Constant Input Attribute Based (and Predicate) Encryption from Evasive and Tensor LWE

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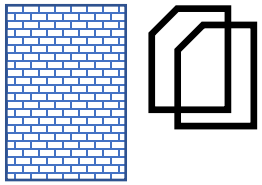
Example

Wants to study the effectiveness of certain medicine on covid patients above 65 years with asthma

Example



The researcher should be able to access only the relevant records

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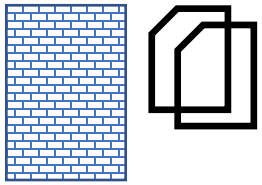

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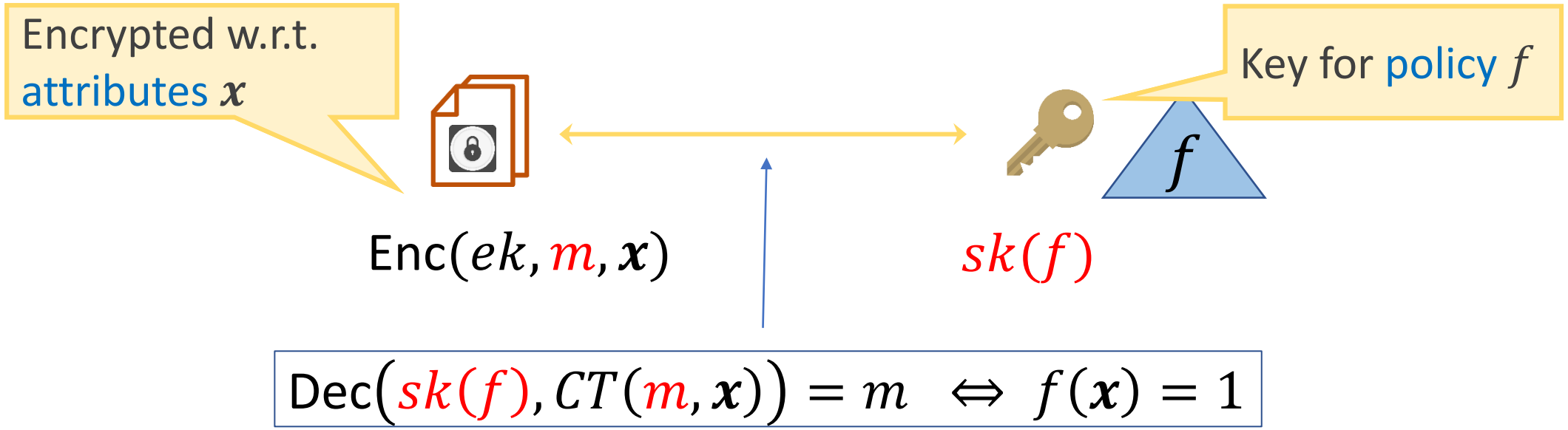
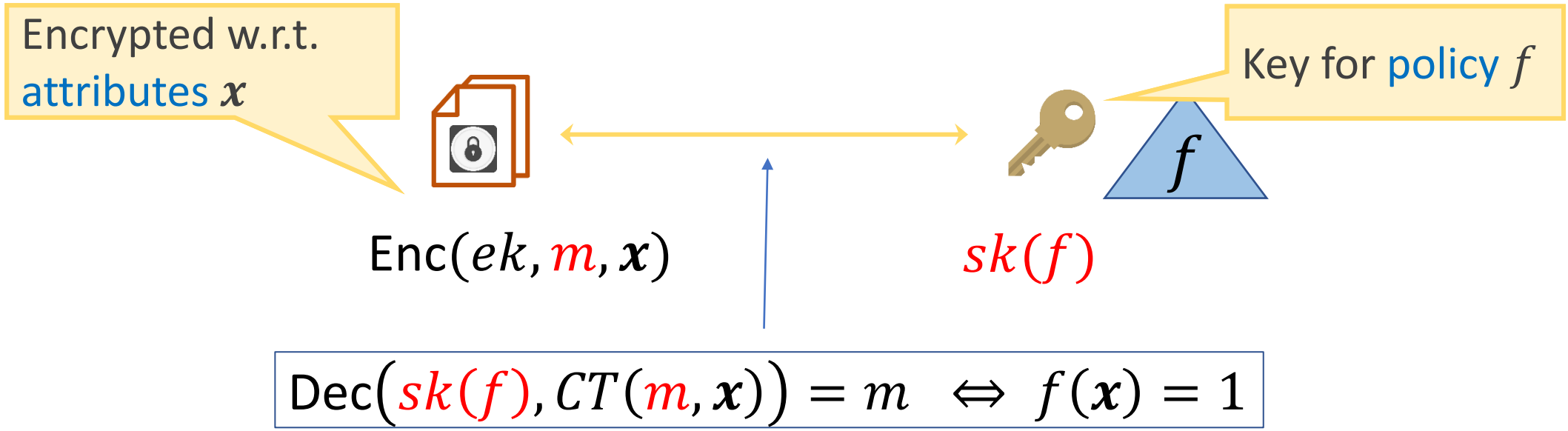
Attribute Based Encryption
ABE

Attribute Based Encryption (ABE)



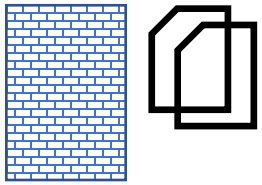
Predicate Encryption (PE)



Predicate Encryption: Ciphertext hides attributes as well

Example

ABE-Enc(, (age, hasCovid, has Asthma))



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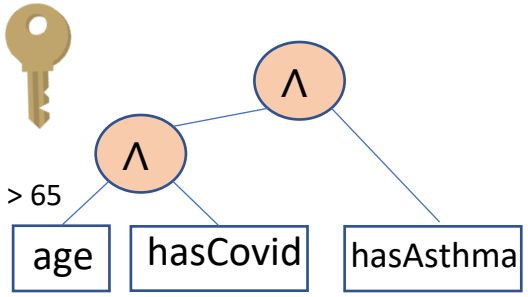

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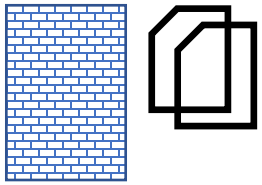

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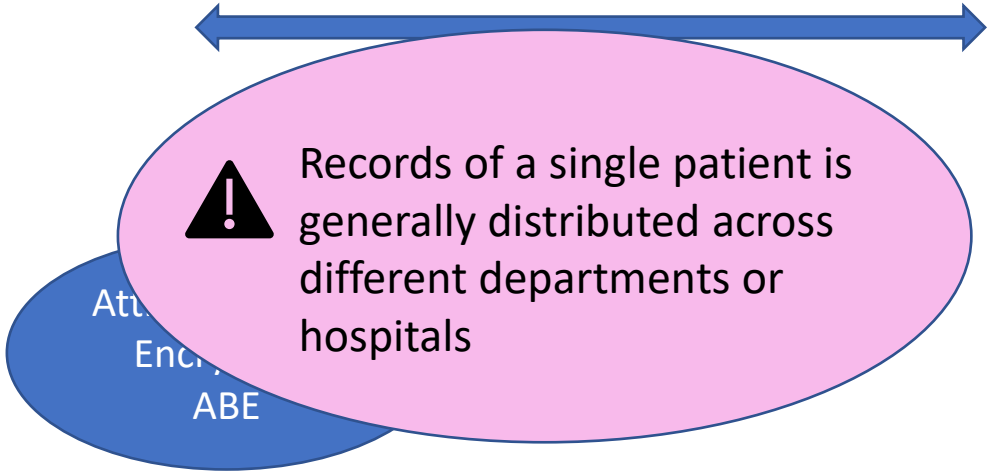

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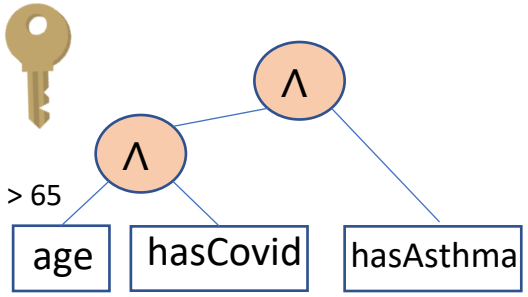

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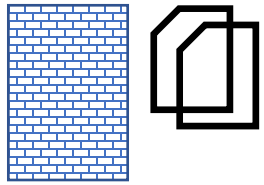
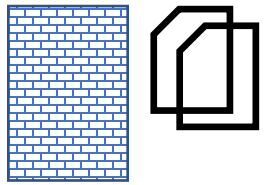


Covid center



Pulmonary Department

The researcher should be able to access only the relevant records

Records of a single patient is generally distributed across different departments or hospitals

Att…
Enc…
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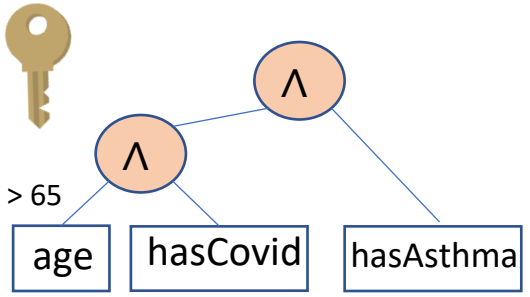

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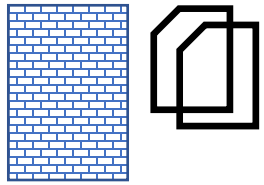

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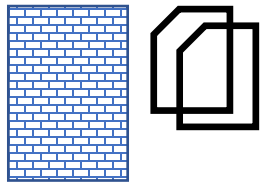

Pulmonary Department

We need ABE/PE in distributed setup

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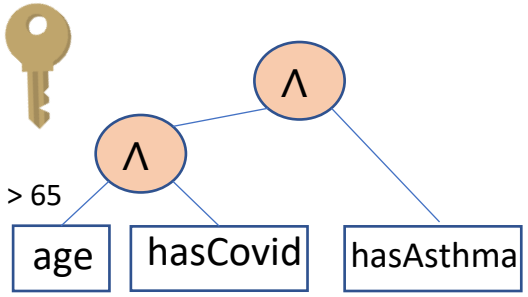
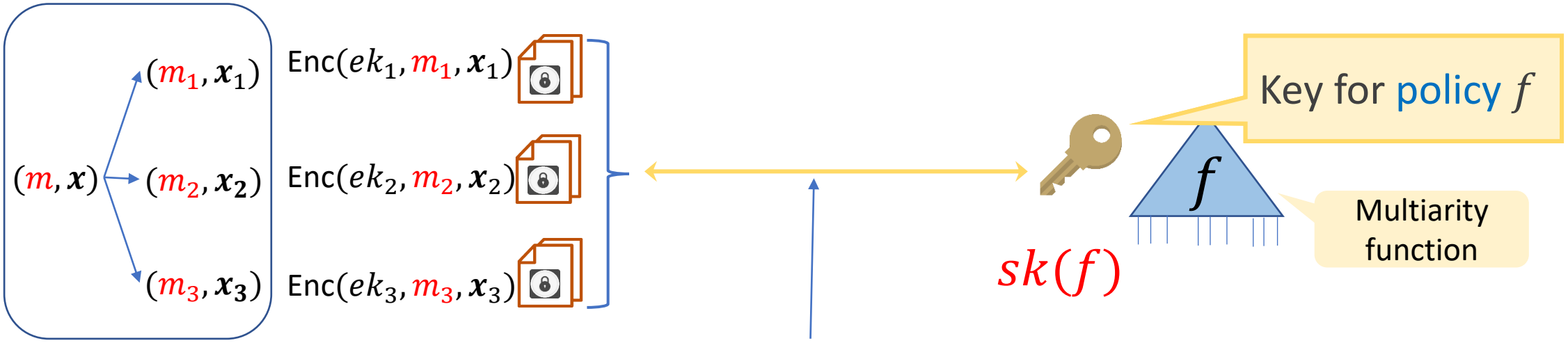


Multi-Input Attribute Based Encryption (miABE)



$$\mathrm{Dec}\big(sk(f), \mathrm{Enc}(m_1, \boldsymbol{x}_1), \mathrm{Enc}(m_2, \boldsymbol{x}_2), \mathrm{Enc}(m_3, \boldsymbol{x}_3)\big) = (m_1, m_2, m_3) \iff f(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = 1$$

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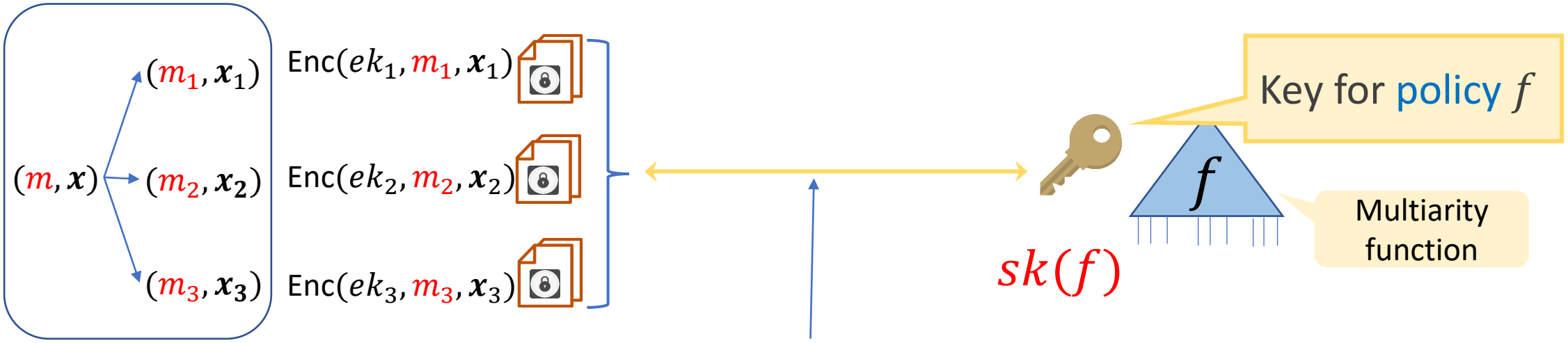

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Related Work

Reference	Function Class	Arity	Assumption
[AYY22]	NC1	2	LWE+pairings
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Collusion Resistant

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Collusion Resistant

[FFMV23] supports conjunctions without collusion resistance from LWE

Our Results

miABE for constant arity

Arity	**Function Class**	**Assumption**
Constant	NC1	evasive LWE
2	P	Evasive and tensor LWE
Constant	P	Evasive and Generalized tensor LWE

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By using [AYY22] compiler, we get Multi Input Predicate Encryption for same settings

Studying tensor LWE: We show that tensor LWE can be reduced to standard LWE in a special case

Fundamental Challenge in Constructing miABE

Two opposite requirements

Multiple encryptors generate the ciphertext components independently

The ciphertext components are independent

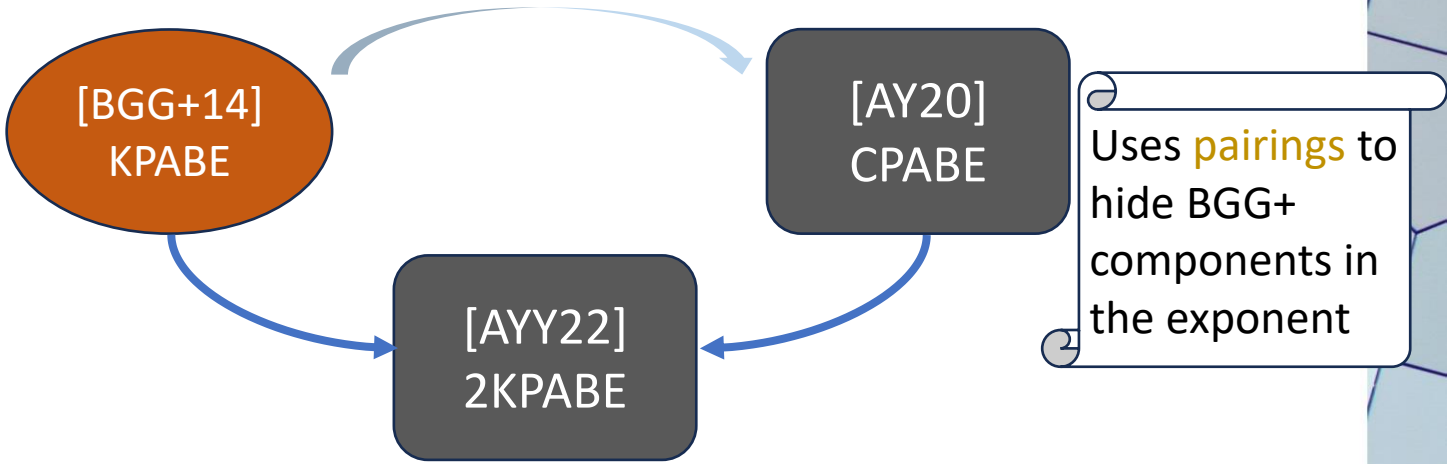
Independently generated components must be joined in a meaningful way

Need correlated ciphertext components for decryption

Pathway


[BGG+14]
KPABE
[AY20]
CPABE
Uses pairings to hide BGG+ components in the exponent

Pathway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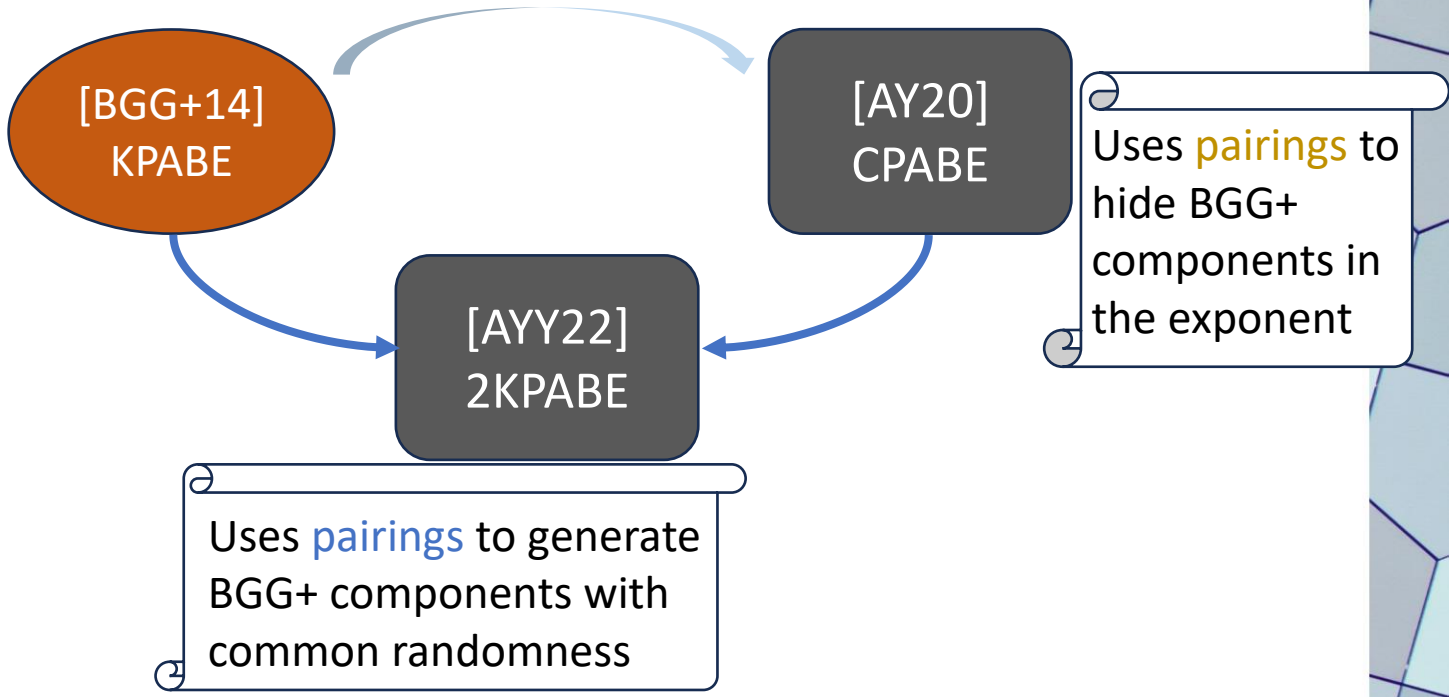
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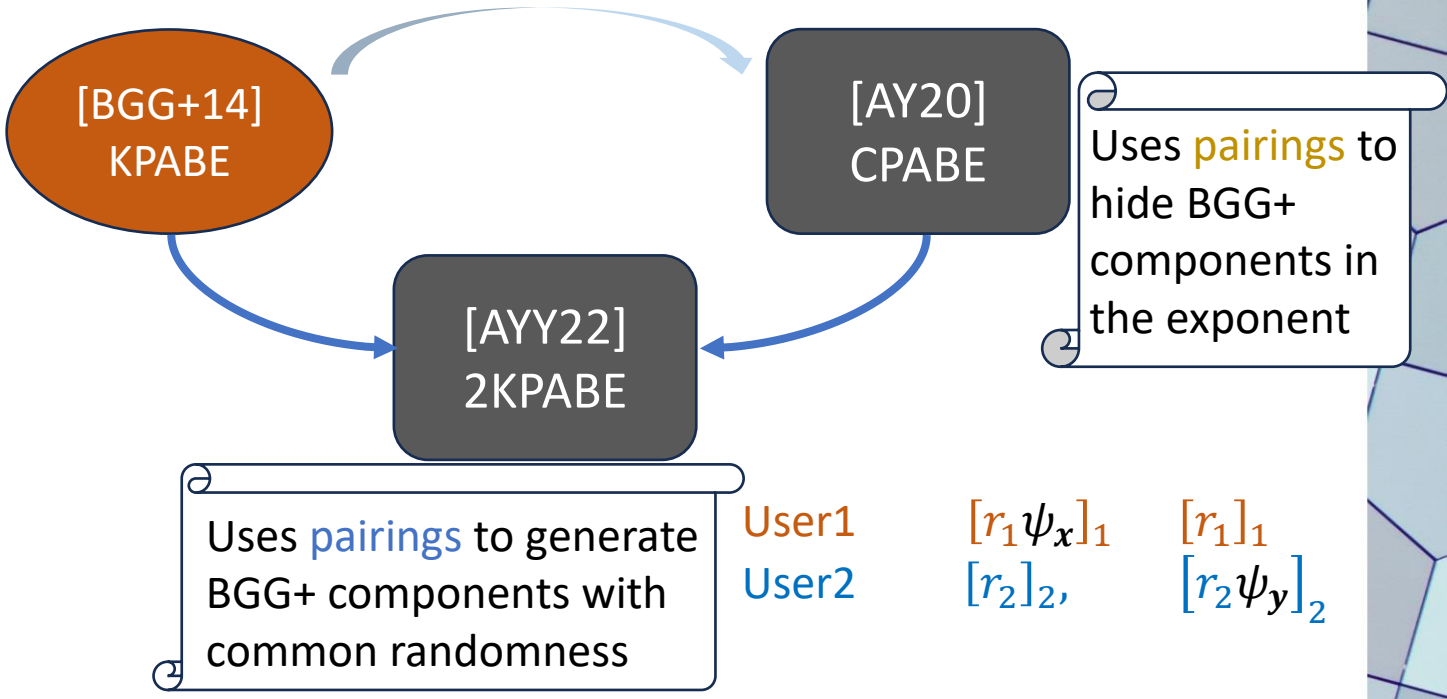
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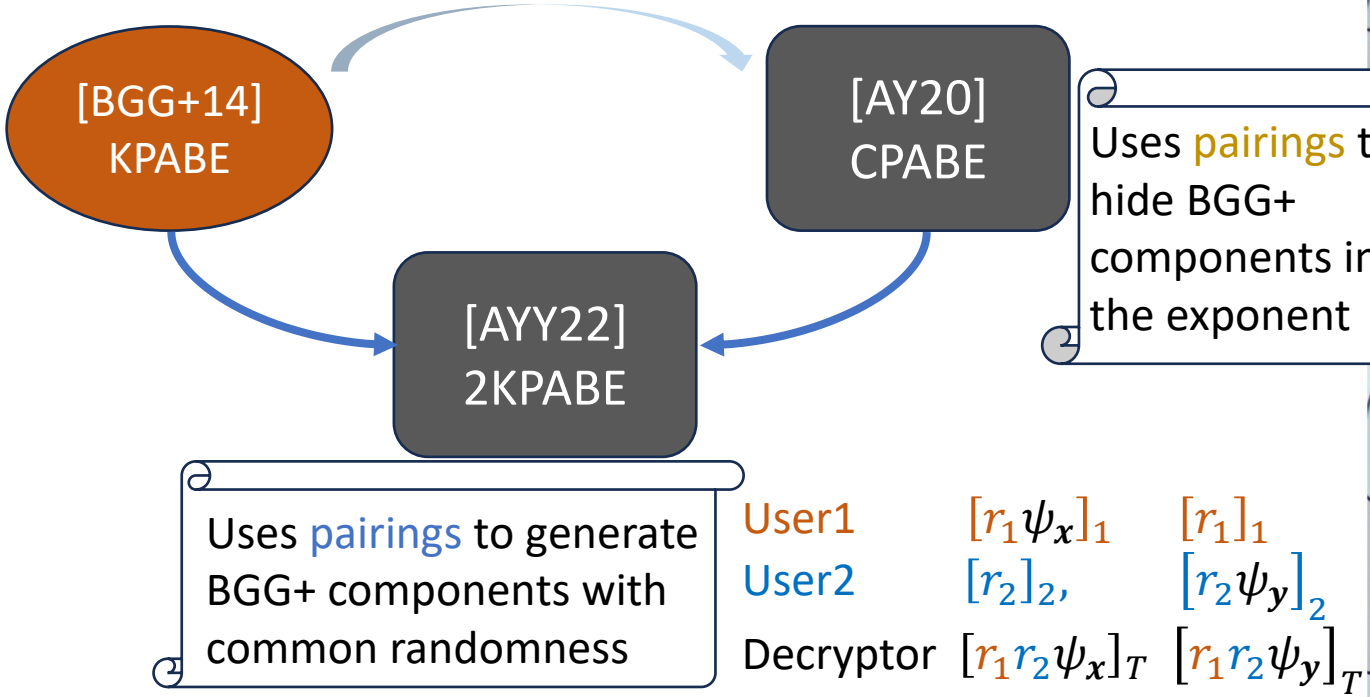
User1
User2
$[r_1\psi_x]_1$
$[r_2]_2$,
$[r_1]_1$
$[r_2\psi_y]_2$

Pathway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2KPABE
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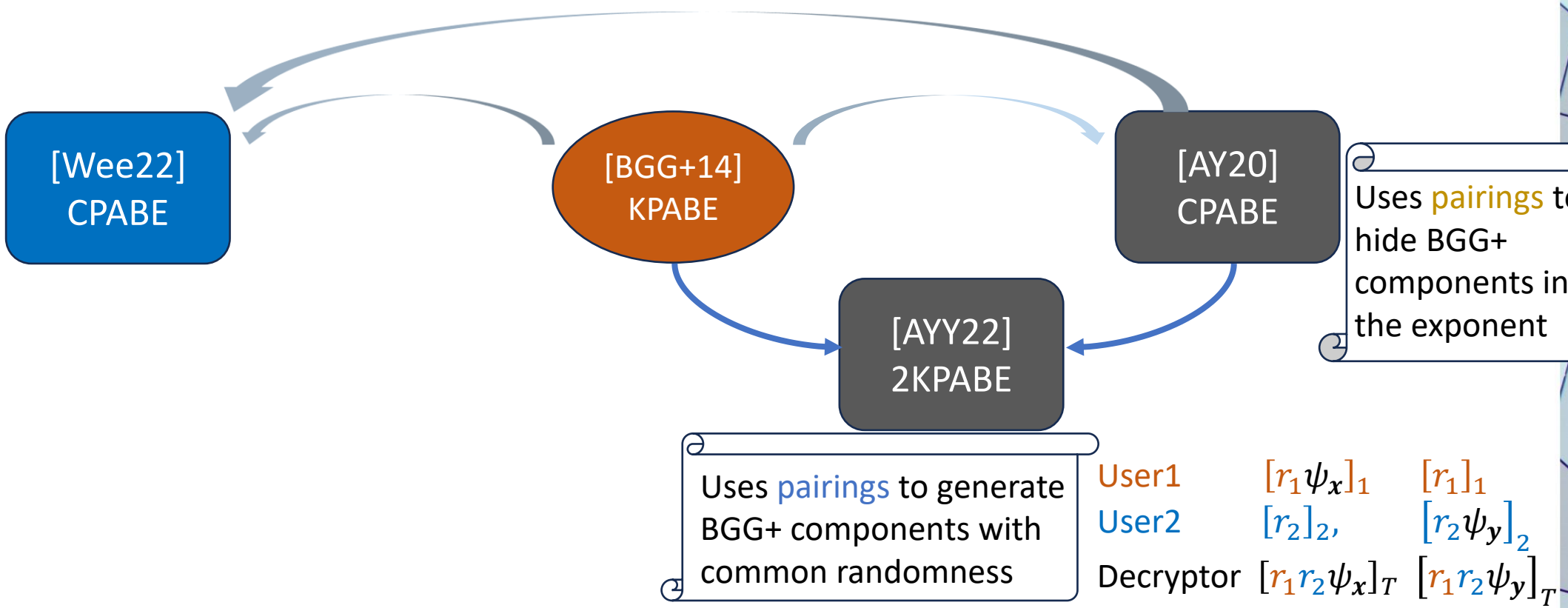
User1 $[r_1\psi_x]_1$ $[r_1]_1$
User2 $[r_2]_2$, $[r_2\psi_y]_2$
Decryptor $[r_1r_2\psi_x]_T$ $[r_1r_2\psi_y]_T$

Pathway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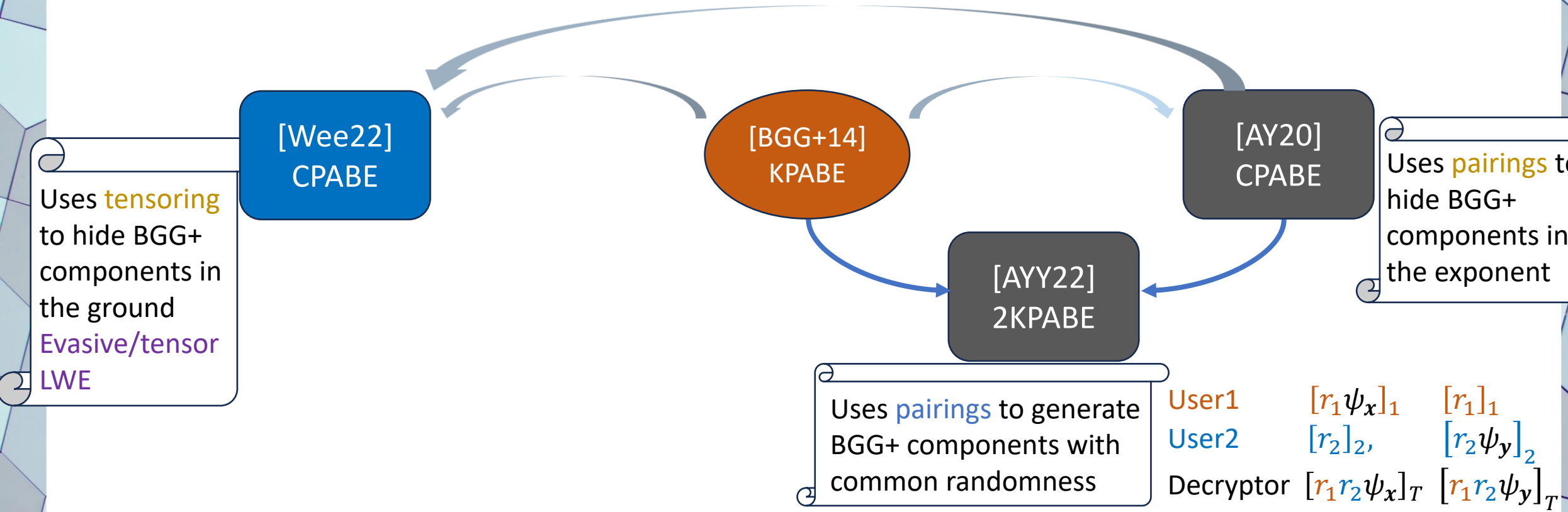
Pathway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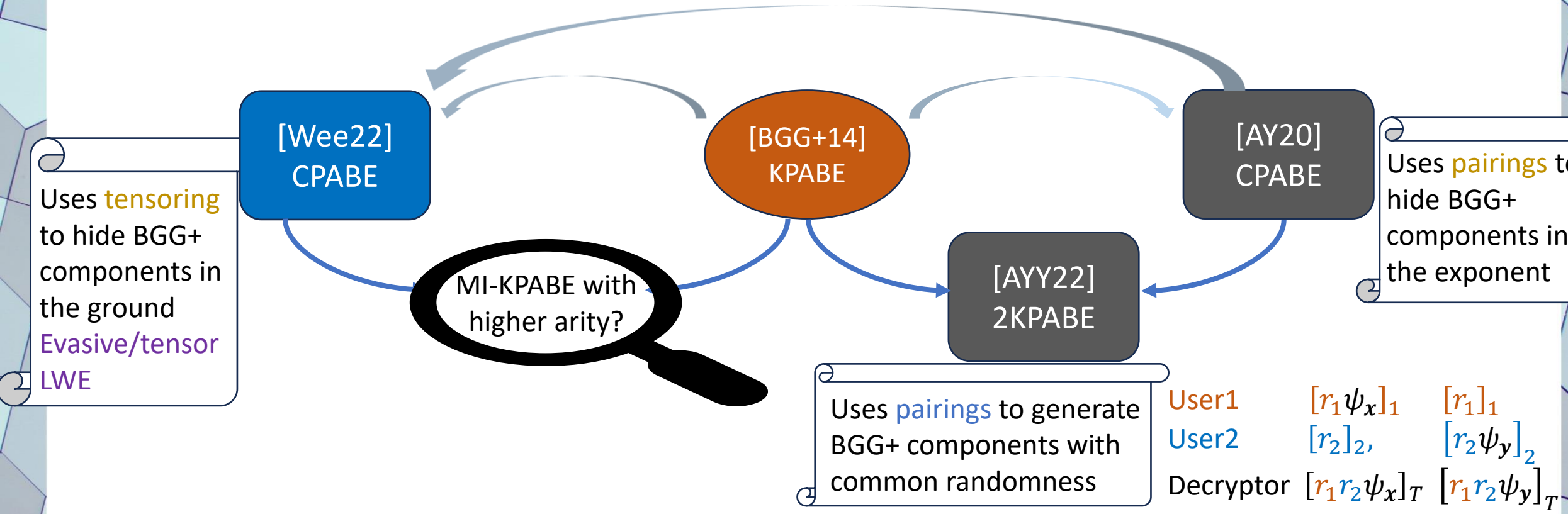
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Evasive/tensor LWE
MI-KPABE with higher arity?
[AYY22] 2KPABE
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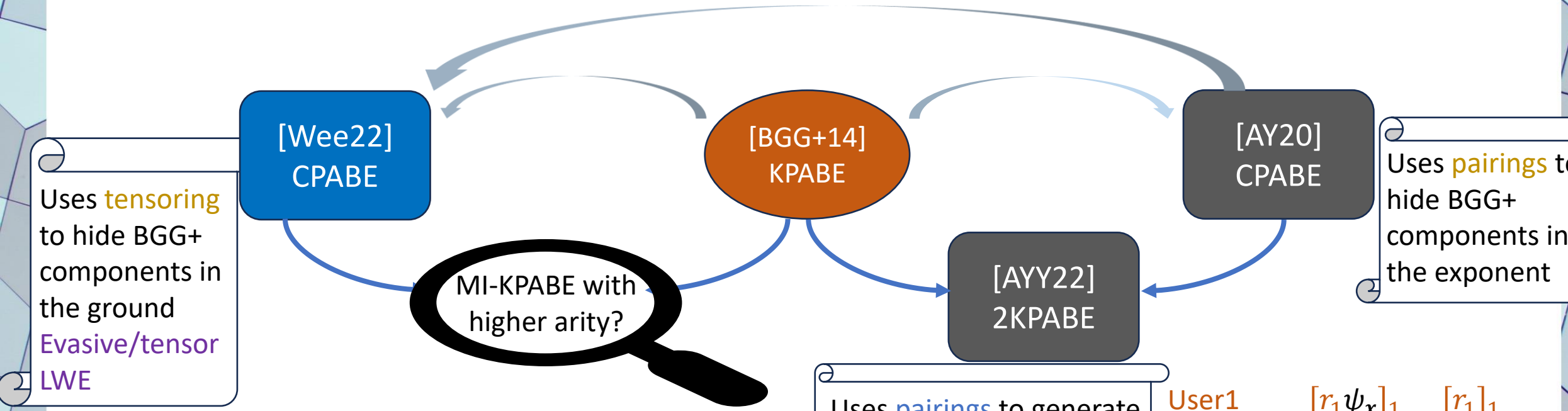

Constant Input KP ABE for NC1 (and P) from Evasive (and tensor) LWE

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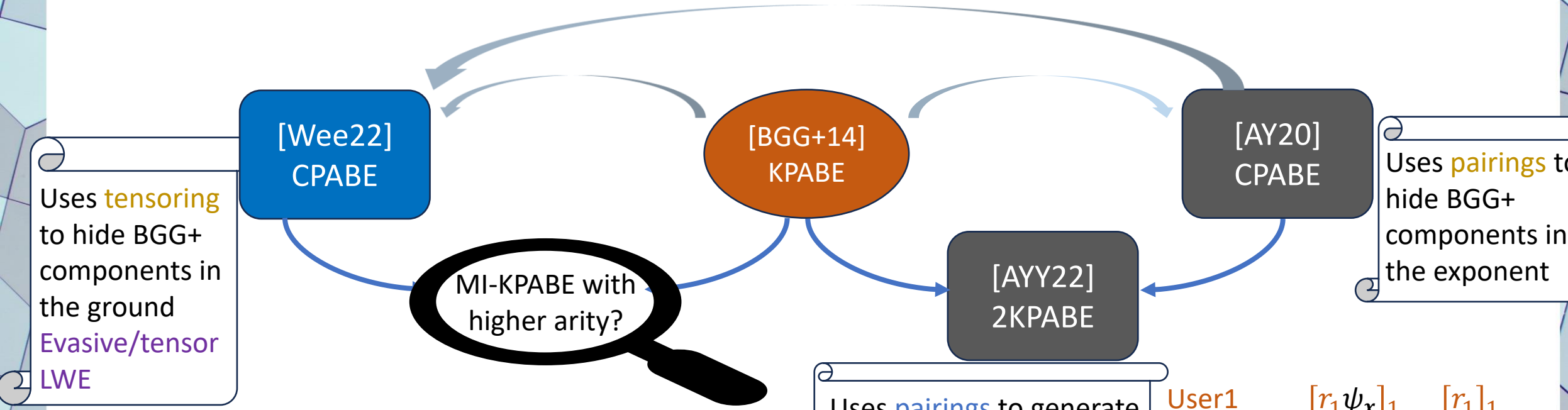

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AYY22 compiler

PE for same settings

Tensor Product

Tensoring

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \boldsymbol{B} \qquad \boldsymbol{A} \otimes \boldsymbol{B} = \begin{pmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} \end{pmatrix}$$

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$$(\boldsymbol{A} \otimes \boldsymbol{I})(\boldsymbol{I} \otimes \boldsymbol{r}^T) = \boldsymbol{A} \otimes \boldsymbol{r}^T$$

$$(\boldsymbol{A} \otimes \boldsymbol{r}^T)\boldsymbol{B} = \boldsymbol{AB} \otimes \boldsymbol{r}^T$$

BGG+14 KPABE Overview

Given $\boldsymbol{A}, \boldsymbol{x}, f$ $\exists$ efficiently computable short matrix $\boldsymbol{H}$ such that

$$(\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G})\boldsymbol{H} = \boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}$$

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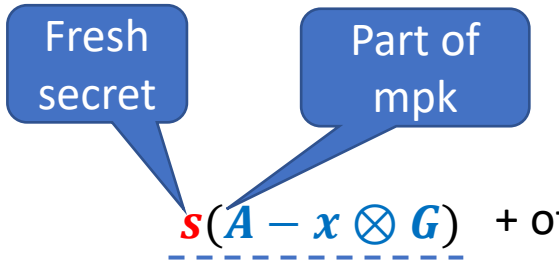

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Fresh secret

Part of mpk

$\boldsymbol{s}(\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G})$ + other terms to embed msg

Right multiplication with $\boldsymbol{H}$ gives $\boldsymbol{s}\boldsymbol{A}_f$ if $f(\boldsymbol{x}) = 0$

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KeyGen(f)

A short preimage of a public vector $\boldsymbol{u}$ wrt $\boldsymbol{A}_f$ to enable recovering the masking term $\boldsymbol{s}\boldsymbol{A}_f$ when $f(\boldsymbol{x}) = 0$

Evasive and Tensor LWE

Evasive LWE

$$(\boldsymbol{B}, \boldsymbol{sB} + \boldsymbol{e}) \approx (\boldsymbol{B}, random)$$ LWE

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Given $\boldsymbol{B}^{-1}(\boldsymbol{P})$, can compute

$$(\boldsymbol{sB} + \boldsymbol{e})\boldsymbol{B}^{-1}(\boldsymbol{P}) = \boldsymbol{sP} + \boldsymbol{e}'$$

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If $(\boldsymbol{B}, \boldsymbol{sB} + \boldsymbol{e}, \boldsymbol{sP} + \boldsymbol{e}') \approx (\boldsymbol{B}, rand, rand)$

Then $\left(\boldsymbol{B}, \boldsymbol{sB} + \boldsymbol{e}, \boldsymbol{B}^{-1}(\boldsymbol{P})\right) \approx (\boldsymbol{B}, rand, \boldsymbol{B}^{-1}(\boldsymbol{P}))$

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Tensor LWE

Correlated BGG+ samples tensored with different random vectors remain pseudorandom

Evasive and Tensor LWE

Evasive LWE

$(\boldsymbol{B}, \boldsymbol{sB} + \boldsymbol{e}) \approx (\boldsymbol{B}, random)$ LWE

Given $\boldsymbol{B}^{-1}(\boldsymbol{P})$, can compute

$$(\boldsymbol{sB} + \boldsymbol{e})\boldsymbol{B}^{-1}(\boldsymbol{P}) = \boldsymbol{sP} + \boldsymbol{e}'$$

Evasive LWE assumes that this is the only way of using $\boldsymbol{B}^{-1}(\boldsymbol{P})$

If $(\boldsymbol{B}, \boldsymbol{sB} + \boldsymbol{e}, \boldsymbol{sP} + \boldsymbol{e}') \approx (\boldsymbol{B}, rand, rand)$

Then $\left(\boldsymbol{B}, \boldsymbol{sB} + \boldsymbol{e}, \boldsymbol{B}^{-1}(\boldsymbol{P})\right) \approx (\boldsymbol{B}, rand, \boldsymbol{B}^{-1}(\boldsymbol{P}))$

Tensor LWE

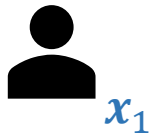
Correlated BGG+ samples tensored with different random vectors remain pseudorandom

$$\boldsymbol{A}, \boldsymbol{s}(\boldsymbol{I} \otimes \boldsymbol{r}_1^T)(\boldsymbol{A} - \boldsymbol{x}_1 \otimes \boldsymbol{G}) + noise, \boldsymbol{r}_1, \cdots, \boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_Q^T\right)\left(\boldsymbol{A} - \boldsymbol{x}_Q \otimes \boldsymbol{G}\right) + noise, \boldsymbol{r}_Q$$
$$\approx_c \quad \boldsymbol{A}, random, \boldsymbol{r}_1, \cdots, random, \boldsymbol{r}_Q$$

Construction Warm-Up

Encryption

$$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$$



Construction Warm-Up

Encryption

$$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$$

BGG+ ciphertext

$$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$$

$\boldsymbol{x}_1$

$\boldsymbol{x}_2$

Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$\boldsymbol{x}_1$

$\boldsymbol{s}(\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G})$

$\boldsymbol{x}_2$

$\boldsymbol{s}(\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G})$

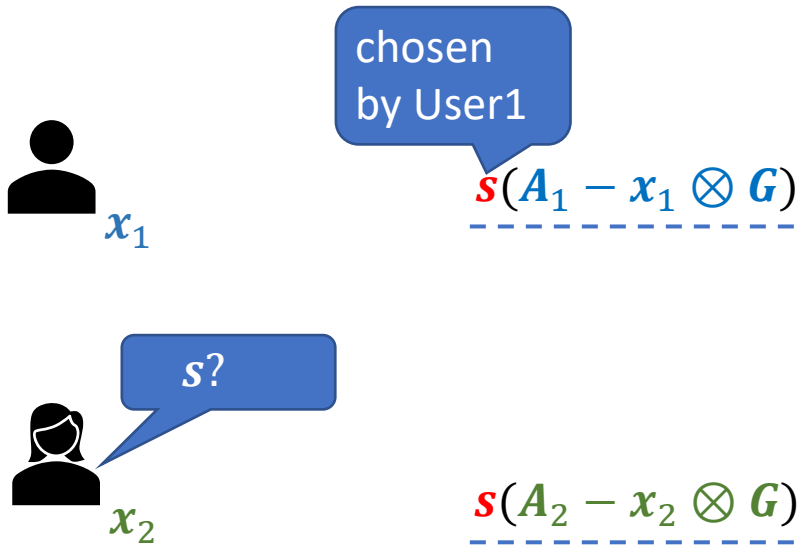
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



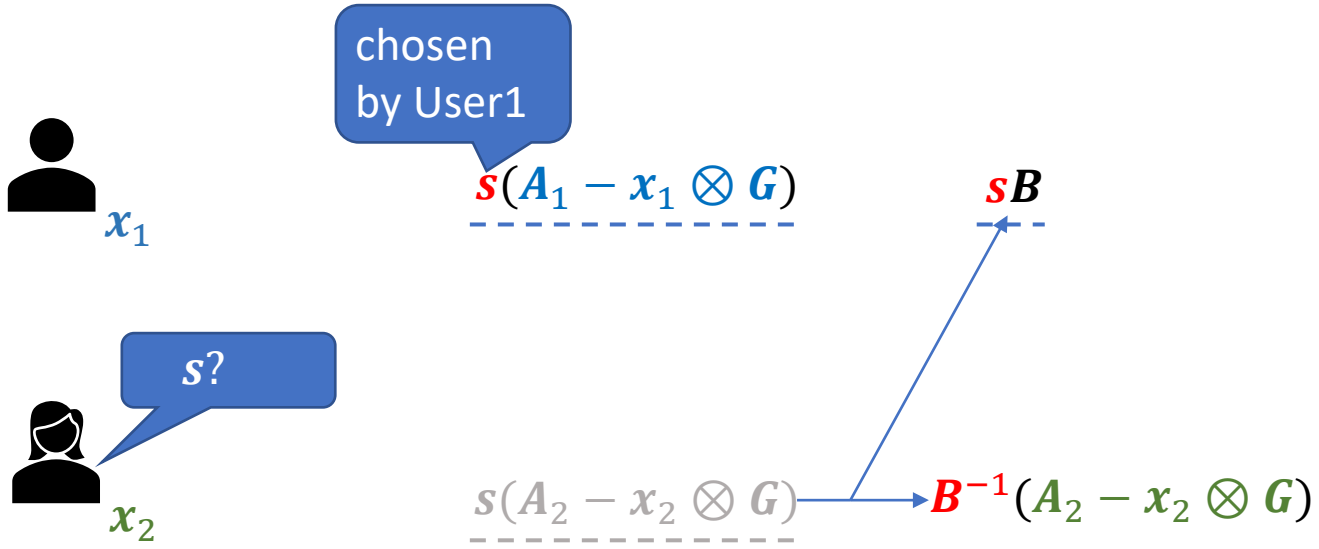
Construction Warm-Up

Encryption

$$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$$

BGG+ ciphertext

$$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$$



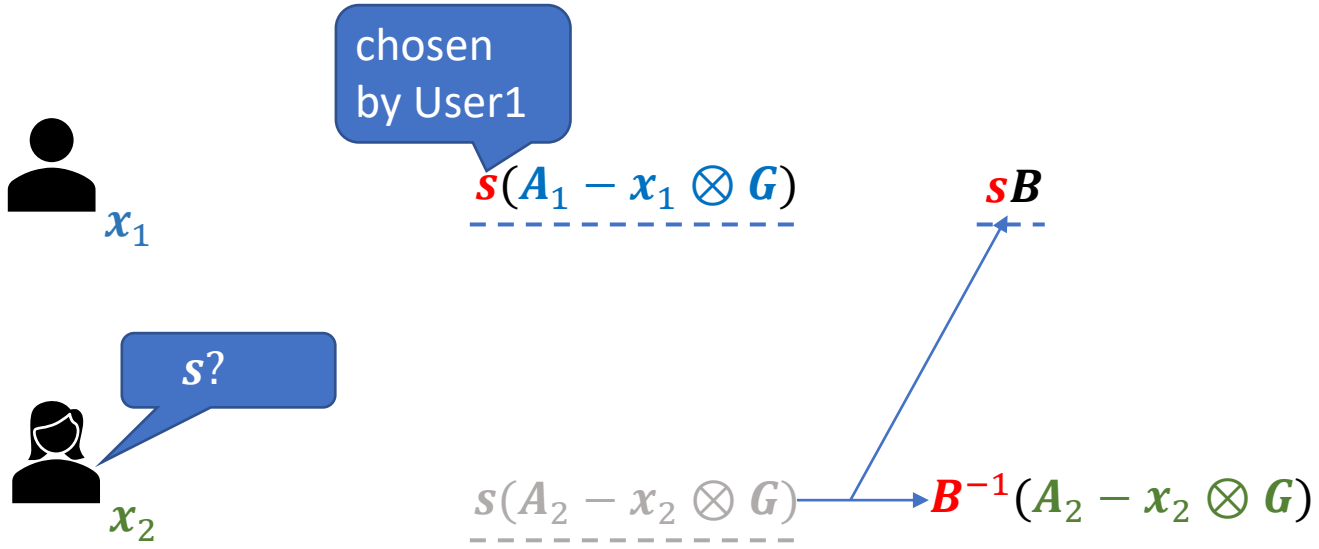
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$

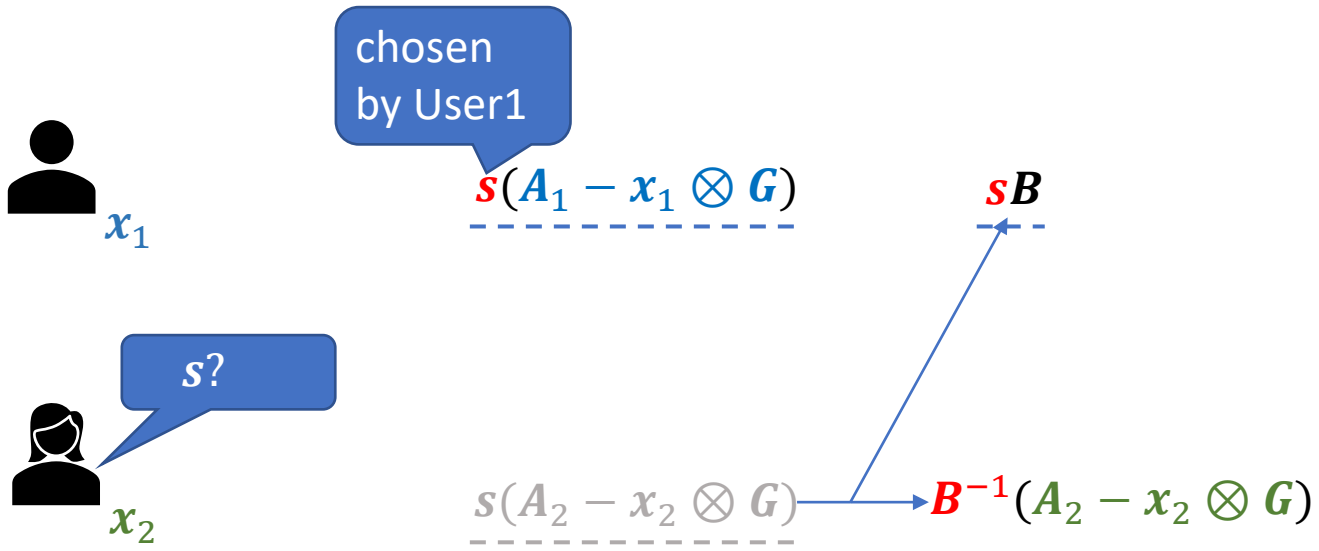
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$

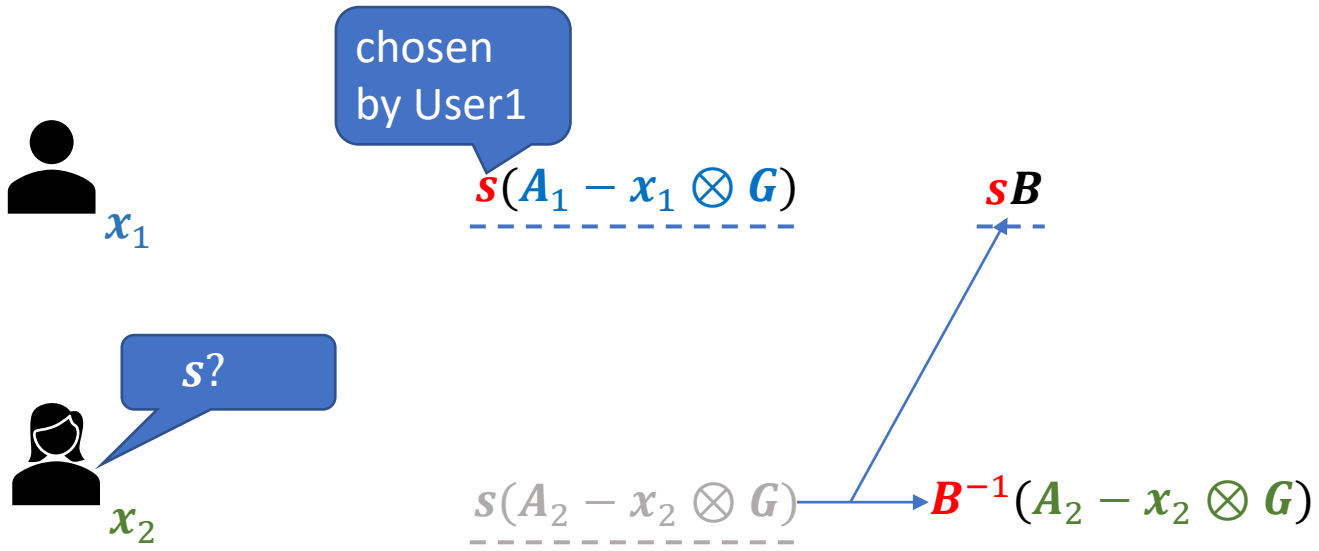
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$

$\boldsymbol{s}(\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G})$, $\boldsymbol{sB}$

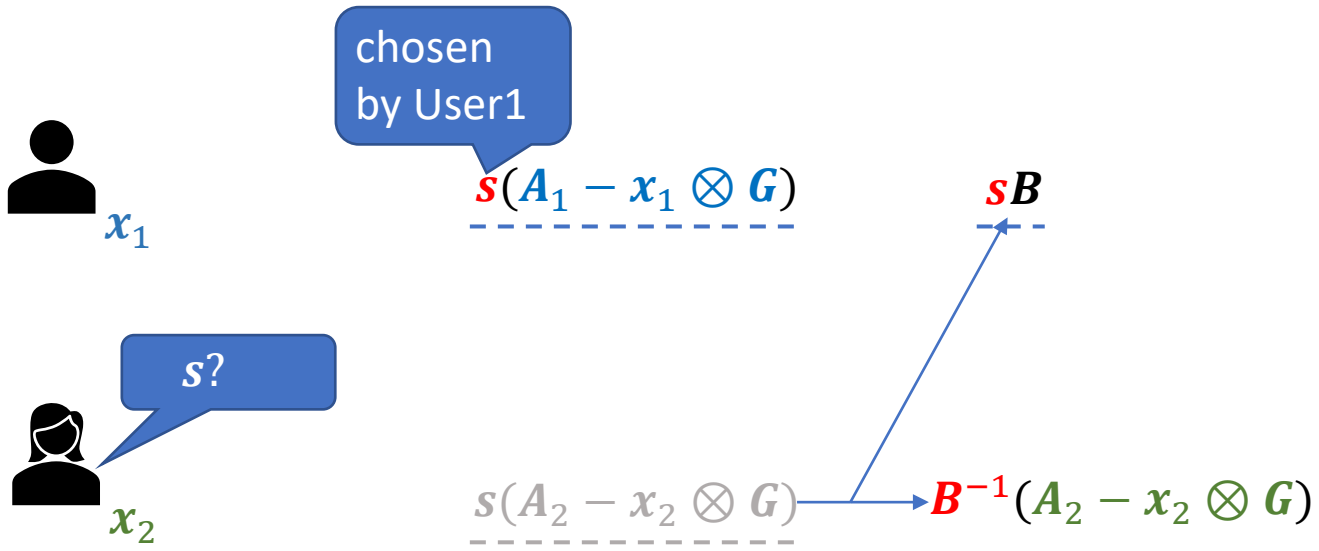
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$

$\boldsymbol{s}(\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G})$, $\boldsymbol{sB}$

$\boldsymbol{B}^{-1}(\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G})$

$\boldsymbol{B}^{-1}(\boldsymbol{A}_2 - \bar{\boldsymbol{x}}_2 \otimes \boldsymbol{G})$

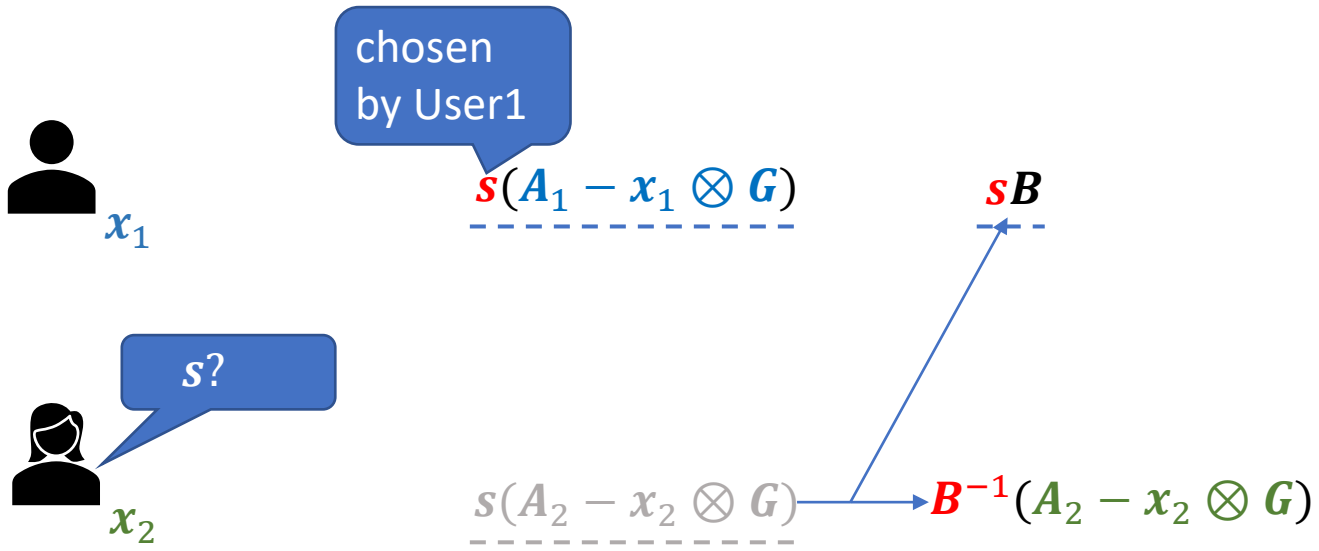
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

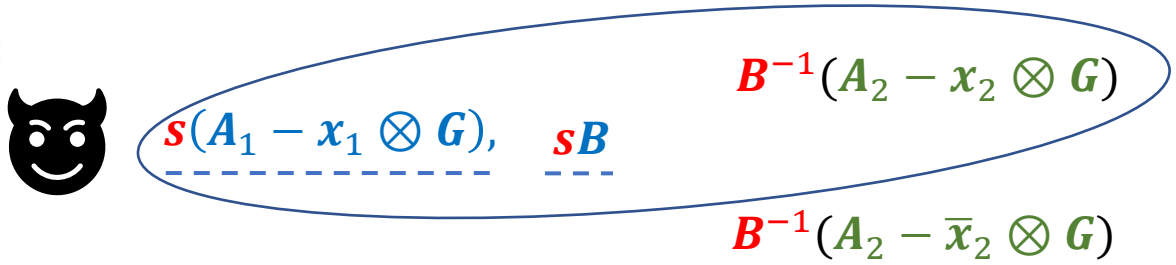
BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$



$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

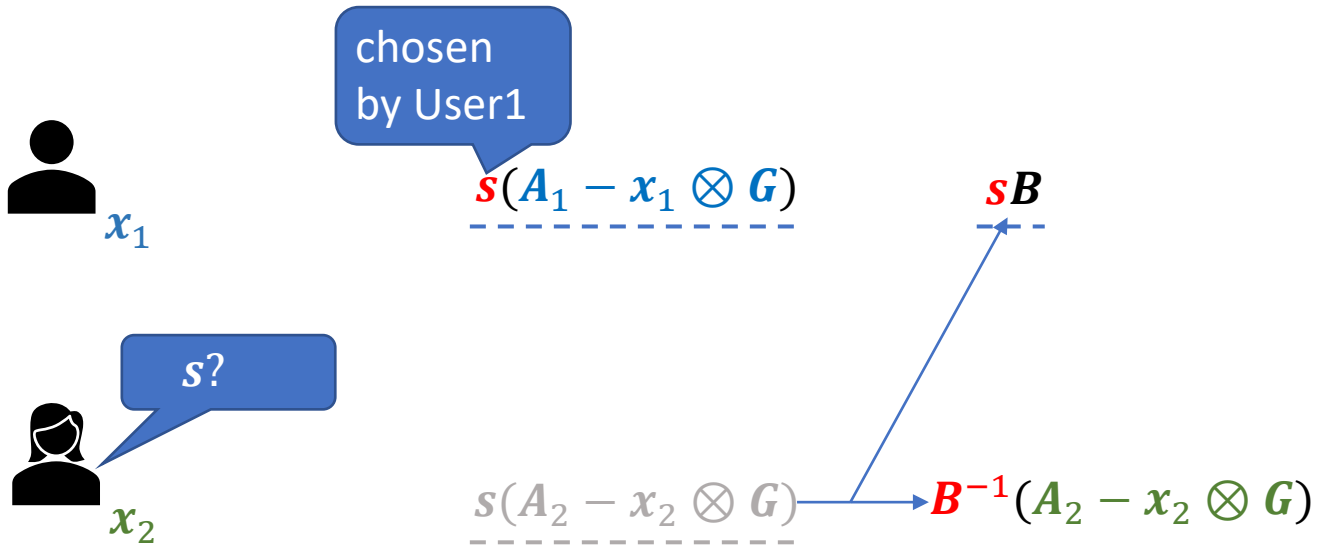
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

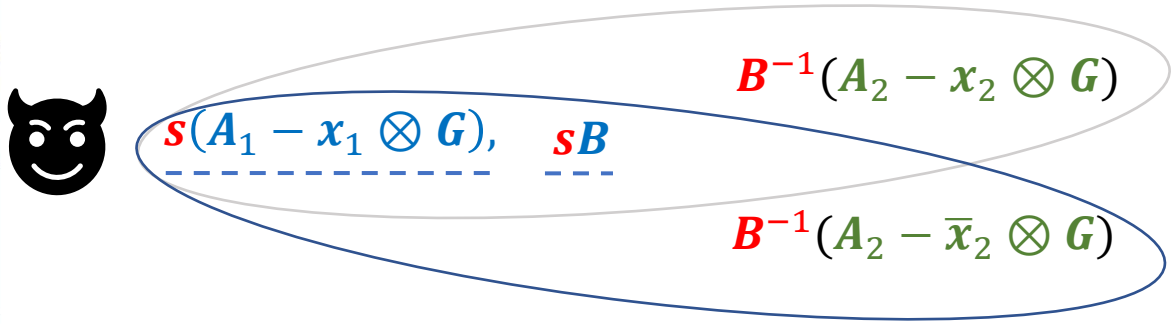
BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$



$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \bar{\boldsymbol{x}}_2) \otimes \boldsymbol{G})$

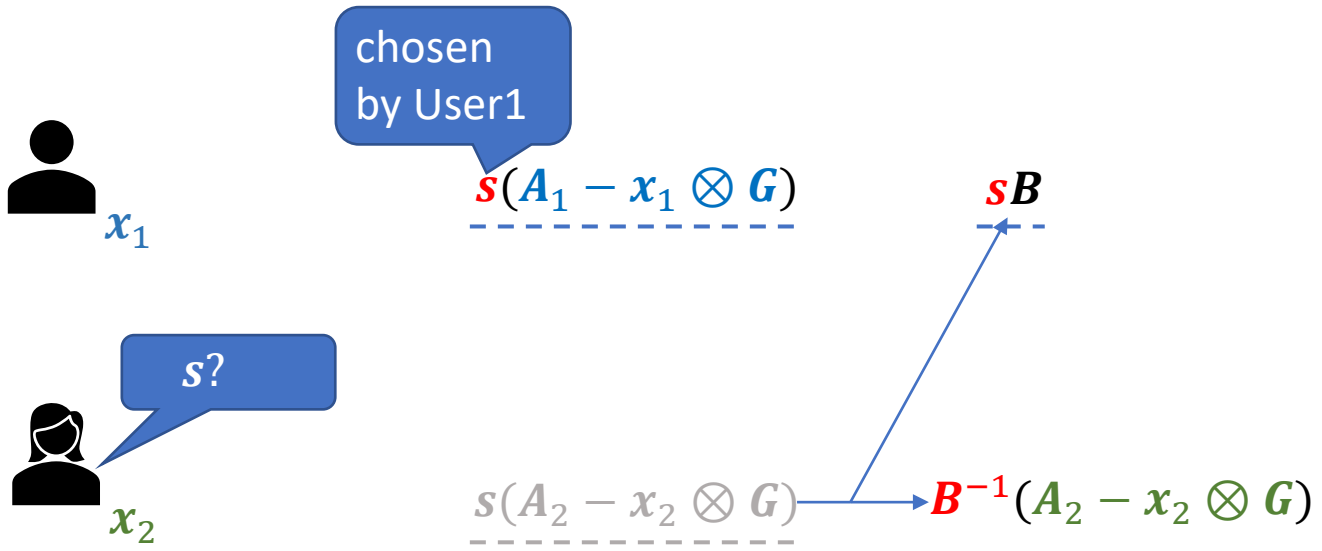
Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

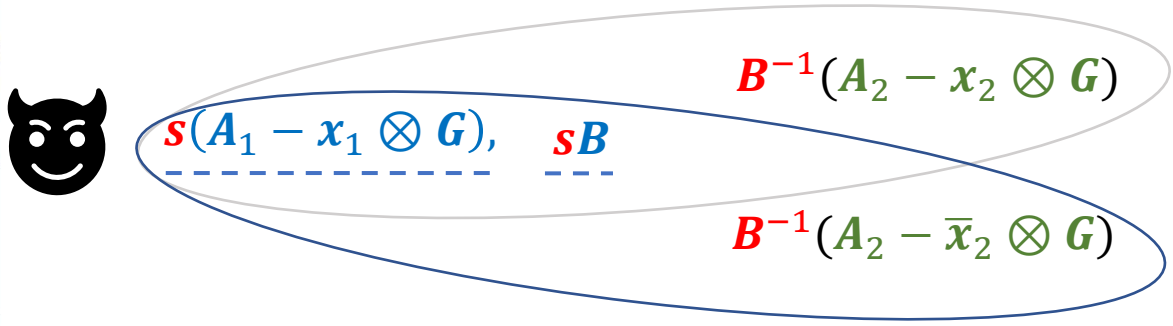
BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$



KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$



$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \bar{\boldsymbol{x}}_2) \otimes \boldsymbol{G})$

Two BGG+ ciphertexts with same secret – Insecure!

Construction Warm-Up

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$\boldsymbol{x}_1$

chosen by User1

$\boldsymbol{s}(\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G})$

$\boldsymbol{sB}$

$\boldsymbol{s}$?

$\boldsymbol{x}_2$

$\boldsymbol{s}(\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G})$ → $\boldsymbol{B}^{-1}(\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G})$

KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{G}\boldsymbol{u}^T)$

Fix [Wee22]

Ensure different random secret $\boldsymbol{s}_i$ for each BGG+ ciphertext as

$\boldsymbol{s}_i = \boldsymbol{s}(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$

Freshly sampled by User 2

$\boldsymbol{s}(\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G})$, $\boldsymbol{sB}$

$\boldsymbol{B}^{-1}(\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G})$

$\boldsymbol{B}^{-1}(\boldsymbol{A}_2 - \bar{\boldsymbol{x}}_2 \otimes \boldsymbol{G})$

$\boldsymbol{s}((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \bar{\boldsymbol{x}}_2) \otimes \boldsymbol{G})$

Two BGG+ ciphertexts with same secret – Insecure!

Construction Attempt 1

Encryption

$$x = (x_1 | x_2)$$

BGG+ ciphertext

$$s\left(I \otimes r_i^T\right)((A_1|A_2) - (x_1|x_2) \otimes G)$$

$$= s(((A_1|A_2) - (x_1|x_2) \otimes G) \otimes r_i^T)$$

$$s\left((A_1 - x_1 \otimes G) \otimes I\right), \quad sB$$

$$B^{-1}\left((A_2 - x_2 \otimes G) \otimes r_i^T\right), \quad r_i^T$$

Construction Attempt 1

Encryption

$$x = (x_1 | x_2)$$

BGG+ ciphertext

$$s\left(I \otimes r_i^T\right)((A_1|A_2) - (x_1|x_2) \otimes G)$$

$$= s(((A_1|A_2) - (x_1|x_2) \otimes G) \otimes r_i^T)$$

x_1

$$s\left((A_1 - x_1 \otimes G) \otimes I\right), \qquad sB$$

$$s\left((A_1 - x_1 \otimes G) \otimes I\right)\left(I \otimes r_i^T\right)$$
$$= s\left((A_1 - x_1 \otimes G) \otimes r_i^T\right)$$

x_2

$$B^{-1}\left((A_2 - x_2 \otimes G) \otimes r_i^T\right), \quad r_i^T$$

Construction Attempt 1

Encryption

$$x = (x_1 | x_2)$$

BGG+ ciphertext

$$s\left(I \otimes r_i^T\right)((A_1|A_2) - (x_1|x_2) \otimes G)$$

$$= s(((A_1|A_2) - (x_1|x_2) \otimes G) \otimes r_i^T)$$

x_1

$$s\left((A_1 - x_1 \otimes G) \otimes I\right), \qquad sB$$

$$s\left((A_1 - x_1 \otimes G) \otimes I\right)\left(I \otimes r_i^T\right)$$
$$= s\left((A_1 - x_1 \otimes G) \otimes r_i^T\right)$$

x_2

$$B^{-1}\left((A_2 - x_2 \otimes G) \otimes r_i^T\right), \quad r_i^T$$

[Wee22] - Homomorphism is preserved even after tensoring

$$\left((A - x \otimes G) \otimes r^T\right)H = (A_f - f(x)G) \otimes r^T$$

Construction Attempt 1

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\right)((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right), \quad \boldsymbol{sB}$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right), \quad \boldsymbol{r}_i^T$

KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{Gu}^T)$

[Wee22] - Homomorphism is preserved even after tensoring

$\left((\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\right)\boldsymbol{H} = (\boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}) \otimes \boldsymbol{r}^T$

Construction Attempt 1

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\right)((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right), \quad \boldsymbol{sB}$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right), \quad \boldsymbol{r}_i^T$

KeyGen(f)

Same as BGG+ key: $\boldsymbol{A}_f^{-1}(\boldsymbol{Gu}^T)$

[Wee22] - Homomorphism is preserved even after tensoring

$\left((\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\right)\boldsymbol{H} = (\boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}) \otimes \boldsymbol{r}^T$

Structured matrix

Proving Security: Cannot apply evasive LWE with $\boldsymbol{A}_f^{-1}(\cdot)$

Construction Attempt 2

Encryption

$$x = (x_1 | x_2)$$

BGG+ ciphertext

$$s\left(I \otimes r_i^T\right)((A_1|A_2) - (x_1|x_2) \otimes G)$$

$$= s(((A_1|A_2) - (x_1|x_2) \otimes G) \otimes r_i^T)$$

KeyGen(f)

x_1

$$s\left((A_1 - x_1 \otimes G) \otimes I\right), \qquad sB$$

$$s\left((A_1 - x_1 \otimes G) \otimes I\right)\left(I \otimes r_i^T\right) = s\left((A_1 - x_1 \otimes G) \otimes r_i^T\right)$$

x_2

$$B^{-1}\left((A_2 - x_2 \otimes G) \otimes r_i^T\right), \quad r_i^T$$

Construction Attempt 2

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\right)((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right), \quad \boldsymbol{sB}$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right), \quad \boldsymbol{r}_i^T$

KeyGen(f)

Modify the key as : $\boldsymbol{B}^{-1}(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I})$

Construction Attempt 2

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\right)((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right), \quad \boldsymbol{sB}$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right), \quad \boldsymbol{r}_i^T$

KeyGen(f)

Modify the key as : $\boldsymbol{B}^{-1}(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I})$

Proving Security: Can now apply evasive LWE

Construction Attempt 2

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)$, $\boldsymbol{sB}$

$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

BGG+ ciphertext

$\boldsymbol{s}\big(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\big)((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{r}_i^T$

KeyGen(f)

Modify the key as : $\boldsymbol{B}^{-1}(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I})$

Proving Security: Can now apply evasive LWE

Prove pseudorandomness of $\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)$, $\boldsymbol{sB}$, $\boldsymbol{s}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{s}\big(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I}\big)$

Construction Attempt 2

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)$, $\boldsymbol{sB}$

$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

BGG+ ciphertext

$\boldsymbol{s}\big(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\big)((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{r}_i^T$

KeyGen(f)

Modify the key as : $\boldsymbol{B}^{-1}(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I})$

Proving Security: Can now apply evasive LWE

Prove pseudorandomness of $\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)$, $\boldsymbol{sB}$, $\boldsymbol{s}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{s}\big(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I}\big)$

Use Tensor LWE assumption, but…

Construction Attempt 2

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)$, $\boldsymbol{sB}$

$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

BGG+ ciphertext

$\boldsymbol{s}\big(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\big)((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1 | \boldsymbol{A}_2) - (\boldsymbol{x}_1 | \boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{r}_i^T$

KeyGen(f)

Modify the key as : $\boldsymbol{B}^{-1}(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I})$

Proving Security: Can now apply evasive LWE

Prove pseudorandomness of $\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big)$, $\boldsymbol{sB}$, $\boldsymbol{s}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{s}\big(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I}\big)$

Use Tensor LWE assumption, but…

misfit

Construction Attempt 2

Encryption

$\boldsymbol{x} = (\boldsymbol{x}_1 | \boldsymbol{x}_2)$

BGG+ ciphertext

$\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^T\right)((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G})$

$= \boldsymbol{s}(((\boldsymbol{A}_1|\boldsymbol{A}_2) - (\boldsymbol{x}_1|\boldsymbol{x}_2) \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T)$

$\boldsymbol{x}_1$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)$, $\boldsymbol{sB}$

$\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)(\boldsymbol{I} \otimes \boldsymbol{r}_i^T)$
$= \boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$

$\boldsymbol{x}_2$

$\boldsymbol{B}^{-1}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{r}_i^T$

KeyGen(f)

Modify the key as : $\boldsymbol{B}^{-1}(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I})$

Proving Security: Can now apply evasive LWE

Prove pseudorandomness of $\boldsymbol{s}\left((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\right)$, $\boldsymbol{sB}$, $\boldsymbol{s}\left((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^T\right)$, $\boldsymbol{s}\left(\boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I}\right)$

Use Tensor LWE assumption, but… misfit

Fix: Hide these terms using LWE samples

Construction Attempt 3

Applyig the Fix

x_1

$$s((A_1 - x_1 \otimes G) \otimes I) \longrightarrow s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$$

sampled by user 1

Part of mpk

Construction Attempt 3

Applyig the Fix

x_1

$$s((A_1 - x_1 \otimes G) \otimes I) \longrightarrow s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$$

$$s(A_f u^T \otimes I) \longrightarrow s(A_f u^T \otimes I) + s_1(D \otimes I)$$

sampled by user 1

Part of mpk

Construction Attempt 3

Applyig the Fix

x_1

$s((A_1 - x_1 \otimes G) \otimes I)$ → $s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$

$s(A_f u^T \otimes I)$ → $s(A_f u^T \otimes I) + s_1(D \otimes I)$

sampled by user 1

Part of mpk

sampled by user 1

Part of mpk

Construction Attempt 3

Applyig the Fix

x_1

$s((A_1 - x_1 \otimes G) \otimes I)$

$s(A_f u^T \otimes I)$

$s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$

$s(A_f u^T \otimes I) + s_1(D \otimes I)$

sampled by user 1

Part of mpk

sampled by user 1

Part of mpk

Construction Attempt 3

Applyig the Fix

x_1

$s((A_1 - x_1 \otimes G) \otimes I)$

$s(A_f u^T \otimes I)$

$s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$

$s(A_f u^T \otimes I) + s_1(D \otimes I)$

sampled by user 1

Part of mpk

sampled by user 1

Part of mpk

Construction Attempt 3

Applyig the Fix

x_1

$s((A_1 - x_1 \otimes G) \otimes I)$ → $s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$

$s(A_f u^T \otimes I)$ → $s(A_f u^T \otimes I) + s_1(D \otimes I)$

sampled by user 1

Part of mpk

sampled by user 1

Part of mpk

Same mask for different functions - insecure

Construction Attempt 3

Applyig the Fix

x_1

$s((A_1 - x_1 \otimes G) \otimes I)$ → $s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$

$s(A_f u^T \otimes I)$ → $s(A_f u^T \otimes I) + s_1(D \otimes I)$

sampled by user 1

Part of mpk

sampled by user 1

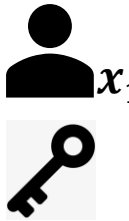
Part of mpk

Same mask for different functions - insecure

Fix: KeyGen must introduce its own randomness

Construction Attempt 3 (Final)

Applyig the Fix



$$s((A_1 - x_1 \otimes G) \otimes I) \longrightarrow s((A_1 - x_1 \otimes G) \otimes I) + s_0(A_0 \otimes I)$$

$$s(A_f u^T \otimes I) \longrightarrow s(A_f u^T \otimes I) + s_1(D \otimes I)$$

sampled by user 1

Part of mpk

sampled by user 1

Part of mpk

Same mask for different functions - insecure

Fix: KeyGen must introduce its own randomness

$$s(A_f u^T \otimes I) + s_1(D \otimes I) \longrightarrow s(A_f u^T \otimes I) + s_1(D \otimes t^T \otimes I)$$

sampled by KeyGen

Evasive LWE Suffices for NC1

Low norm

$$\boldsymbol{s}((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}) + noise \longrightarrow \boldsymbol{s}\left((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise$$

Evasive LWE Suffices for NC1

Low norm

$$s((A_i - x_i \otimes G) \otimes r_i^T) + noise \longrightarrow s\left((A_i - x_i \otimes I) \otimes r_i^T\right) + noise$$

$$\left((A - x \otimes I) \otimes r^T\right)H = (A_f - f(x)G) \otimes r^T$$

Evasive LWE Suffices for NC1

$$s((A_i - x_i \otimes G) \otimes r_i^T) + noise \longrightarrow s\left((A_i - x_i \otimes I) \otimes r_i^T\right) + noise$$

Low norm

$$\left((A - x \otimes I) \otimes r^T\right)H = (A_f - f(x)G) \otimes r^T$$

Low norm if $f \in NC1$

Evasive LWE Suffices for NC1

$$\boldsymbol{s}((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}) + noise \longrightarrow \boldsymbol{s}\left((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise$$

Low norm

$$= \boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$$\left((\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{I}) \otimes \boldsymbol{r}^T\right)\boldsymbol{H} = (\boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}) \otimes \boldsymbol{r}^T$$

Low norm if $f \in NC1$

Evasive LWE Suffices for NC1

$$\boldsymbol{s}((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}) + noise \longrightarrow \boldsymbol{s}\left((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise$$

Low norm

$$= \boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$$\approx (\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$$\left((\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{I}) \otimes \boldsymbol{r}^T\right)\boldsymbol{H} = (\boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}) \otimes \boldsymbol{r}^T$$

Low norm if $f \in NC1$

Evasive LWE Suffices for NC1

Low norm

$$\boldsymbol{s}((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}) + noise \longrightarrow \boldsymbol{s}\left((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise$$

$$= \boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$$\approx \left(\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise\right)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

Fresh random secret

$$\approx \boldsymbol{s}_i(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$$\left((\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{I}) \otimes \boldsymbol{r}^T\right)\boldsymbol{H} = (\boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}) \otimes \boldsymbol{r}^T$$

Low norm if $f \in NC1$

Evasive LWE Suffices for NC1

$$\boldsymbol{s}((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{G}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}) + noise \longrightarrow \boldsymbol{s}\left((\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise$$

Low norm

$$= \boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$$\approx (\boldsymbol{s}\left(\boldsymbol{I} \otimes \boldsymbol{r}_i^{\boldsymbol{T}}\right) + noise)(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

Fresh random secret

$$\approx \boldsymbol{s}_i(\boldsymbol{A}_i - \boldsymbol{x}_i \otimes \boldsymbol{I}) + noise$$

$\approx$ random (from LWE)

$$\left((\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{I}) \otimes \boldsymbol{r}^T\right)\boldsymbol{H} = (\boldsymbol{A}_f - f(\boldsymbol{x})\boldsymbol{G}) \otimes \boldsymbol{r}^T$$

Low norm if $f \in NC1$

Summary and Open Problems

We constructed constant arity ABE from evasive and tensor LWE

Evasive LWE suffices for NC1 circuits

We also studied tensor LWE assumption and show new implications

Summary and Open Problems

We constructed constant arity ABE from evasive and tensor LWE

Evasive LWE suffices for NC1 circuits

We also studied tensor LWE assumption and show new implications

Open Problems

Construction of constant arity miABE from standard LWE

Going beyond constant arity

Supporting corruptions

Thank You!

Final Construction

User1$(\boldsymbol{x}_1, m)$

$$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big) + \boldsymbol{s}_0(\boldsymbol{A}_0 \otimes \boldsymbol{I}),$$

$$(\boldsymbol{s}, \boldsymbol{s}_0, \quad)\boldsymbol{B}, \qquad \text{if } m = 0, \text{ else random}$$

Final Construction

User1$(\boldsymbol{x}_1, m)$

$$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big) + \boldsymbol{s}_0(\boldsymbol{A}_0 \otimes \boldsymbol{I}),$$

$$(\boldsymbol{s}, \boldsymbol{s}_0, \quad)\boldsymbol{B}, \qquad \text{if } m = 0, \text{ else random}$$

User2$(\boldsymbol{x}_2)$

$$\boldsymbol{B}^{-1} \begin{pmatrix} \big((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\big) \\ (\boldsymbol{A}_0 \otimes \boldsymbol{r}^T) \end{pmatrix}$$

Final Construction

User1$(\boldsymbol{x}_1, m)$

$$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big) + \boldsymbol{s}_0(\boldsymbol{A}_0 \otimes \boldsymbol{I}),$$

$$(\boldsymbol{s}, \boldsymbol{s}_0, \boldsymbol{s}_1)\boldsymbol{B}, \qquad \text{if } m = 0, \text{ else random}$$

KeyGen(f)

$$\boldsymbol{B}^{-1}\begin{pmatrix} \boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I} \\ \boldsymbol{0} \\ (\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{I}) \end{pmatrix},$$

User2$(\boldsymbol{x}_2)$

$$\boldsymbol{B}^{-1}\begin{pmatrix} \big((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\big) \\ (\boldsymbol{A}_0 \otimes \boldsymbol{r}^T) \end{pmatrix}$$

Final Construction

User1$(\boldsymbol{x}_1, m)$

$$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big) + \boldsymbol{s}_0(\boldsymbol{A}_0 \otimes \boldsymbol{I}),$$

$$(\boldsymbol{s}, \boldsymbol{s}_0, \boldsymbol{s}_1)\boldsymbol{B}, \qquad \text{if } m = 0, \text{ else random}$$

KeyGen(f)

$$\boldsymbol{B}^{-1}\begin{pmatrix} \boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I} \\ \boldsymbol{0} \\ (\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{I}) \end{pmatrix},$$

Recovering the mask $\boldsymbol{s}_1(\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{r}^T)$

User2$(\boldsymbol{x}_2)$

$$\boldsymbol{B}^{-1}\begin{pmatrix} \big((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\big) \\ (\boldsymbol{A}_0 \otimes \boldsymbol{r}^T) \end{pmatrix}$$

Final Construction

User1$(\boldsymbol{x}_1, m)$

$$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big) + \boldsymbol{s}_0(\boldsymbol{A}_0 \otimes \boldsymbol{I}),$$

$$(\boldsymbol{s}, \boldsymbol{s}_0, \boldsymbol{s}_1)\boldsymbol{B}, \qquad \text{if } m = 0, \text{ else random}$$

KeyGen(f)

$$\boldsymbol{B}^{-1}\begin{pmatrix} \boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I} \\ \boldsymbol{0} \\ (\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{I}) \end{pmatrix},$$

User2$(\boldsymbol{x}_2)$

$$\boldsymbol{B}^{-1}\begin{pmatrix} \big((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\big) \\ (\boldsymbol{A}_0 \otimes \boldsymbol{r}^T) \end{pmatrix}$$

Recovering the mask $\boldsymbol{s}_1(\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{r}^T)$

$$= \boldsymbol{s}_1(\boldsymbol{I} \otimes \boldsymbol{r}^T)(\boldsymbol{D} \otimes \boldsymbol{t}^T)$$

Final Construction

User1$(\boldsymbol{x}_1, m)$

$$\boldsymbol{s}\big((\boldsymbol{A}_1 - \boldsymbol{x}_1 \otimes \boldsymbol{G}) \otimes \boldsymbol{I}\big) + \boldsymbol{s}_0(\boldsymbol{A}_0 \otimes \boldsymbol{I}),$$

$$(\boldsymbol{s}, \boldsymbol{s}_0, \boldsymbol{s}_1)\boldsymbol{B}, \qquad \text{if } m = 0, \text{ else random}$$

KeyGen(f)

$$\boldsymbol{B}^{-1}\begin{pmatrix} \boldsymbol{A}_f \boldsymbol{u}^T \otimes \boldsymbol{I} \\ \boldsymbol{0} \\ (\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{I}) \end{pmatrix},$$

User2$(\boldsymbol{x}_2)$

$$\boldsymbol{B}^{-1}\begin{pmatrix} \big((\boldsymbol{A}_2 - \boldsymbol{x}_2 \otimes \boldsymbol{G}) \otimes \boldsymbol{r}^T\big) \\ (\boldsymbol{A}_0 \otimes \boldsymbol{r}^T) \end{pmatrix}$$

Recovering the mask $\boldsymbol{s}_1(\boldsymbol{D} \otimes \boldsymbol{t}^T \otimes \boldsymbol{r}^T)$

$$= \boldsymbol{s}_1(\boldsymbol{I} \otimes \boldsymbol{r}^T)(\boldsymbol{D} \otimes \boldsymbol{t}^T)$$

$$= \boldsymbol{s}_1\,(\boldsymbol{I} \otimes \boldsymbol{r}^T)\boldsymbol{C}\,\boldsymbol{C}^{-1}(\boldsymbol{D} \otimes\ \boldsymbol{t}^T)$$

Final Construction

User1(x_1, m)

$$s\big((A_1 - x_1 \otimes G) \otimes I\big) + s_0(A_0 \otimes I),$$
$$(s, s_0, s_1)B, \qquad \text{if } m = 0, \text{ else random}$$

KeyGen(f)

$$B^{-1}\begin{pmatrix} A_f u^T \otimes I \\ 0 \\ (D \otimes t^T \otimes I) \end{pmatrix}, \quad C^{-1}(D \otimes t^T \otimes I)$$

User2(x_2)

$$B^{-1}\begin{pmatrix} \big((A_2 - x_2 \otimes G) \otimes r^T\big) & & \\ & (A_0 \otimes r^T) & \\ & & (C \otimes r^T) \end{pmatrix}$$

Recovering the mask $s_1(D \otimes t^T \otimes r^T)$

$$= s_1(I \otimes r^T)(D \otimes t^T)$$
$$= s_1\,(I \otimes r^T)C\,C^{-1}(D \otimes\ t^T)$$

Tensor Product

Tensoring

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \boldsymbol{B} \qquad \boldsymbol{A} \otimes \boldsymbol{B} = \begin{pmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} \end{pmatrix}$$

Tensor Product

Tensoring

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \boldsymbol{B} \qquad \boldsymbol{A} \otimes \boldsymbol{B} = \begin{pmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} \end{pmatrix}$$

$$(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = \boldsymbol{AC} \otimes \boldsymbol{BD}$$

Tensor Product

Tensoring

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \boldsymbol{B} \qquad \boldsymbol{A} \otimes \boldsymbol{B} = \begin{pmatrix} a_{11}\boldsymbol{B} & a_{12}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & a_{22}\boldsymbol{B} \end{pmatrix}$$

$$(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = \boldsymbol{AC} \otimes \boldsymbol{BD}$$

$$(\boldsymbol{A} \otimes \boldsymbol{I})(\boldsymbol{I} \otimes \boldsymbol{r}^T) = \boldsymbol{A} \otimes \boldsymbol{r}^T$$

$$(\boldsymbol{A} \otimes \boldsymbol{r}^T)\boldsymbol{B} = \boldsymbol{AB} \otimes \boldsymbol{r}^T$$