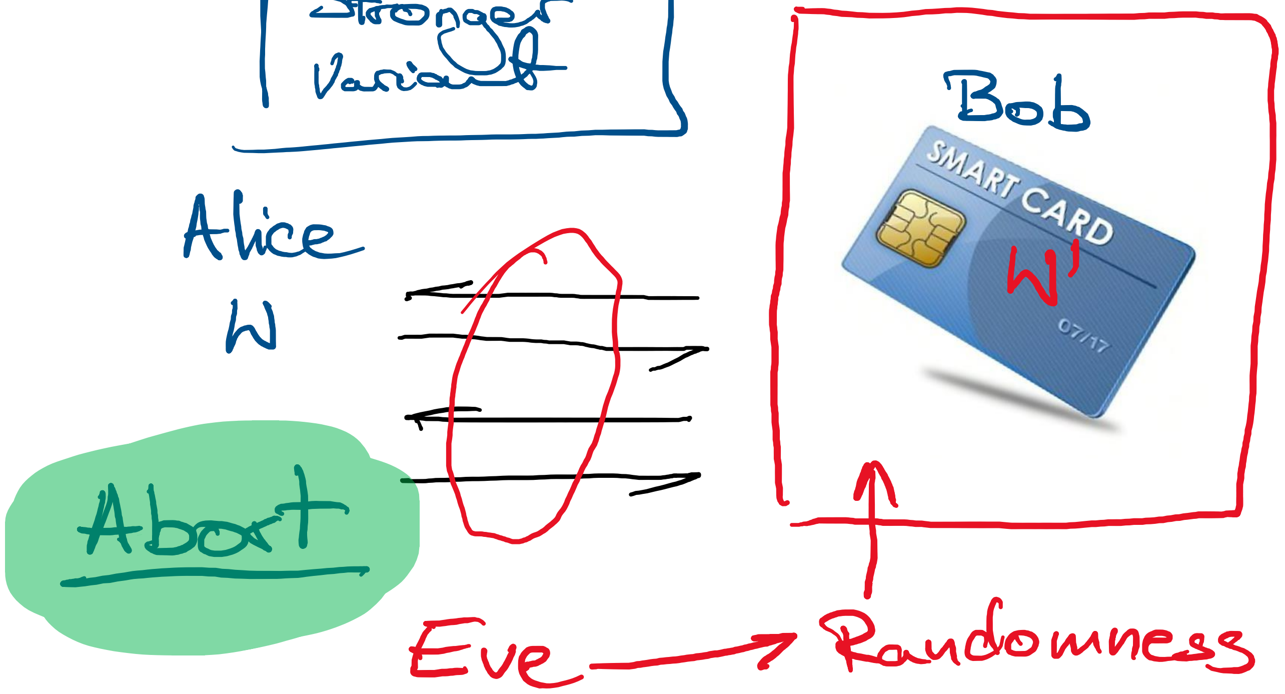
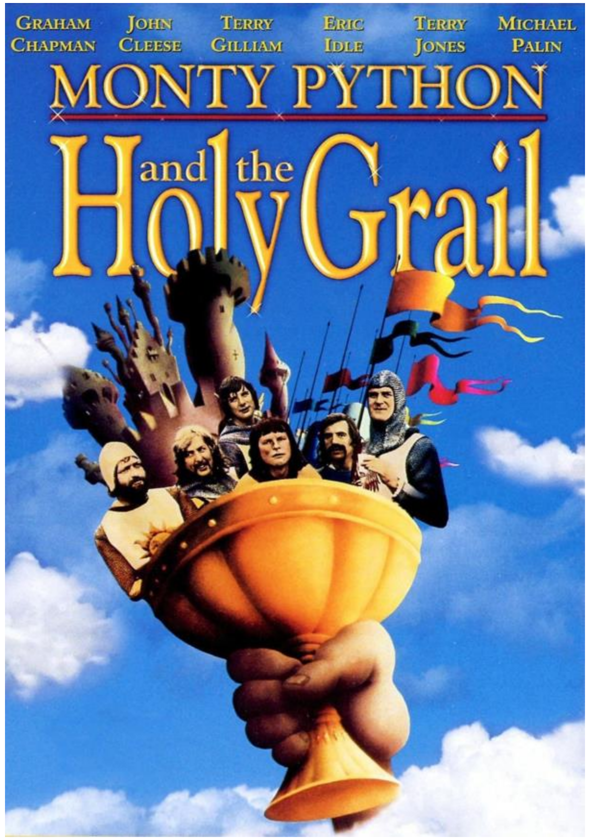


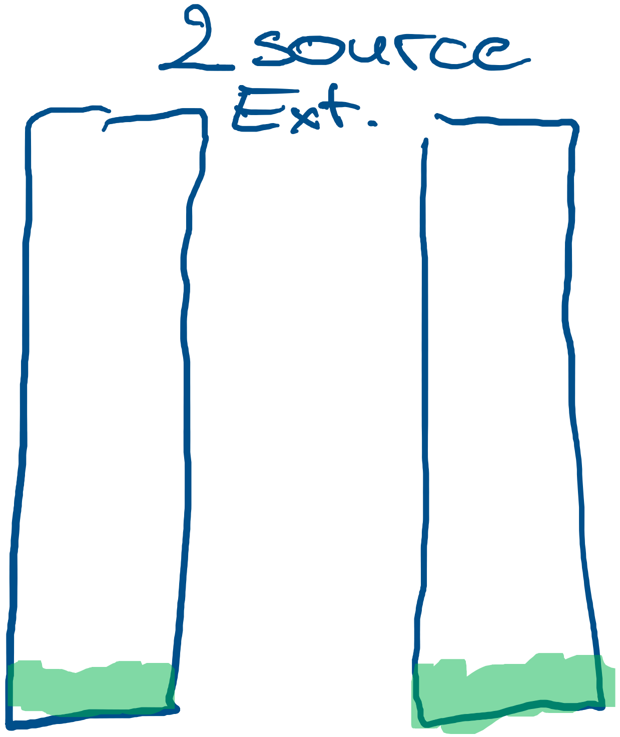
2nmExt

Why I care

Why I care


GRAHAM CHAPMAN
JOHN CLEESE
TERRY GILLIAM
ERIC IDLE
TERRY JONES
MICHAEL PALIN
MONTY PYTHON
and the Holy Grail

Why I care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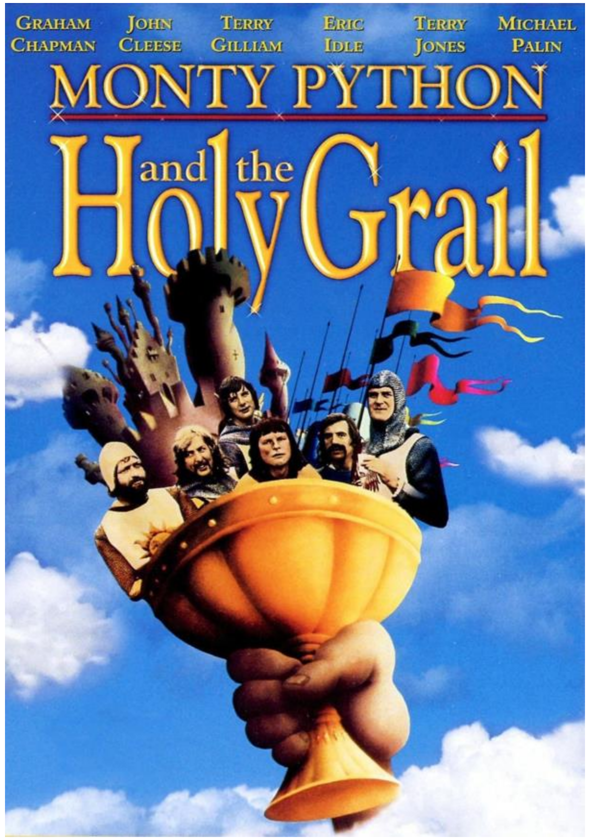
Ext.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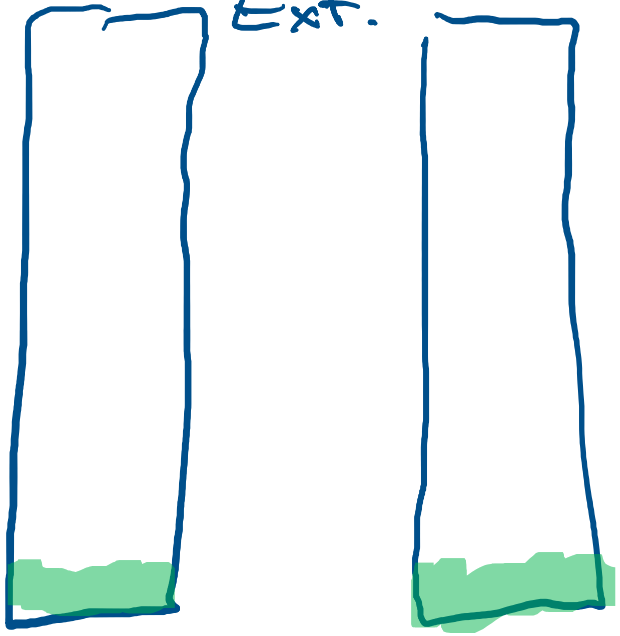
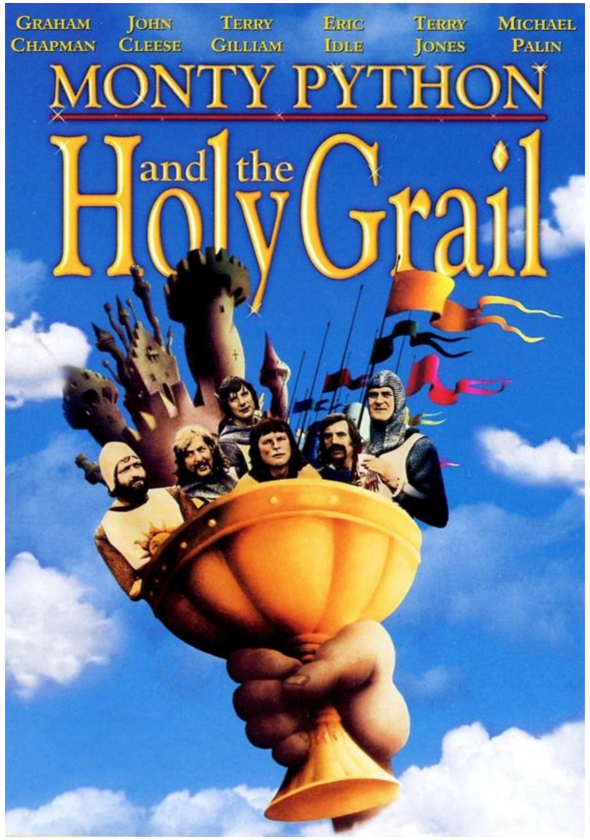
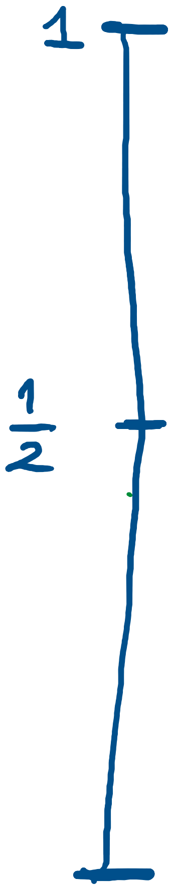




Entropy Rate



Why I care

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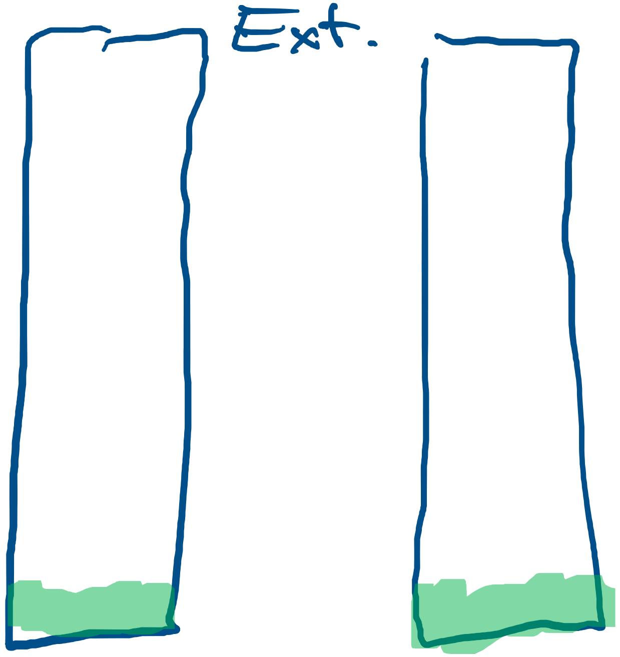

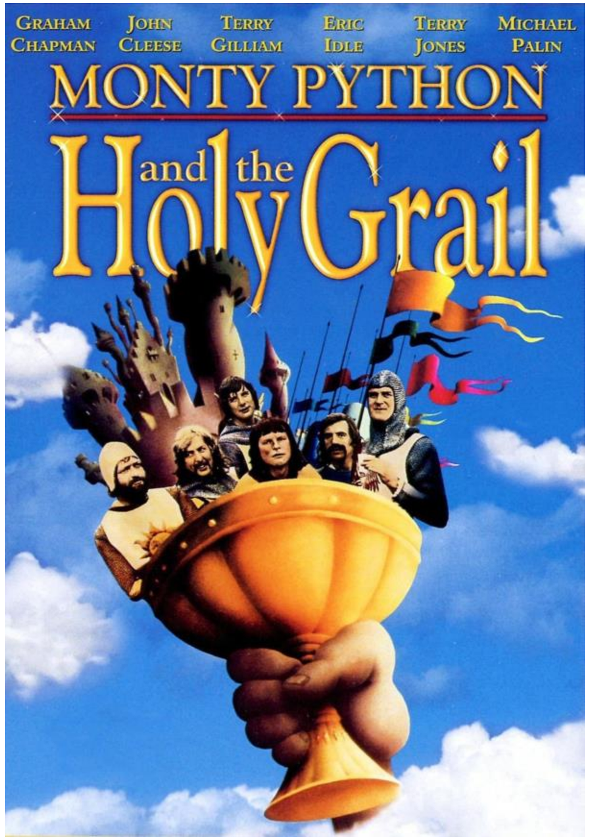

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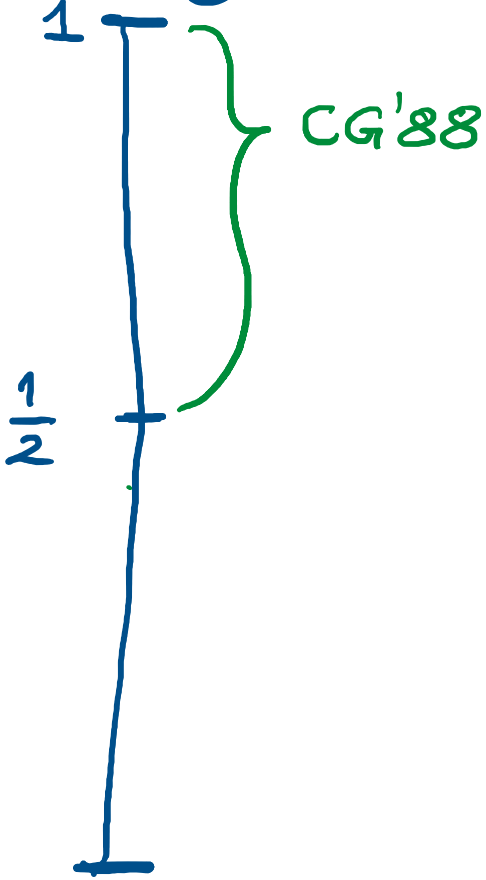

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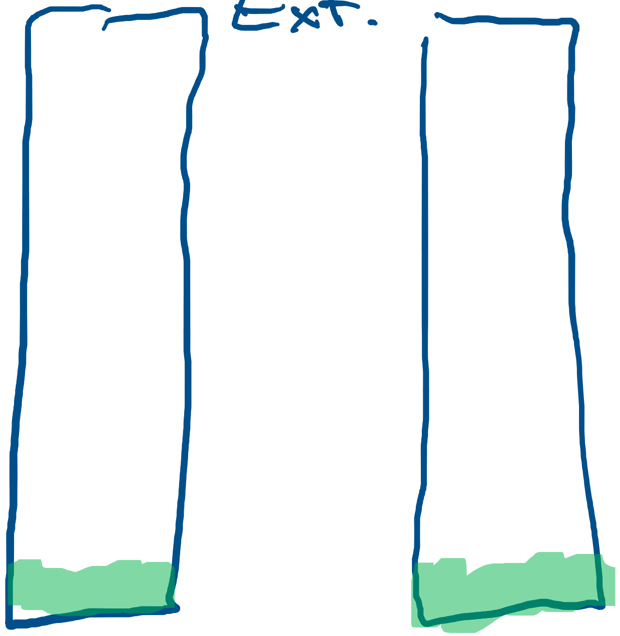

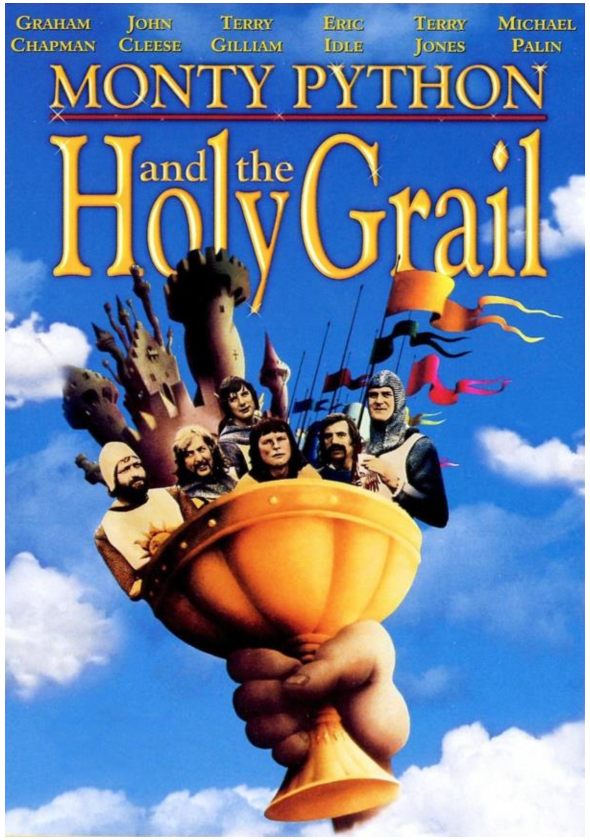

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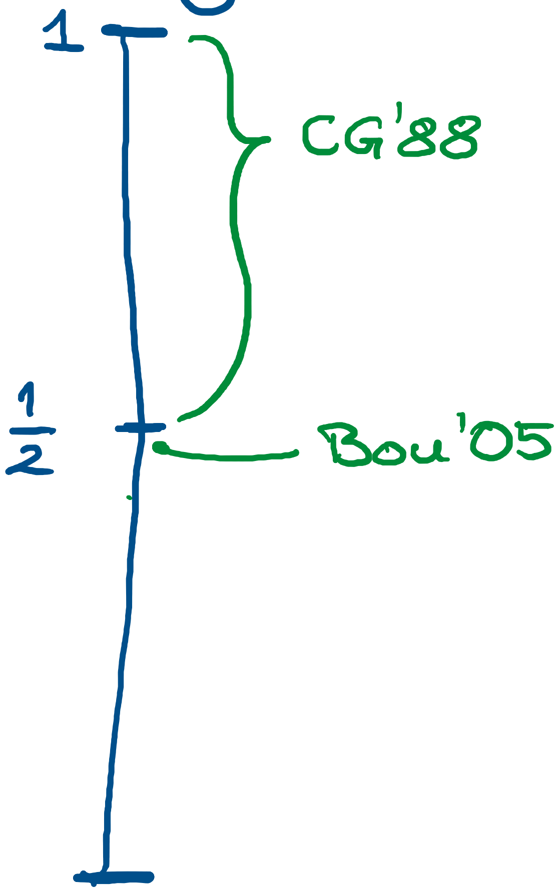

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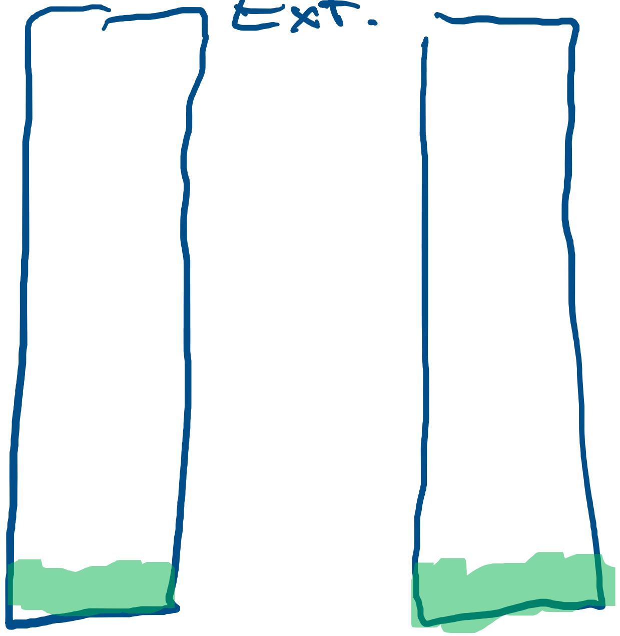

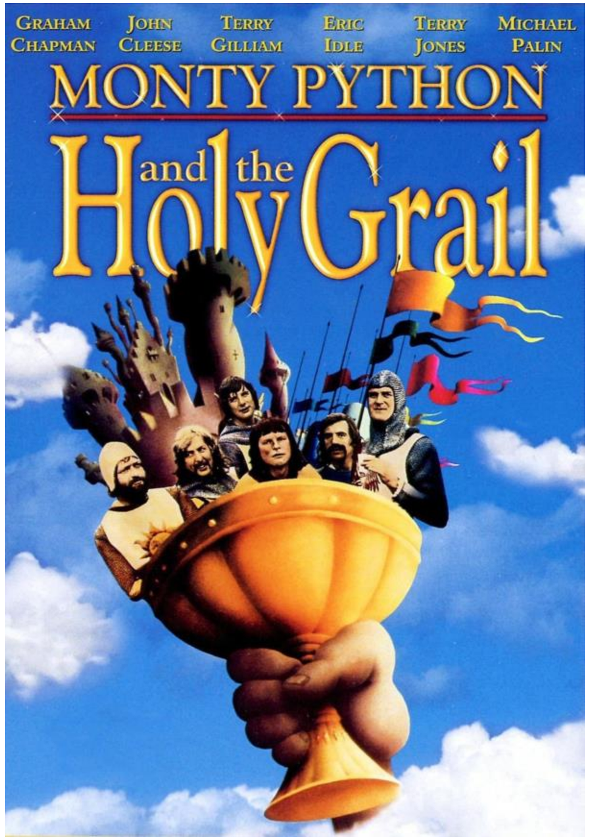

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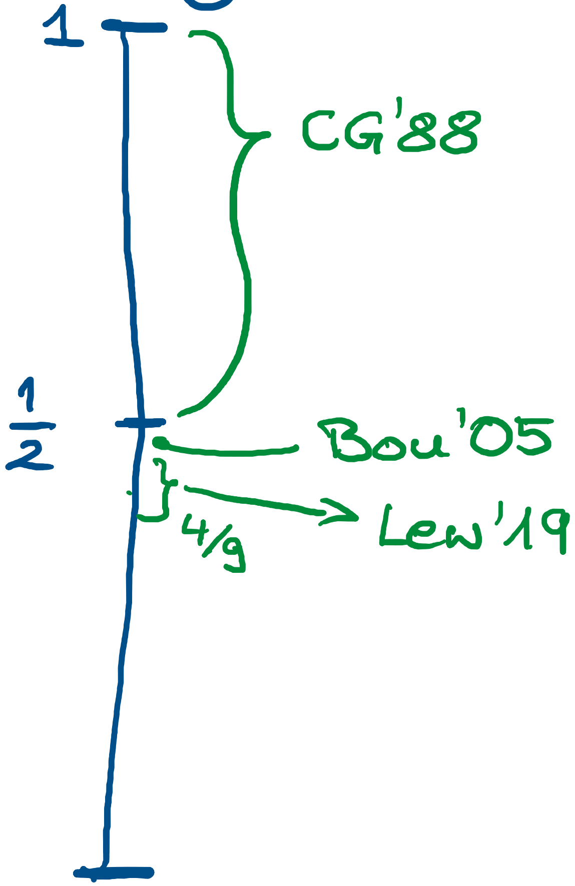

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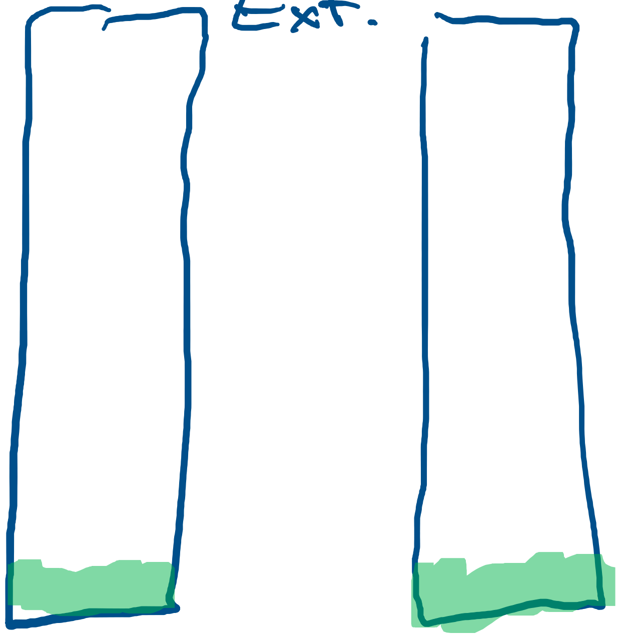

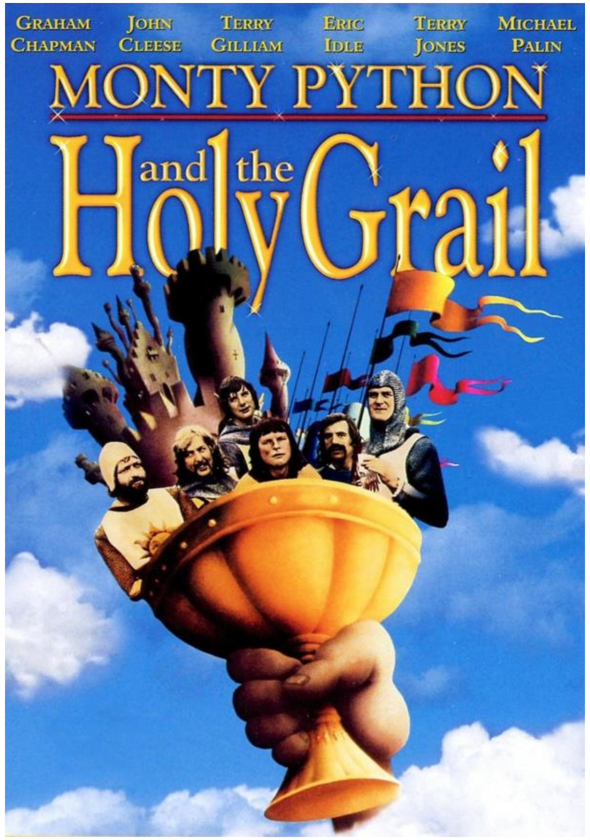

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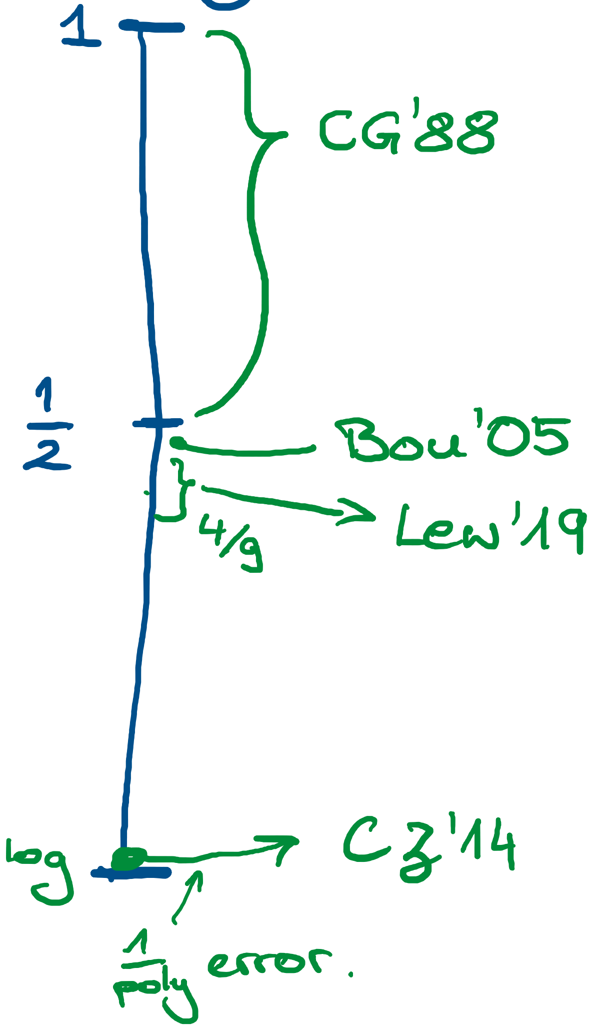

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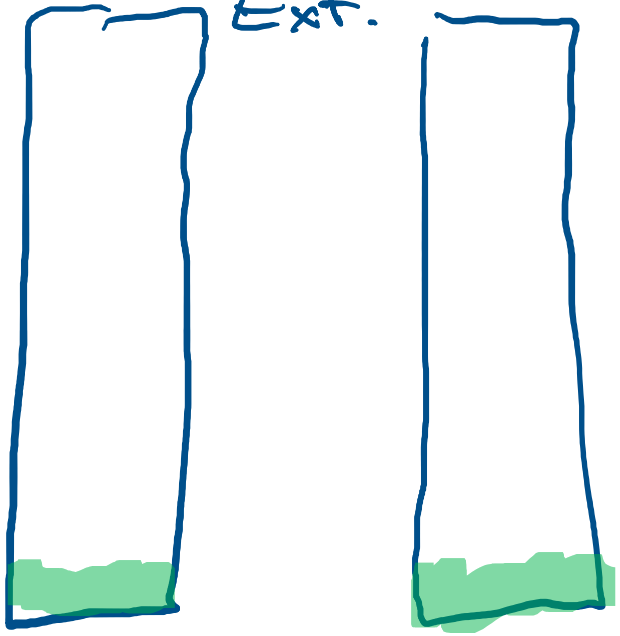
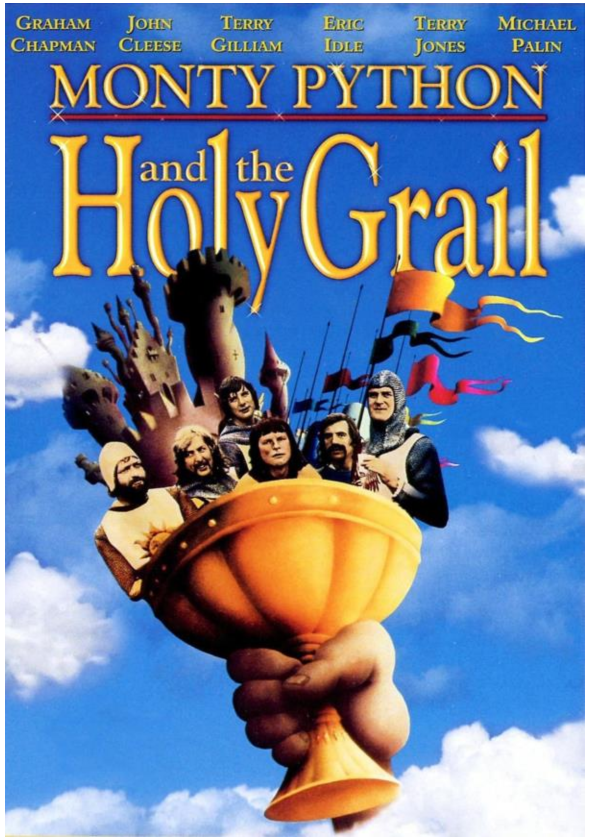
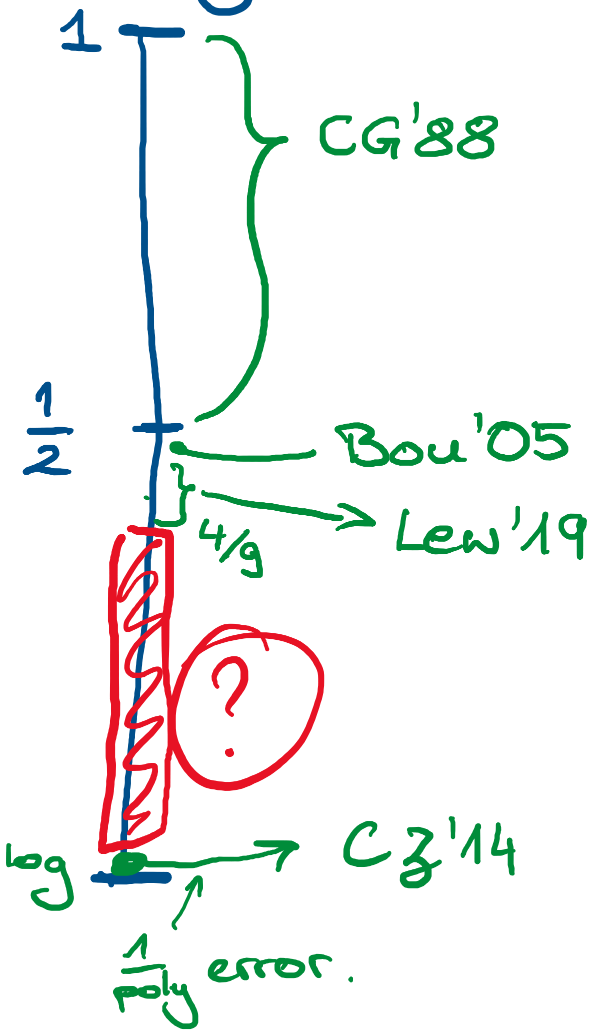
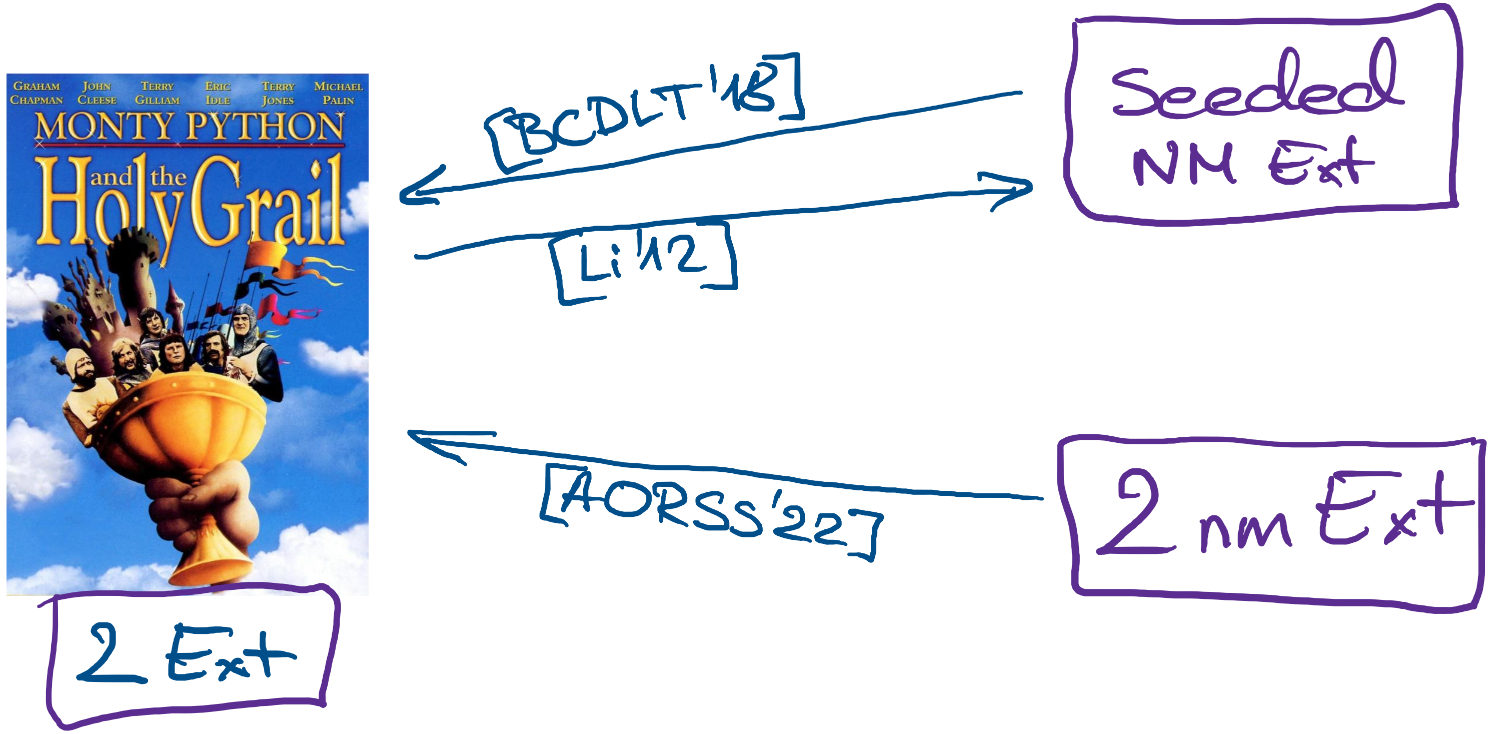




Entropy Rate



Connections



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Seeded NM Ext

[BCDLT'18]

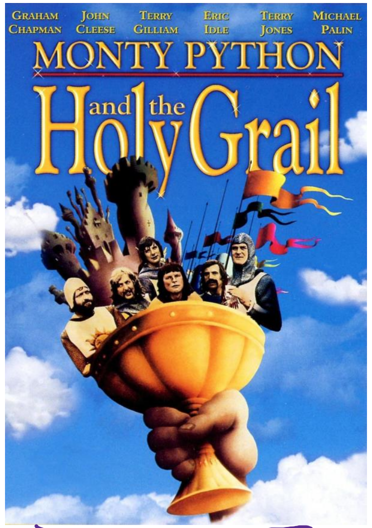
[Li'12]

2 nm Ext

[AORSS'22]

2 Ext

Connections



2 Ext

[BCDLT'18]

Seeded NM Ext

[Li'12]

[AORSS'22]

2 nm Ext

$(1-\frac{1}{T})$ entropy threshold

this paper gets $\left(1-\frac{1}{2T+3}\right)$

Thank You

22.30