Publicly-Verifiable Deletion via Target-Collapsing Functions

James Bartusek (UC Berkeley)
Dakshita Khurana (UIUC)
Alexander Poremba (Caltech)

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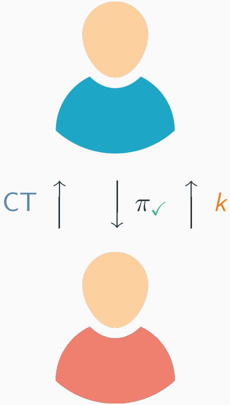
Caltech

Can we prove that data has been deleted?

Quantum Encryption with Certified Deletion (BI'20)

Bob



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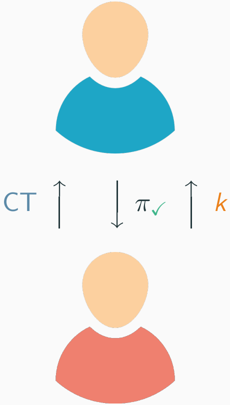

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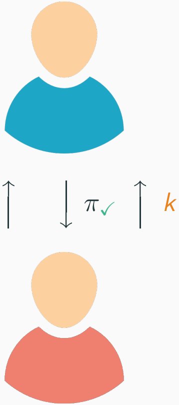

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> Publicly-verifiable deletion?
>
> Anyone should be able to verify a certificate π to determine whether CT was successfully deleted.

Publicly-Verifiable Deletion

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THIS WORK: PVD under standard assumptions.

- Initiate the study of (certified-everlasting) target-collapsing hashes:
  - Proof of *Strong Gaussian-Collapsing Conjecture* (implies PVD under LWE/SIS)
  - Proof that the public-key encryption scheme by Hhan, Morimae, Yamawaka (Eurocrypt'23) enables PVD (implies PVD under non-abelian group actions)
- Generic compiler for cryptosystems with PVD from target-collapsing hashes (for example, assuming *almost-regular one-way functions*)

Candidate construction:

Dual-Regev public-key scheme with PVD

Poremba (ITCS '23)

Gaussian superpositions

MAIN IDEA: Quantum reduction from SIS to LWE [Reg'05,SSTX'09]. Let $\mathbf{A} \in \mathbb{Z}_q^{n\times m}$.

$$|\psi\rangle = \sum_{\mathbf{x}\in\mathbb{Z}_q^m} \rho_\sigma(\mathbf{x}) \, |\mathbf{x}\rangle \otimes |\mathbf{A}\cdot\mathbf{x} \pmod{q}\rangle, \qquad \rho_\sigma(\mathbf{x}) = \exp(-\pi\|\mathbf{x}\|^2/\sigma^2).$$

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Measure the second register, which results in an outcome $\mathbf{y} \in \mathbb{Z}_q^n$.

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Coset state

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Coset state

Superposition of short vectors $\mathbf{x}$
subject to the constraint that

$$\mathbf{A}\mathbf{x} = \mathbf{y} \pmod{q}.$$

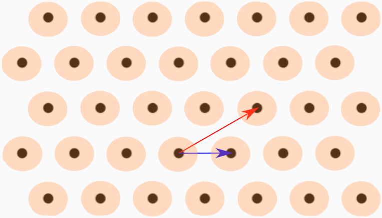
(solutions to inhomogenous SIS problem)

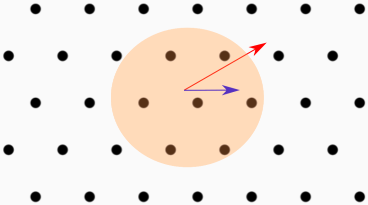
Duality of Gaussian States

$$\sum_{\mathbf{s}\in\mathbb{Z}_q^n}\sum_{\mathbf{e}\in\mathbb{Z}_q^m}\rho_{q/\sigma}(\mathbf{e})\,\omega_q^{-\langle\mathbf{s},\mathbf{y}\rangle}\,|\mathbf{s}^\mathsf{T}\mathbf{A}+\mathbf{e}^\mathsf{T}\rangle \quad\overset{\text{FT}}{\longleftrightarrow}\quad \sum_{\substack{\mathbf{x}\in\mathbb{Z}_q^m:\\ \mathbf{A}\mathbf{x}=\mathbf{y}\pmod q}}\rho_\sigma(\mathbf{x})\,|\mathbf{x}\rangle$$

Primal state

Dual state





Duality of Gaussian States

Use primal domain to *encrypt* messages.

$$\sum_{\mathbf{s}\in\mathbb{Z}_q^n}\sum_{\mathbf{e}\in\mathbb{Z}_q^m}\rho_{q/\sigma}(\mathbf{e})\,\omega_q^{-\langle\mathbf{s},\mathbf{y}\rangle}\left|\mathbf{s}^\intercal\mathbf{A}+\mathbf{e}^\intercal\right\rangle$$

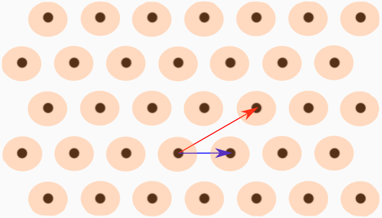
Primal state

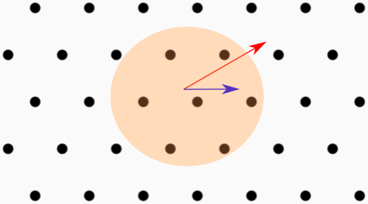
$$\xleftrightarrow{\text{FT}}$$

Use dual domain to *prove* deletion.

$$\sum_{\substack{\mathbf{x}\in\mathbb{Z}_q^m:\\ \mathbf{A}\mathbf{x}=\mathbf{y}\pmod q}}\rho_\sigma(\mathbf{x})\,|\mathbf{x}\rangle$$

Dual state





Dual-Regev public-key encryption with PVD [Poremba, ITCS'23]

- $\mathsf{KeyGen}(1^\lambda)$ generate $\mathsf{pk} = \mathbf{A} \in \mathbb{Z}_q^{n\times m}$ and $\mathsf{sk} = \mathbf{t}$ with $\mathbf{A}\cdot\mathbf{t} = \mathbf{0} \pmod q$.
- $\mathsf{Enc}_{\mathsf{pk}}(b)$ generate a verification key $\mathsf{vk} \leftarrow (\mathbf{A}, \mathbf{y} \in \mathbb{Z}_q^n)$ and ciphertext

$$|\mathsf{CT}\rangle \leftarrow \sum_{\mathbf{s}\in\mathbb{Z}_q^n} \sum_{\mathbf{e}\in\mathbb{Z}_q^m} \rho_{q/\sigma}(\mathbf{e})\,\omega_q^{-\langle \mathbf{s},\mathbf{y}\rangle} |\mathbf{s}^\intercal\mathbf{A} + \mathbf{e}^\intercal + (0,\ldots,0,b\cdot\lfloor q/2\rfloor)\rangle.$$

- $\mathsf{Dec}_{\mathsf{sk}}(|\mathsf{CT}\rangle)$ measure the ciphertext in the computational basis with outcome $\mathbf{c} \in \mathbb{Z}_q^m$ and round $\mathbf{c}^\intercal \cdot \mathsf{sk} \in \mathbb{Z}_q$ with respect to 0 and $\lfloor \frac{q}{2} \rfloor$.

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To delete the ciphertext, apply the quantum Fourier transform resulting in

$$|\widehat{\mathsf{CT}}\rangle = \sum_{\substack{\mathbf{x} \in \mathbb{Z}_q^m: \\ \mathbf{A}\mathbf{x} = \mathbf{y} \pmod q}} \rho_\sigma(\mathbf{x})\, \omega_q^{\langle \mathbf{x}, (0, \dots, 0, b \cdot \lfloor \frac{q}{2} \rfloor) \rangle} |\mathbf{x}\rangle$$

and measure to obtain a *short* solution π to the ISIS problem $\mathbf{A} \cdot \pi = \mathbf{y}$ (public verification!)

Security

Certified Deletion Experiment

$\text{CD-EXP}_{\mathcal{A}}(b)$:

1. Sample a matrix $\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n\times m}$.
2. Generate a pair $(\mathbf{y}, |\psi_{b,\mathbf{y}}\rangle)$ with

$$|\psi_{b,\mathbf{y}}\rangle = \sum_{\mathbf{x}\in\mathbb{Z}_q^m \text{ s.t. } \mathbf{Ax}=\mathbf{y}} \rho_\sigma(\mathbf{x})\ \omega_q^{\langle \mathbf{x},(0,\ldots,0,b\cdot\lfloor \frac{q}{2}\rfloor)\rangle} |\mathbf{x}\rangle.$$

3. Run $\mathcal{A}(\mathbf{A}, \mathbf{y}, |\psi_{b,\mathbf{y}}\rangle)$ to produce certificate π and residual state ρ.
4. If π is short and $\mathbf{A}\cdot\pi = \mathbf{y}$, output ρ. Else, output $|\perp\rangle\langle\perp|$.

Security: For any QPT $\mathcal{A}$: $\mathrm{TD}\big(\text{CD-EXP}_{\mathcal{A}}(0), \text{CD-EXP}_{\mathcal{A}}(1)\big) \le \text{negl}.$

(Certified-Everlasting) Target-Collapsing Hashes

Certified-Everlasting Gaussian-Collapsing

CEGC-EXP$_{\mathcal{A}}(b)$:

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2. Generate a pair $(\mathbf{y}, |\psi_{\mathbf{y}}\rangle_{\mathsf{X}})$ with

$$|\psi_{\mathbf{y}}\rangle_{\mathsf{X}} = \sum_{\mathbf{x}\in\mathbb{Z}_q^m \text{ s.t. } \mathbf{A}\mathbf{x}=\mathbf{y}} \rho_\sigma(\mathbf{x})\,|\mathbf{x}\rangle_{\mathsf{X}}.$$

3. If $b = 1$, additionally measure register X in computational basis.
4. Run $\mathcal{A}(\mathbf{A}, \mathbf{y}, \mathsf{X})$ to produce certificate π and residual state ρ.
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Proof idea:
- use entanglement via $|+\rangle_{\mathsf{C}}$: superposition of non-measured/measured register X.
- wait until $\mathcal{A}$ replies with π, then perform projective measurement on register C.

Generalization: Certified-Everlasting Target-Collapsing

Let $\mathcal{H} = \{h : \mathcal{X} \to \mathcal{Y}\}_h$ be a hash family, $\mathcal{D}$ be a distribution and $\mathcal{M}$ be a measurement.

Certified-Everlasting $(\mathcal{D}, \mathcal{M})$-Target-Collapsing

- Ajtai hash function $\to$ random hash h in $\mathcal{H}$
- Gaussian distribution $\to$ distribution specified by $\mathcal{D}$.
- Computational basis measurement of X $\to$ measurement specified by $\mathcal{M}$.

$$|\psi\rangle_{\mathsf{XY}} = \sum_{x \in \mathcal{X}} \sqrt{\mathcal{D}(x)}\, |x\rangle_{\mathsf{X}} \otimes |h(x)\rangle_{\mathsf{Y}}.$$

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Main Theorem: If the hash function family $\mathcal{H}$ satisfies

- $(\mathcal{D}, \mathcal{M})$-target-collision-resistance
- $(\mathcal{D}, \mathcal{M})$-target-collapsing

then $\mathcal{H}$ is certified-everlasting $(\mathcal{D}, \mathcal{M})$-target-collapsing.

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- $(\mathcal{D}, \mathcal{M})$-target-collision-resistance $\quad$ ($\leftarrow$ quantum generalization of classical TCR)
- $(\mathcal{D}, \mathcal{M})$-target-collapsing $\quad$ ($\leftarrow$ weakening of collapsing property [Unruh'16])

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- We introduce a natural weakening of *collapsing* called target-collapsing.
- We show that hash functions which satisfy basic (non-everlasting) security properties *automatically* satisfy certified-everlasting target-collapsing.
- We use our framework to prove that the encryption schemes of Poremba (ITCS'23) and Hhan, Morimae and Yamakawa (Eurocrypt'23) enable PVD.
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Open Problems:

- investigate the relationship between target-collapsing and target-collision-resistance, and related notions.
- new cryptographic applications of target-collapsing hashes.

Questions?