Tighter QCCA-Secure Key Encapsulation Mechanism with Explicit Rejection in the Quantum Random Oracle Model

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Overview

Background

NIST's post-quantum cryptography (PQC) standardization

- Public key encryption (PKE)
- Key encapsulation mechanism (KEM)
- Digital signatures (DS)

KEM constructions in the PQC standardization

KEM variants of Fujisaki-Okamoto (FO) transformation [HHK17]

- FO-like transformations: $\mathsf{FO}^{\not\perp}$, $\mathsf{FO}^{\perp}$, $\mathsf{FO}_m^{\not\perp}$, $\mathsf{FO}_m^{\perp}$, $\mathsf{QFO}_m^{\not\perp}$ and $\mathsf{QFO}_m^{\perp}$
- Modular FO transformations: $\mathsf{U}^{\not\perp}$, $\mathsf{U}^{\perp}$, $\mathsf{U}_m^{\not\perp}$, $\mathsf{U}_m^{\perp}$, $\mathsf{QU}_m^{\not\perp}$ and $\mathsf{QU}_m^{\perp}$

OW/IND-CPA-secure PKE⇒IND-CCA-secure KEM (ROM)

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OW/IND-CPA-secure PKE $\Rightarrow$ IND-CCA-secure KEM (ROM)

- $\perp$ (explicit rejection type): Decapsulation algorithm outputs $\perp$ for an invalid ciphertext
- $\not\perp$ (implicit rejection type): Decapsulation algorithm outputs a pseudorandom value for an invalid ciphertext

KEM constructions in the PQC standardization

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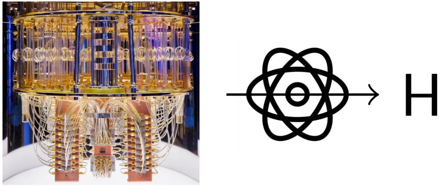
OW/IND-CPA-secure PKE⇒IND-CCA-secure KEM (ROM)

- $\perp$ (explicit rejection type): Decapsulation algorithm outputs $\perp$ for an invalid ciphertext
- $\not\perp$ (implicit rejection type): Decapsulation algorithm outputs a pseudorandom value for an invalid ciphertext
- m: Encapsulation algorithm outputs $G(m^*)$ as the key
- Without m: Encapsulation algorithm outputs $G(m^*, c^*)$ as the key

Quantum random oracle model [BDF$^+$11]



- Quantum computer can compute hash function H on an arbitrary superposition of inputs.

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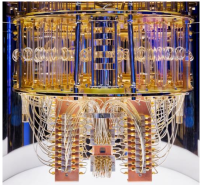
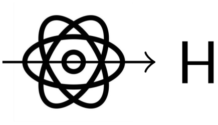
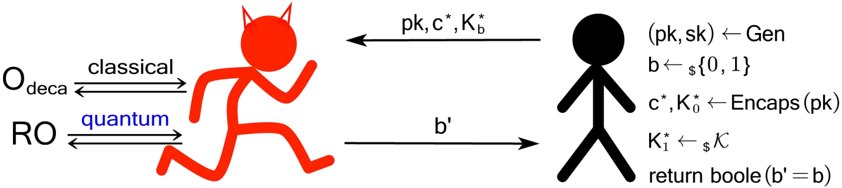




- Quantum computer can compute hash function H on an arbitrary superposition of inputs.
- The ROM should be lifted to the quantum random oracle model (QROM)

IND-CCA and IND-qCCA security [BZ13]

IND-CCA game of KEM (QROM)



IND-CCA and IND-qCCA security [BZ13]

Indistinguishability under quantum chosen-ciphertext attacks (IND-qCCA)

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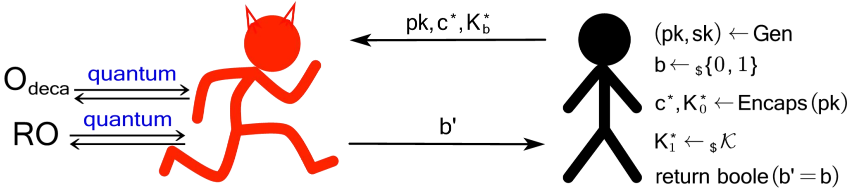

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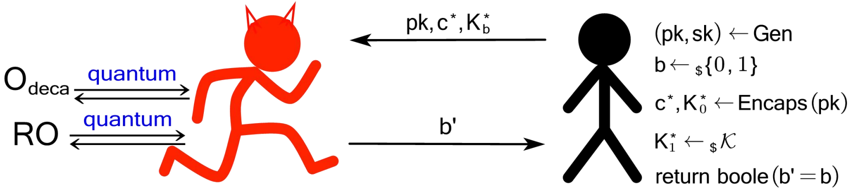

IND-qCCA security *implies* IND-CCA security (QROM)

Motivation - 1

Appendix A of [HHM22] (eprint 2022/365):

- Security of $\mathsf{FO}_m^{\perp}$ implies security of all the remaining FO-like transformations

  $\mathsf{FO}^{\not\perp}$, $\mathsf{FO}^{\perp}$, $\mathsf{FO}_m^{\not\perp}$, $\mathsf{FO}_m^{\perp}$, $\mathsf{QFO}_m^{\not\perp}$ and $\mathsf{QFO}_m^{\perp}$

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Transformation	Underlying security	Achieved security	Requirement	Security bound(≈)
$\mathsf{FO}_m^{\perp}$ [DFMS22]	OW-CPA	IND-CCA	γ-spread	$q \cdot \sqrt{\epsilon} + q^{1.5} q_D \cdot \sqrt[4]{2^{-\gamma}} + q^2\sqrt{\delta}$
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$\mathsf{FO}^{\not\perp}$ [KSS+20]	IND-CPA	IND-CCA	η-injective	$q^2 \cdot \epsilon + q \cdot \sqrt{\eta} + \cdots$

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Can we give a security reduction of $\mathsf{FO}_m^{\perp}$ that avoids the quadratic security loss and has a tighter security bound (QROM)?

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Does security of $\mathrm{FO}_m^{\not\perp}$ imply security of $\mathrm{FO}_m^{\perp}$?

Our results - 1

Can we give a security reduction of $\mathsf{FO}_m^{\perp}$ that avoids the quadratic security loss and has a tighter security bound (QROM)?

Theorem 1 (IND-CPA PKE $\overset{\text{QROM}}{\Rightarrow}$ IND-qCCA KEM, informal)

If P is δ-correct and γ-spread, for any IND-qCCA adversary $\mathcal{A}$ against $\mathsf{FO}_m^{\perp}[P,H,G]$, issuing at most q_D queries to the decapsulation oracle Deca, q queries to the random oracles, there exists an IND-CPA adversary $\mathcal{B}$ against P such that

$$\mathrm{Adv}^{\mathsf{IND\text{-}qCCA}}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}(\mathcal{A}) \leq q^2 \cdot \mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{P}}(\mathcal{B}) + q \cdot \sqrt{\delta} + q_D \cdot 2^{-\gamma/2}$$

Our results - 2

Does security of $\mathsf{FO}_m^{\not\perp}$ imply security of $\mathsf{FO}_m^{\perp}$?

- If the underlying PKE scheme is γ-spread, then the IND-qCCA security of $\mathsf{FO}_m^{\not\perp}$ implies the IND-qCCA security of $\mathsf{FO}_m^{\perp}$ (QROM)

Core theorem

Theorem 2 (IND-CPA KEM $\overset{\text{QROM}}{\Rightarrow}$ IND-qCCA KEM, informal)

If P is δ-correct and γ-spread, for any IND-qCCA adversary $\mathcal{A}$ against $\mathsf{FO}_m^{\perp}[\mathsf{P},\mathsf{H},\mathsf{G}]$, issuing at most q_D queries to the decapsulation oracle Deca, q queries to the random oracles, there exists an IND-CPA adversary $\mathcal{B}$ against $\mathsf{FO}_m^{\perp}[\mathsf{P},\mathsf{H},\mathsf{G}]$ such that

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Question 1

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- $\mathrm{Adv}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}^{\mathsf{IND\text{-}qCCA}}(\mathcal{A}) \leq \mathrm{Adv}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}^{\mathsf{IND\text{-}CPA}}(\mathcal{A}_1) + q \cdot \sqrt{\delta} + q_D \cdot 2^{-\gamma/2}$ Our Theorem 2

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- Semi-classical O2H [AHU19], Measure-Rewind-Measure O2H [KSS+20]

$$\mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}(\mathcal{A}_1) \leq q^2 \cdot \mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{P}}(\mathcal{B})$$

Question 2

Does security of $\mathsf{FO}_m^{\not\perp}$ imply security of $\mathsf{FO}_m^{\perp}$?

Theorem 3 (IND-qCCA $\mathsf{FO}_m^{\not\perp} \Rightarrow$ IND-qCCA $\mathsf{FO}_m^{\perp}$, informal)

If P is δ-correct and γ-spread, for any IND-qCCA adversary $\mathcal{A}$ against $\mathsf{FO}_m^{\perp}$[P,H,G], issuing at most q_D queries to the decapsulation oracle Deca, q queries to the random oracles, there exists an IND-qCCA adversary $\mathcal{B}$ against $\mathsf{FO}_m^{\not\perp}$[P,H,G] such that

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- $\mathrm{Adv}^{\mathsf{IND\text{-}qCCA}}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}(\mathcal{A}) \leq \mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}(\mathcal{A}_1) + q \cdot \sqrt{\delta} + q_D \cdot 2^{-\gamma/2}$ Our Theorem 2
- $\mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{FO}_m^{\perp}[\mathsf{P,H,G}]}(\mathcal{A}_1) = \mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{FO}_m^{\not\perp}[\mathsf{P,H,G}]}(\mathcal{A}_2)$

Question 2

Theorem 3 (IND-qCCA $\mathrm{FO}_m^{\not\perp} \Rightarrow$ IND-qCCA $\mathrm{FO}_m^{\perp}$, informal)

If P is δ-correct and γ-spread, for any IND-qCCA adversary $\mathcal{A}$ against $\mathrm{FO}_m^{\perp}[P,H,G]$, issuing at most q_D queries to the decapsulation oracle Deca, q queries to the random oracles, there exists an IND-qCCA adversary $\mathcal{B}$ against $\mathrm{FO}_m^{\not\perp}[P,H,G]$ such that

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Proof outline of Theorem 2

Theorem 2 (IND-CPA KEM $\overset{\text{QROM}}{\Longrightarrow}$ IND-qCCA KEM, informal)

If P is δ-correct and γ-spread, for any IND-qCCA adversary $\mathcal{A}$ against $\mathsf{FO}_m^{\perp}[P,H,G]$, issuing at most q_D queries to the decapsulation oracle Deca, q queries to the random oracles, there exists an IND-CPA adversary $\mathcal{B}$ against $\mathsf{FO}_m^{\perp}[P,H,G]$ such that

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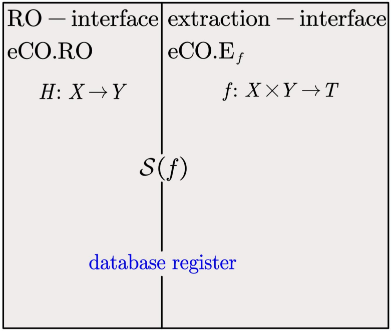
- Simulate the decapsulation oracle Deca by using the extractable RO simulator [DFMS22]
- Change the simulation of Deca by a sequences of games: Game 1-Game 4
- Game 4 can be rewritten to an IND-CPA game of $\mathsf{FO}_m^{\perp}$.

The extractable RO-simulator [DFMS22]

$\mathsf{S}(f) = \{\mathsf{eCO.RO}, \mathsf{eCO.E}_f\}$: A general version of the compressed standard oracle [Zha19]



eCO.RO: compressed standard oracle

$\mathrm{eCO.E}_f : |t, D, z\rangle \mapsto |t, D, z \oplus x\rangle$

x is the smallest value that satisfies $f(x, D(x)) = t$

Game 1

$\mathcal{S}(f_1) = \{\mathsf{eCO.RO}, \mathsf{eCO.E}_{f_1}\}$

$(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{Gen}$
$\mathrm{b} \leftarrow_{\$} \{0, 1\}$
$\mathrm{m}^* \leftarrow_{\$} \mathcal{M}$
$\mathrm{c}^* = \mathrm{Enc}_{\mathrm{pk}}(\mathrm{m}^*, \mathrm{H}(\mathrm{m}^*))$
$\mathrm{K}_0^* = \mathrm{G}(\mathrm{m}^*),\ \mathrm{K}_1^* \leftarrow_{\$} \mathcal{K}$

$\mathrm{b}' \leftarrow \mathcal{A}^{\mathrm{H},\mathrm{G},\mathrm{Deca}}(\mathrm{pk}, \mathrm{c}^*, \mathrm{K}_\mathrm{b}^*)$

$\mathrm{return\ boole}(\mathrm{b}' = \mathrm{b})$

H: eCO.RO

Deca: Query $\mathsf{eCO.E}_{f_1}$
Query G
Query $\mathsf{eCO.E}_{f_1}$ again

$$\left|\mathrm{Adv}^{\mathsf{IND\text{-}qCCA}}_{\mathsf{FO}^{\perp}_{m}[\mathsf{P},\mathsf{H},\mathsf{G}]}(\mathcal{A}) - \Pr[1 \leftarrow \mathsf{Game}\ 1]\right| \leq q_D \cdot 2^{-\gamma/2}$$

Game 2

$\mathcal{S}(f_2) = \{\mathsf{eCO.RO}, \mathsf{eCO.E}_{f_2}\}$

$$
\begin{array}{|l|}
\hline
(\mathrm{pk},\mathrm{sk}) \leftarrow \mathrm{Gen} \\
\mathrm{b} \leftarrow_{\$} \{0,1\} \\
\mathrm{m}^* \leftarrow_{\$} \mathcal{M} \\
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H: eCO.RO

Deca: Query $\mathsf{eCO.E}_{f_2}$

Query G

Query $\mathsf{eCO.E}_{f_2}$ again

Compressed oracle O2H lemma [CMSZ19, eprint 2023/792]

$$|\Pr[1 \leftarrow \text{Game } 1] - \Pr[1 \leftarrow \text{Game } 2]| \leq q \cdot \sqrt{\delta}$$

Game 3

$$|\Pr[1 \leftarrow \text{Game } 2] - \Pr[1 \leftarrow \text{Game } 3]| \leq q_D \cdot 2^{-\gamma/2}$$

$(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{Gen}$
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$\mathrm{c}^* = \mathrm{Enc}_{\mathrm{pk}}(\mathrm{m}^*, \mathrm{O}(\mathrm{m}^*))$
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$\mathrm{b}' \leftarrow \mathcal{A}^{\mathrm{H},\mathrm{G},\mathrm{Deca}}(\mathrm{pk}, \mathrm{c}^*, \mathrm{K}_\mathrm{b}^*)$

return $\mathrm{boole}(\mathrm{b}' = \mathrm{b})$

$$\mathrm{H}:\ |\mathrm{x},\mathrm{y}\rangle|\mathrm{D}\rangle \mapsto \begin{cases} \mathrm{eCO.RO}|\mathrm{x},\mathrm{y}\rangle|\mathrm{D}\rangle & \text{if } \mathrm{x} \neq \mathrm{m}^* \\ |\mathrm{x},\mathrm{y} \oplus \mathrm{O}(\mathrm{m}^*)\rangle|\mathrm{D}\rangle & \text{if } \mathrm{x} = \mathrm{m}^* \end{cases}$$

Deca: Query $\mathsf{eCO.E}_{f_2}$
Query G
Query $\mathsf{eCO.E}_{f_2}$ again

Game 3

$$|\Pr[1 \leftarrow \text{Game } 2] - \Pr[1 \leftarrow \text{Game } 3]| \leq q_D \cdot 2^{-\gamma/2}$$

$(\mathrm{pk},\mathrm{sk}) \leftarrow \mathrm{Gen}$

$\mathrm{b} \leftarrow_{\$} \{0,1\}$

$\mathrm{m}^* \leftarrow_{\$} \mathcal{M}$

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$\mathrm{K}_0^* = \mathrm{G}(\mathrm{m}^*),\ \mathrm{K}_1^* \leftarrow_{\$} \mathcal{K}$

$\mathrm{b}' \leftarrow \mathcal{A}^{\mathrm{H},\mathrm{G},\mathrm{Deca}}(\mathrm{pk}, \mathrm{c}^*, \mathrm{K}_\mathrm{b}^*)$

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Query G

Query $\mathsf{eCO.E}_{f_2}$ again

$$\mathsf{x} = \mathsf{m}^* \iff \mathsf{Enc}_{\mathsf{pk}}(\mathsf{x}, \mathsf{O}(\mathsf{x})) = \mathsf{c}^*$$

Game 4

$$|\mathsf{Pr}[1 \leftarrow \mathsf{Game}\ 3] = \mathsf{Pr}[1 \leftarrow \mathsf{Game}\ 4]| \leq 2\delta$$

$(\mathrm{pk},\mathrm{sk}) \leftarrow \mathrm{Gen}$
$\mathrm{b} \leftarrow_{\$} \{0,1\}$
$\mathrm{m}^* \leftarrow_{\$} \mathcal{M}$
$\mathrm{c}^* = \mathrm{Enc}_{\mathrm{pk}}(\mathrm{m}^*, \mathrm{O}(\mathrm{m}^*))$
$\mathrm{K}_0^* = \mathrm{G}(\mathrm{m}^*),\ \mathrm{K}_1^* \leftarrow_{\$} \mathcal{K}$

$\mathrm{b}' \leftarrow \mathcal{A}^{\mathrm{H,G,Deca}}(\mathrm{pk}, \mathrm{c}^*, \mathrm{K}_{\mathrm{b}}^*)$

$\mathrm{return\ boole}(\mathrm{b}' = \mathrm{b})$

$$\mathrm{H}\colon |\mathrm{x},\mathrm{y}\rangle|\mathrm{D}\rangle \mapsto \begin{cases} \mathsf{eCO.RO}|\mathrm{x},\mathrm{y}\rangle|\mathrm{D}\rangle & \text{if } \mathrm{Enc}_{\mathrm{pk}}(\mathrm{x},\mathrm{O}(\mathrm{x})) \neq \mathrm{c}^* \\ |\mathrm{x},\mathrm{y} \oplus \mathrm{O}(\mathrm{x})\rangle|\mathrm{D}\rangle & \text{if } \mathrm{Enc}_{\mathrm{pk}}(\mathrm{x},\mathrm{O}(\mathrm{x})) = \mathrm{c}^* \end{cases}$$

Deca: Query $\mathsf{eCO.E}_{f_2}$
Query G
Query $\mathsf{eCO.E}_{f_2}$ again

Rewrite Game 4 to a CPA game

$(\mathsf{pk},\mathsf{sk}) \leftarrow \mathsf{Gen}$
$\mathsf{b} \leftarrow_{\$} \{0,1\}$
$\mathsf{m}^* \leftarrow_{\$} \mathcal{M}$
$\mathsf{c}^* = \mathsf{Enc}_{\mathsf{pk}}(\mathsf{m}^*, \mathsf{O}(\mathsf{m}^*))$
$\mathsf{K}_0^* = \mathsf{G}(\mathsf{m}^*),\ \mathsf{K}_1^* \leftarrow_{\$} \mathcal{K}$

$\mathsf{b}' \leftarrow \mathcal{B}^{\mathsf{O},\mathsf{G}}(\mathsf{pk},\mathsf{c}^*,\mathsf{K}_\mathsf{b}^*)$

$\mathsf{return\ boole}(\mathsf{b}' = \mathsf{b})$

$\mathcal{S}(f_2) = \{\mathsf{eCO.RO}, \mathsf{eCO.E}_{f_2}\}$

$\mathsf{b}' \leftarrow \mathcal{A}^{\mathsf{H},\mathsf{G},\mathsf{Deca}}(\mathsf{pk},\mathsf{c}^*,\mathsf{K}_\mathsf{b}^*)$

$$\mathrm{H:}\ |x,y\rangle|D\rangle \mapsto \begin{cases} \mathrm{eCO.RO}|x,y\rangle|D\rangle & \text{if } \mathrm{Enc}_{\mathrm{pk}}(x,O(x)) \neq c^* \\ |x,y \oplus O(x)\rangle|D\rangle & \text{if } \mathrm{Enc}_{\mathrm{pk}}(x,O(x)) = c^* \end{cases}$$

Deca: Query $\mathsf{eCO.E}_{f_2}$
Query G
Query $\mathsf{eCO.E}_{f_2}$ again

$$\Pr[1 \leftarrow \mathsf{Game}\ 4] = \mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{FO}^{\perp}_{m}[\mathsf{P},\mathsf{O},\mathsf{G}]}(\mathcal{B}) = \mathrm{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{FO}^{\perp}_{m}[\mathsf{P},\mathsf{H},\mathsf{G}]}(\mathcal{B})$$

Summary

- We give an IND-qCCA security reduction of $\mathsf{FO}_m^{\perp}$ in the QROM that avoids the quadratic security loss and has a tighter security bound

$$\mathrm{Adv}_{\mathsf{FO}_m^{\perp}[\mathsf{P},\mathsf{H},\mathsf{G}]}^{\mathsf{IND\text{-}qCCA}}(\mathcal{A}) \leq q^2 \cdot \mathrm{Adv}_{\mathsf{P}}^{\mathsf{IND\text{-}CPA}}(\mathcal{B}) + q \cdot \sqrt{\delta} + q_D \cdot 2^{-\gamma/2}$$

- In the QROM, if the underlying PKE scheme is γ-spread,

IND-qCCA security of $\mathsf{FO}_m^{\not\perp}$ *implies* IND-qCCA security of $\mathsf{FO}_m^{\perp}$

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Thank you for your attention! Full version: eprint 2023/862

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