Malicious-Secure Structure-Aware Private Set Intersection

Gayathri Garimella
Mike Rosulek
Jaspal Singh

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Private Set Intersection (PSI)

PKC [Mea86, CT10], OT ext [PSZ14, PSSZ15, KKRT16, RR17, PRTY19, CM20, PRTY20], VOLE[RS21, RR22]…

Alice

input A

{p, r, i, v, a, t, e}

special case of two-party secure computation

Bob

input B

{s, e, c, u, r, i, t, y}

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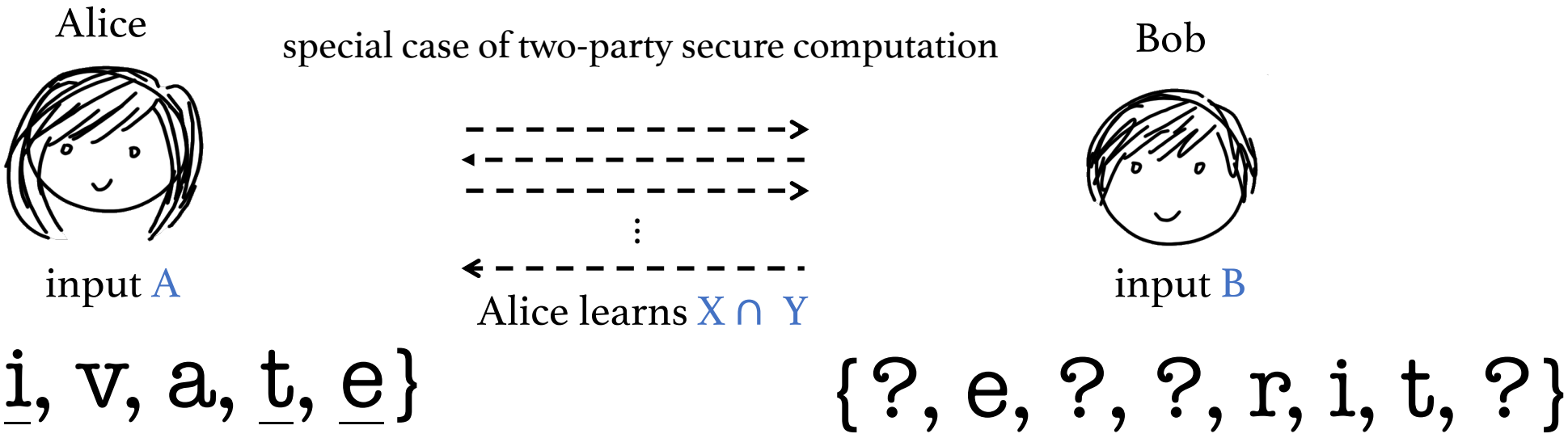

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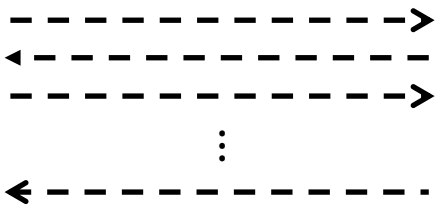

Alice learns X ∩ Y

Bob

input B

{p, r, i, v, a, t, e}

{?, e, ?, ?, r, i, t, ?}

Bob learns nothing extra

structure-aware Private Set Intersection (sa-PSI)

a variant of PSI where Alice's input has a publicly known structure

Examples - interval, ball or union of balls in some metric space, ...

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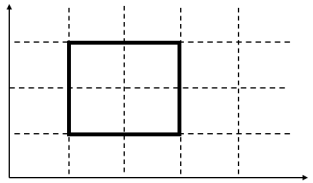
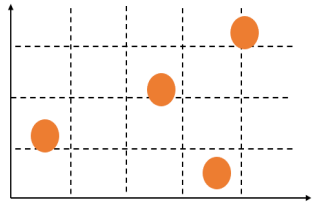


input B



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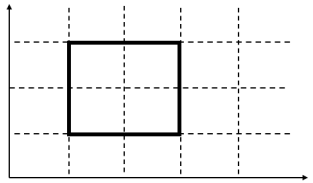

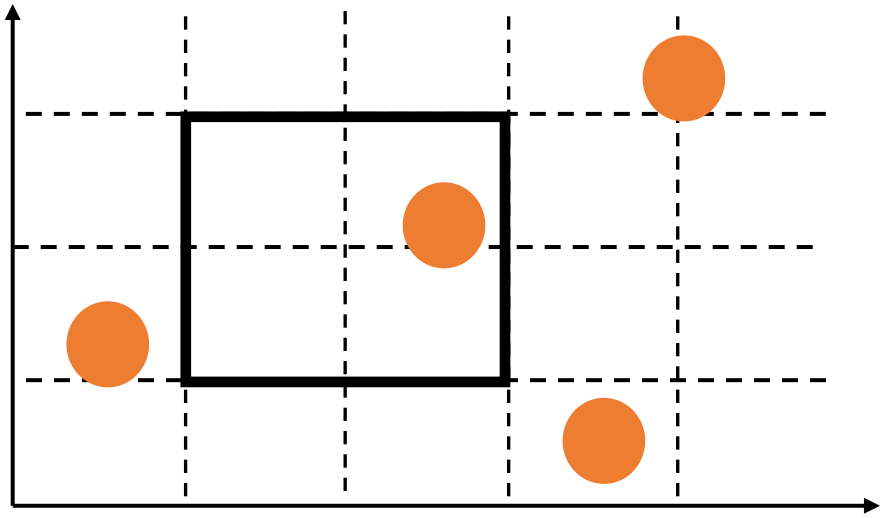

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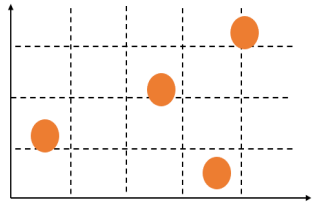

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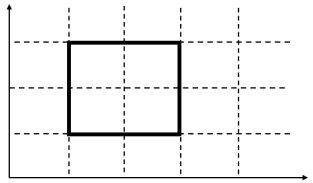

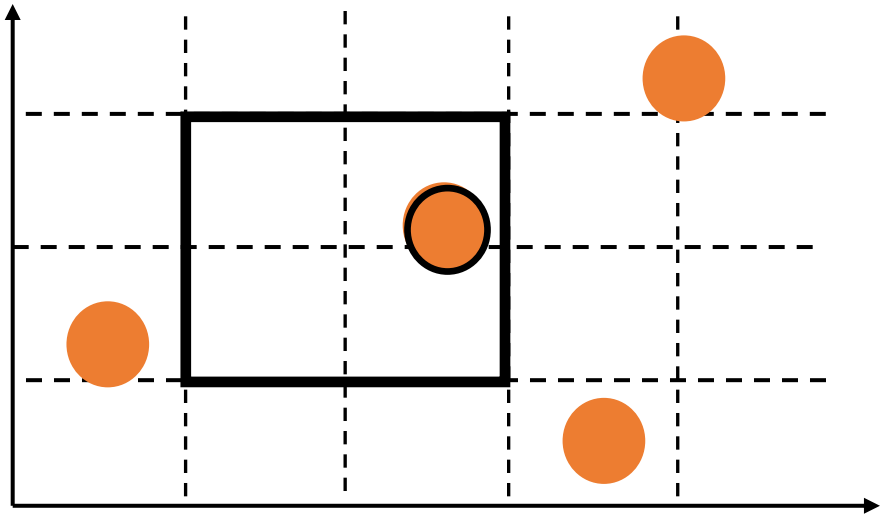

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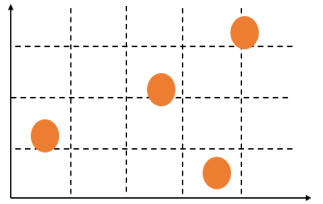

Alice learns X ∩ Y = {all points inside square}

Motivating example

privacy-preserving ride hailing service

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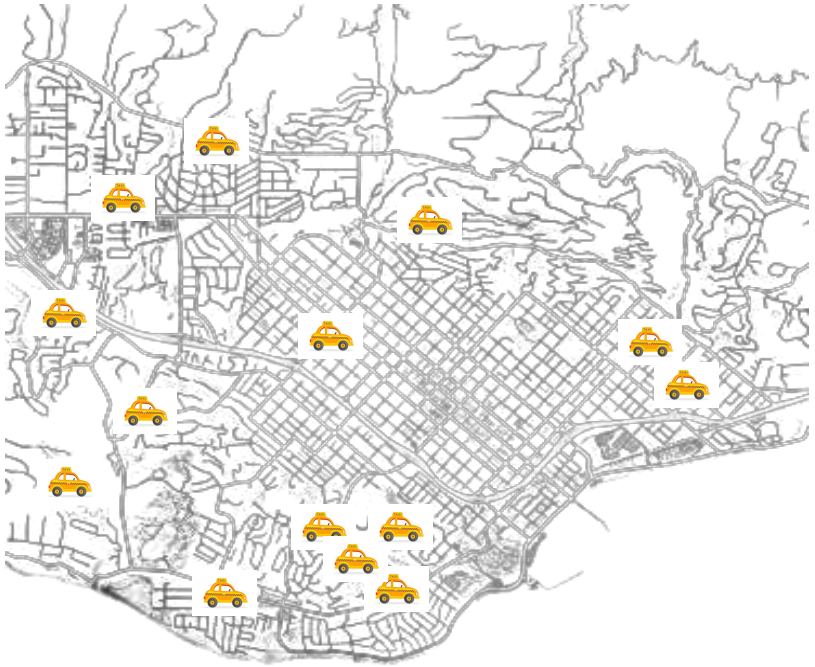

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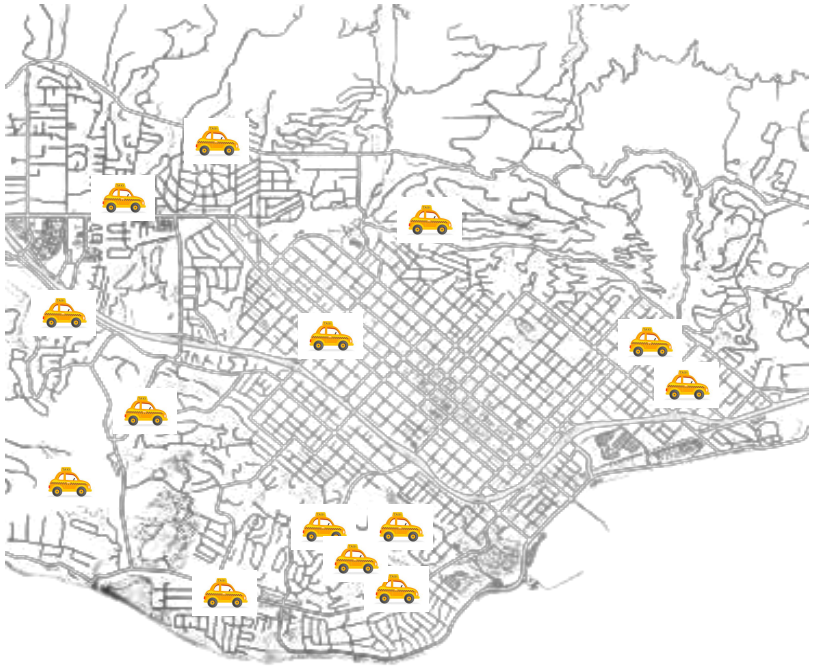

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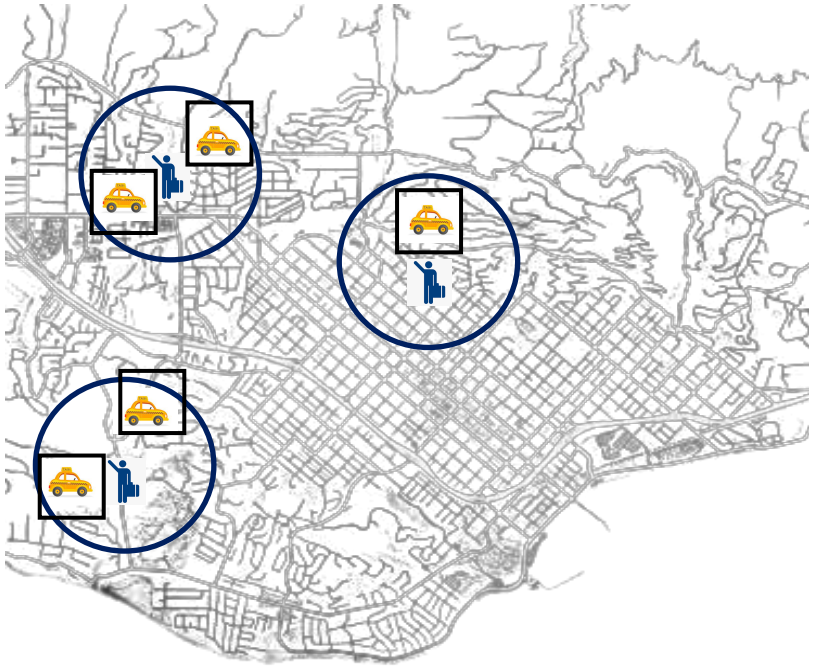

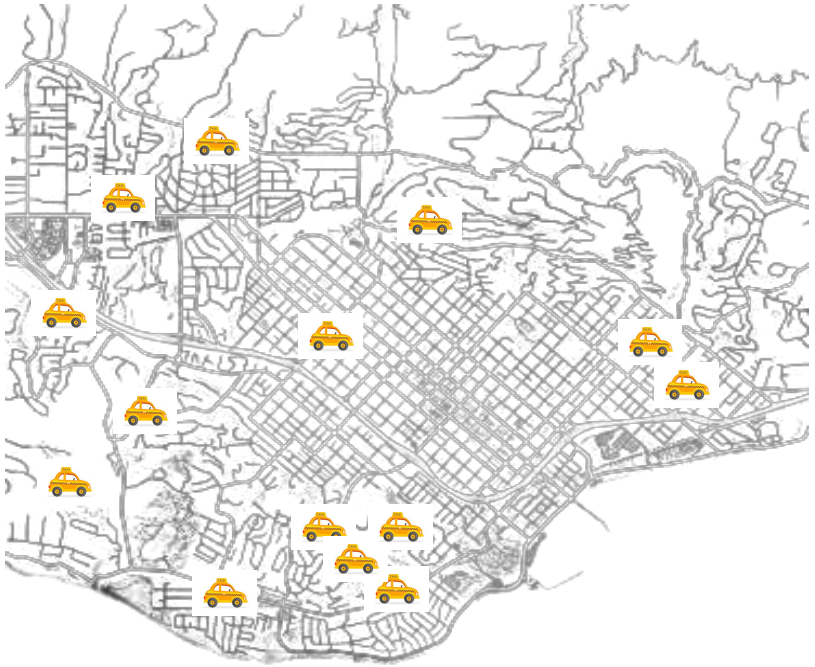

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- Naïve solution: Alice's enumerates her structured input A and runs plain PSI

communication cost $O((|A| + |B|).\kappa)$

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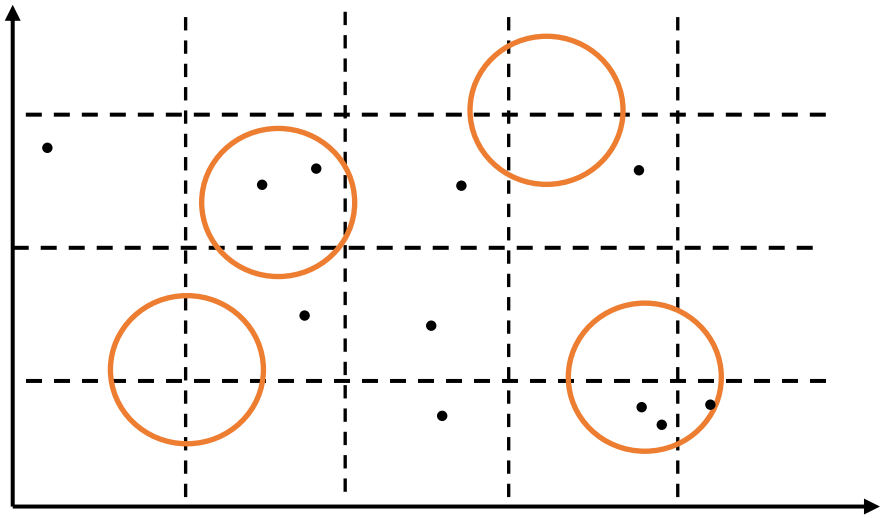

communication scales with total volume $|A|$ of balls in Alice's input

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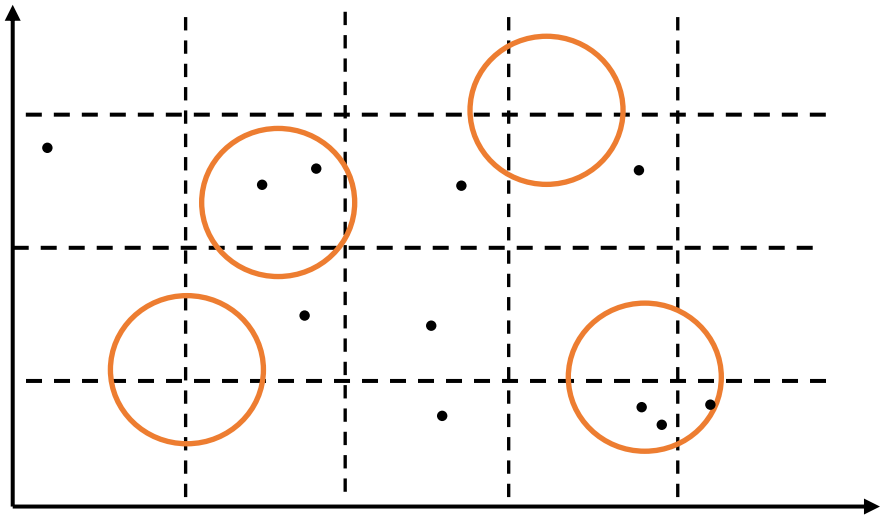

instead, can communication scale with # of balls (description size) of Alice's input?

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$$\text{communication cost } O((|A| + |B|).\kappa)$$

- [GarimellaRosulekSingh'22]: If Alice's input A can be compactly represented by boolean Function Secret Sharing with share size σ , we get semi-honest structure-aware PSI

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boolean Function Secret Sharing (bFSS)

[BoyleGilboaIshai15] – style FSS for set membership in A function

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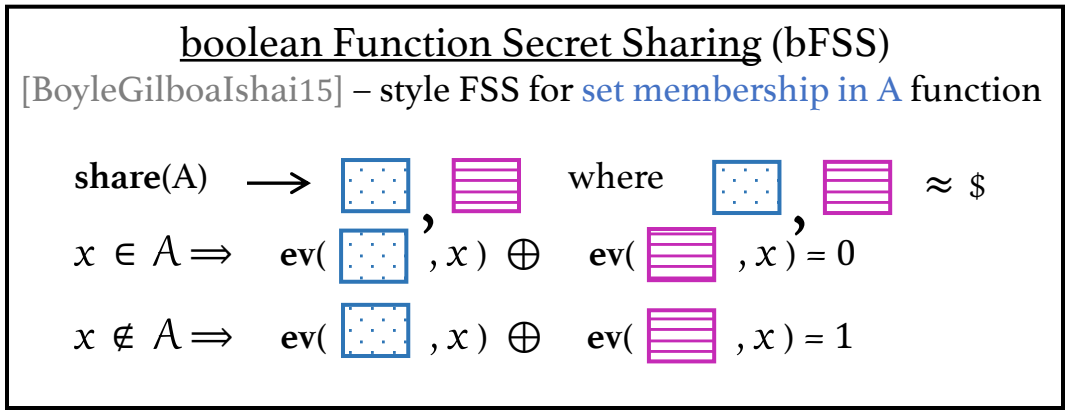

Research Question

Can we design malicious-secure structure-aware PSI?

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Why is [GRS'22] not malicious-secure?

How can we fix this?

How does [GRS'22] work?

assumptions: OT-hybrid (Oblivious Transfer[Rabin81]), random oracle model

input A

1. generates κ instances of bFSS shares

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$\mathbf{share}(A) \longrightarrow$ 

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$\vdots$

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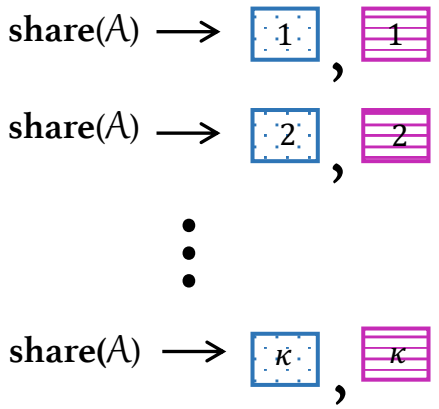
2. picks κ choice bits to learn  or 



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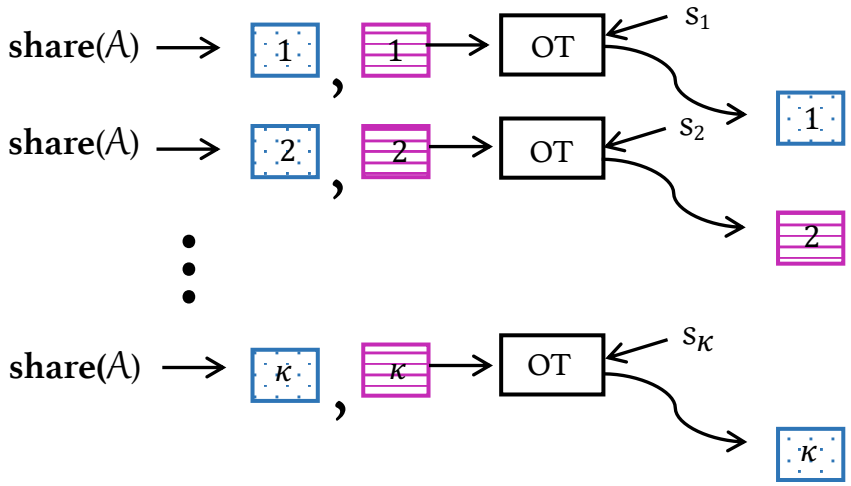

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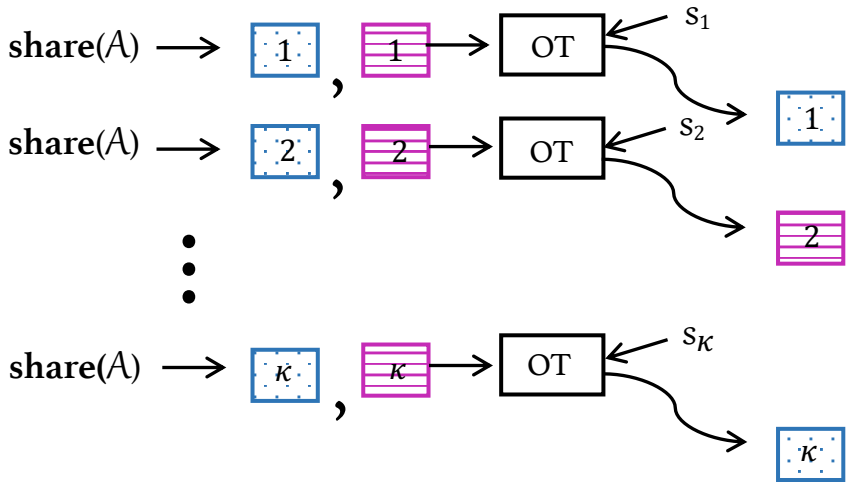

$$F(x) = \mathbf{H}(\mathbf{ev}(\boxed{1}, x) \| \mathbf{ev}(\boxed{2}, x) \| \cdots \mathbf{ev}(\boxed{\kappa}, x))$$

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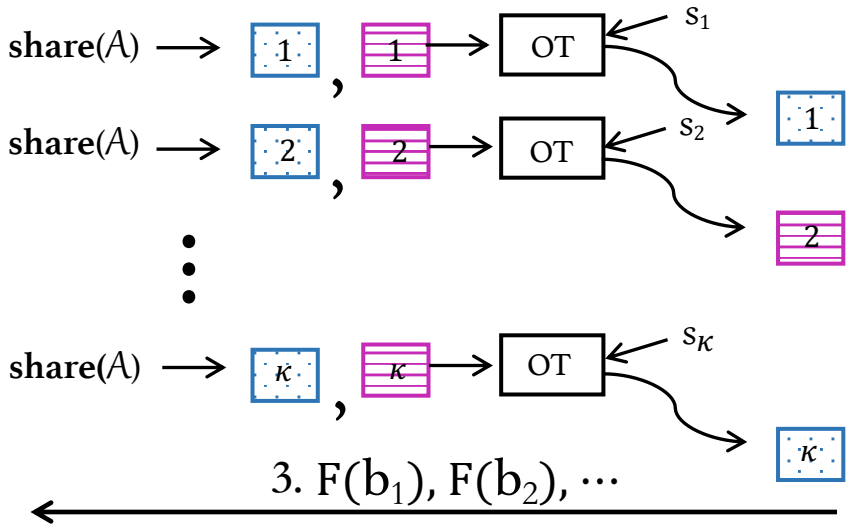

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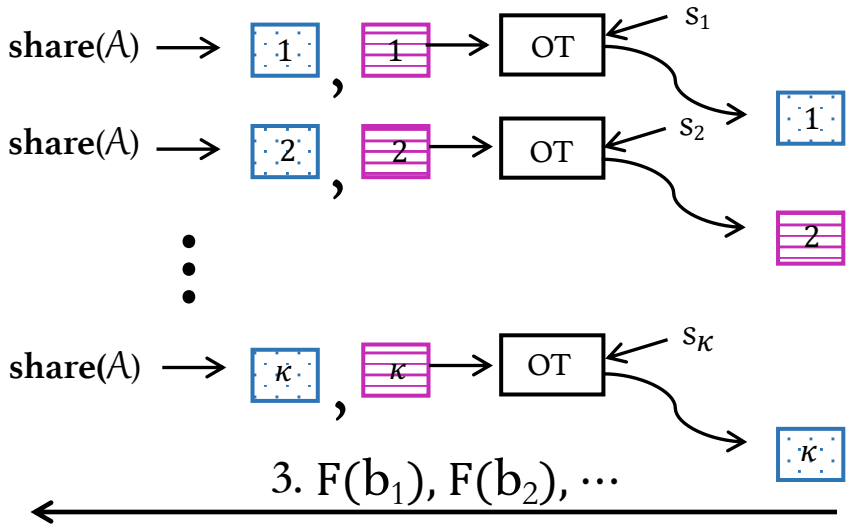

if $(x \in A)$ $\Rightarrow$ Alice can compute $F(x)$

if $(x \notin A)$ $\Rightarrow$ $F(x) \approx \$\$$

$x \in A \implies \mathbf{ev}($  $, x) = \mathbf{ev}($  $, x)$

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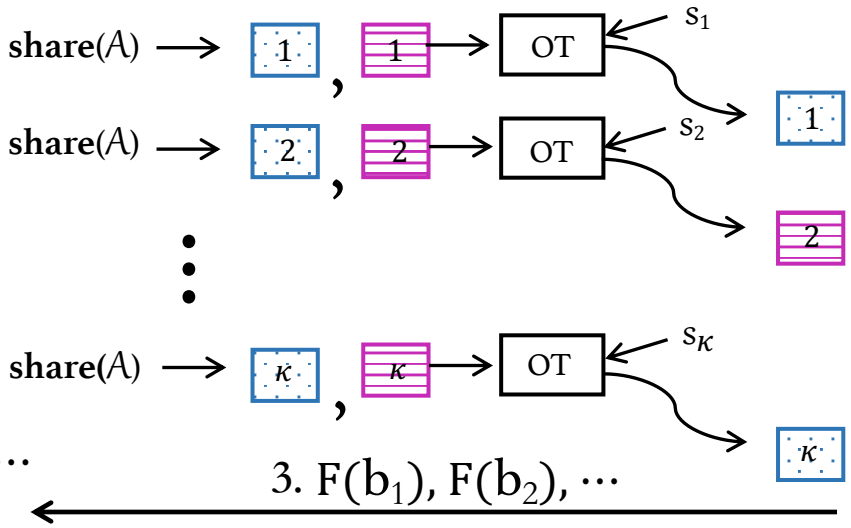

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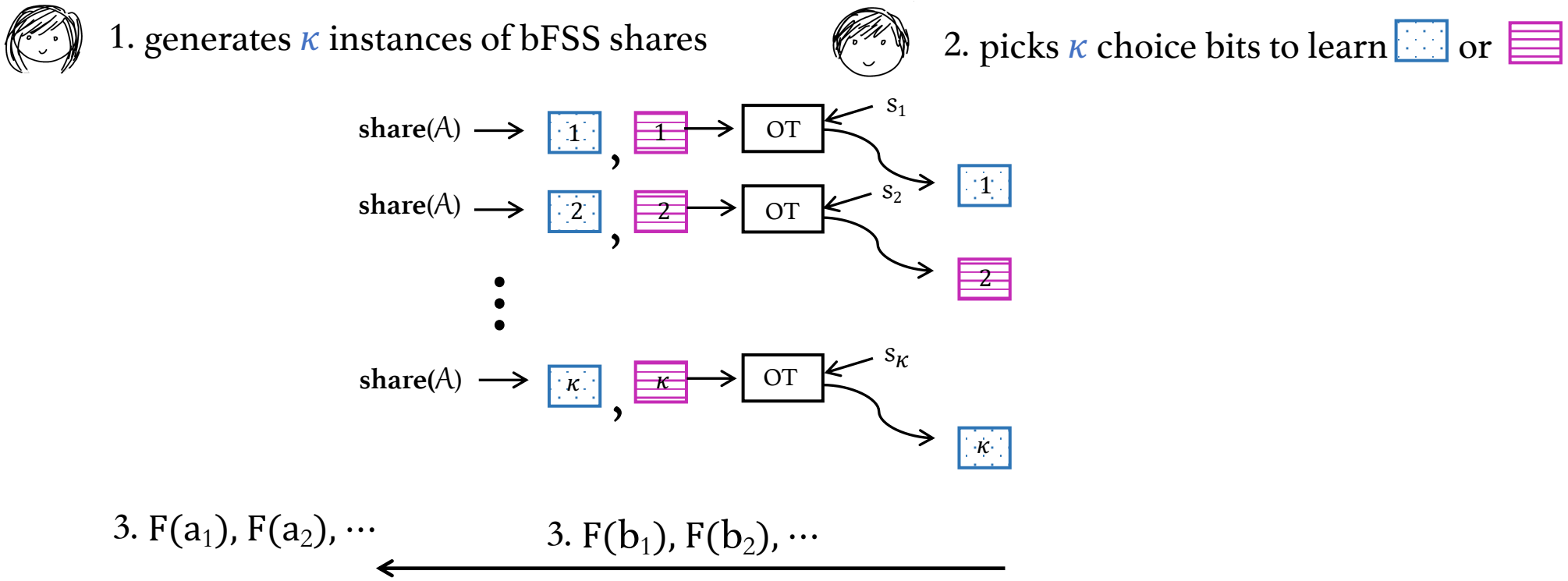

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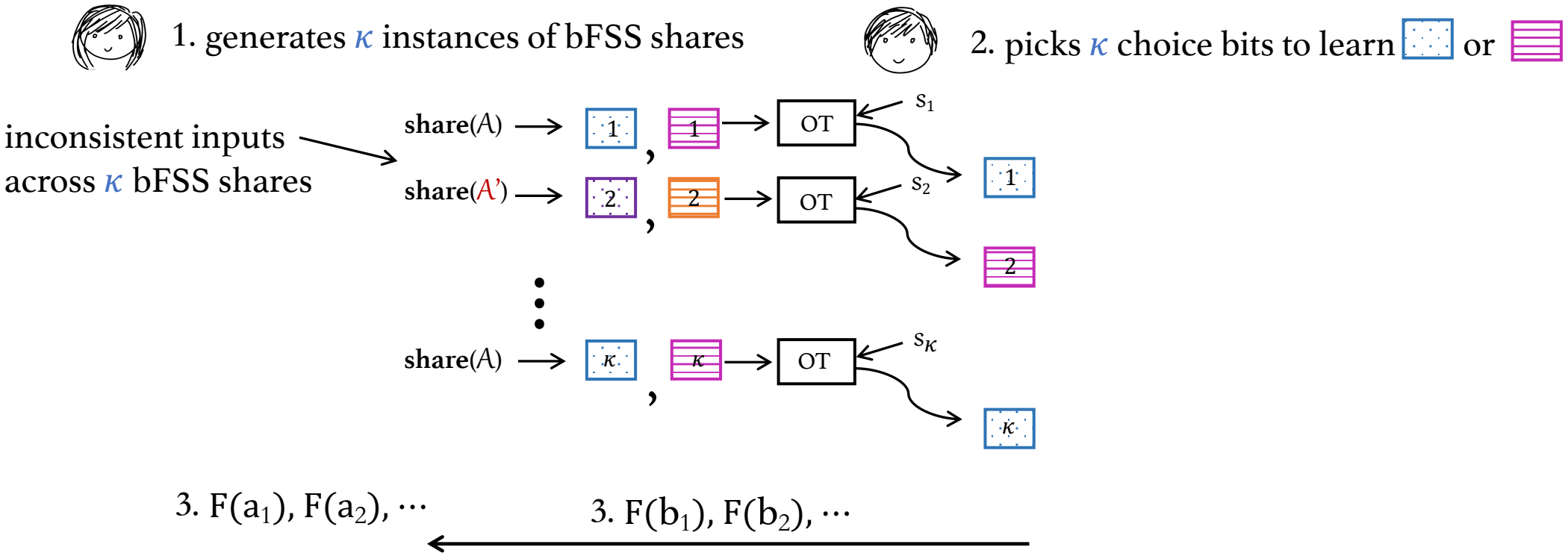

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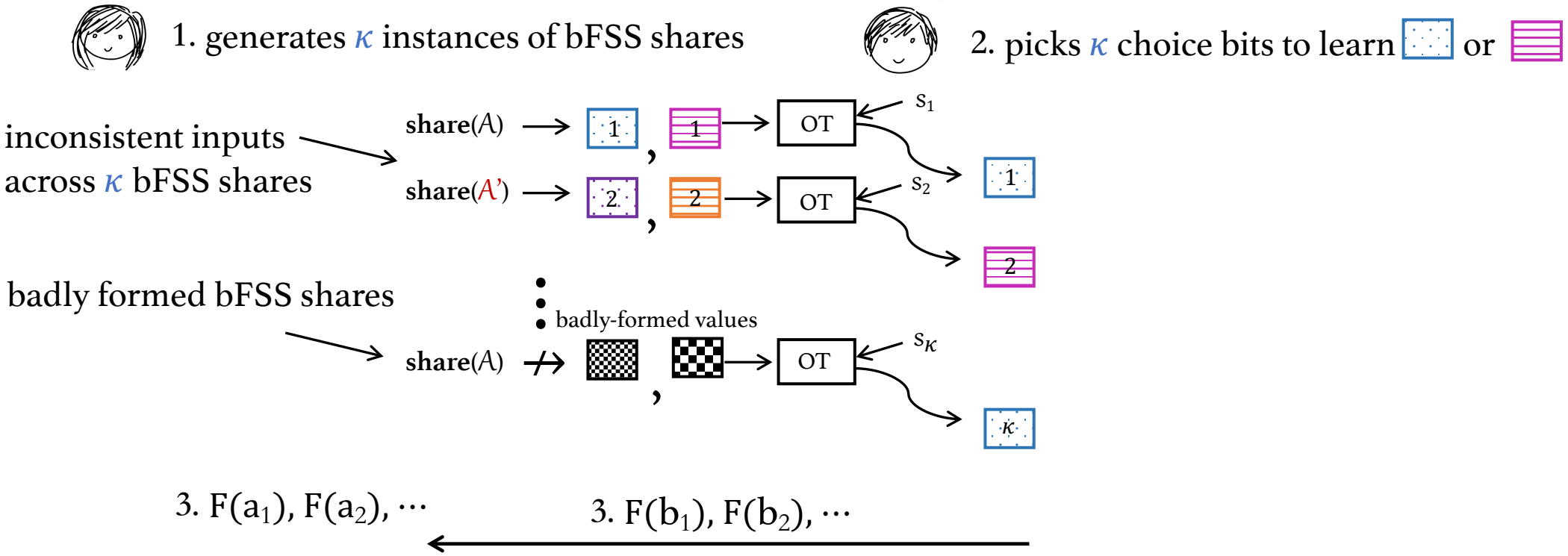

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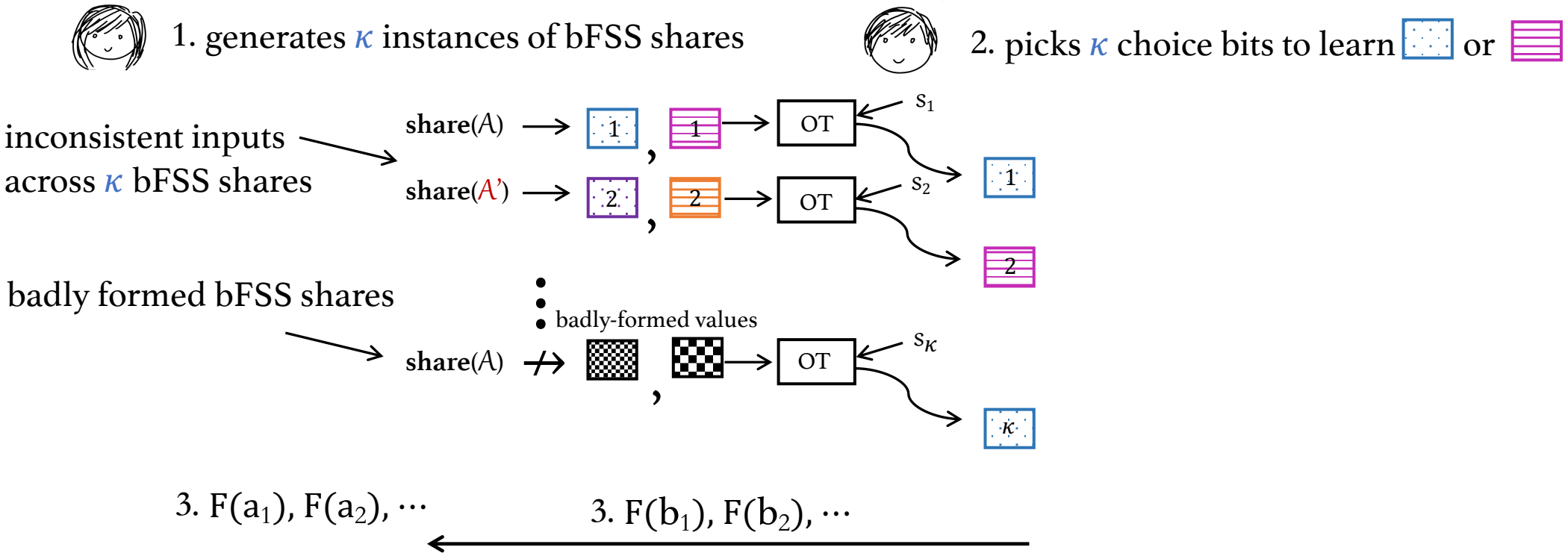
malicious-secure sa-PSI reduces to consistency-of-bFSS problem



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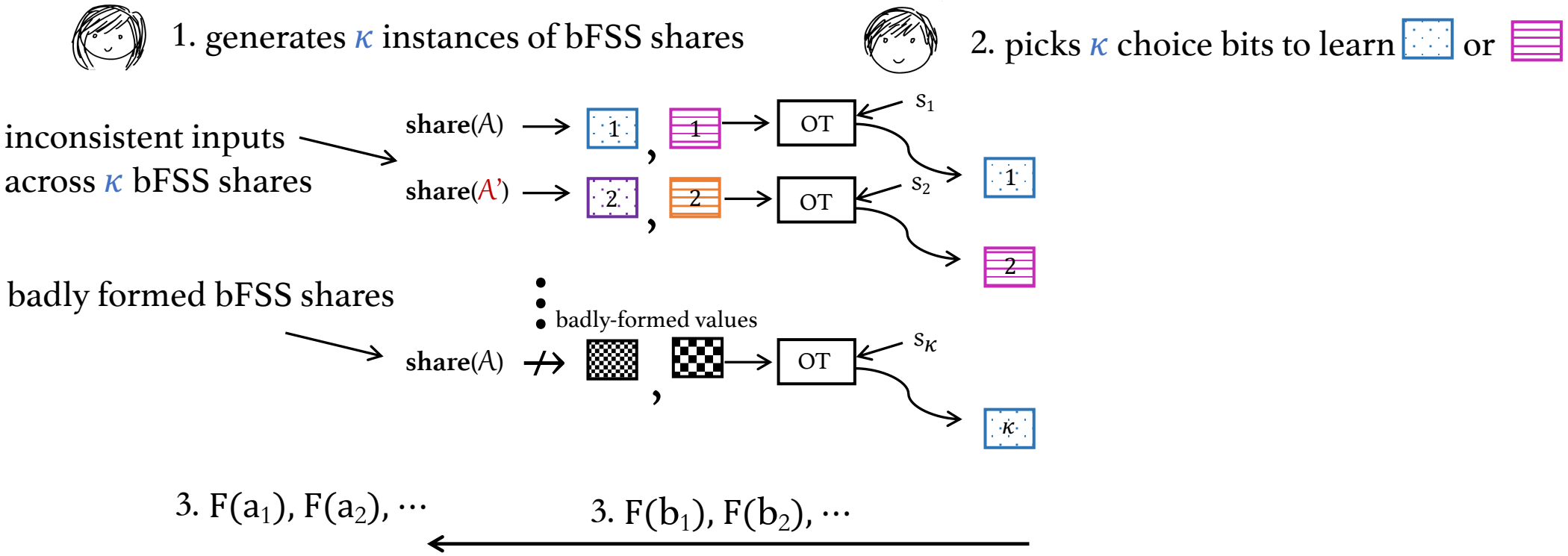
Challenge: enforce that Alice encodes same valid set across all κ bFSS shares



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Cut-and-choose: allow Bob to open and verify both bFSS shares for some subset of 't' instances

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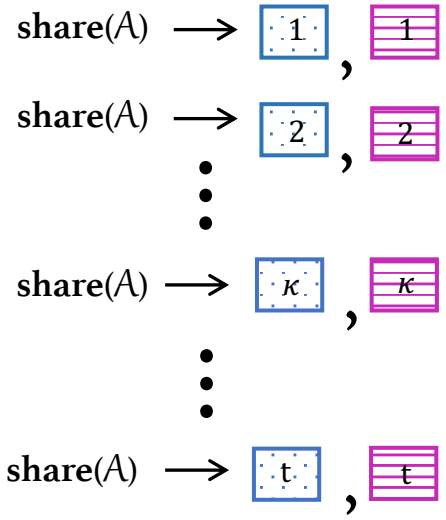

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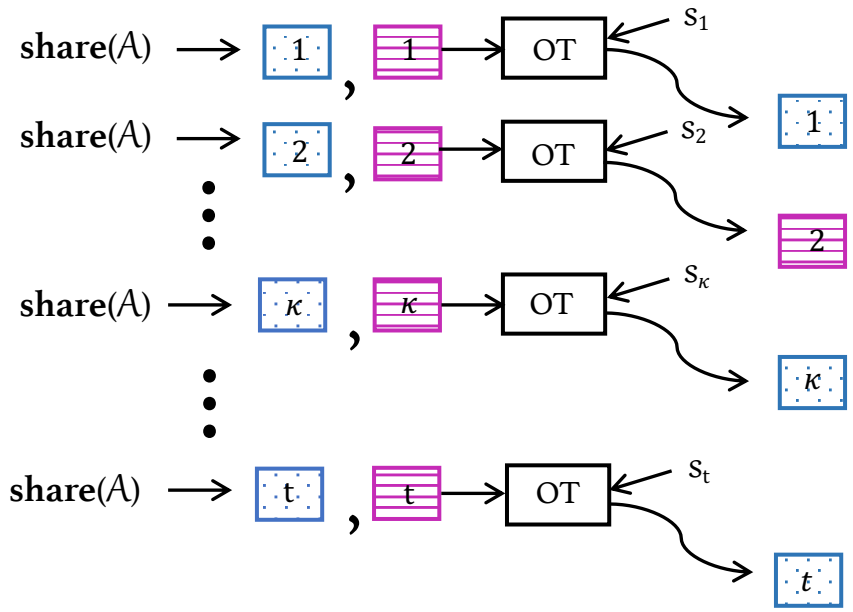

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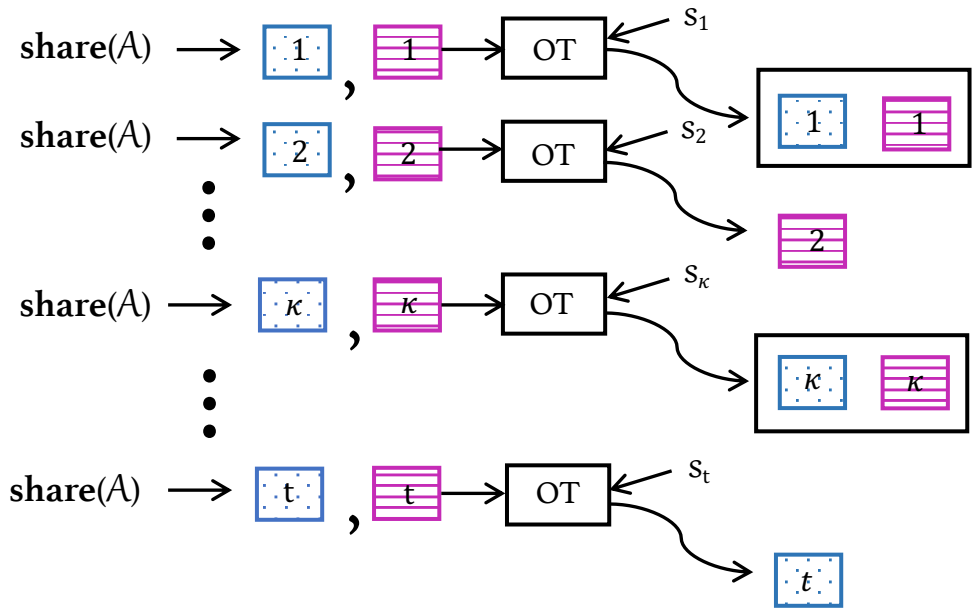

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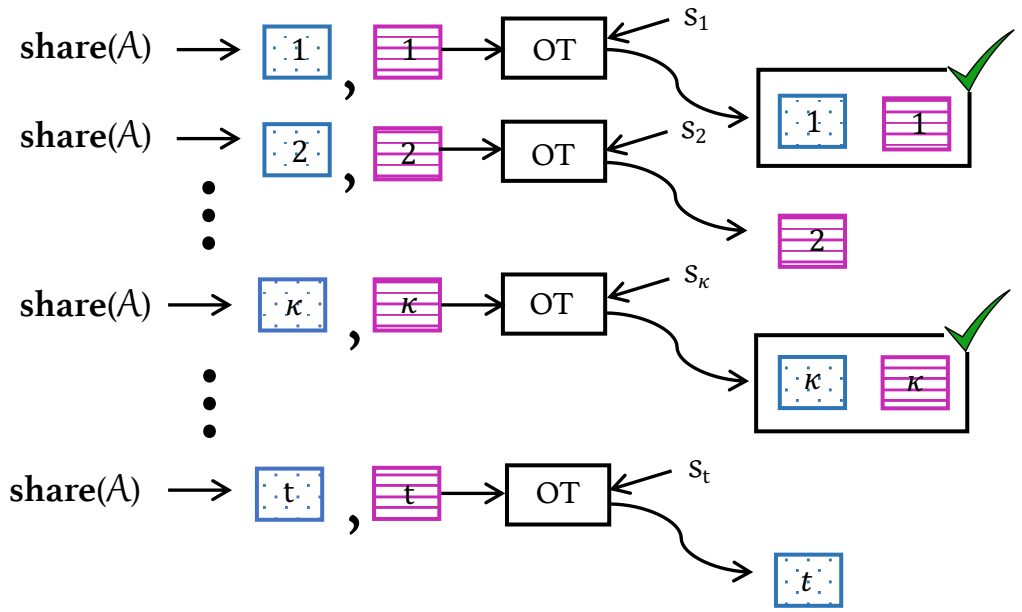

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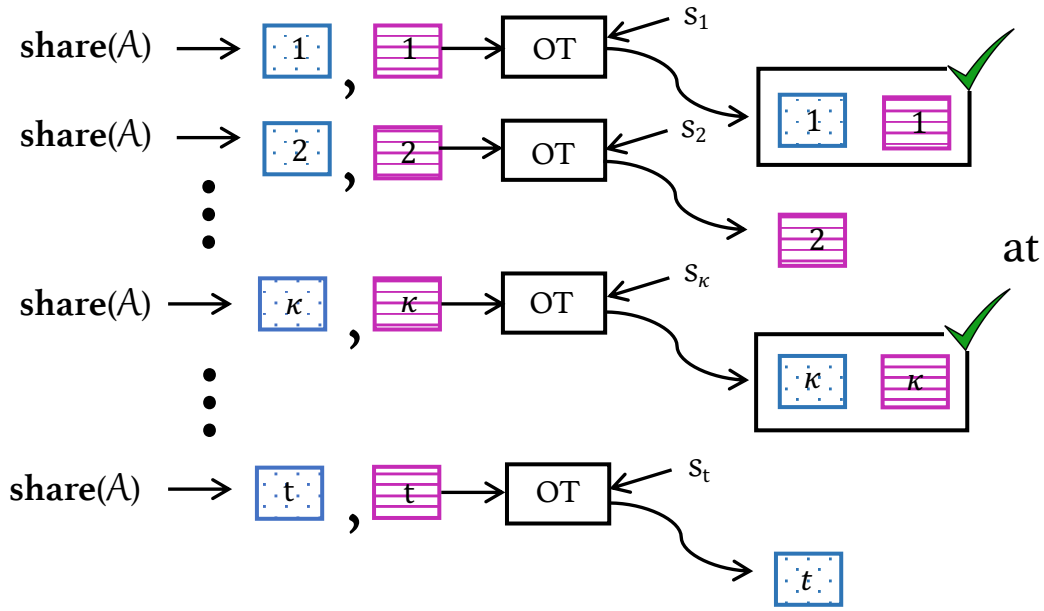

cut-and-choose parameters:
at least κ unopened shares are valid


4. checks if : { **share**$^{-1}$(1 , 1) = **share**$^{-1}$(i , i) = $\cdots$}
all shares of **check indices** encode the **same** underlying input set

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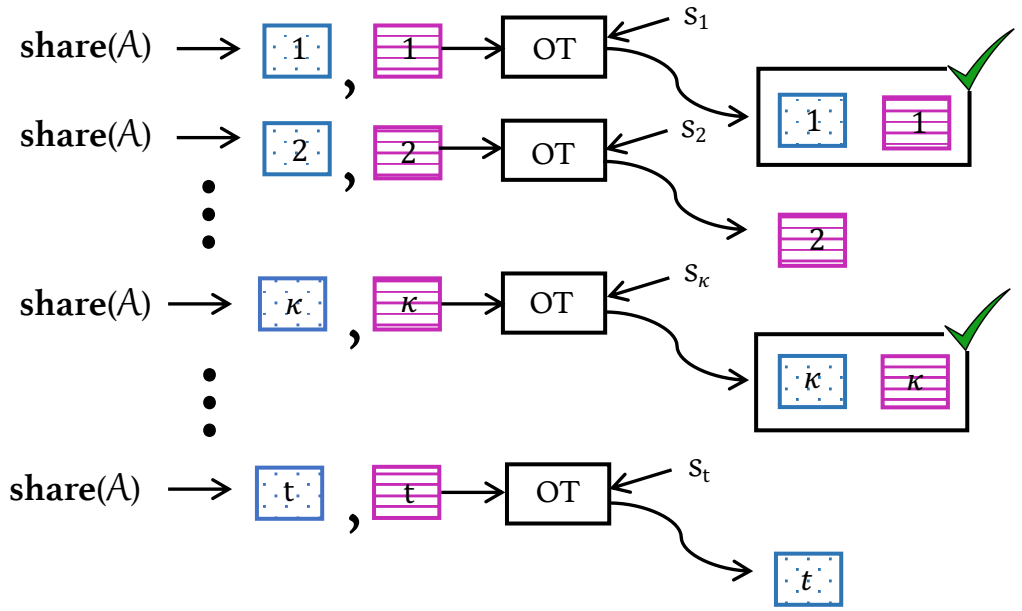

Bob can re-construct Alice's input
violates **input privacy!!!**


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derandomizable bFSS

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homomorphic commitments

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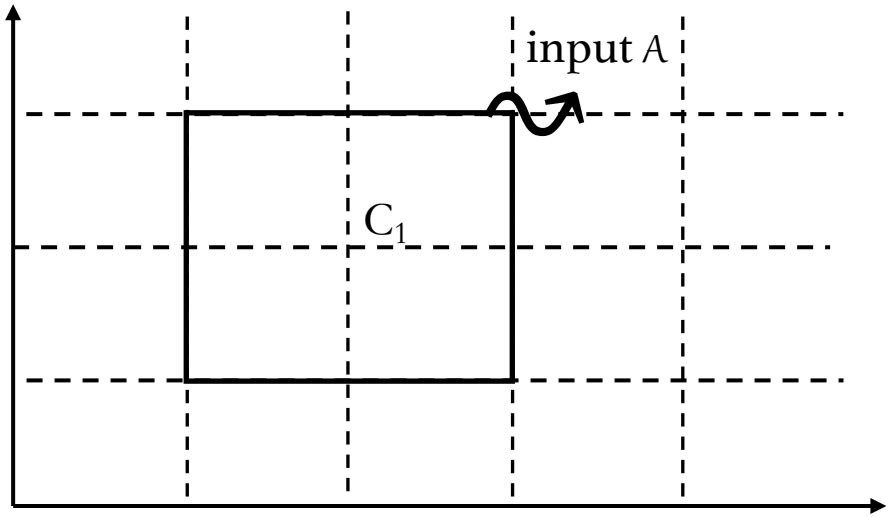
given input $A \in \mathcal{S}$ from a class of structured sets

structured input example: square



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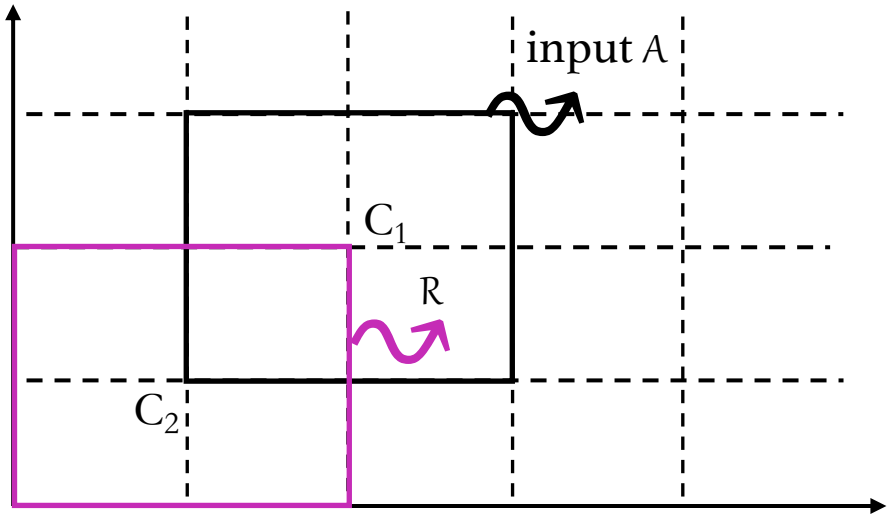

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offset = A - R

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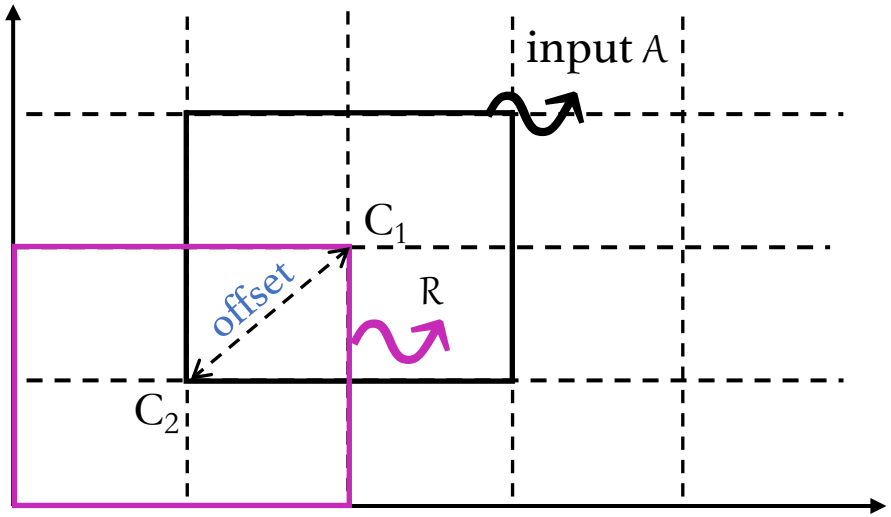
encode(A) → center of square C_1

offset = C_1 - C_2



Derandomizable bFSS

we formalize a new primitives derandomizable Function Secret Sharing with following properties

[BoyleGilboaIshai19]: FSS with derandomizability for "offset" families

derandomizable Function Secret Sharing (drFSS)

given input $A \in \mathcal{S}$ from a class of structured sets

Rshare() $\longrightarrow$  ,  , $\mathbb{R}$ where  ,  $\approx$ \$

offset = A - $\mathbb{R}$

$x \in A \Longrightarrow$ **Deval**( , offset , x) $\oplus$ **Deval**( , offset , x) = 0

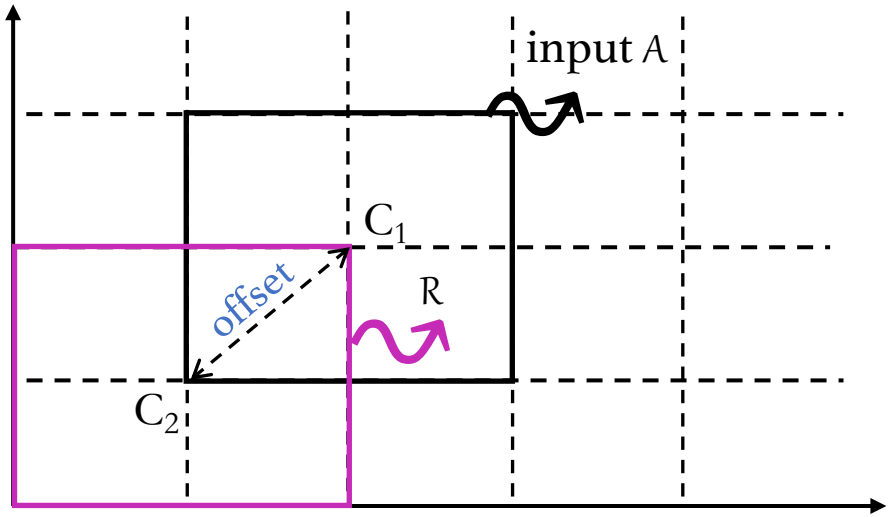
$x \notin A \Longrightarrow$ **Deval**( , offset , x) $\oplus$ **Deval**( , offset , x) = 1

Privacy guarantee:  , offset $\approx$ \$ and  , offset $\approx$ \$

structured input example: square

encode(A) $\rightarrow$ center of square C_1

offset = C_1 - C_2



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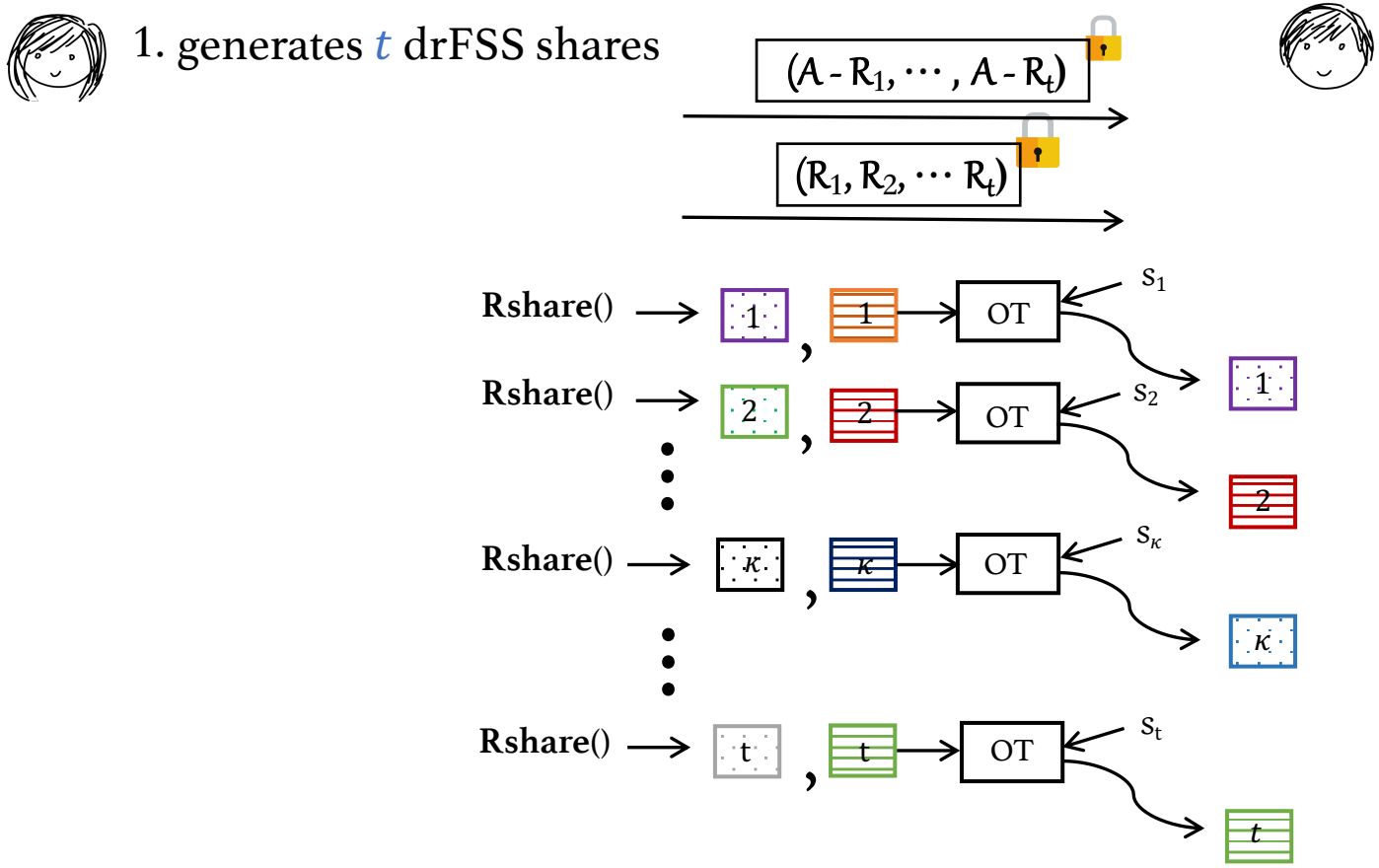

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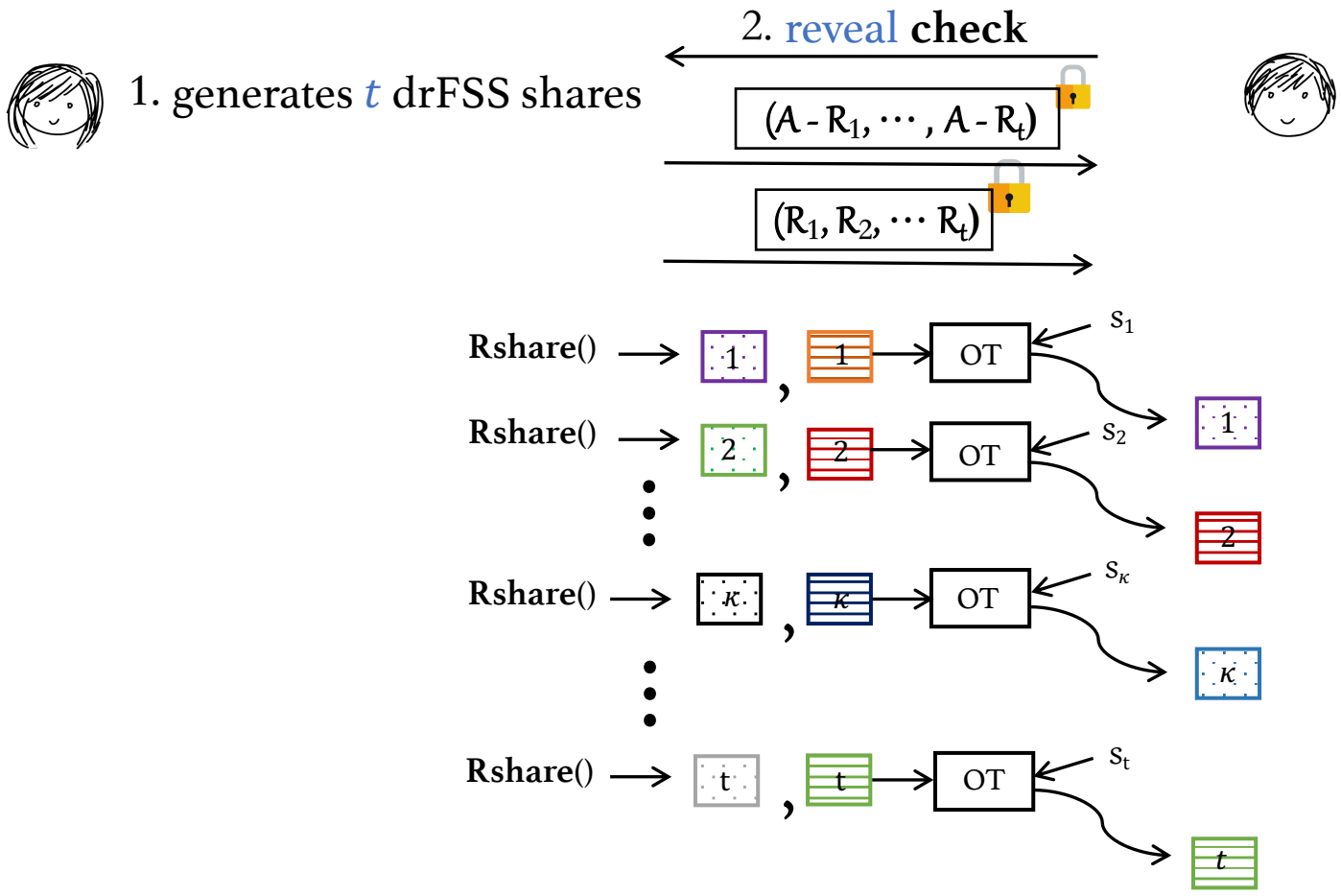

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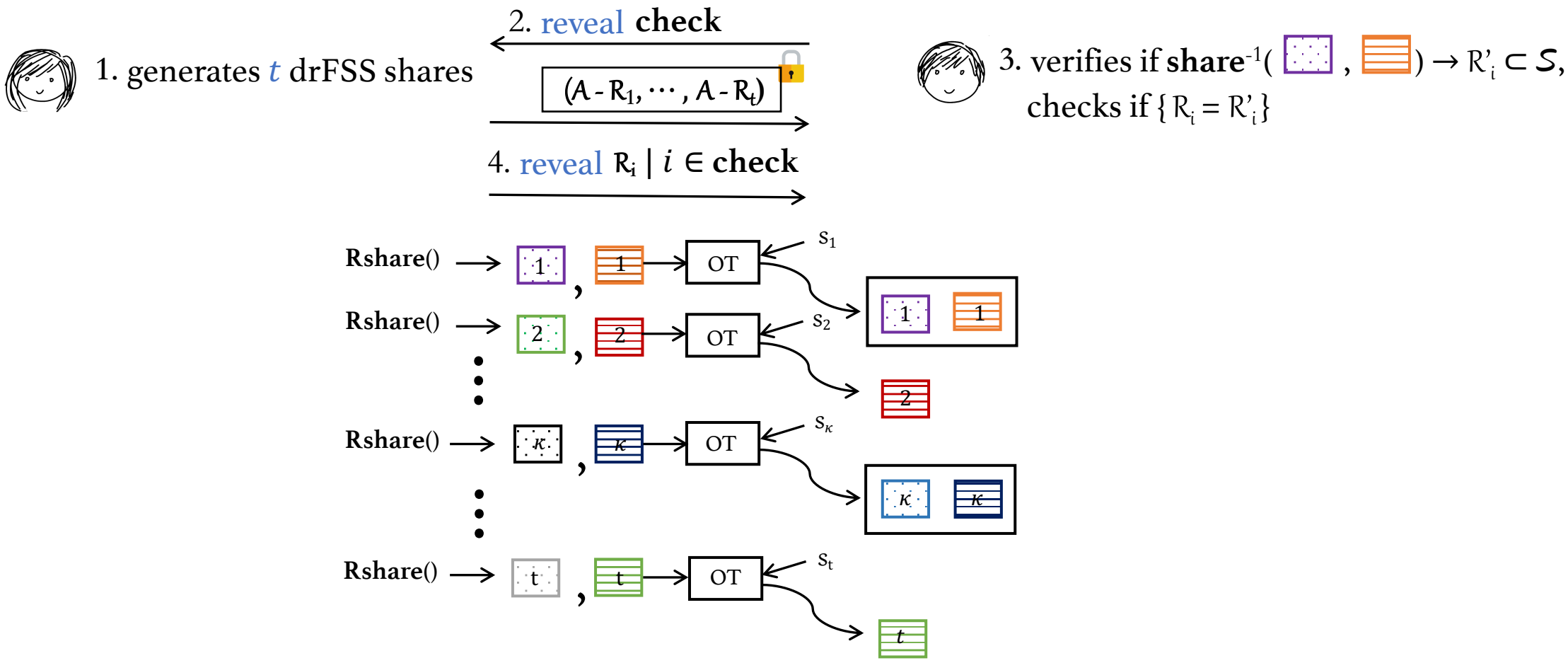

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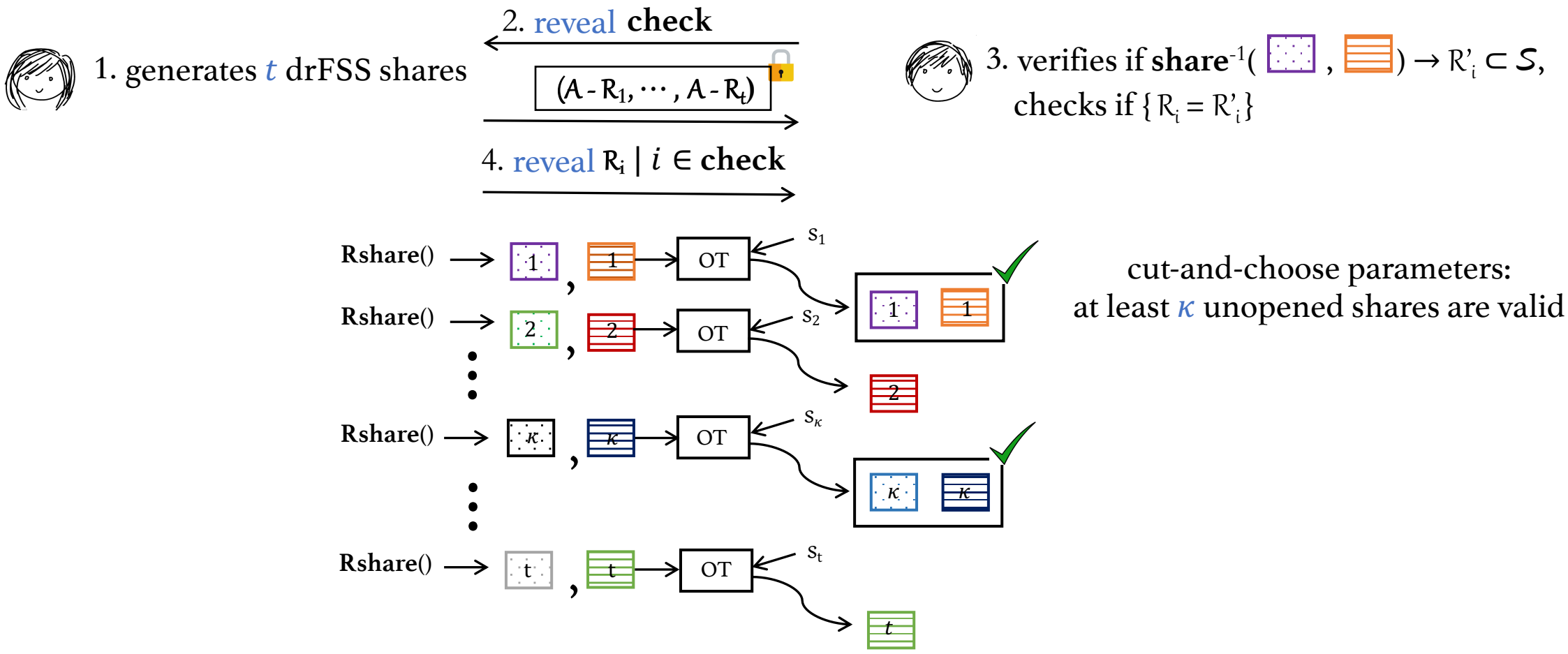

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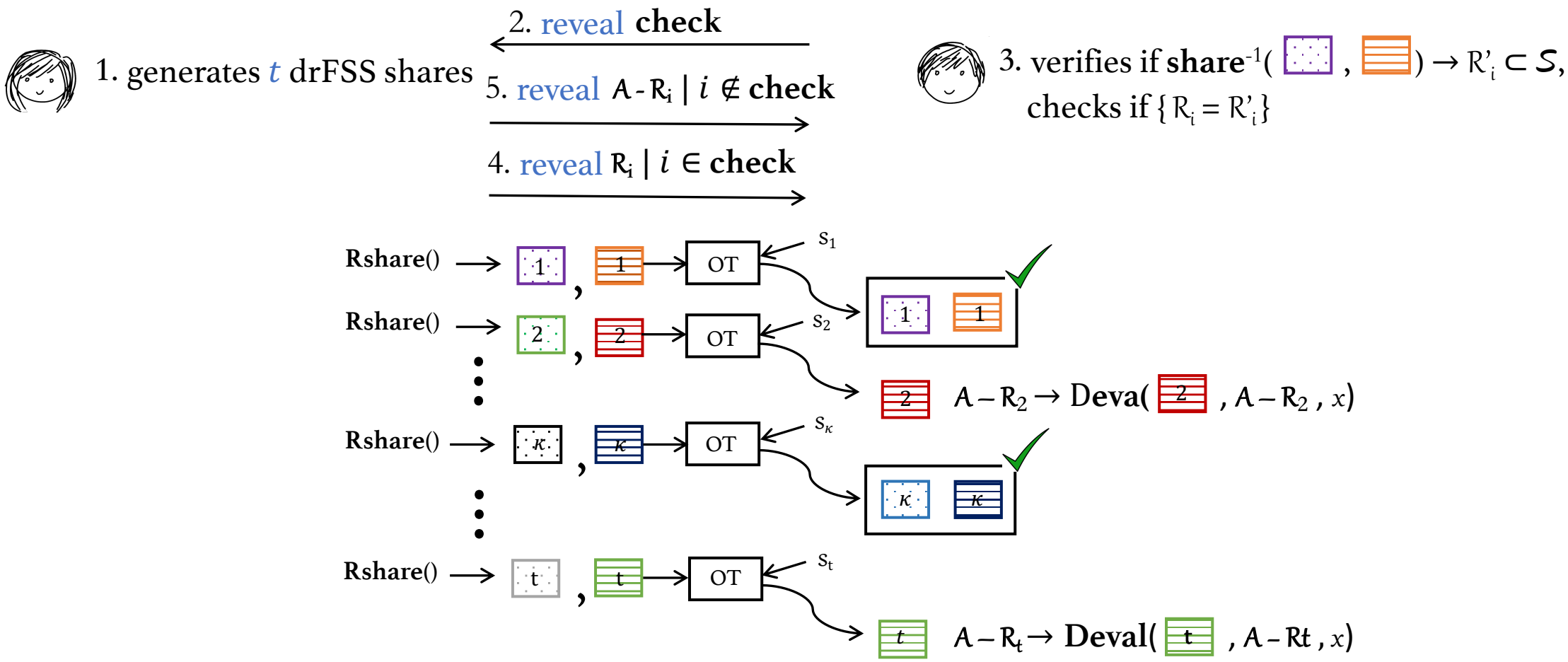

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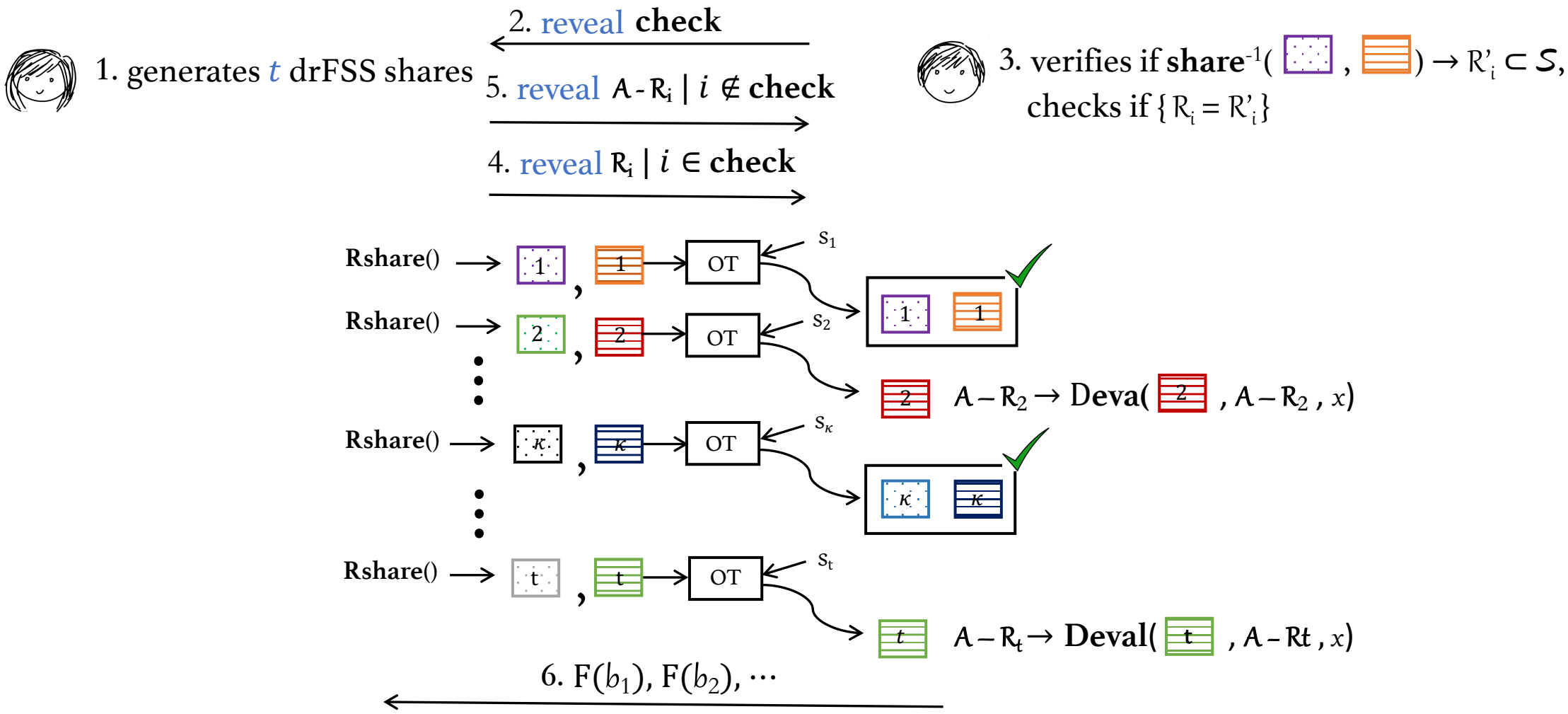

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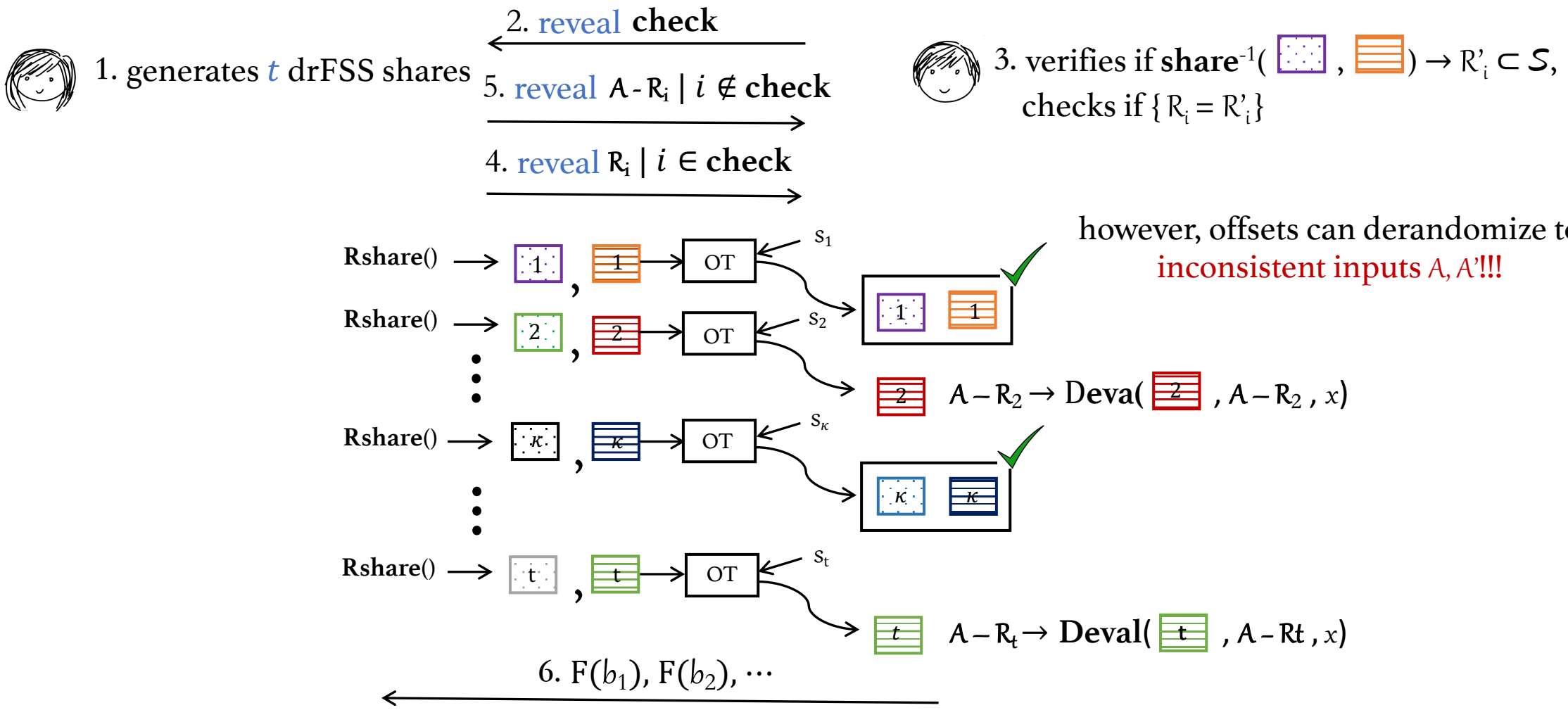

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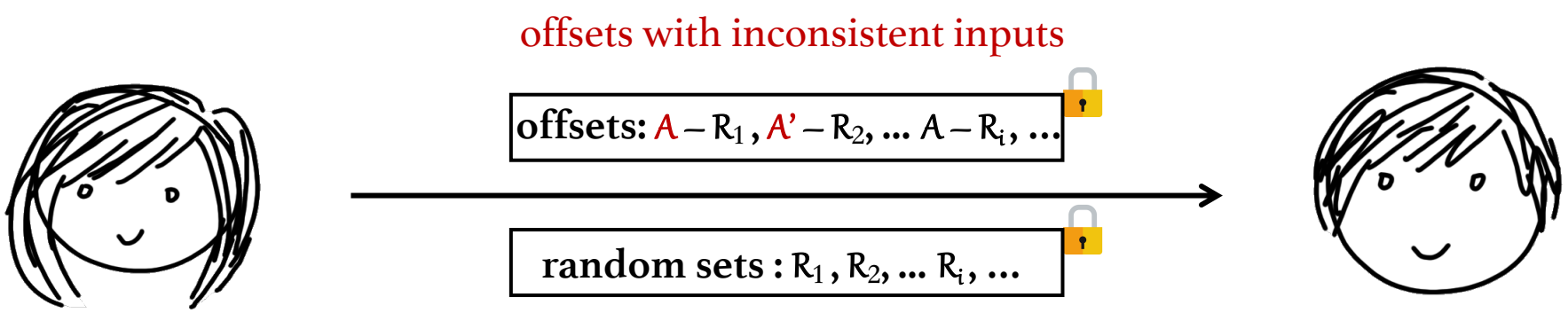
How can we ensure that offsets de-randomize evaluation back to the same input?



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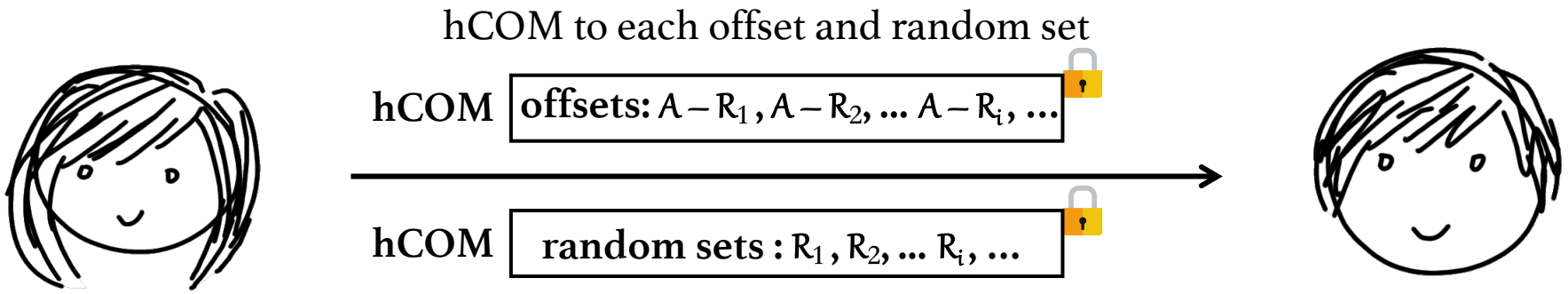

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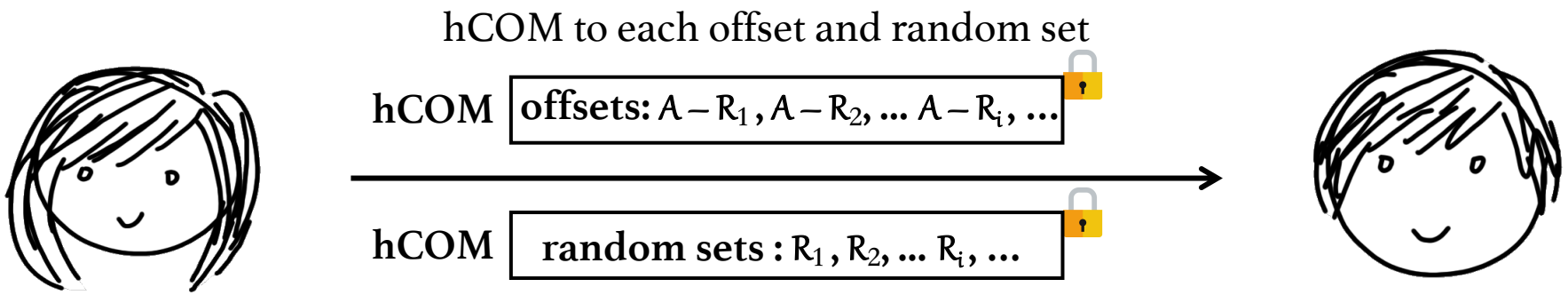
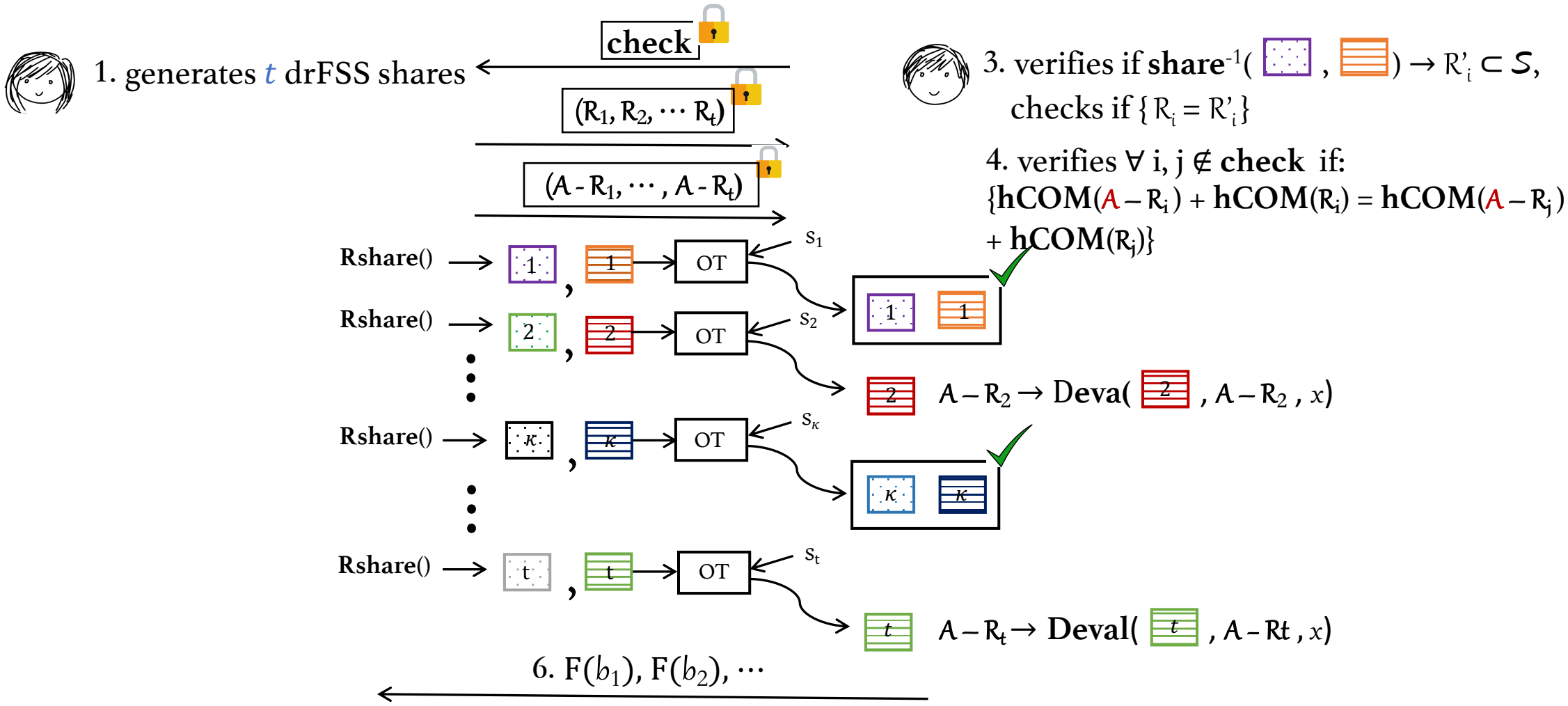


Before evaluating Bob verifies $\forall$ i, j $\notin$ **check** if:

$$\{\mathbf{hCOM}(A - R_i) + \mathbf{hCOM}(R_i) = \mathbf{hCOM}(A - R_j) + \mathbf{hCOM}(R_j)\}$$

Protocol Snapshot

assumptions: drFSS, committed OT, hCOM, random oracle model



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✓ more generally, show that derandomizable FSS can instantiate semi-honest structure-aware PSI

Future directions

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- Can we have structure-aware PSI where party with unstructured input learns result?
- What if both Alice and Bob have structured inputs?
- Can we design drFSS for union of balls in other metric spaces like hamming distance, L-2 ?

Questions?

How does [GRS'22] work?

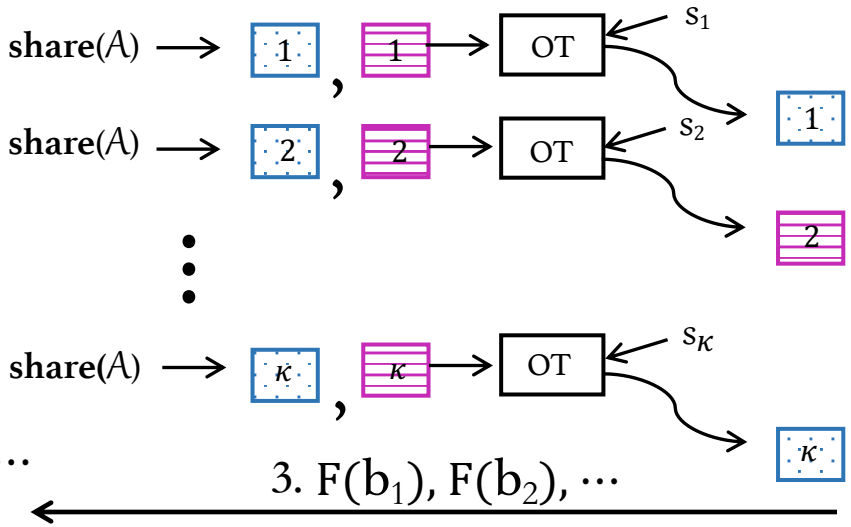
assumptions: OT-hybrid (Oblivious Transfer[Rabin81]), random oracle model

input A

1. generates κ instances of bFSS shares

input B

2. picks κ choice bits to learn  or 



$$(x \in A \implies \mathbf{ev}(\,\square_1\,, x) = \mathbf{ev}(\,\square_2\,, x))$$

$$F(x) = \mathbf{H}(\mathbf{ev}(\,\square_1\,, x) \| \mathbf{ev}(\,\square_2\,, x) \| \cdots \mathbf{ev}(\,\square_\kappa\,, x))$$
$$= \mathbf{H}(\mathbf{ev}(\,\square_1\,, x) \| \mathbf{ev}(\,\square_2\,, x) \| \cdots \mathbf{ev}(\,\square_\kappa\,, x))$$

Bob computes $F(x)$ on all his inputs

$$F(x) = \mathbf{H}(\mathbf{ev}(\,\square_1\,, x) \| \mathbf{ev}(\,\square_2\,, x) \| \cdots \mathbf{ev}(\,\square_\kappa\,, x))$$

construction	dFSS?	share size	eval cost
disjoint balls: basic FSS [BGI16]	yes	$O(n\kappa u^d)$	$O(\boldsymbol{n}u^d)$
disjoint balls: [GRS22]	no	$O(n(4\log\delta)^d\kappa)$	$O((2\log\delta)^d)$
disjoint balls: ours	yes	$O(n\kappa(ud+(\log\delta)^d))$	$O((2\log\delta)^d)$
balls with centers $> 4\delta$ apart: [GRS22]	no	$O(nd2^d\kappa\log\delta)$	$O(d\log\delta)$
balls with centers $> 8\delta$ apart: ours	yes	$O(nd\kappa\log\delta)$	$O(d\log\delta)$
axis-disjoint balls: [GRS22]	no	$O(nd\kappa\log\delta)$	$O(d\log\delta)$

Figure 1: FSS share size for n balls (ℓ_∞ norm) of radius δ in d dimensions, over u-bit integers. Evaluation time is for evaluating on one point.

structured input set	Communication cost (in GB)	
	[GRS22] (semi-honest secure)	Ours (malicious secure)
disjoint union of ℓ_∞ balls	101.1	662.4
union of balls with centers $> 8\delta$ apart	13.2	85.6

Figure 2: PSI concrete communication cost comparison with [GRS22]. Here Alice's structured set (of size $\approx 10^7$) contains $n = 2700$ ℓ_∞ balls in 2 dimensions, each with radius $\delta = 30$ in universe of size 2^{32} along each dimension, and Bob inputs an unstructured set of size 10^6.

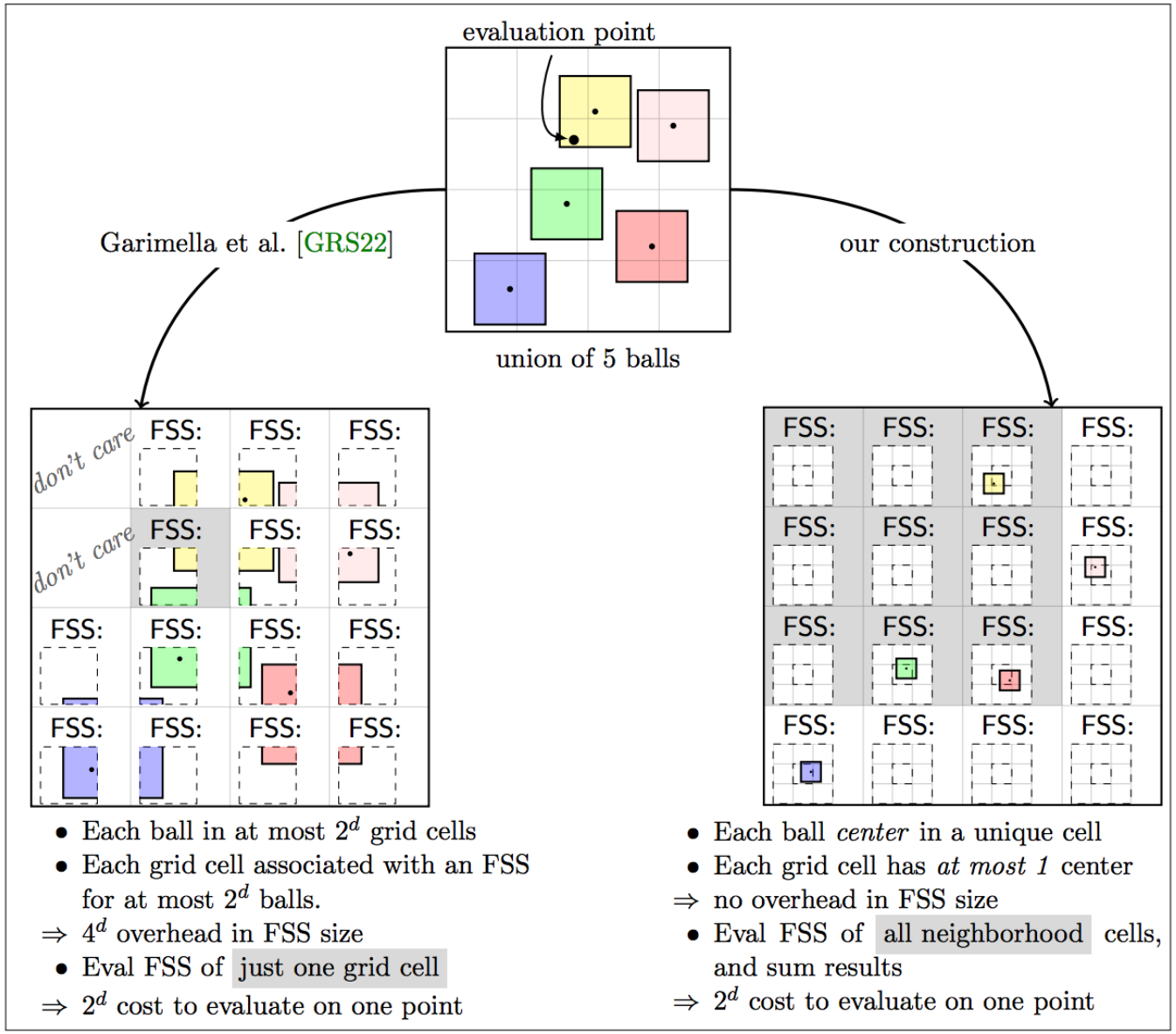


Figure 8: Illustration of the construction of Garimella et al. [GRS22] and our improved construction, for the union of n balls.