

# The Power of Undirected Rewindings for Adaptive Security

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ETH Zürich

# Motivation

Selective Security

$\mathcal{E}$

$\mathcal{A}$

Adaptive Security

$\mathcal{E}$

$\mathcal{A}$

Corruptions

RoR-challenge

# Motivation

Selective Security

$\mathcal{E}$

$\mathcal{A}$

$\xrightarrow[\text{chall}]{\text{cor1, cor2, cor3}}$

$\mathcal{E}$  Adaptive Security  $\mathcal{A}$

# Motivation

Selective Security

$\mathcal{E}$

$\mathcal{A}$

$\overleftarrow{cor_1, cor_2, cor_3}$   
chal

$\overrightarrow{ke_1, ke_2, ke_3}$

Adaptive Security

# Motivation

## Selective Security

$\mathcal{E}$   $\mathcal{A}$

$\overleftarrow{\text{cor}_1, \text{cor}_2, \text{cor}_3}$   
 $\text{chal}$

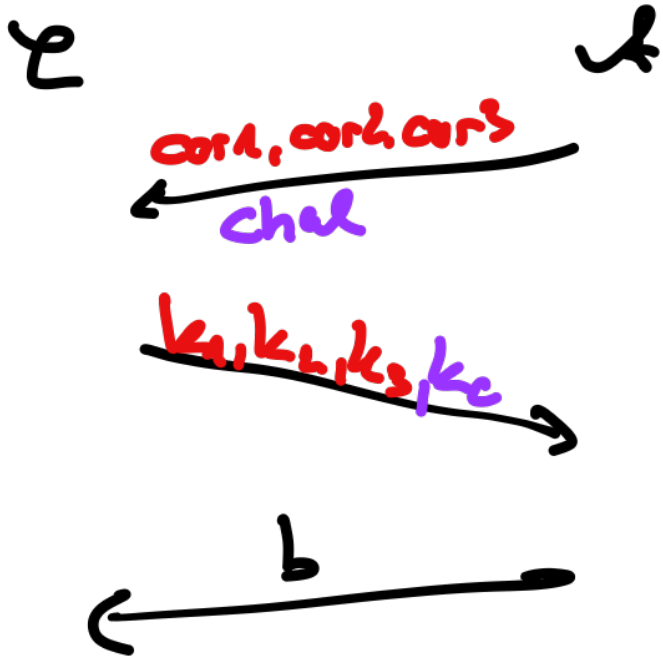
$\overrightarrow{\text{ke}_1, \text{ke}_2, \text{ke}_3}$

$\overleftarrow{b}$

## Adaptive Security

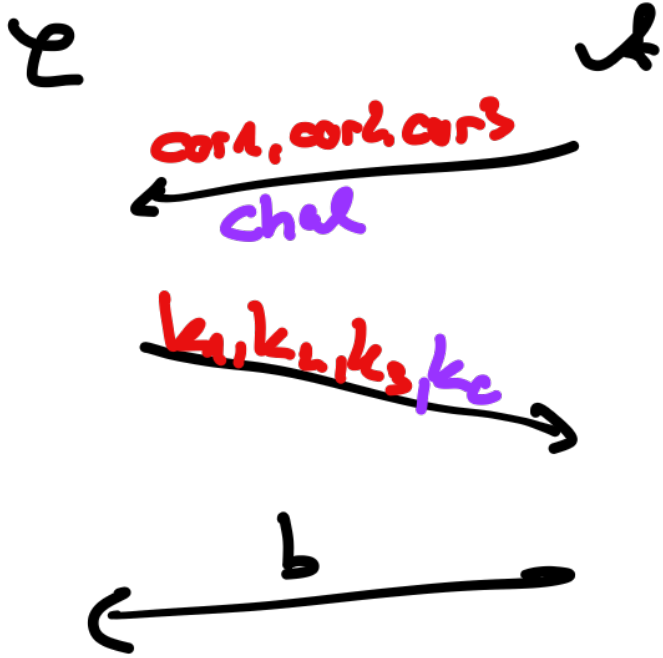
# Motivation

## Selective Security

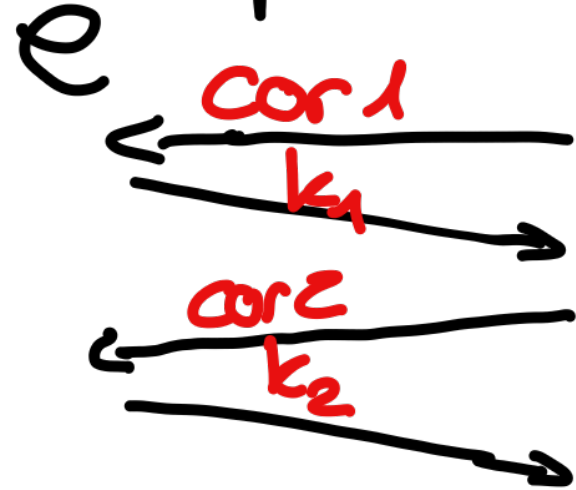


# Motivation

## Selective Security

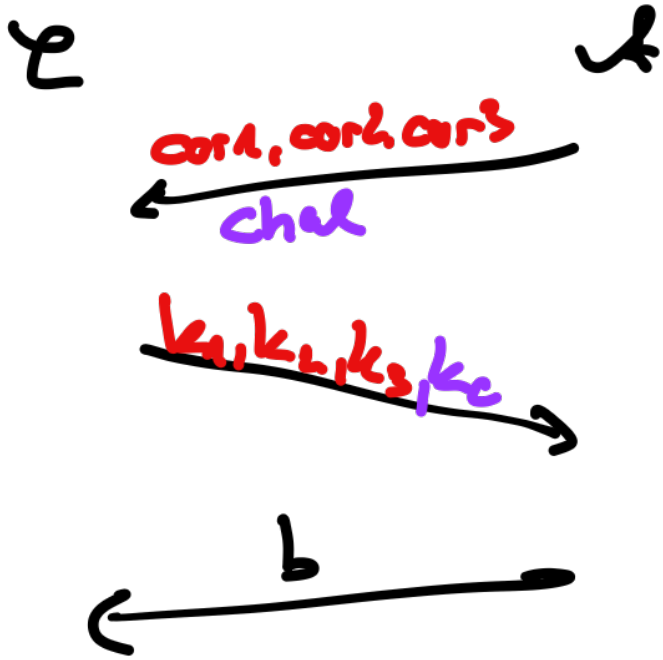


## Adaptive Security

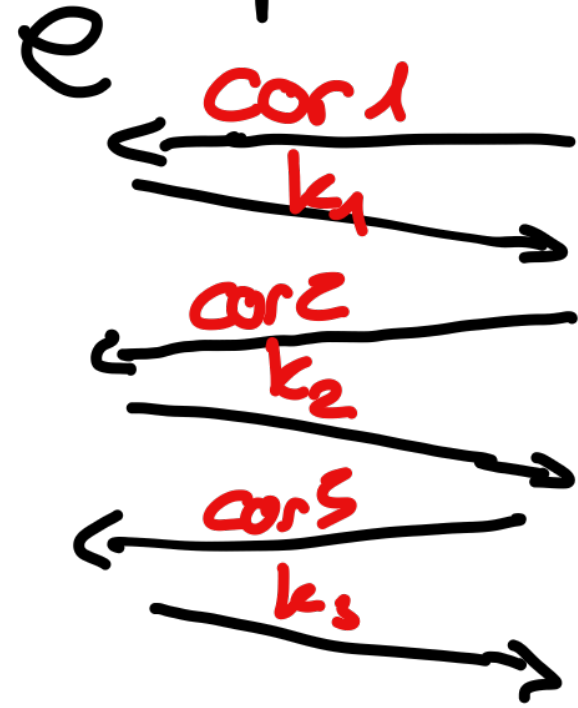


# Motivation

## Selective Security



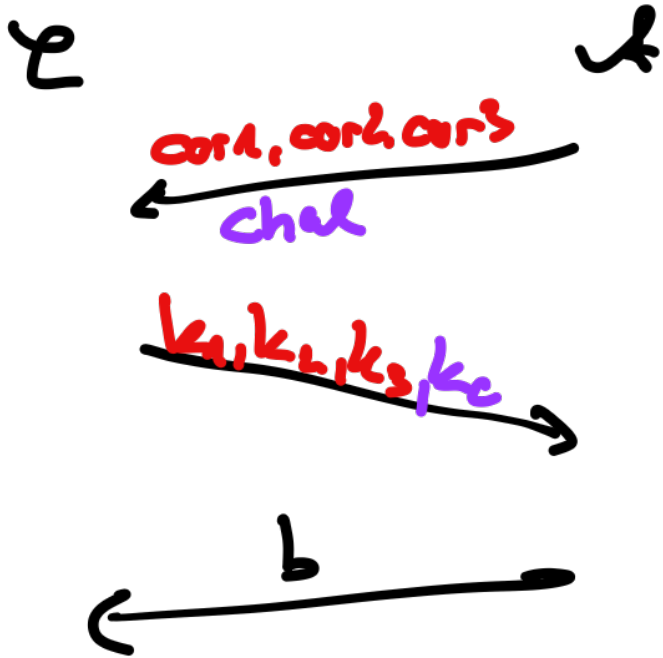
## Adaptive Security



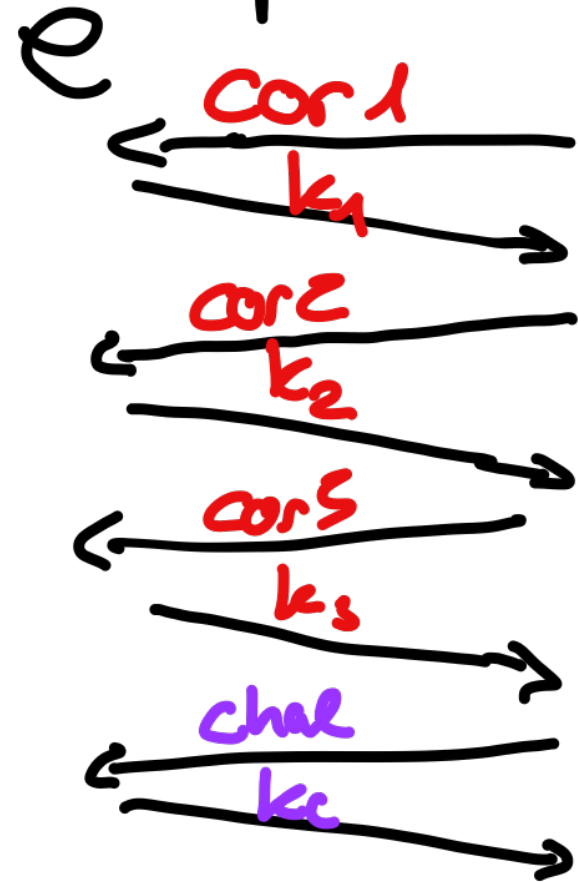


# Motivation

## Selective Security

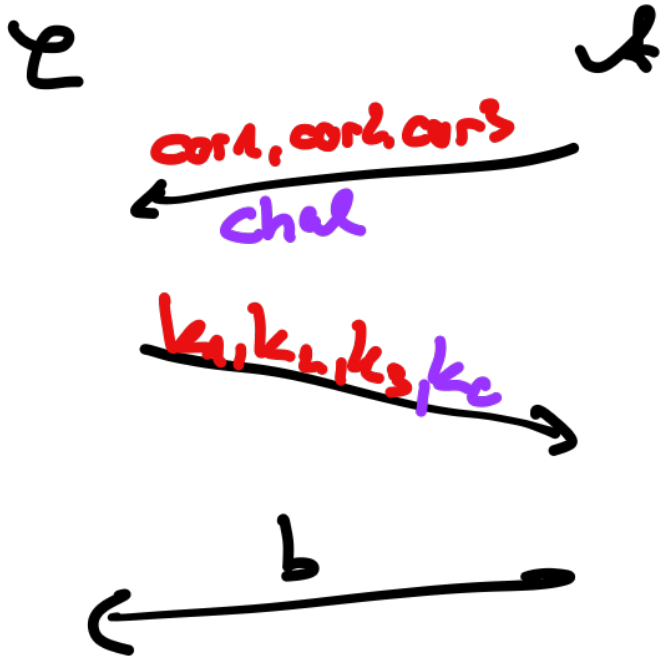


## Adaptive Security

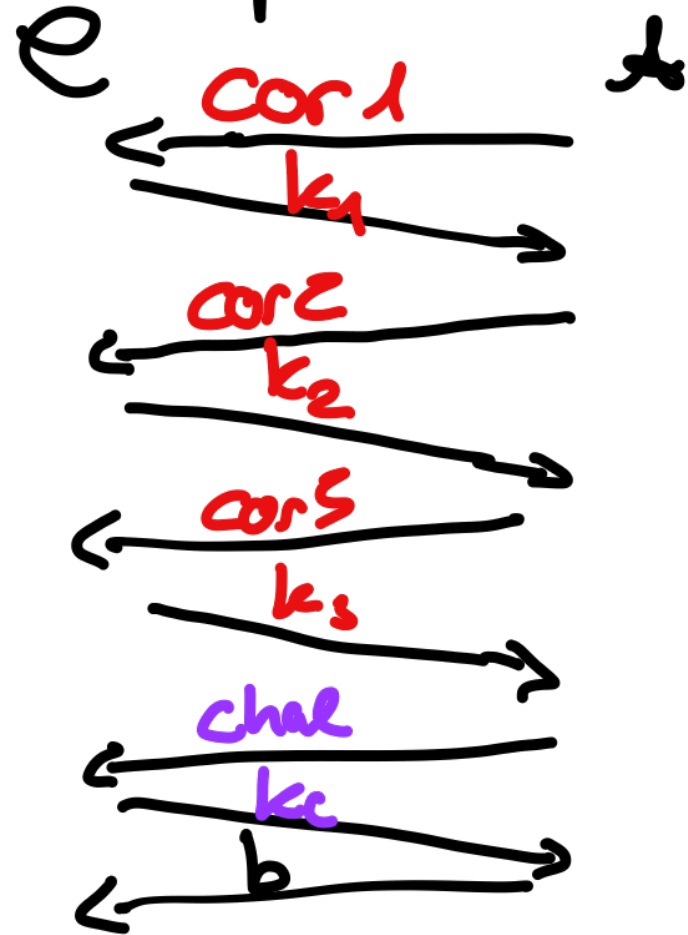


# Motivation

## Selective Security

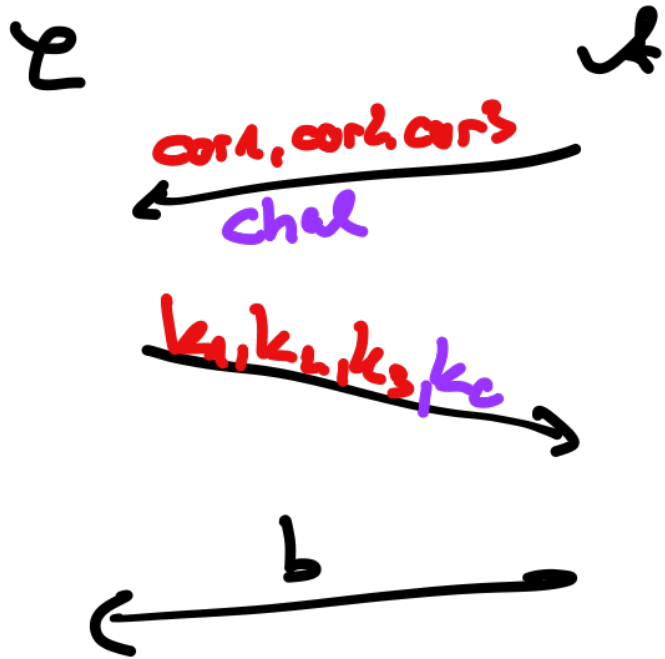


## Adaptive Security



# Motivation

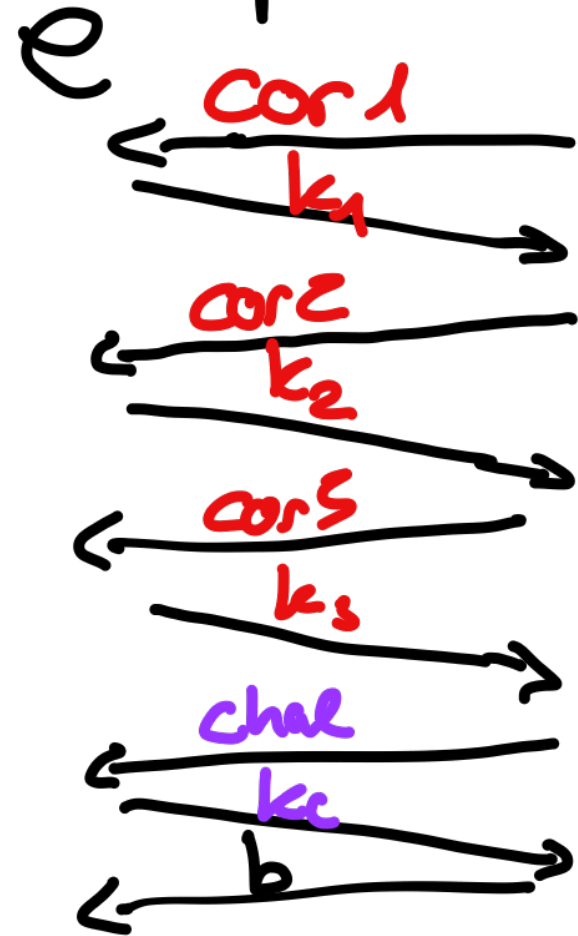
## Selective Security



easier to prove

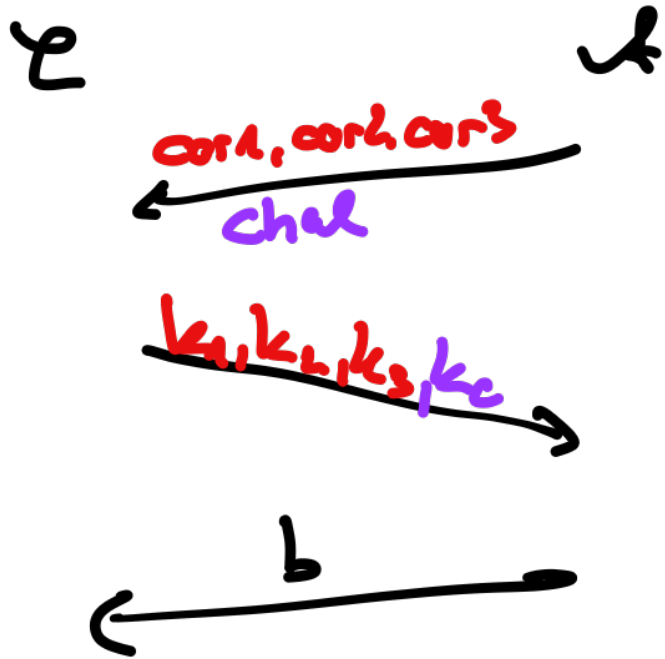
weaker notion

## Adaptive Security



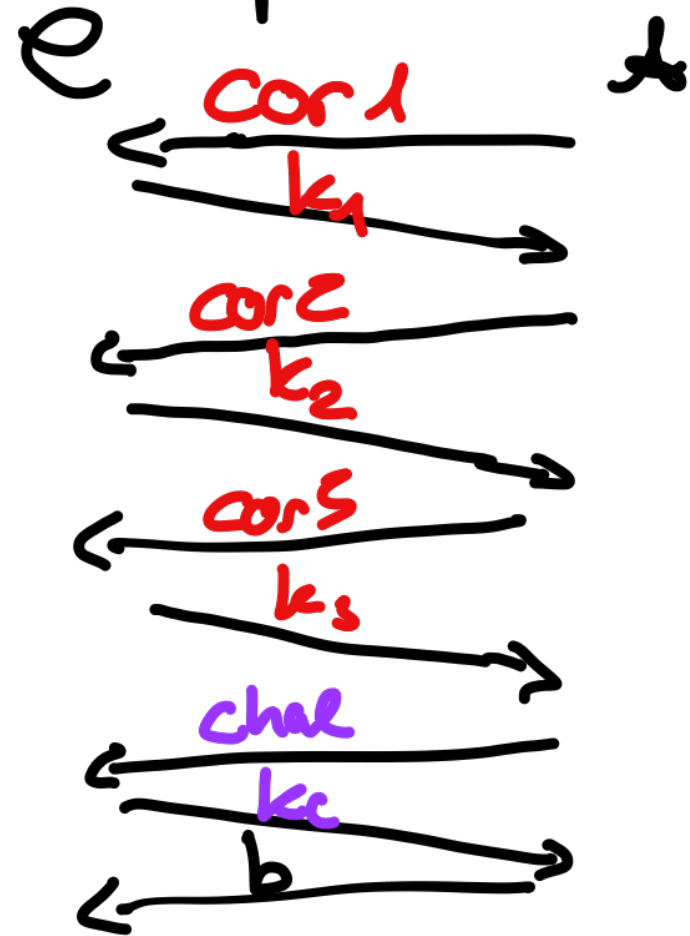
# Motivation

## Selective Security



easier to prove  
weaker notion

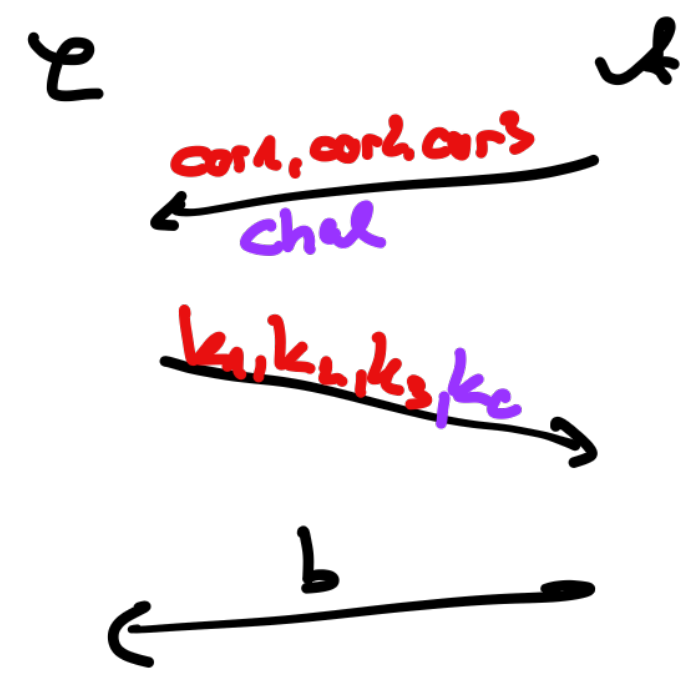
## Adaptive Security



harder to prove  
stronger notion

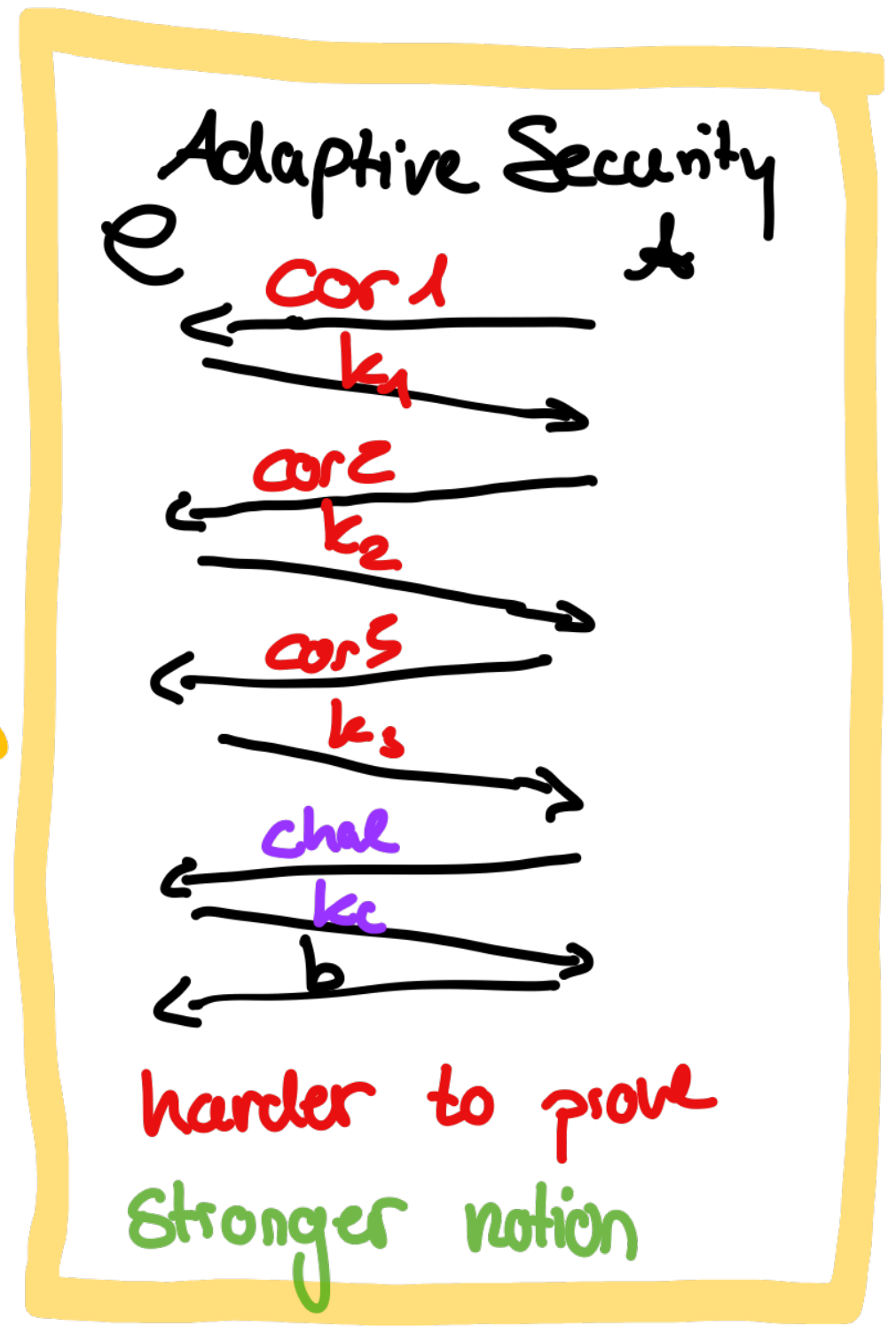
# Motivation

## Selective Security



easier to prove  
weaker notion

This talk



harder to prove  
stronger notion

PRFs

$$F_k : \{0,1\}^n \rightarrow \{0,1\}^m$$

PRFs

$$F_k : \{0,1\}^n \rightarrow \{0,1\}^m$$

Security:

$$k \leftarrow \mathcal{K}$$

$$\Pr [A^{R(\cdot)} = 1] - \Pr [A^{F_k(\cdot)} = 1] \leq \text{negl}$$

# Prefix-Constrained PRF

$$F_k : \{0,1\}^n \rightarrow \{0,1\}^m$$

Constrained keys  $k_x$  *evaluate if  
input has prefix  $x$*



# Prefix-Constrained PRF

$$F_k : \{0,1\}^n \rightarrow \{0,1\}^m$$

Constrained keys  $k_x$  evaluate if input has prefix  $x$

Security:

$\mathcal{A}$  gets to

Corrupt keys  
request challenge

$$\Pr[\mathcal{A}^{\text{Corrupt}(\cdot), R(\cdot)} = 1] - \Pr[\mathcal{A}^{\text{Corrupt}(\cdot), F_k(\cdot)} = 1] \leq \text{negl}$$

# Our contributions

- **adaptive** security of **CCM PC-PRF**  
based on security of **PRG** with **polynomial**  
loss
- **adaptive** security of **LKH (Multicast Encryption)**  
based on **IND-CPA** security of underlying  
encryption with **polynomial** loss

Main technique: **undirected rewinding**

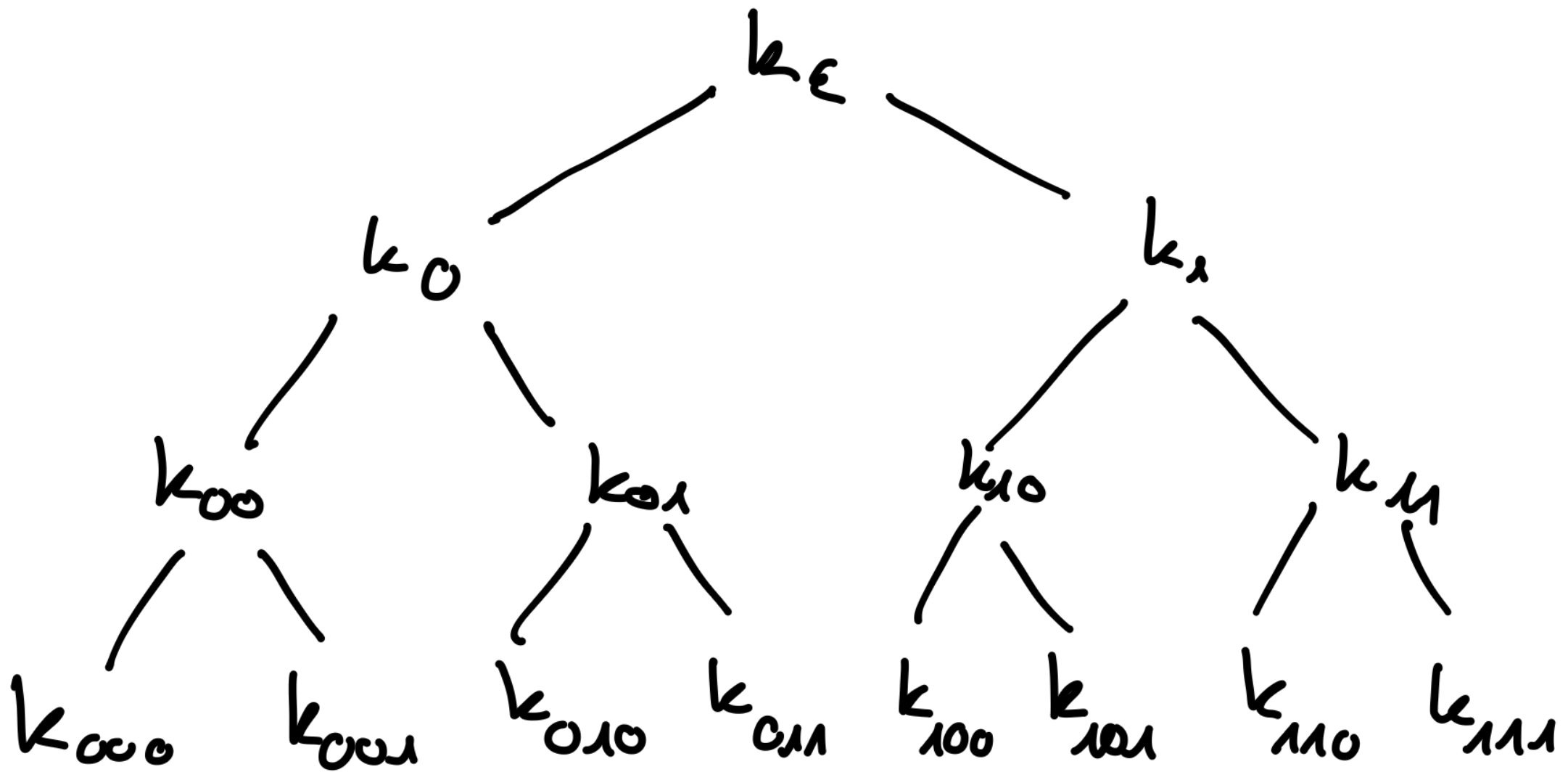
# Our contributions

↳ This talk

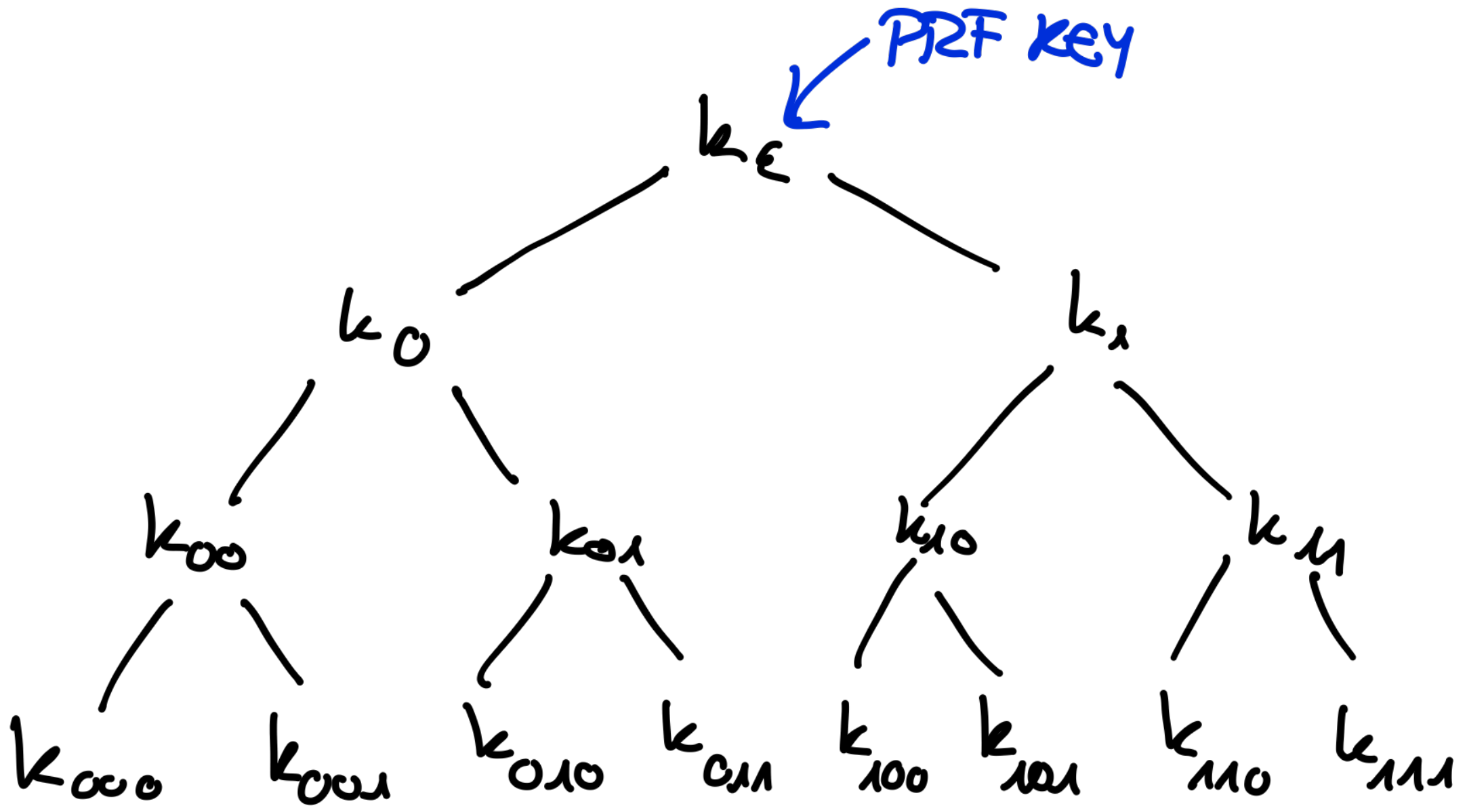
- **adaptive** security of **CCM PC-PRF**  
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# The GGM PRF

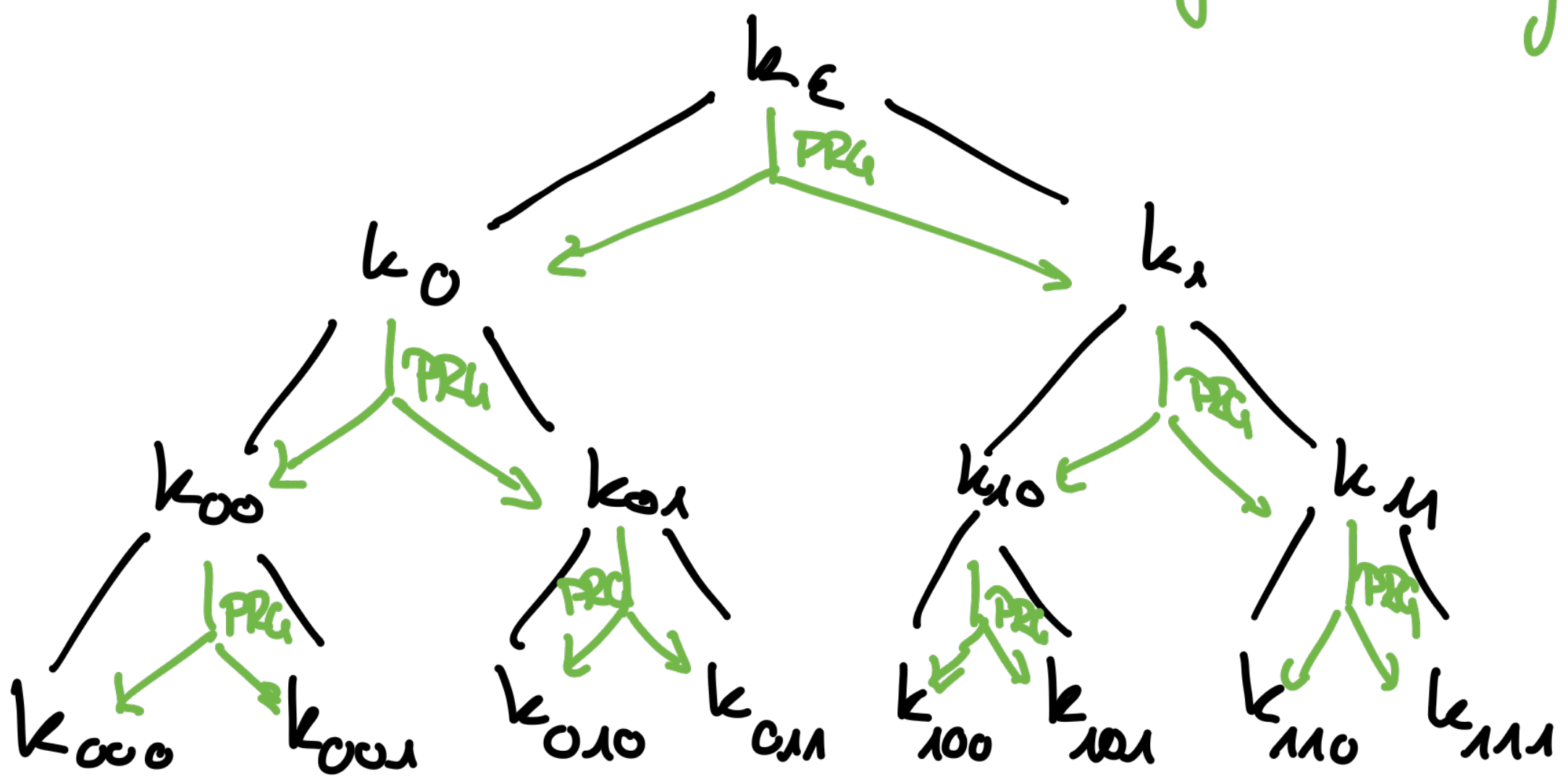


# The GCM PRF

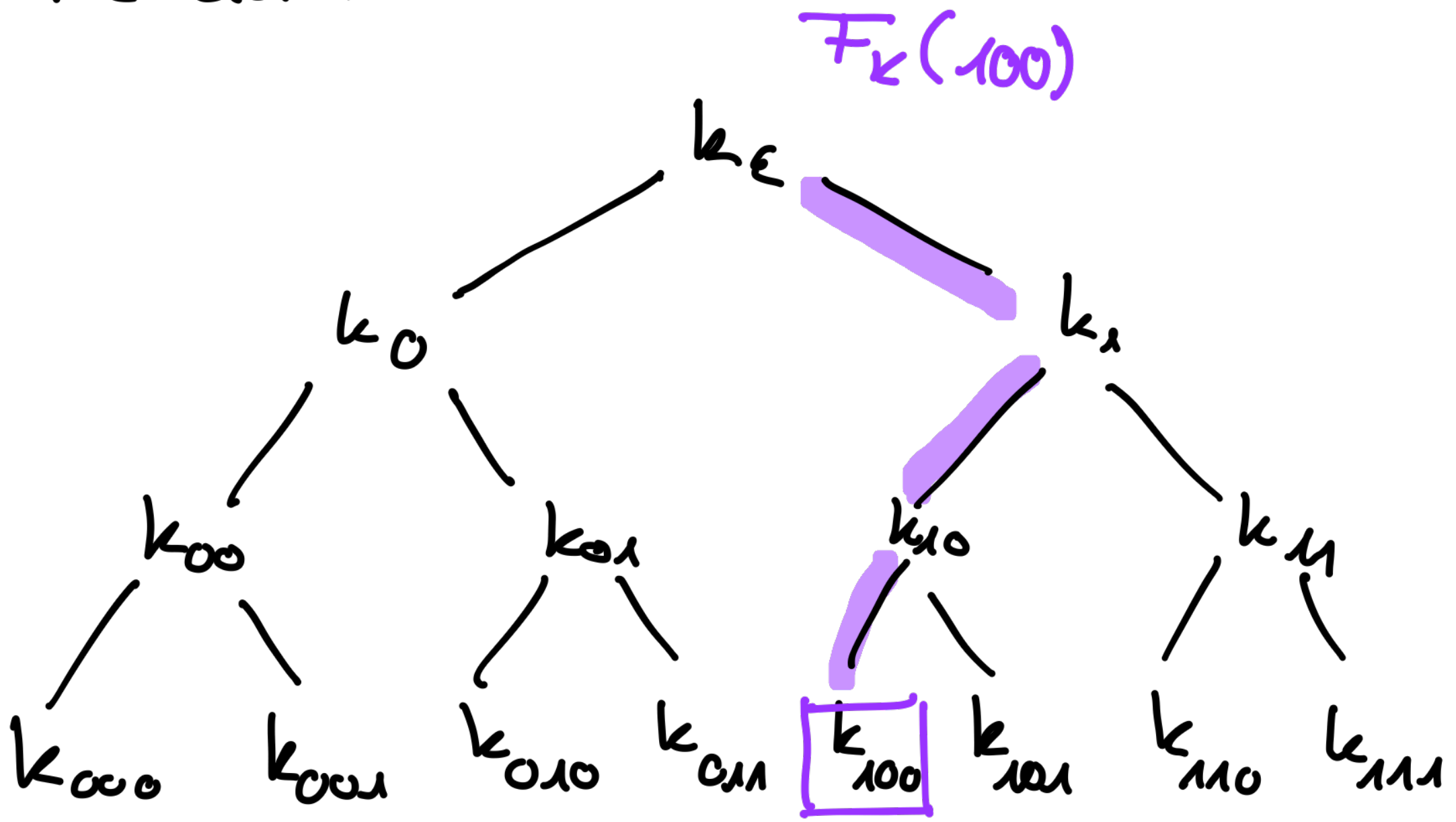


# The GCM PRF

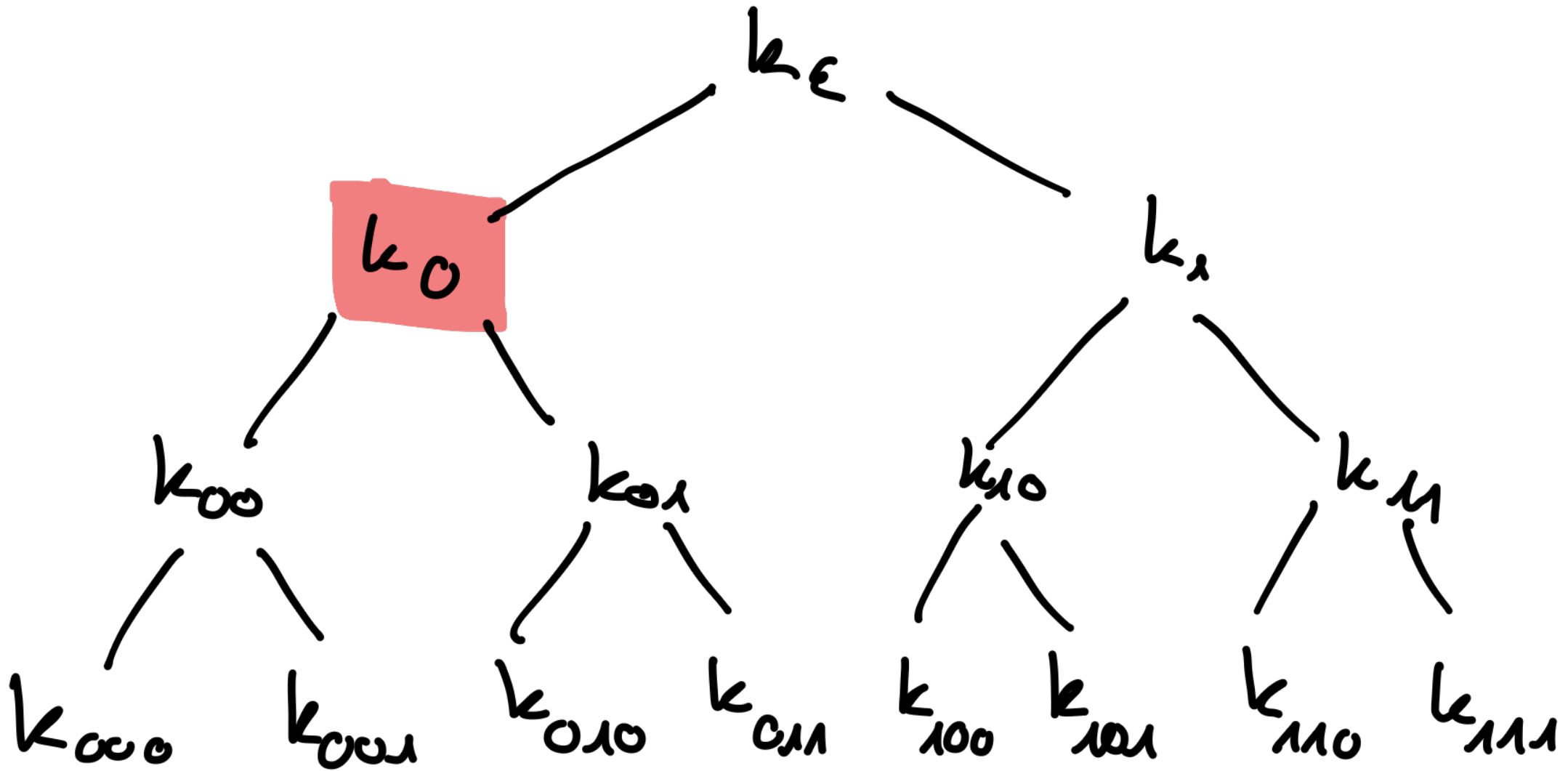
PRC: length doubling



# The GGM PRF

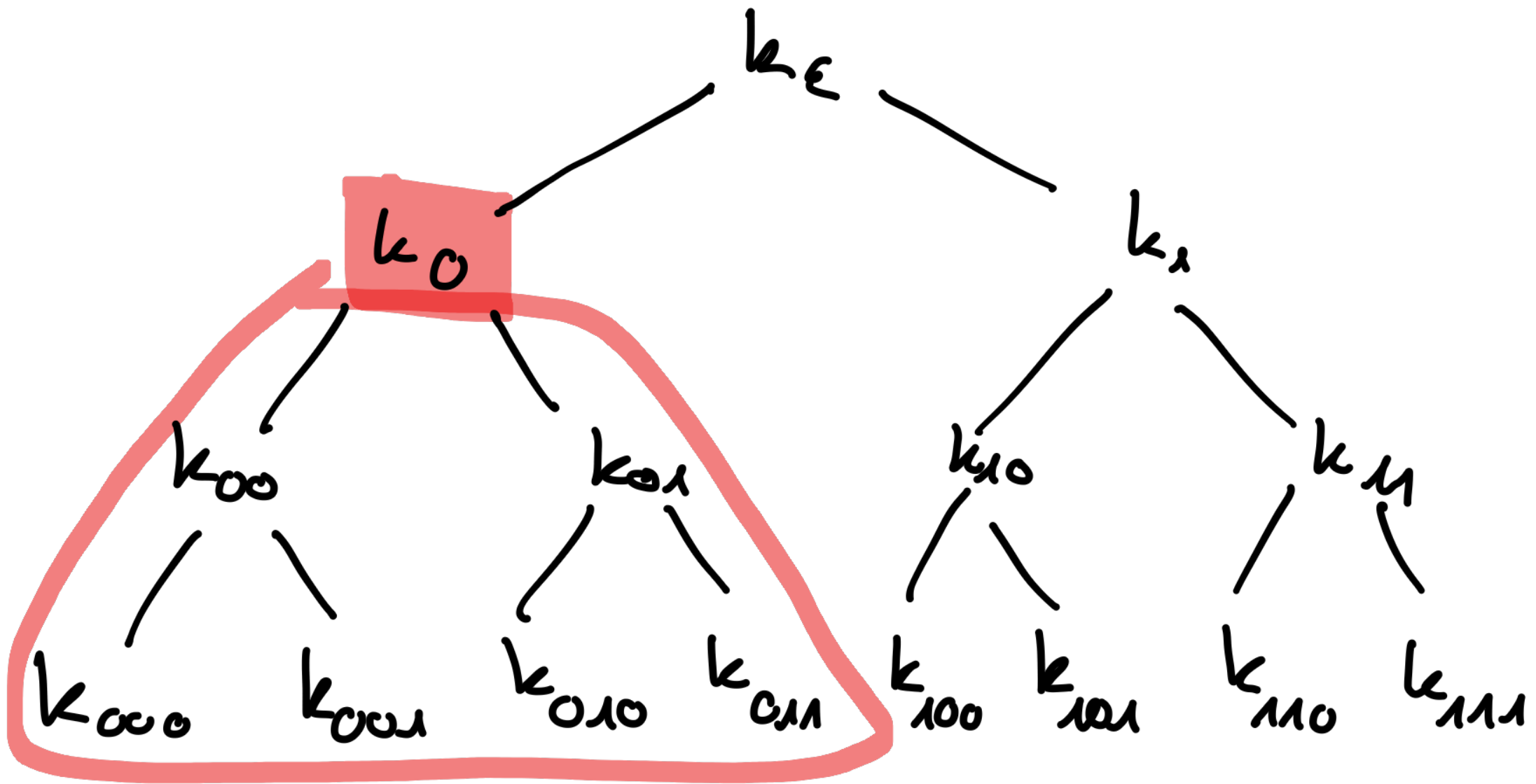


# The GGM PRF as a TC-PRF

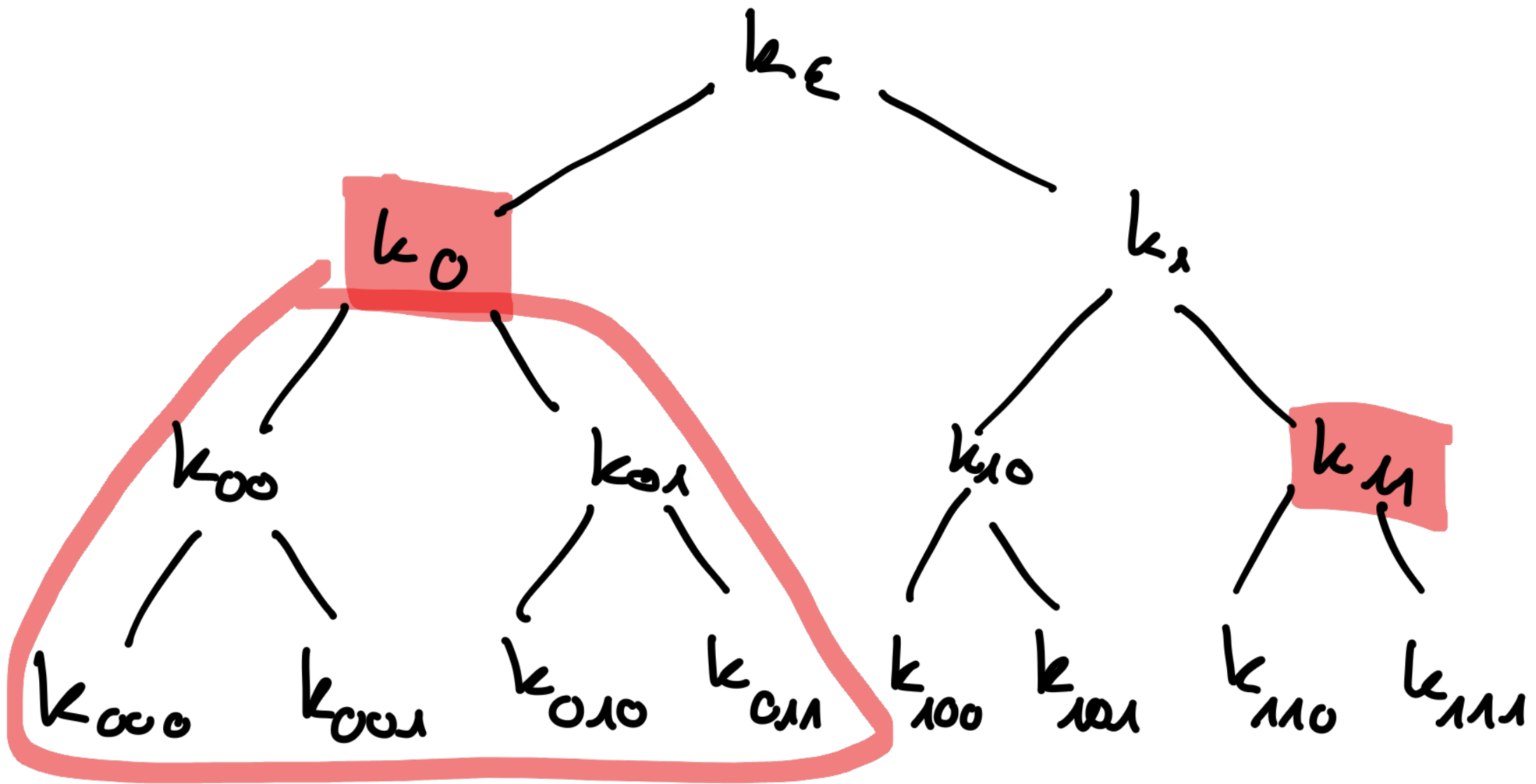




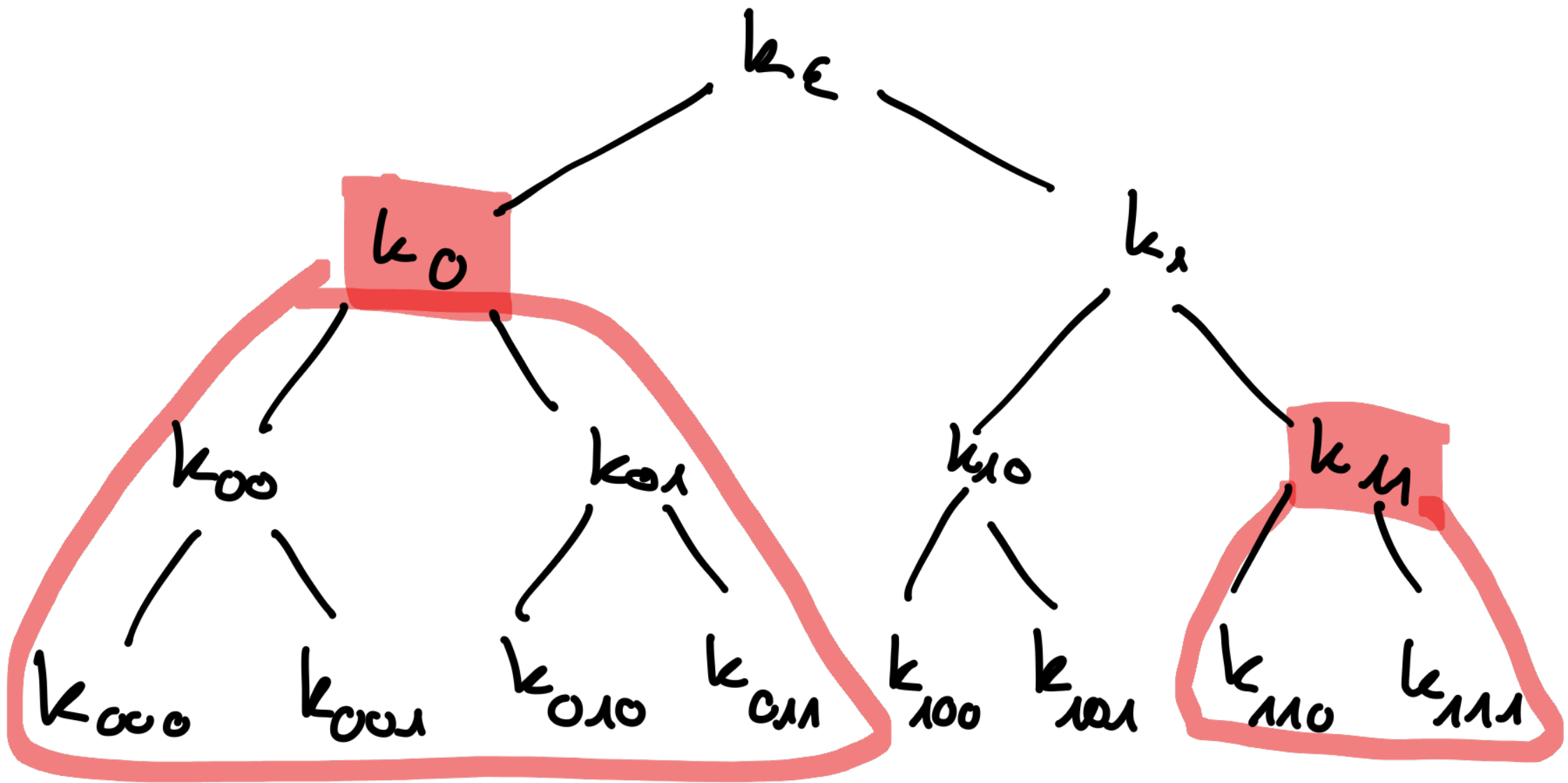
# The GGM PRF as a PC-PRF



# The GGM PRF as a TC-PRF



# The GGM PRF as a PC-PRF



# PRG Security

$$\text{PRG} : \{0,1\}^n \rightarrow \{0,1\}^{2n}$$

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$$\Pr[A^{\text{chal}}(1^\lambda) = 1] - \Pr[A^{\text{chal}}(1^\lambda) = 1] \leq \text{negl}(\lambda)$$

# PRG Security

$$\text{PRG} : \{0,1\}^n \rightarrow \{0,1\}^{2n}$$

output:  $y \in \{0,1\}^{2n}$



$$\Pr[A^{\text{chal}}(1^n) = 1] - \Pr[A^{\text{chal}}(1^n) = 1] \leq \text{negl}$$

# PRG Security

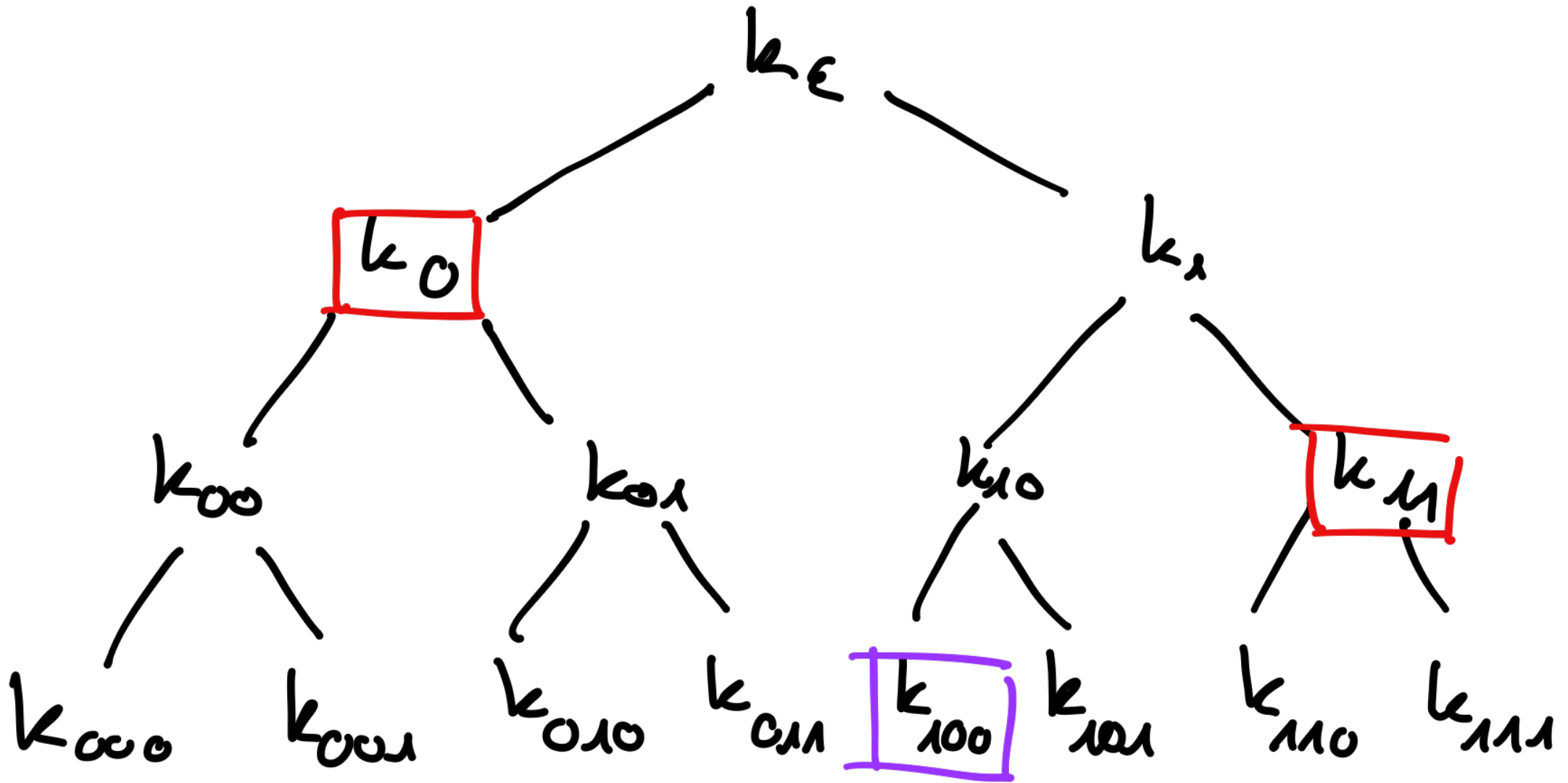
$$\text{PRG} : \{0,1\}^n \rightarrow \{0,1\}^{2n}$$

$$\text{output: } \gamma \leftarrow \{0,1\}^{2n} \quad s \leftarrow \{0,1\}^n$$

$$\text{output: } \gamma := \text{PRG}(s)$$

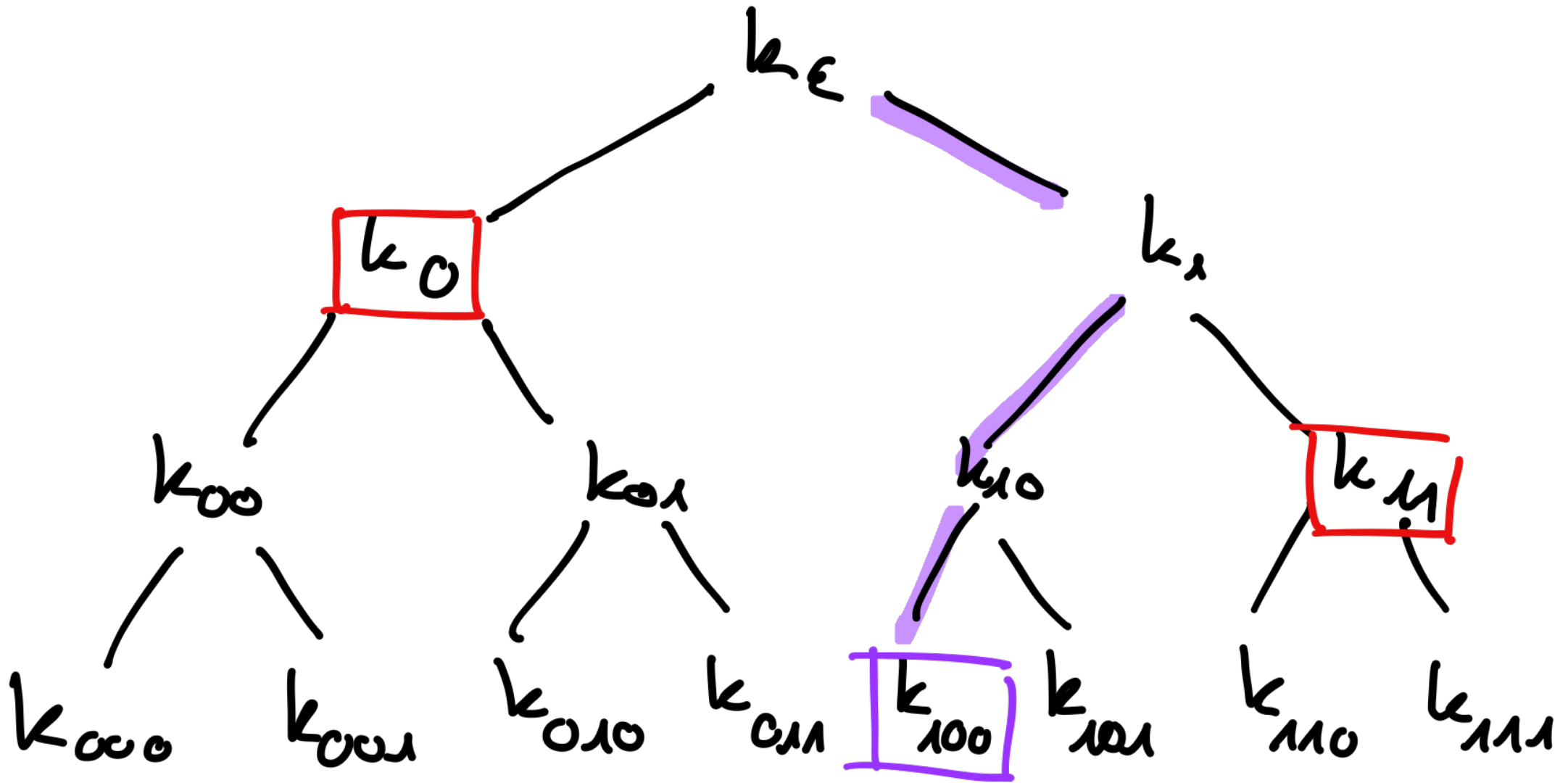
$$\Pr[A^{\text{chal}}(\gamma) = 1] - \Pr[A^{\text{chal}}(s) = 1] \leq \text{negl}(\lambda)$$

# Selective Security

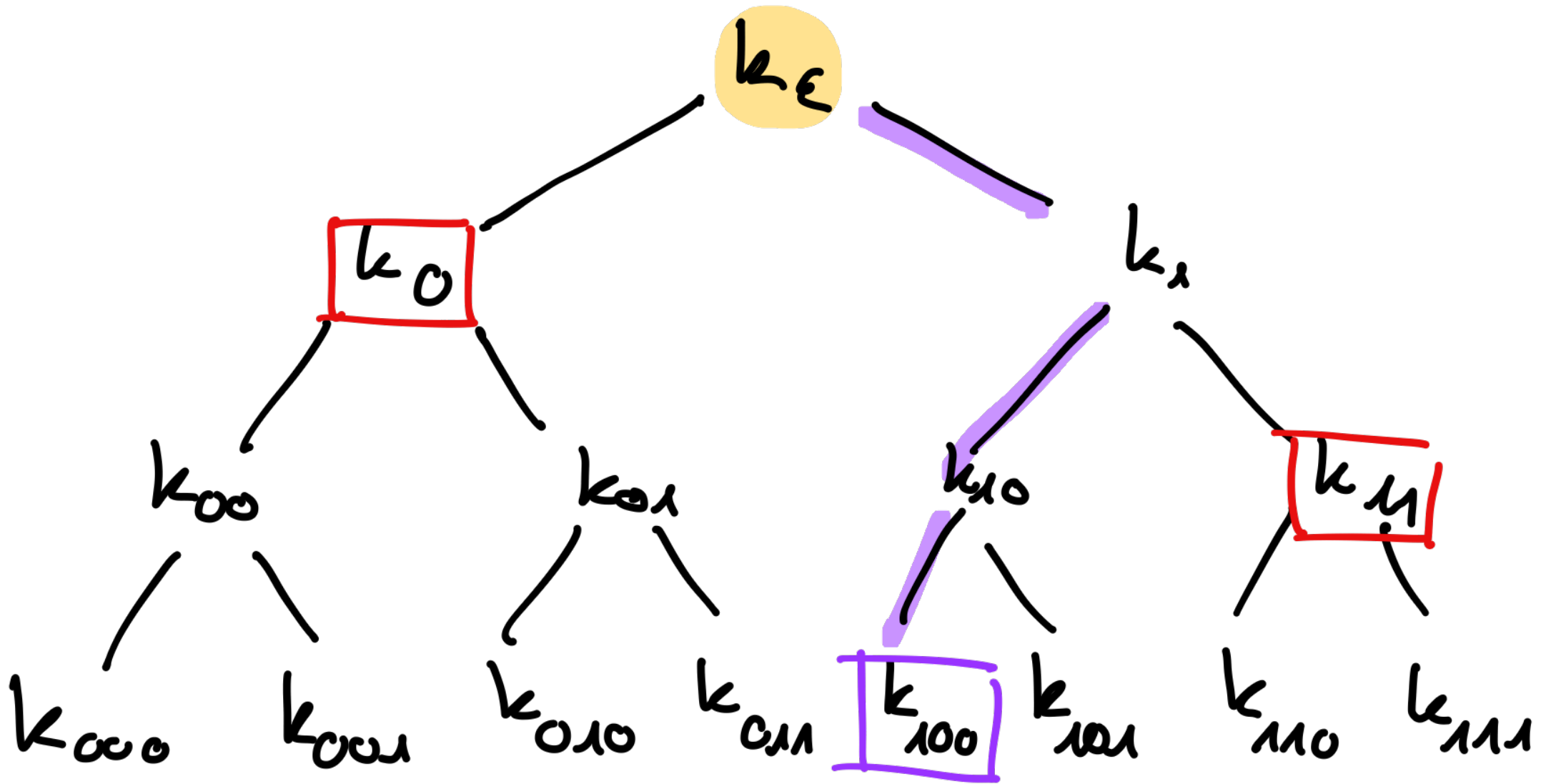




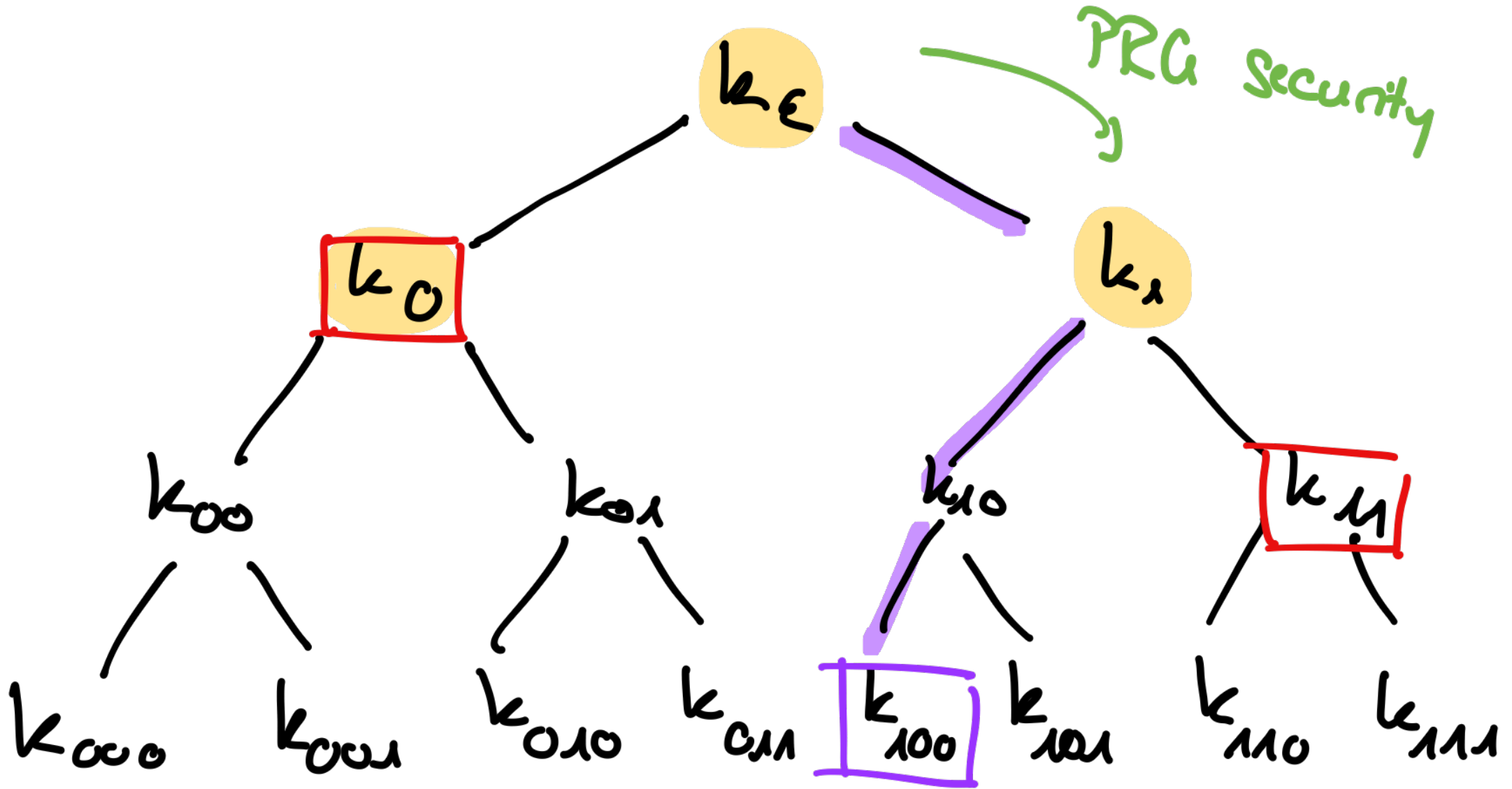
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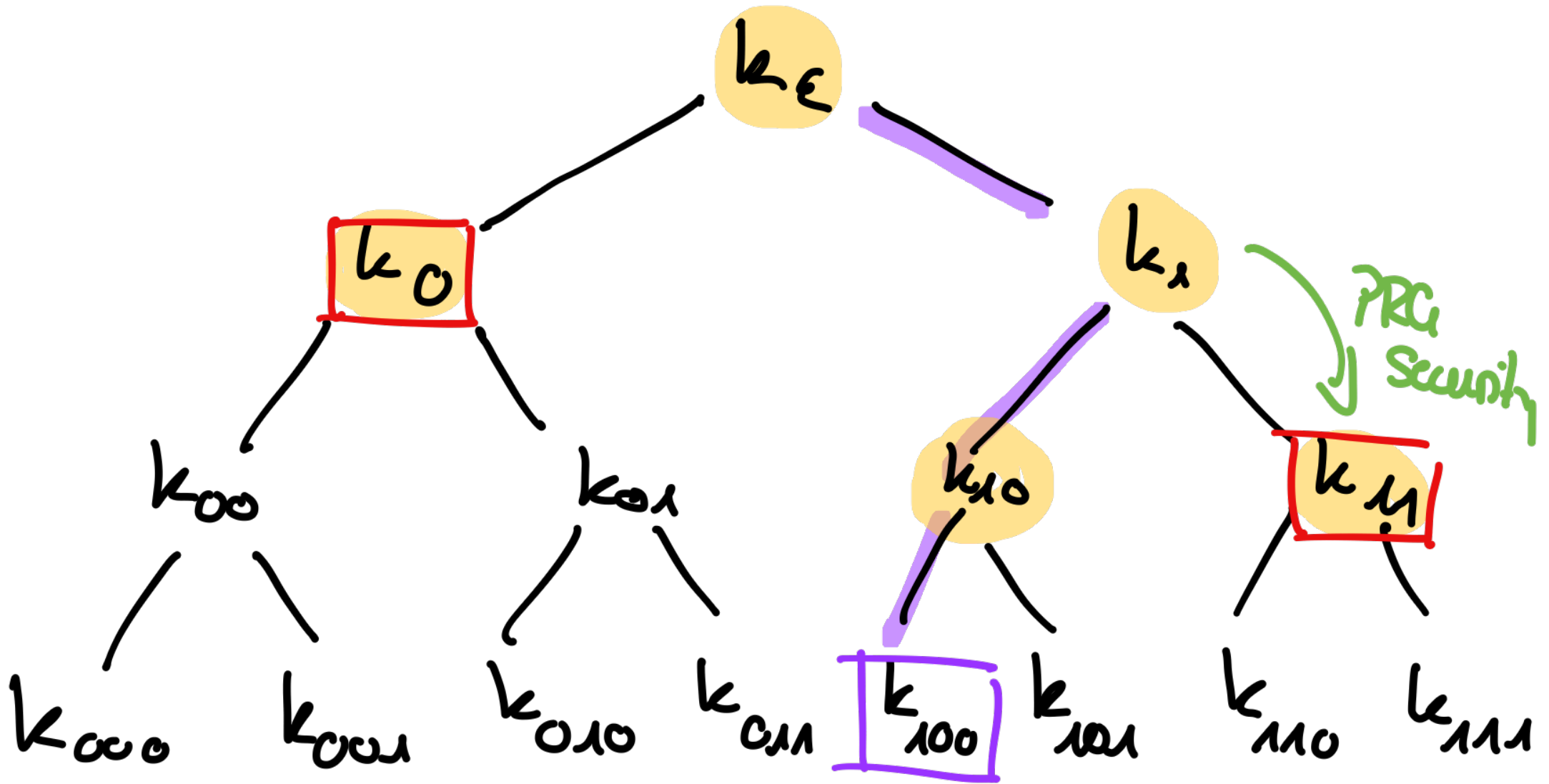
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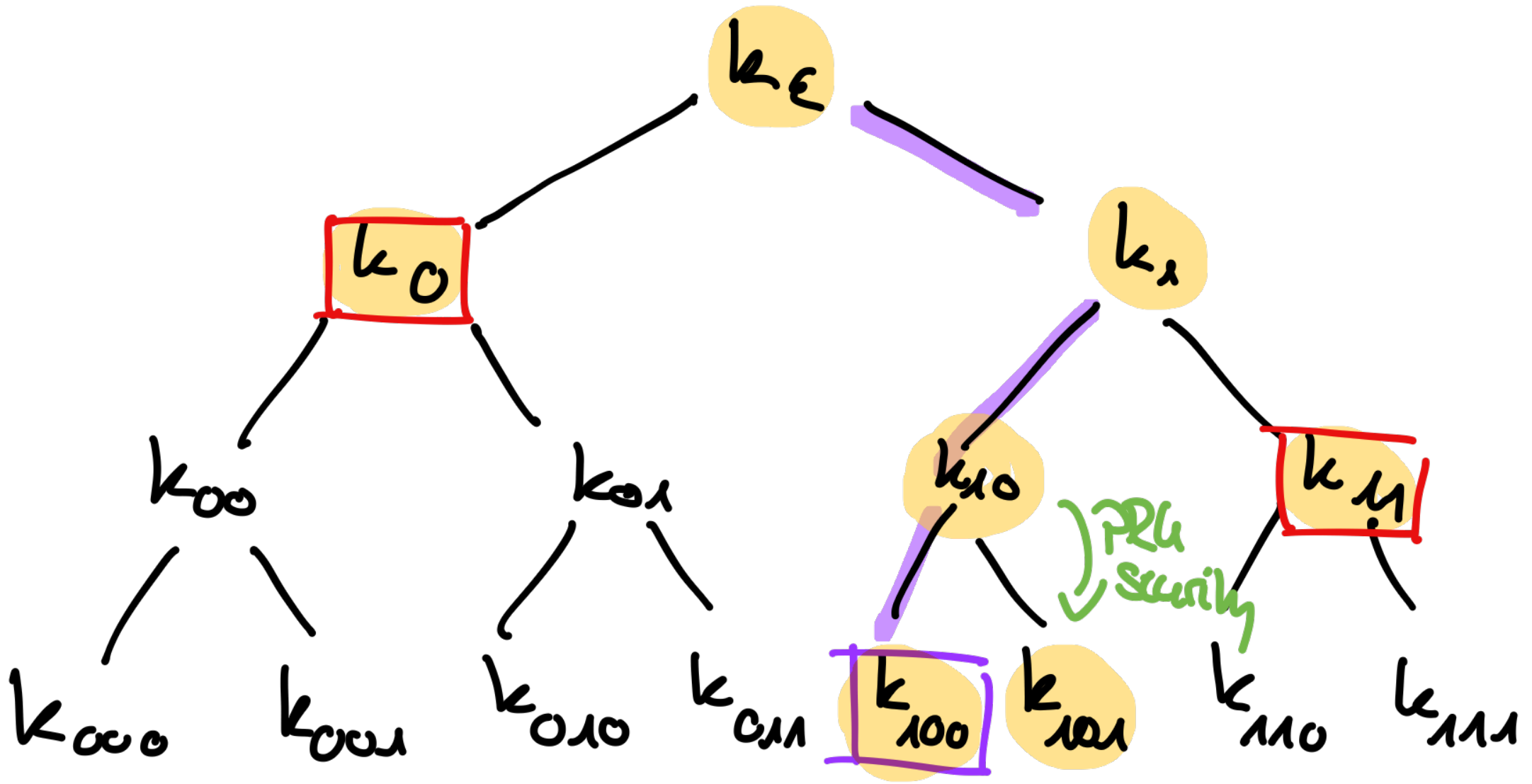
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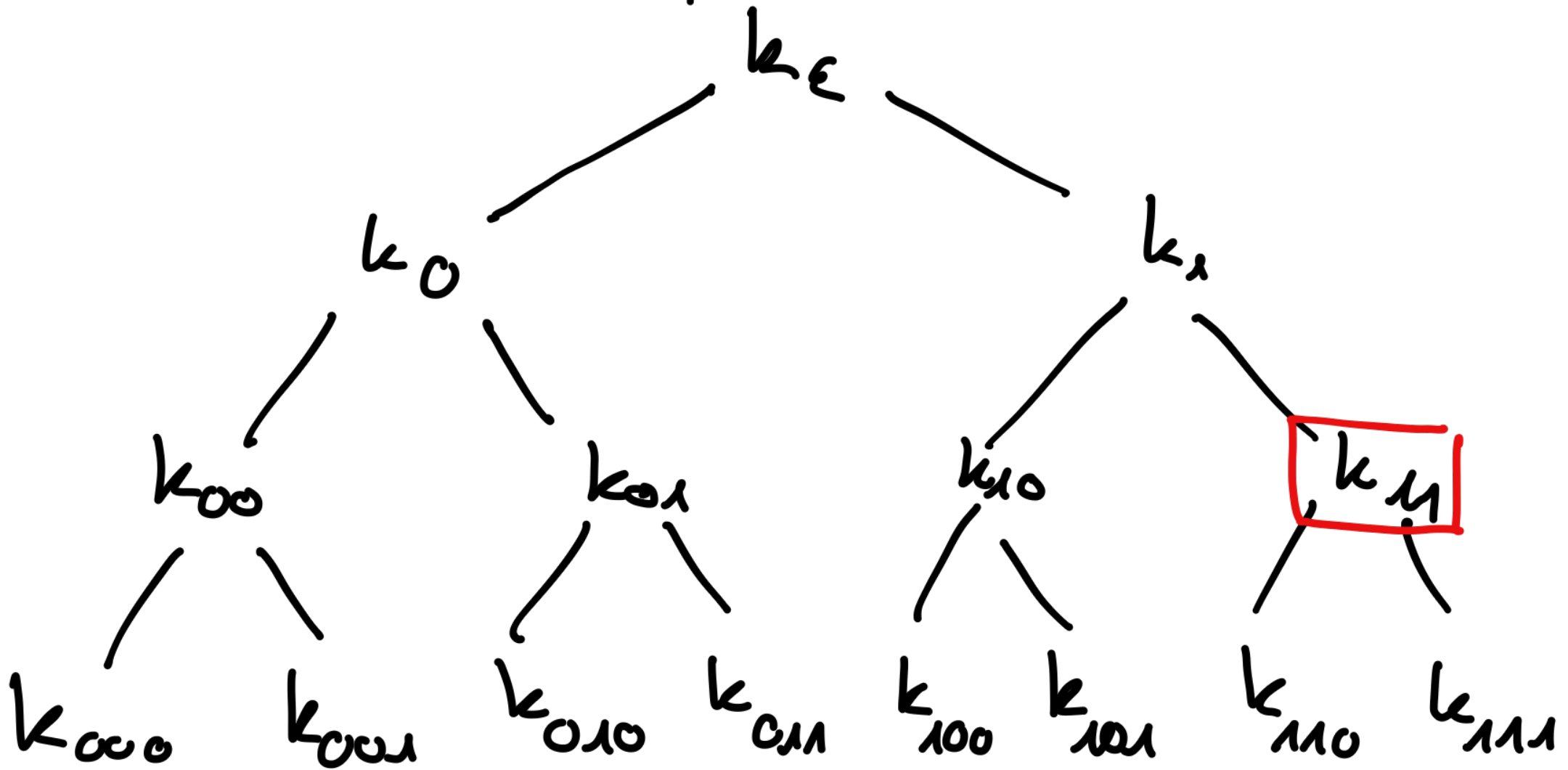
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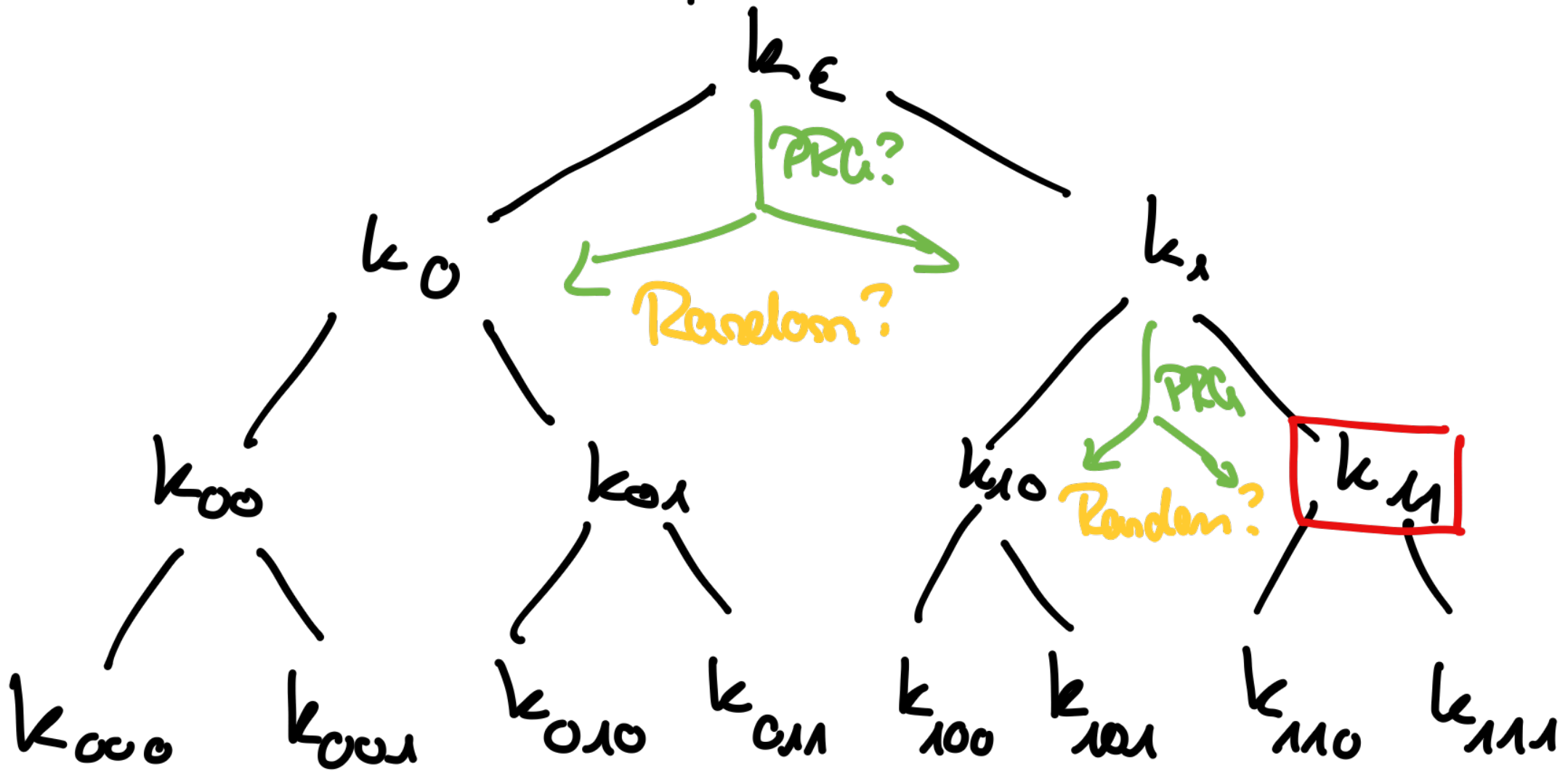
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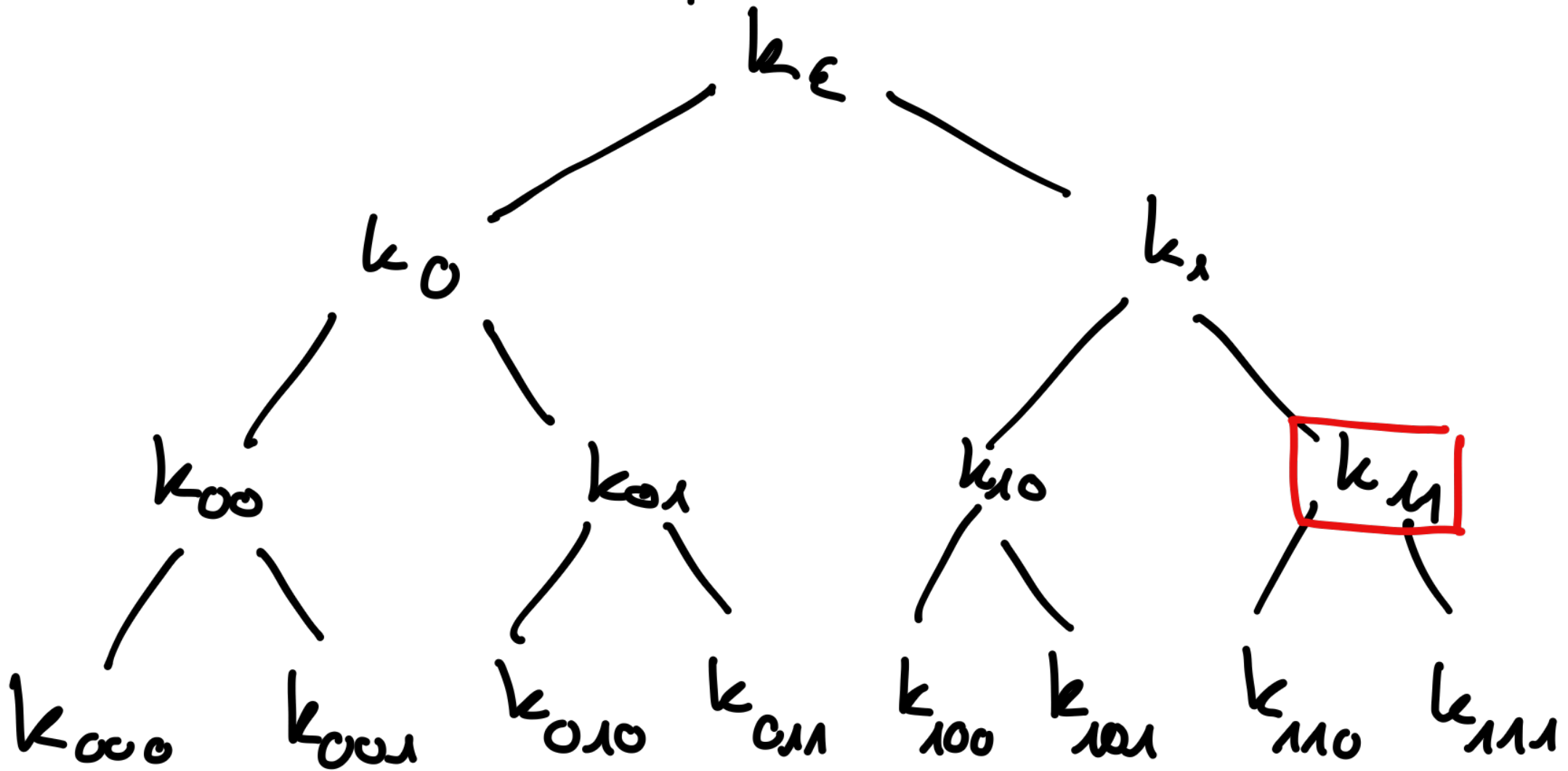
# Adaptive Security Challenges



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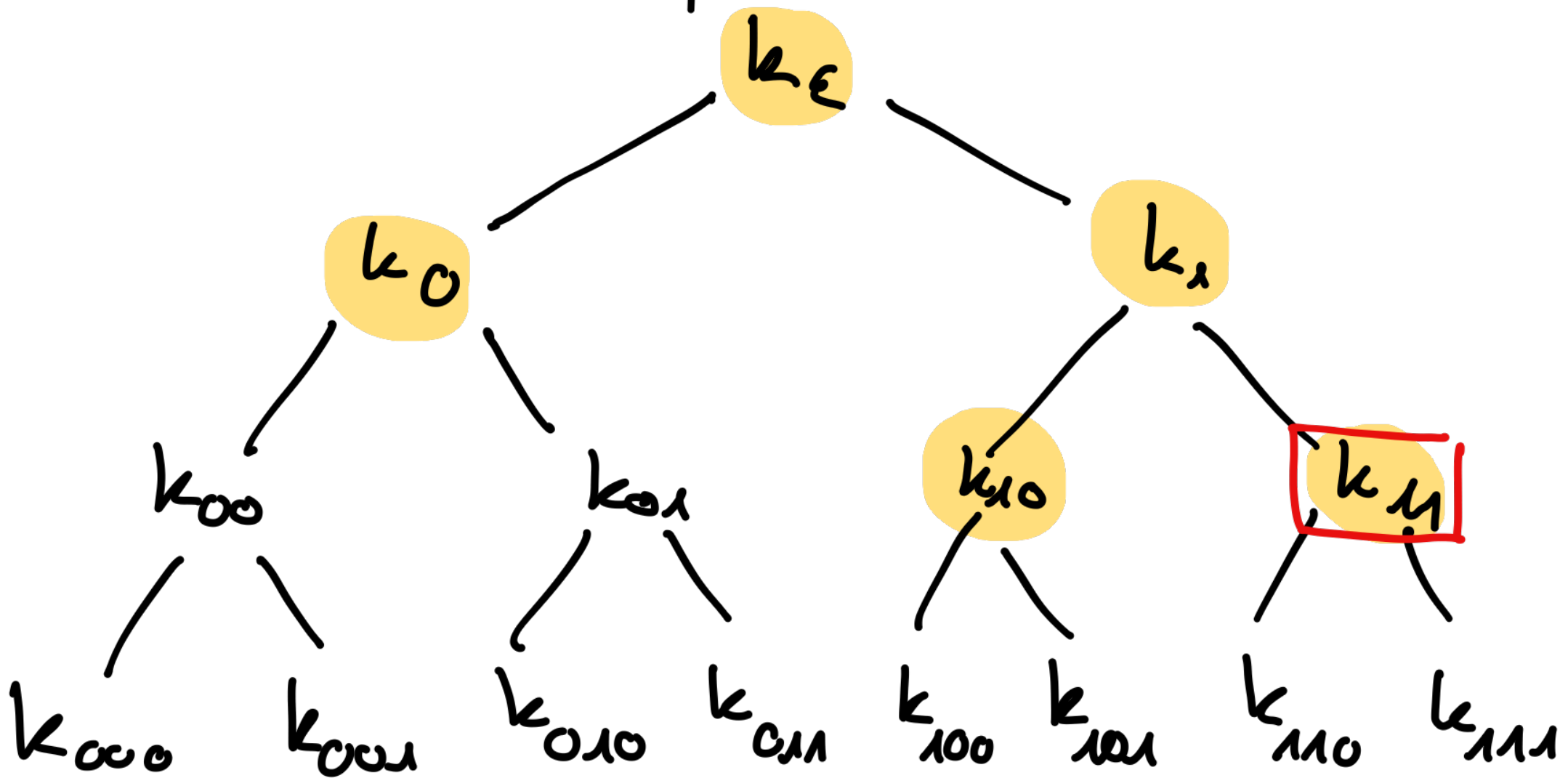
# Adaptive Security Challenges



challenge here?

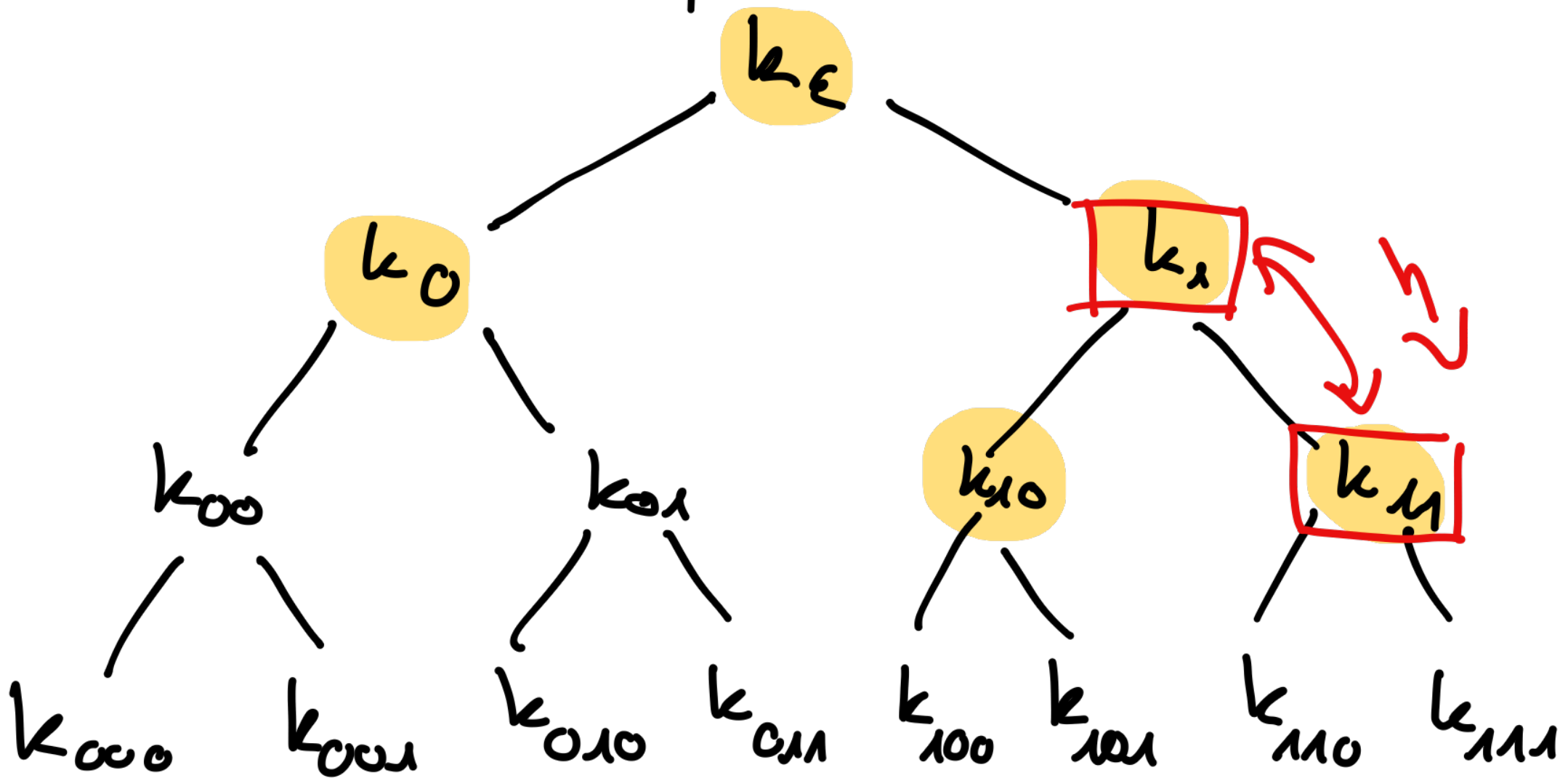


# Adaptive Security Challenges



challenge here?

# Adaptive Security Challenges



challenge here?

# Impossibility Result [KKPW21]

Any straight-line reduction proving adaptive security for the GGM PC-PRF based on the security of the underlying PRG loses a superpolynomial factor in the input size  $n$ .

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Any straight-line reduction proving adaptive security for the GGM PC-PRF based on the security of the underlying PRG loses a superpolynomial factor in the input size  $n$ .

rewinding!

# Impossibility Result [KKPW21]

rewinding!

Any straight-line reduction proving adaptive security for the GGM PC-PRF based on the

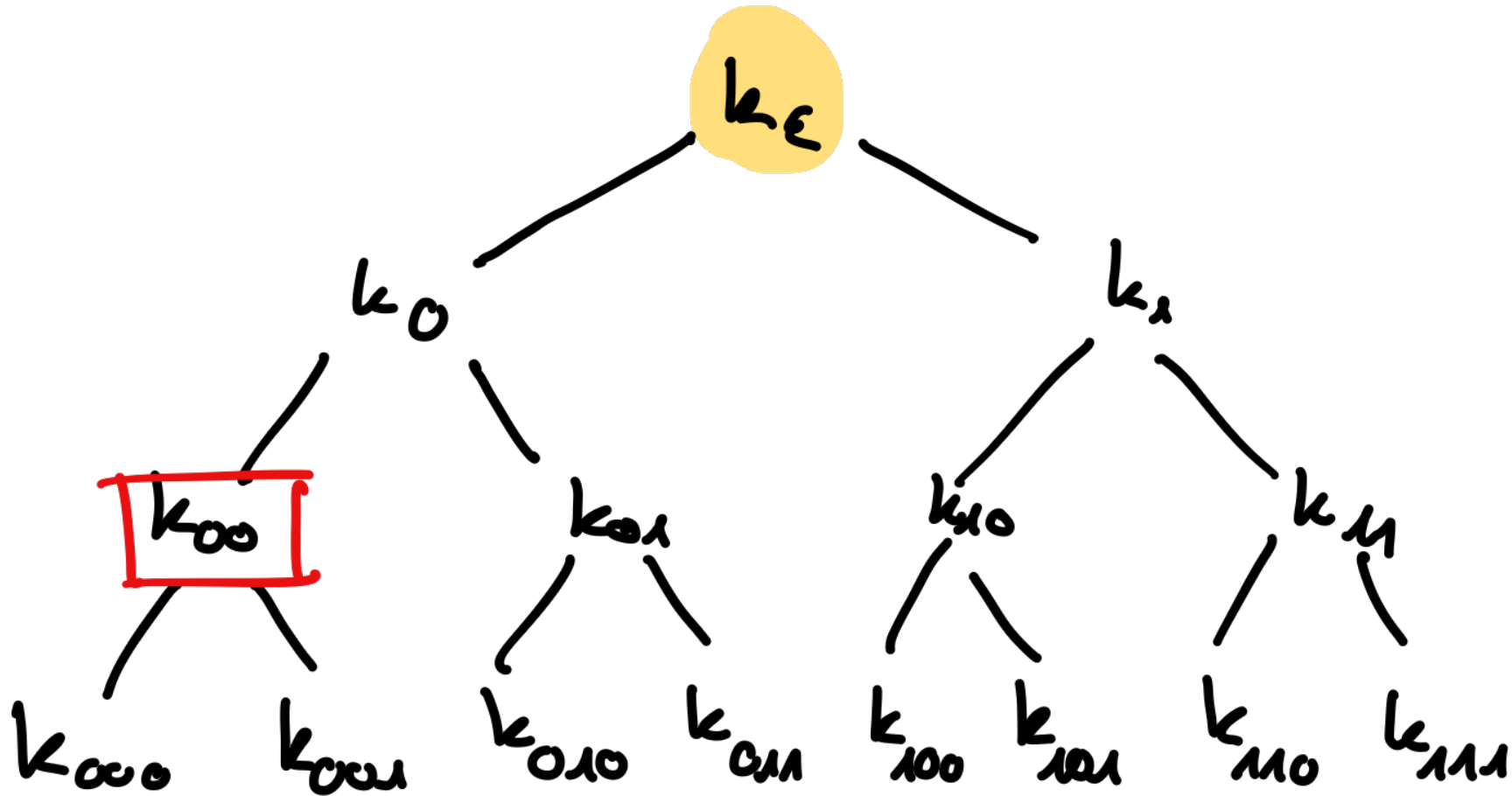
security of the underlying PRG loses a ~~superpolynomial~~ factor in the input size  $n$ .

- polynomial -  
- , , , ,

# Adversarial views

Corrupt  $k_{00}$

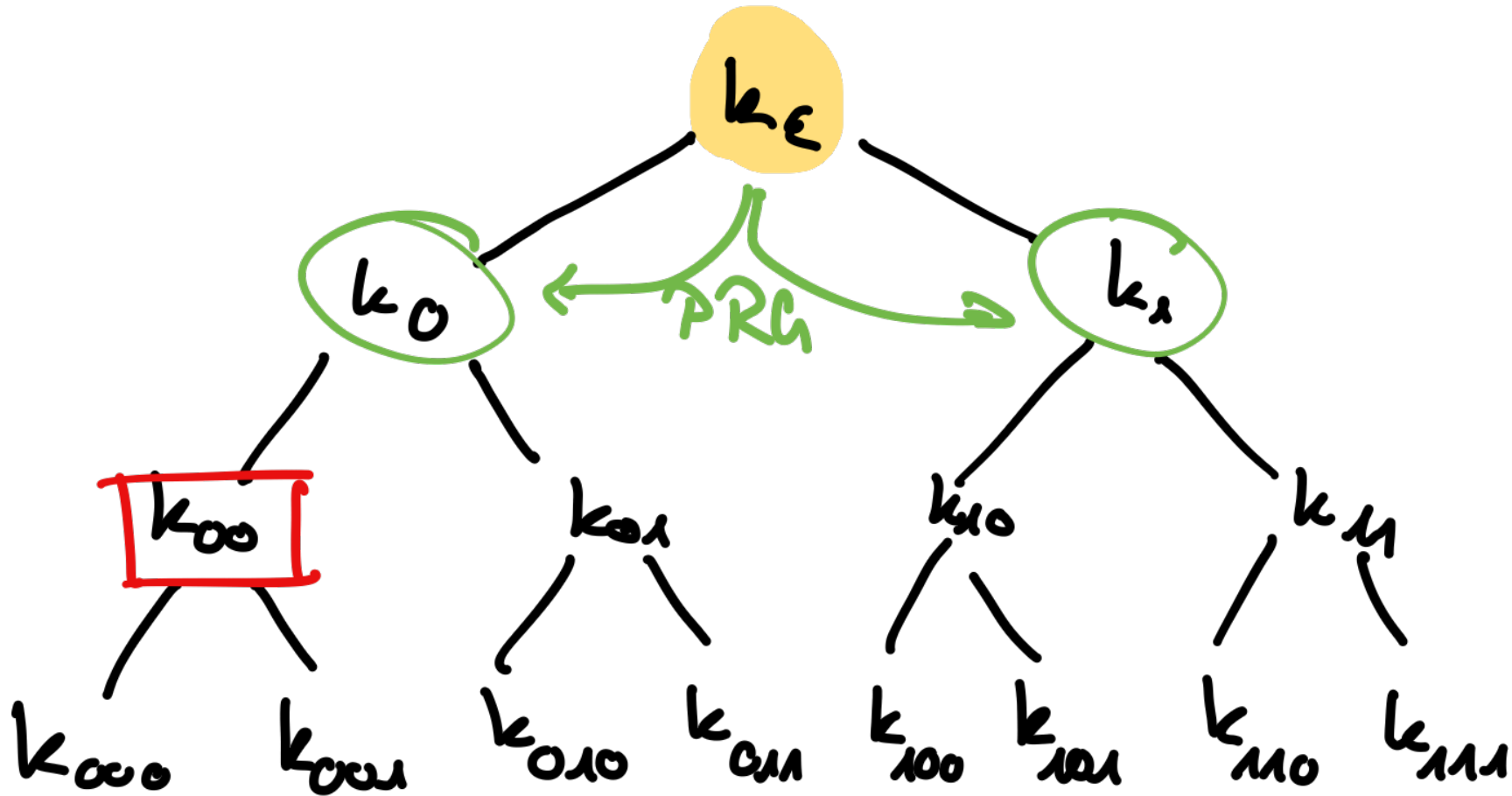
- Corruptions
- honest PRG
- random values



# Adversarial views

- Corruptions
- honest PRG
- random values

Corrupt  $k_{00}$   
PRG  $\varepsilon$



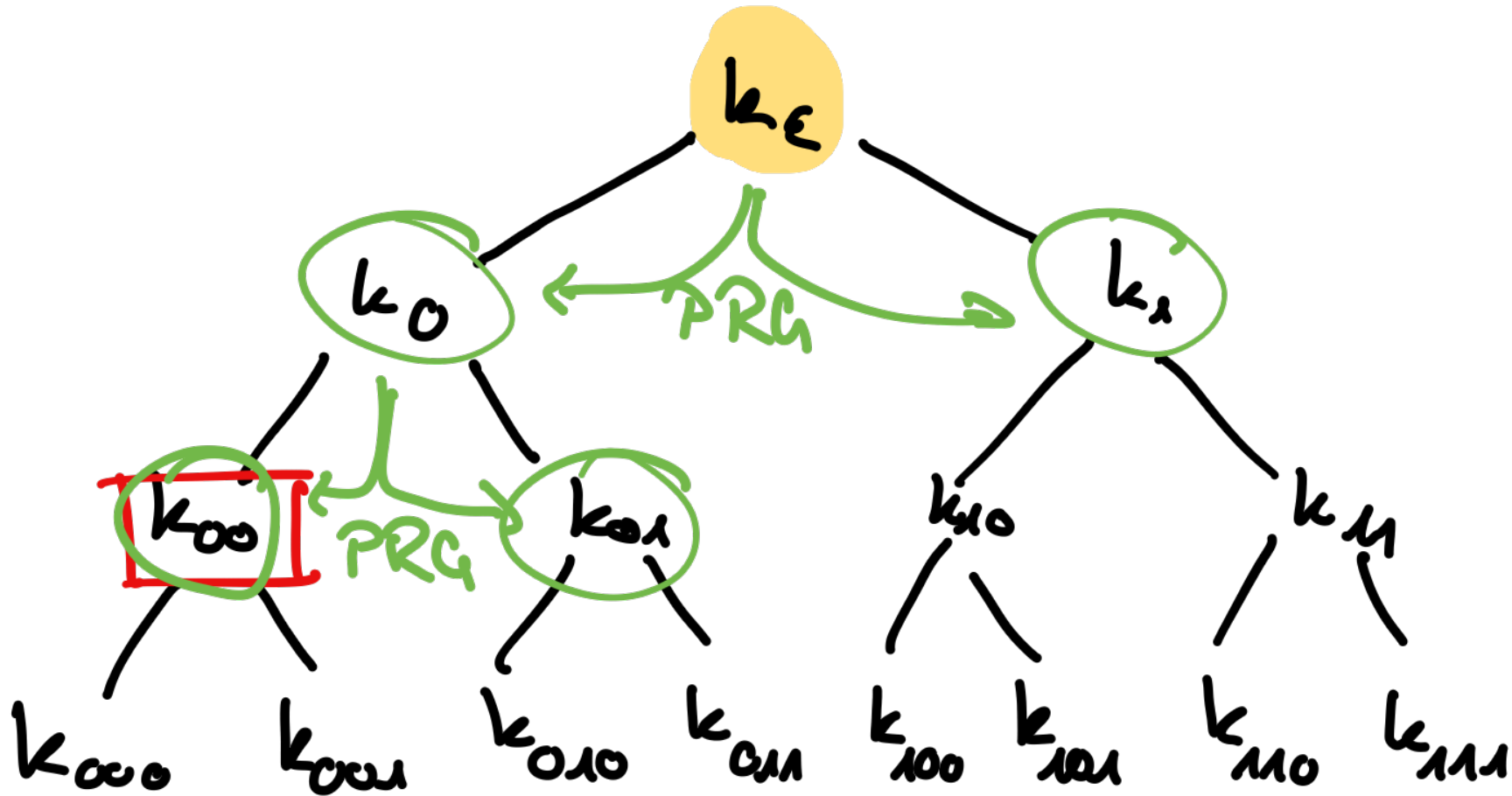
# Adversarial views

- Corruptions
- honest PRG
- random values

Corrupt  $k_{00}$

PRG  $\varepsilon$

PRG  $0$





# Adversarial views

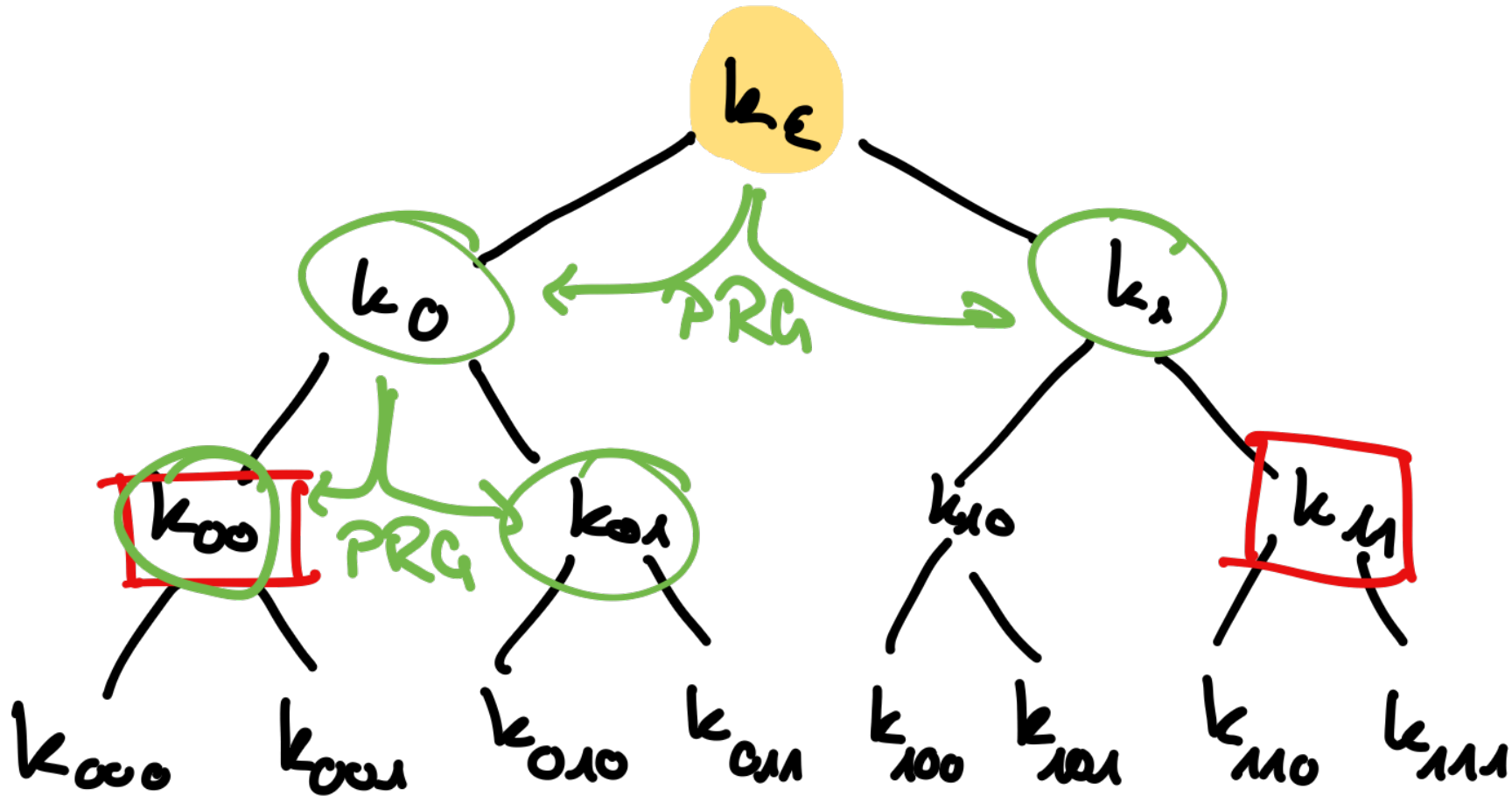
- Corruptions
- honest PRG
- random values

Corrupt  $k_{00}$

PRG  $\varepsilon$

PRG  $\circ$

Corrupt  $k_{11}$



# Adversarial views

- Corruptions
- honest PRG
- random values

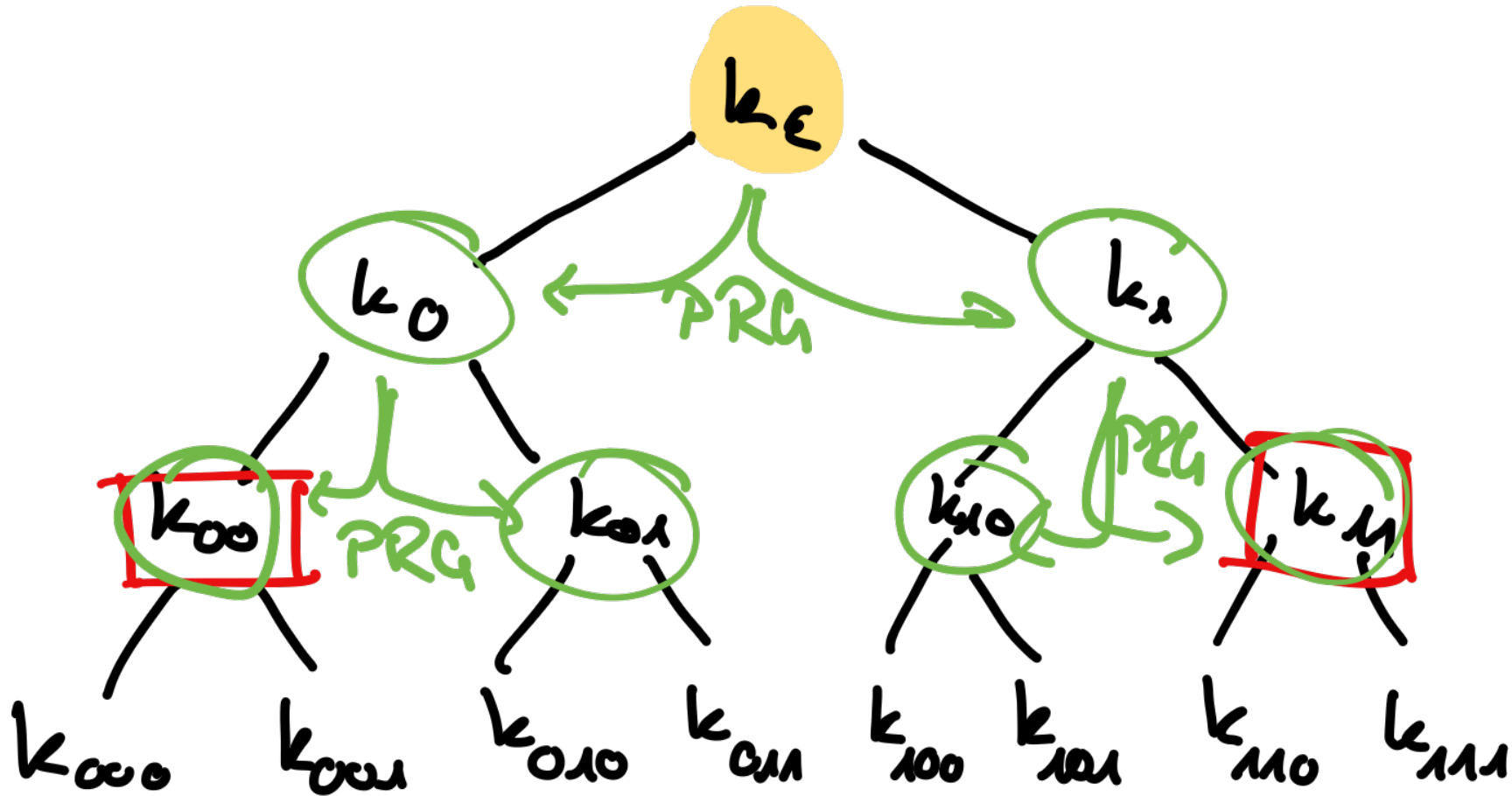
Corrupt  $k_{00}$

PRG  $\epsilon$

PRG  $0$

Corrupt  $k_{11}$

PRG  $1$



# Adversarial views

- Corruptions
- honest PRG
- random values

Corrupt  $k_{00}$

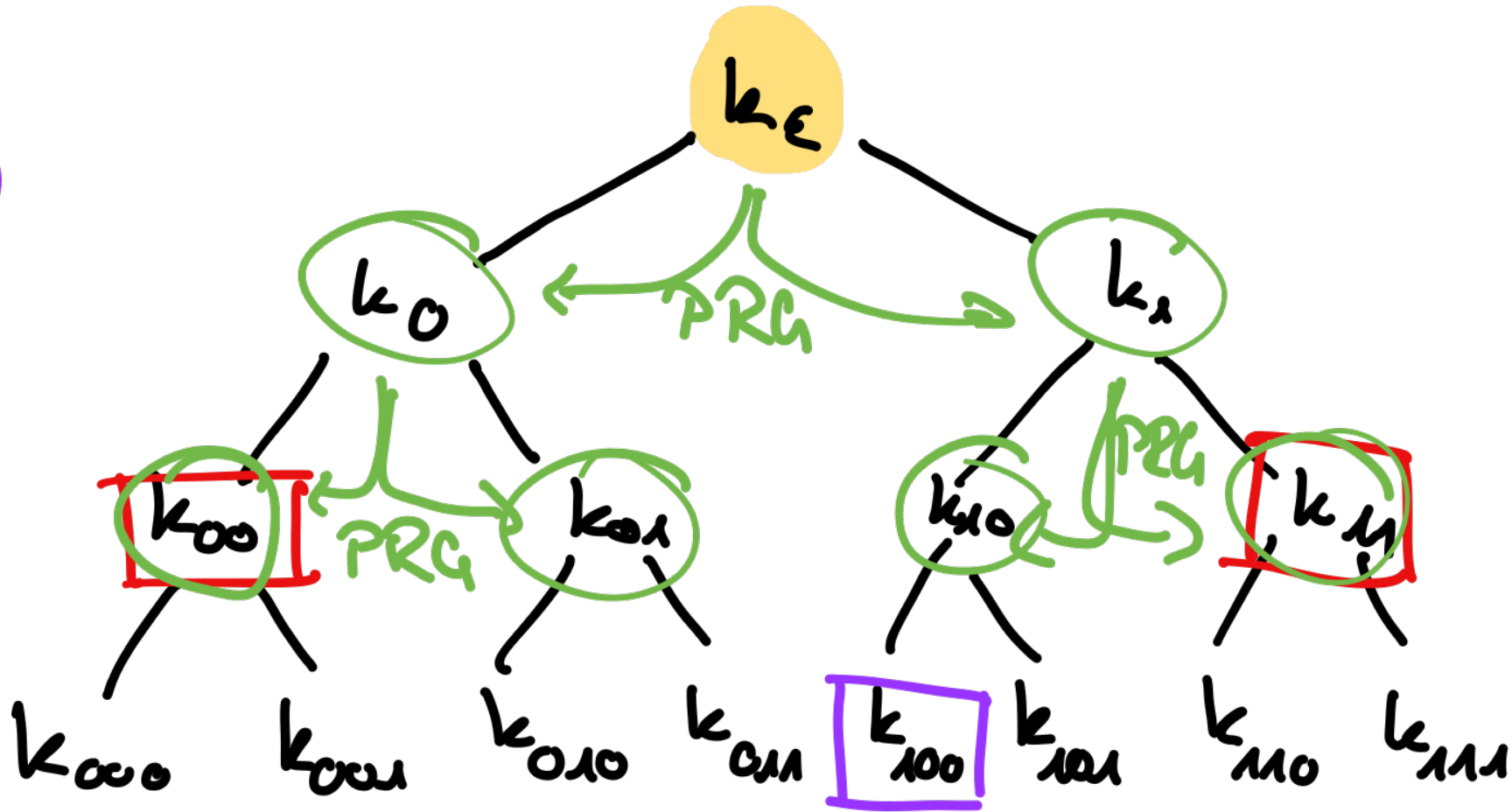
PRG  $\epsilon$

PRG  $0$

Corrupt  $k_{11}$

PRG  $1$

Challenge  $100$



# Adversarial views

- Corruptions
- honest PRG
- random values

Corrupt  $k_{00}$

PRG  $\varepsilon$

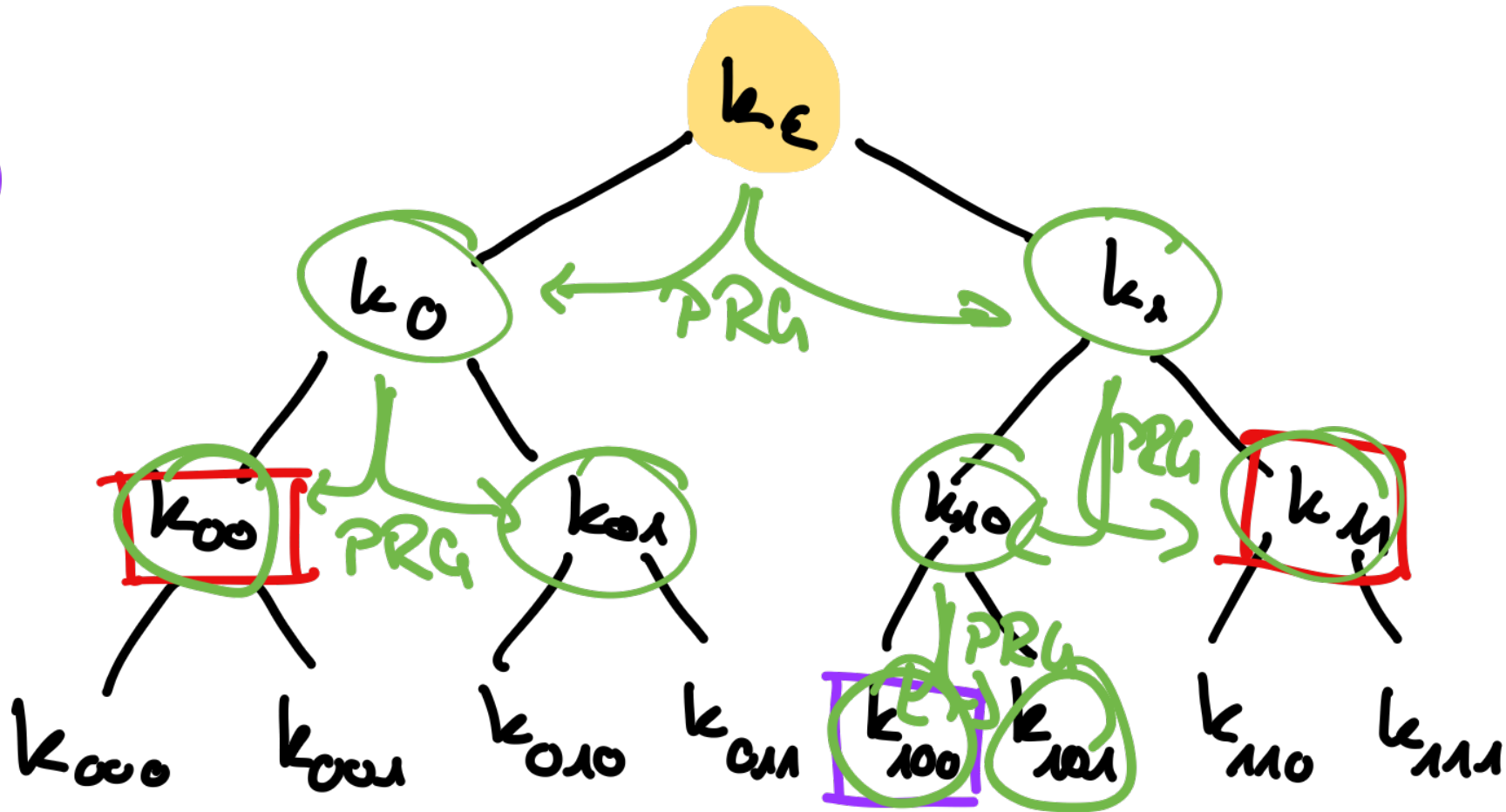
PRG  $0$

Corrupt  $k_{11}$

PRG  $1$

Challenge  $100$

PRG  $10$



# Adversarial views

- Corruptions
- honest PRG
- random values
- rewinding index

Corrupt  $k_{00}$  ←

PRG  $\varepsilon$

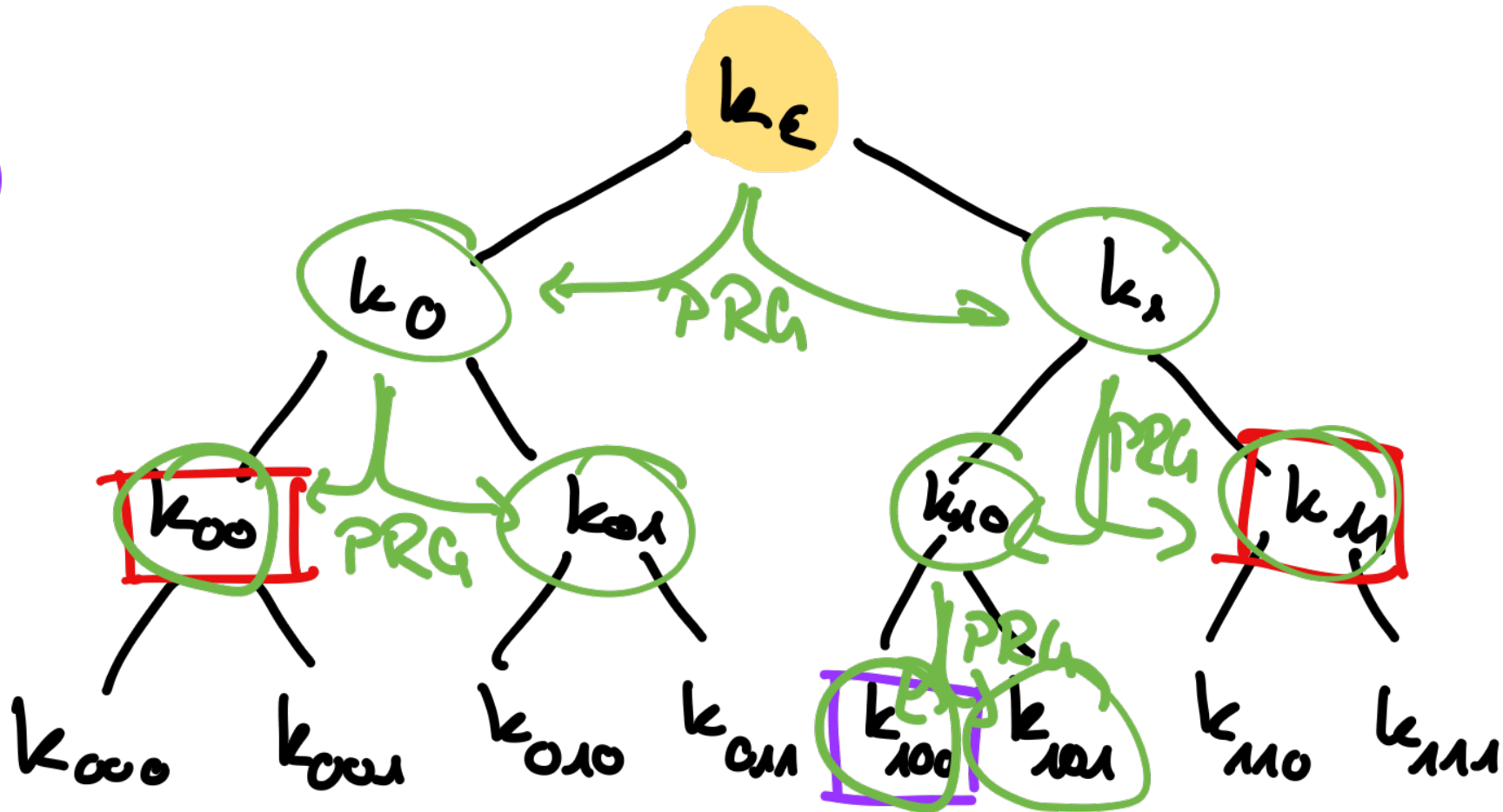
PRG 0

Corrupt  $k_{11}$

PRG 1

Challenge 100

PRG 10



# Adversarial views

- Corruptions
- honest PRG
- random values
- ↔ rewinding index
- relevant index

Corrupt  $k_{00}$

PRG  $\varepsilon$

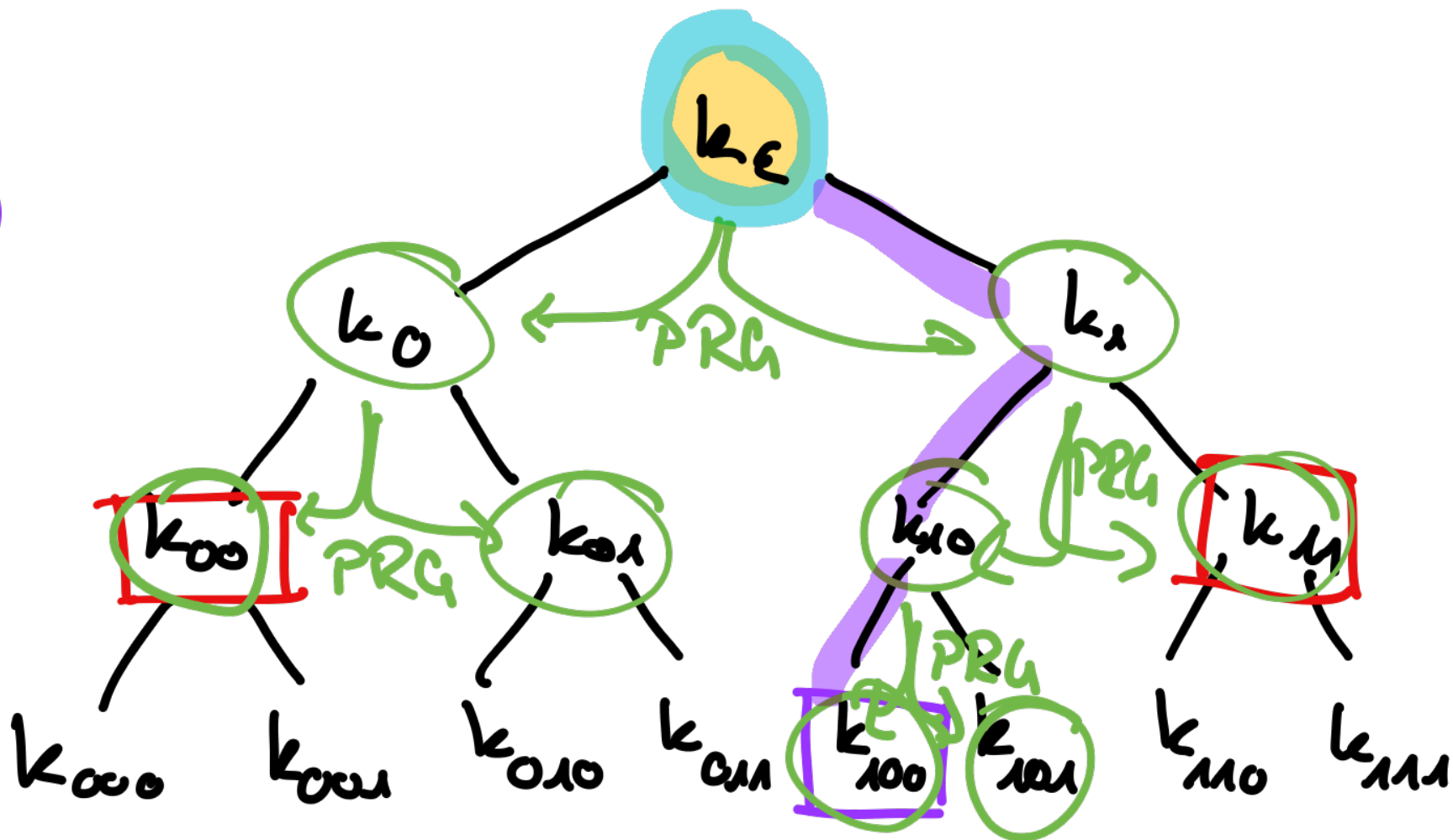
PRG 0

Corrupt  $k_{11}$

PRG 1

Challenge 100

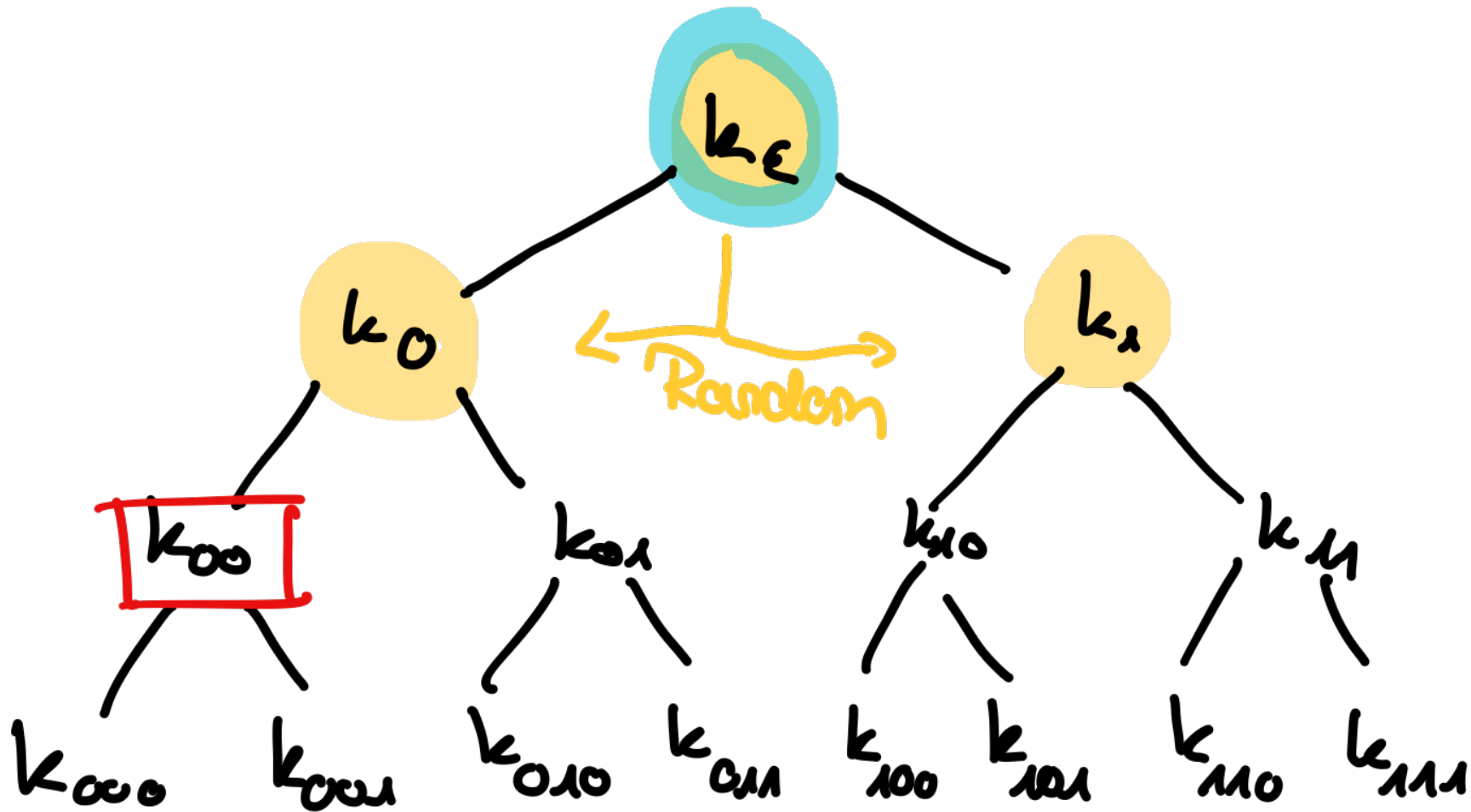
PRG 10



# Adversarial views

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- ↔ rewinding index
- relevant index

Corrupt  $k_{00}$   
PRG  $\varepsilon$



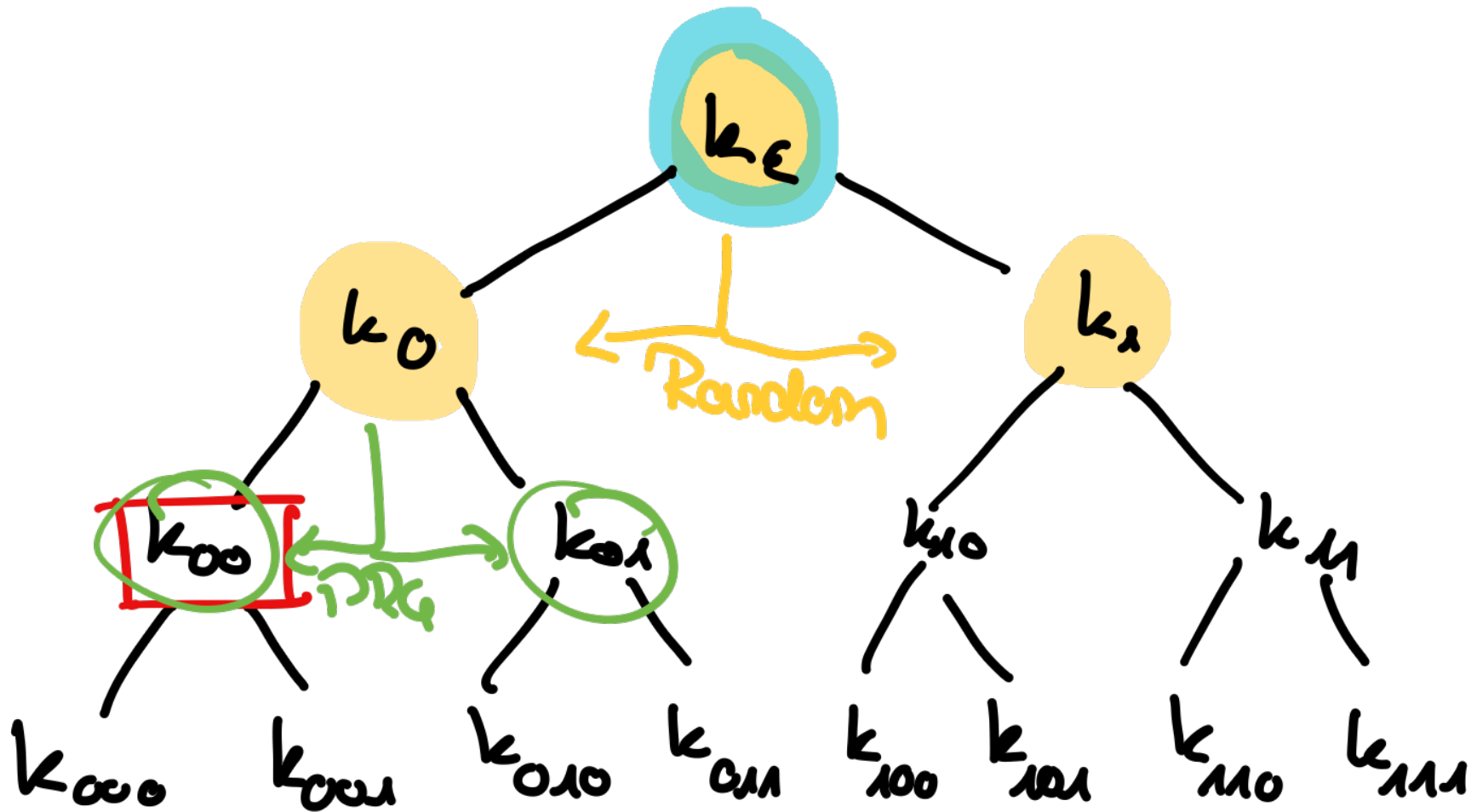
# Adversarial views

- Corruptions
- honest PRG
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- relevant index

Corrupt  $k_{00}$

PRG  $\varepsilon$

PRG  $0$





# Adversarial views

- Corruptions
- honest PRG
- random values
- ↔ rewinding index
- relevant index

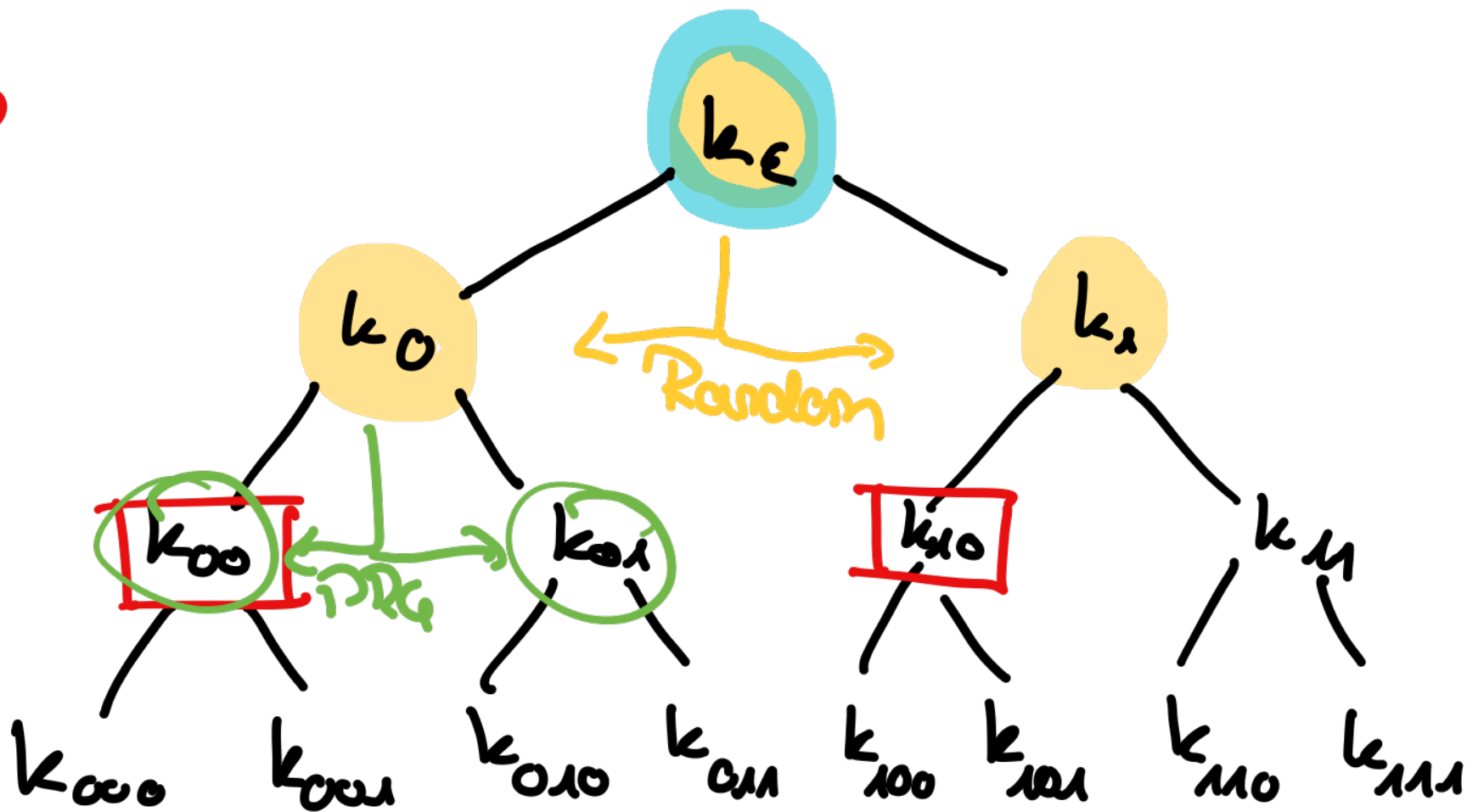
Corrupt  $k_{00}$

PRG  $\epsilon$



PRG 0

Corrupt  $k_{10}$



# Adversarial views

- Corruptions
- honest PRG
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- relevant index

Corrupt  $k_{00}$

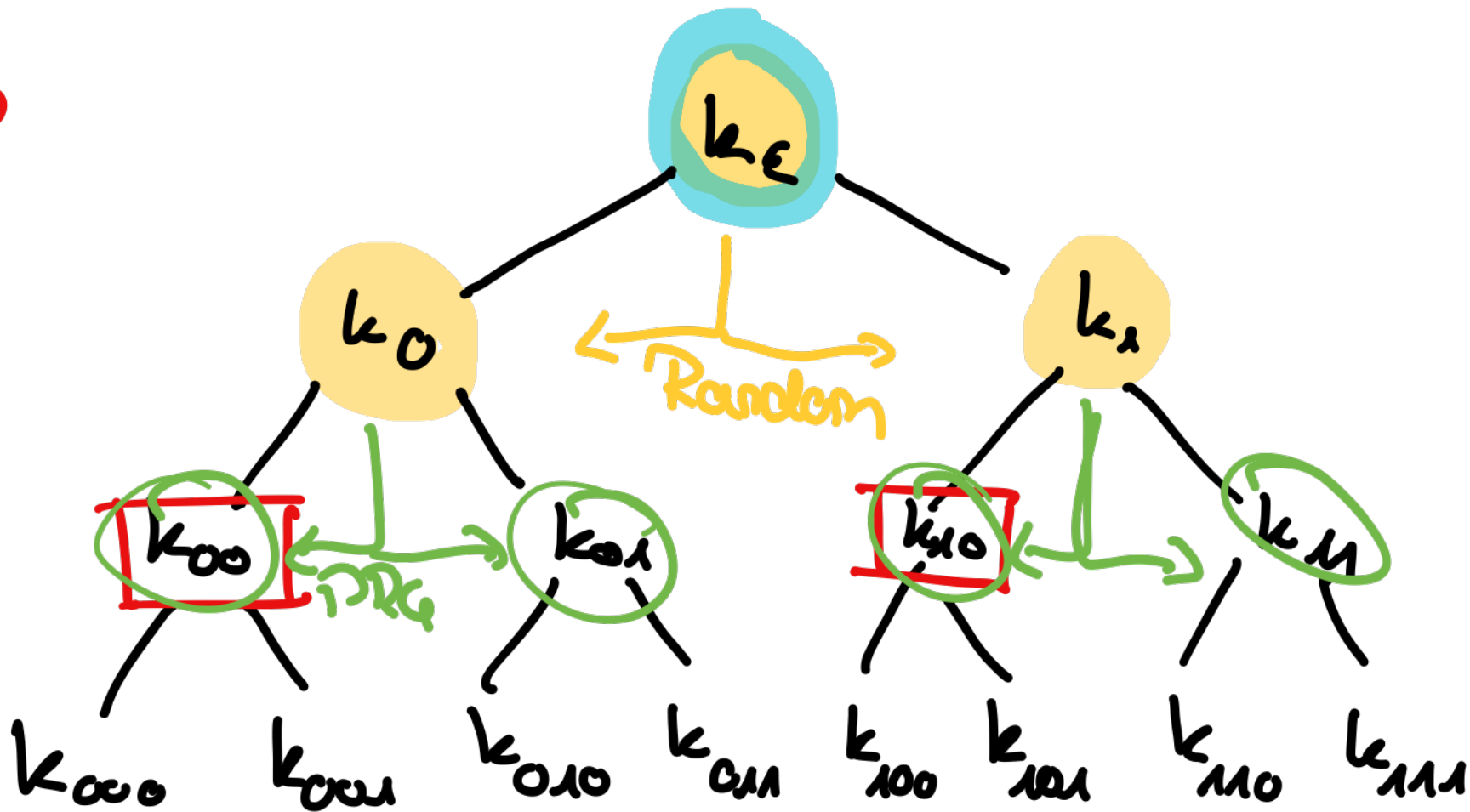
PRG  $\epsilon$



PRG 0

Corrupt  $k_{10}$

PRG 1



# Adversarial views

- Corruptions
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- relevant index

Corrupt  $k_{00}$

PRG  $\epsilon$

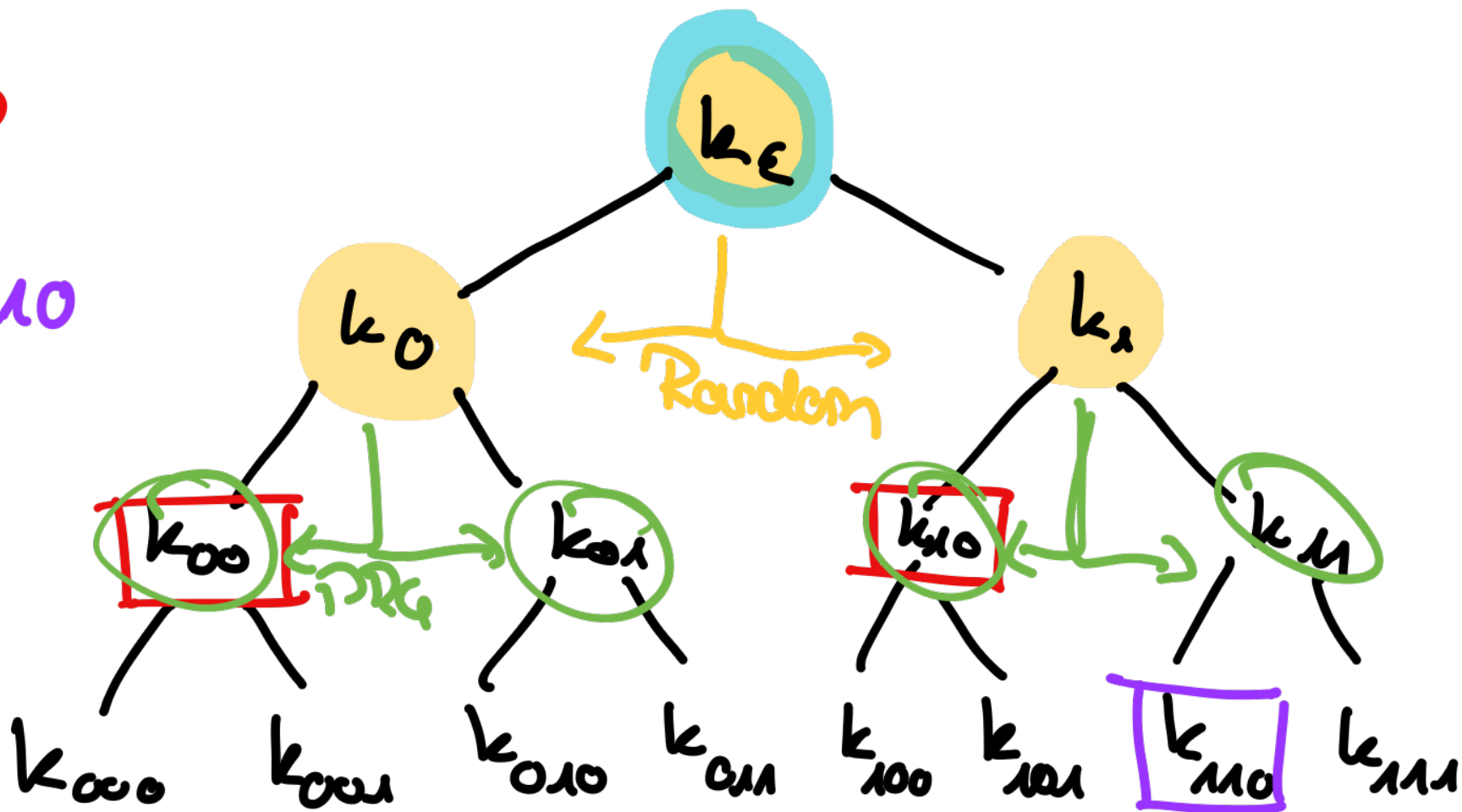


PRG 0

Corrupt  $k_{10}$

PRG 1

challenge  $k_{10}$



# Adversarial views

- Corruptions
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Corrupt  $k_{00}$

PRG  $\epsilon$



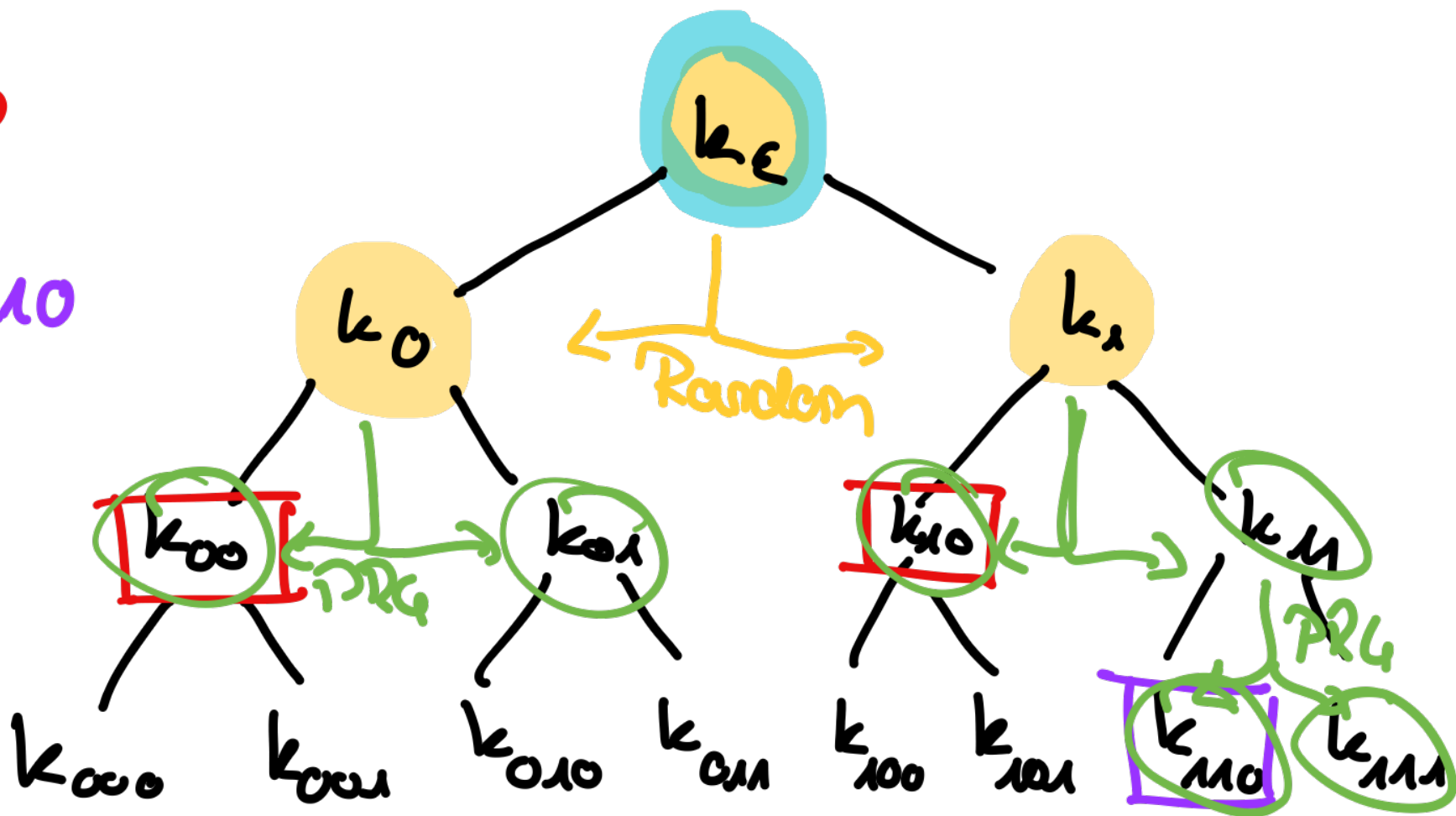
PRG 0

Corrupt  $k_{10}$

PRG 1

challenge  $k_{110}$

PRG  $M$



# Adversarial views

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Corrupt  $k_{00}$

PRG  $\epsilon$



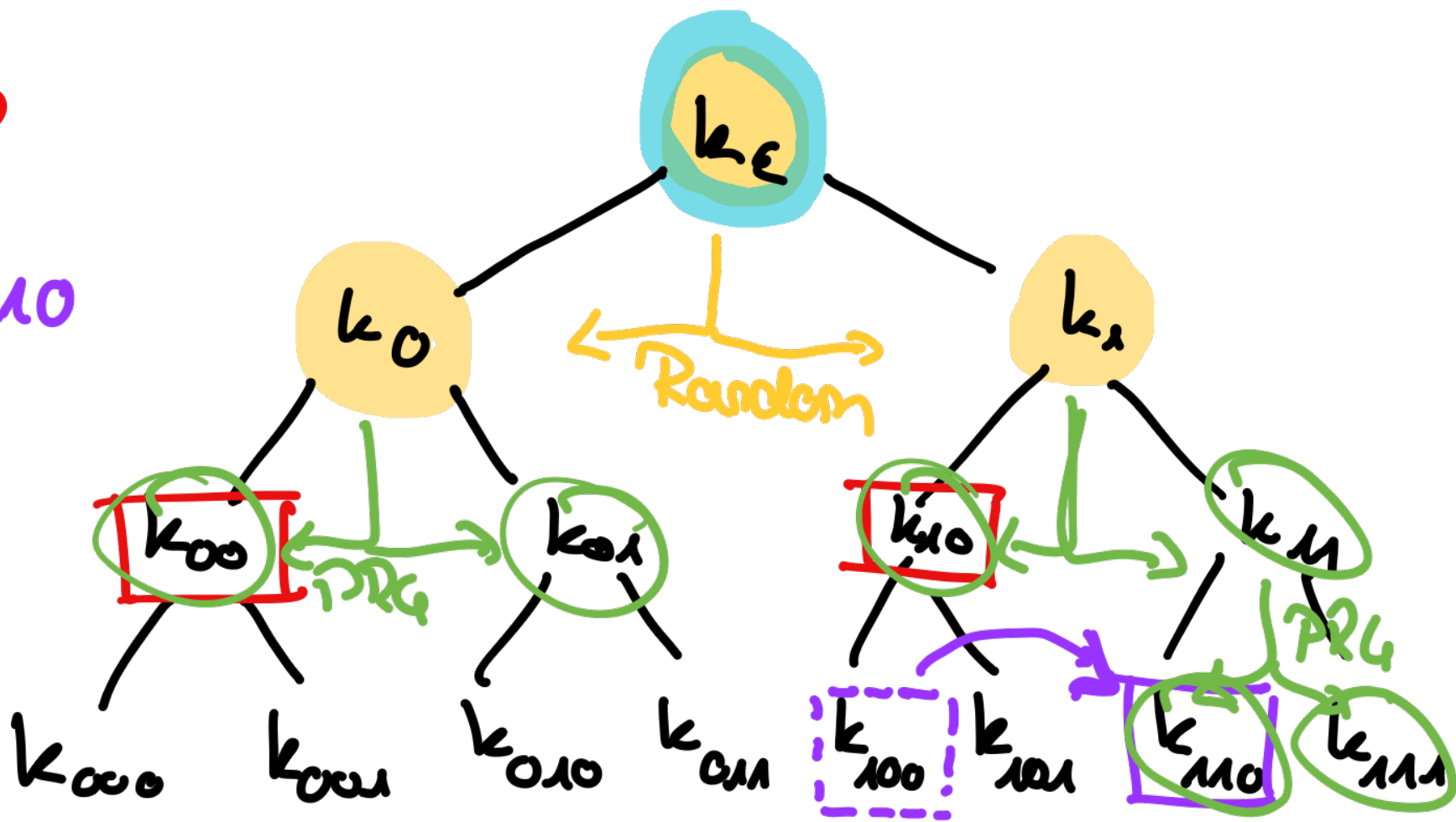
PRG 0

Corrupt  $k_{10}$

PRG 1

challenge  $k_{10}$

PRG  $M$



# Adversarial views

- Corruptions
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Corrupt  $k_{00}$

PRG  $\epsilon$



PRG 0

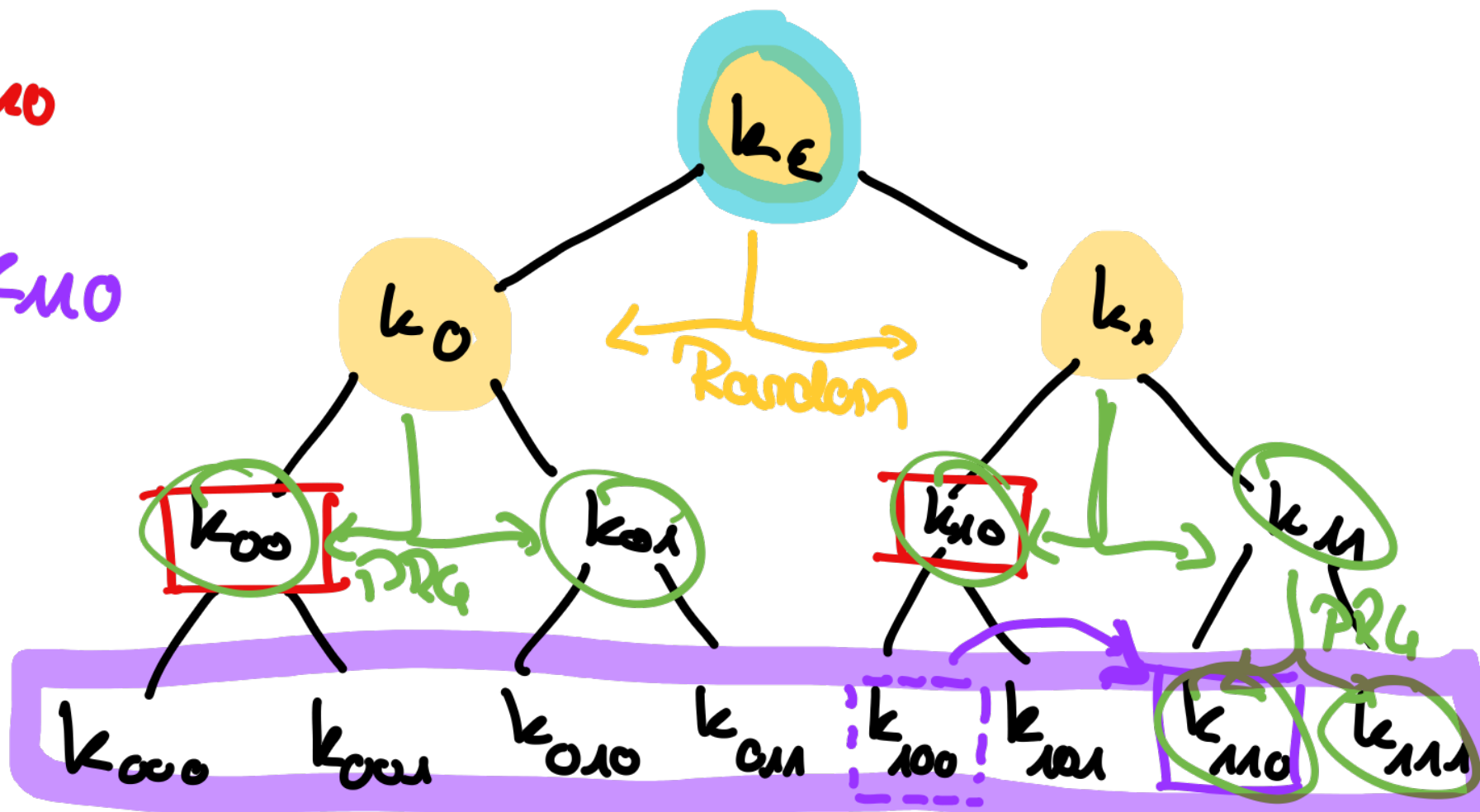
Corrupt  $k_{10}$

PRG 1

challenge  $k_{10}$

PRG  $M$

exponential



# Adversarial views

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Corrupt  $k_{00}$

PRG  $\epsilon$

PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

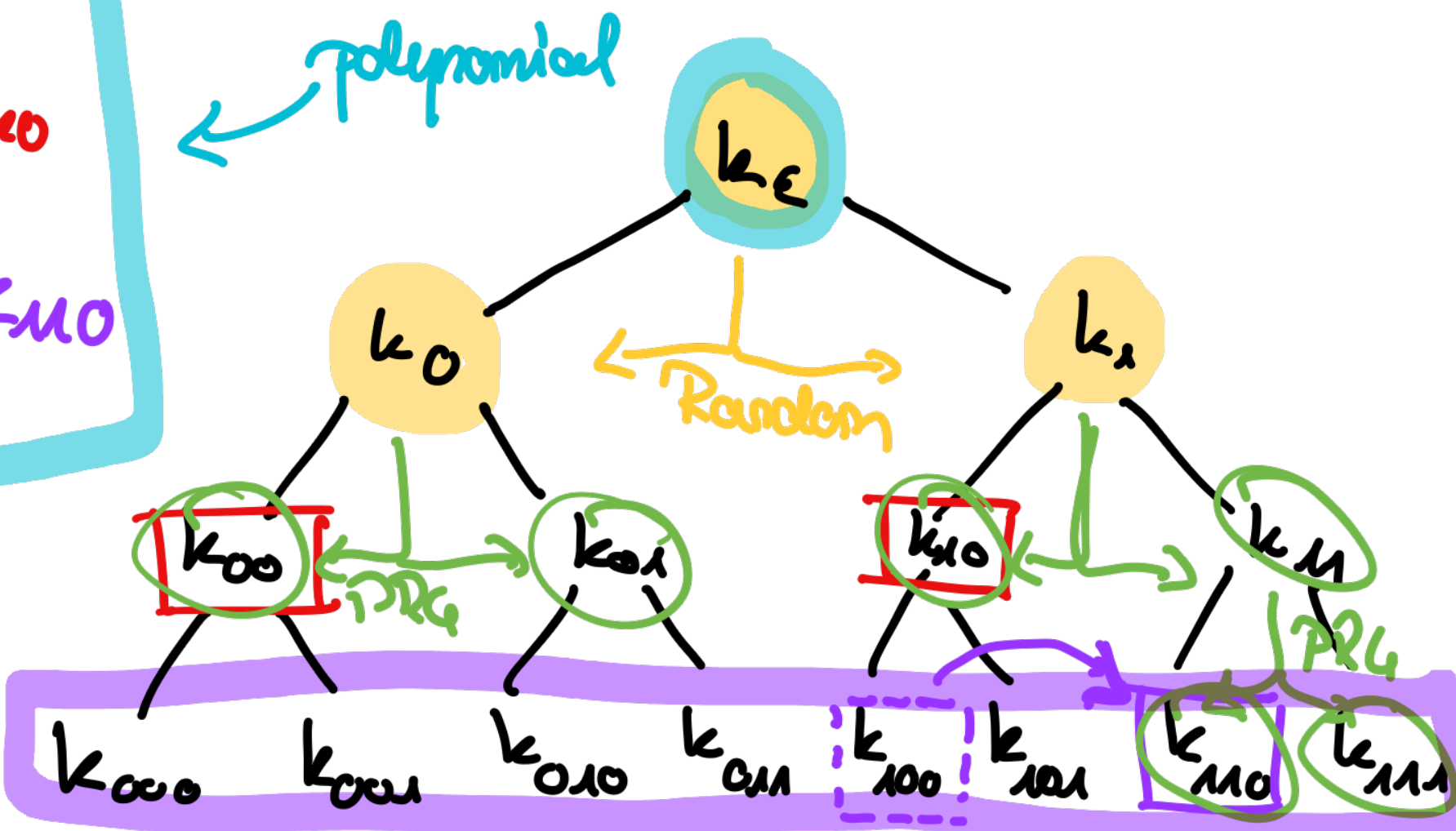
challenge  $k_{10}$

PRG  $M$



polynomial

exponential



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Corrupt  $k_{00}$

PRG  $\epsilon$

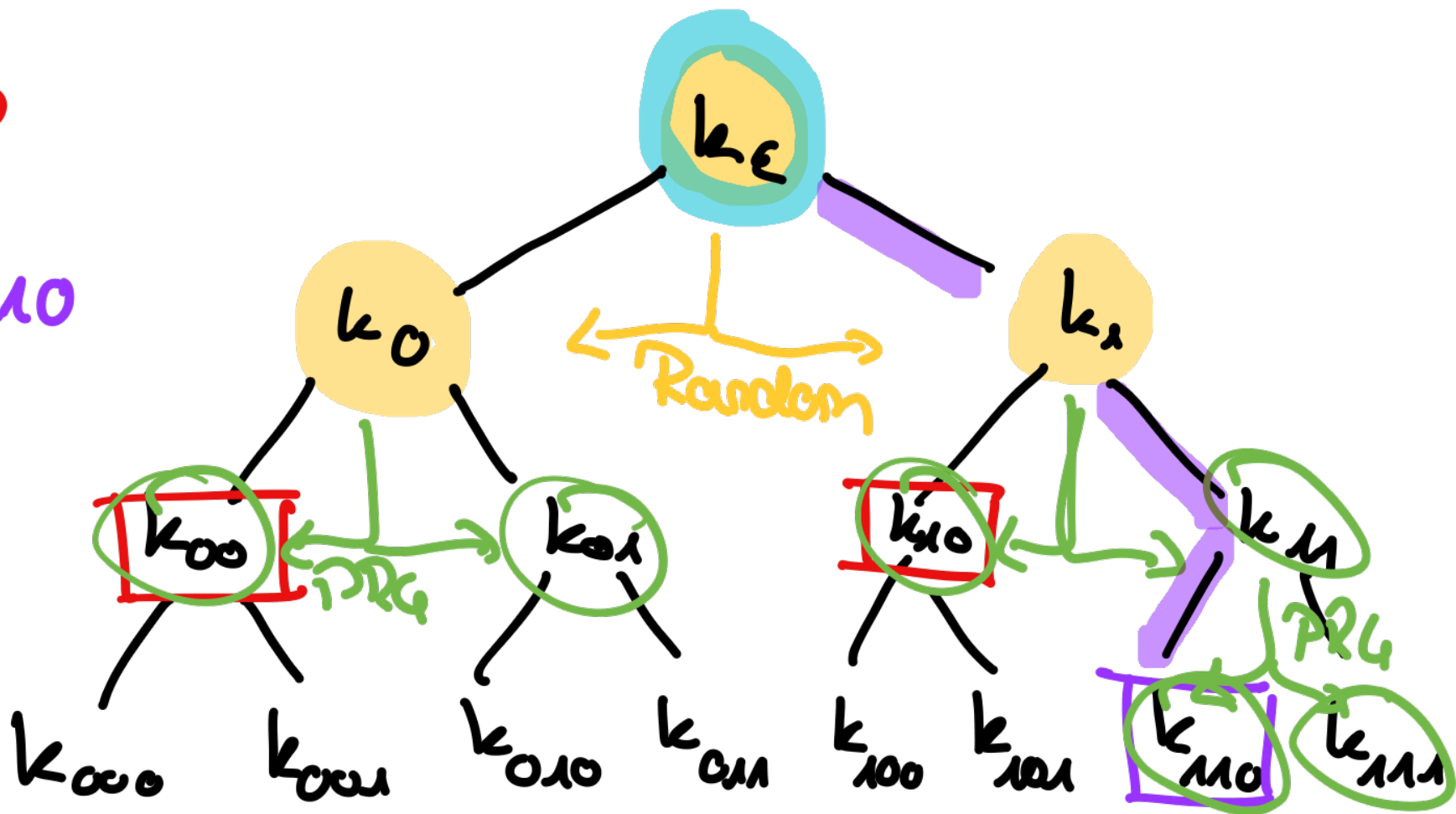
PRG 0

Corrupt  $k_{10}$

PRG 1

challenge  $k_{10}$

PRG  $M$





# Adversarial views

- Corruptions
- honest PRG
- random values
- ← rewinding index
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Corrupt  $k_{00}$

PRG  $\epsilon$

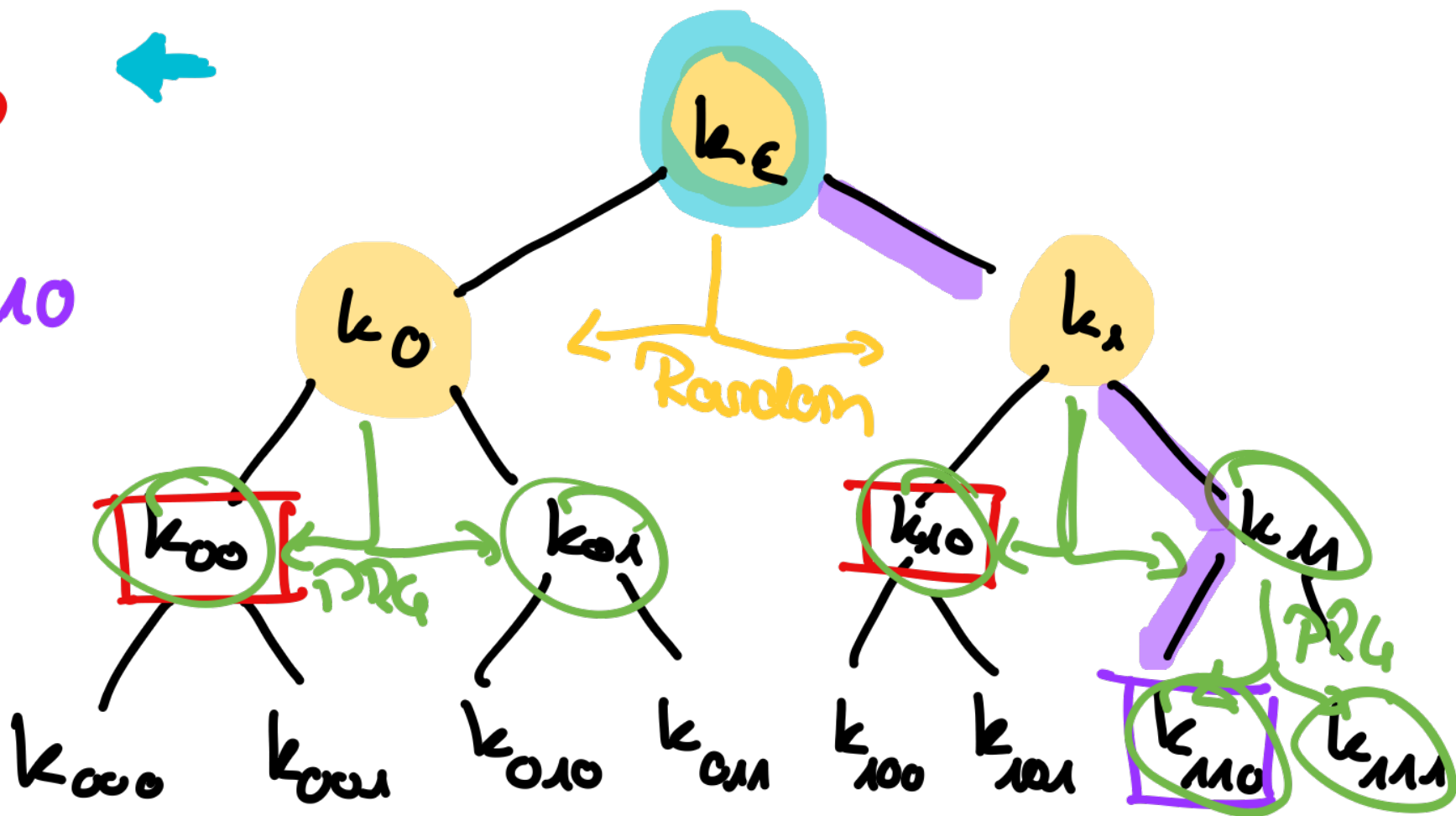
PRG 0

Corrupt  $k_{10}$

PRG 1

challenge  $k_{10}$

PRG  $M$



# Adversarial views

- Corruptions
- honest PRG
- random values
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Corrupt  $k_{00}$

PRG  $\varepsilon$

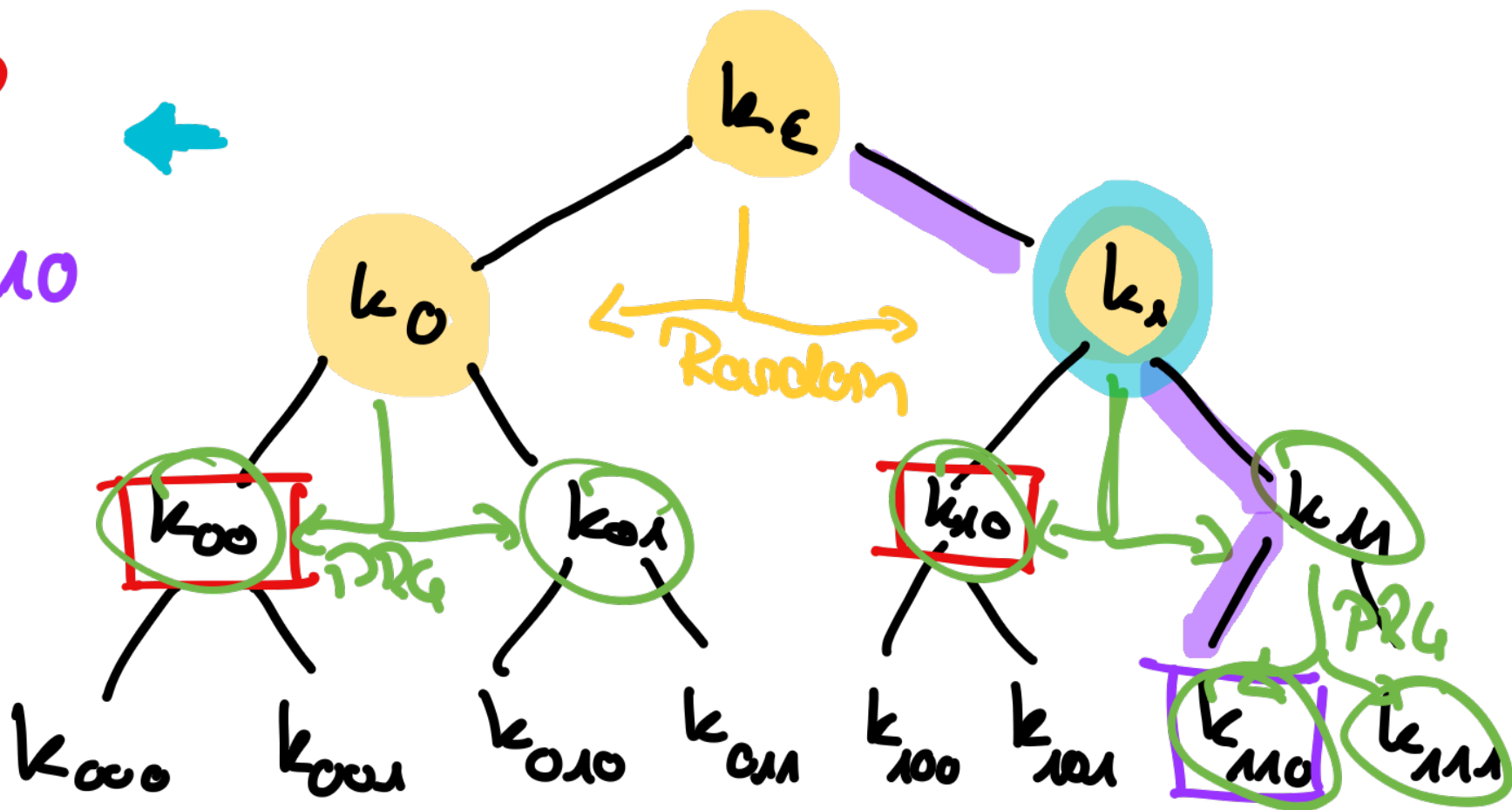
PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

challenge  $k_{10}$

PRG  $11$



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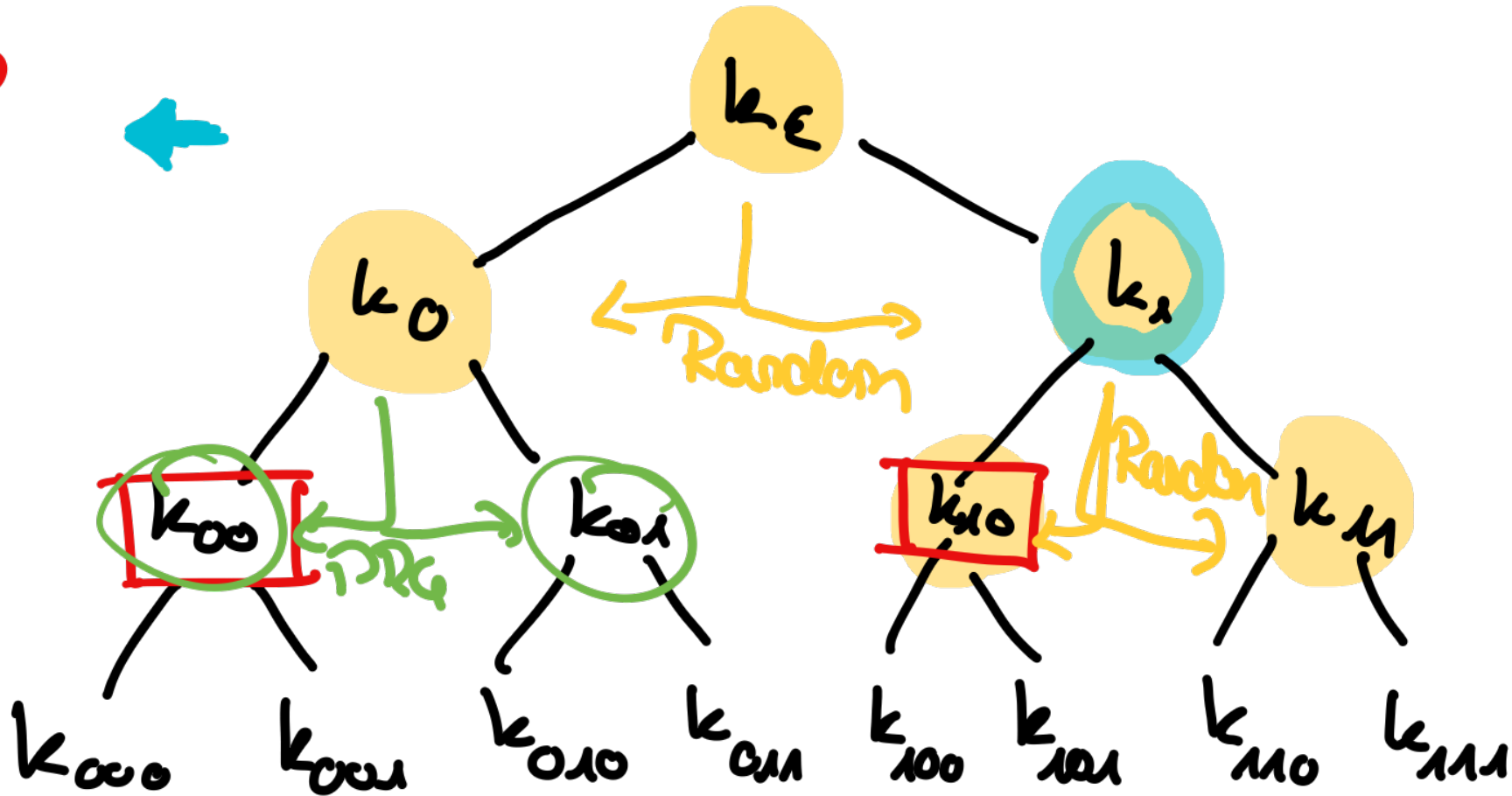
Corrupt  $k_{00}$

PRG  $\varepsilon$

PRG  $0$

Corrupt  $k_{10}$

PRG  $1$



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- ← rewinding index
- relevant index

Corrupt  $k_{00}$

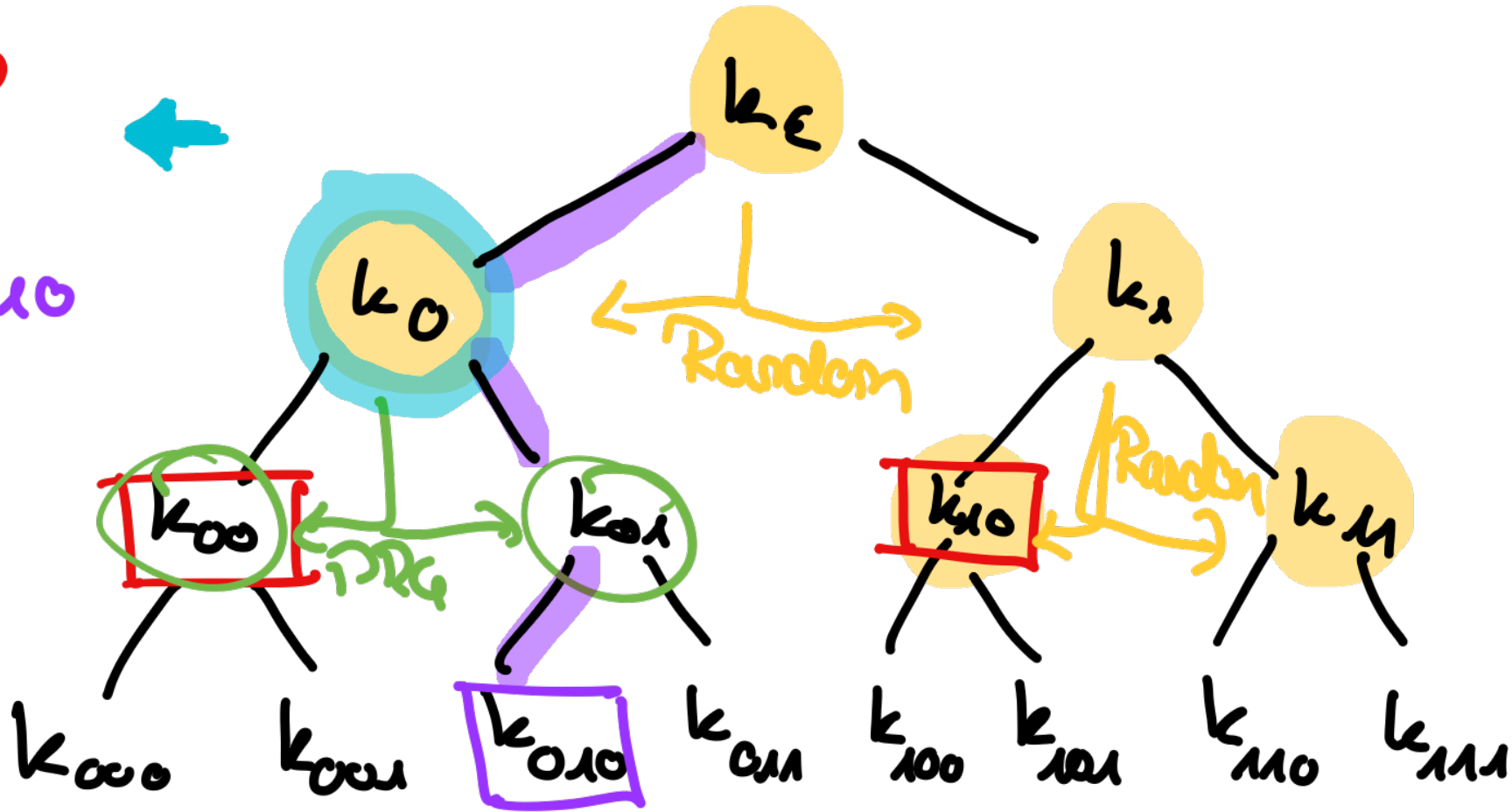
PRG  $\varepsilon$

PRG 0

Corrupt  $k_{10}$

PRG 1

Challenge  $k_{010}$



# Adversarial views

- Corruptions
- honest PRG
- random values
- ← rewinding index
- relevant index

Corrupt  $k_{00}$

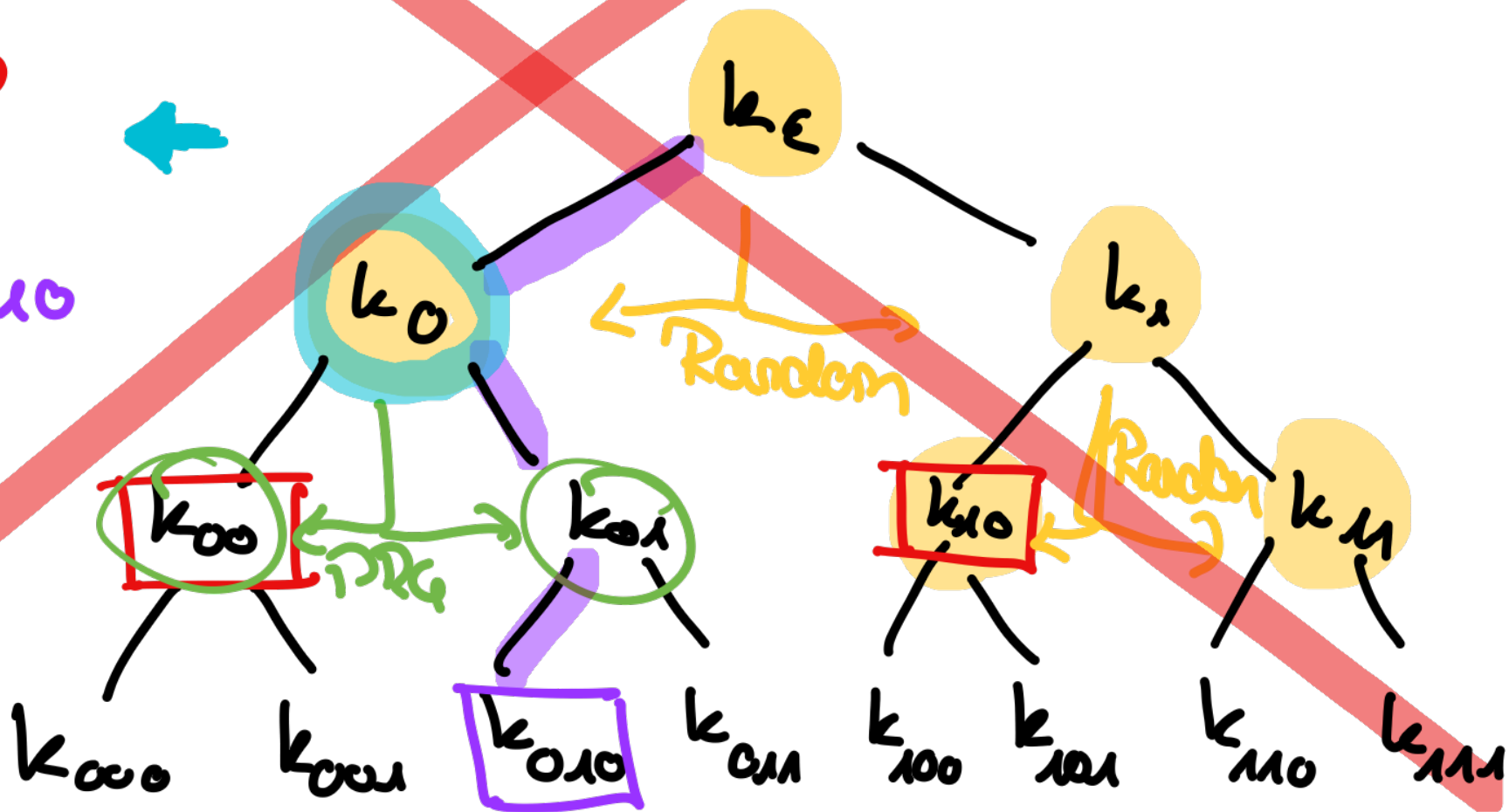
PRG  $\varepsilon$

PRG 0

Corrupt  $k_{10}$

PRG 1

challenge  $k_{010}$



# Adversarial views

- Corruptions
- honest PRG
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- ← rewinding index
- relevant index

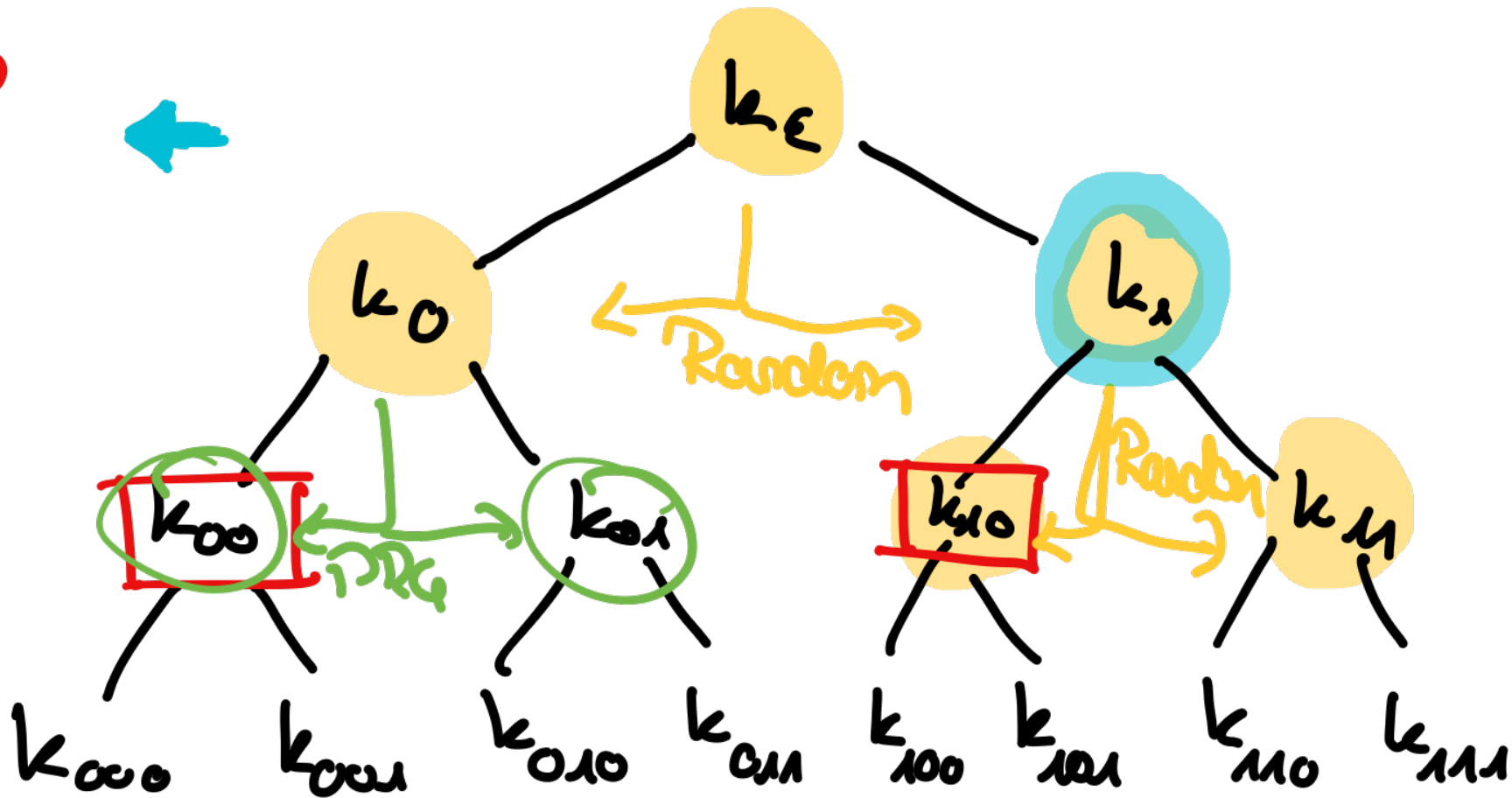
Corrupt  $k_{00}$

PRG  $\varepsilon$

PRG  $0$

Corrupt  $k_{10}$

PRG  $1$



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- honest PRG
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Corrupt  $k_{00}$

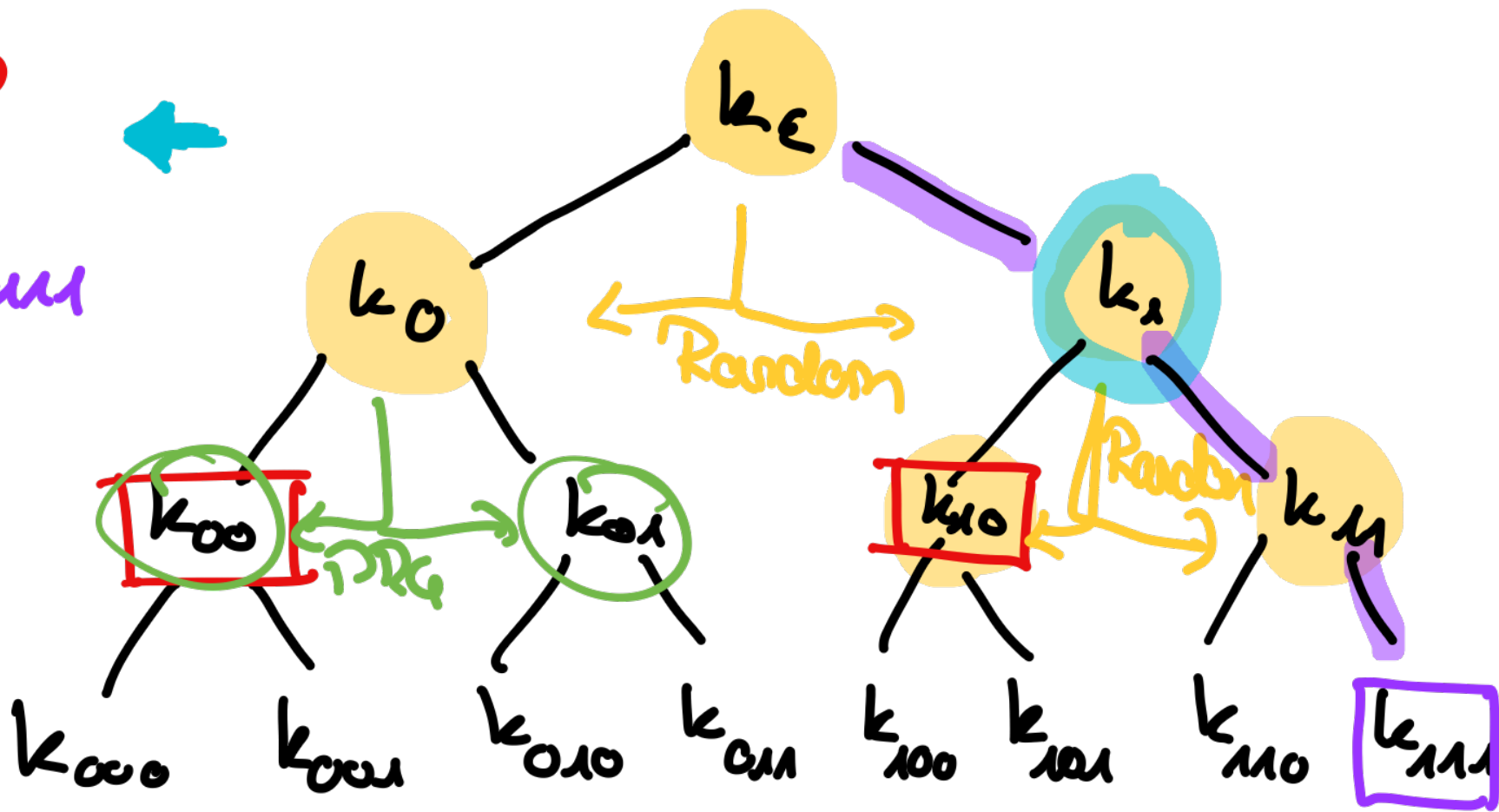
PRG  $\varepsilon$

PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

challenge  $k_{11}$



# Adversarial views

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- honest PRG
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Corrupt  $k_{00}$

PRG  $\varepsilon$

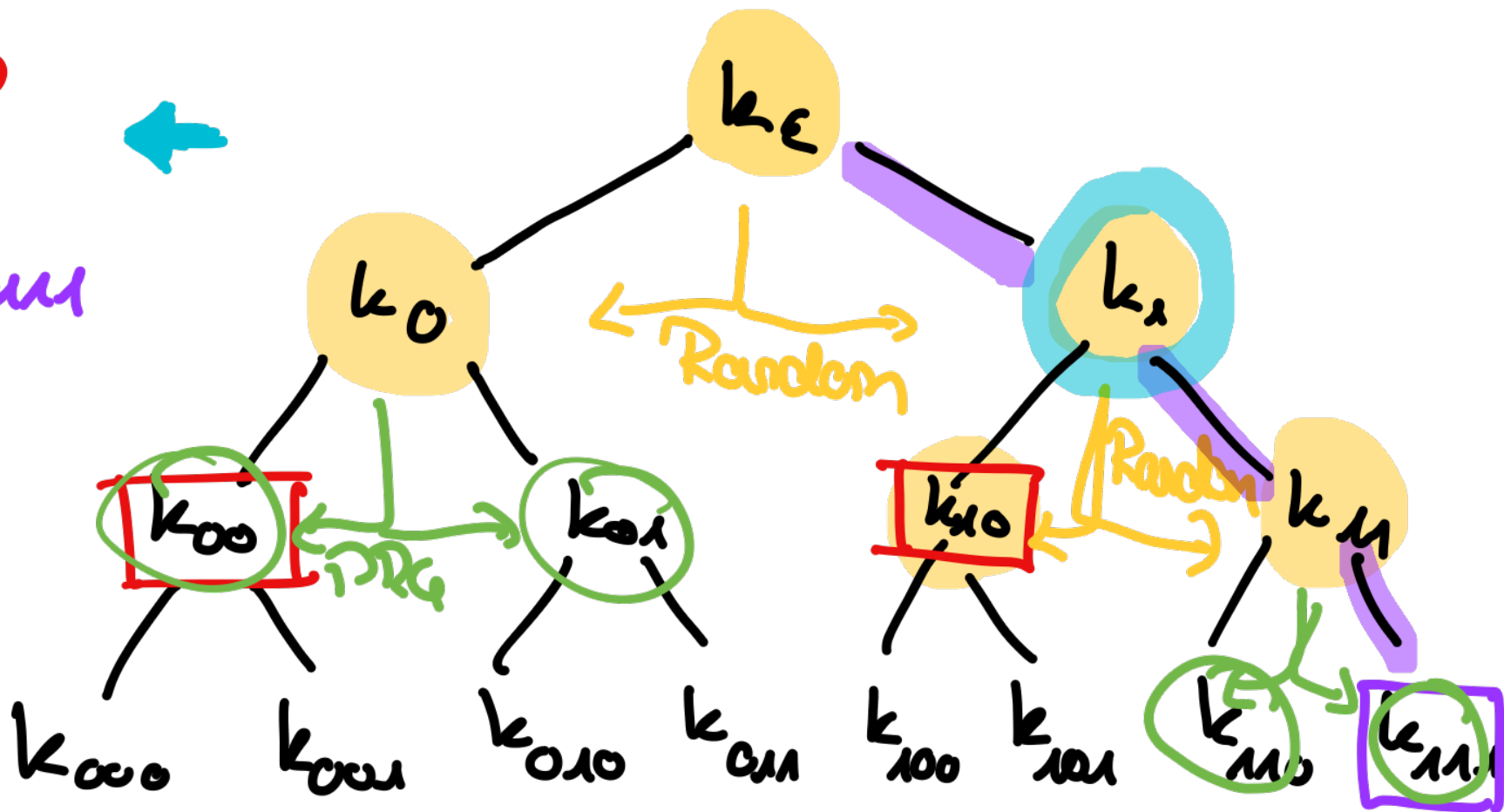
PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

Challenge  $k_{11}$

PRG  $11$





# Adversarial views

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Corrupt  $k_{00}$

PRG  $\varepsilon$

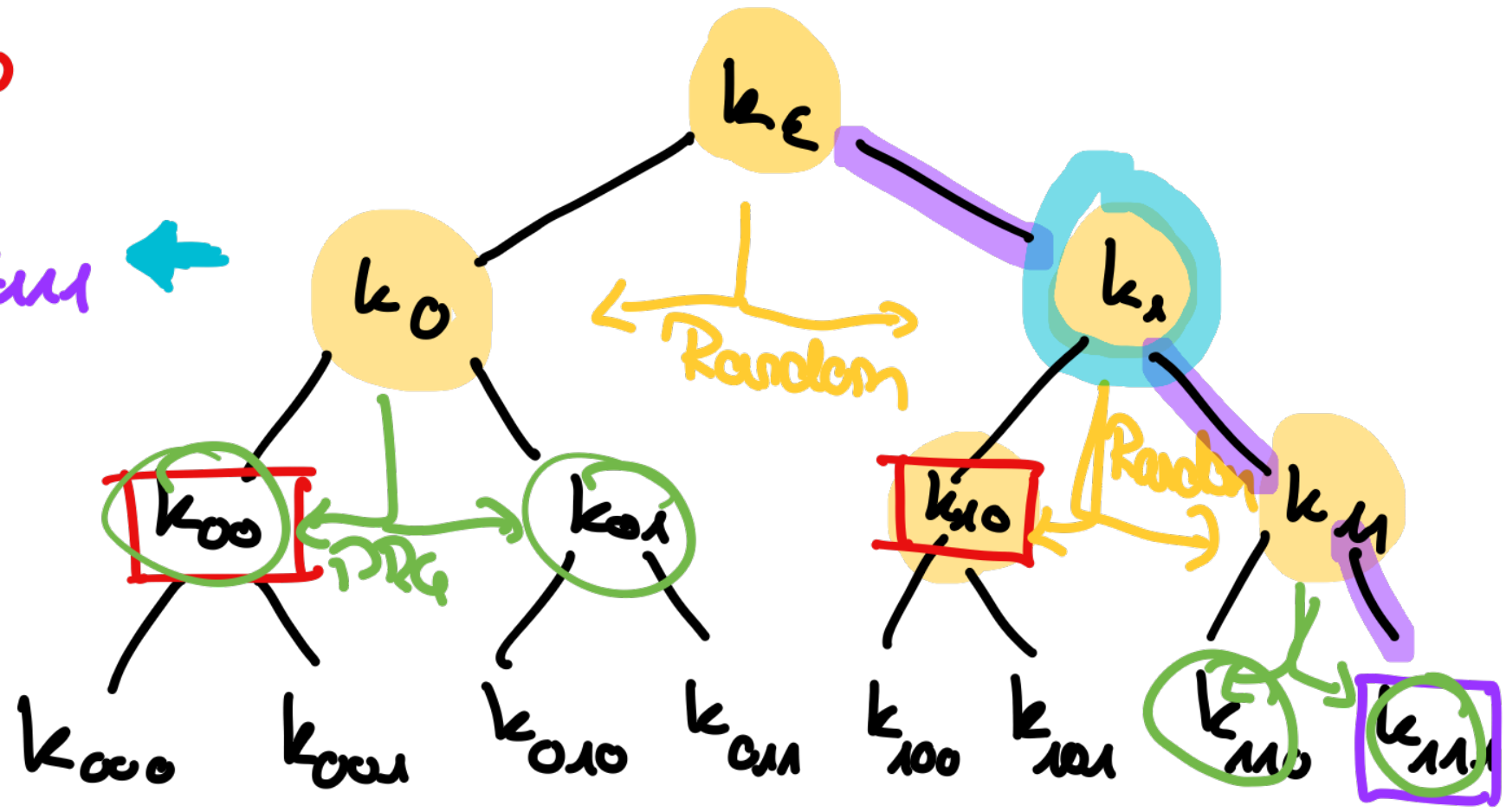
PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

Challenge  $k_{11}$

PRG  $11$



# Adversarial views

- Corruptions
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- random values
- ↔ rewinding index
- relevant index

Corrupt  $k_{00}$

PRG  $\varepsilon$

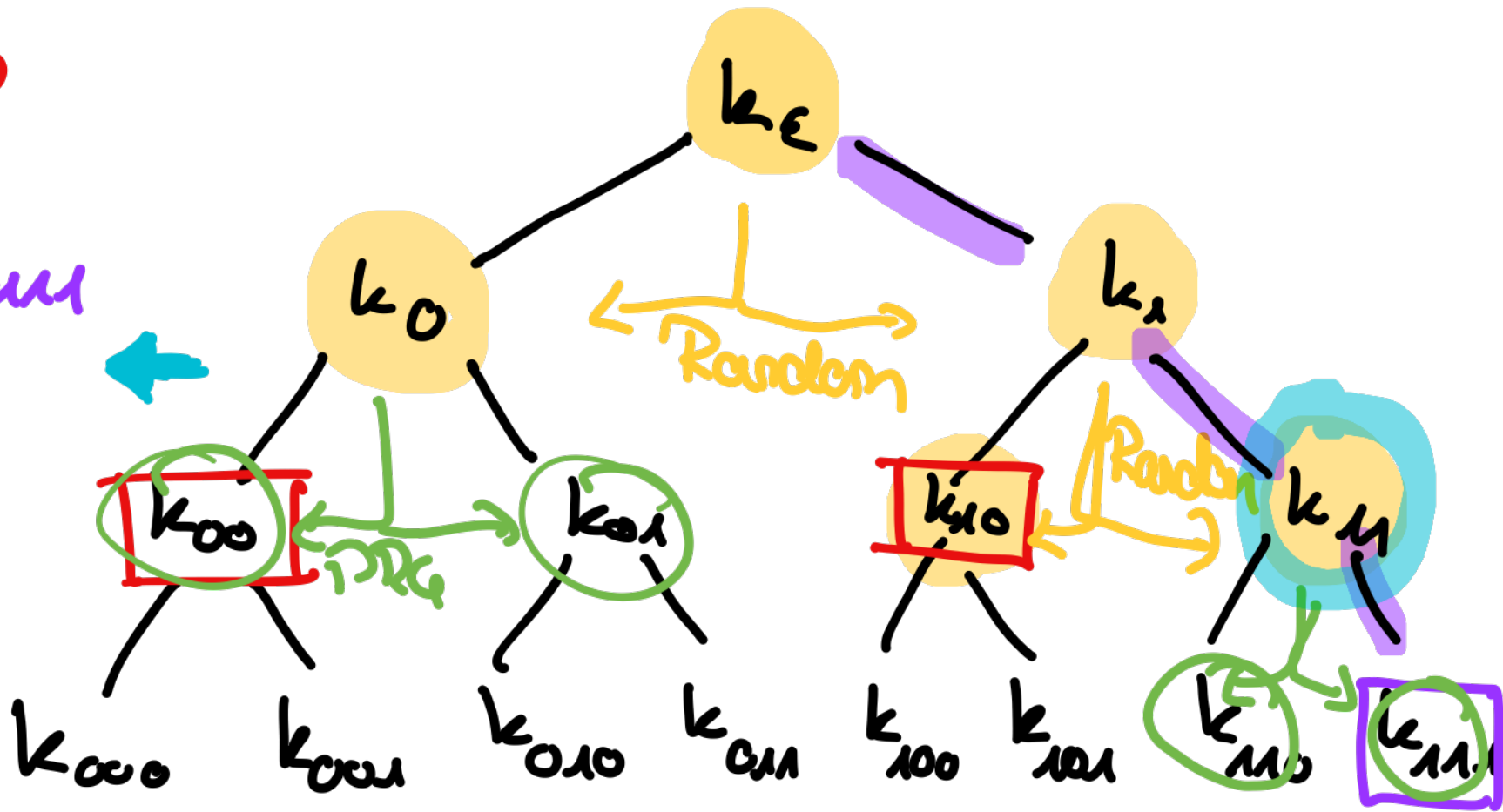
PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

Challenge  $k_{11}$

PRG  $11$



# Adversarial views

- Corruptions
- honest PRG
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- ← rewinding index
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Corrupt  $k_{00}$

PRG  $\varepsilon$

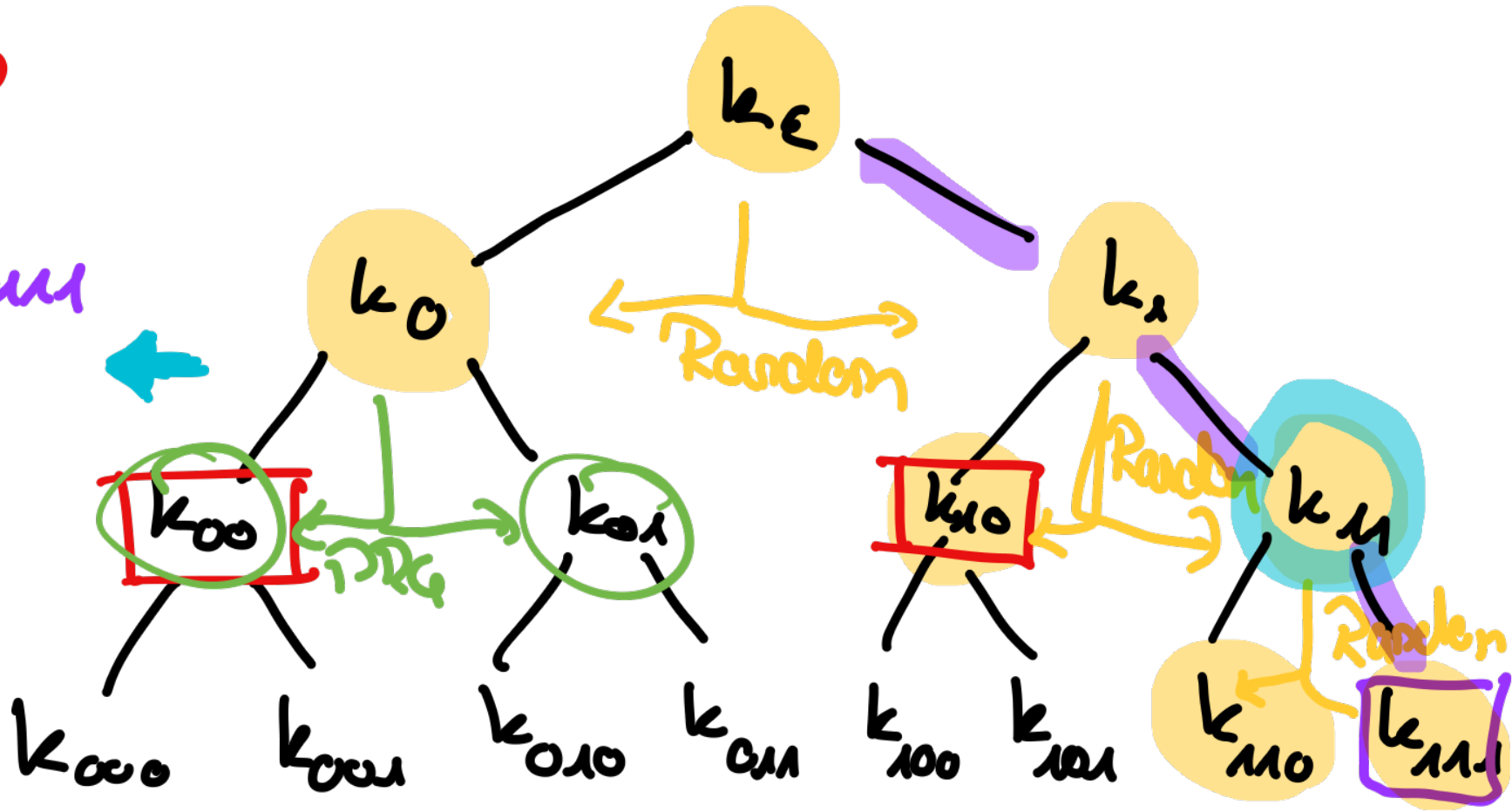
PRG  $0$

Corrupt  $k_{10}$

PRG  $1$

Challenge  $k_{11}$

PRG  $11$



# Conclusion

- undirected rewinding
- PC-PRF security of UCUM PRF with  $\tilde{\text{poly}}$  loss from PRG
- security of LKH (Multicast Encryption) with  $\tilde{\text{poly}}$  loss from IND-CPA

# Conclusion

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## Open Questions

- Other Applications?
- Generic Applicability?

# Conclusion

- undirected rewinding
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Questions?

