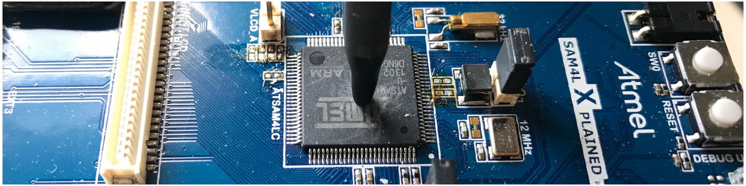

Prouff & Rivain's Security Proof of Masking, Revisited

Tight Bounds in the Noisy Leakage Model

Loïc Masure   François-Xavier Standaert

CRYPTO 2023, Santa Barbara, August 21st

https://eprint.iacr.org/2023/883

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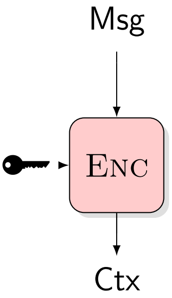
Context : Side-Channel Analysis (SCA)



: N bits

Black-box cryptanalysis: 2^N

Context : Side-Channel Analysis (SCA)

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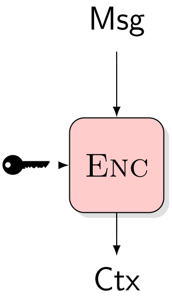

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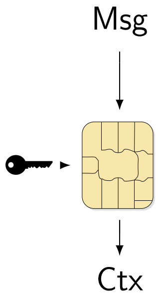

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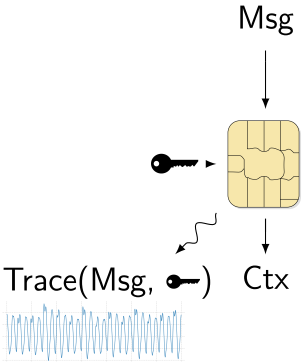

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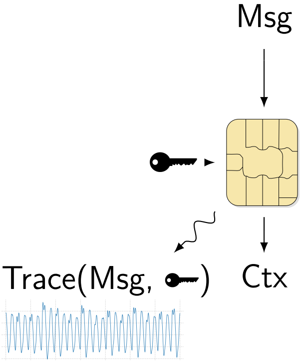

: N bits

Black-box cryptanalysis: 2^N

Side-Channel Analysis: $2^n \cdot \frac{N}{n}, n \ll N$

Trace : power, EM, acoustics, runtime, ...

The Counter-Measure: Masking

Masking, aka *MPC on silicon*: linear secret sharing over a finite field $(\mathbb{F}, \star, \cdot)$

Y (secret)

The Counter-Measure: Masking

Masking, aka *MPC on silicon*: linear secret sharing over a finite field $(\mathbb{F}, \star, \cdot)$

$$\begin{tikzcd}
 & & Y\,(\text{secret}) & & \\
 & & \star \arrow[u] & & \\
Y_1 \arrow[urr] & & Y_2 \arrow[u] & \cdots & Y_d \arrow[ull]
\end{tikzcd}$$

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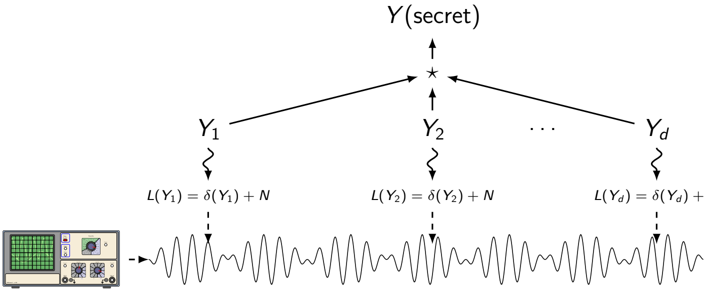

Masking amplifies noise [1]

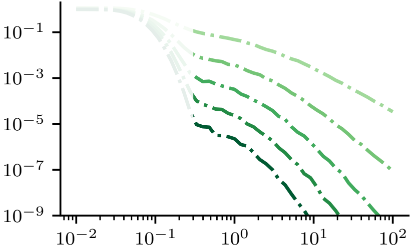

Figure: $\mathrm{MI}(Y; Trace)$ vs. σ^2, $2 \leq d \leq 6$

Cst gap between each curve (log scale)

$\iff$

exponential security w.r.t. #shares d

Simulation: $L(Y_i) = hw(Y_i) + \mathcal{N}(0; \sigma^2)$,
hw = Hamming weight

[1] Chari et al., "Towards Sound Approaches to Counteract Power-Analysis Attacks"

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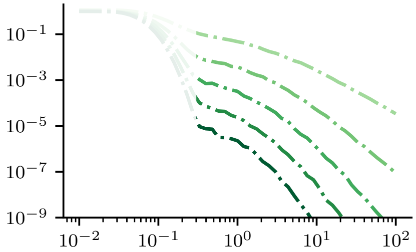

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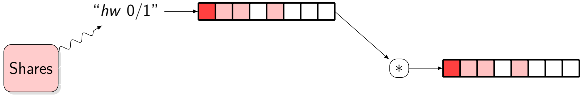
Explanation: Masking $\iff$ convolution



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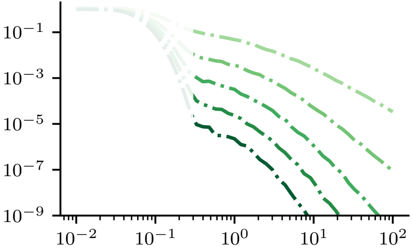

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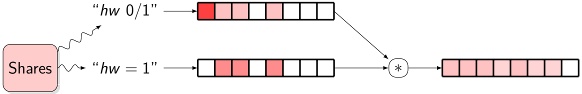

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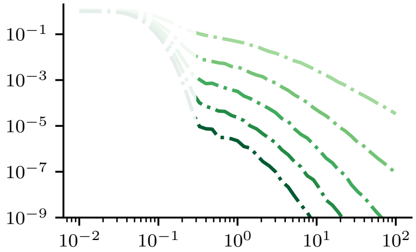

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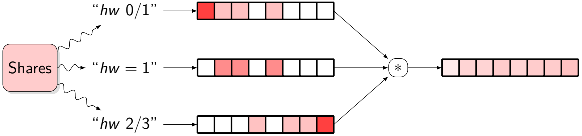

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Leakage from encoding $\longrightarrow$ leakage from computations

So far we have considered one secret leaking from one encoding ...

[2] Ishai, Sahai, and Wagner, "Private Circuits: Securing Hardware against Probing Attacks"; Rivain and Prouff, "Provably Secure Higher-Order Masking of AES"

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$\to$ Sequence of (linear) additions and multiplications

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Multiplication over secret sharing

A

B

Multiplication over secret sharing

$$\begin{array}{cc|cccc} & & & & \mathbf{A} & \\ & & A_0 & A_1 & \dots & A_d \\ \hline & B_0 & & & & \\ \mathbf{B} & B_1 & & & & \\ & \vdots & & & & \\ & B_d & & & & \end{array}$$

Multiplication over secret sharing

$$
\begin{array}{c|cccc}
 & & & \mathbf{A} & \\
 & A_0 & A_1 & \dots & A_d \\
B_0 & A_0 \cdot B_0 & A_1 \cdot B_0 & \dots & A_d \cdot B_0 \\
B_1 & A_0 \cdot B_1 & A_1 \cdot B_1 & \dots & A_d \cdot B_1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
B_d & A_0 \cdot B_d & A_1 \cdot B_d & \dots & A_d \cdot B_d
\end{array}
$$

B

1.Cross-products

Multiplication over secret sharing

$$
\begin{array}{c|ccccc}
 & & & \mathbf{A} & & \\
 & A_0 & A_1 & \dots & A_d & \\
B_0 & A_0 \cdot B_0 & A_1 \cdot B_0 - {\color{blue}R_{0,1}} & \dots & A_d \cdot B_0 - {\color{green}R_{0,d}} & \\
B_1 & A_0 \cdot B_1 + {\color{blue}R_{0,1}} & A_1 \cdot B_1 & \dots & A_d \cdot B_1 - {\color{red}R_{1,d}} & \text{2. Refreshing} \\
\mathbf{B} \quad \vdots & \vdots & \vdots & \ddots & \vdots & \\
B_d & A_0 \cdot B_d + {\color{green}R_{0,d}} & A_1 \cdot B_d + {\color{red}R_{1,d}} & \dots & A_d \cdot B_d &
\end{array}
$$

Multiplication over secret sharing

$$
\begin{array}{c|cccccc|l}
 & & & \mathbf{A} & & & & \\
 & & A_0 & A_1 & \dots & A_d & & \\
 & B_0 & A_0 \cdot B_0 & A_1 \cdot B_0 - {\color{blue}R_{0,1}} & \dots & A_d \cdot B_0 - {\color{green}R_{0,d}} & & \\
\mathbf{B} & B_1 & A_0 \cdot B_1 + {\color{blue}R_{0,1}} & A_1 \cdot B_1 & \dots & A_d \cdot B_1 - {\color{red}R_{1,d}} & & \text{3. Compression} \\
 & \vdots & \vdots & \vdots & \ddots & \vdots & & \\
 & B_d & A_0 \cdot B_d + {\color{green}R_{0,d}} & A_1 \cdot B_d + {\color{red}R_{1,d}} & \dots & A_d \cdot B_d & & \\
 & & \Sigma_0 & \Sigma_1 & \dots & \Sigma_d & &
\end{array}
$$

Multiplication over secret sharing

$$\begin{array}{c|ccccc|c}
 & & & \mathbf{A} & & & \\
 & & A_0 & A_1 & \dots & A_d & \\
 & B_0 & A_0 \cdot B_0 & A_1 \cdot B_0 - R_{0,1} & \dots & A_d \cdot B_0 - R_{0,d} & \\
\mathbf{B} & B_1 & A_0 \cdot B_1 + R_{0,1} & A_1 \cdot B_1 & \dots & A_d \cdot B_1 - R_{1,d} & \text{3. Compression} \\
 & \vdots & \vdots & \vdots & \ddots & \vdots & \\
 & B_d & A_0 \cdot B_d + R_{0,d} & A_1 \cdot B_d + R_{1,d} & \dots & A_d \cdot B_d & \\
\downarrow & & \sum_0 & \sum_1 & \dots & \sum_d & \downarrow
\end{array}$$

The $\sum_i$ form a secure sharing of $A \cdot B$ against a *d-probing* adversary

Multiplication over secret sharing

$$
\begin{array}{cccccc}
 & & & \mathbf{A} & & \\
 & & A_0 & A_1 & \cdots & A_d \\
 & B_0 & A_0 \cdot B_0 & A_1 \cdot B_0 - R_{0,1} & \cdots & A_d \cdot B_0 - R_{0,d} \\
\mathbf{B} & B_1 & A_0 \cdot B_1 + R_{0,1} & A_1 \cdot B_1 & \cdots & A_d \cdot B_1 - R_{1,d} \\
 & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & B_d & A_0 \cdot B_d + R_{0,d} & A_1 \cdot B_d + R_{1,d} & \cdots & A_d \cdot B_d \\
 & & \sum_0 & \sum_1 & \cdots & \sum_d
\end{array}
$$

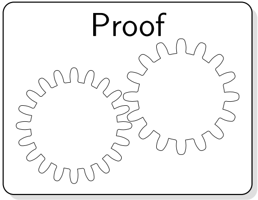
3. Compression

The $\sum_i$ form a secure sharing of $A \cdot B$ against a *d-probing* adversary

Probing model

A t-probing adversary can reveal a subset of t intermediate calculations. The target is secure if the subset is independent of the secret.

Proving Security in the Probing Model



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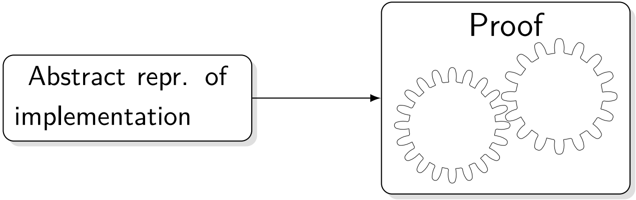

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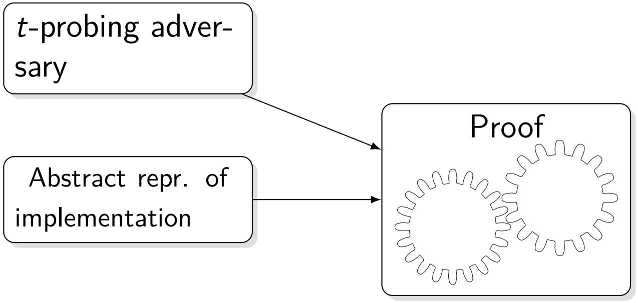

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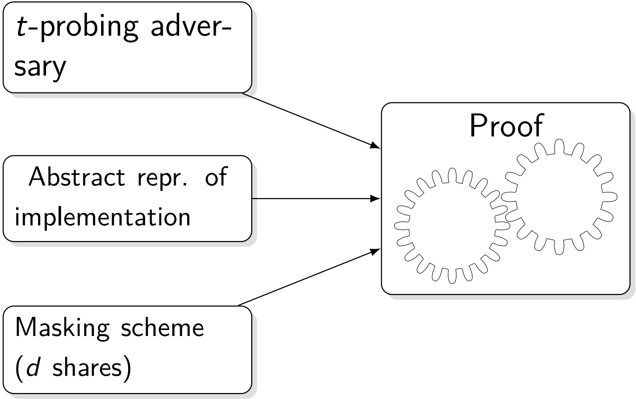

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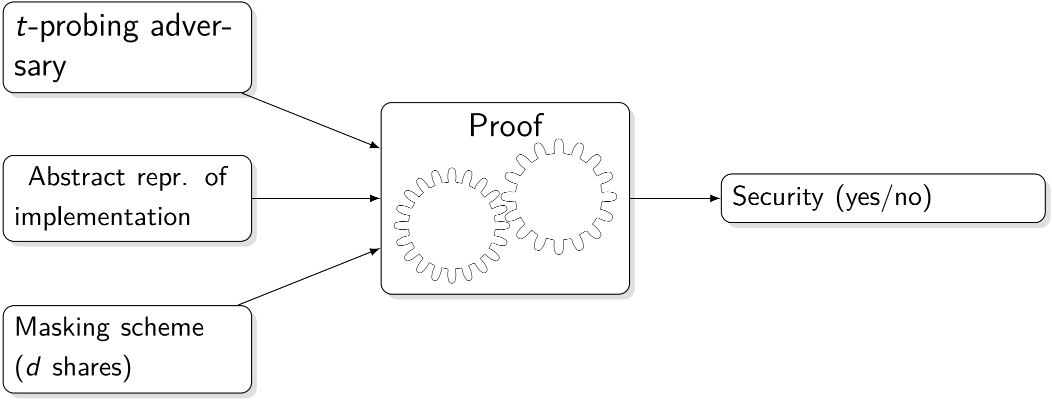

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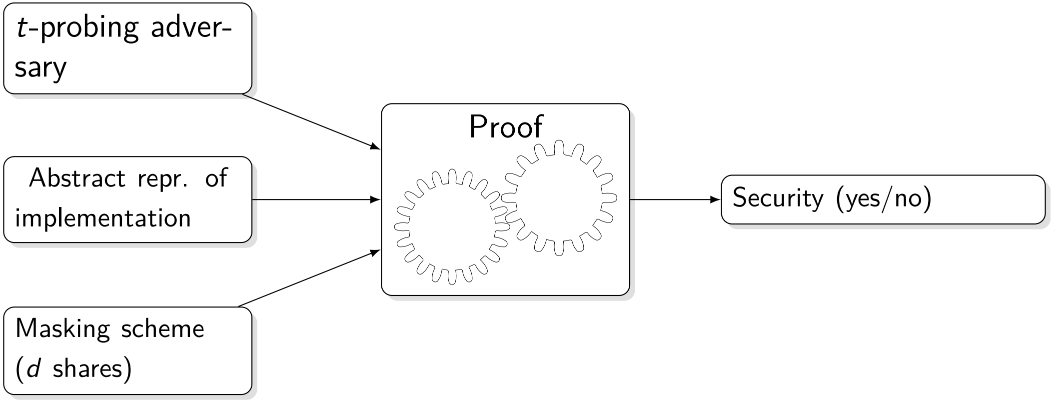
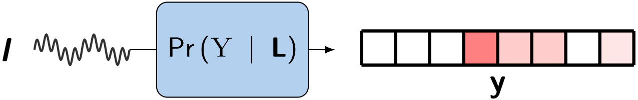


Very practical to verify (using formal verification tools), but unrealistic adversary

The Noisy Leakage Model

In this model, for each intermediate computation, the adversary gets a probability distribution about its operands:



[3] D: Kullback - Leibler (KL) divergence, total variation, Euclidean norm, ...

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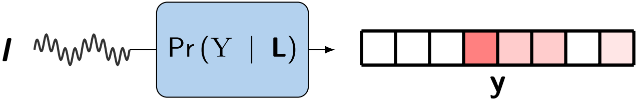
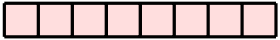


If, the adversary gets:



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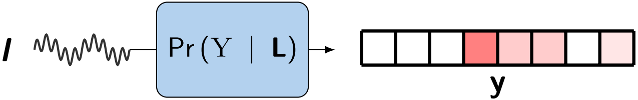

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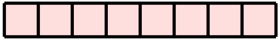

Very noisy
Sensitive computation unpredictable

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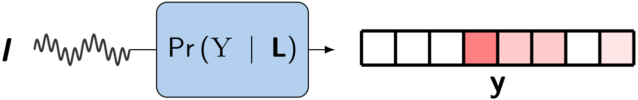

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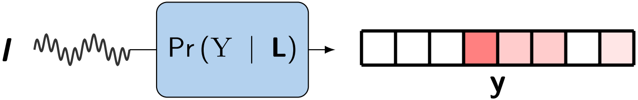

If, the adversary gets:

Low-noise

Exact prediction of the sensitive computation

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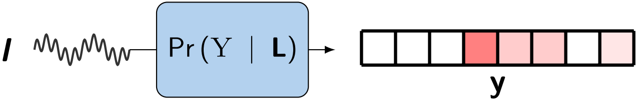
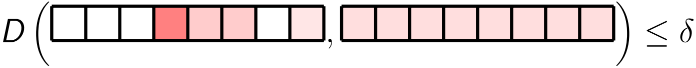


δ-NOISY ADVERSARY

All the p.m.f.s accessed by the adversary are δ-close[3] to the uniform:



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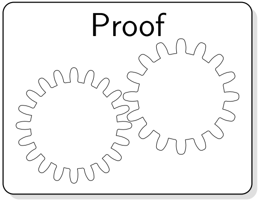
Proving the Masking Security



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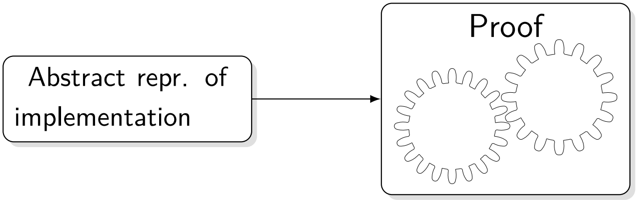

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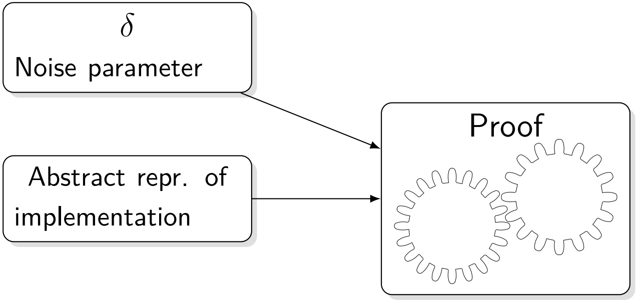

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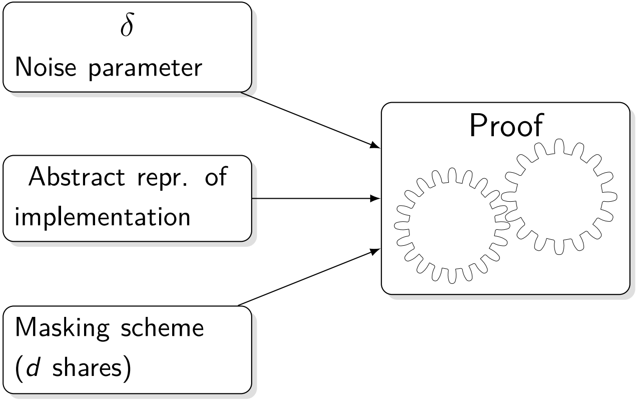

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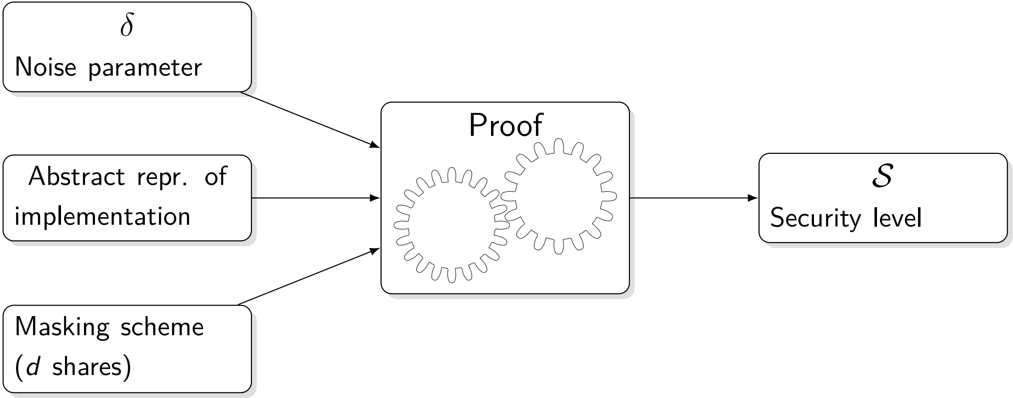

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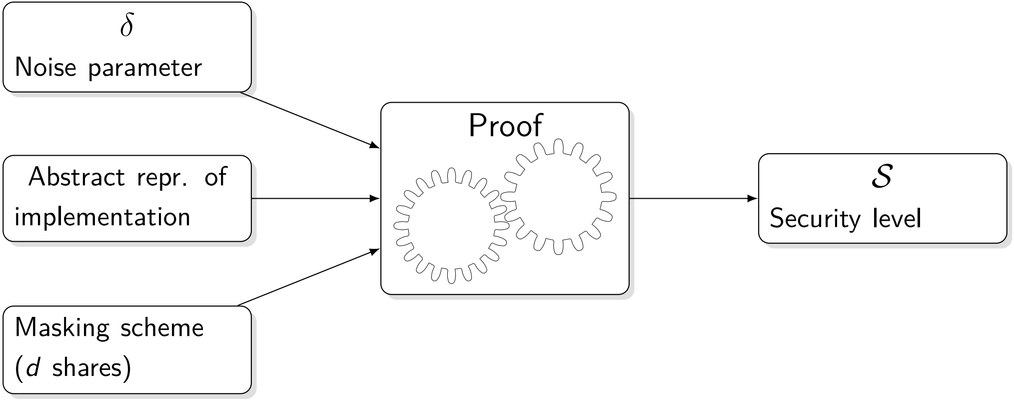
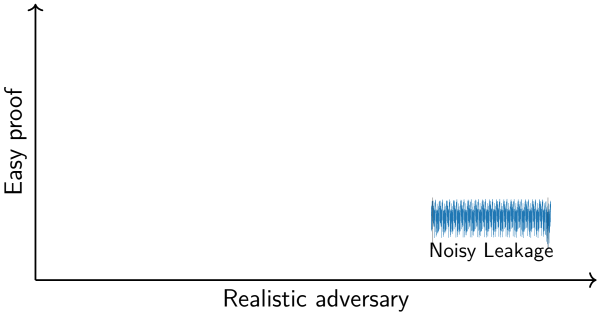


"Any attack requires $\mathcal{S}$ queries "

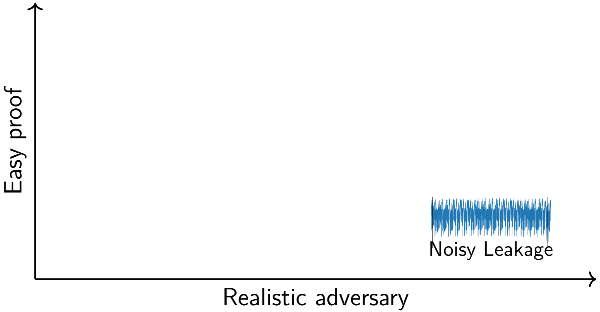
The Link between both Models



The Link between both Models

"Any successful adversary requires $\mathcal{S} = \Omega\left(\left(\frac{1}{\delta d |\mathbb{F}|}\right)^d\right)$ queries"

$\rightarrow$ Direct proof, with significant artifacts, with restricting assumptions[4]



[4] Prouff and Rivain, "Masking against Side-Channel Attacks: A Formal Security Proof".

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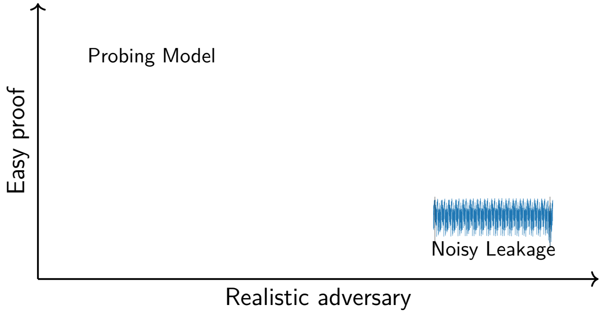

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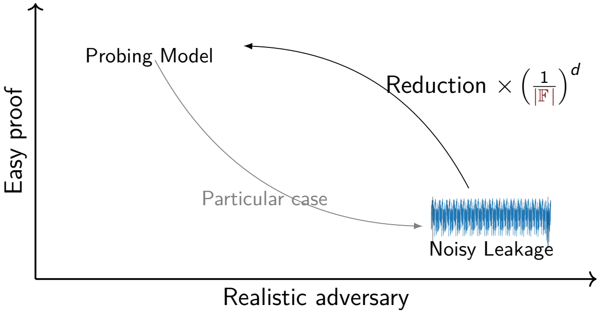
"Any successful adversary requires $\mathcal{S} = \Omega\left(\left(\frac{1}{\delta d |\mathbb{F}|}\right)^d\right)$ queries"

$\rightarrow$ Reduction, with significant artifacts, without restricting assumptions[4]



[4] Duc, Dziembowski, and Faust, "Unifying Leakage Models: From Probing Attacks to Noisy Leakage".

Our Work

"Any successful adversary requires $\mathcal{S} = \Omega\left(\left(\frac{1}{\delta d}\right)^{d}\right)$ queries"

Direct proof, without significant artifacts, with restricting assumptions[5]

[5] Almost identical to the ones of Prouff & Rivain

First Improvement: the Noise Amplification Bound

One bottleneck in P&R's proof is the bound over *one* encoding

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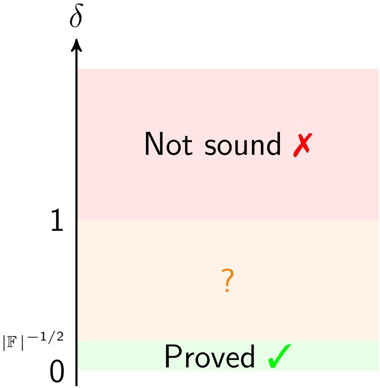
$$\mathrm{EN}\left(Secret; Leaky\ \ Encoding\right) \leq \left(\delta \cdot \sqrt{|\mathbb{F}|}\right)^{d}$$

Two issues with EN:

$\rightarrow$ Field-size dependent

$\rightarrow$ Does not extend well to leakage over computations

$\rightarrow$



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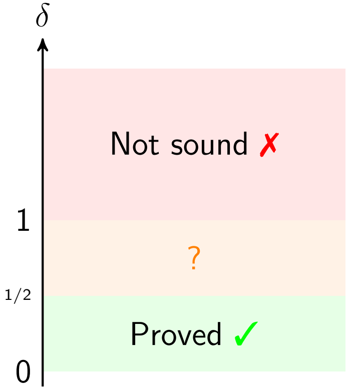
$$\mathrm{SD}\left(\mathit{Secret}; \mathit{Leaky\ \ Encoding}\right) \leq (\delta \cdot 2)^{d}$$

One issue with SD: [6]

$\rightarrow$ Not Field-size dependent

$\rightarrow$ Does not extend well to leakage over computations

$\rightarrow$



[6] Dziembowski, Faust, and Skórski, "Optimal Amplification of Noisy Leakages"

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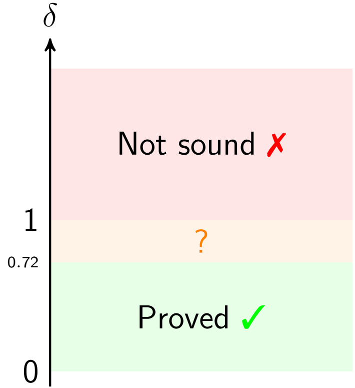
$$\mathrm{MI}\left(Secret; Leaky\ \ Encoding\right) \leq 0.72 \cdot \left(\frac{\delta}{0.72}\right)^{d}$$



New bounds for MI: [6]

→ Not Field-size dependent

→ **This paper**: Extends well to leakage over computations

→

[6] Béguinot *et al.*, COSADE 2023, following Masure *et al.* at CARDIS'23 and Ito *et al.* at CCS'23)

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Required to get independent shares $\rightarrow$ reduction to random walks

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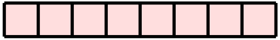
Our observation: If $\mathrm{Y} =$ ,

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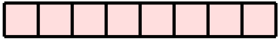
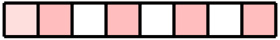
Our observation: If $\mathrm{Y} =$ , then $\mathrm{Y}^3 \approx$ 
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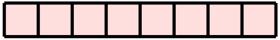
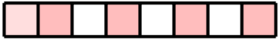
Our observation: If $\mathrm{Y} =$ , then $\mathrm{Y}^3 \approx$ 
Y^3 not that far from uniform $\implies$ specific (refined) reduction

Pros: reasonable quasi-constant factor overhead
Cons: requires monomial SBoxes (ok for AES, not for DES)

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Future works:

- Relaxing the assumptions of direct proofs?
- Direct proofs for other masking schemes, *e.g.* table-based?

Thanks!

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