Reusable Secure Computation in the Plain Model

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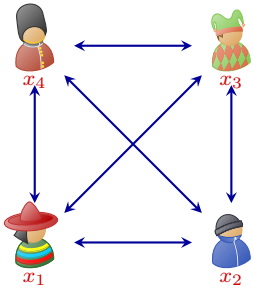
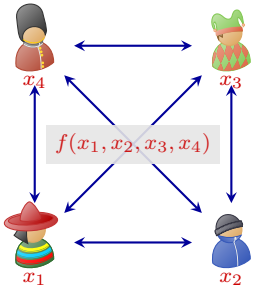
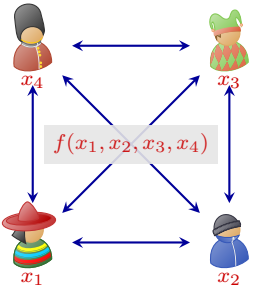
UC Berkeley

Aug 2023 @ Crypto'23


x_4
x_3
x_1
x_2


x_4
x_3
$f(x_1, x_2, x_3, x_4)$
x_1
x_2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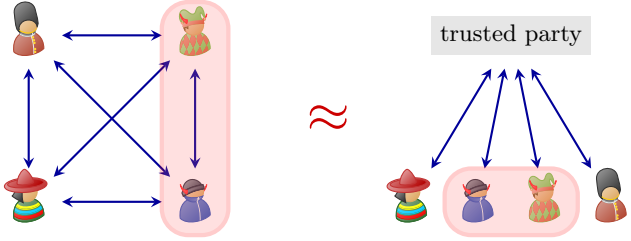
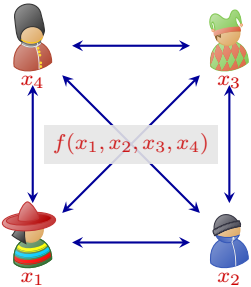
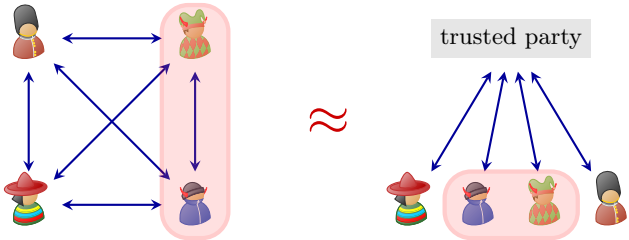
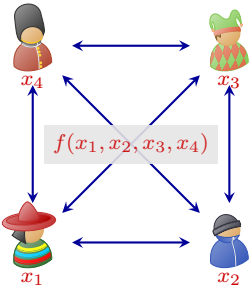

trusted party

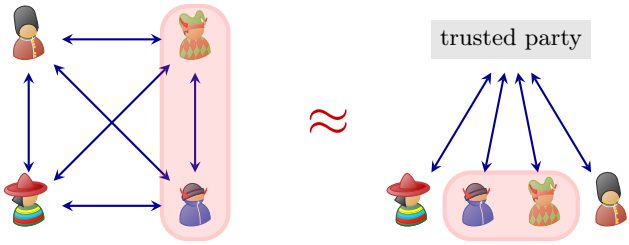


Secure Multiparty Computation

- plain model
- dishonest majority
- malicious security (black-box simulation)
- polynomial-time simulator







Secure Multiparty Computation

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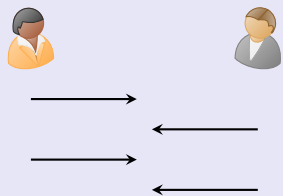
Objective

Construct round-optimal protocol.

Lower bound

- 2PC unidirectional message
- 4 rounds [Katz-Ostrovsky'04]



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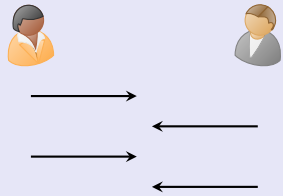
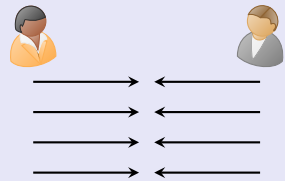


- MPC Simultaneous message
- 4 rounds
  [Garg-Mukherjee-Pandey-Polychroniadou'16]



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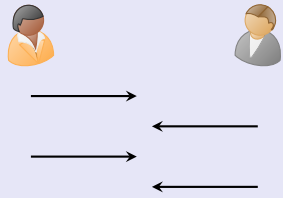

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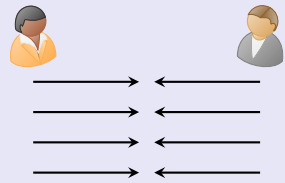


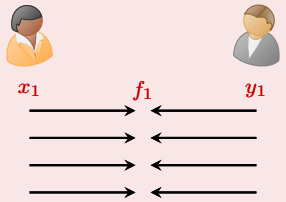
Upper bound

[Yao86, BMR90, KO04, IPS08, IKOPS11, ORS15, GMPP16, BHP17, ACJ17, BL18, GS18, BGJKKS18, HHPV18, FMV19, CCGJO20]

Matching upper bound with minimal assumption (4-round OT)

Reusable Setting

Suppose Alice and Bob want to continuously evaluate multiple functions $f_1(x_1, y_1),\ f_2(x_2, y_2), \ldots$



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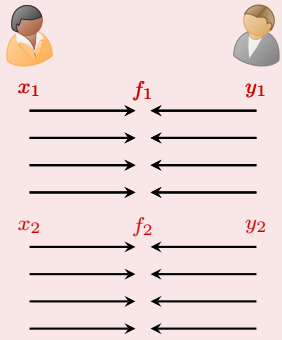

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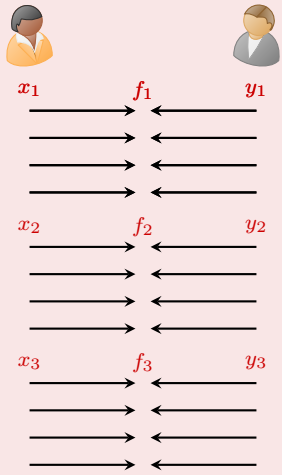

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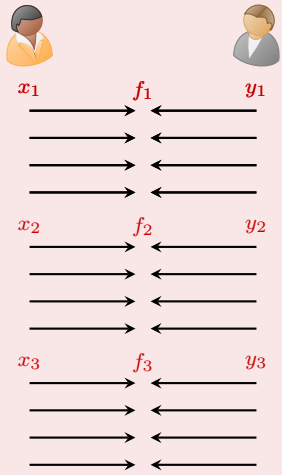
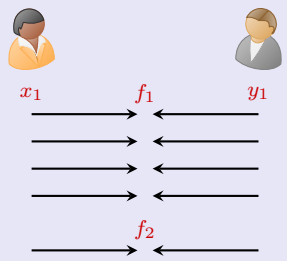


Can we reuse the previous interactions to reduce the number of rounds?

Optimal Round Complexity for Different Modes of Reusability

if both inputs and function change

if only the function changes



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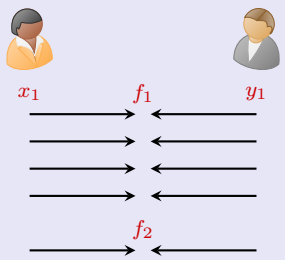

The first three rounds can be reused!

Optimal Round Complexity for Different Modes of Reusability

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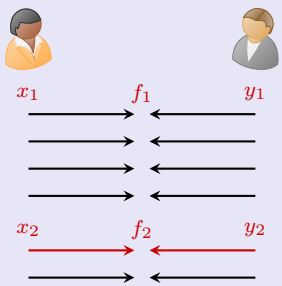
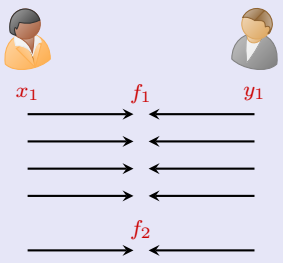


if only the function changes



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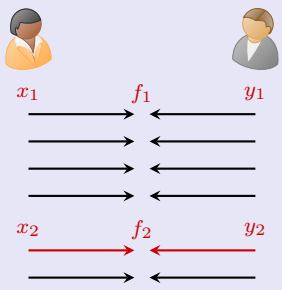
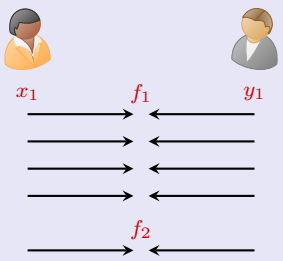


Residual attack if only one round of interaction.

if only the function changes



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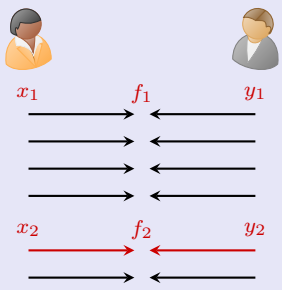
Optimal Round Complexity for Different Modes of Reusability

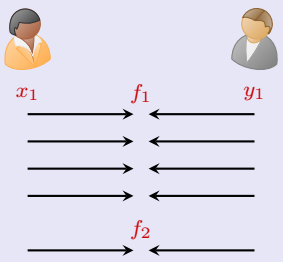
if both inputs and function change



Residual attack if only one round of interaction.

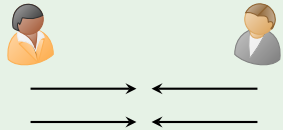
The first two rounds can be reused!

if only the function changes



The first three rounds can be reused!

Our Model for Round-optimal Reusable Secure Computation



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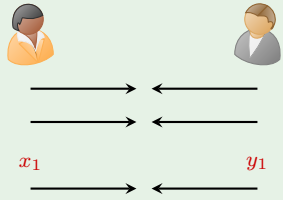

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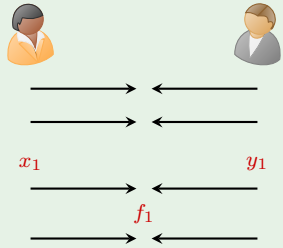

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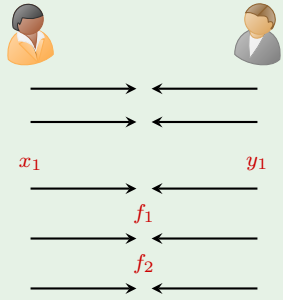

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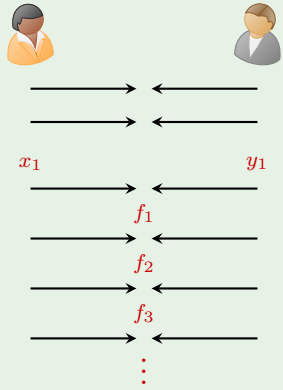

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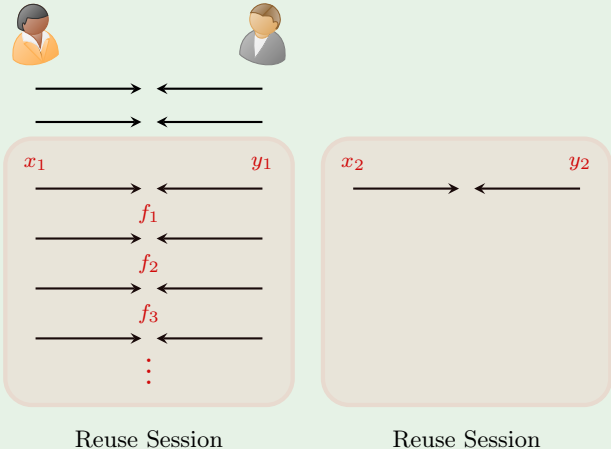

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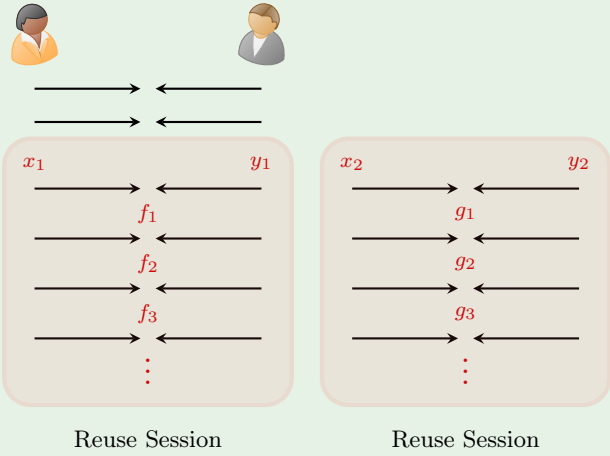

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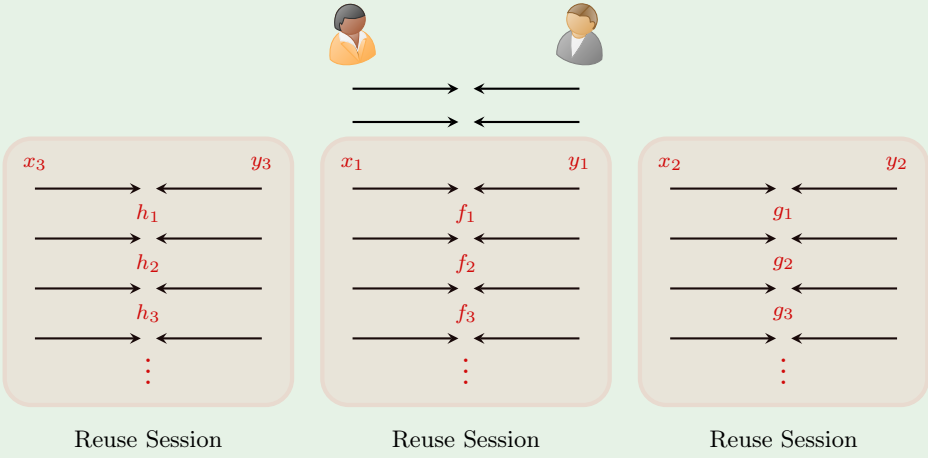

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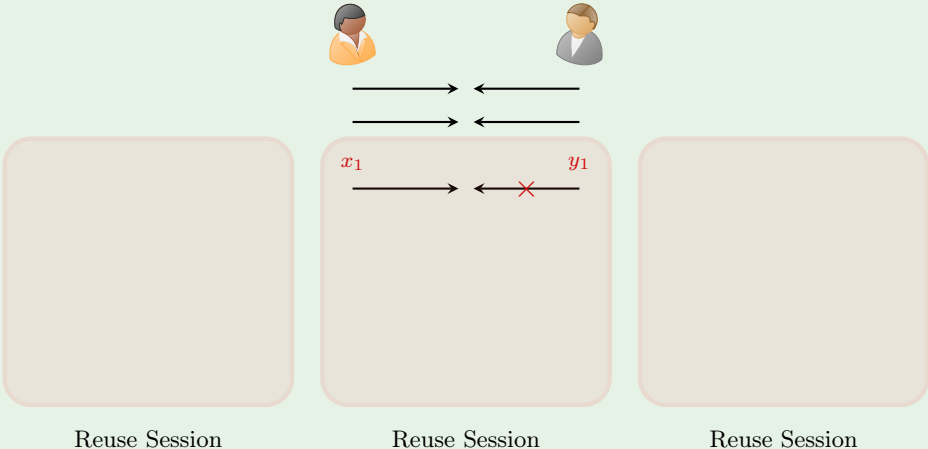

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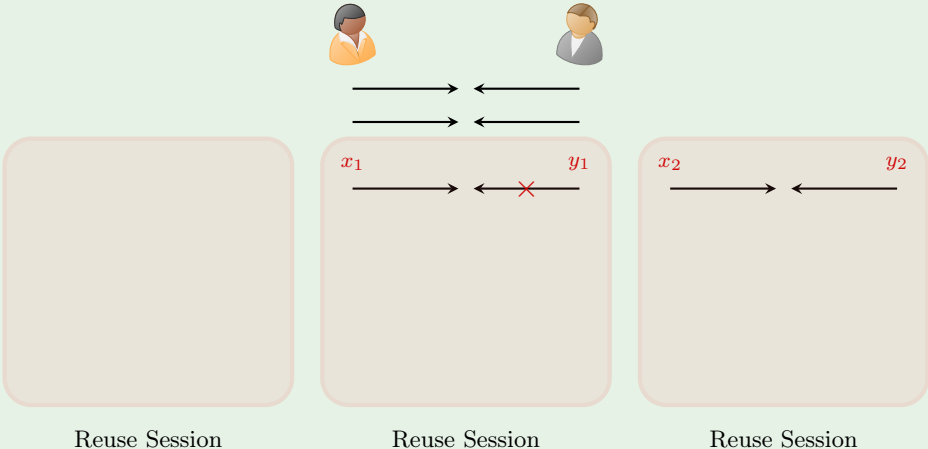

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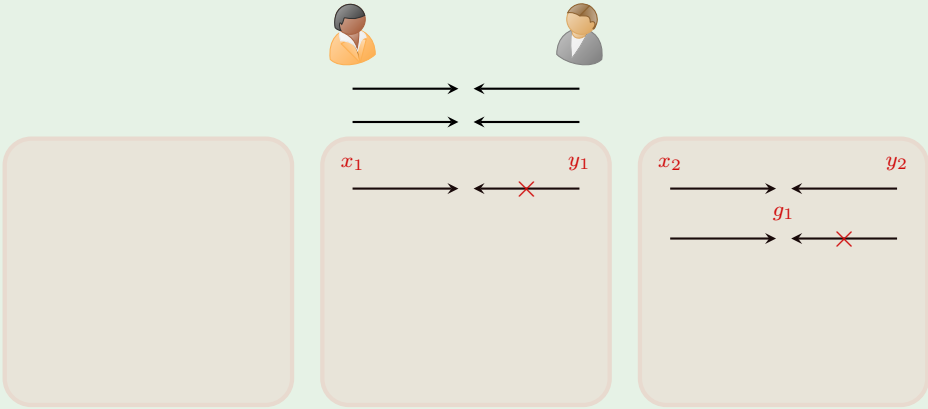

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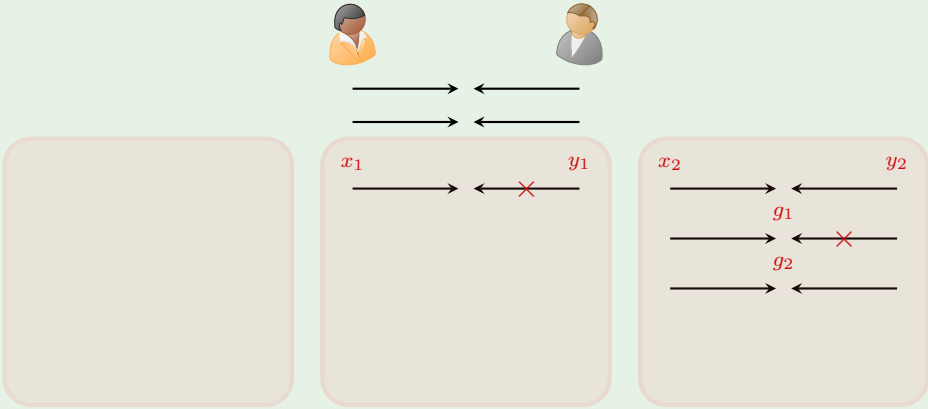

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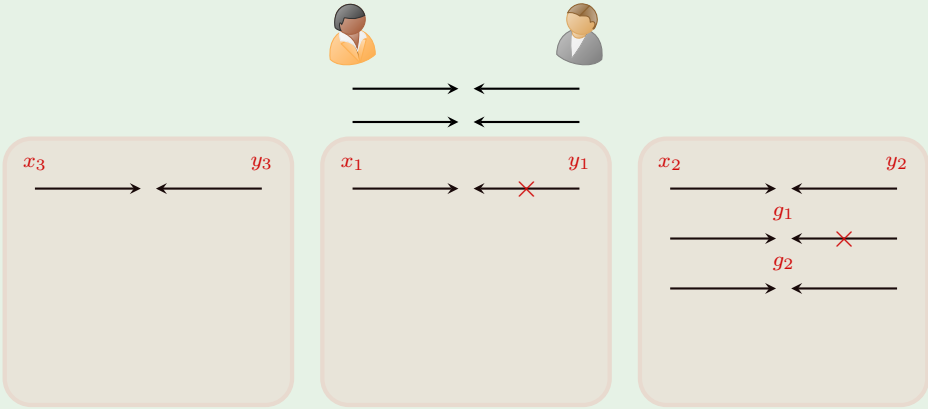

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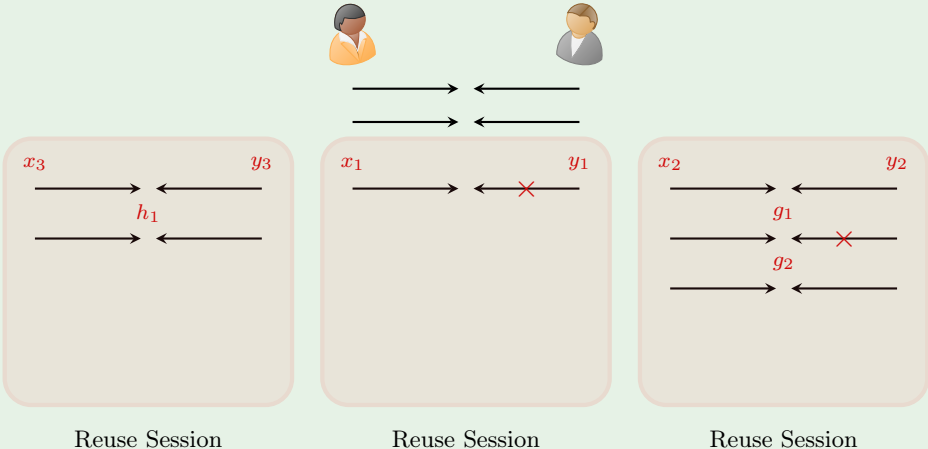

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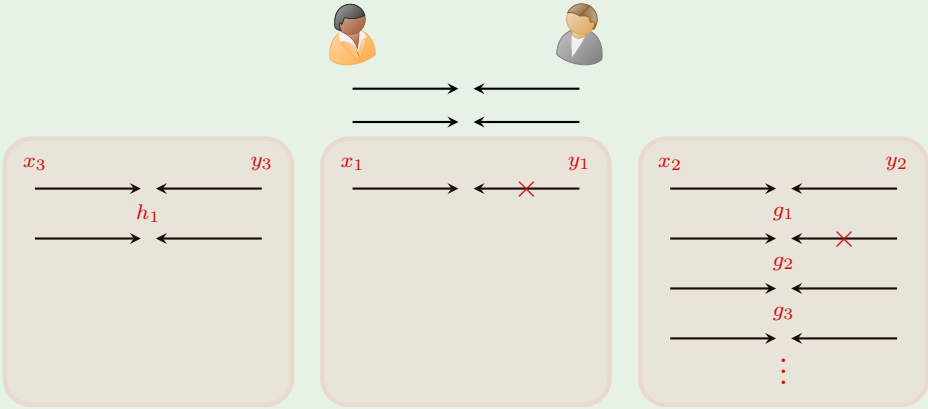

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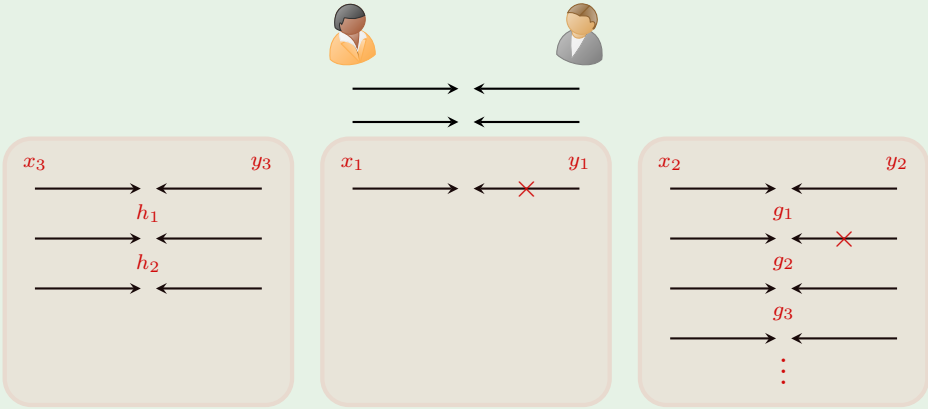

Related Work — Reusable MPC with trusted setup

[Benhamouda-Lin'20, Bartusek-Garg-Masny-Mukherjee'20, Ananth-Jain-Jin-Malavolta'21'22, Benhamouda-Jain-Komargodski-Lin'21, Bartusek-Garg-Srinivasan-Zhang'22]

- In the CRS model
- Two-round malicious-secure MPC protocol
- First round can be reused

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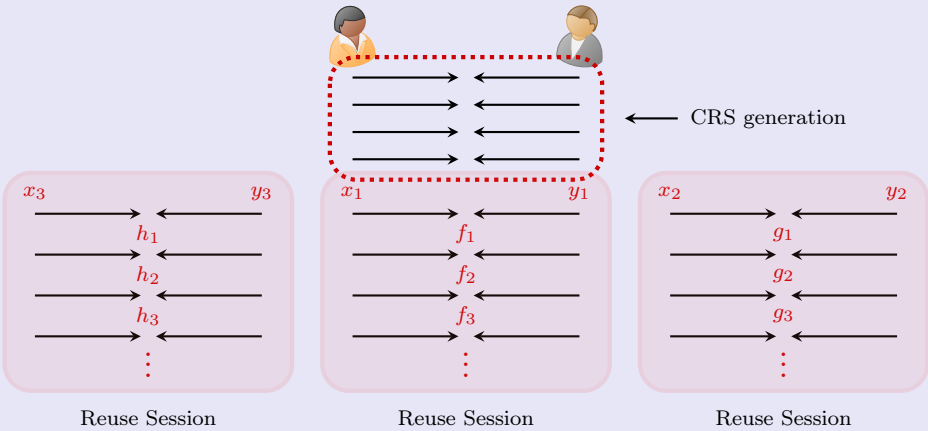

Related Work

Reusable MPC in the plain model

[Fernando-Jain-Komargodski'23]

- Plain model
- Two rounds where the first round can be reused
- Super-polynomial-time Simulator

Our Results

Reusable 2PC

(DDH or QR) + ZAP ===> Four-round Reusable 2PC

ZAP

[Dwork-Naor'00] Two-round public coin witness indistinguishable proof.

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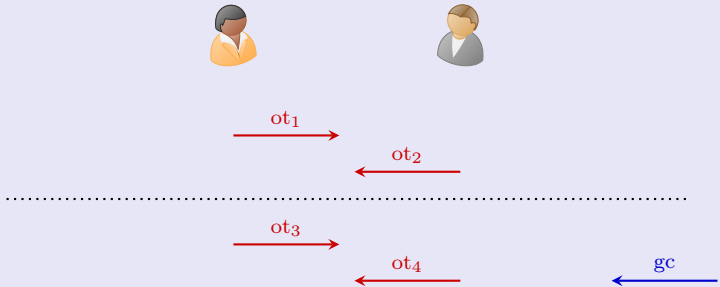
Reusable MPC

Four-round Reusable 2PC/OT + (semi-malicious) Two-round Reusable MPC + ZAP ===> Four-round Reusable MPC

Semi-malicious Two-round Reusable MPC

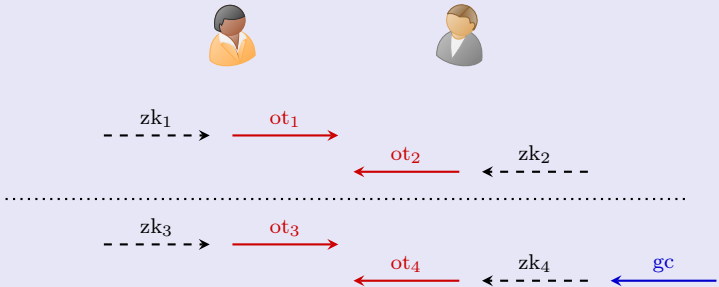
DDH [BGMM20], pairing [BL20], LWE [AJJM20, AJJM21, BJKL21], LPN [BGSZ22]

Technical Challenges: 2PC



- GC: garbled circuit
- OT: four-round oblivious transfer (simulation security for malicious receiver; indistinguishability-based security for malicious senders)

Technical Challenges: 2PC



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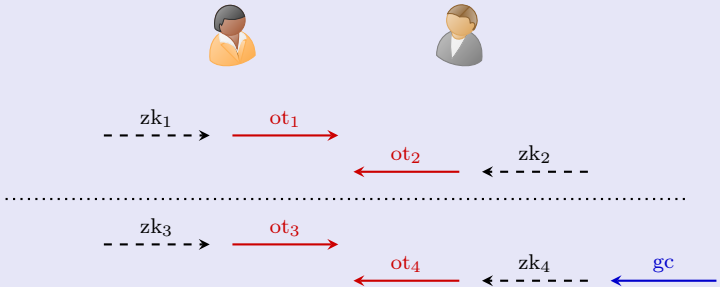

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ZK and OT need to be reusably secure!

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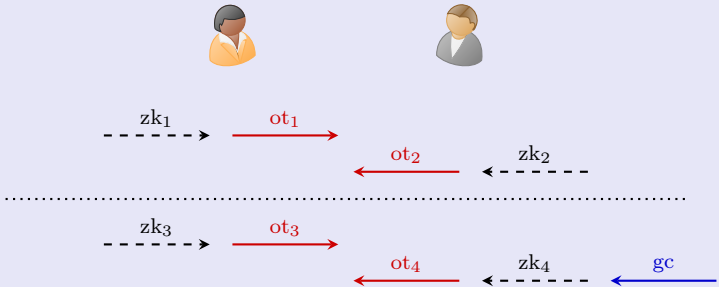

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Issue

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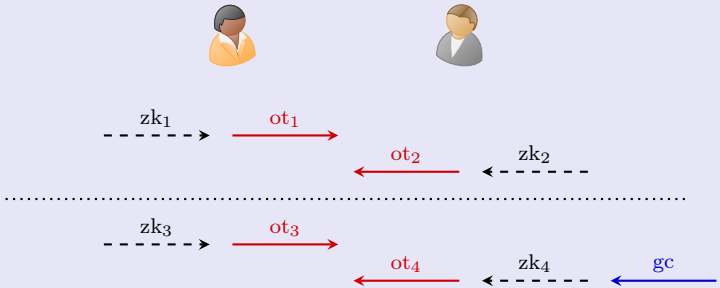
- A four-round OT that both the receiver and the sender may *change input*.

Technical Challenges: 2PC



- GC: garbled circuit
- OT: four-round oblivious transfer (simulation security for malicious receiver; indistinguishability-based security for malicious senders)
- ZK: zero-knowledge protocol (proof of knowledge)

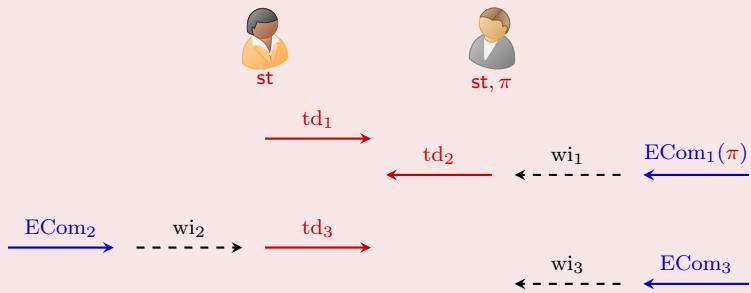
Issue

ZK and OT need to be reusably secure!

- A four-round OT that both the receiver and the sender may *change input*.
- A four-round ZK that the prover may send multiple fourth-round messages proving different statements

Reusable Zero-knowledge [Feige-Lapidot-Shamir'90]

- TD: trapdoor generation protocol
- ECom: extractable commitment scheme
- WI: witness indistinguishable proof



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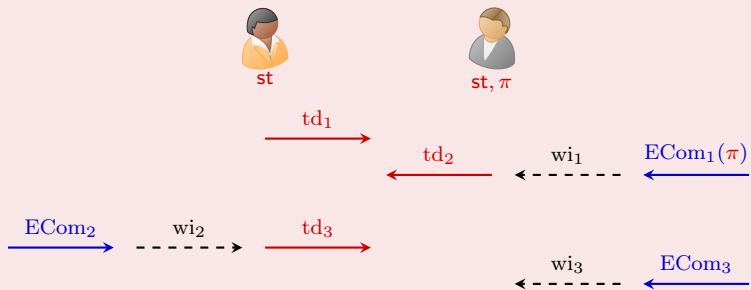

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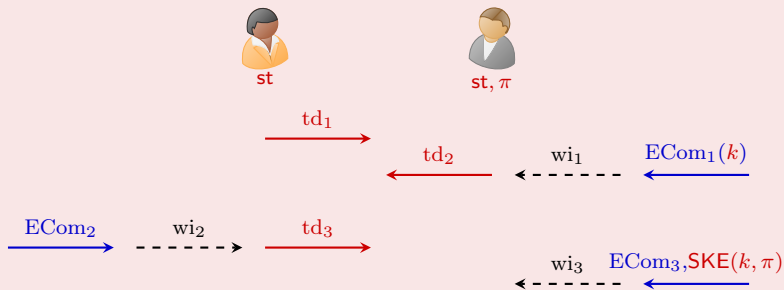

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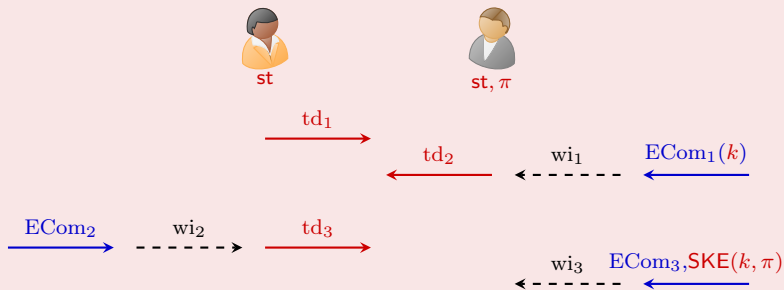

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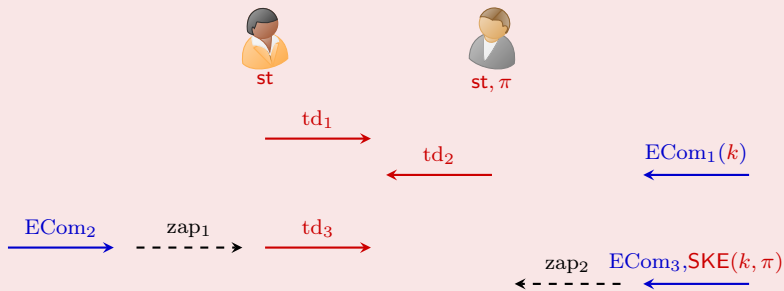

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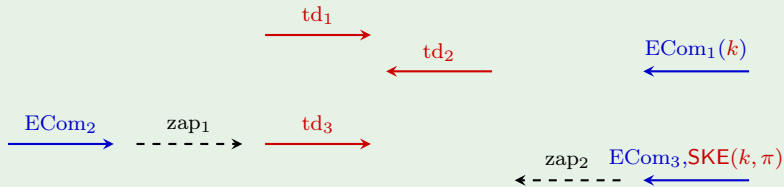

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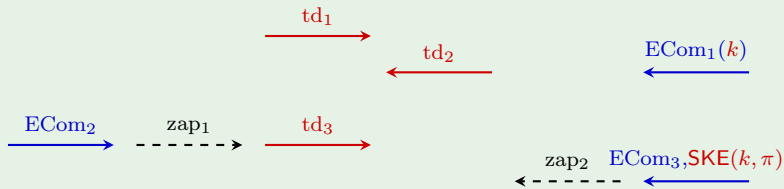

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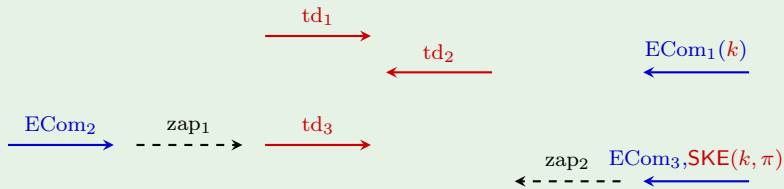

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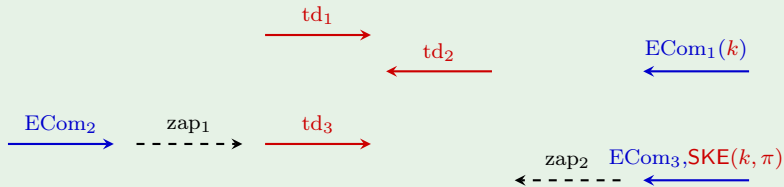

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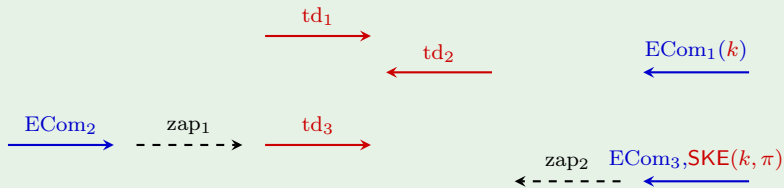

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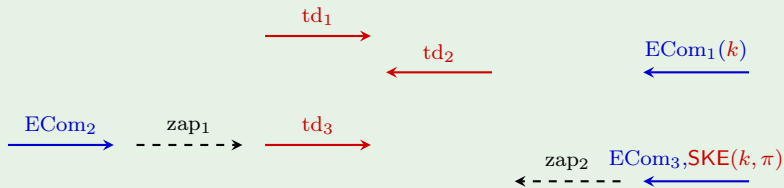

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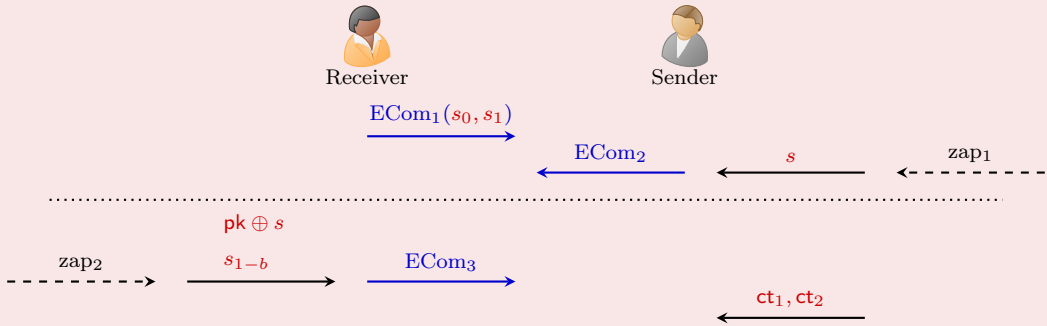
  - not an issue in simultaneous message model/MPC

Reusable OT: both sender and receiver may change input

[Katz-Ostrovsky'04,Ostrovsky-Richelson-Scafuro'15, Friolo-Masny-Venturi'19]



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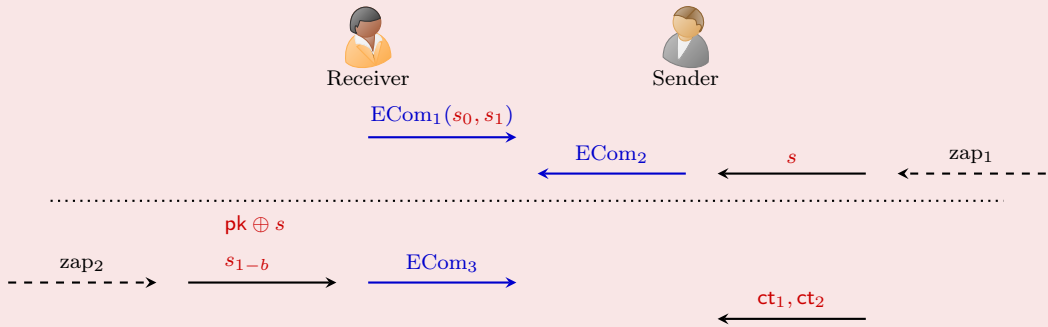

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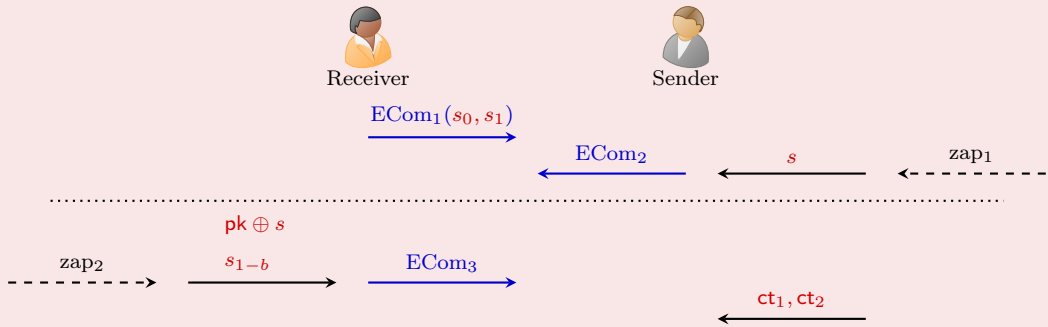

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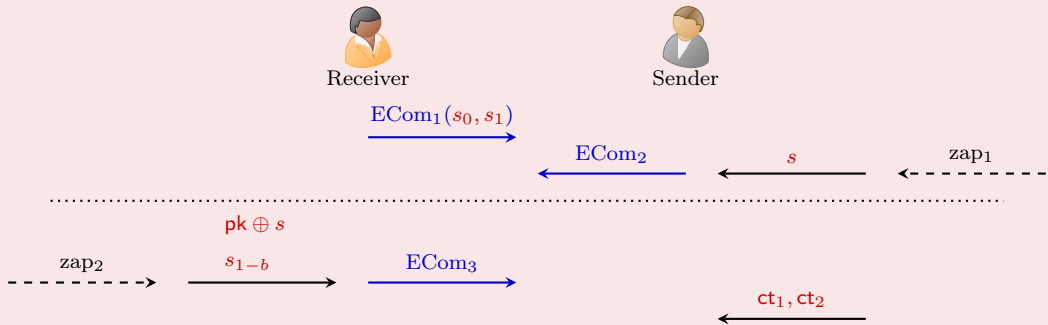

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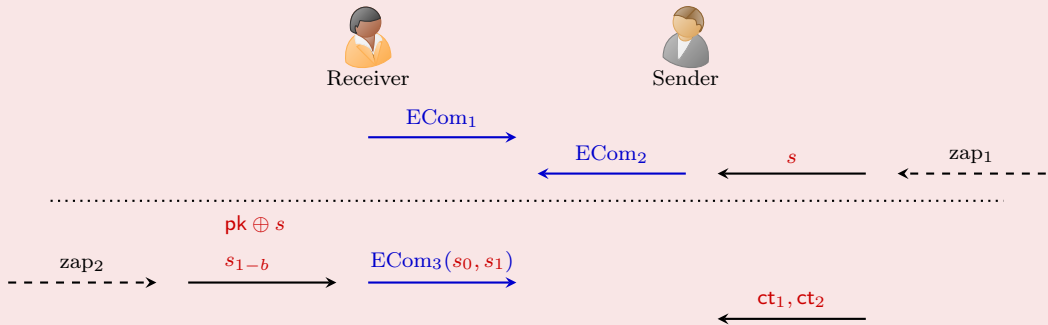

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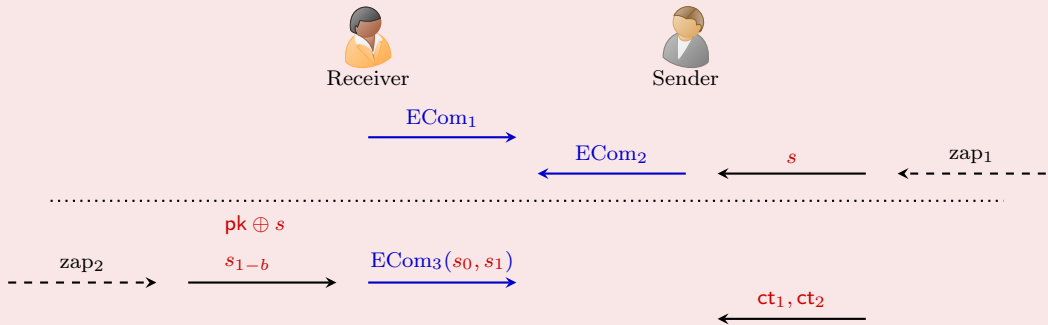

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- Need to reconcile between giving the receiver too much freedom and too little freedom

Naor's bit commitment scheme [Naor'91]

s

$\begin{cases} b = 0 & \mathsf{PRG}(\mathsf{sd}) \\ b = 1 & \mathsf{PRG}(\mathsf{sd}) \oplus s \end{cases}$

sd

- Insecure (equivocal) if $\mathsf{PRG}(\mathsf{sd}) \oplus s = \mathsf{PRG}(\mathsf{sd}')$

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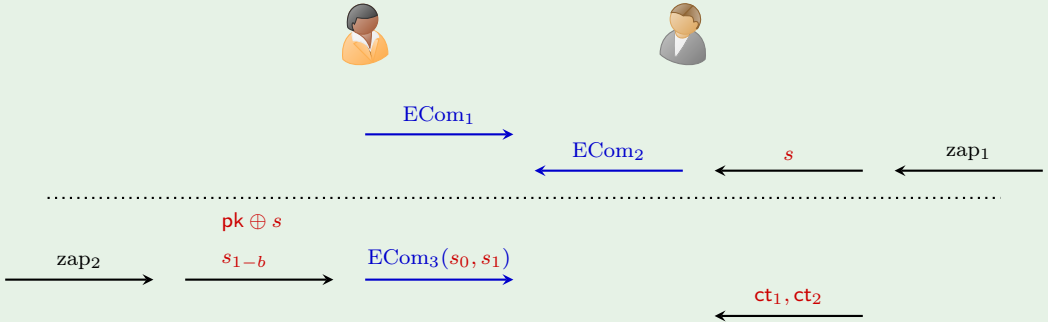
- If $\mathsf{PRG} : \{0,1\}^\lambda \to \{0,1\}^{3\lambda}$, $2^\lambda \times 2^\lambda$ choices of $\mathsf{PRG}(\mathsf{sd}) \oplus \mathsf{PRG}(\mathsf{sd}')$ and $2^{3\lambda}$ choices of s.

Reusable OT


ECom_1
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zap_1
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zap_2
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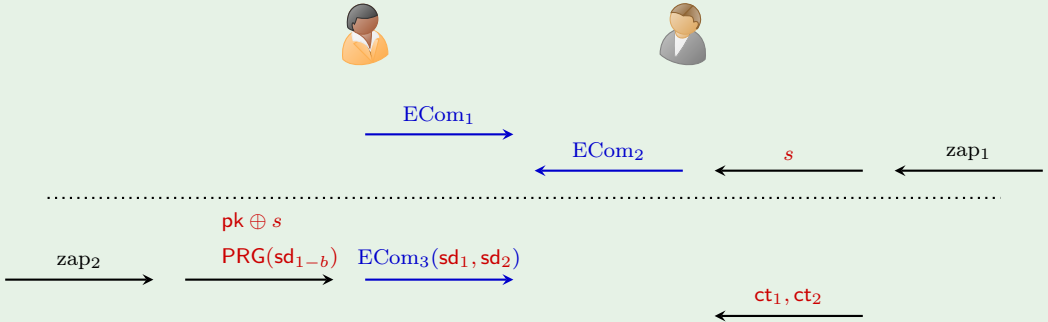
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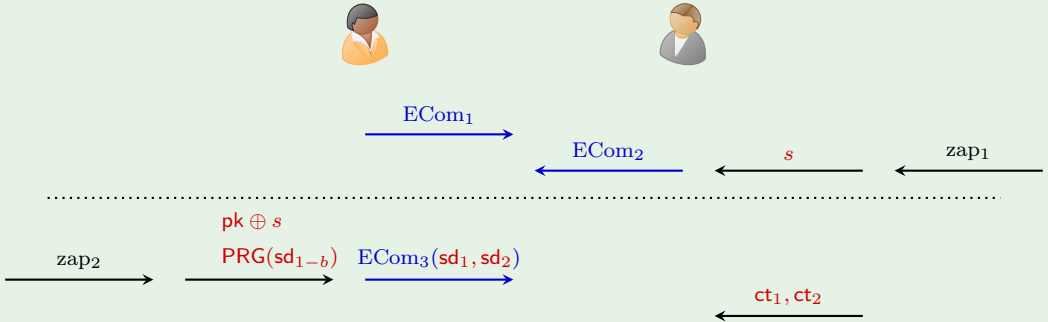

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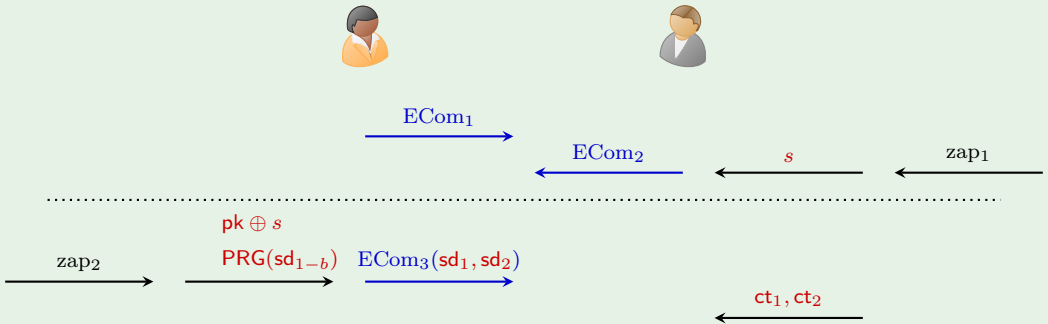

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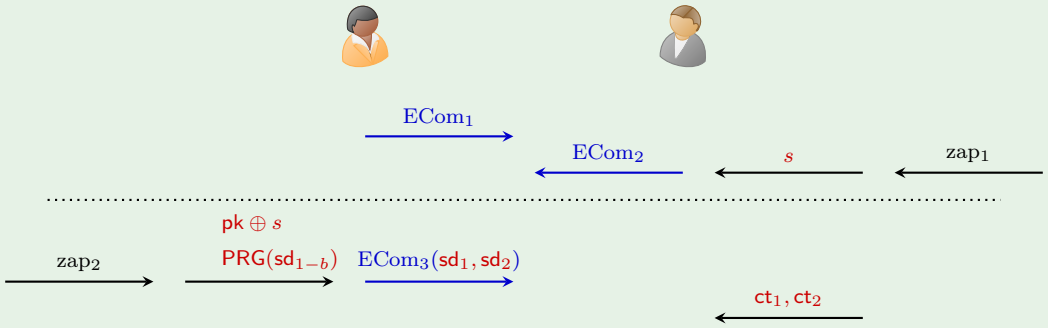

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- A special kind of PKE
  - pseudorandom public key
  - valid public keys are scarce
  - maliciously chosen invalid public keys still hide the message

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2. valid public keys are scarce
3. maliciously chosen invalid public keys still hide the message

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Special PKE scheme from DDH

- Public key domain $\begin{pmatrix} g & g^a \\ g^b & g^c \end{pmatrix}$.
- Valid public key $c = ab$, invalid public key $c \neq ab$
- Encryption

$$(u, v) \cdot \begin{pmatrix} g & g^a \\ g^b & g^c \end{pmatrix} \oplus (0, \mathsf{msg})$$

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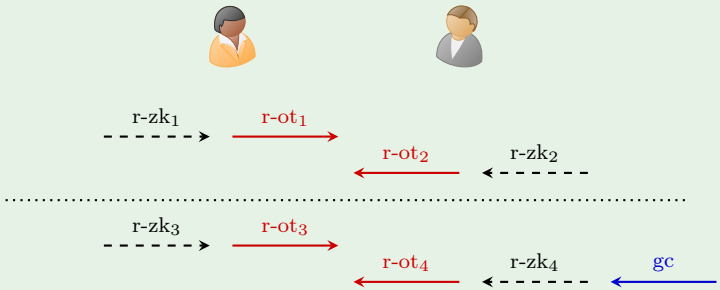
We also show how to construct it from SSP-OT and QR.

Reusable 2PC



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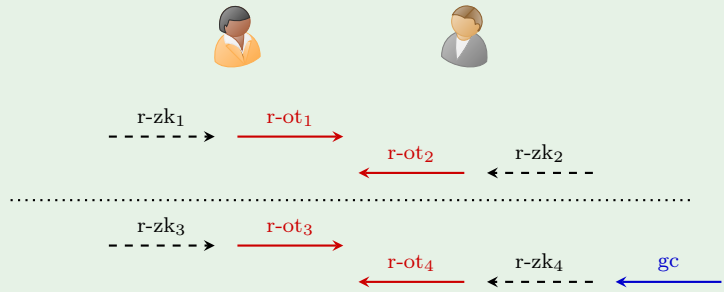
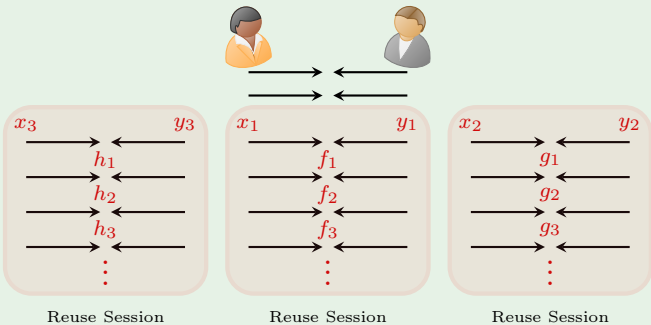


Reusable MPC

- Based on appropriate adaptations of [Choudhuri-Ciampi-Goyal-Jain-Ostrovsky'20]
- Replace OT and ZK with our r-OT and r-ZK
- Additionally, we need two-round reusable semi-malicious MPC

Summary



- Reusable 2PC from
  - DDH or QR
  - ZAP
- Reusable MPC from
  - Reusable 2pc
  - (semi-malicious) two-round reusable MPC
  - ZAP

Thanks! Questions? ia.cr/2023/1006