Simple Tests of Quantumness Also Certify Qubits

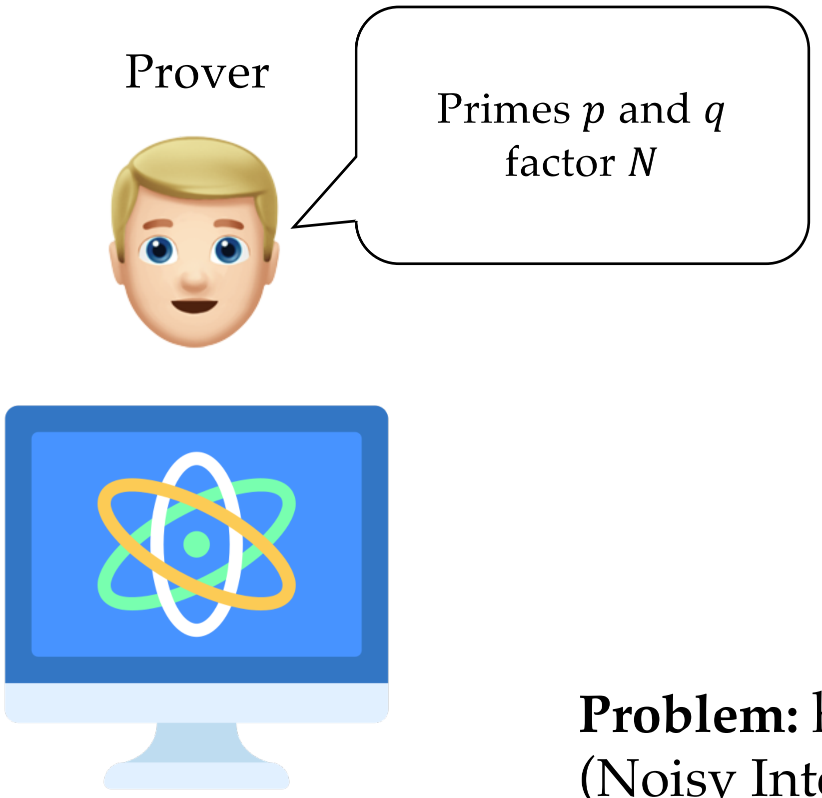
Zvika Brakerski (Weizmann Institute of Science)
Alexandru Gherghiu (Chalmers University of Technology)
Gregory D Kahanamoku-Meyer (University Of California, Berkeley)
Eitan Porat (Weizmann Institute of Science)
Thomas Vidick (Weizmann Institute of Science)

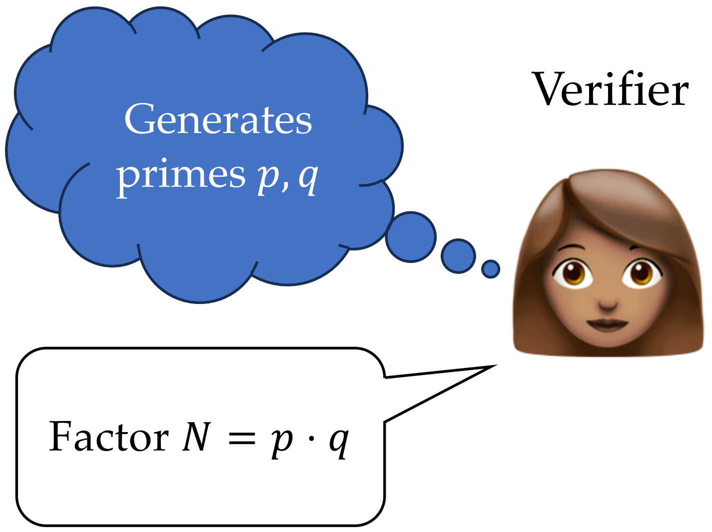
Quantum Supremacy (Test of Quantumness)

- Perform computations that outperforms classical computers.
- A need for efficiently-verifiable quantum advantage.

Google Sycamore

Example of Proof of Quantumness [Shor'94]





Problem: hard to implement on NISQ (Noisy Intermediate-Scale Quantum) Computers.

Previous Works

- BCMVV'18[1] – Proof of Quantumness + Certifiable Randomness based on LWE using **adaptive-hardcore bit**.
  - Requires an aggressive setting of parameters for LWE which hampers practical implementation.
- YZ'22[2] – Proofs of Quantumness + Certifiable Randomness in the **random oracle model**.
- Recently, two proposals of protocols in the standard model with milder computational assumptions KCVY'21[3] and KLVY'22[4].

1. A Cryptographic Test of Quantumness and Certifiable Randomness from a Single Quantum Device, Z. Brakerski, P. Christiano, U. Mahadev, U. Vazirani, T. Vidick, 2018
2. Verifiable Quantum Advantage without Structure, T. Yamakawa, M. Zhandry, 2022
3. Classically-Verifiable Quantum Advantage from a Computational Bell Test, G. Kahanamoku-Meyer, S. Choi, U. Vazirani, N. Yao.
4. Quantum Advantage from Any Non-Local Game, Y. Tauman Kalai, A. Lombardi, V. Vaikuntanathan, L. Yang

Beyond Quantum Supremacy

- Suppose we have a NISQ computer which achieves Quantum Supremacy
- Could we make it generate certifiable randomness?
- Could we delegate computation to the Quantum computer?
- Qubit Certification - a useful building block for quantum verification protocols.

Qubit Certification

- Could we verify that the quantum computer has a qubit?
- What does it mean to “have” a qubit?

Qubit Certification

- **Operational view of Qubits*:** the prover has a triplet $(|\psi\rangle, X, Z)$ where X and Z are binary observables which ”approximately anti-commute” on $|\psi\rangle$.
- Could we use existing proofs of Quantumness as tests for qubits?
- Our Answer: Yes!

* Course FSMP, Fall’20: Interactions with Quantum Devices, Thomas Vidick, 2022

Our Results

- For a specific class of protocols, we show:
  - A quantum soundness barrier against quantum cheating provers (vs classical soundness).
  - Provers that approach the quantum soundness barrier *must perform anti-commuting measurements* (a qubit test).
- NZ'23 show related results for the KLVY'22 protocol. Prove how it can be used to get a protocol *for delegation of quantum computation*.

Our Protocol Template

Prover

$|\psi_{\texttt{trans}}\rangle$
quantum state

PHASE 1

⋮

$\texttt{flag} \in \{\texttt{acc, rej, cont}\}$

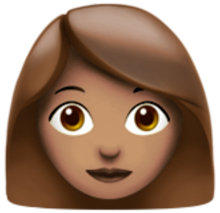
if $\texttt{flag = cont:}$

$m \in \{0,1\}$ *challenge*

PHASE 2

$b \in \{0,1\}$ *response*

Verifier



 $\texttt{rand}$

$\texttt{trans}$ *transcript of the protocol*

verifier accepts if $b = b_m^{\texttt{correct}}(\texttt{rand, trans})$

Soundness for classical provers – Sketch

- **Parity hardness:** prove that it is hard for classical provers to compute $b_0^{\text{correct}} \oplus b_1^{\text{correct}}$ w.p. $\geq \frac{1}{2} + \varepsilon$

- **Reduce soundness to parity hardness:**
  Assume adversary succeeds w.p. $\geq \frac{3}{4} + \varepsilon$.

  Run Phase 1 of the protocol.

$m = 0$ *challenge*

b_0 *response*

$m = 1$ *challenge*

b_1 *response*

$$b_0 \oplus b_1 \approx b_0^{\text{correct}} \oplus b_1^{\text{correct}}$$

Computing Parity in the Quantum World

- **Problem:** Quantum computers cannot perform rewinding…
- Could they somehow compute the parity with some noticeable advantage?

Modeling Quantum Provers

- For each $m \in \{0,1\}$ (challenge bit) the prover performs a set projective measurement on its state $|\psi_{\texttt{trans}}\rangle$

$$\{\Pi_b^m\}$$

Challenge bit

Response bit

Parity Algorithm

(Algorithm $\mathcal{A}_1$)

- Execute Phase 1 of the protocol template to obtain (trans, $|\psi_{\texttt{trans}}\rangle$)

(Algorithm $\mathcal{A}_2$)

- b_0 = measurement of $\mathcal{H}_P$ using $\{\Pi_0^0, \Pi_1^0\}$.
- b_1 = measurement of $\mathcal{H}_P$ using $\{\Pi_0^1, \Pi_1^1\}$.
- Return $b_0 \oplus b_1$.

Soundness for quantum provers - sketch

- Prove that it is hard (**quantum**) to compute the parity of both challenges: $b_0^{\text{correct}} \oplus b_1^{\text{correct}}$
- **Quantum Analogue:** Show that a quantum adversary that achieves $\cos^2\left(\frac{\pi}{8}\right) + \varepsilon$ success probability, using the parity algorithm can compute parities.

Parity Hardness → Quantum Soundness

No classical (quantum) polynomial time algorithm **guesses** $\hat{b}_0 \oplus \hat{b}_1$ with non-negligible advantage

→

Then no classical (quantum) polynomial-time prover **succeeds in the protocol template** with probability larger than 75% (resp. $\cos^2(\pi/8)$ $\approx$ 85%) by more than a negligible amount

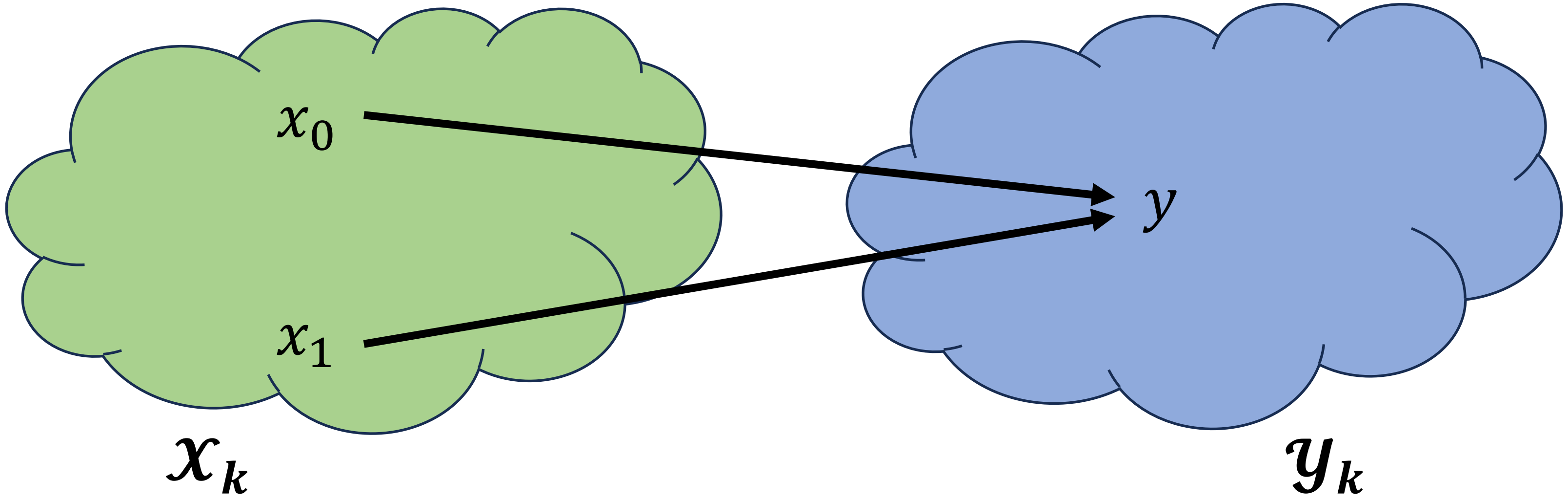
Qubit Test

- The quantum soundness result gives us a qubit test
- If a prover approaches the soundness barrier, then the measurements the prover performs must be close to anti-commuting

Example: KCVY Protocol (modified)

Trapdoor claw-free functions

- Keyed functions $f_k\colon \mathcal{X}_k \to \mathcal{Y}_k$ with trapdoor t_k



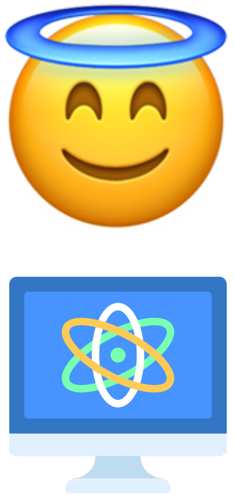
- **Hard (quantum)** to find a **claw** (x_0, x_1) such that $f_k(x_0) = f_k(x_1)$
- Given trapdoor t_k, for each y easy to find $f_k(x_0) = f_k(x_1) = y$

Trapdoor claw-free functions – cntd.

- Efficiently generate superposition
$$\frac{1}{\sqrt{|\mathcal{X}_k|}}\sum_{x}|x\rangle|f_k(x)\rangle$$
- Efficiently distinguish between the preimages x_0 and x_1

KCVY Protocol - Simplified

Honest Prover



PHASE 1

1. Generates
   $\sum_x |x\rangle_{\mathcal{X}} |f_k(x)\rangle_{\mathcal{Y}}$

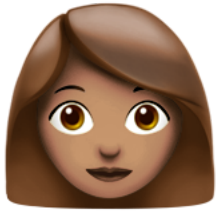
2. Measures $\mathcal{Y}$ register
   $(|x_0\rangle_{\mathcal{X}} + |x_1\rangle_{\mathcal{X}})|y\rangle_{\mathcal{Y}}$

3. Sends y to the verifier.

Generates k, t_k for a TCF

Verifier

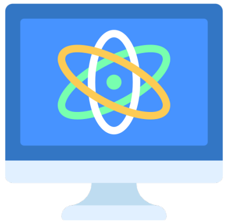
k = Key for TCF



KCVY Protocol - Simplified

Honest Prover

PHASE 1



1. Computes ancilla bit

   $$|0\rangle|x_0\rangle_{\mathcal{X}} + |1\rangle|x_1\rangle_{\mathcal{X}}$$

2. Using ancilla

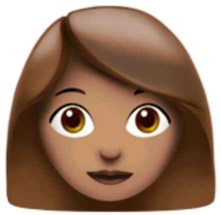
   $$|0\rangle|x_0\rangle_{\mathcal{X}}|r_0 \cdot x_0\rangle \quad + |1\rangle|x_1\rangle_{\mathcal{X}}|r_1 \cdot x_1\rangle$$

3. Uncomputes ancilla

   $$|x_0\rangle_{\mathcal{X}}|r_0 \cdot x_0\rangle + |x_1\rangle_{\mathcal{X}}|r_1 \cdot x_1\rangle$$

Generates k, t_k for a TCF

Verifier



k = Key for TCF

$r_0, r_1 \leftarrow_R \{0,1\}^n$

KCVY Protocol - Simplified

Generates

k, t_k for a TCF

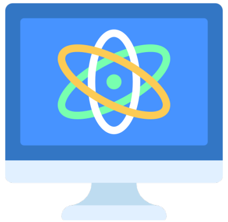
Verifier

Honest Prover

PHASE 1

3. Computes Hadamard on $\mathcal{X}$ register

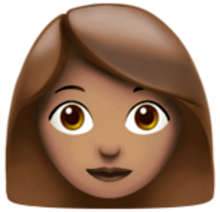
$$\sum_d |d\rangle_{\mathcal{X}} \left((-1)^{d\cdot x_0} |r_0 \cdot x_0\rangle + (-1)^{d\cdot x_1} |r_1 \cdot x_1\rangle \right)$$



4. Measures $\mathcal{X}$ register

$$|d\rangle_{\mathcal{X}} \left((-1)^{d\cdot x_0} |r_0 \cdot x_0\rangle + (-1)^{d\cdot x_1} |r_1 \cdot x_1\rangle \right)$$

5. Sends d to the verifier



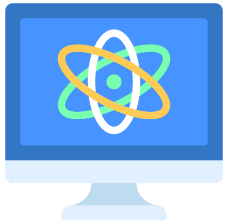
k = Key for TCF

$r_0, r_1 \leftarrow_R \{0,1\}^n$

KCVY Protocol - Simplified

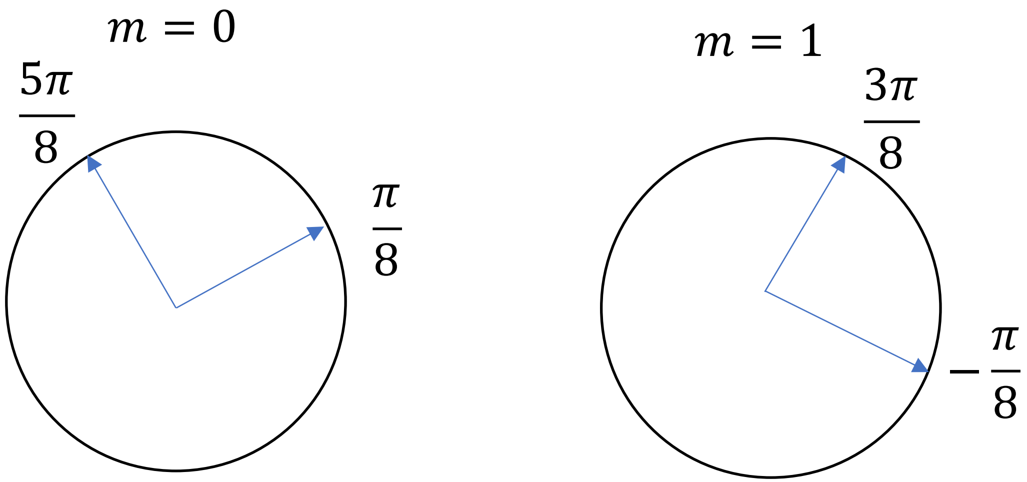
PHASE 2

Honest Prover



Holds the state

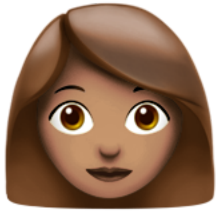
$$|\psi\rangle = |r_0 \cdot x_0\rangle + (-1)^{d\cdot(x_0 \oplus x_1)}|r_1 \cdot x_1\rangle$$



Sends b the outcome of the measurement.

Generates k, t_k for a TCF

Verifier



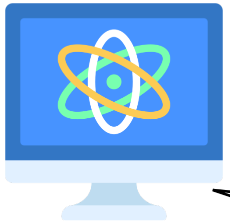
k = Key for TCF

$r_0, r_1 \leftarrow_R \{0,1\}^n$

$m \in \{0,1\}$

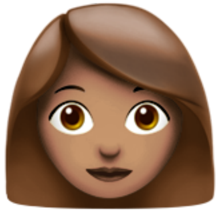
KCVY Protocol - Simplified

Honest Prover



PHASE 1

Verifier



Generates k, t_k for a TCF

k = Key for TCF

$y \in \mathcal{Y}_k$

Generate claw and measure y

$r_0, r_1 \leftarrow_R \{0,1\}^n$

$d \in \{0,1\}^n$

Multiply by r_0, r_1
& Perform Hadamard measurement

PHASE 2

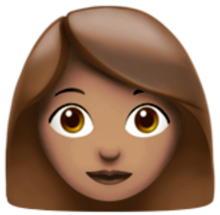
$m \in \{0,1\}$

$b \in \{0,1\}$

Challenge-Response

KCVY Protocol - Simplified

Verifier



Accept if b is the "expected" measurement outcome

Using trapdoor t_k can find x_0 and x_1

Computes $b_m^{\text{correct}}(x_0, x_1, r_0, r_1, d)$

Accepts if $b = b_m^{\text{correct}}$

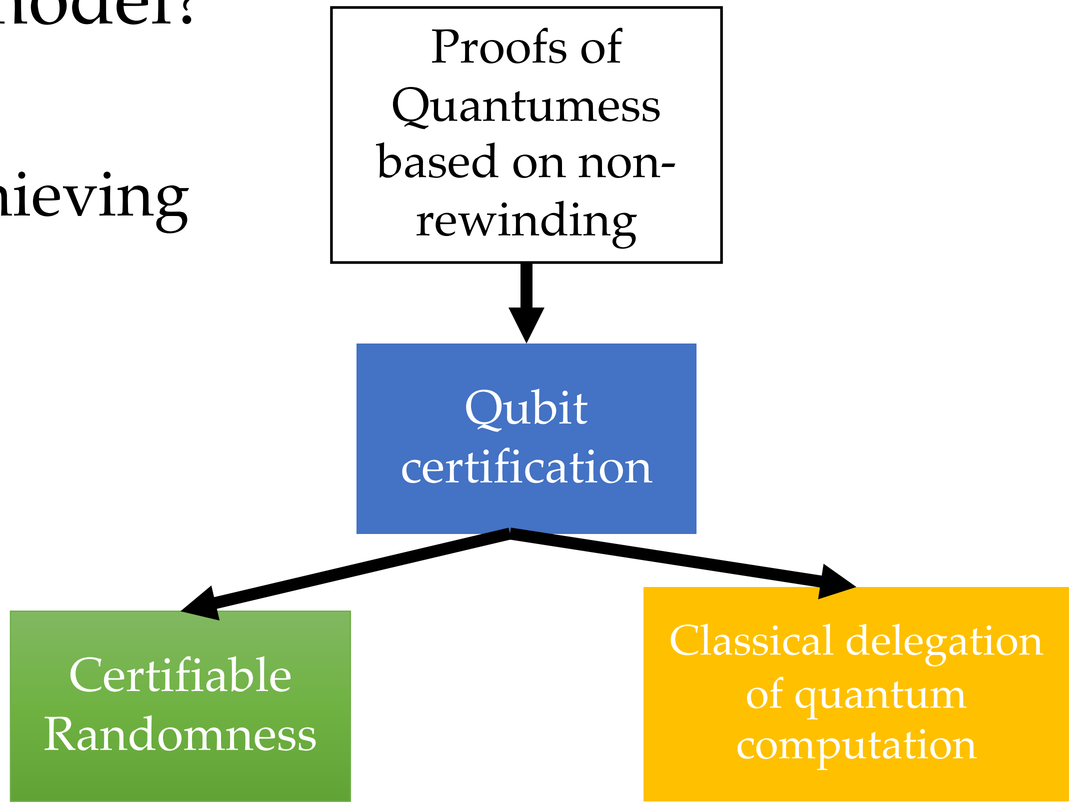
Post-Quantum TCF → Hardness of parity

Post-Quantum Trapdoor claw-free functions $\xrightarrow{\textit{Quantum Goldreich-Levin}^{*}}$ Hardness of Computing Parity

* A quantum Goldreich-Levin theorem with cryptographic applications, Mark Adcock, Richard Cleve, 2002

Open Questions

- Could we generalize our approach to the tests of quantumness in BCMVV'18 and the ones that operate in the random oracle model?
- A hierarchy of "capabilities"
  - What is the minimal basis for achieving these capabilities?



Q & A