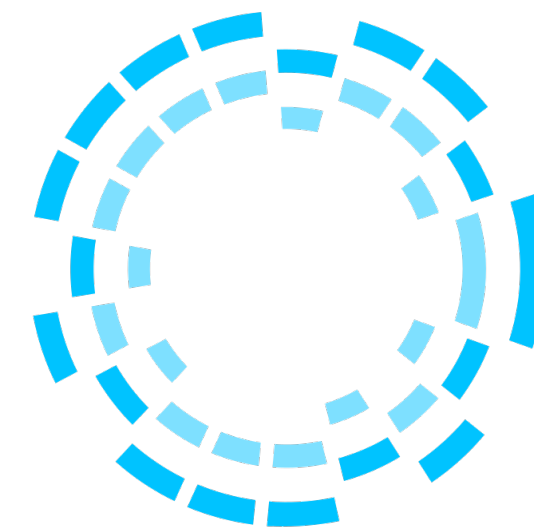


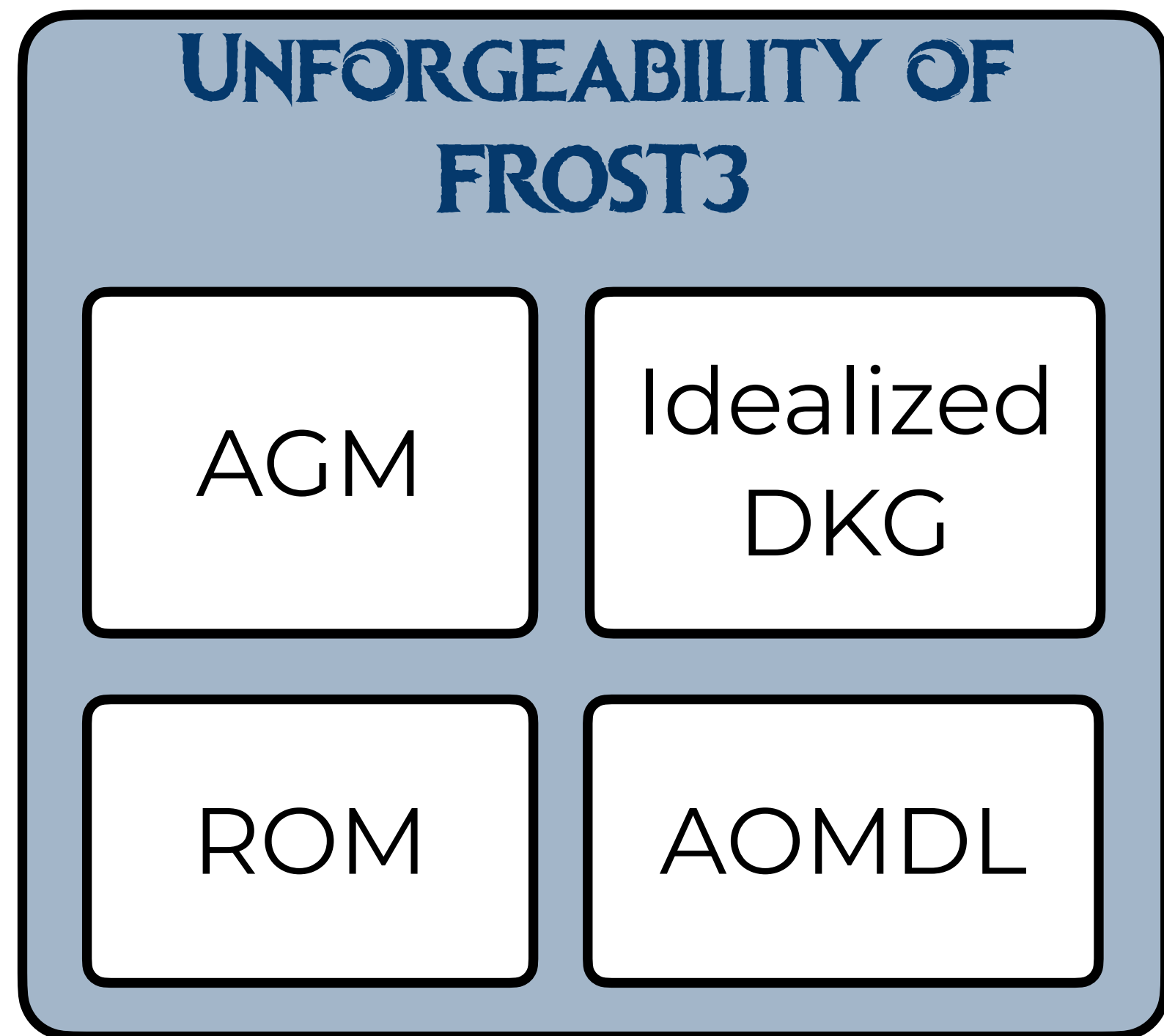
Practical Schnorr Threshold Signatures Without the Algebraic Group Model

Hien Chu¹, Paul Gerhart¹, Tim Ruffing², and Dominique Schröder¹

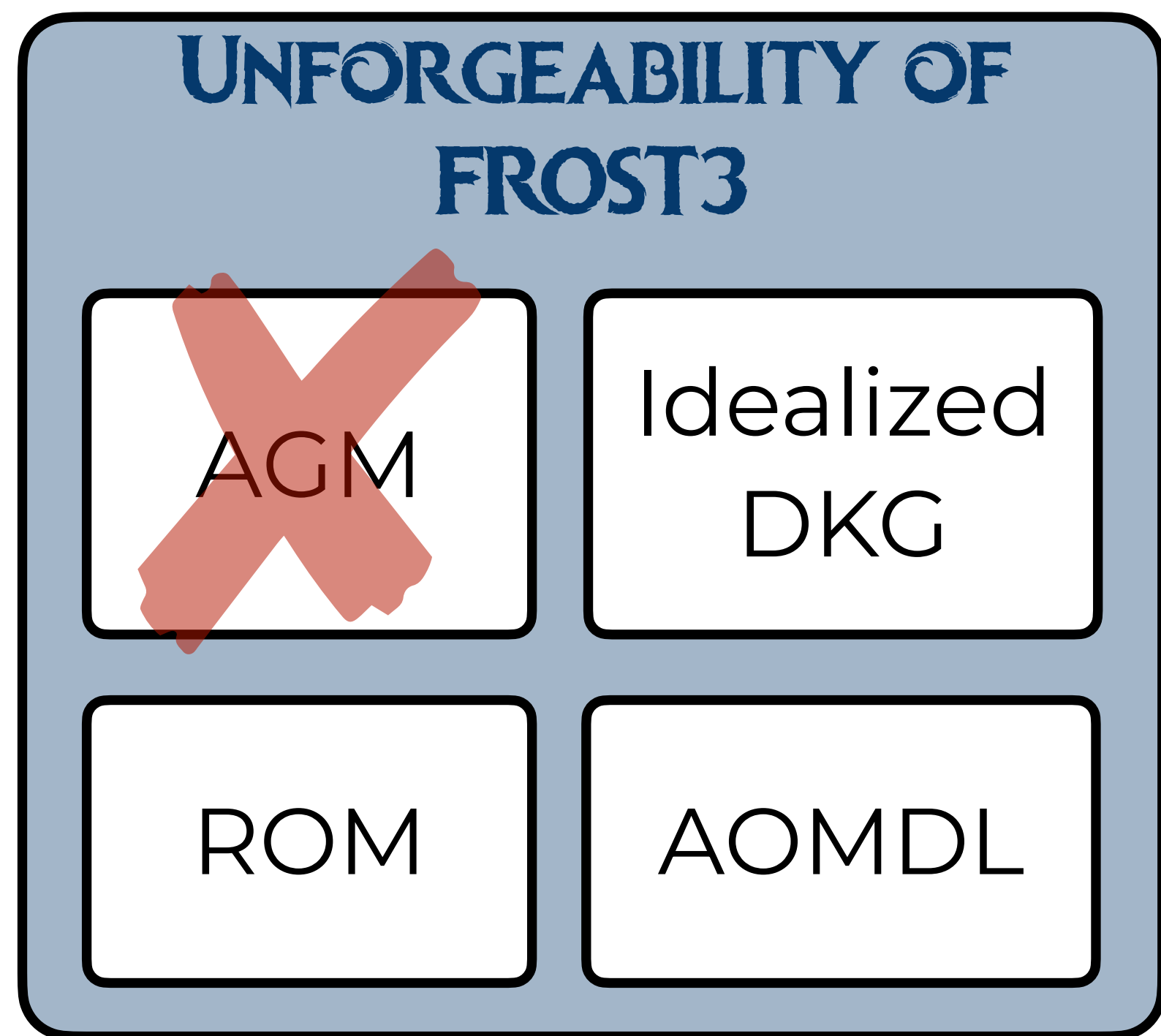


Blockstream²

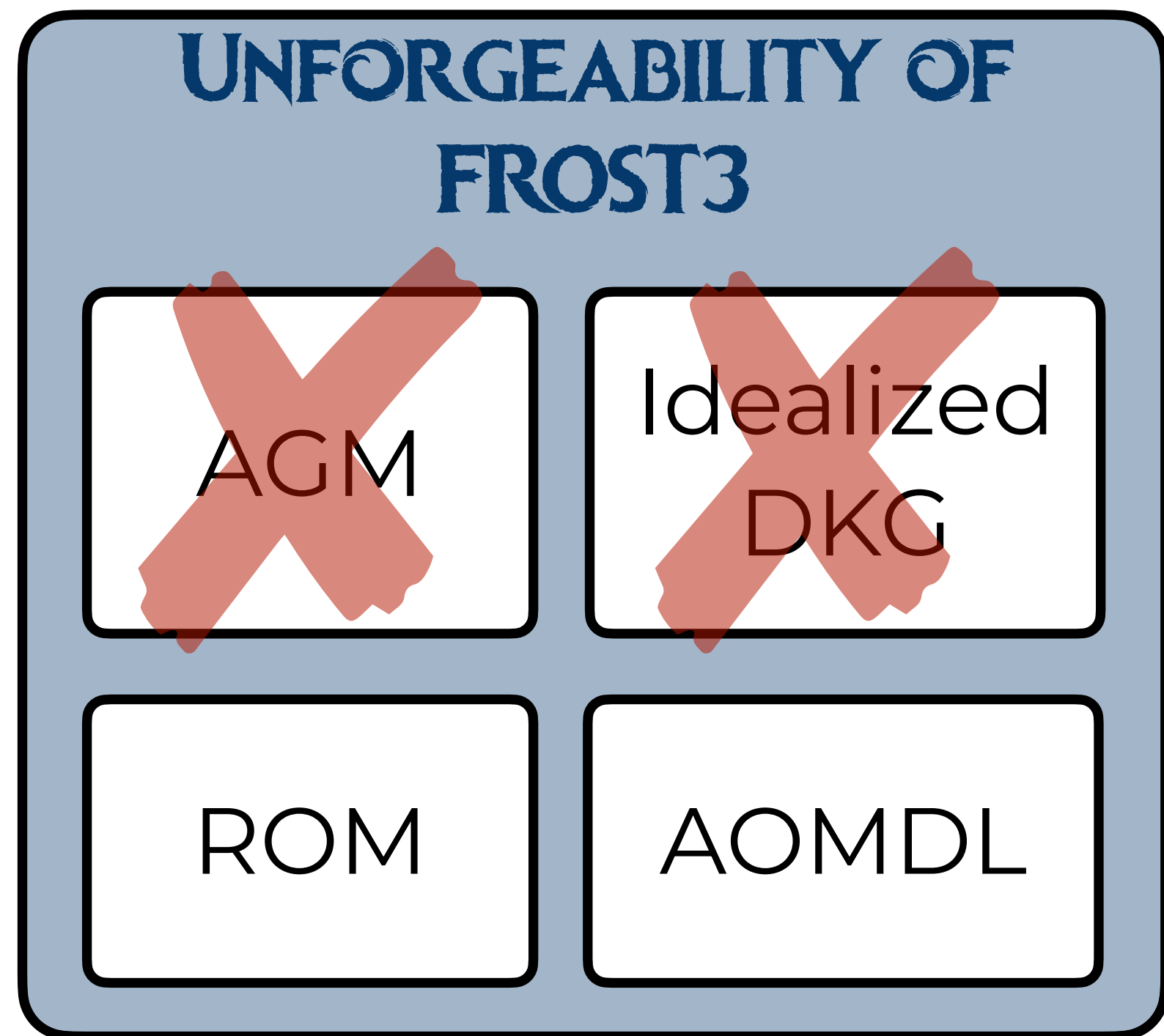
Our Contributions



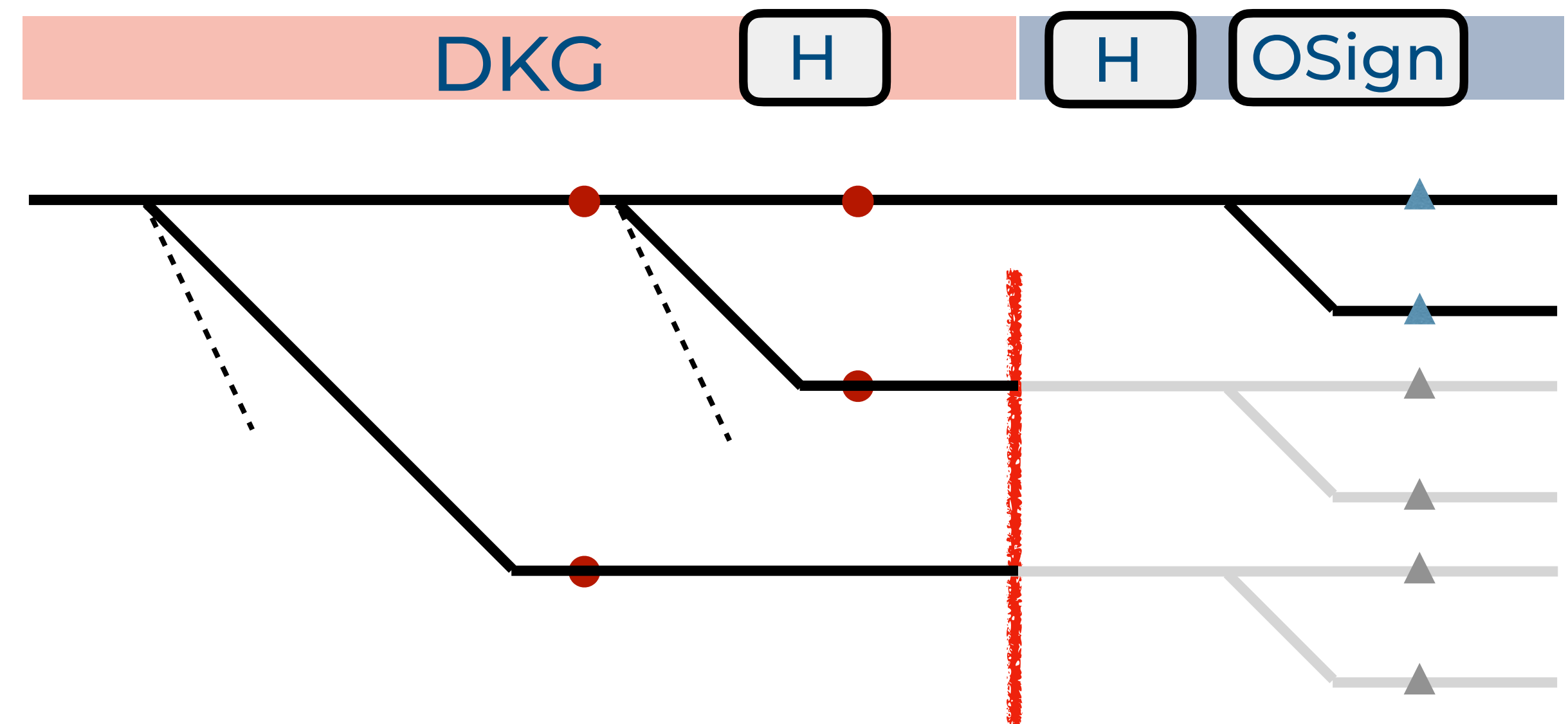
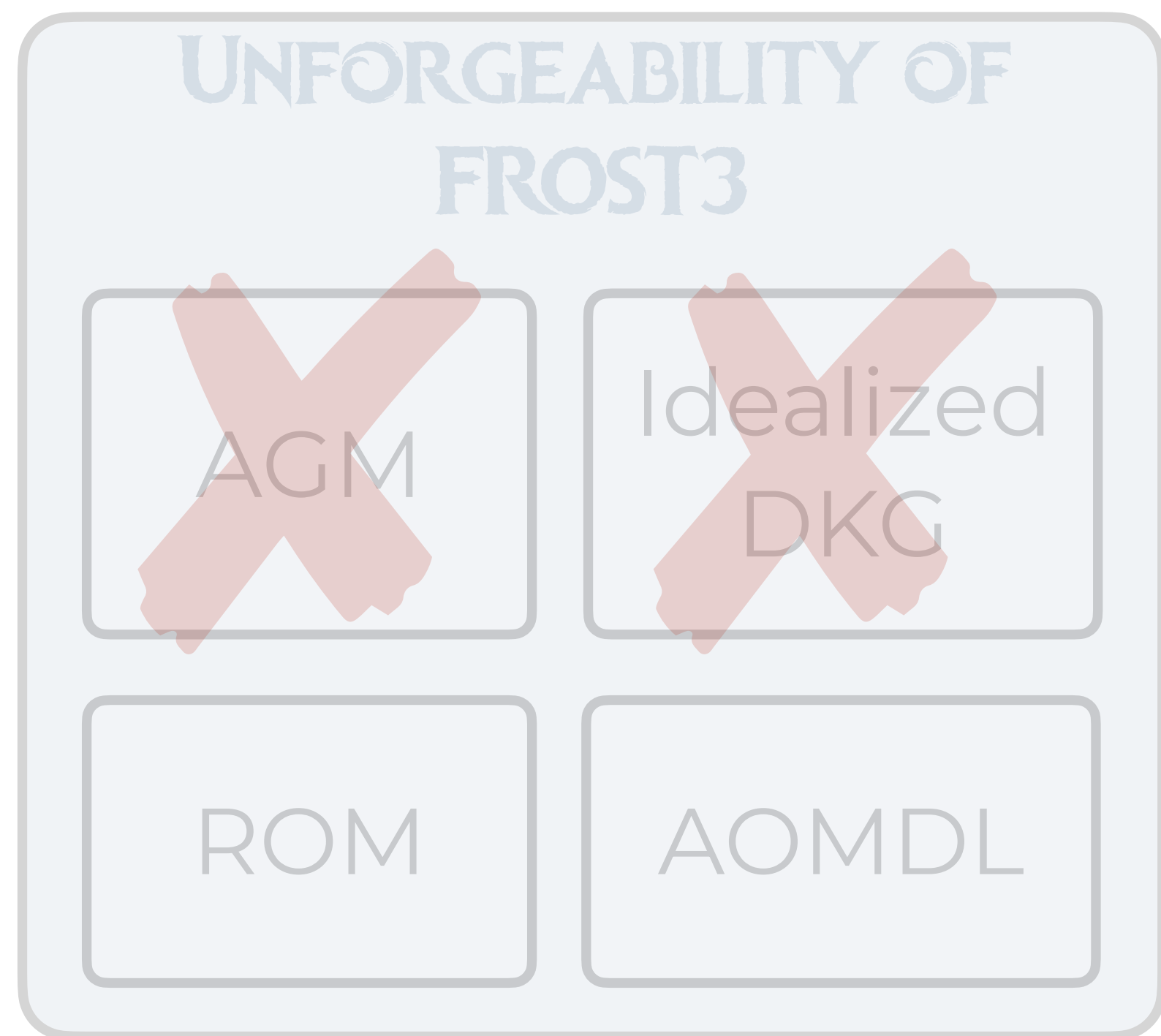
Our Contributions



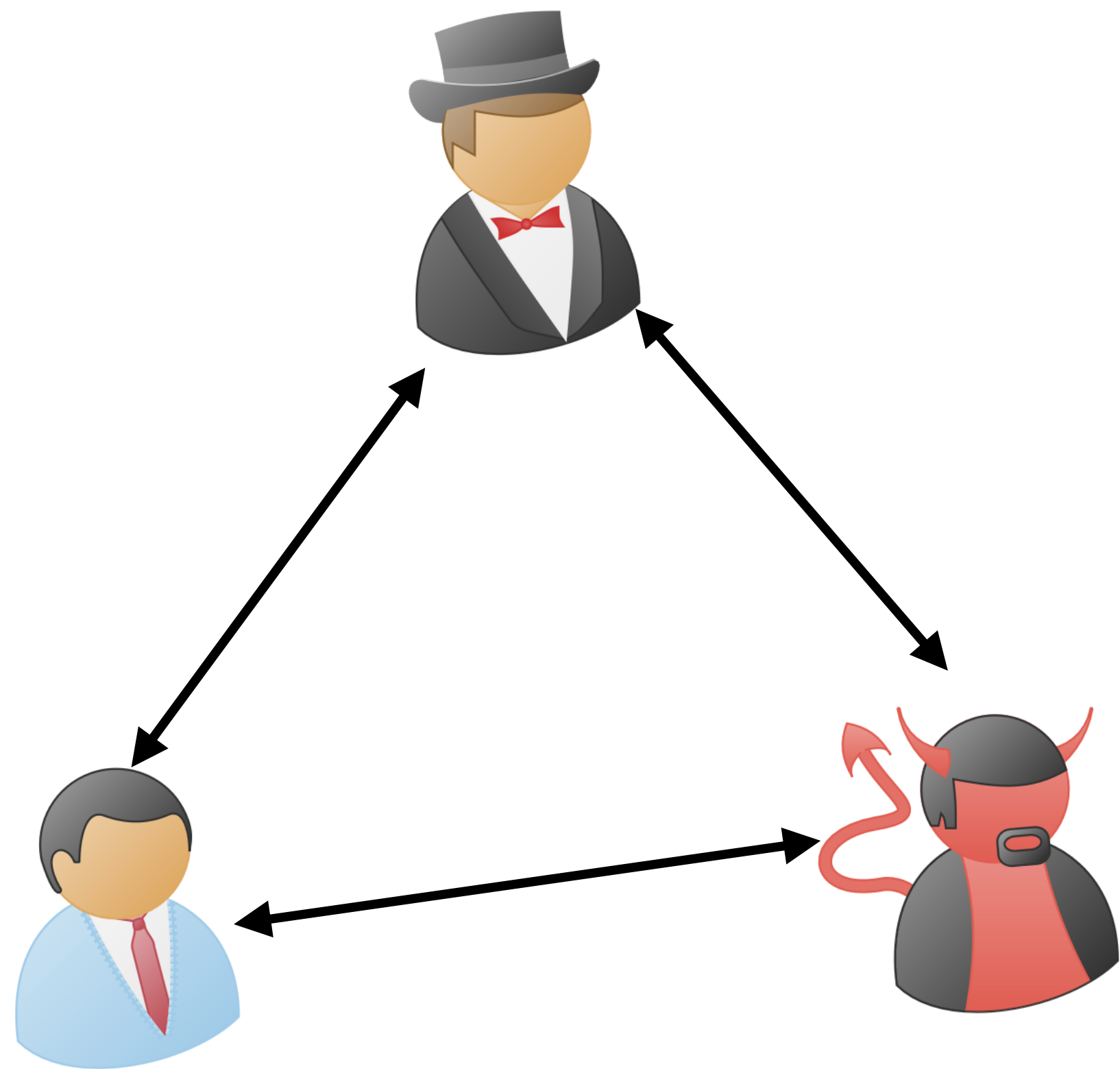
Our Contributions



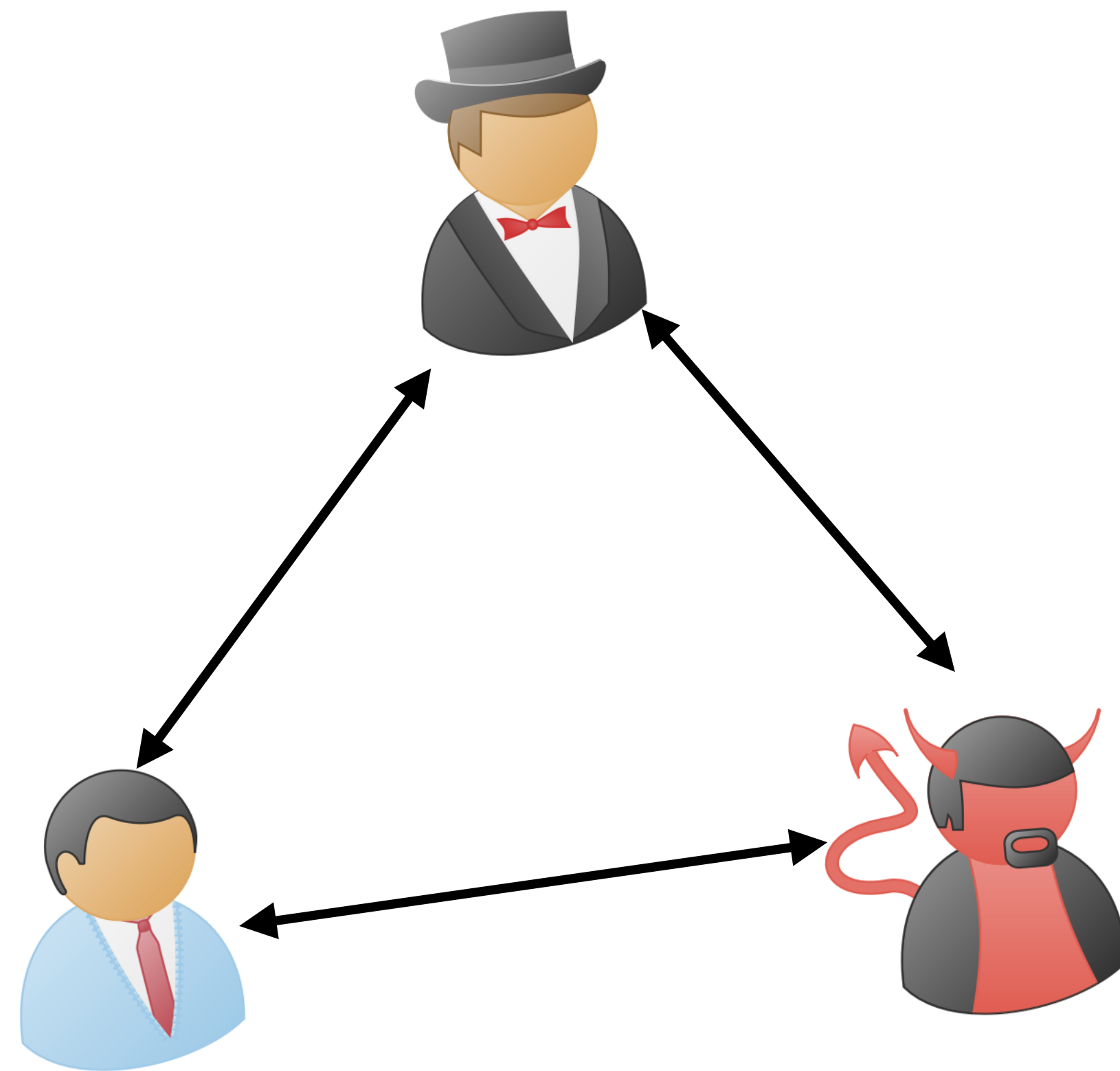
Our Contributions



Threshold Signatures [DES88, DF90]

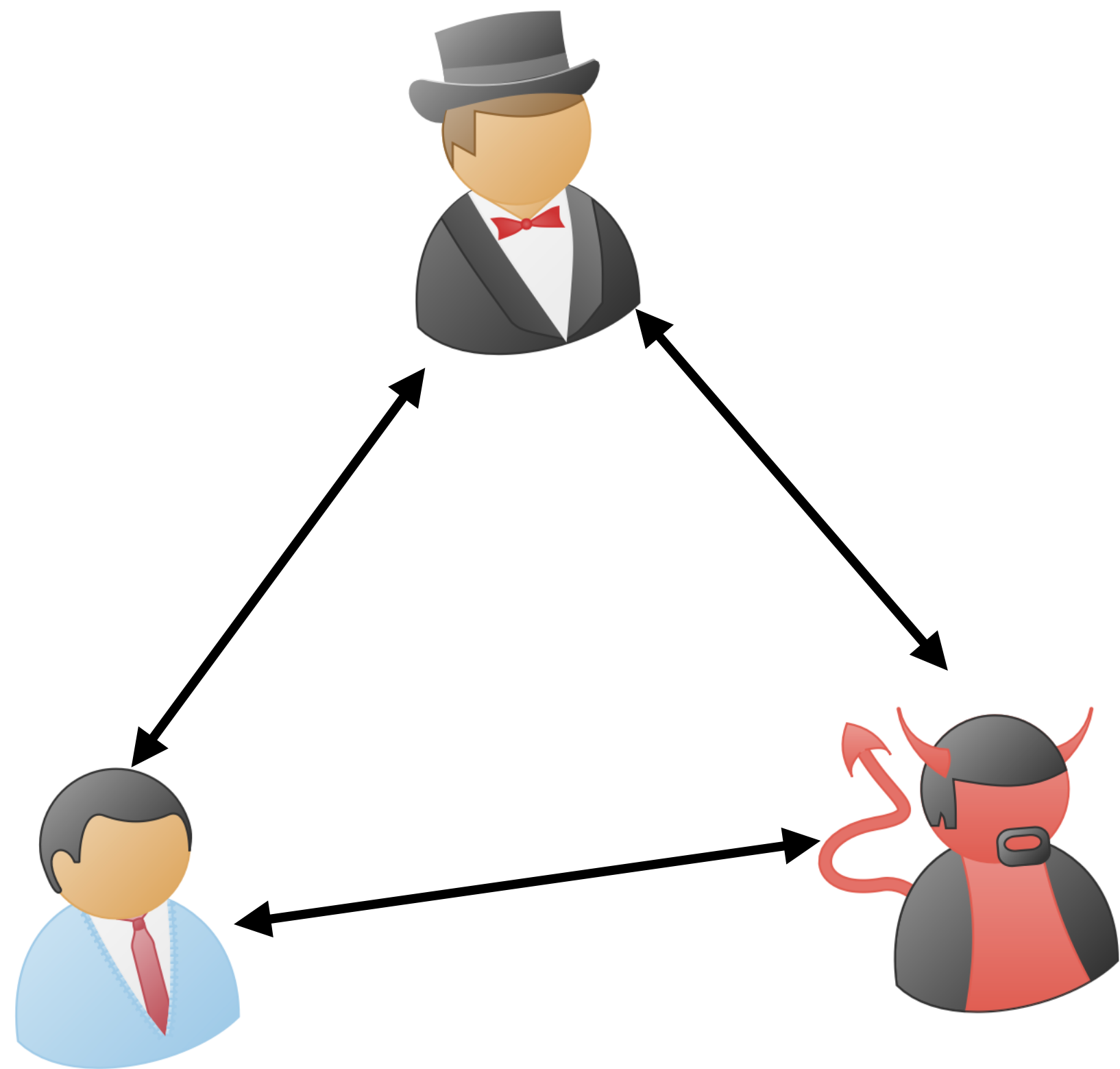


Threshold Signatures [DES88, DF90]



$m =$ LET'S PROVE FROST3

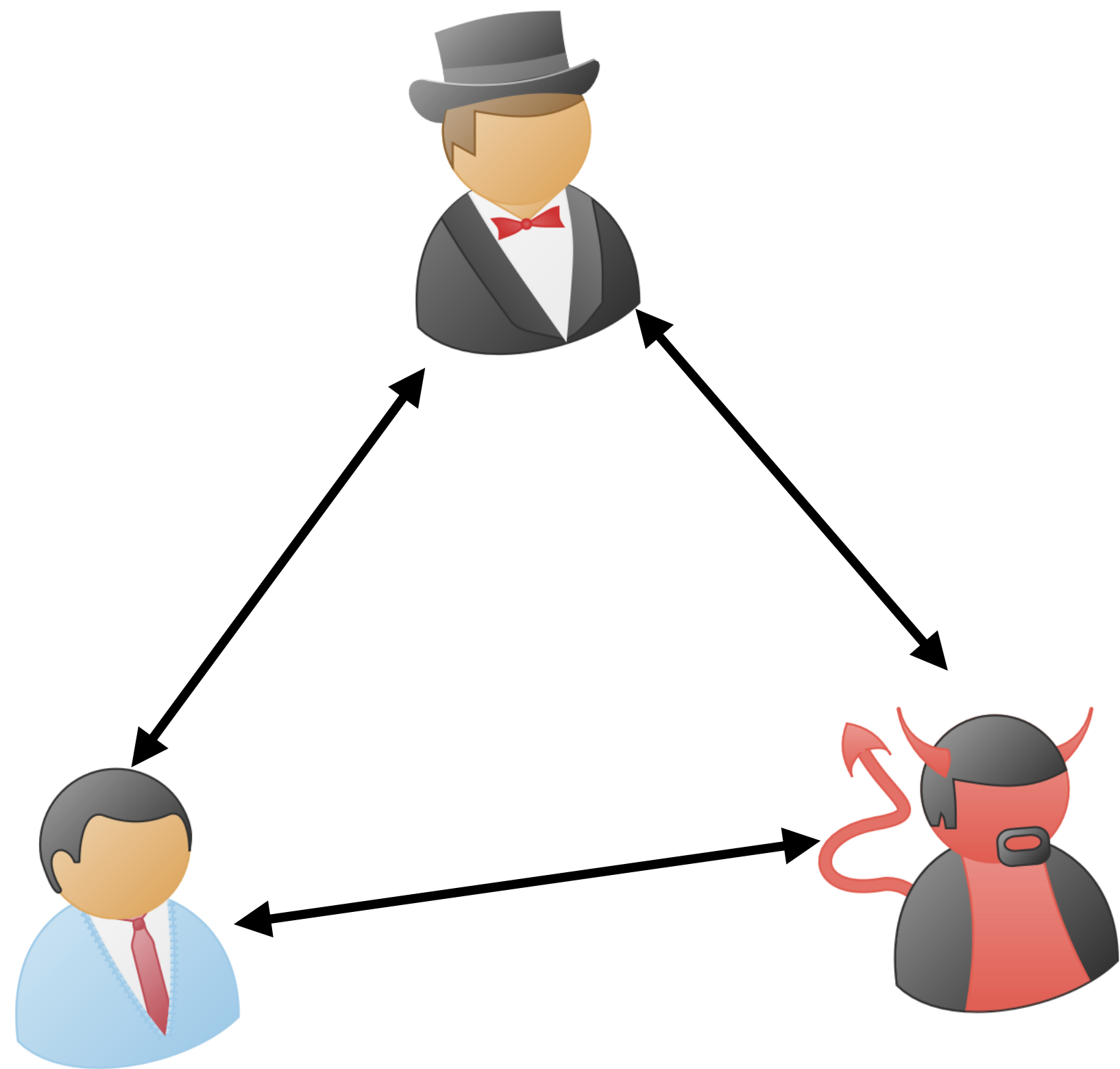
Threshold Signatures [DES88, DF90]



$m = \text{LET'S PROVE FROST3}$

$$(\sigma_1, \sigma_2, \sigma_3) \xrightarrow{\text{Aggregate}} \sigma = (R, s)$$

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$(\sigma_1, \sigma_2, \sigma_3) \xrightarrow{\text{Aggregate}} \sigma = (R, s)$

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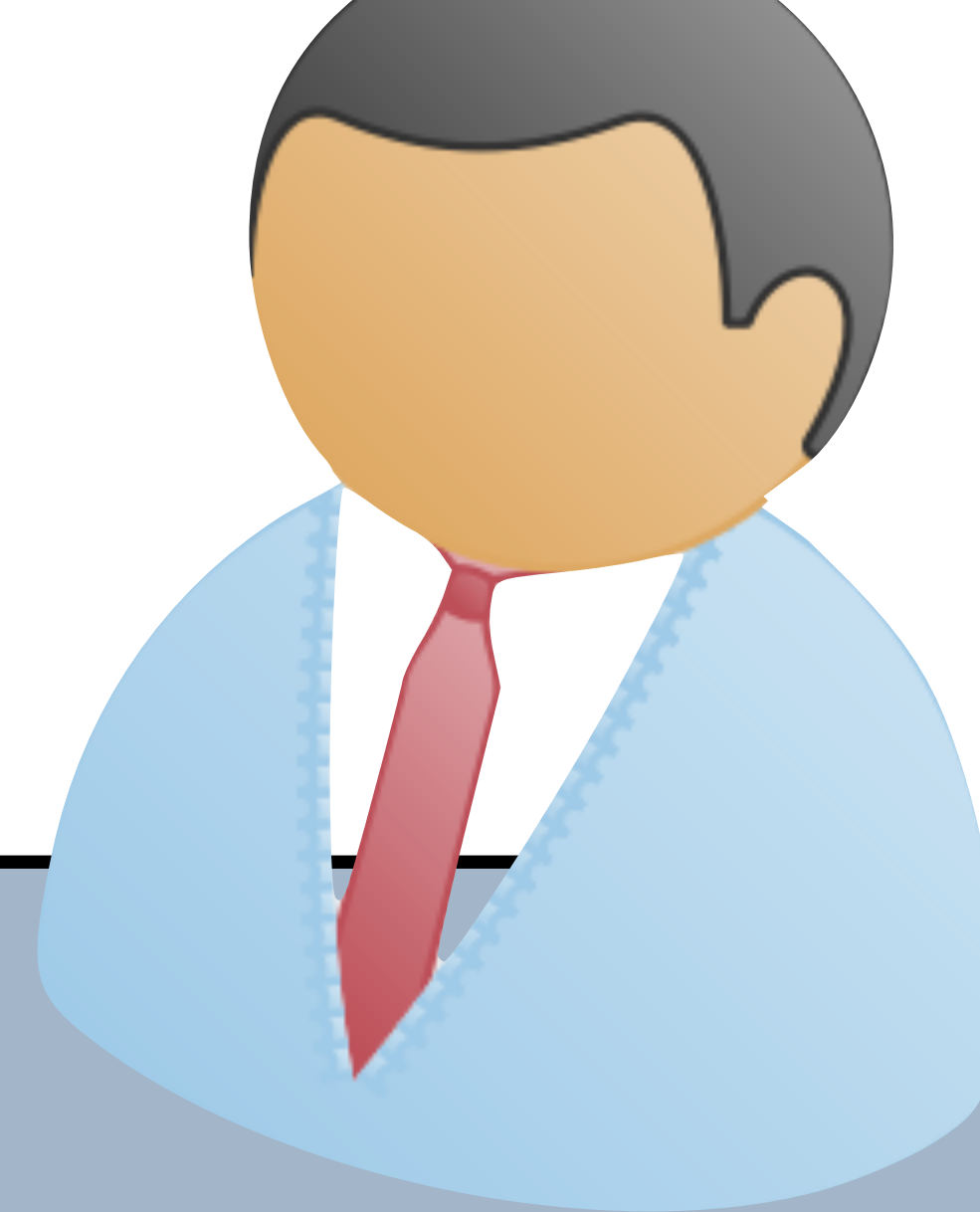
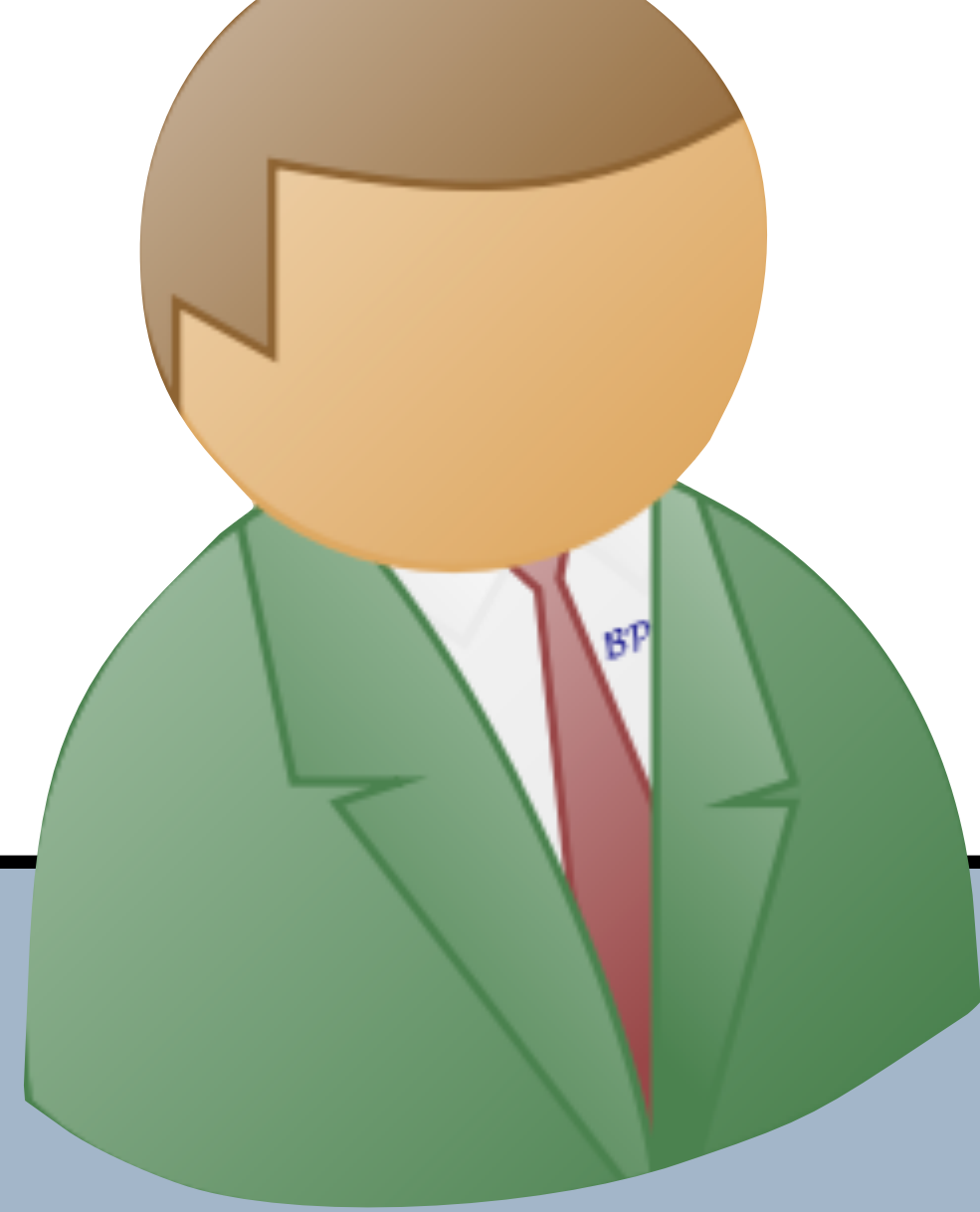
$R \stackrel{?}{=} g^s \text{pk}^{-c}$

FROST

Komlo and Goldberg
SAC 2020



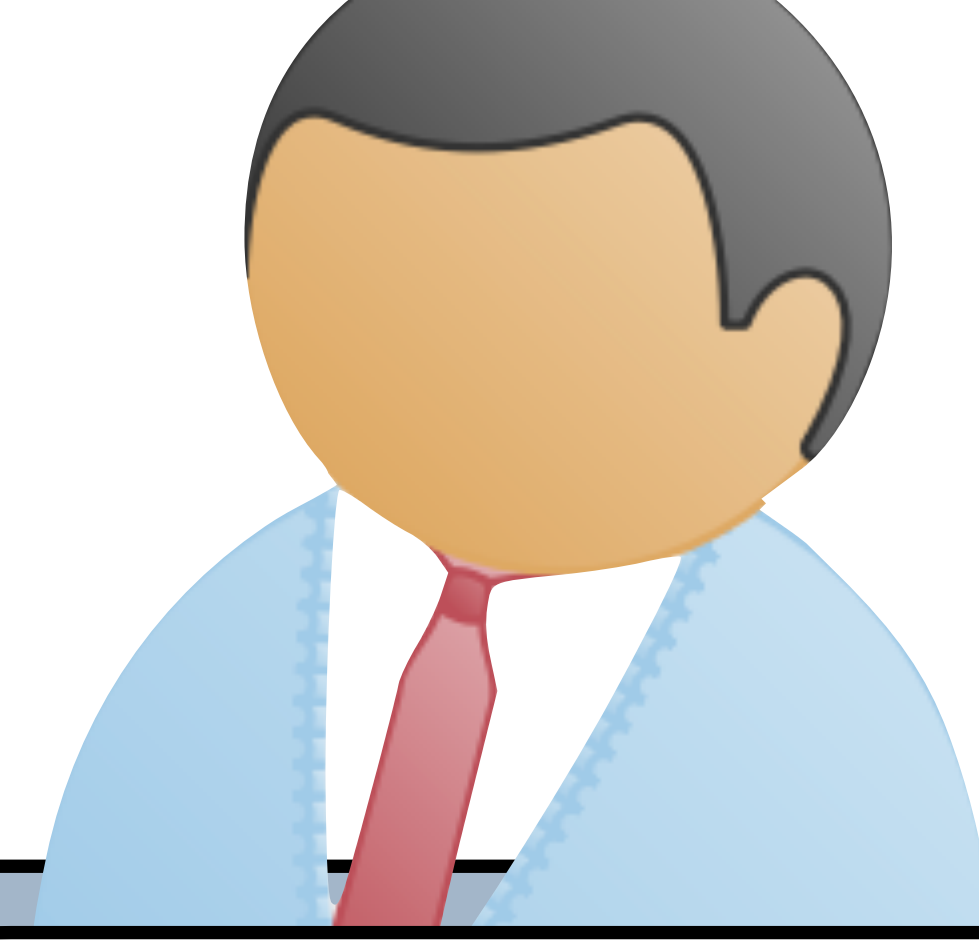
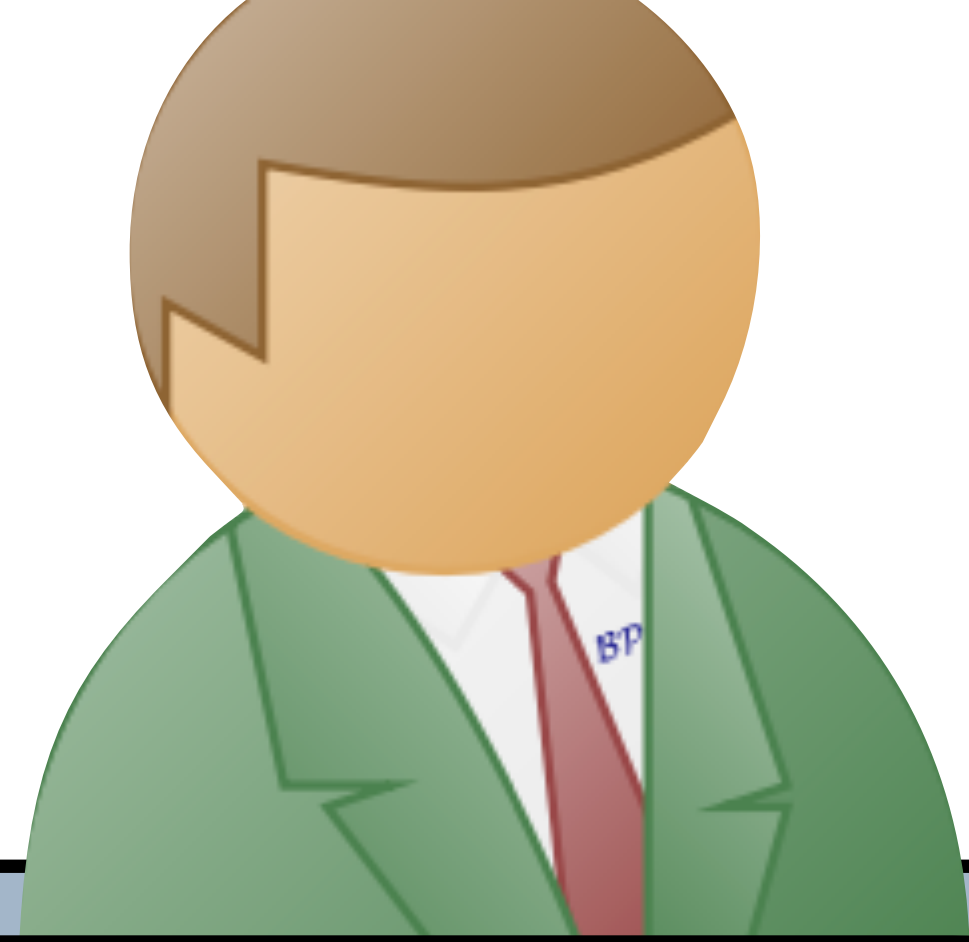
FROST3



Preprocessing

Signing

FROST3



PreRound(pk)

$\rho_i := (D_i, E_i)$

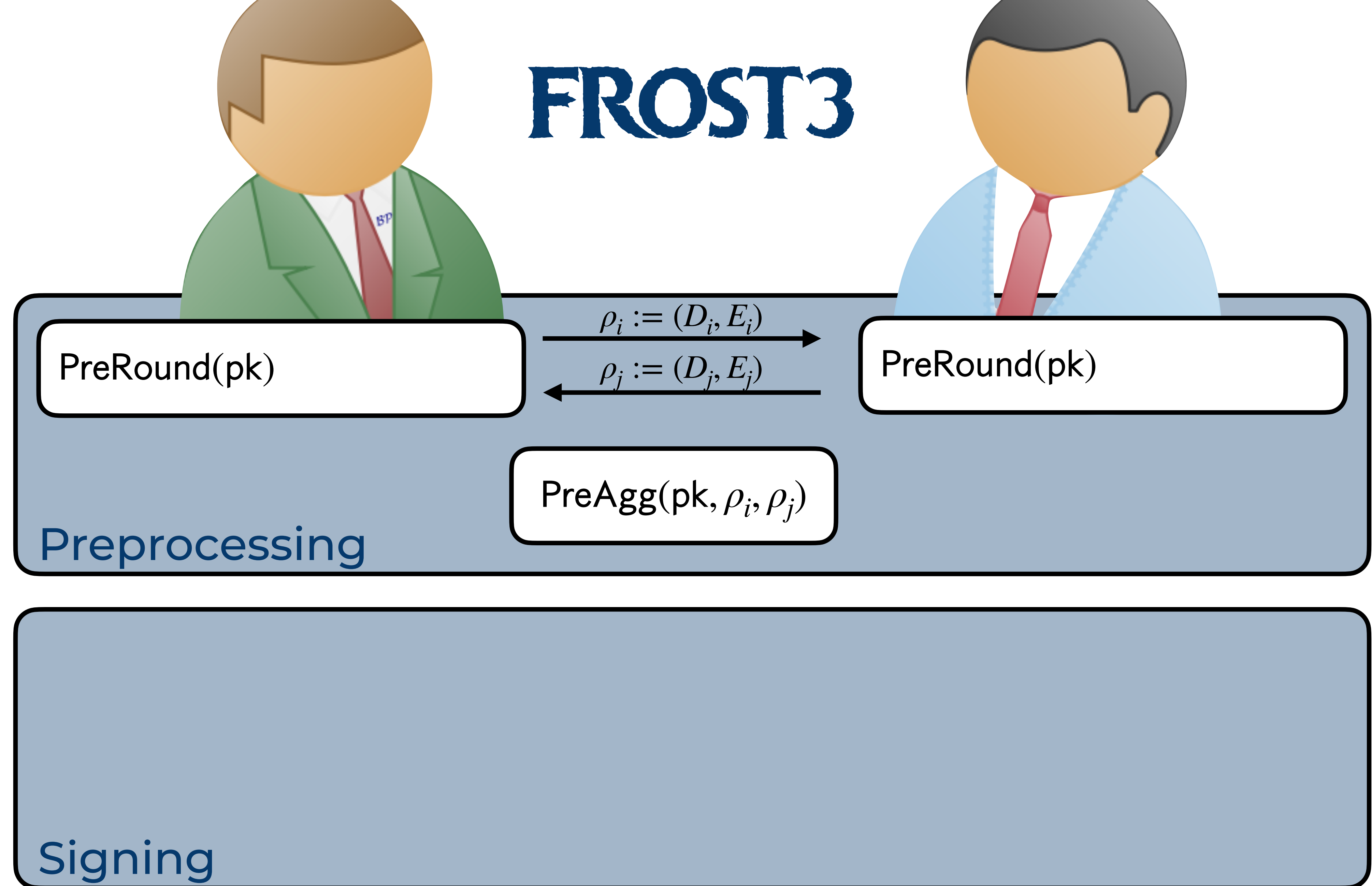
$\rho_j := (D_j, E_j)$

PreRound(pk)

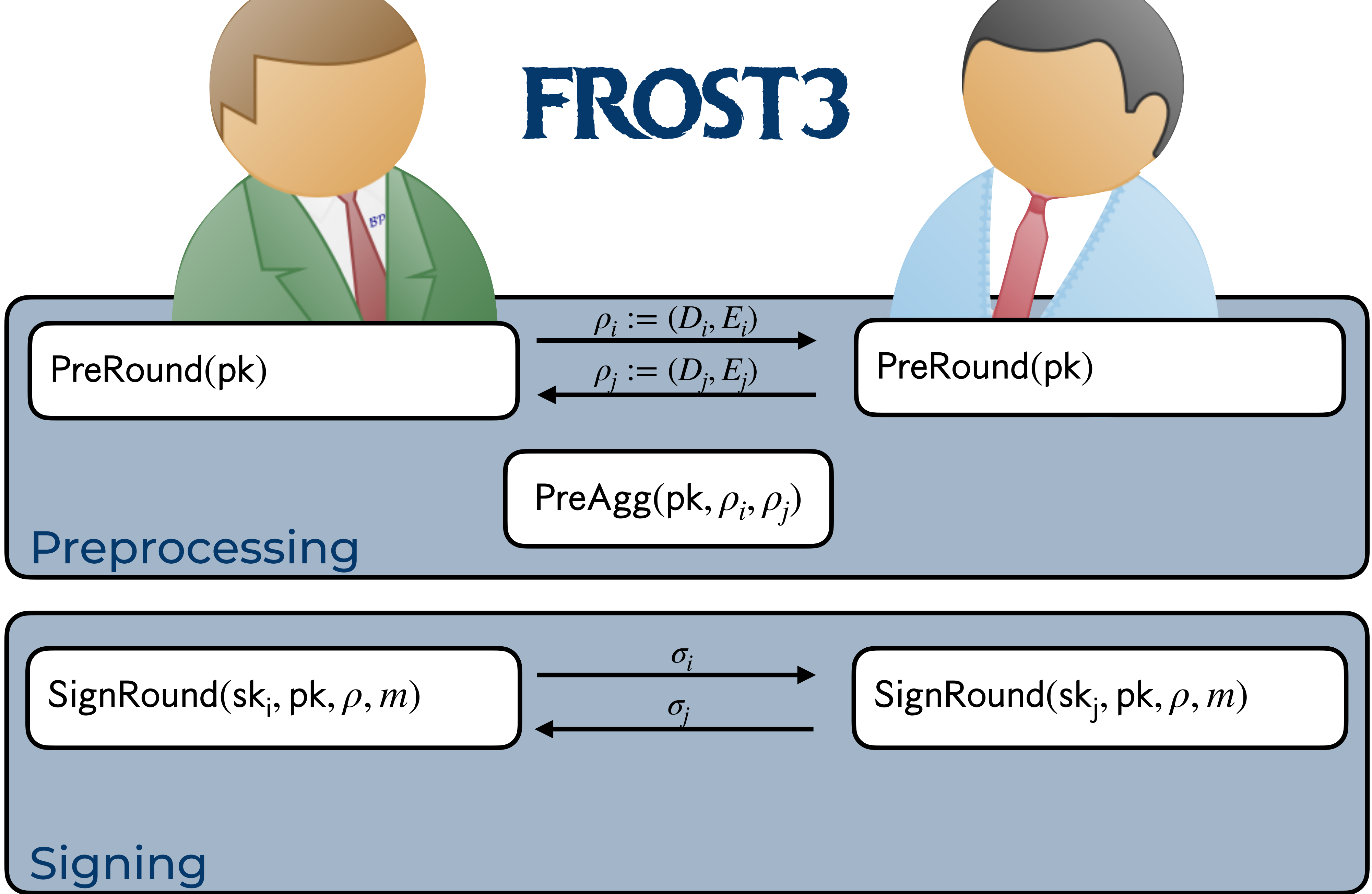
Preprocessing

Signing

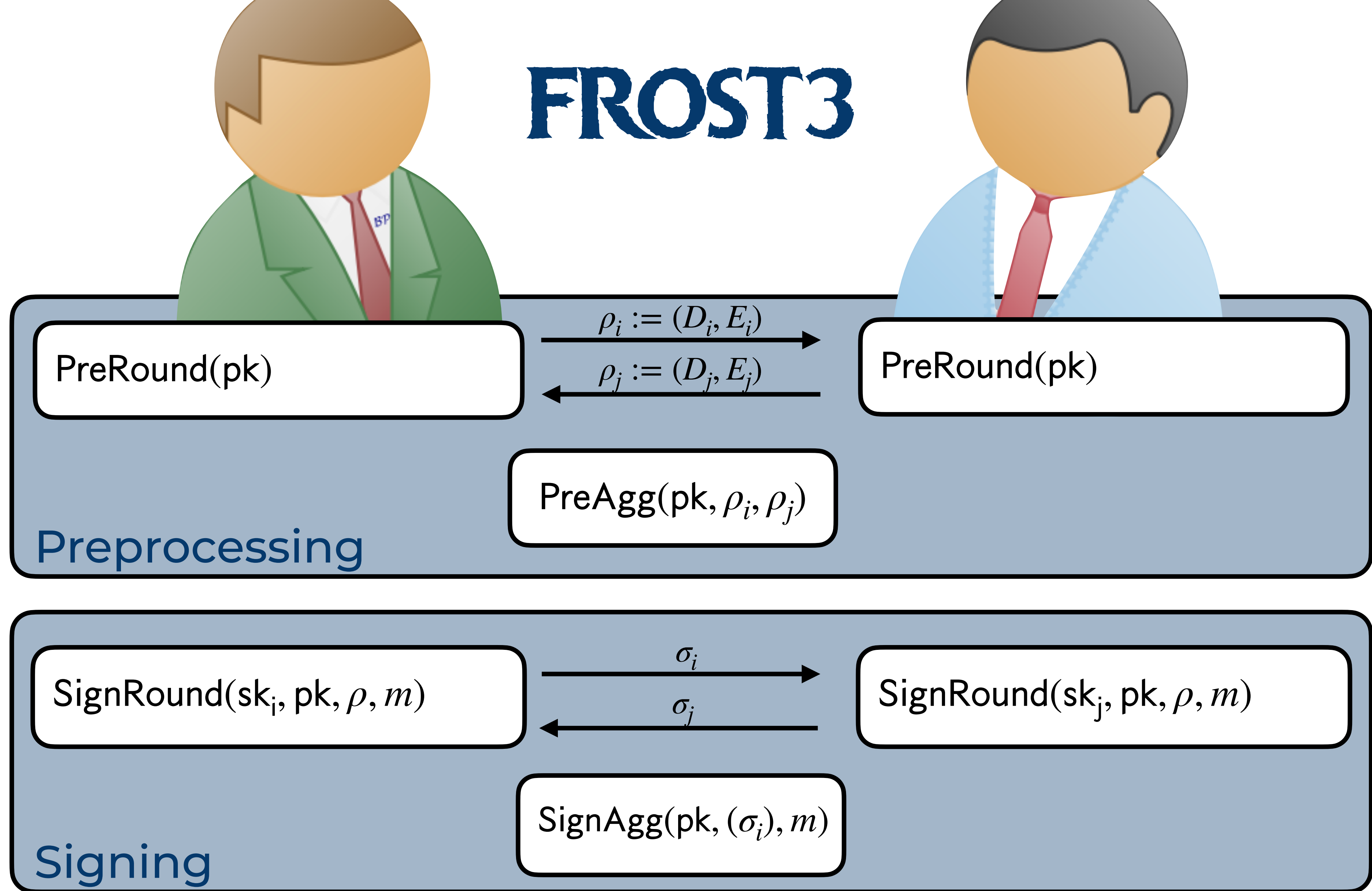
FROST3



FROST3



FROST3





FROST3 PREPROCESSING

(SIMPLIFIED)



FROST3 PREPROCESSING

(SIMPLIFIED)

PreRound(pk)

$$d_i \leftarrow^{\$} \mathbb{Z}_p, \quad e_i \leftarrow^{\$} \mathbb{Z}_p$$

$$D_i \leftarrow g^{d_i}, \quad E_i \leftarrow g^{e_i}$$

$$\rho_i \leftarrow (D_i, E_i)$$

(D_i, E_i)



FROST3 PREPROCESSING

(SIMPLIFIED)

PreRound(pk)

$$\begin{aligned}d_i &\leftarrow^{\$} \mathbb{Z}_p, & e_i &\leftarrow^{\$} \mathbb{Z}_p \\D_i &\leftarrow g^{d_i}, & E_i &\leftarrow g^{e_i} \\ \rho_i &\leftarrow (D_i, E_i)\end{aligned}$$

(D_i, E_i)

(D_j, E_j)

PreRound(pk)

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Blockstream



FROST3 PREPROCESSING

(SIMPLIFIED)

PreRound(pk)

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(D_i, E_i)

(D_j, E_j)

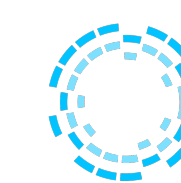
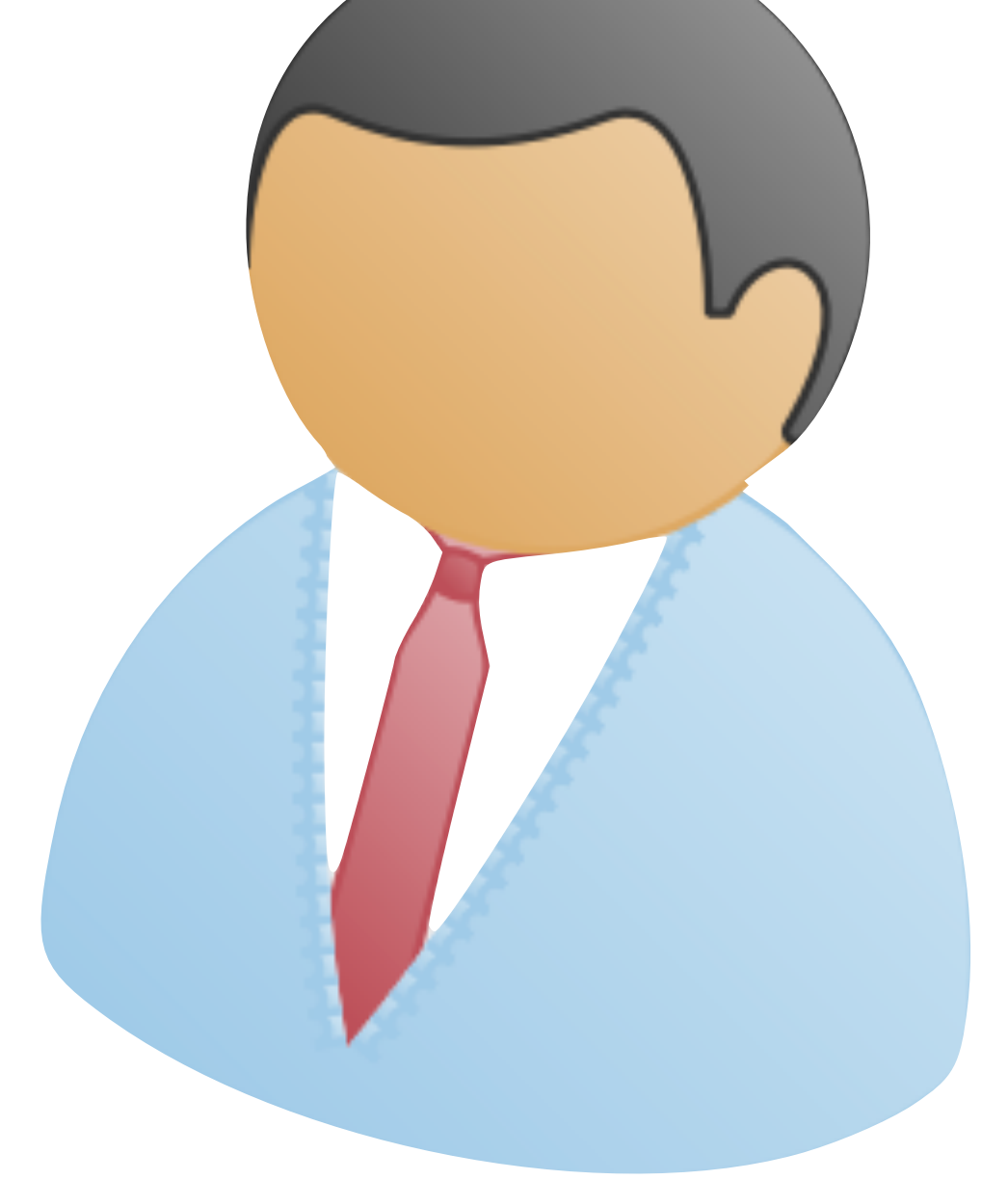
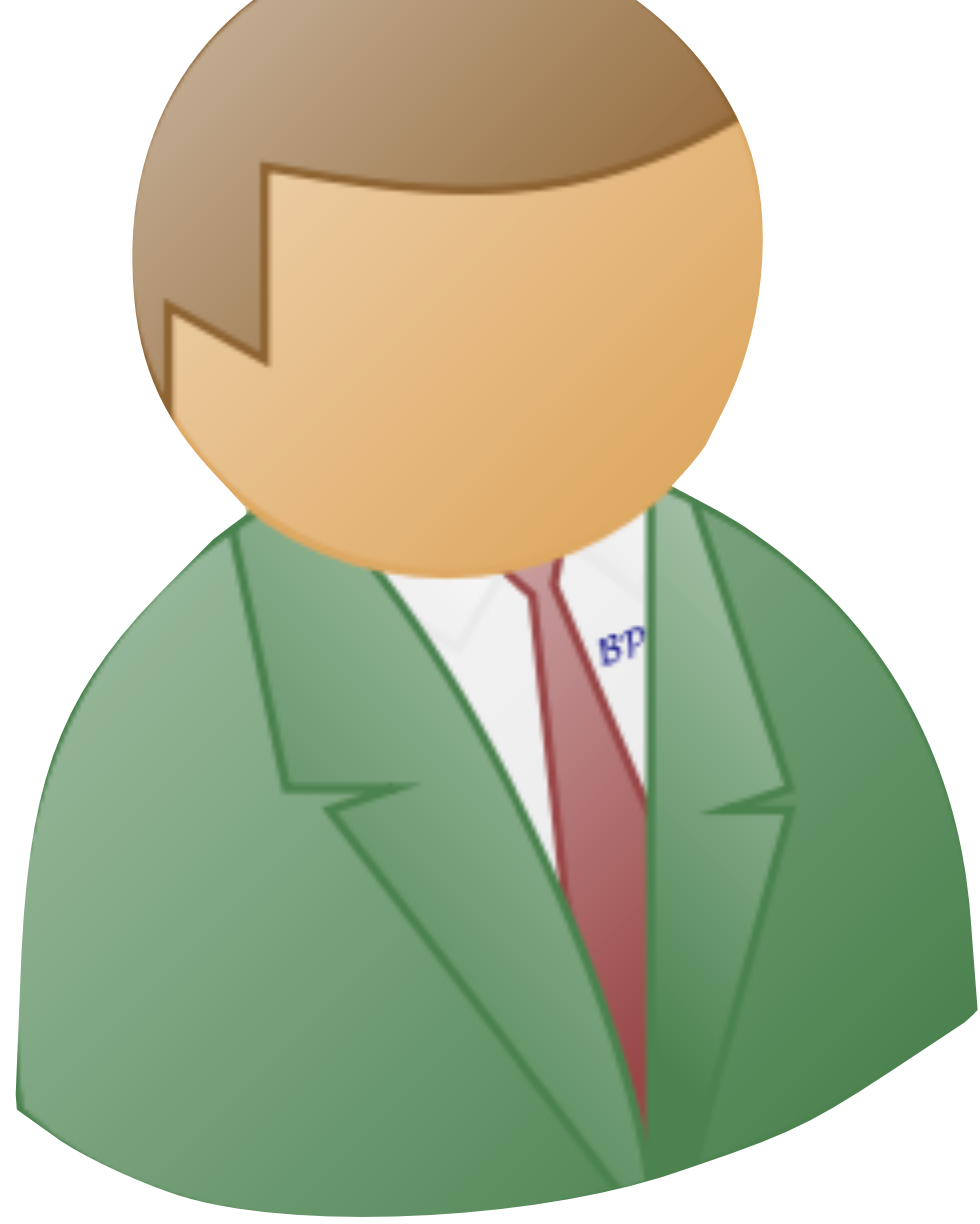
PreRound(pk)

$$\begin{aligned}d_j &\leftarrow^{\$} \mathbb{Z}_p, & e_j &\leftarrow^{\$} \mathbb{Z}_p \\ D_j &\leftarrow g^{d_j}, & E_j &\leftarrow g^{e_j} \\ \rho_j &\leftarrow (D_j, E_j)\end{aligned}$$

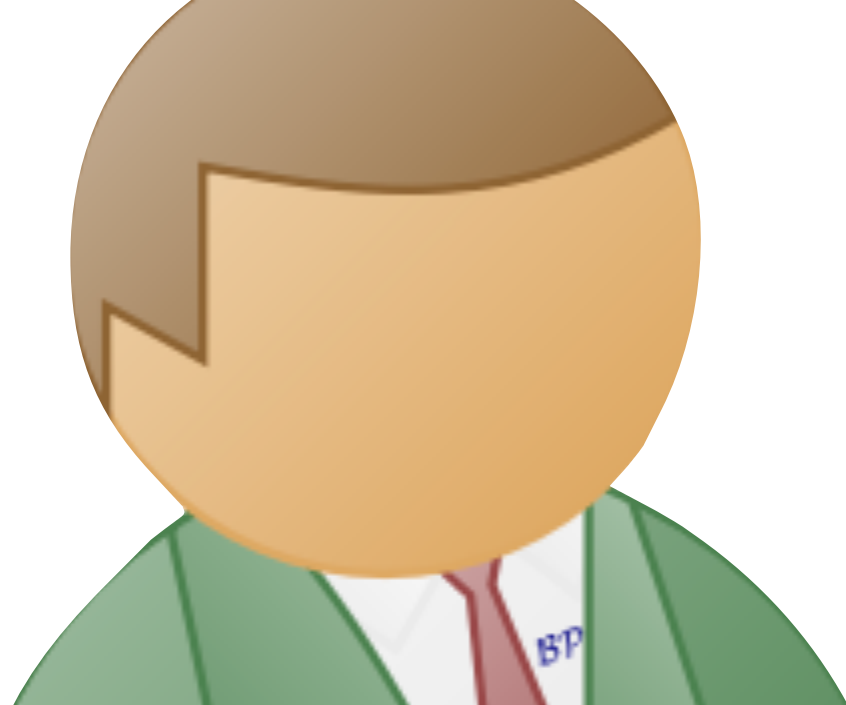
PreAgg(pk, ρ_i, ρ_j)

$$D := D_i \cdot D_j \quad E := E_i \cdot E_j$$

FROST3 SIGNING



FROST3 SIGNING



SignRound($sk_i, pk, S, state_i, \rho, m$)

$b \leftarrow H_{non}(pk, S, \rho, m)$

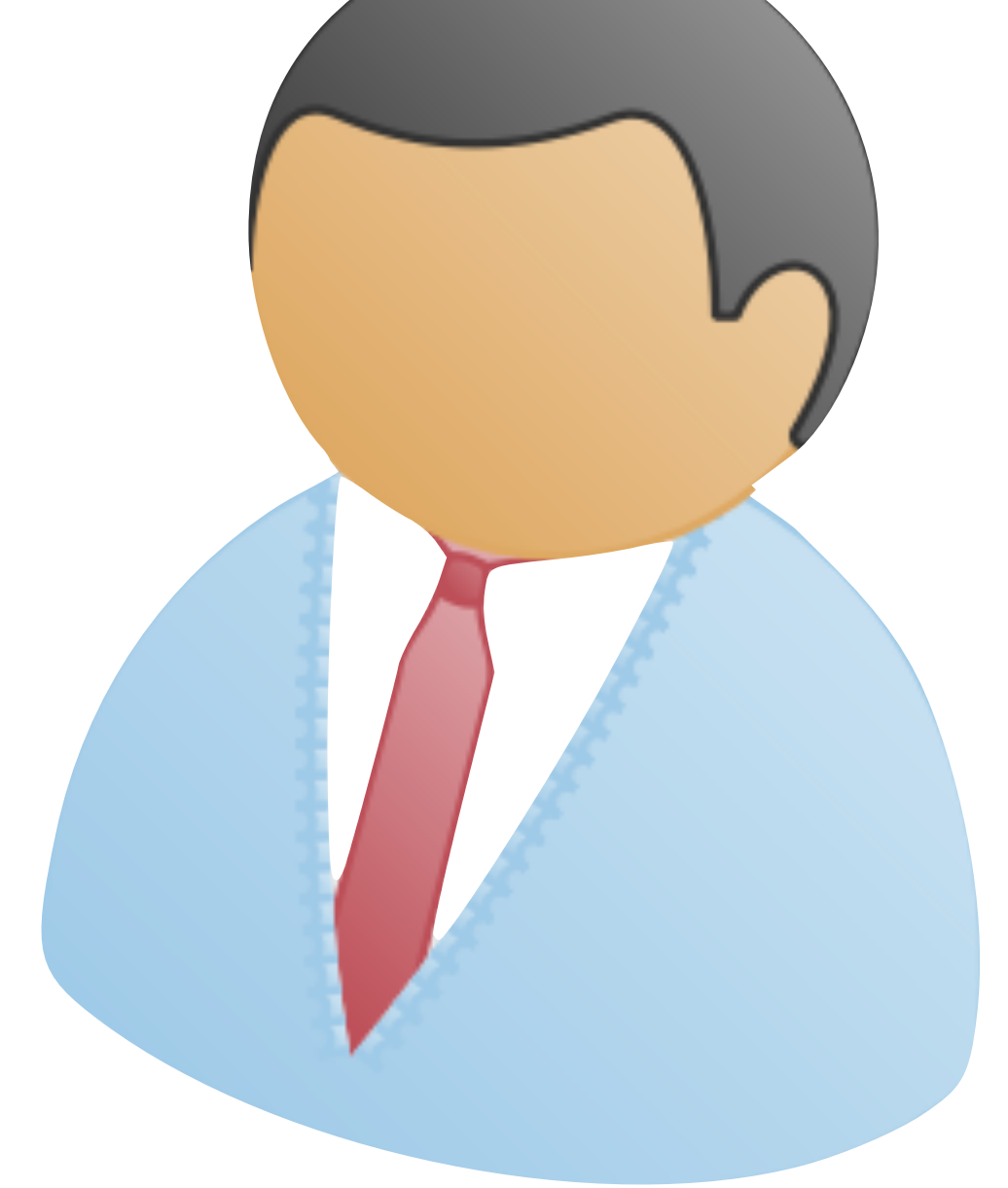
$R \leftarrow DE^b$

$c \leftarrow H_{sig}(X, R, m)$

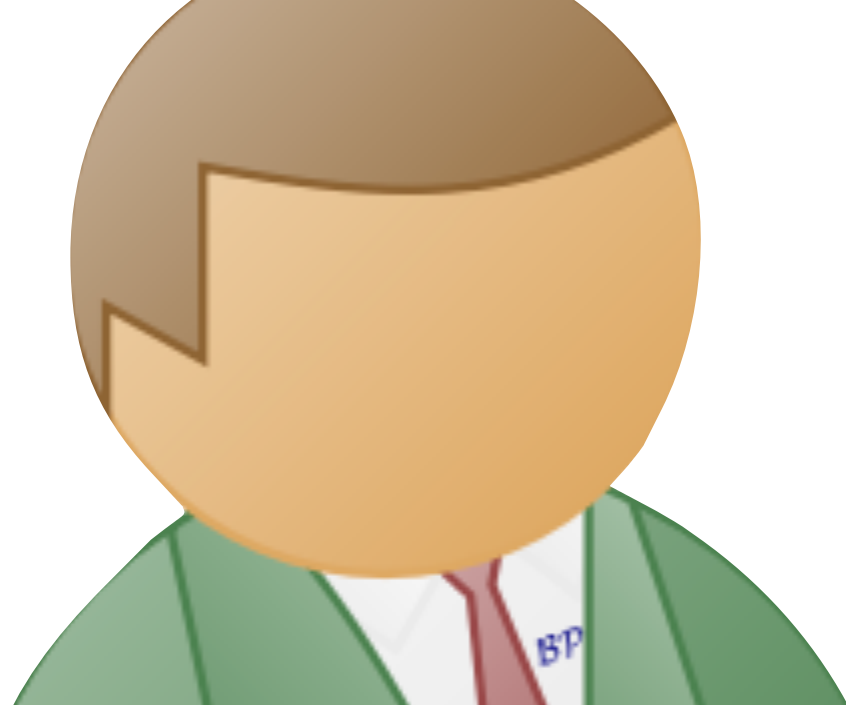
$\Lambda_i \leftarrow \text{Lagrange}(S, i)$

$\sigma_i \leftarrow d_i + be_i + c\Lambda_i x_i$

σ_i



FROST3 SIGNING



SignRound($sk_i, pk, S, state_i, \rho, m$)

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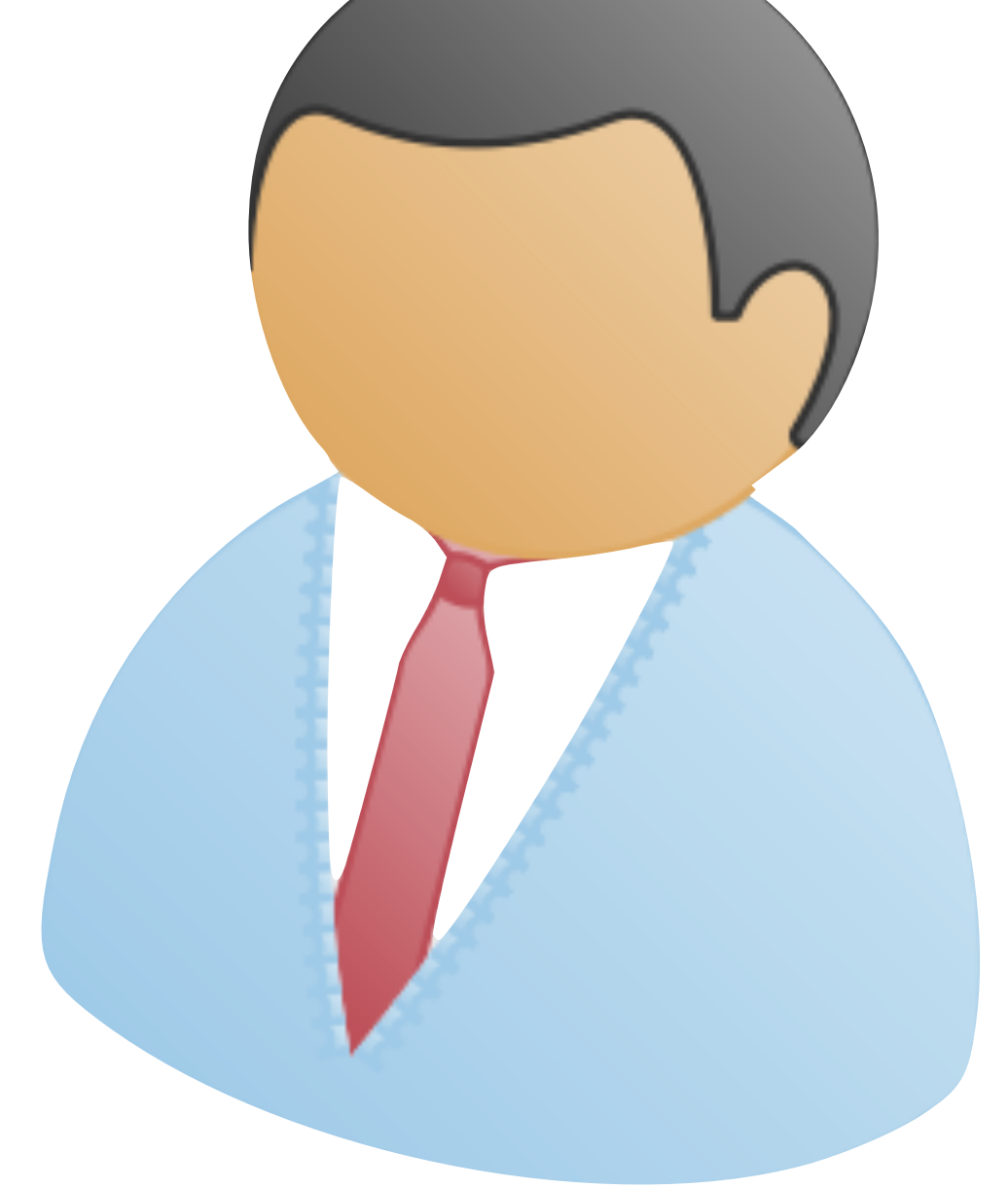
$R \leftarrow DE^b \quad r = d + be$

$c \leftarrow H_{sig}(X, R, m)$

$\Lambda_i \leftarrow \text{Lagrange}(S, i)$

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SignAgg($pk, \rho, \sigma_i, \sigma_j, S, m$)

$b \leftarrow H_{non}(X, S, \rho, m)$
 $R \leftarrow DE^b$

$s' \leftarrow \sigma_i + \sigma_j$
 $\sigma \leftarrow (R, s')$



Blockstream



FROST3 SIGNING

SignRound($sk_i, pk, S, state_i, \rho, m$)

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σ_j

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$b \leftarrow H_{non}(X, S, \rho, m)$
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$s' \leftarrow \sigma_i + \sigma_j = (d + be) + cx = r + cx$
 $\sigma \leftarrow (R, s)$



Blockstream



FROST3 SIGNING

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 $\sigma \leftarrow (R, s) = (g^r, r + cx)$

The History of Proving **FROST** Secure

The History of Proving **FROST** Secure

FROST1

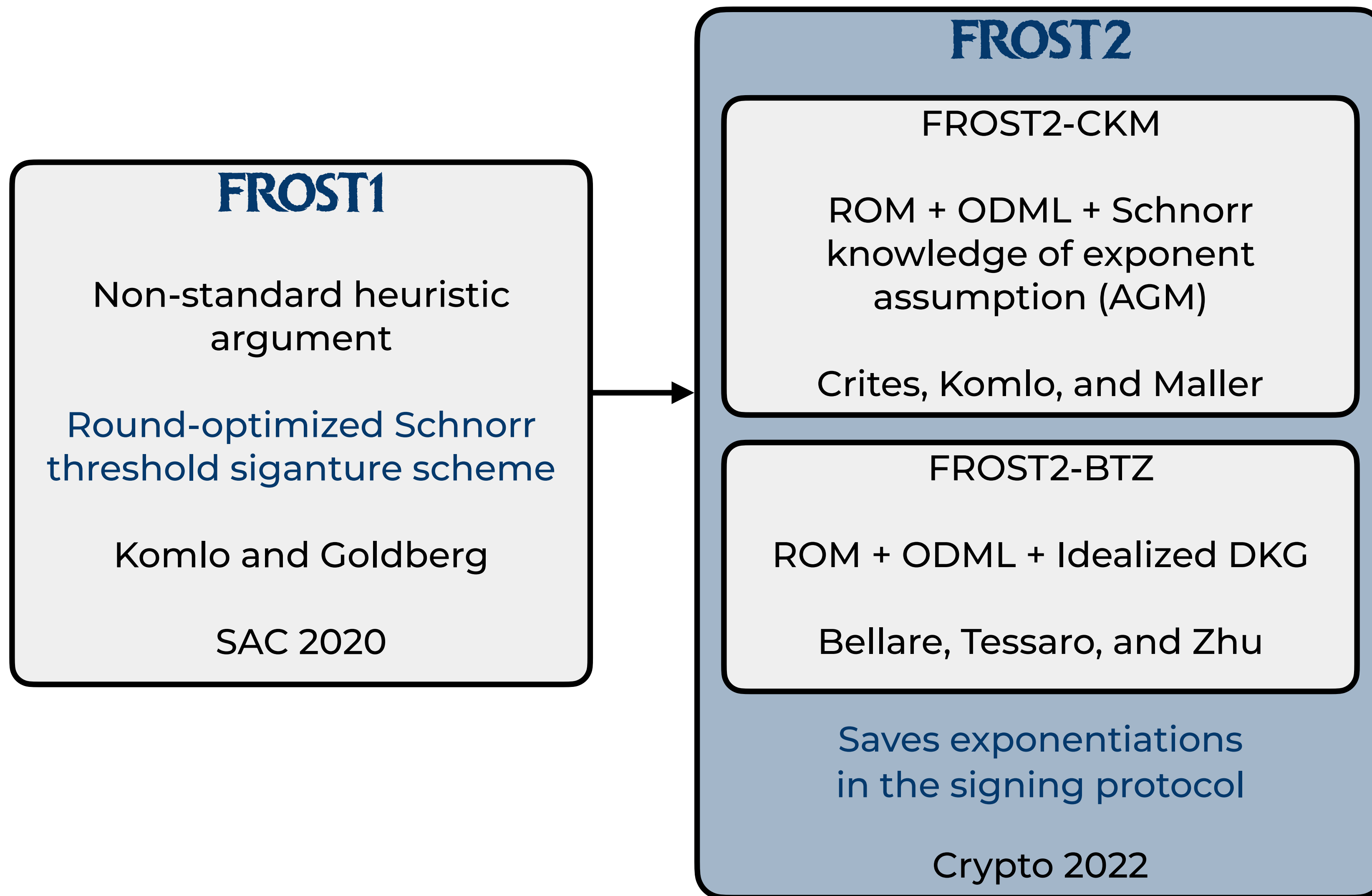
Non-standard heuristic
argument

Round-optimized Schnorr
threshold signature scheme

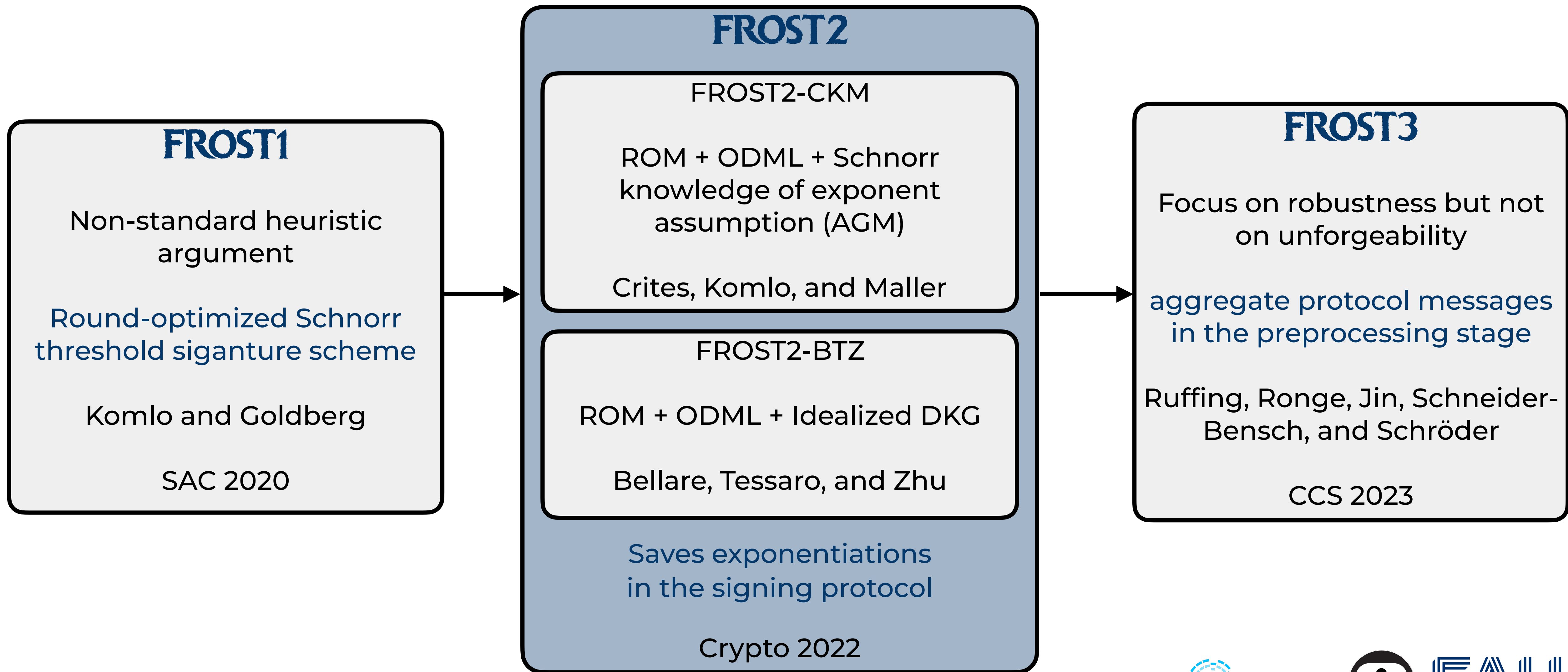
Komlo and Goldberg

SAC 2020

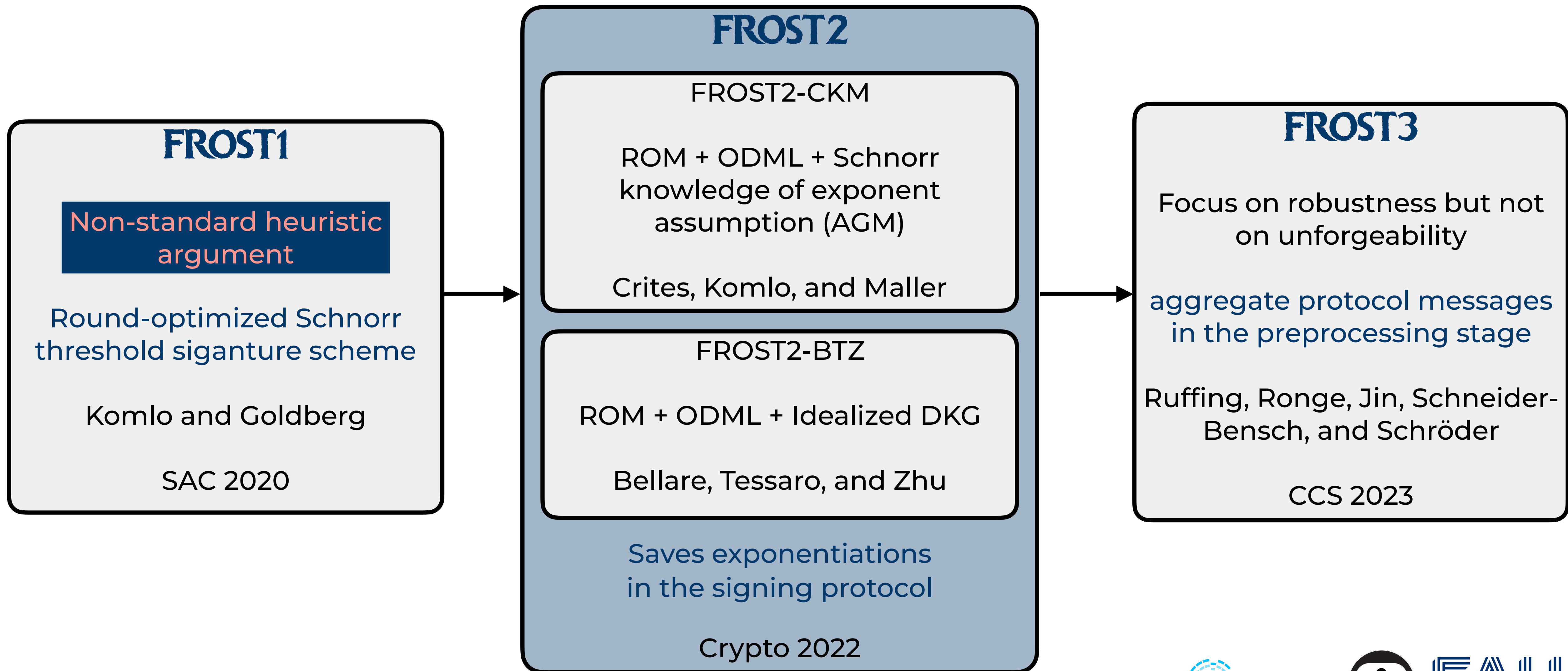
The History of Proving **FROST** Secure



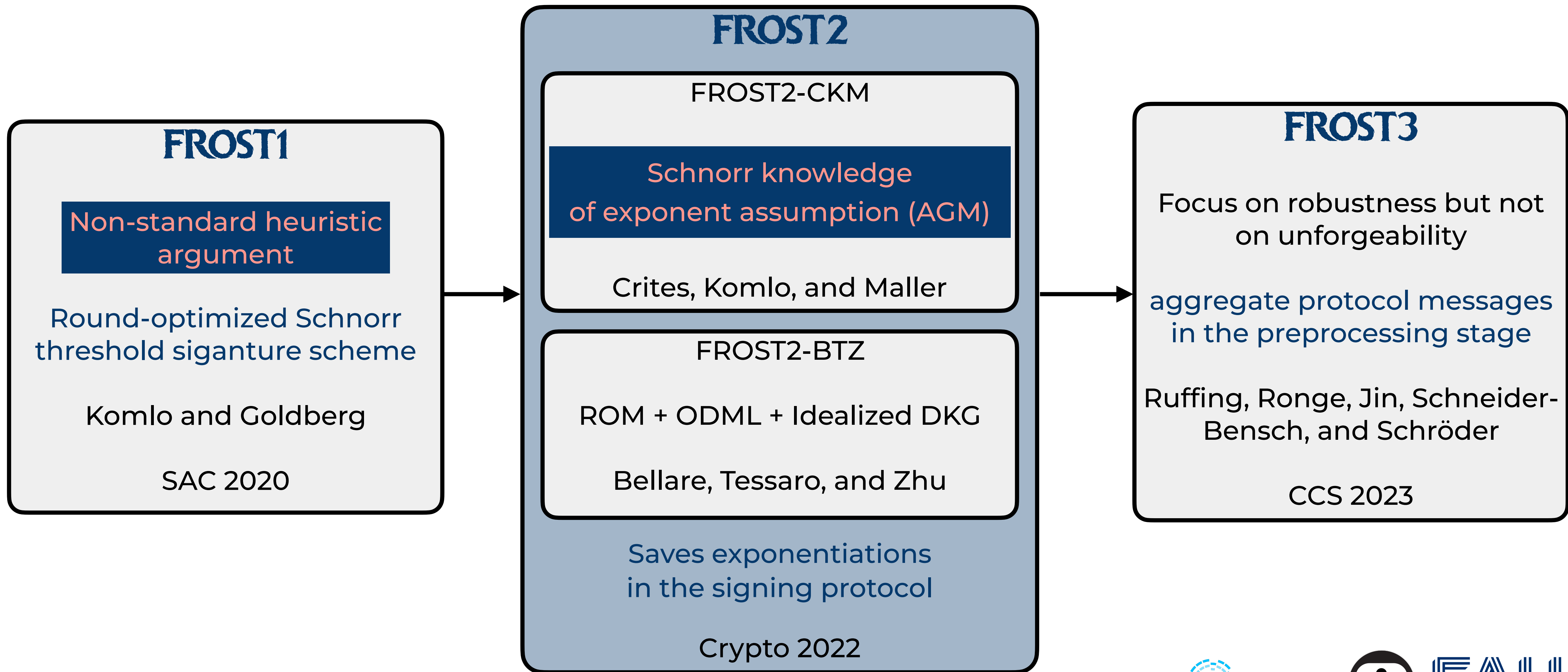
The History of Proving **FROST** Secure



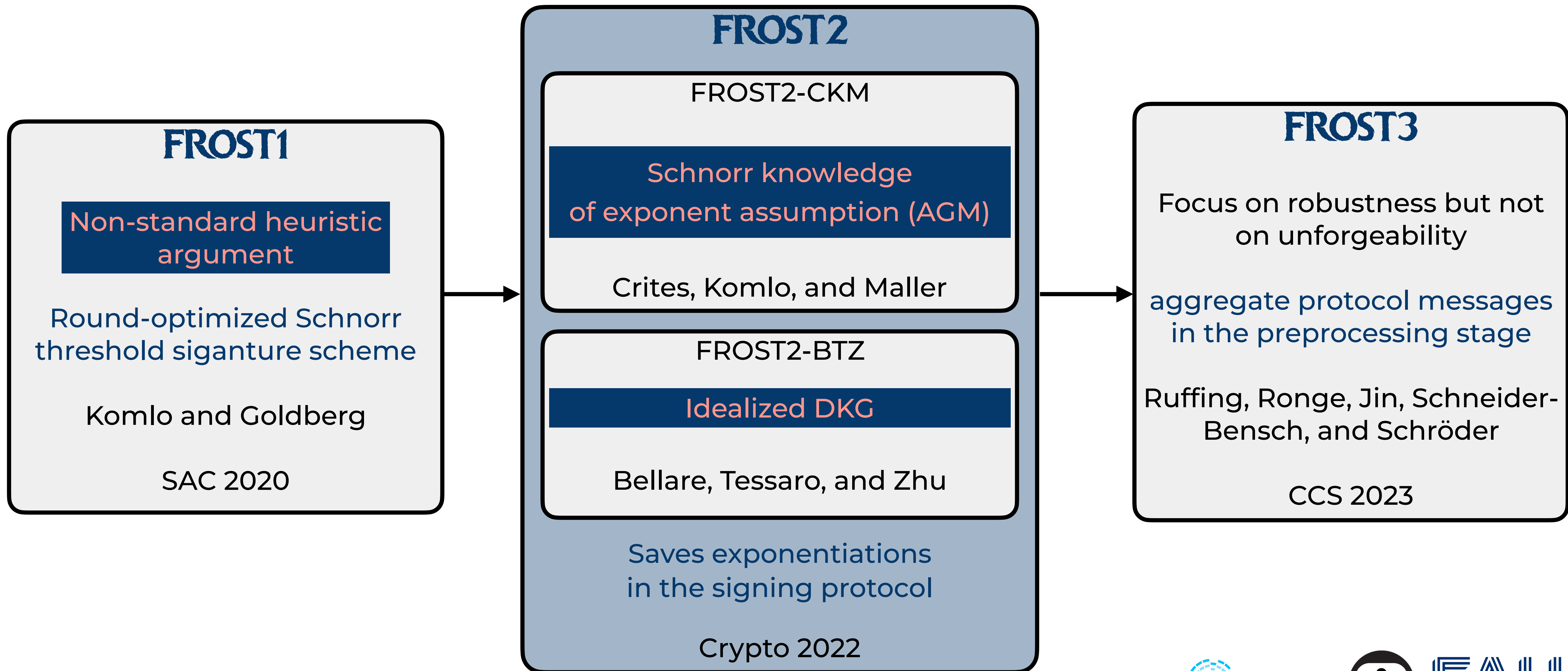
The History of Proving **FROST** Secure



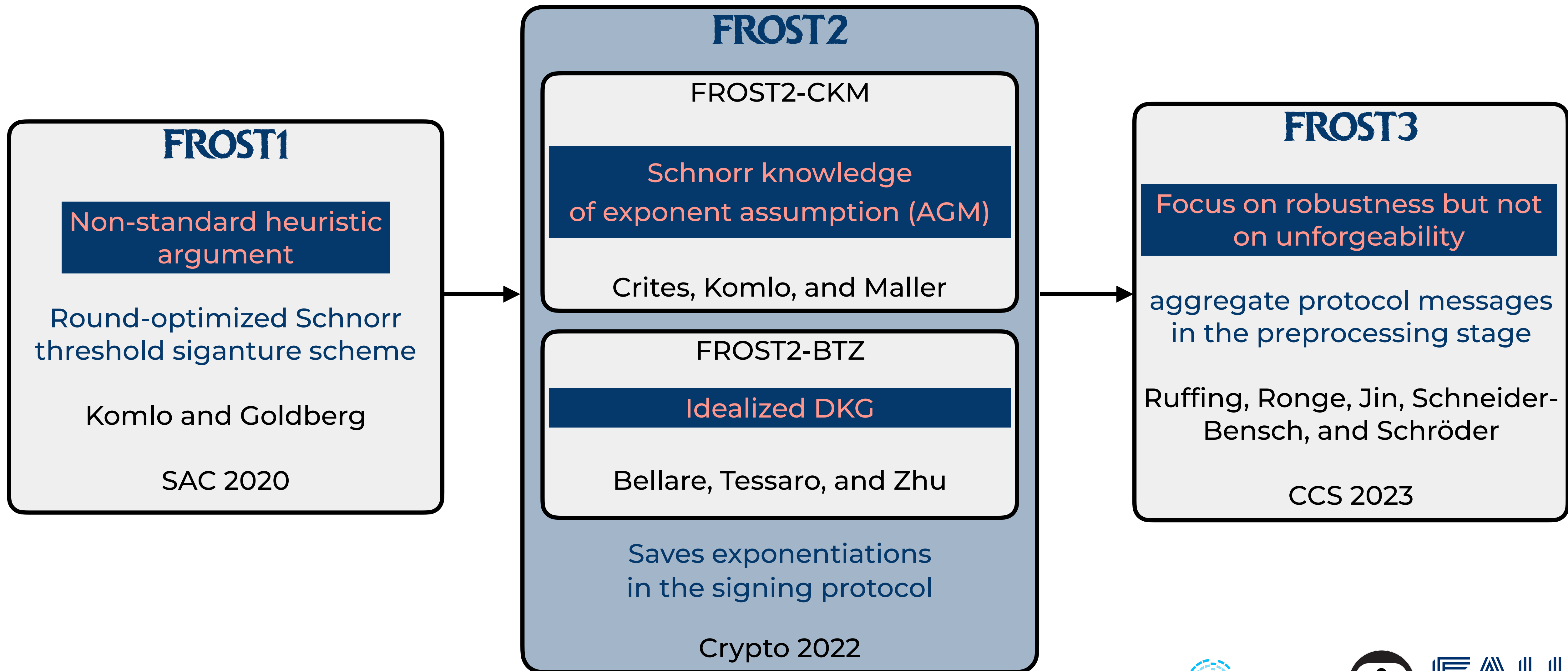
The History of Proving **FROST** Secure



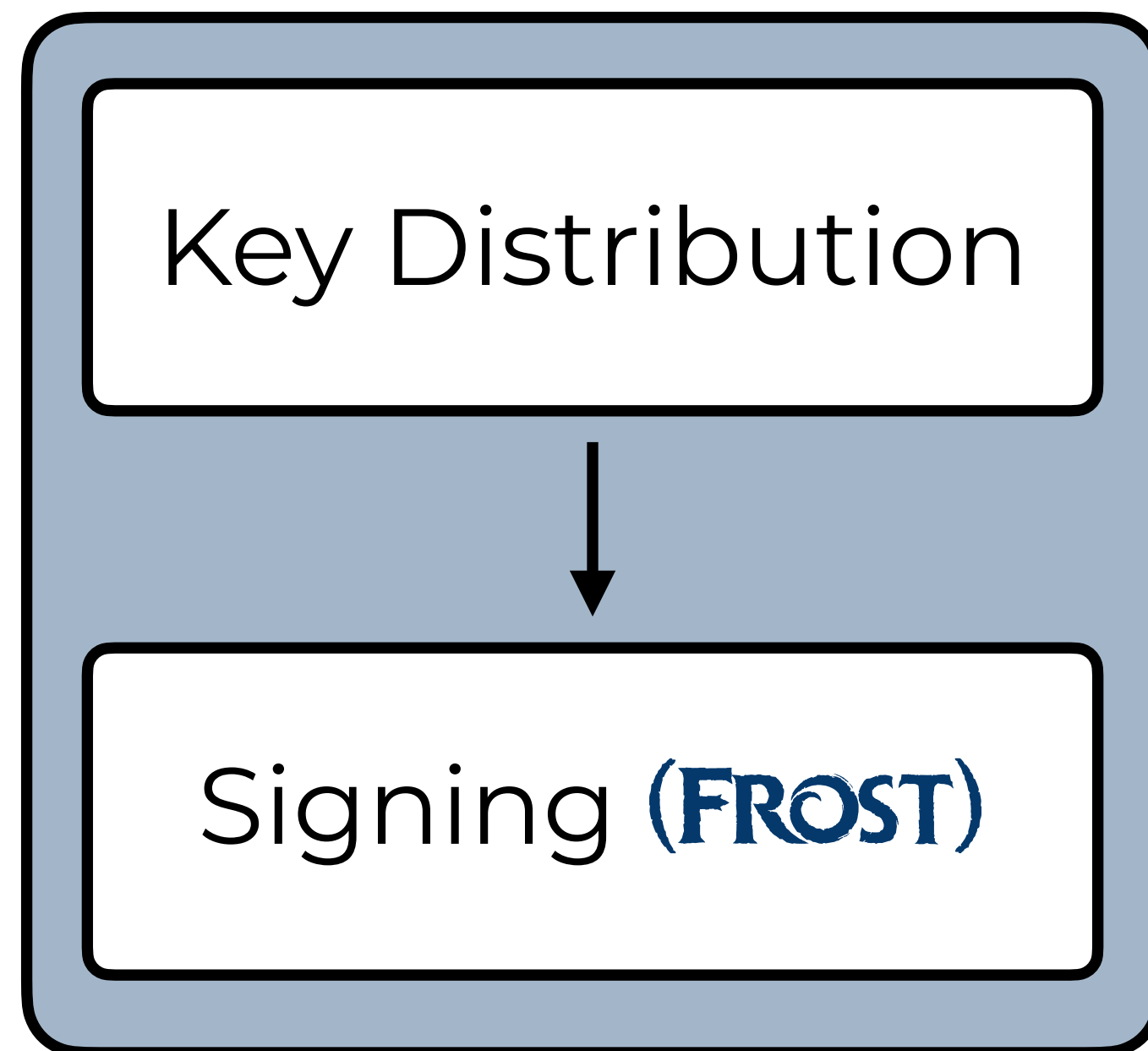
The History of Proving **FROST** Secure



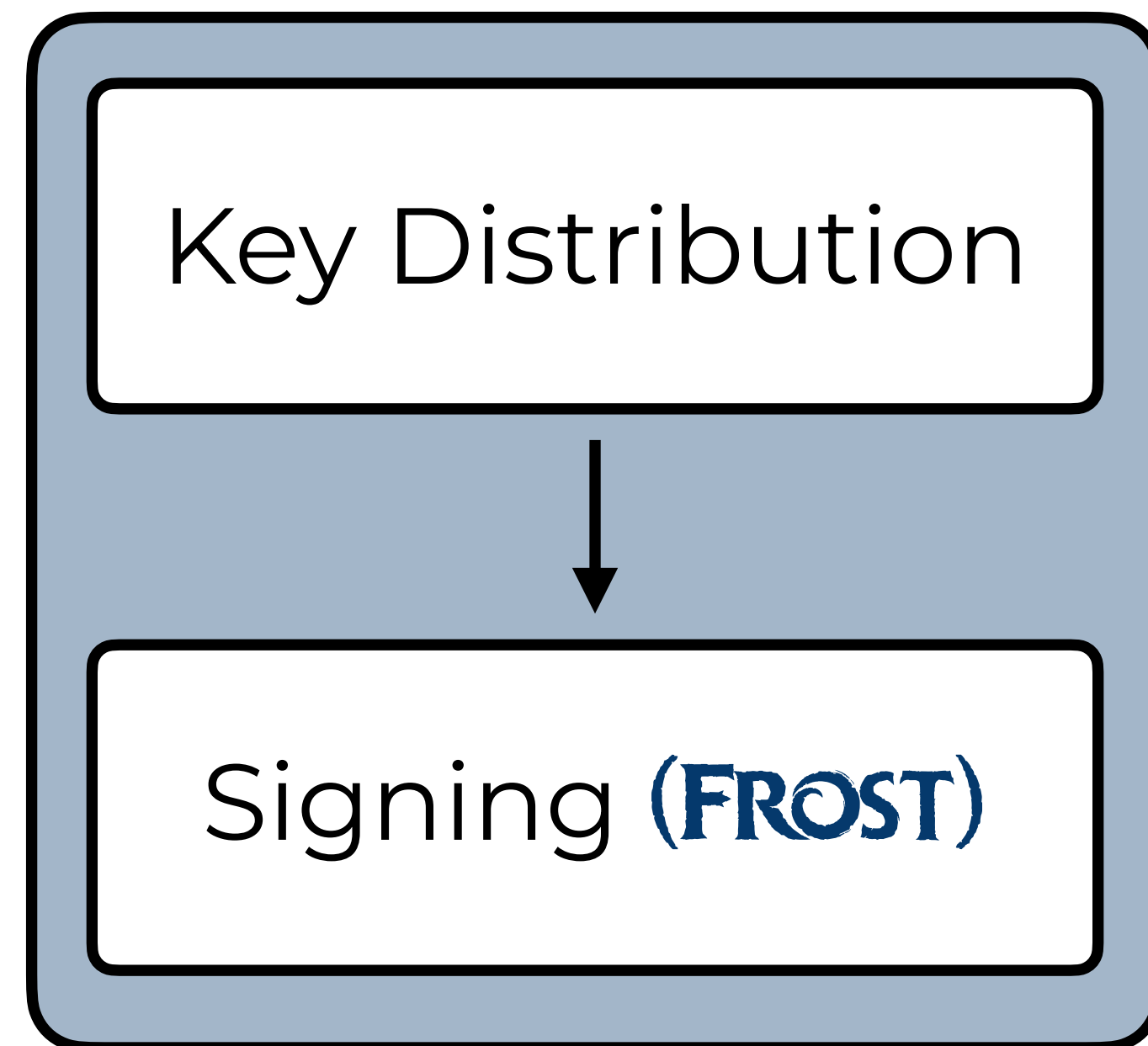
The History of Proving **FROST** Secure



Idealized DKGs [GJKR99,GJKR07,KGS23]

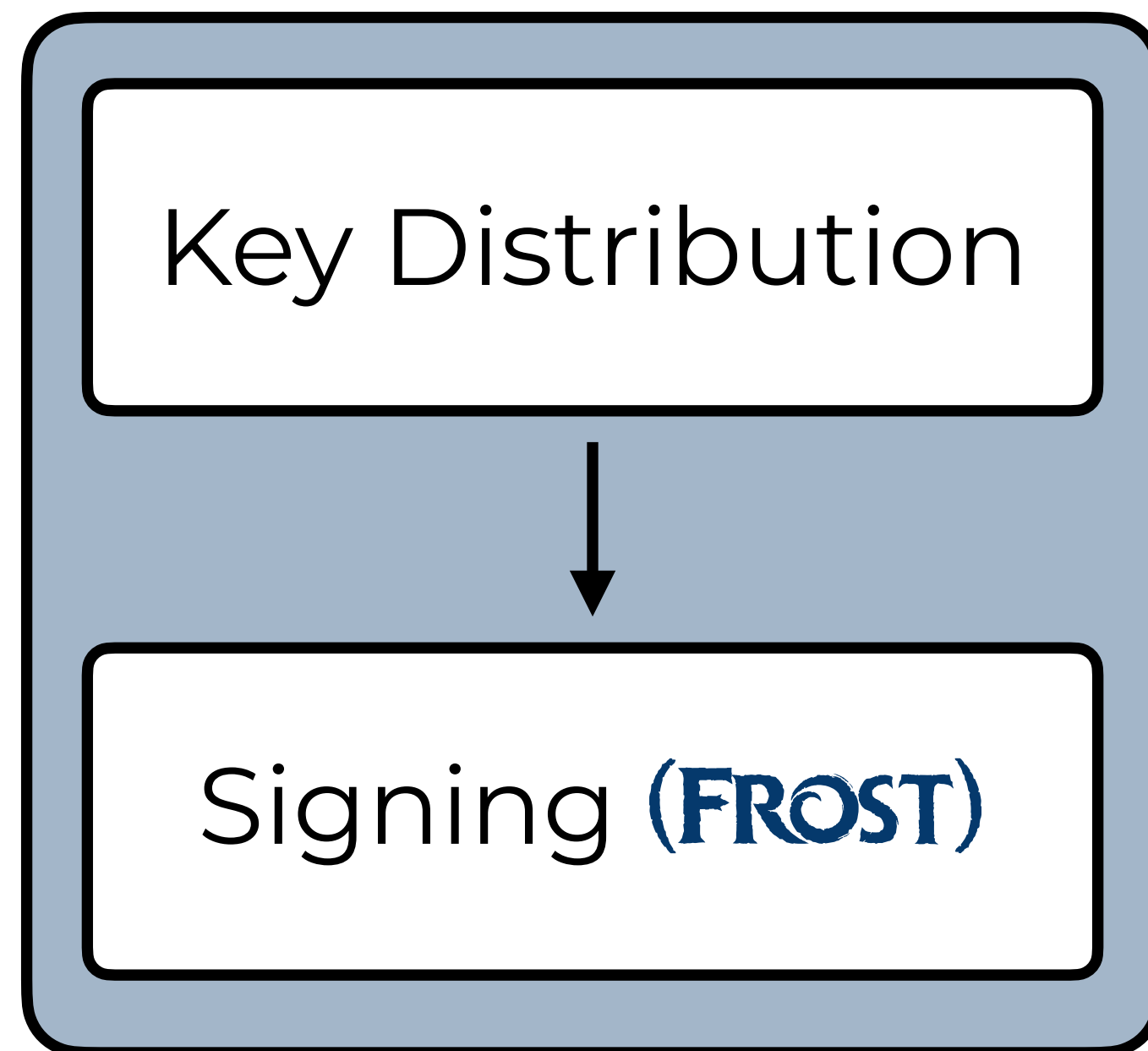


Idealized DKGs [GJKR99,GJKR07,KGS23]



- ✘ No fully simulatable DKG in the dishonest majority setting ($n/2 \leq t - 1$)

Idealized DKGs [GJKR99,GJKR07,KGS23]



- ✘ No fully simulatable DKG in the dishonest majority setting ($n/2 \leq t - 1$)
- ✘ Simulatable DKGs are not as efficient as the Pedersen DKG

Pedersen DKG with PoP [KG20, CKM21]

$$f_i(Z) = a_{i,0} + a_{i,1}Z + \dots + a_{i,t-1}Z^{t-1}$$

$$f(Z) = \sum_{i=1}^n f_i(Z)$$

$$x_i = f(i), pk = g^{f(0)}$$

Pedersen DKG with PoP [KG20, CKM21]

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$$A_{i,k} = g^{a_{i,k}}$$

$$PoP = (R, s)$$



Blockstream



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- ✓ Additive sharing of a secret key
 $x = x_1 + \dots + x_n = a_{1,0} + \dots + a_{n,0}$

Pedersen DKG with PoP [KG20, CKM21]

$$f_i(Z) = a_{i,0} + a_{i,1}Z + \dots + a_{i,t-1}Z^{t-1}$$

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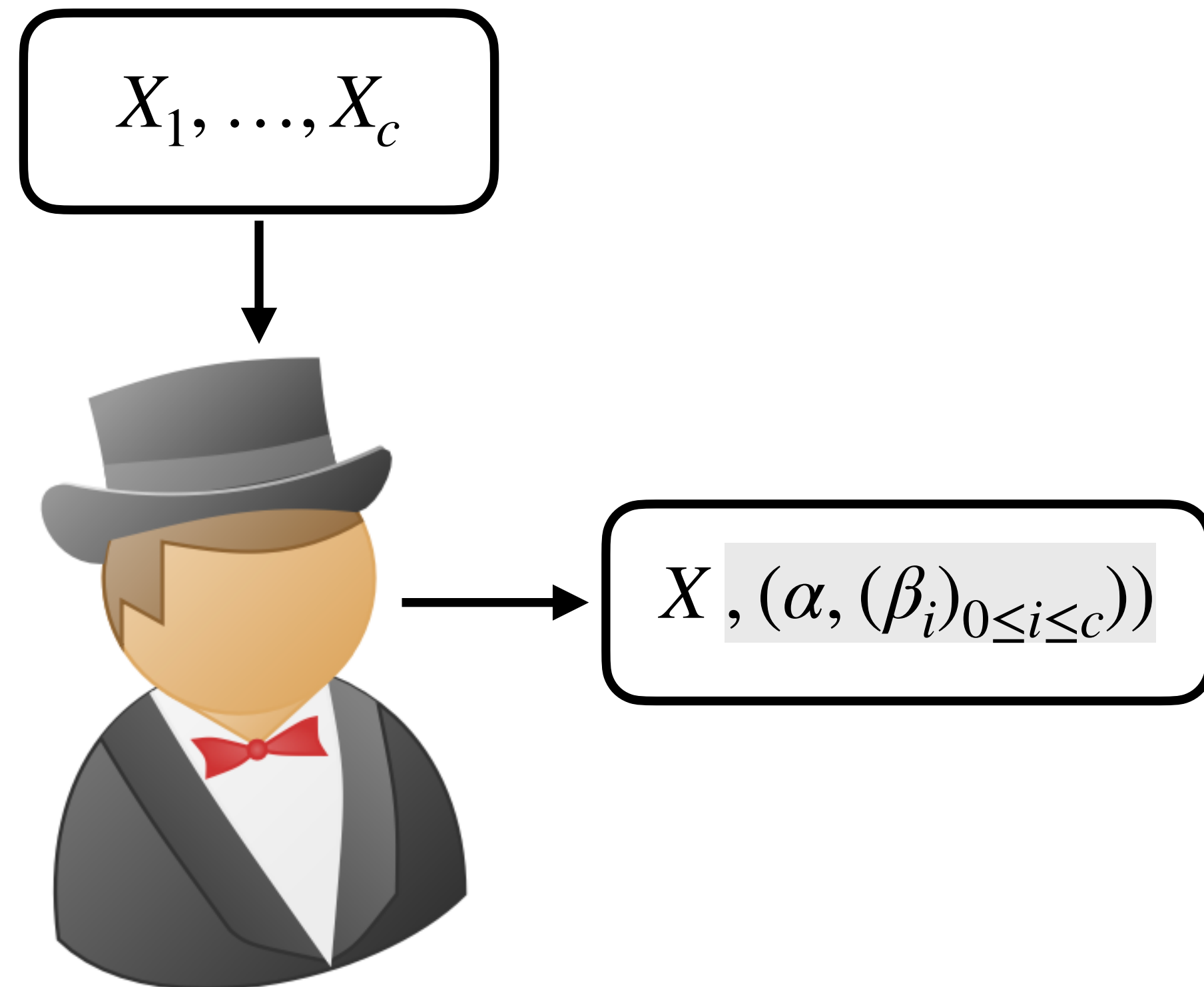
$$A_{i,k} = g^{a_{i,k}}$$

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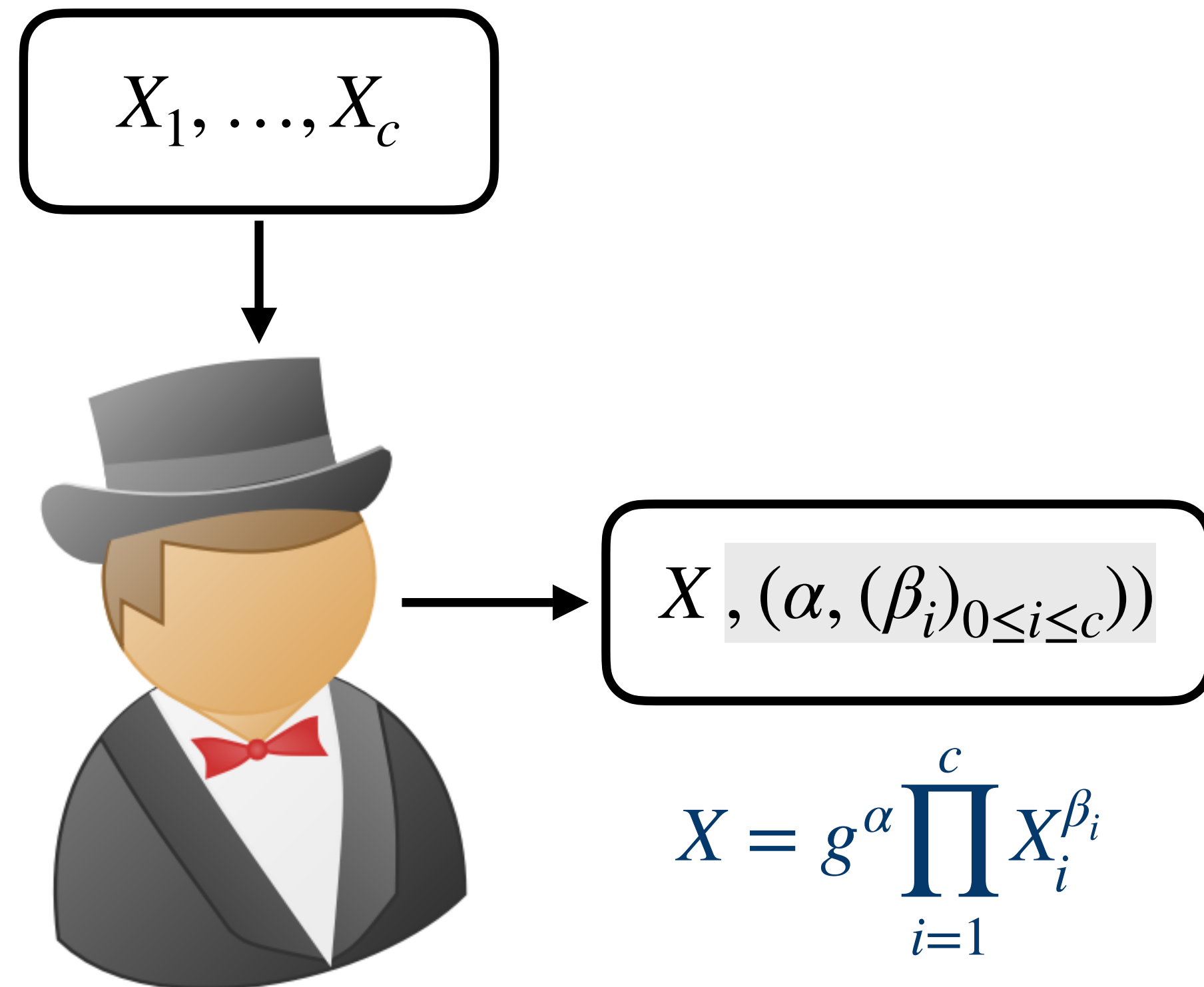
- ✓ Additive sharing of a secret key
 $x = x_1 + \dots + x_n = a_{1,0} + \dots + a_{n,0}$
- ✗ Not fully simulatable DKG due to a possible shift [GJKR06]

The AGM

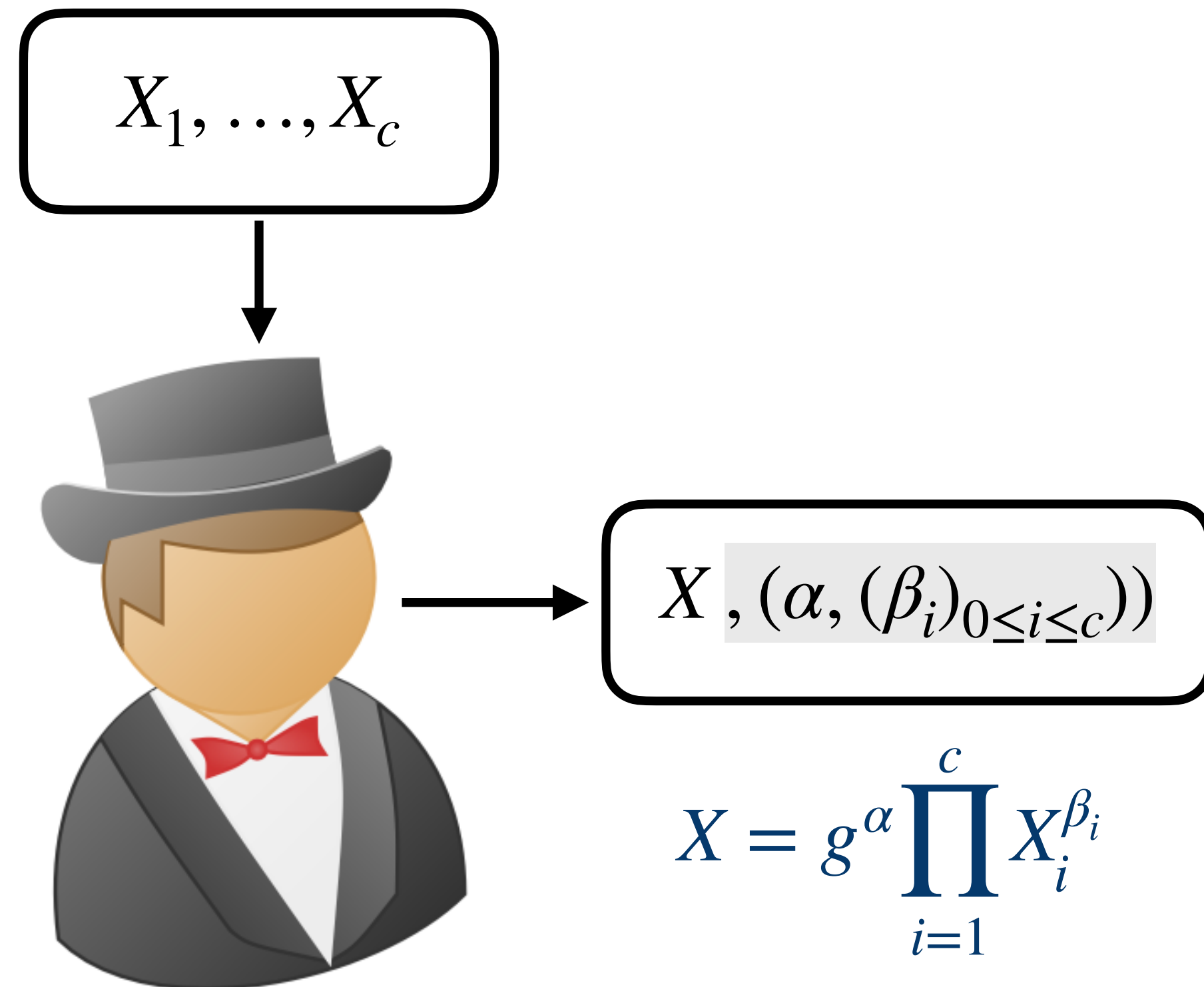
[PV05, FKL18]



The AGM [PV05, FKL18]

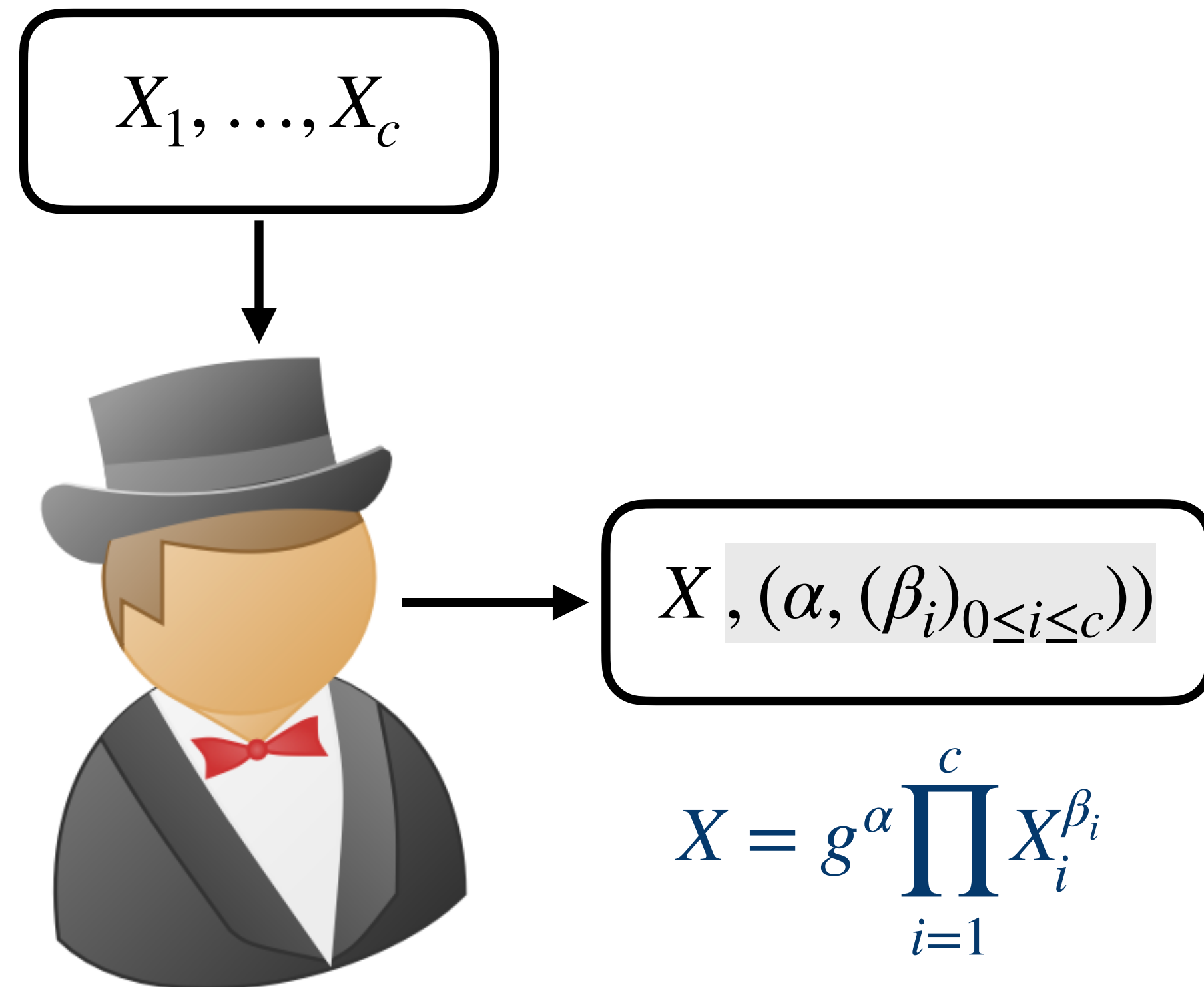


The AGM [PV05, FKL18]



✘ “Hardness in the AGM may not imply hardness in the GGM” [KZZ22]

The AGM [PV05, FKL18]



- ✘ “Hardness in the AGM may not imply hardness in the GGM” [KZZ22]
- ✘ “[...] The AGM is restricted to the Type Safe model games.” [Zha22]

AOMDL Assumption [NRS21]

$\text{ODLog}(X, (\alpha, (\beta_i)_{1 \leq i \leq c}))$

$q \leftarrow q + 1$

return $\alpha + \sum_{i=1}^c \beta_i x_i$

return $\log_g(X)$

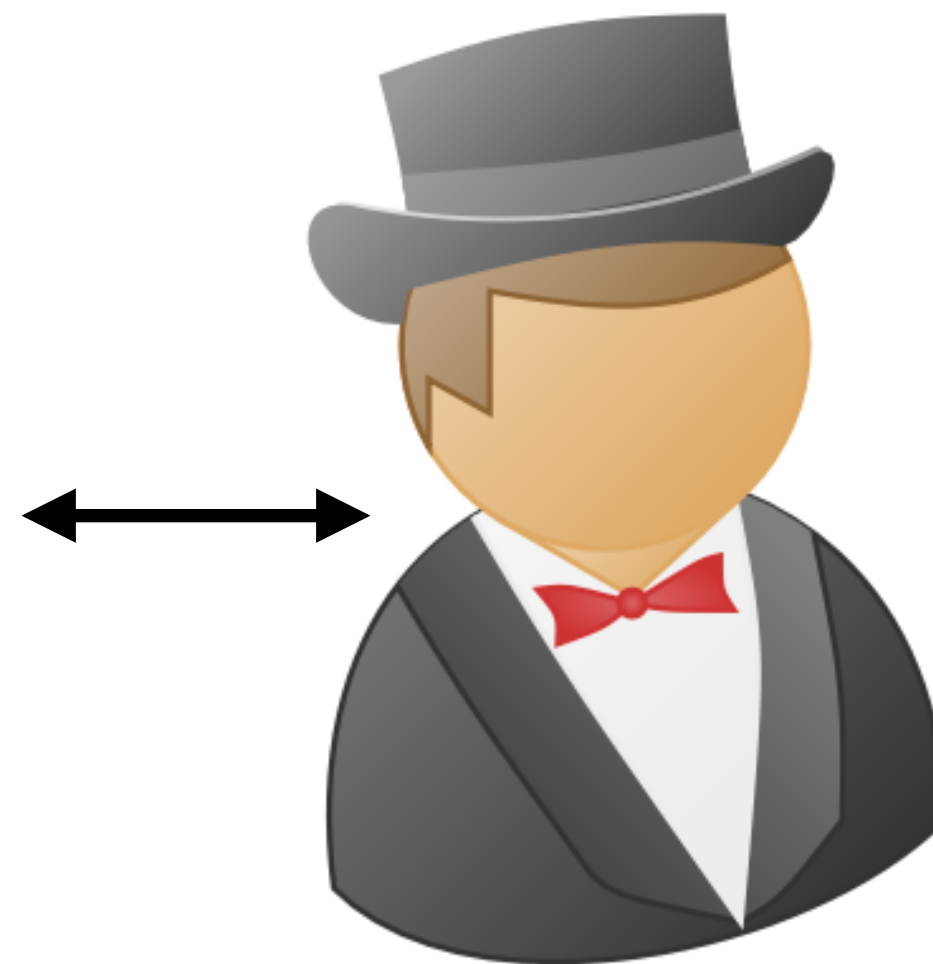
OCH

$c \leftarrow c + 1$

$x_c \leftarrow^{\$} \mathbb{Z}_q$

$X \leftarrow g^{x_c}$

return X



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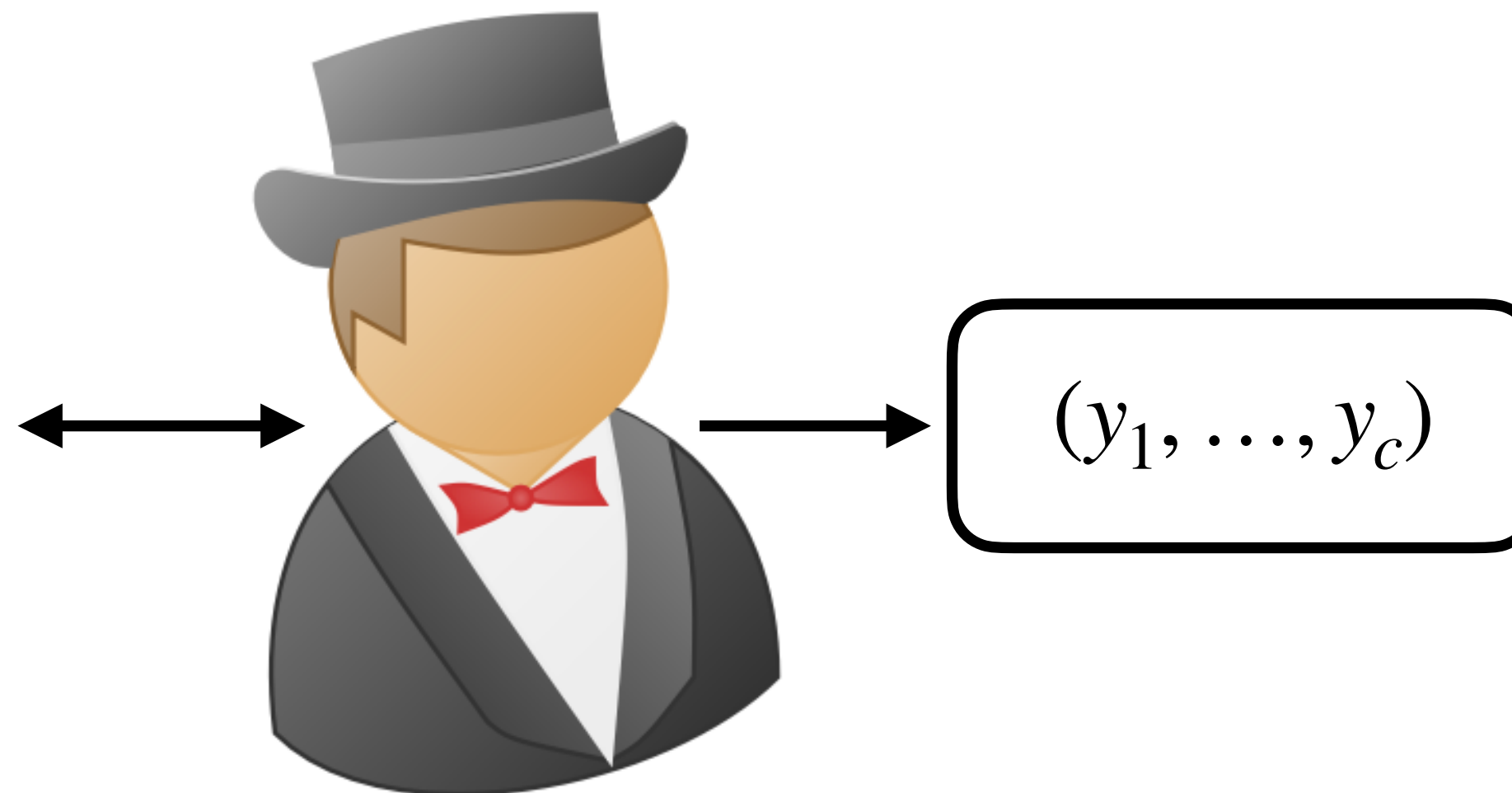
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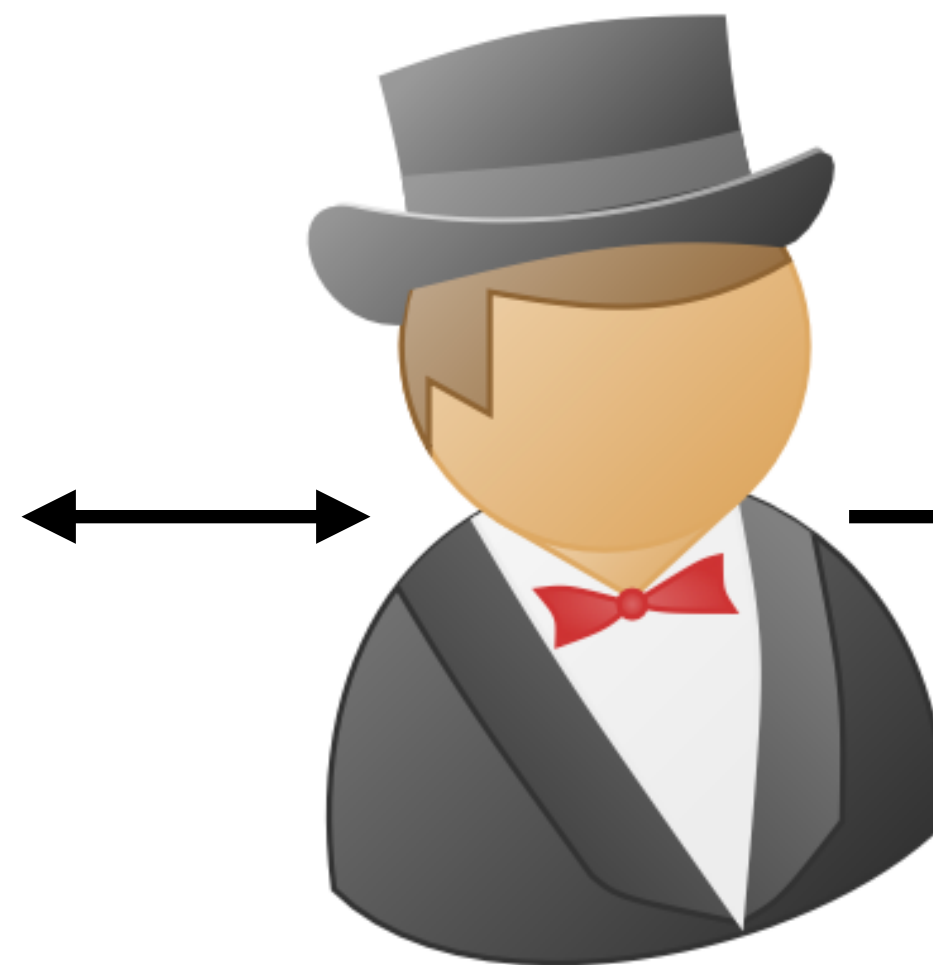
OCH

$c \leftarrow c + 1$

$x_c \leftarrow^{\$} \mathbb{Z}_q$

$X \leftarrow g^{x_c}$

return X



(y_1, \dots, y_c)

$$(q < c) \wedge \bigwedge_{i=1}^c x_i = y_i$$

From **FROST** to **OLAF**

(**H**OPEFULLY) **O**NE **L**AST **F**ROST



From FROST to OLAF

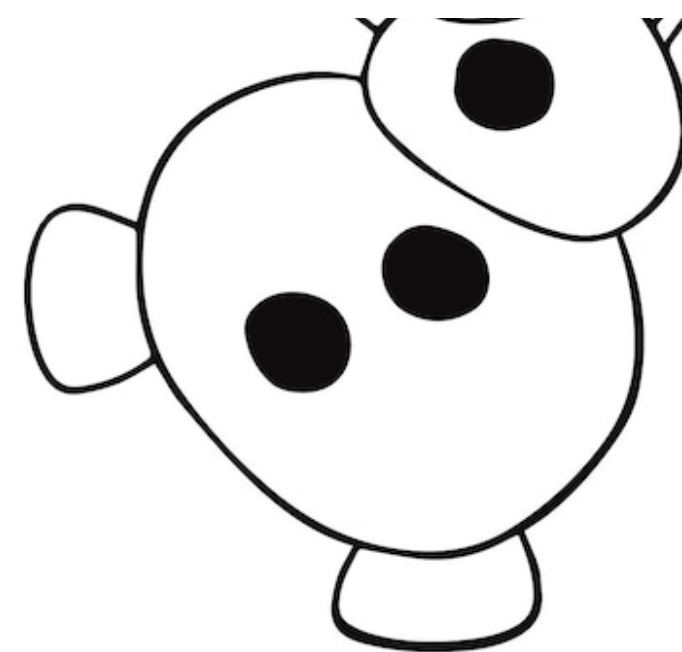


FROST3

From **FROST** to **OLAF**



+



FROST3

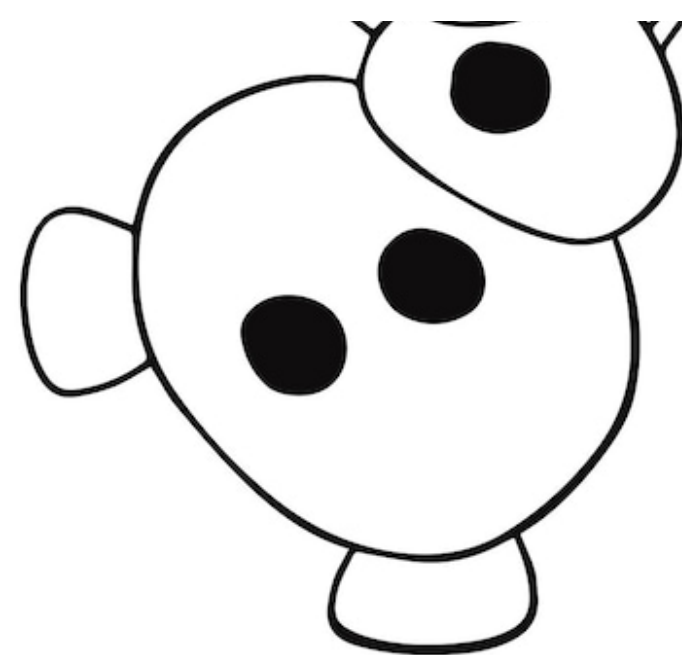
(Simplified)
Pedersen DKG

From FROST to OLAF



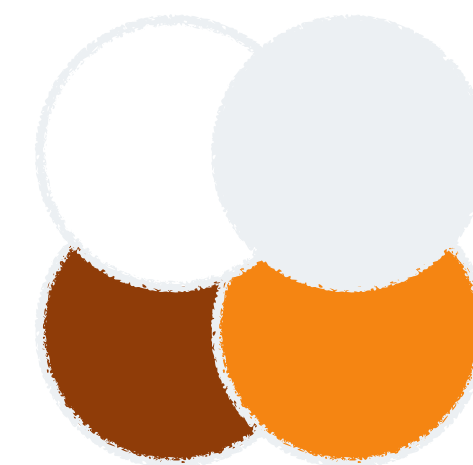
FROST3

+



(Simplified)
Pedersen DKG

+



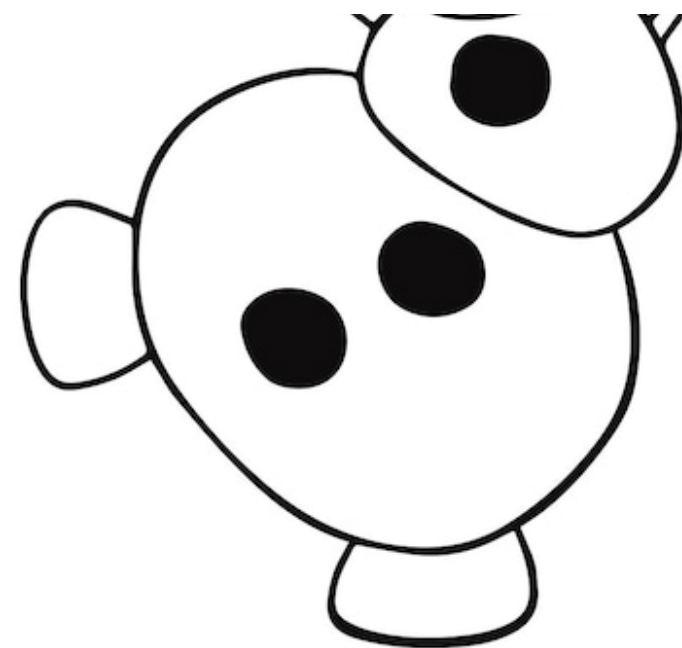
Avoid the AGM
and idealized DKGs

From FROST to OLAF



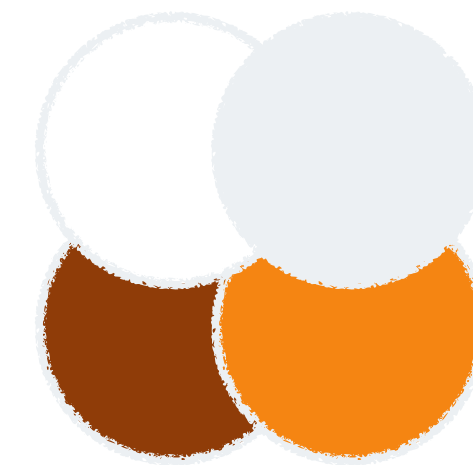
FROST3

+



(Simplified)
Pedersen DKG

+



Avoid the AGM
and idealized DKGs

=



Security Proof for **OLAF**

Our Unforgeability Model

Similar to **TS-UF-0** of [BTZ22]

An honest party accepts NO signing queries **before** key setup (DKG) has finished for it



A

Our Unforgeability Model

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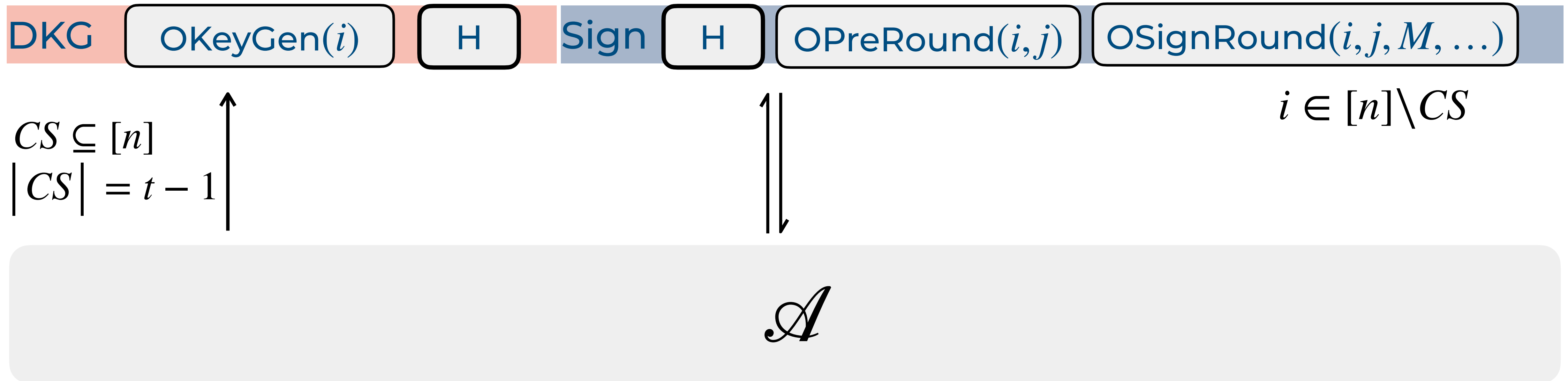
$$\left. \begin{array}{l} CS \subseteq [n] \\ |CS| = t - 1 \end{array} \right\} \uparrow$$

A

Our Unforgeability Model

Similar to **TS-UF-0** of [BTZ22]

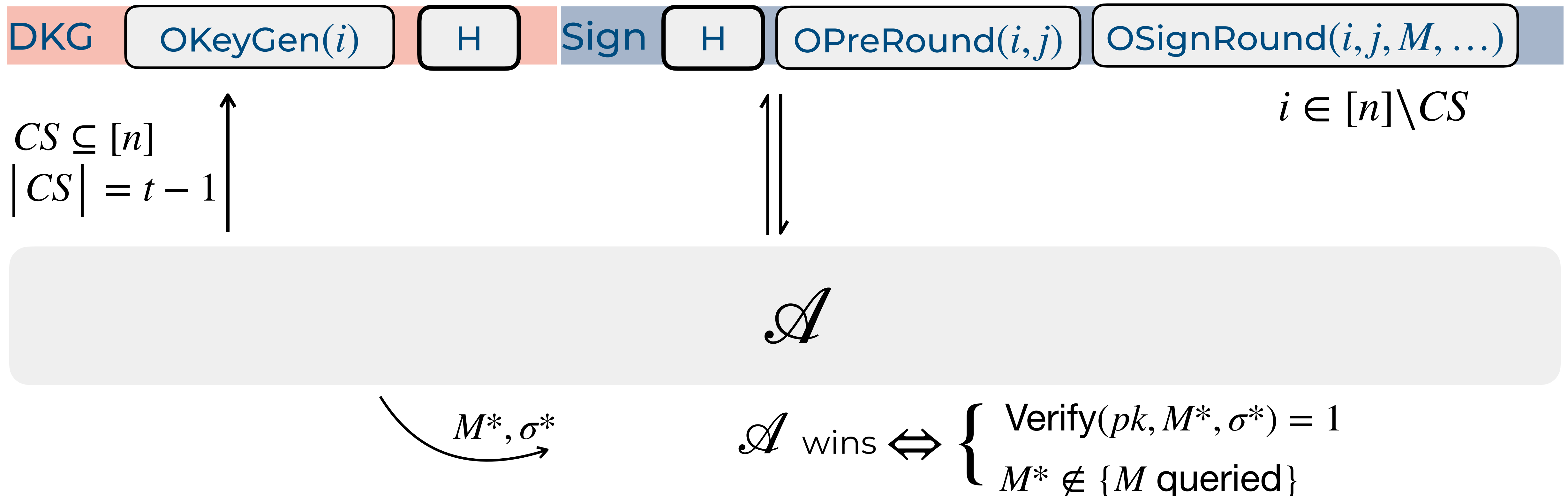
An honest party accepts NO signing queries **before** key setup (DKG) has finished for it



Our Unforgeability Model

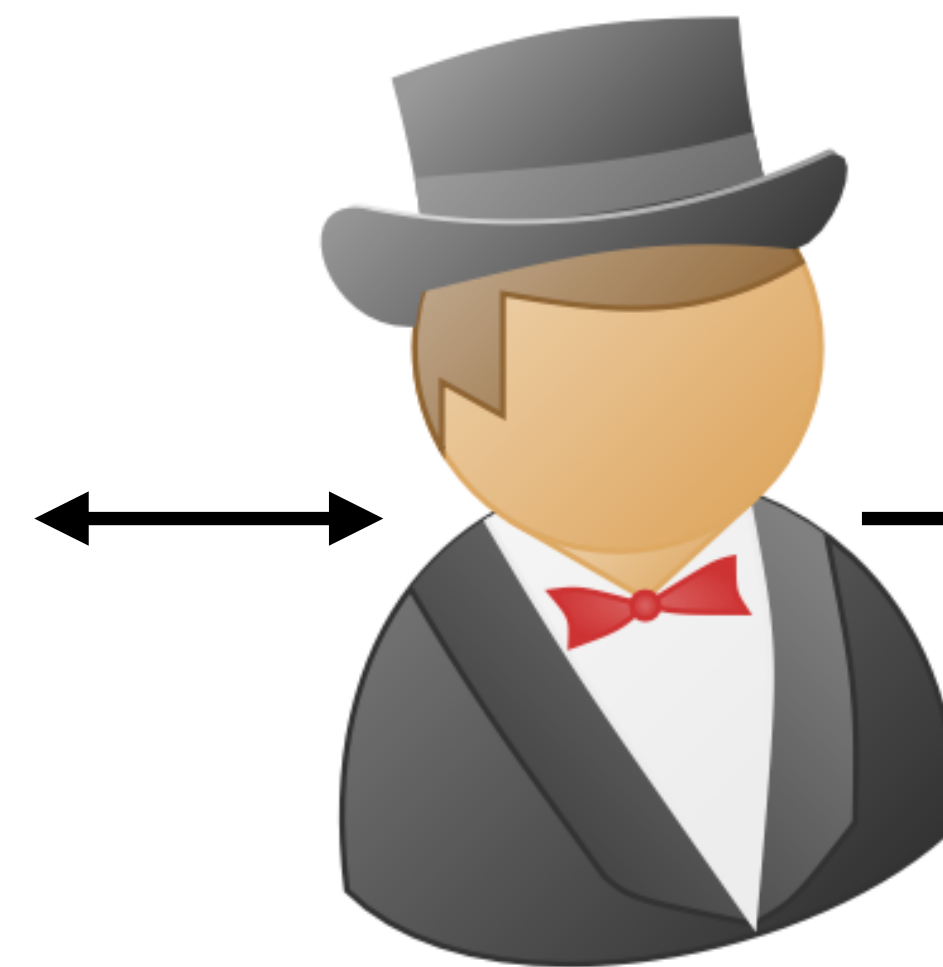
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AOMDL Assumption [NRS21]

$\text{ODLog}(X, (\alpha, (\beta_i)_{1 \leq i \leq c}))$ <hr/> $q \leftarrow q + 1$ $\text{return } \alpha + \sum_{i=1}^c \beta_i x_i$ $\text{return } \log_g(X)$	OCH <hr/> $c \leftarrow c + 1$ $x_c \leftarrow \$ \mathbb{Z}_q$ $X \leftarrow g^{x_c}$ $\text{return } X$
---	--



(y_1, \dots, y_c)

$$(q < c) \wedge \bigwedge_{i=1}^c x_i = y_i$$

Proper (A)OMDL proof has less DLog queries than challenges.



Counting DLog queries could be the most tricky part.

AOMDL Proof for Single-Signer Schnorr

OCH

Challenge Oracle

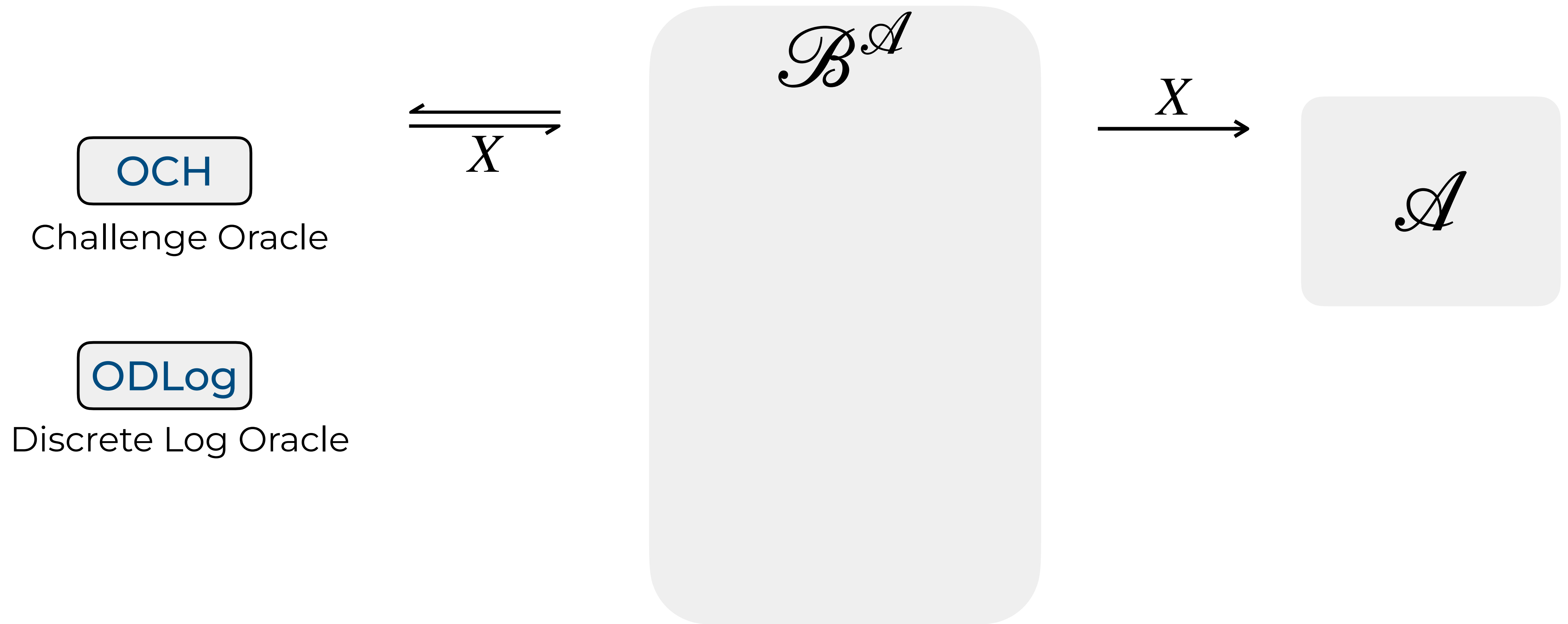
ODLog

Discrete Log Oracle

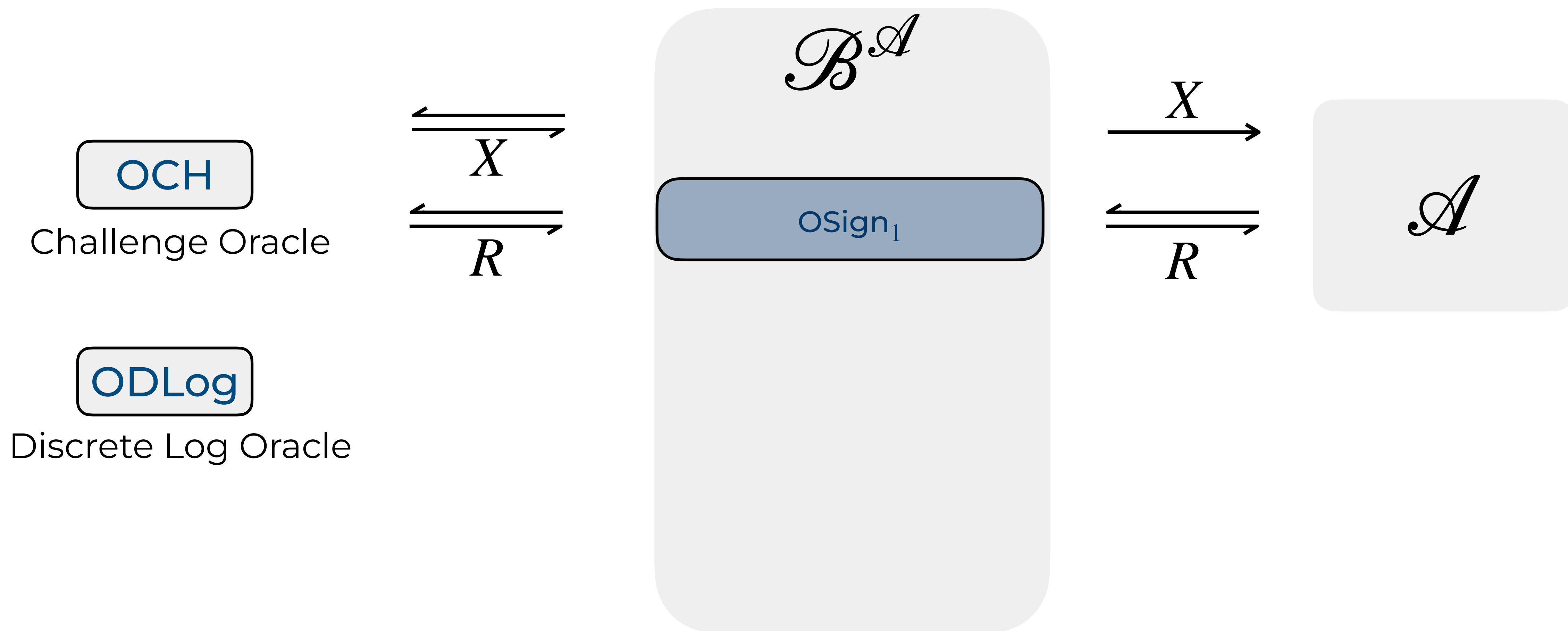
B^A

A

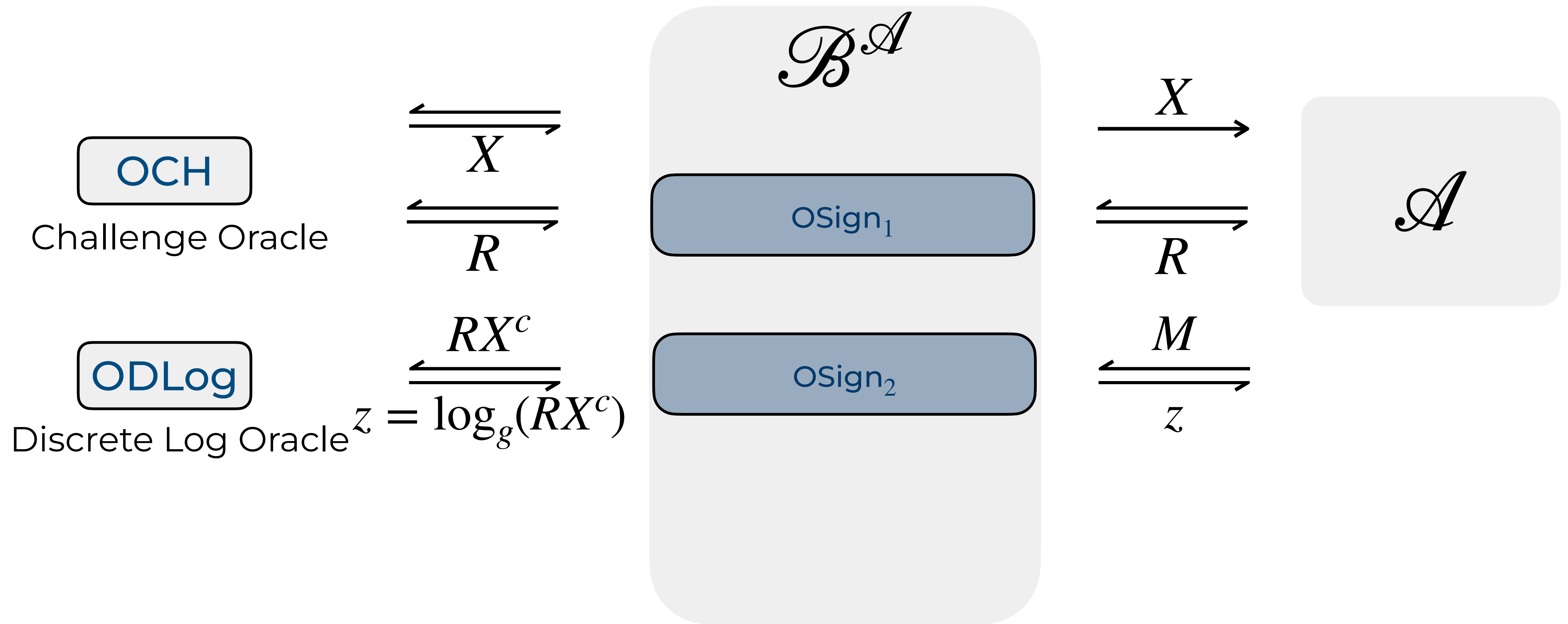
AOMDL Proof for Single-Signer Schnorr



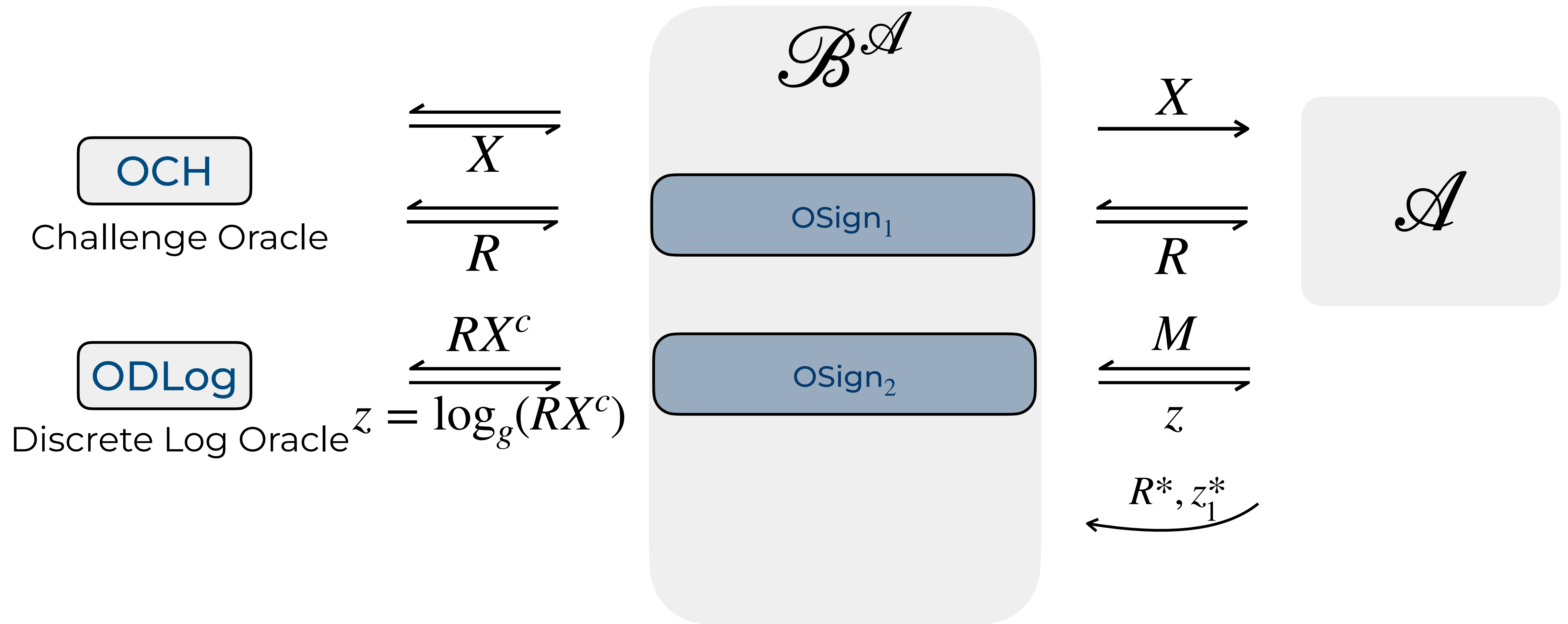
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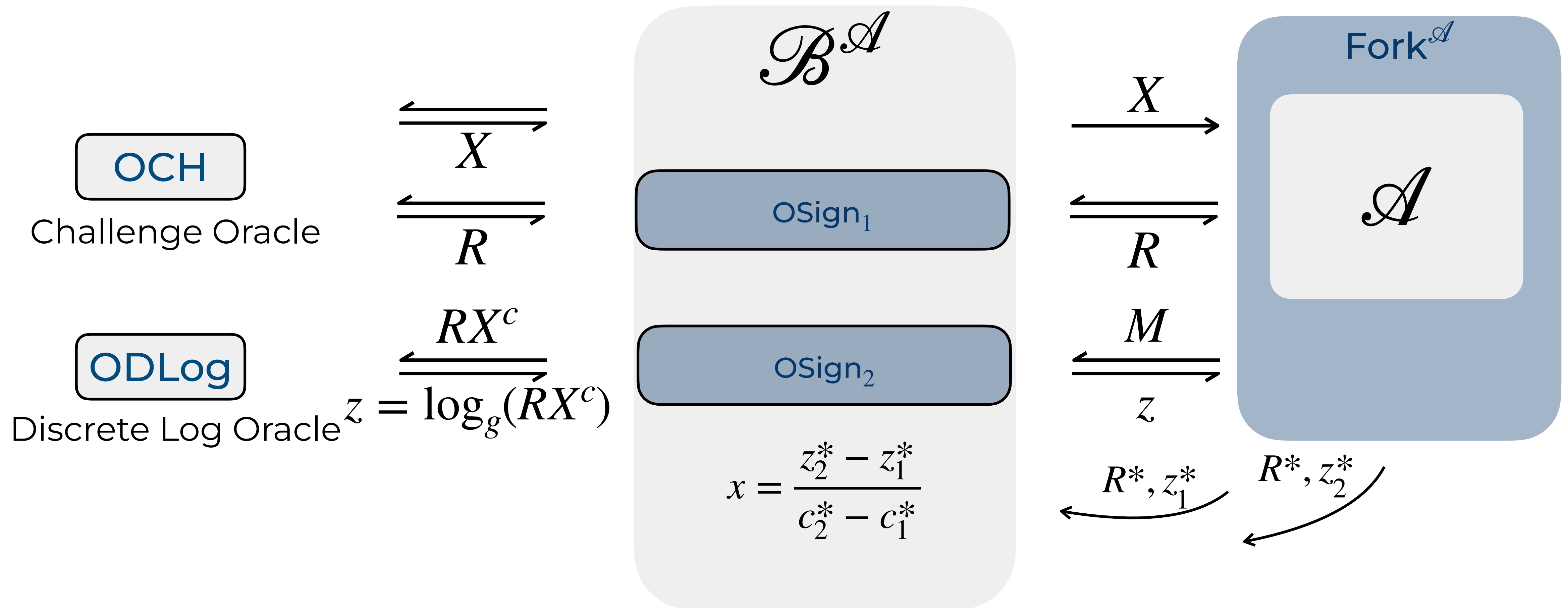
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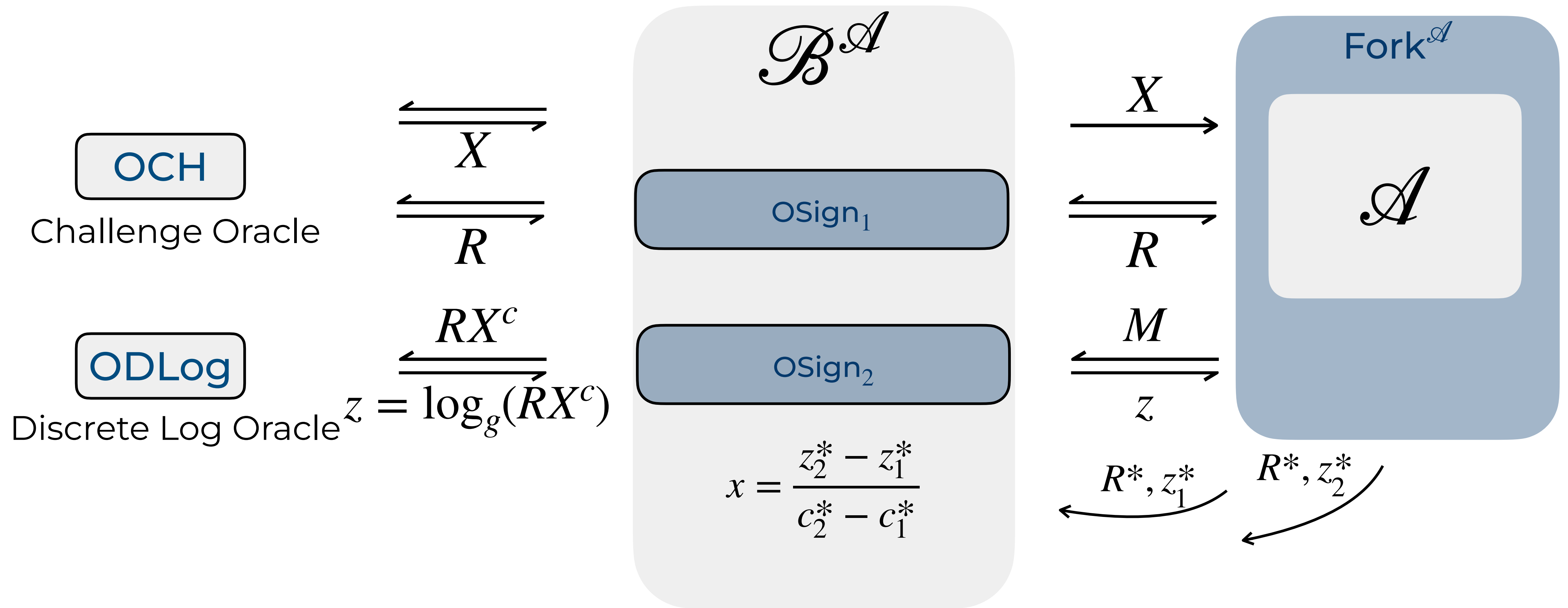
AOMDL Proof for Single-Signer Schnorr



AOMDL Proof for Single-Signer Schnorr



AOMDL Proof for Single-Signer Schnorr



$$\#\text{ODLog} = \#\text{OSign}_2 \leq \text{OSign}_1 = \#\text{OCH} - 1$$

AOMDL Proof for Threshold Schnorr

Proof of single-signer Schnorr is well understood

1

2



AOMDL Proof for Threshold Schnorr

Proof of single-signer Schnorr is well understood

- 1 Simple simulation of signing queries
- 2



AOMDL Proof for Threshold Schnorr

Proof of single-signer Schnorr is well understood

- 1 Simple simulation of signing queries
- 2 Rely on forking [PS00], [BN06], optimal tightness [Seurin11, FJS19]



AOMDL Proof for Threshold Schnorr

Techniques do NOT carry over to the threshold setting

1

2

x_i 's: signing key shares



Blockstream

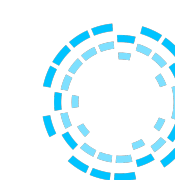


AOMDL Proof for Threshold Schnorr

Techniques do NOT carry over to the threshold setting

- 1 Using **two** nonces D, E instead of one nonce (see FROST [KG20])
To answer two signing queries on **two executions** for same D, E
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Blockstream

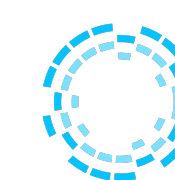


AOMDL Proof for Threshold Schnorr

Techniques do NOT carry over to the threshold setting

- 1 Using **two** nonces D, E instead of one nonce (see FROST [KG20])
To answer two signing queries on **two executions** for same D, E
- 2 Consider **Pedersen DKG**
Reduction needs to know each x_i in $\sum_i x_i = x$ instead of just x to solve AOMDL

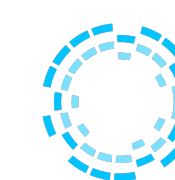
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Blockstream



We only have enough DLog queries for **two** executions of signing.

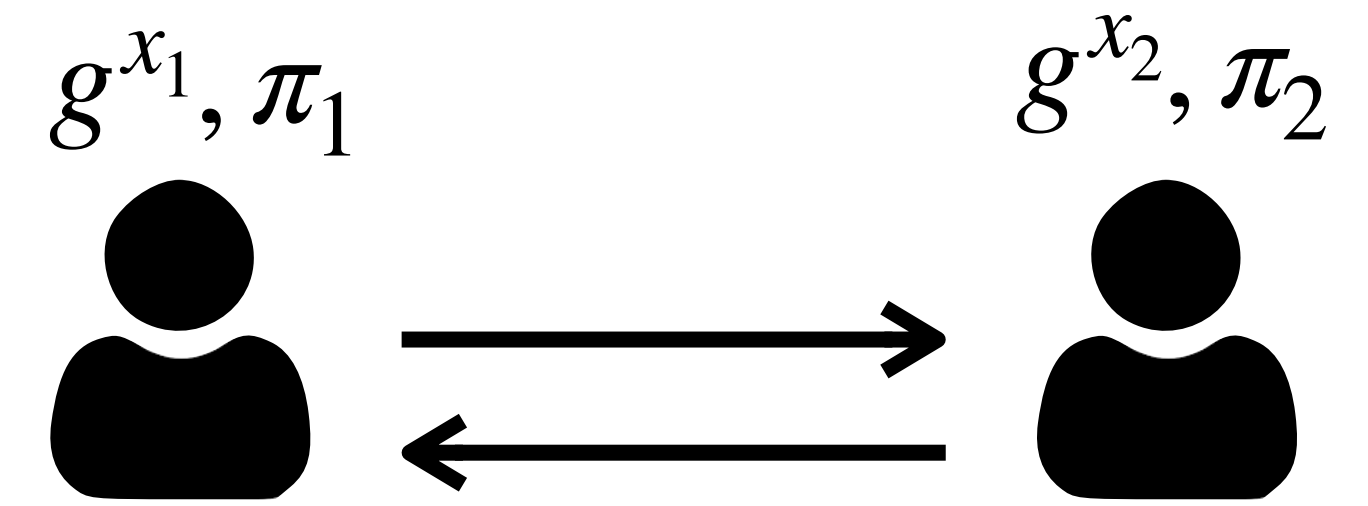


Our Proof Strategy for OLAF

- 1 Use Pedersen DKG with **proofs of possession (PoPs)** as in [CKM21]

PoP is Schnorr-like proof of knowledge

- 2 Use a novel **forking proof technique** to avoid AGM



x_i 's: signing key shares

π_i 's: proof of possession

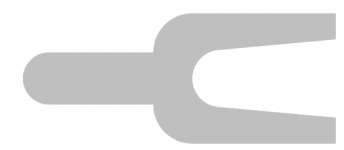
g^{x_i} 's: public key shares

Existing Forking Techniques: Overview

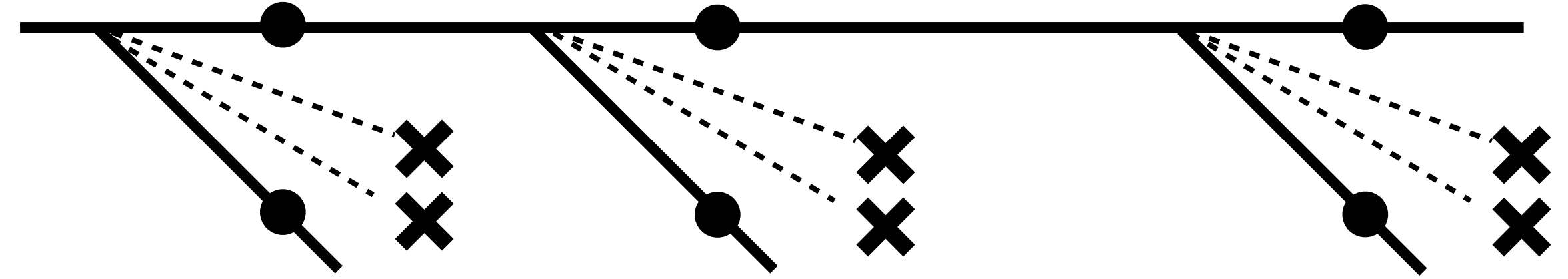


[Bellare-Neven-05] bi-forking lemma $\epsilon' = \epsilon^2/q$

Existing Forking Techniques: Overview



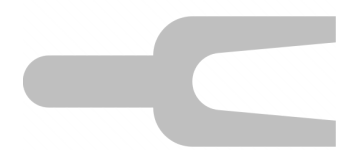
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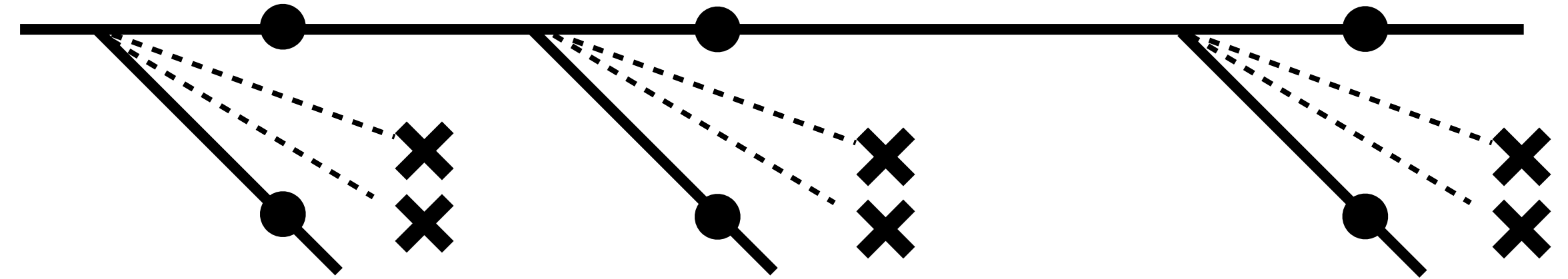
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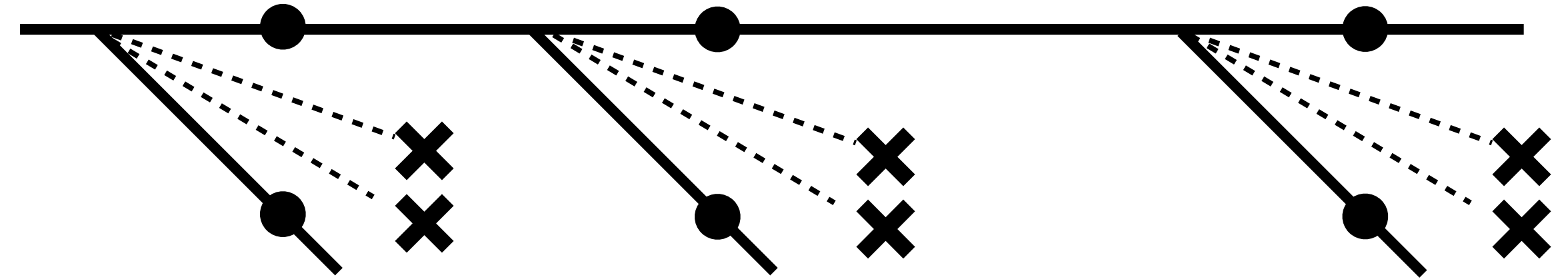


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Existing Forking Techniques: Overview



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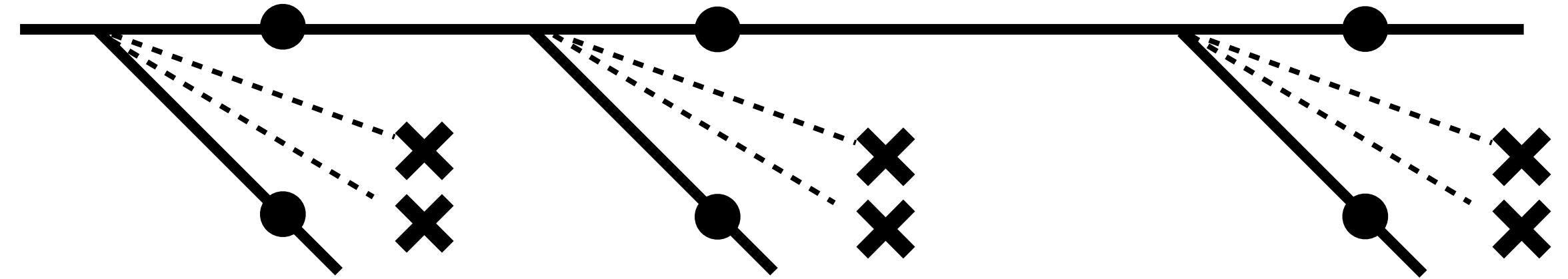
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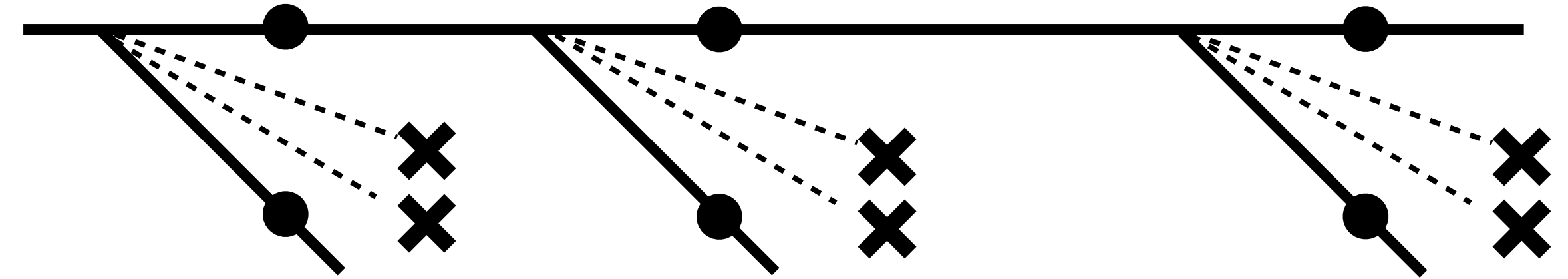
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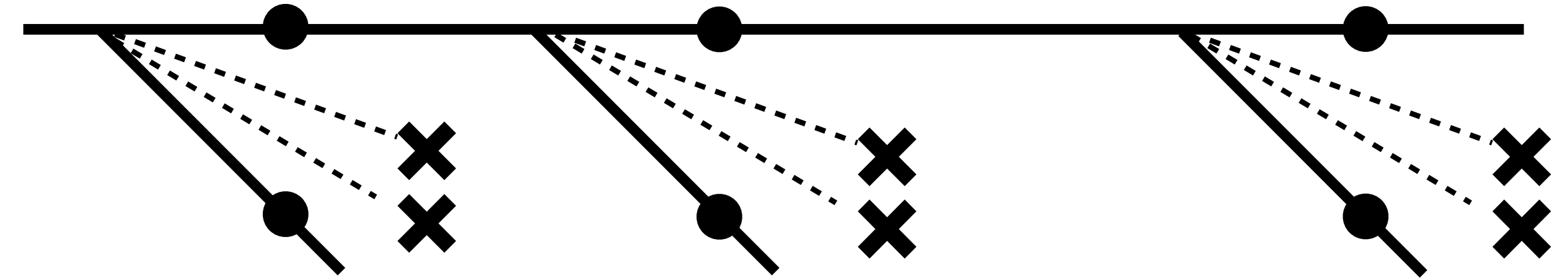
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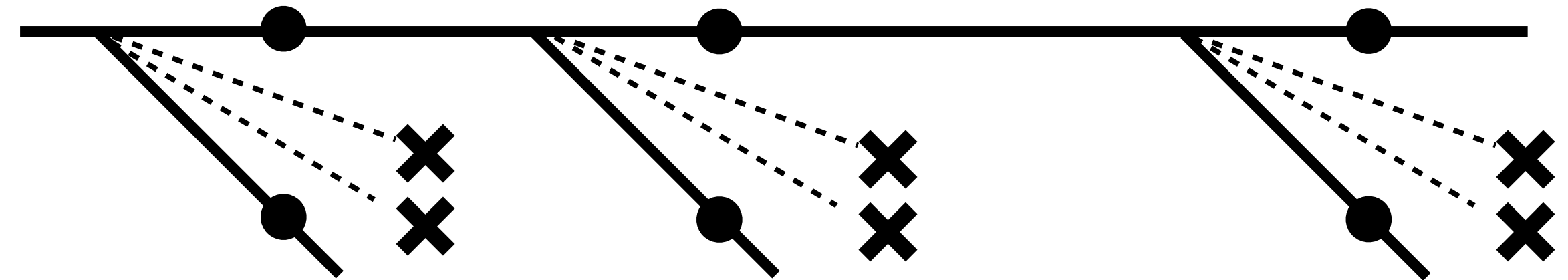
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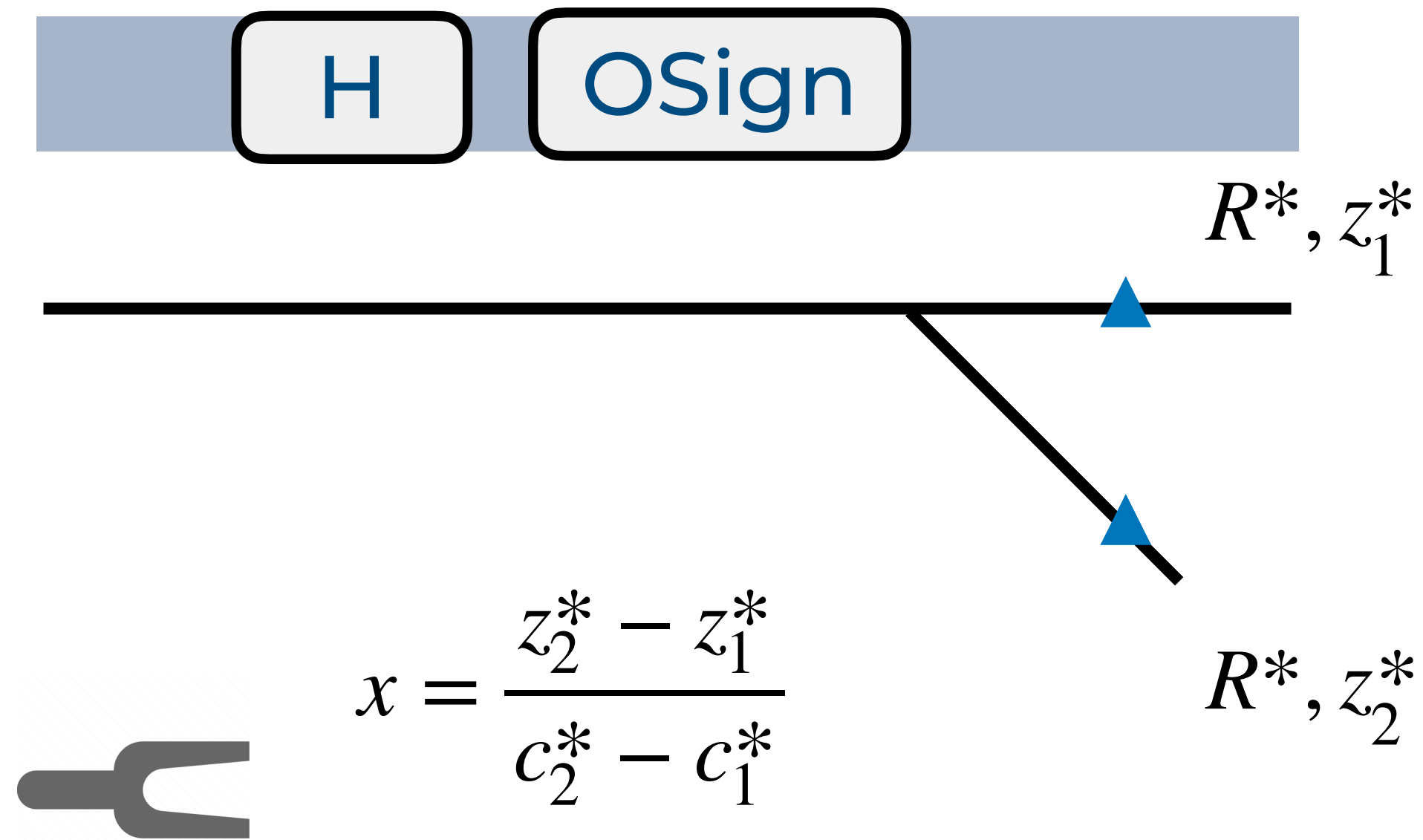
- ✓ Multiple extraction points
- ✗ Polynomial number of executions



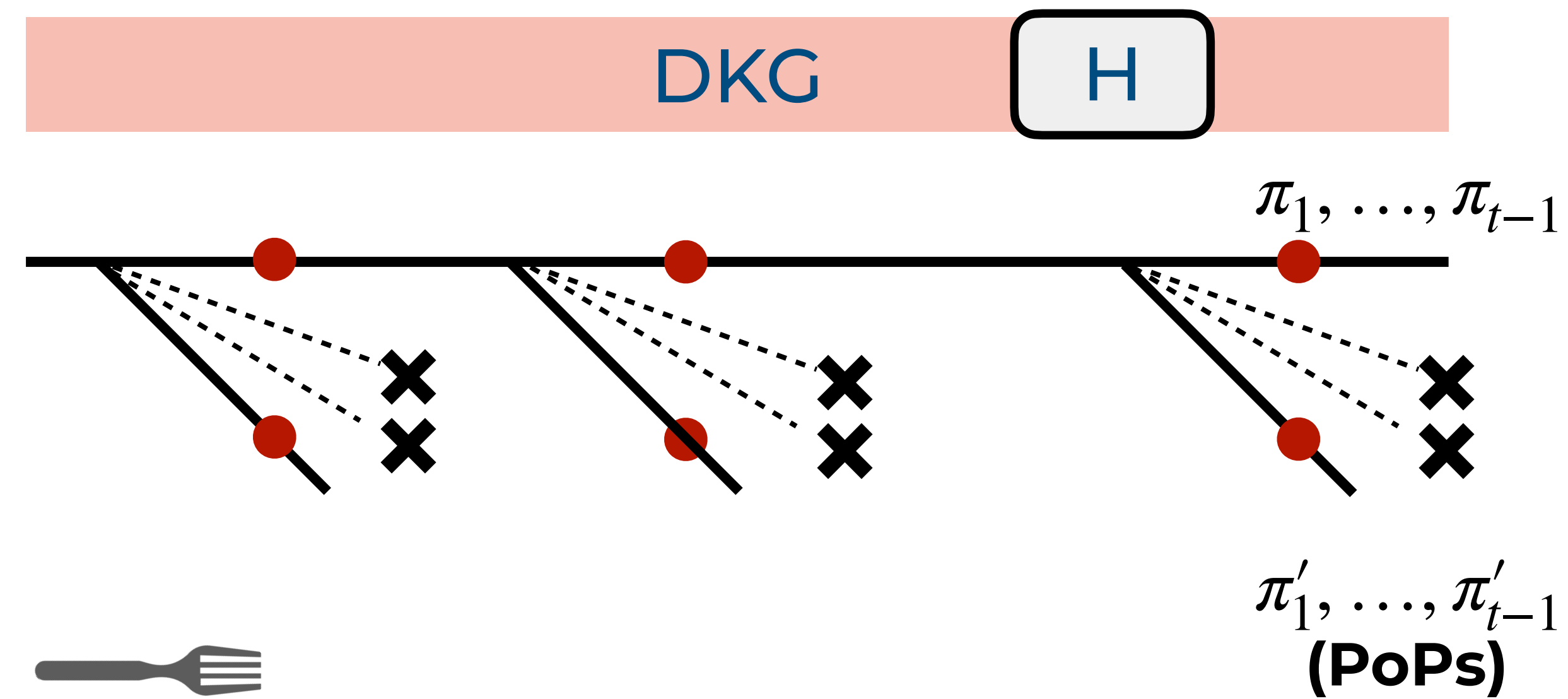
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Existing Forking Techniques: In Our Proof

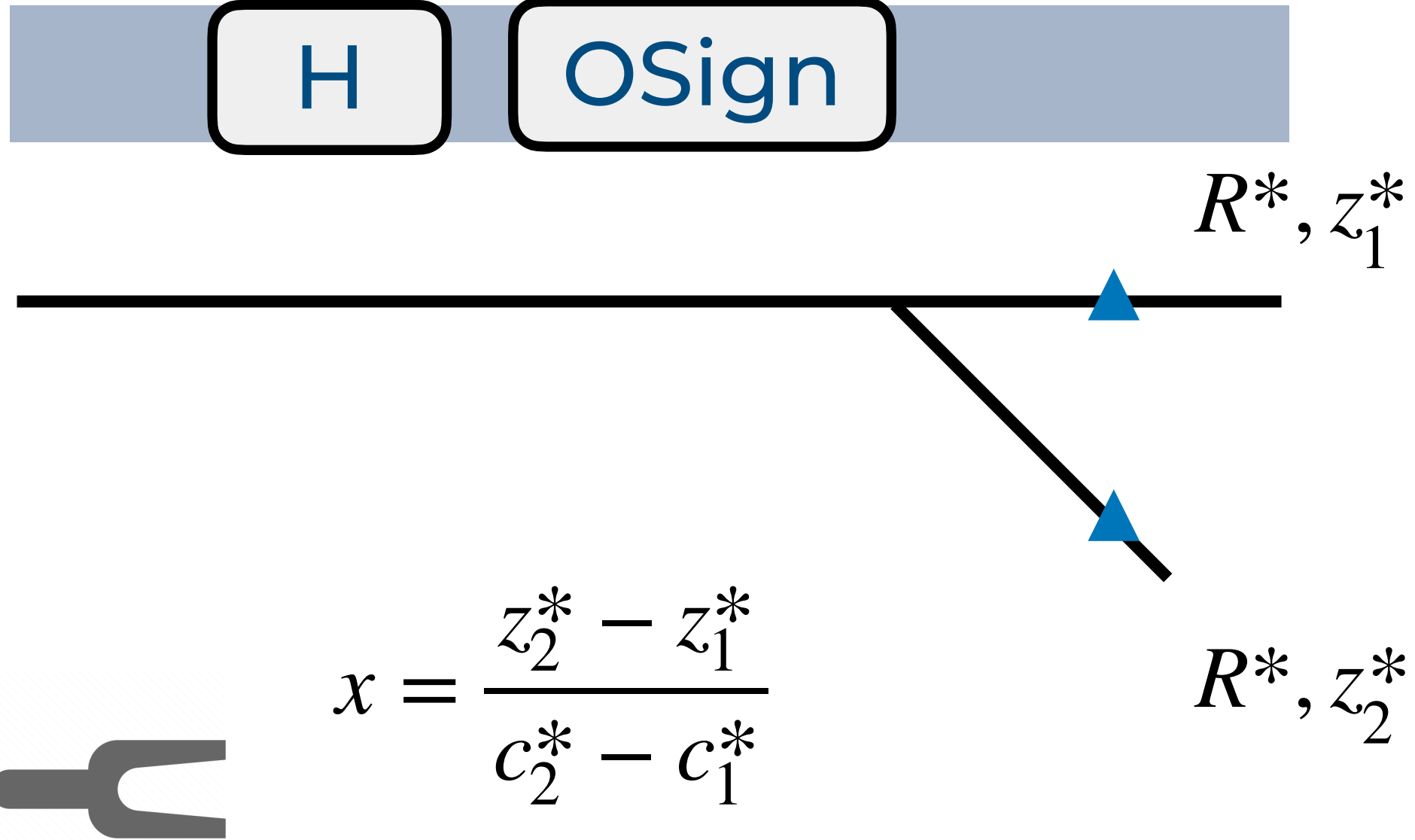


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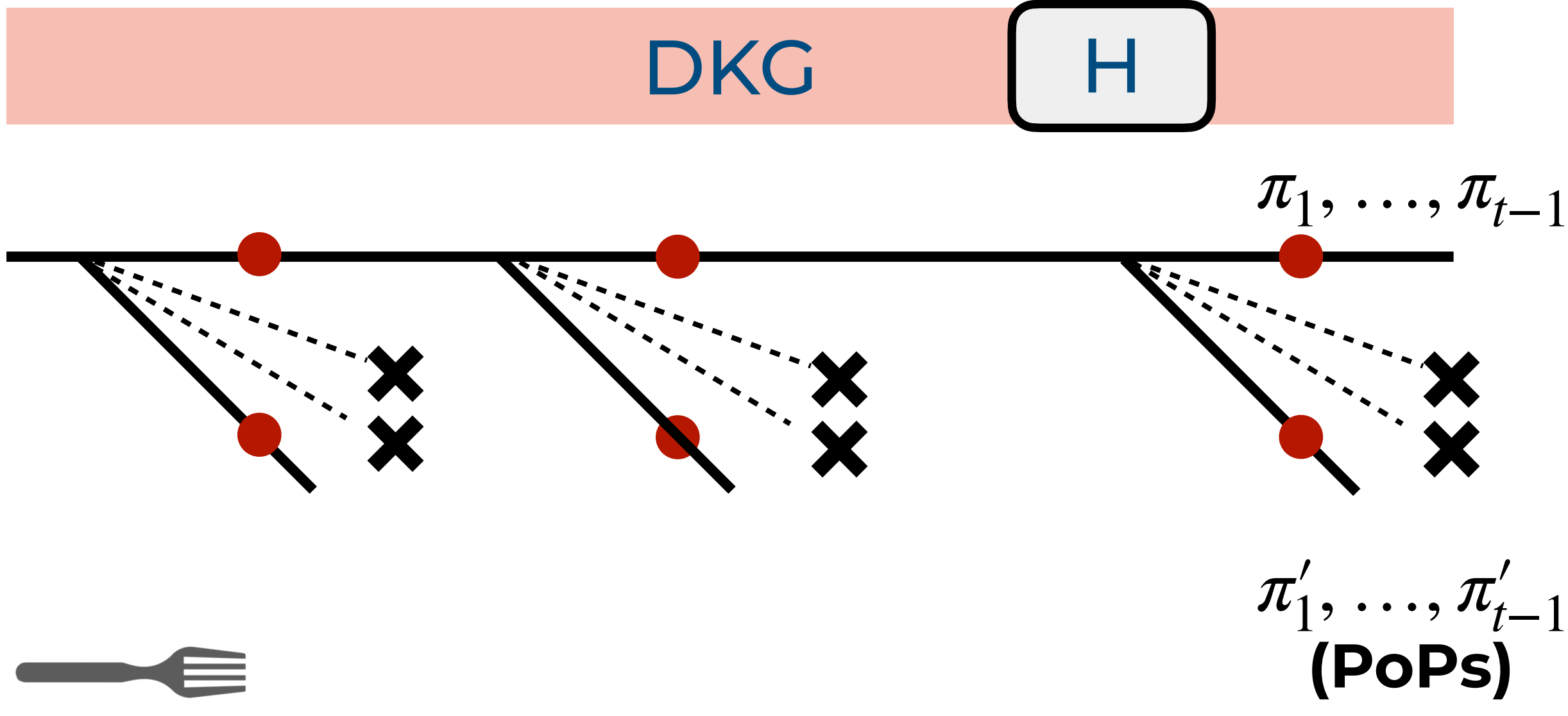


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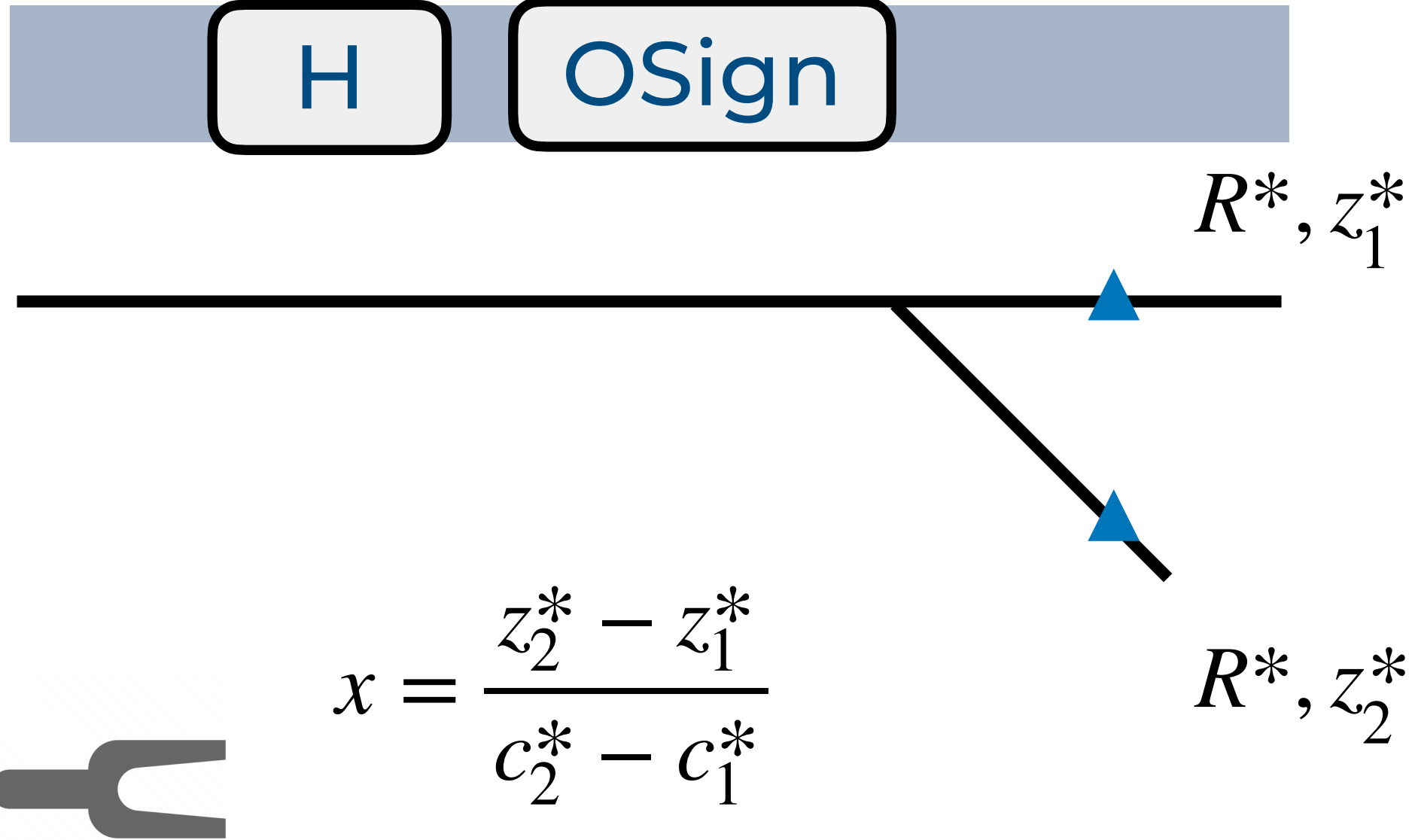
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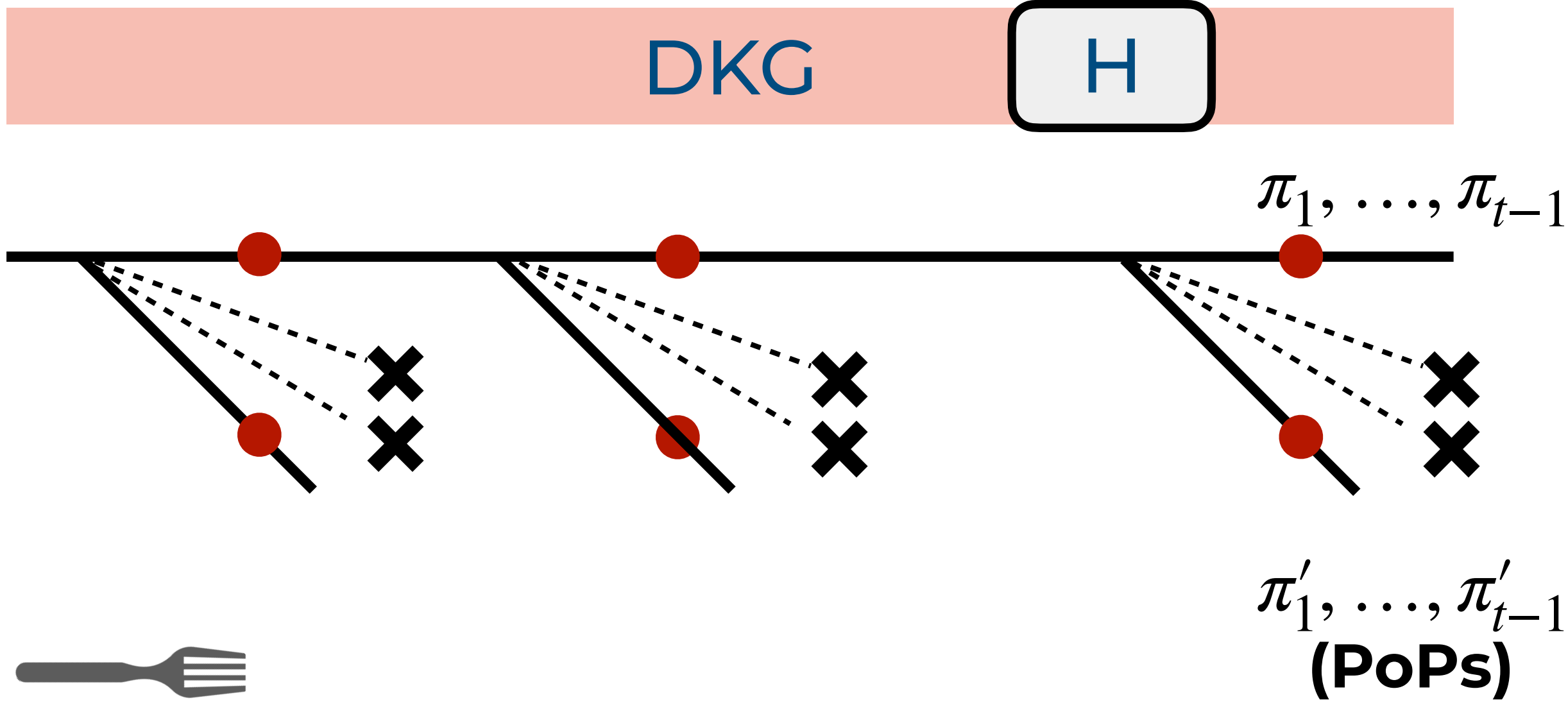
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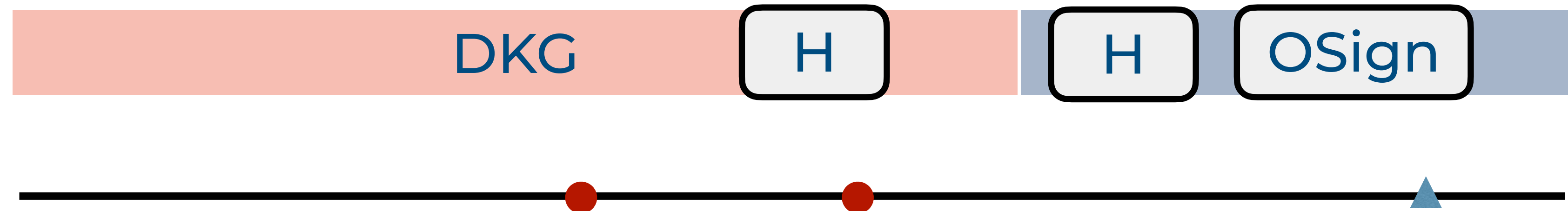
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➔ Can extract x_1, \dots, x_{t-1} from PoPs

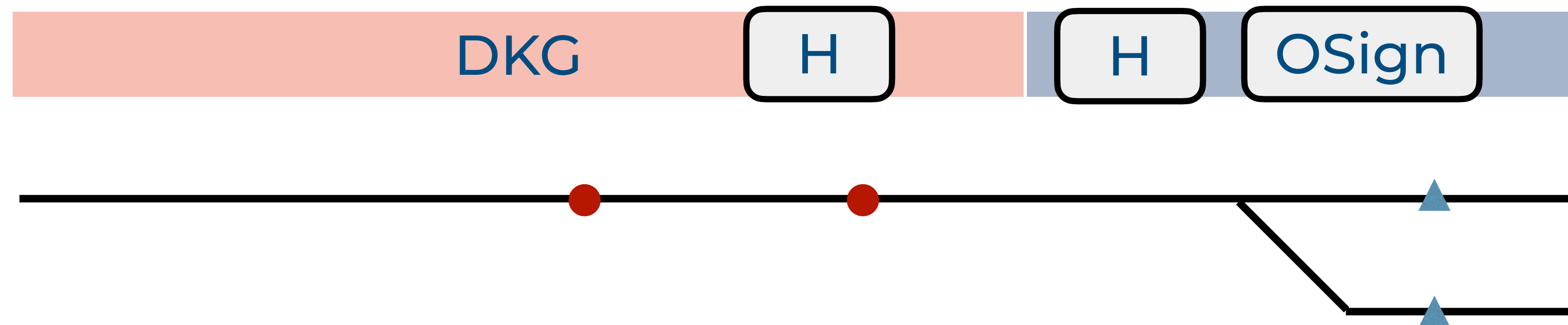
The Mixed Forking



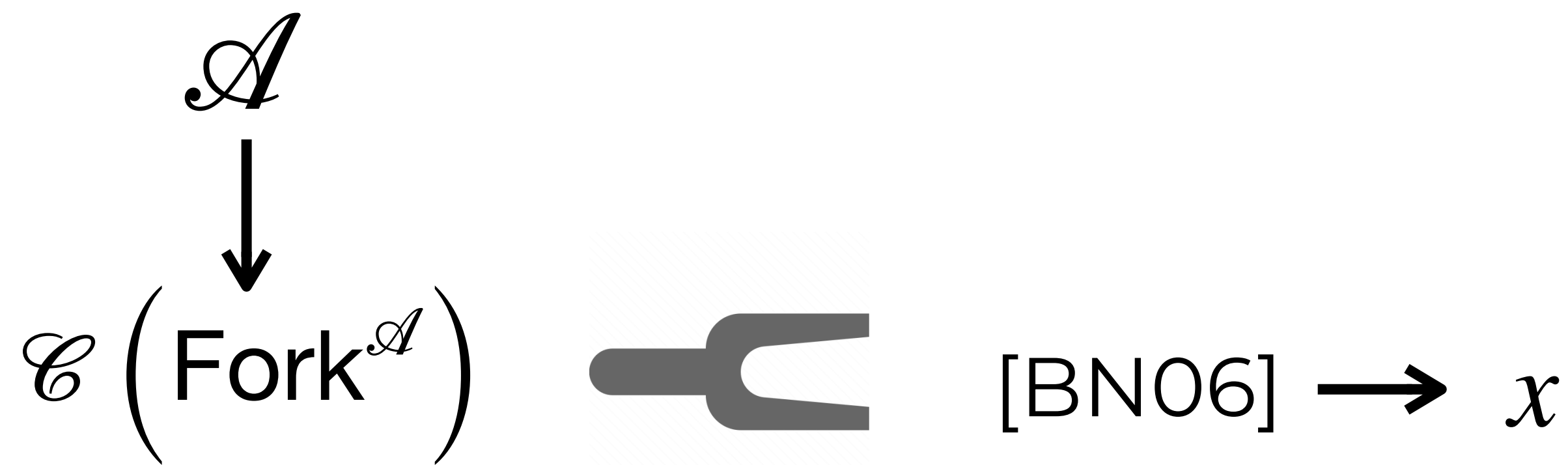
Two-step approach

A

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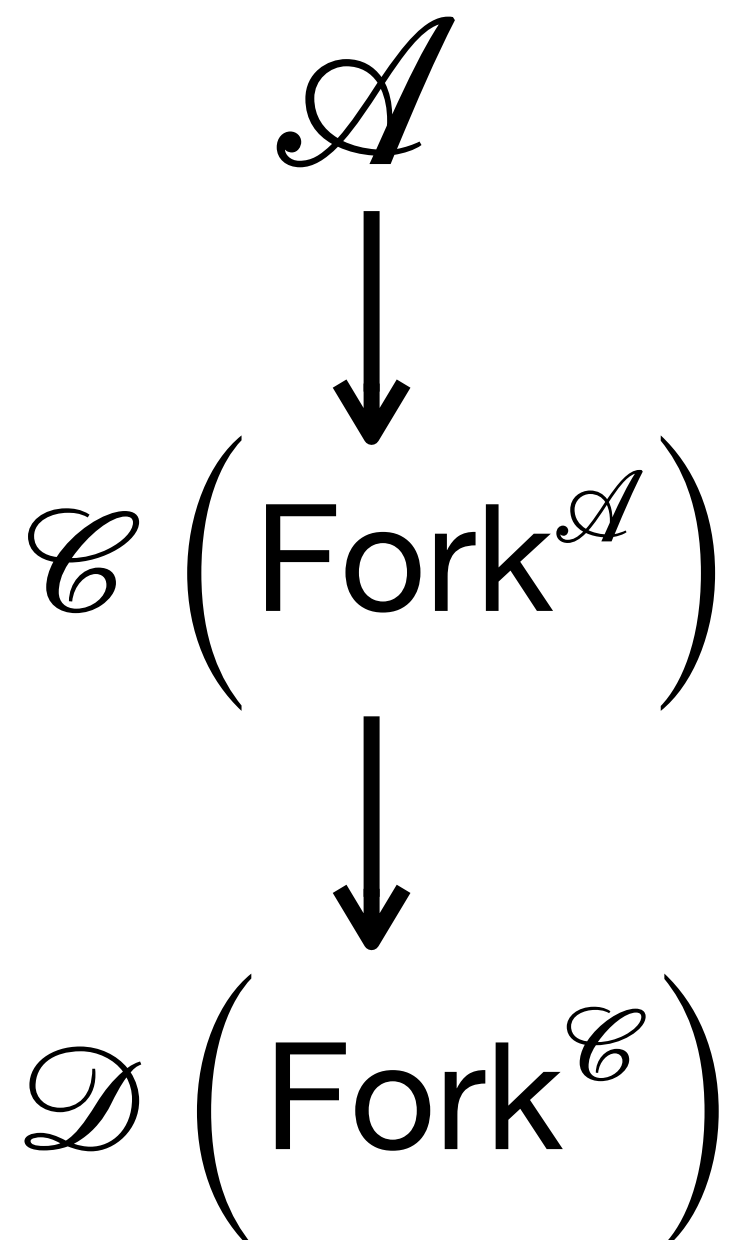
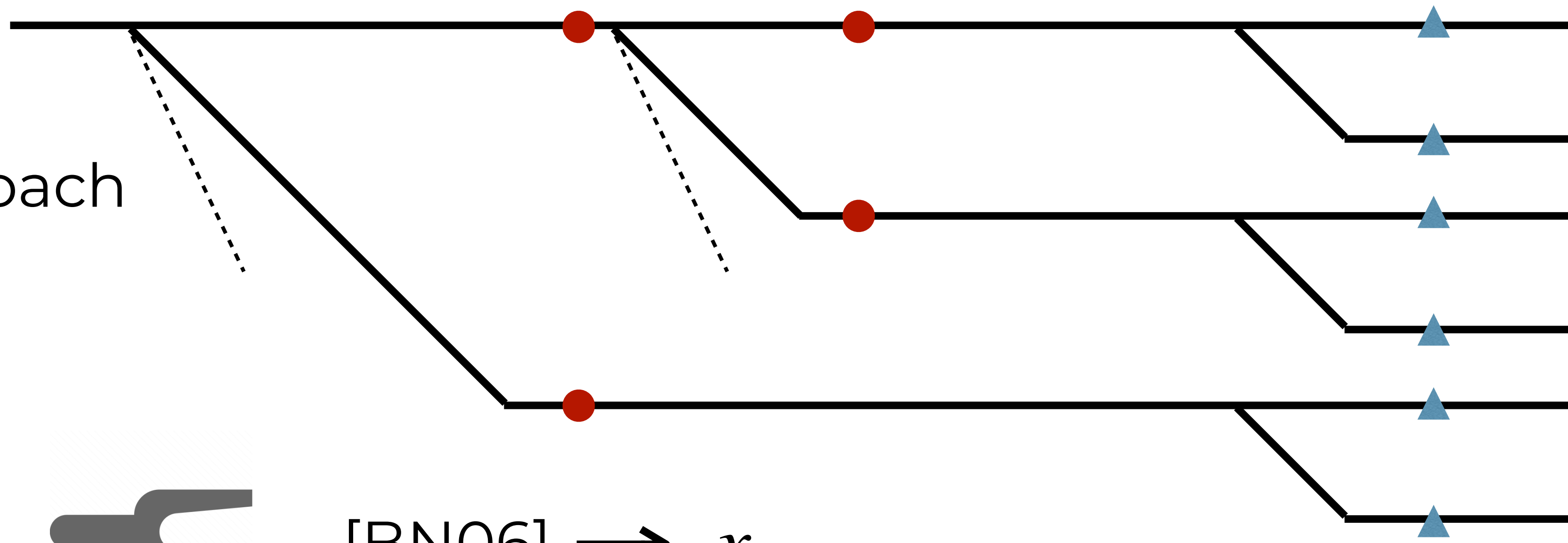
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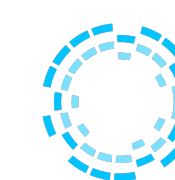


[BN06] → x

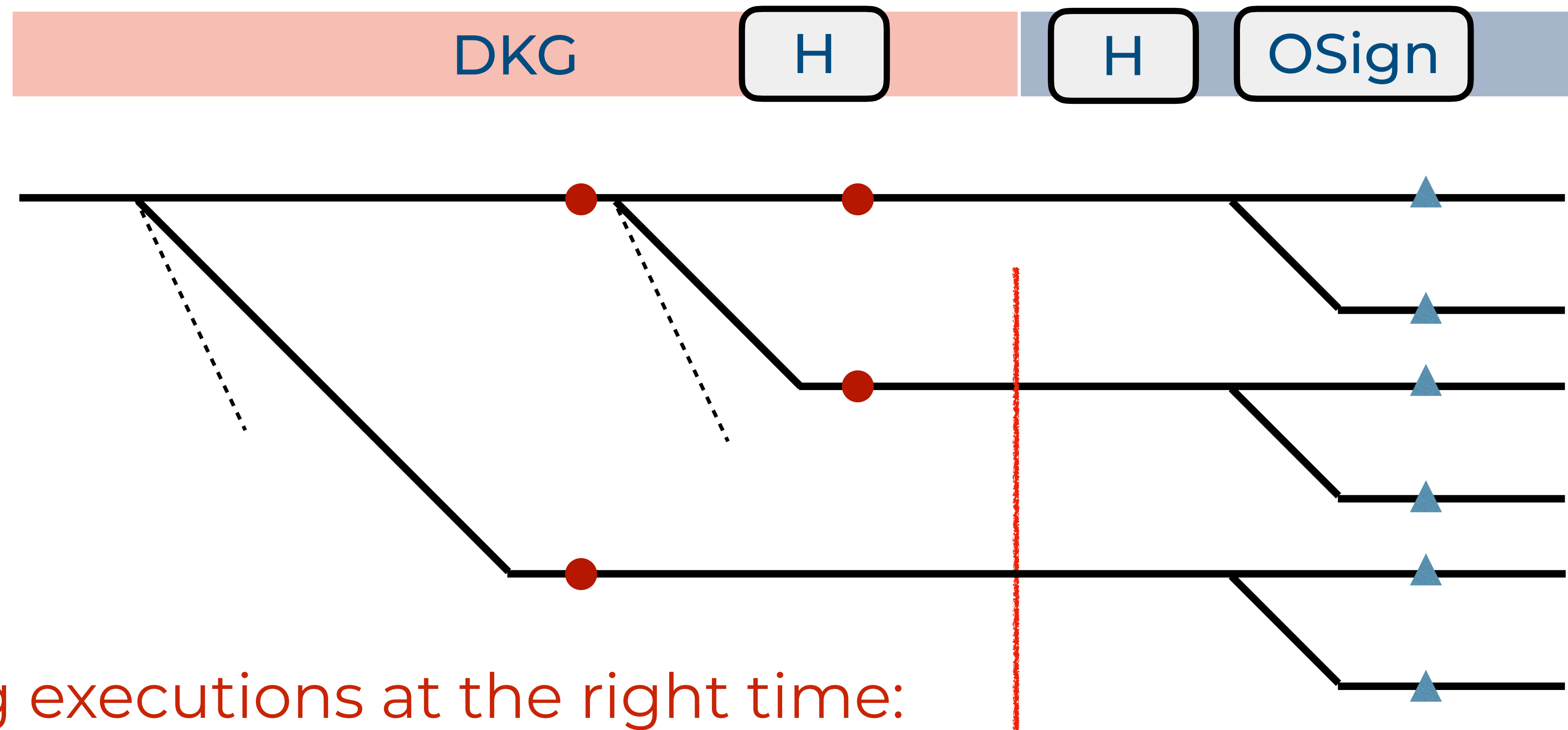


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Reducing Number of DL Queries

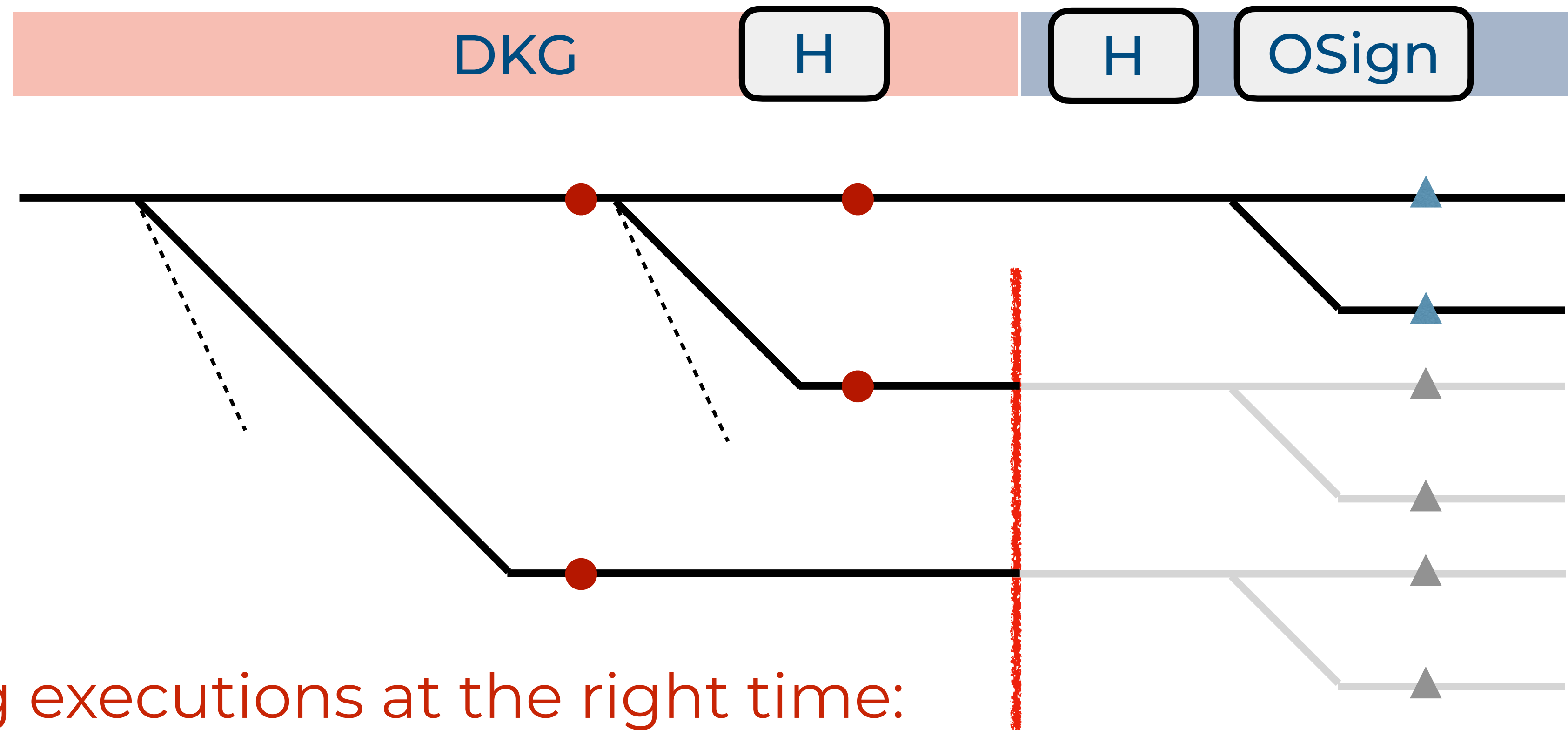


Pruning executions at the right time:

right after the DKG completion

then $\mathcal{D}' \approx \mathcal{D}$ but only **two executions** of \mathcal{A} needs signing simulation

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 - In **AOMDL + ROM**



Our Work:

Practical Schnorr Threshold Signatures without the Algebraic Group Model

Hien Chu, Paul Gerhart, Tim Ruffing, Dominique Schröder

CRYPTO 2023



For Robustness, see:

ROAST: Robust Asynchronous Schnorr Threshold Signatures

Tim Ruffing, Viktoria Ronge, Elliott Jin, Jonas Schneider-Bensch, and Dominique Schröder

CCS 2023