

# SECURITY - PRESERVING DISTRIBUTED SAMPLERS

HOW TO GENERATE ANY CRS IN  
ONE ROUND WITHOUT RANDOM ORACLES

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WATERS

UNIVERSITY OF TEXAS  
AT AUSTIN

—  
NTT RESEARCH

MARK  
ZHANDRY

NTT RESEARCH

TRUSTED SETUP

**WHO?**  
hard to agree  
upon one

**STRUCTURED  
OUTPUTS**  
cannot be implemented  
using random  
oracles

**LARGE  
COMMUNICATION**  
e.g. correlated  
randomness

**TRUSTED SETUP**

**ALWAYS  
ONLINE**  
e.g. correlated  
randomness or  
non-reusable  
CRSs

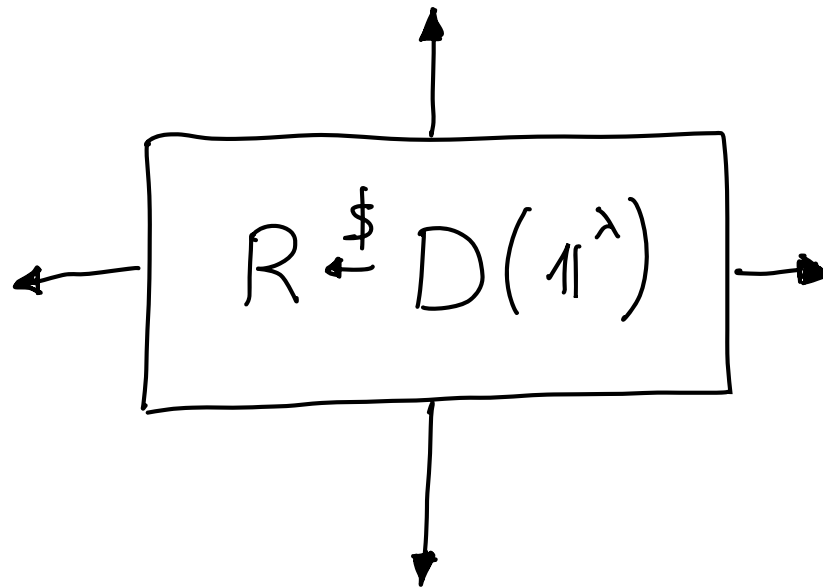
**SINGLE POINT  
OF FAILURE!**

# DISTRIBUTED SAMPLERS

[EC: ABRAM, SCHOLL, YAKOUBOV 22]

$P_m$

$P_1$

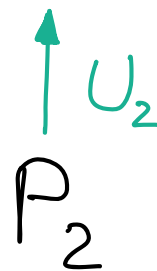
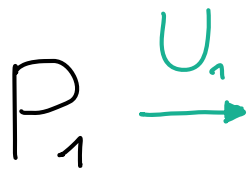
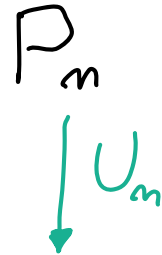


$P_i$

$P_2$

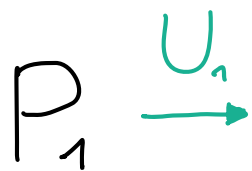
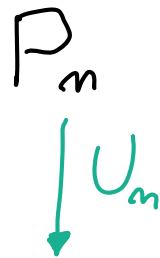
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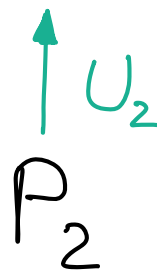
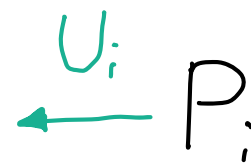


# DISTRIBUTED SAMPLERS

[EC: ABRAM, SCHOLL, YAKOUBOV 22]



$R \leftarrow \text{Sample}(U_1, \dots, U_m)$



# PREVIOUS WORK

POSITIVE RESULTS  
[EC: ABRAM, SCHOLL, YAKOUBOV22]

## SEMI-HONEST DISTRIBUTED SAMPLERS

- any efficient distribution  $D(\mathbb{1}^\lambda)$
- in the plain model
- dishonest majority
- $iO$  + multi-Key FHE

# PREVIOUS WORK

[ POSITIVE RESULTS  
[EC: ABRAM, SCHOLL, YAKOUBOV22]

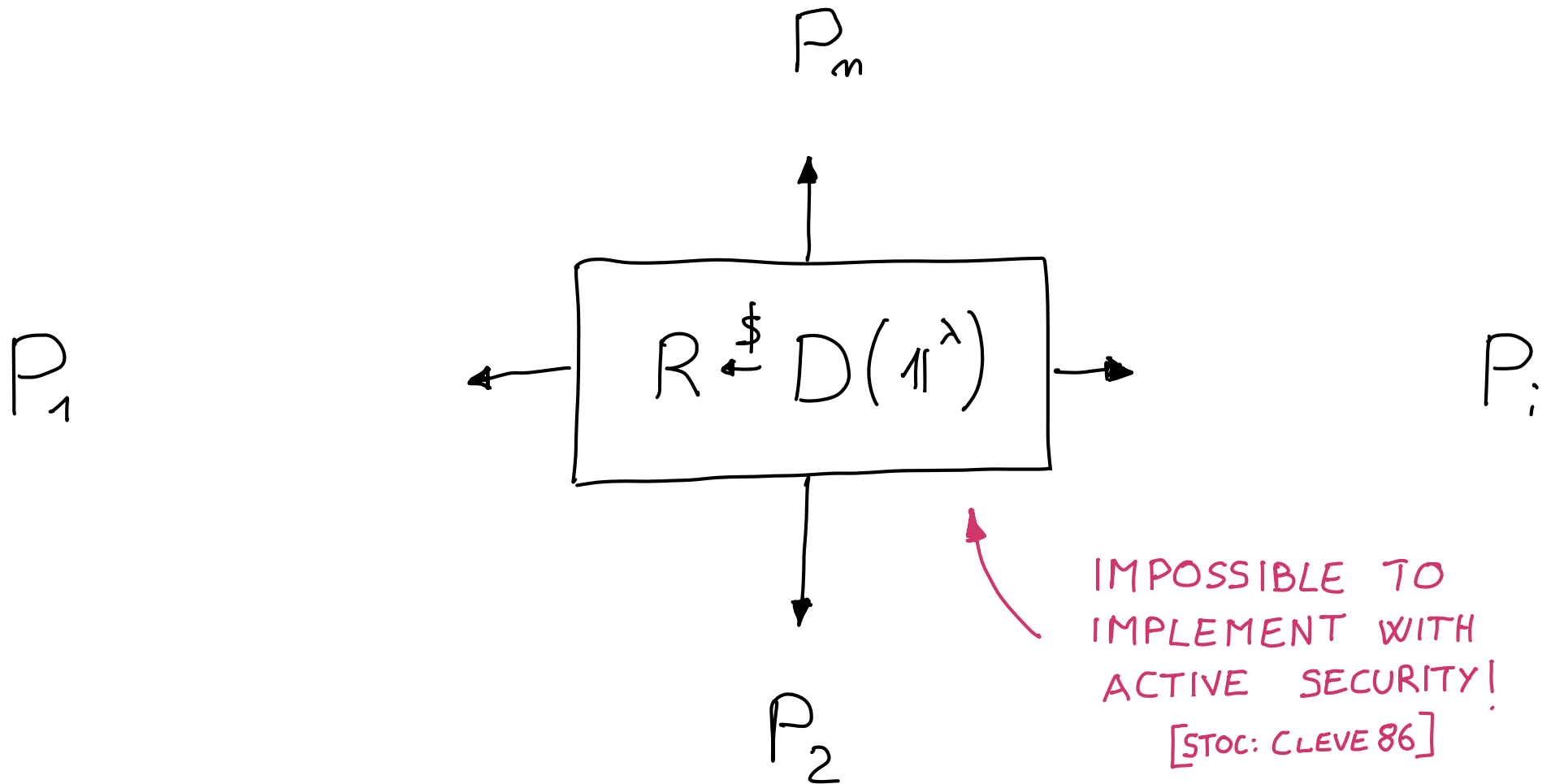
ACTIVE SECURITY ?



# PREVIOUS WORK

POSITIVE RESULTS

[EC: ABRAM, SCHOLL, YAKOUBOV22]



IMPOSSIBLE TO  
IMPLEMENT WITH  
ACTIVE SECURITY!  
[STOC: CLEVE86]

# PREVIOUS WORK

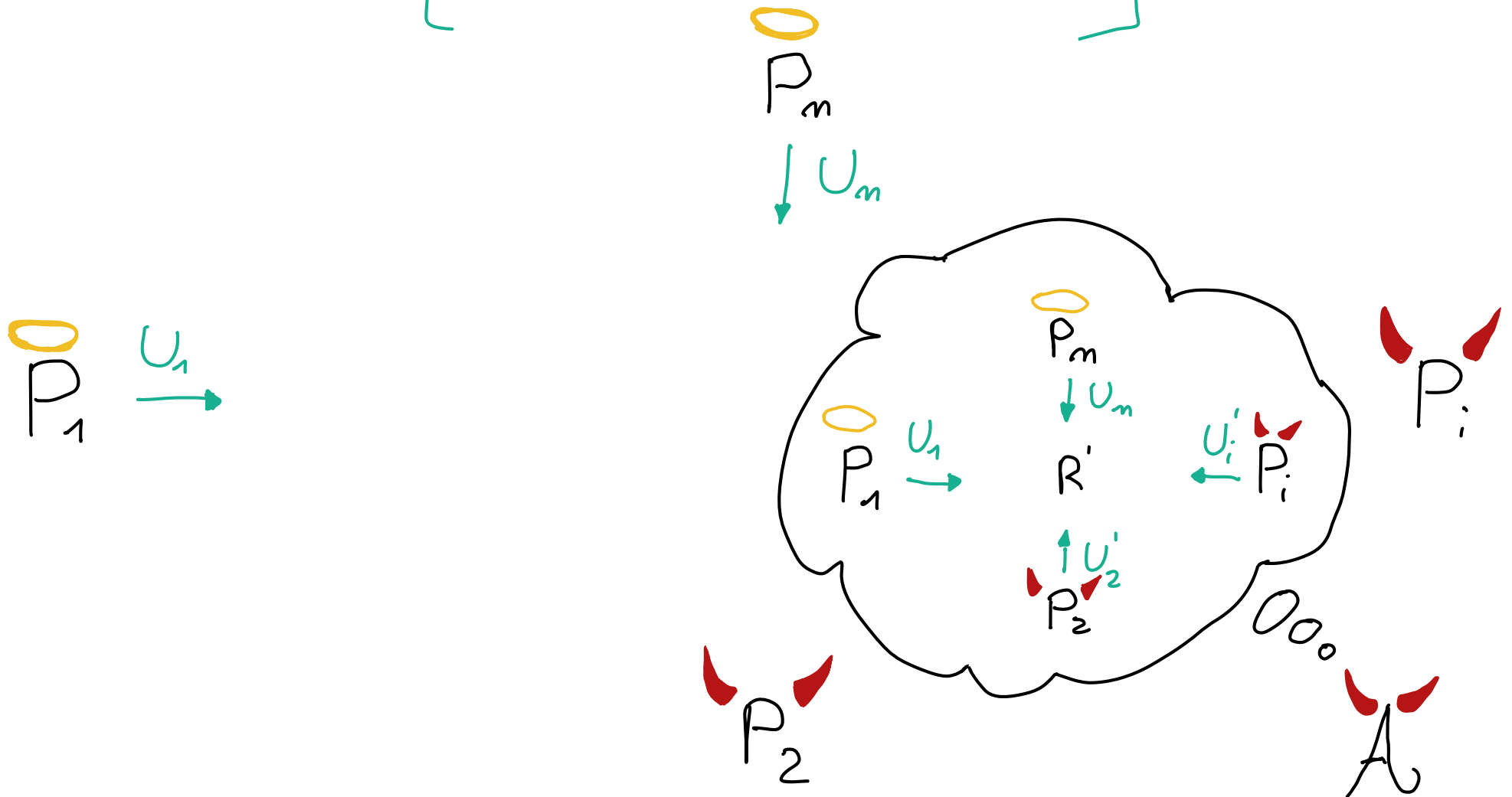
[ POSITIVE RESULTS ]  
[ EC: ABRAM, SCHOLL, YAKUBOV22 ]



# PREVIOUS WORK

POSITIVE RESULTS

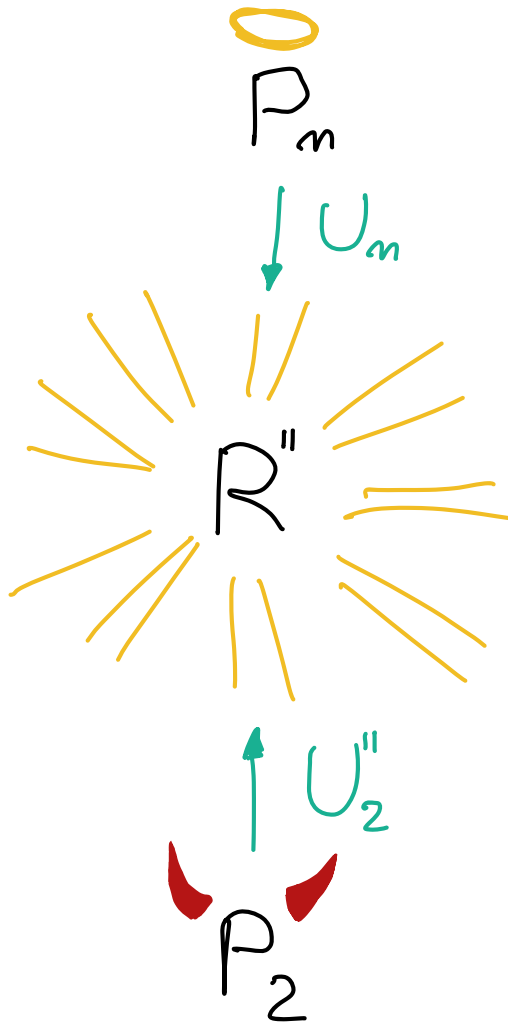
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# PREVIOUS WORK

POSITIVE RESULTS

[EC: ABRAM, SCHOLL, YAKUBOV22]



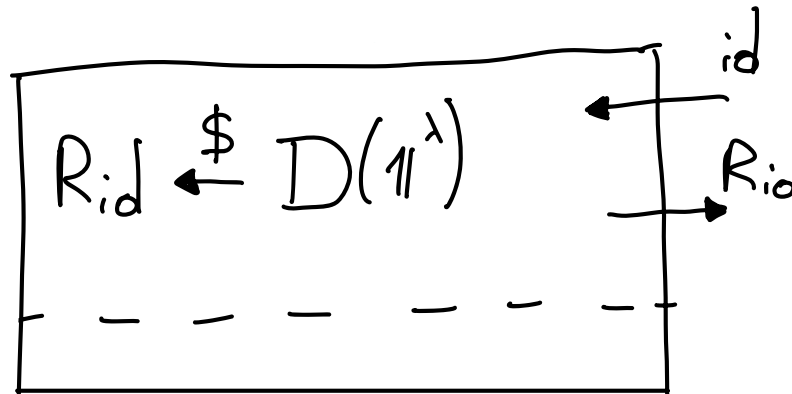
# PREVIOUS WORK

POSITIVE RESULTS  
[EC: ABRAM, SCHOLL, YAKUBOV22]

0

$P_m$

$P_1$



$A$

$P_i$

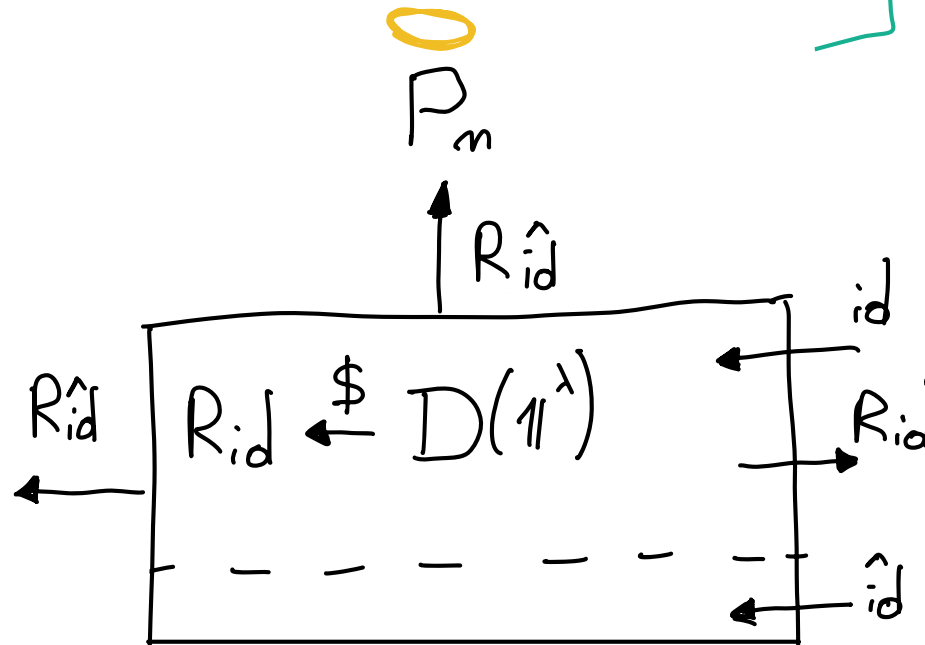
$P_2$

# PREVIOUS WORK

POSITIVE RESULTS

[EC: ABRAM, SCHOLL, YAKUBOV22]

$\overline{D}_1$



$\overline{A}$

$\overline{P}_i$

$\overline{P}_2$

# PREVIOUS WORK

POSITIVE RESULTS  
[EC: ABRAM, SCHOLL, YAKOUBOV22]

## ACTIVE DISTRIBUTED SAMPLERS

- any efficient distribution  $D(\mathbb{1}^n)$
- in the programmable RO model
- dishonest majority, static corruption
- $iO$  + multi-Key FHE + NIZKs

# PREVIOUS WORK

NEGATIVE RESULTS  
[EPRINT: ABRAM, OBREMSKI, SCHOLL 23]



# PREVIOUS WORK

NEGATIVE RESULTS  
[EPRINT: ABRAM, OBREMSKI, SCHOLL 23]

## THEOREM

Suppose that  $H_\infty(D) = \omega(\log \lambda)$ .

Then, any actively secure distributed sampler for  $D(1^\lambda)$   
needs a CRS.

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NEGATIVE RESULTS  
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NEGATIVE RESULTS  
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## SUMMARY

WITHOUT RANDOM ORACLE, ACTIVELY SECURE  
DISTRIBUTED SAMPLERS CANNOT BE BETTER THAN  
THE TRUSTED SETUP!

# OUR CONTRIBUTION

NEW DEFINITIONS OF ACTIVE  
DISTRIBUTED SAMPLERS THAT DON'T  
NEED RANDOM ORACLES

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HARDNESS-PRESERVING  
DISTRIBUTED SAMPLERS

INDISTINGUISHABILITY-PRESERVING  
DISTRIBUTED SAMPLERS

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preserving the hardness of  
search games with efficient  
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INDISTINGUISHABILITY-PRESERVING  
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# OUR CONTRIBUTION

NEW DEFINITIONS OF ACTIVE  
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## HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

preserving the hardness of  
search games with efficient  
challenger.

## INDISTINGUISHABILITY-PRESERVING DISTRIBUTED SAMPLERS

preserving the functionality  
of the compiled protocol  
if certain conditions are  
satisfied.

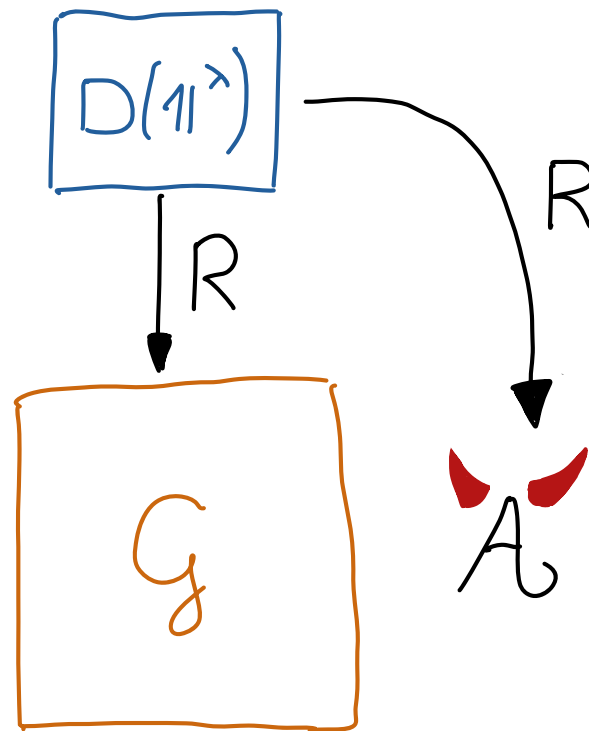
# HARDNESS - PRESERVING DISTRIBUTED SAMPLERS



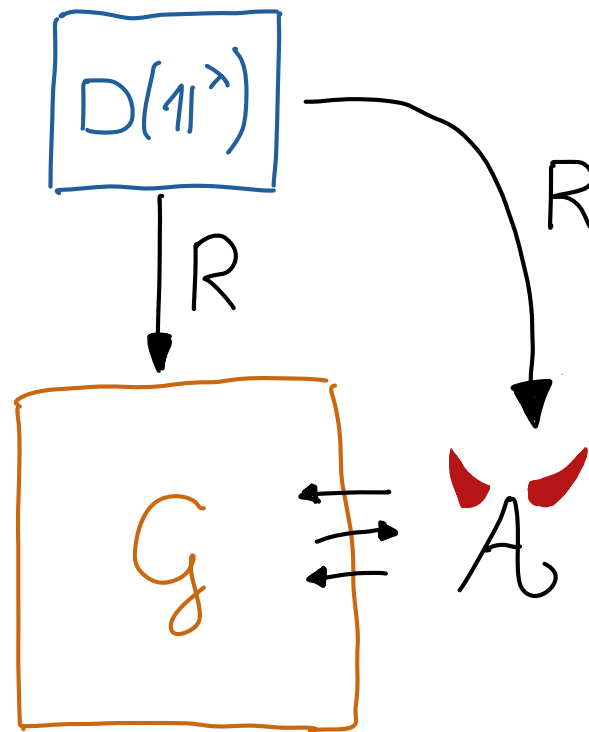
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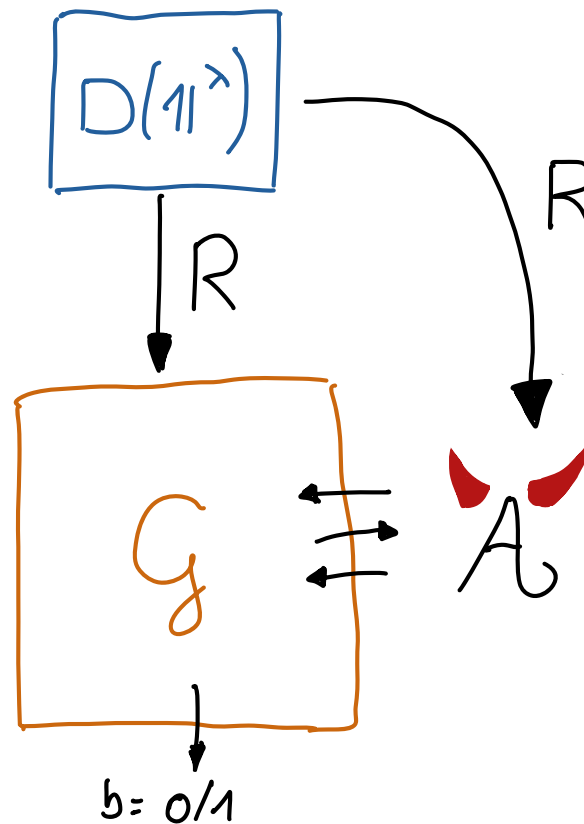
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REAL WORLD

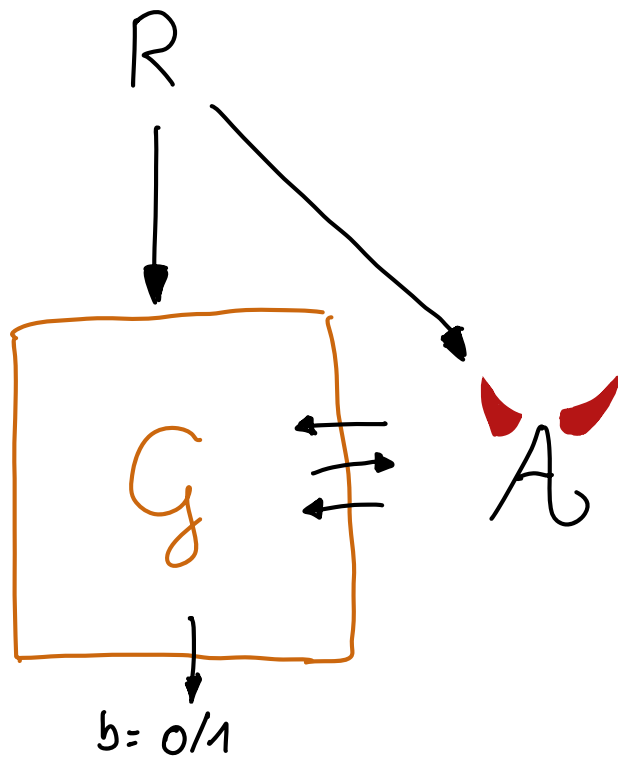
IDEAL WORLD



# HARDNESS - PRESERVING DISTRIBUTED SAMPLERS

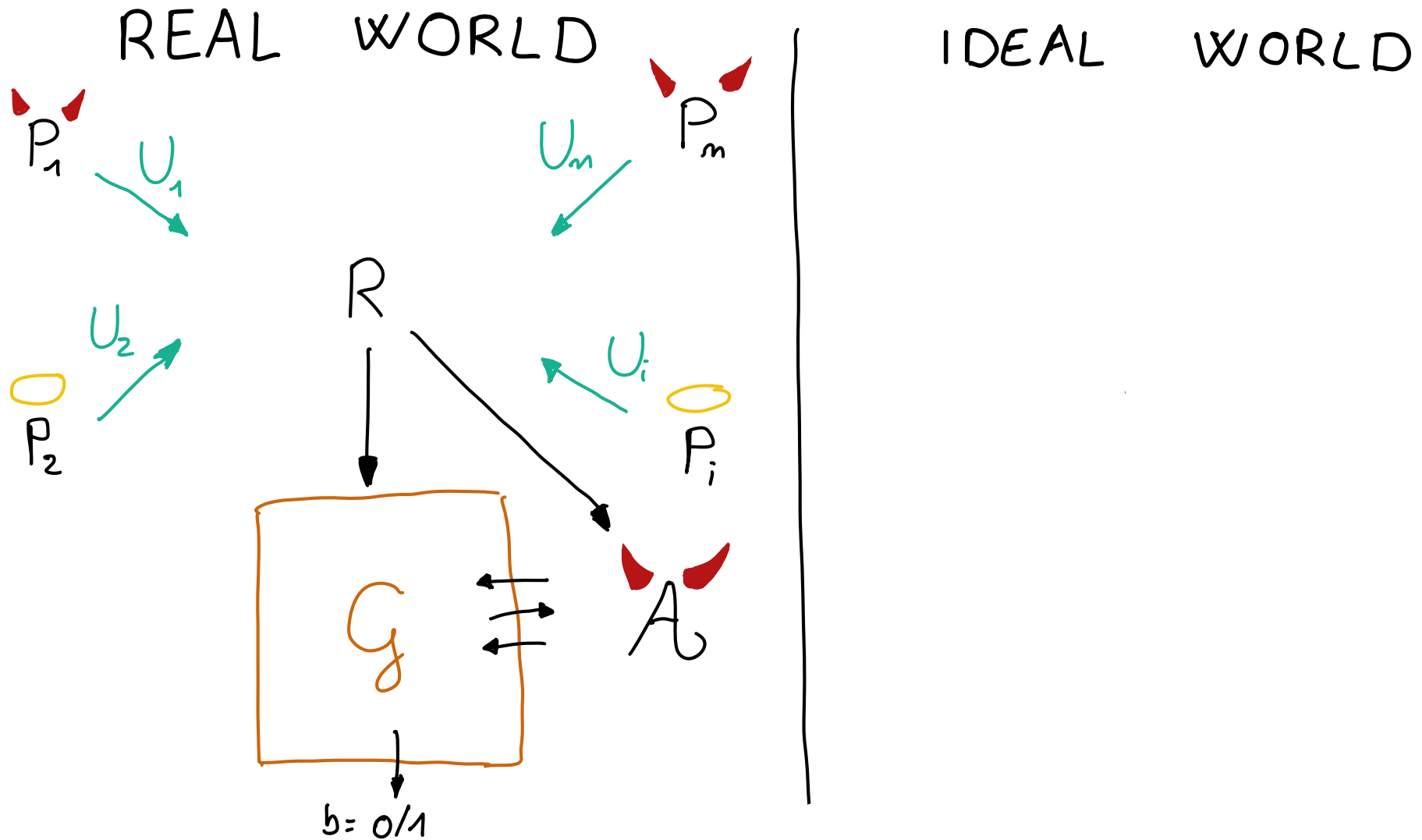
REAL WORLD

IDEAL WORLD

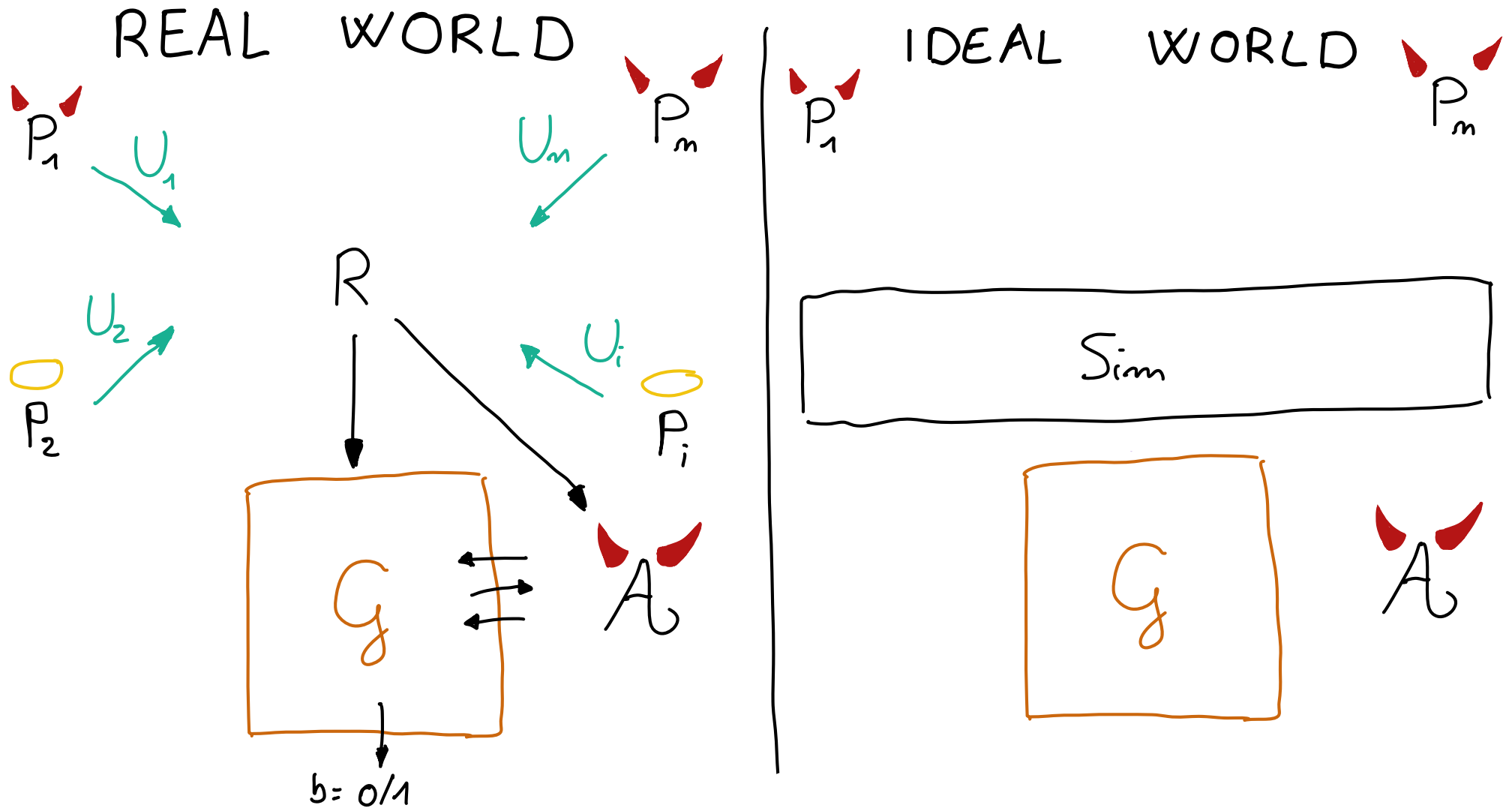




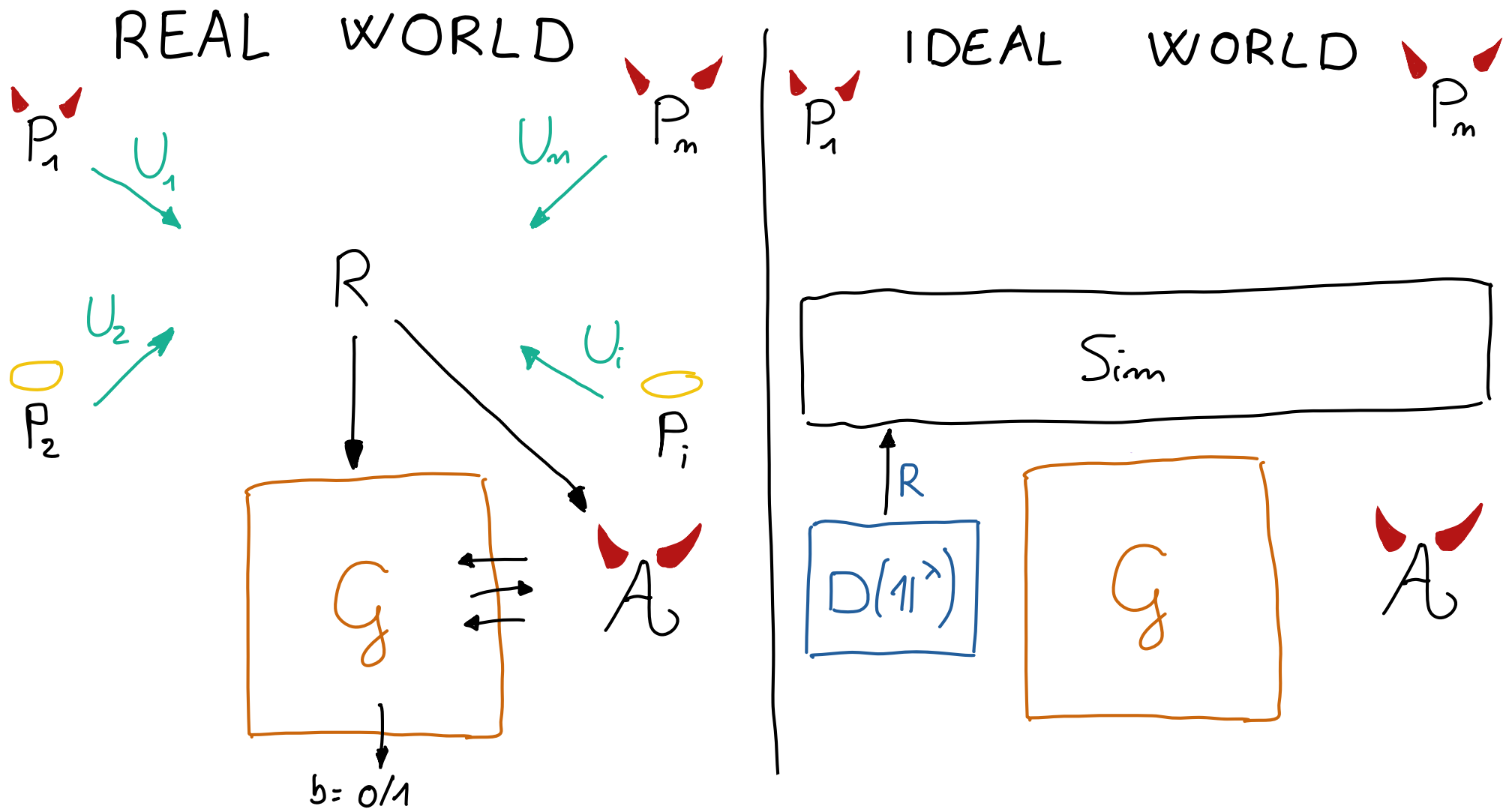
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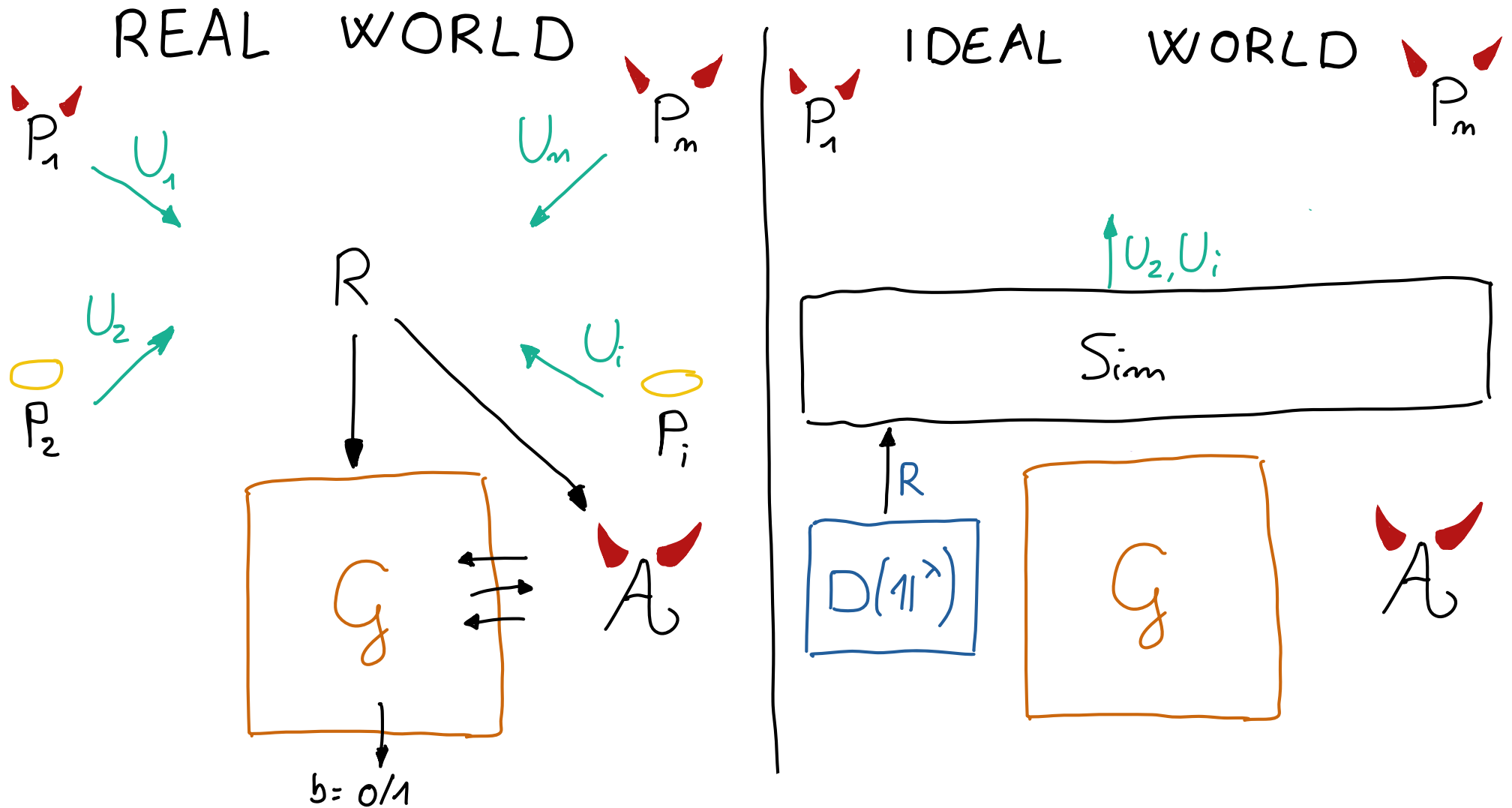
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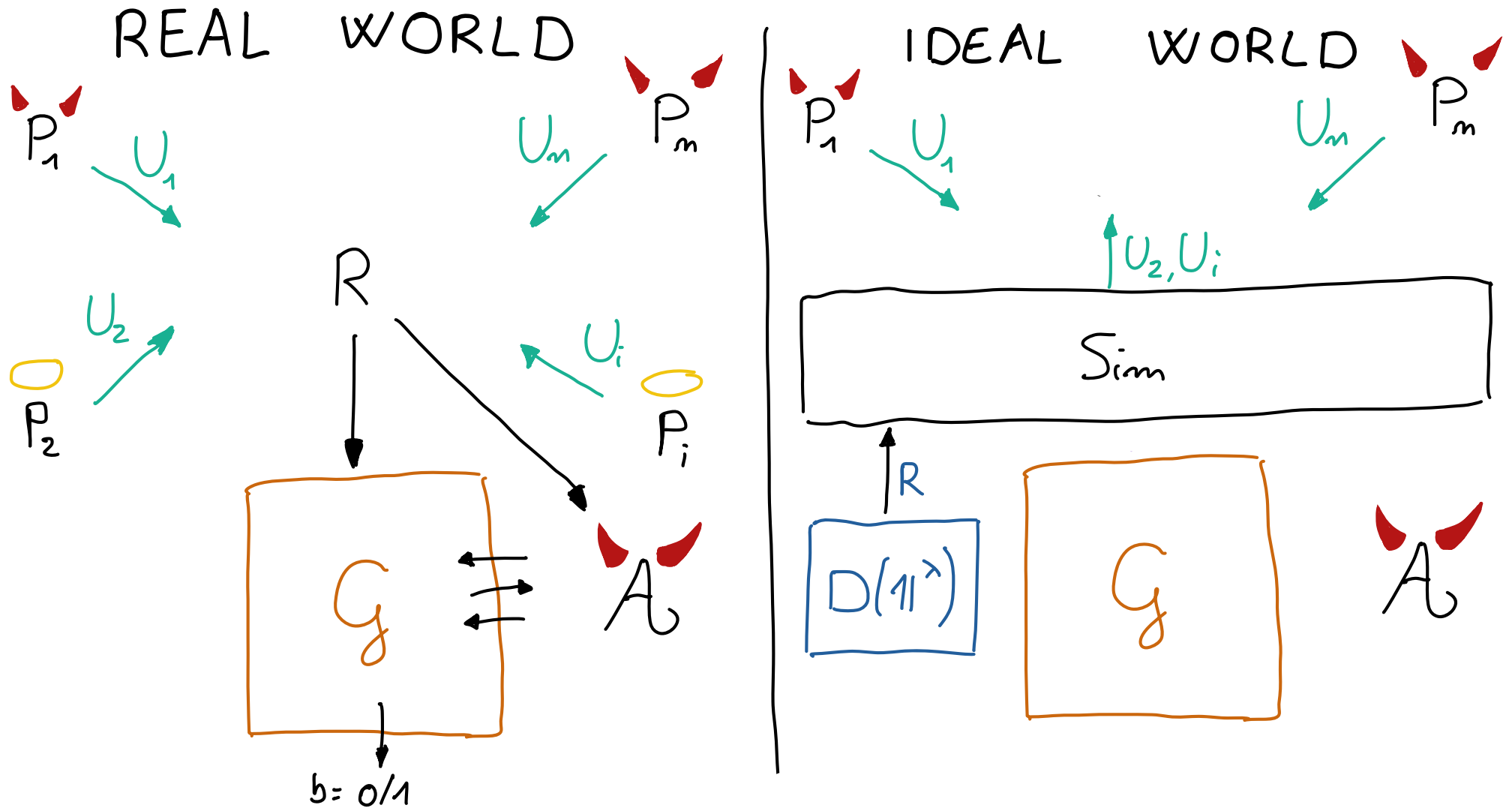
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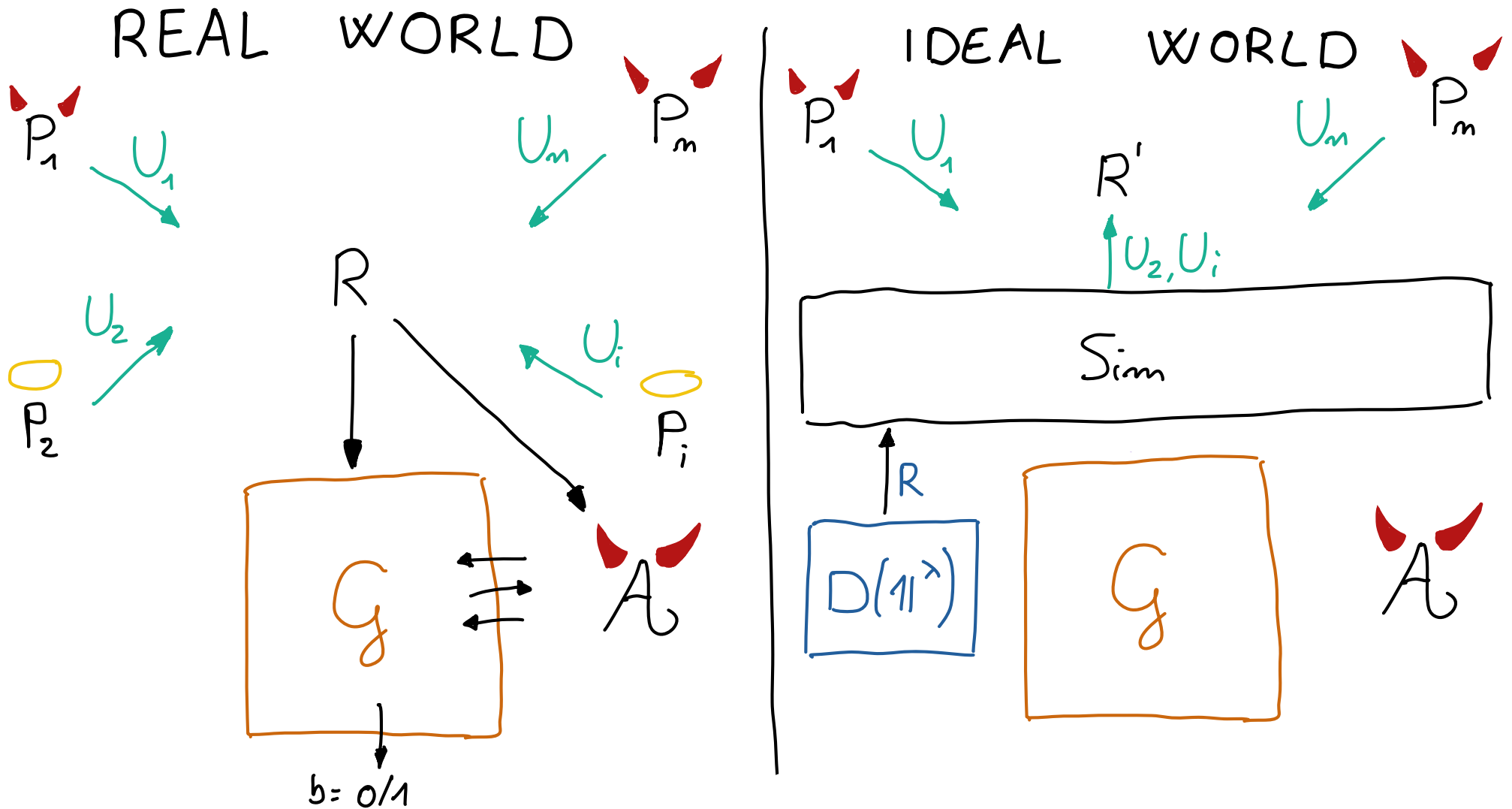
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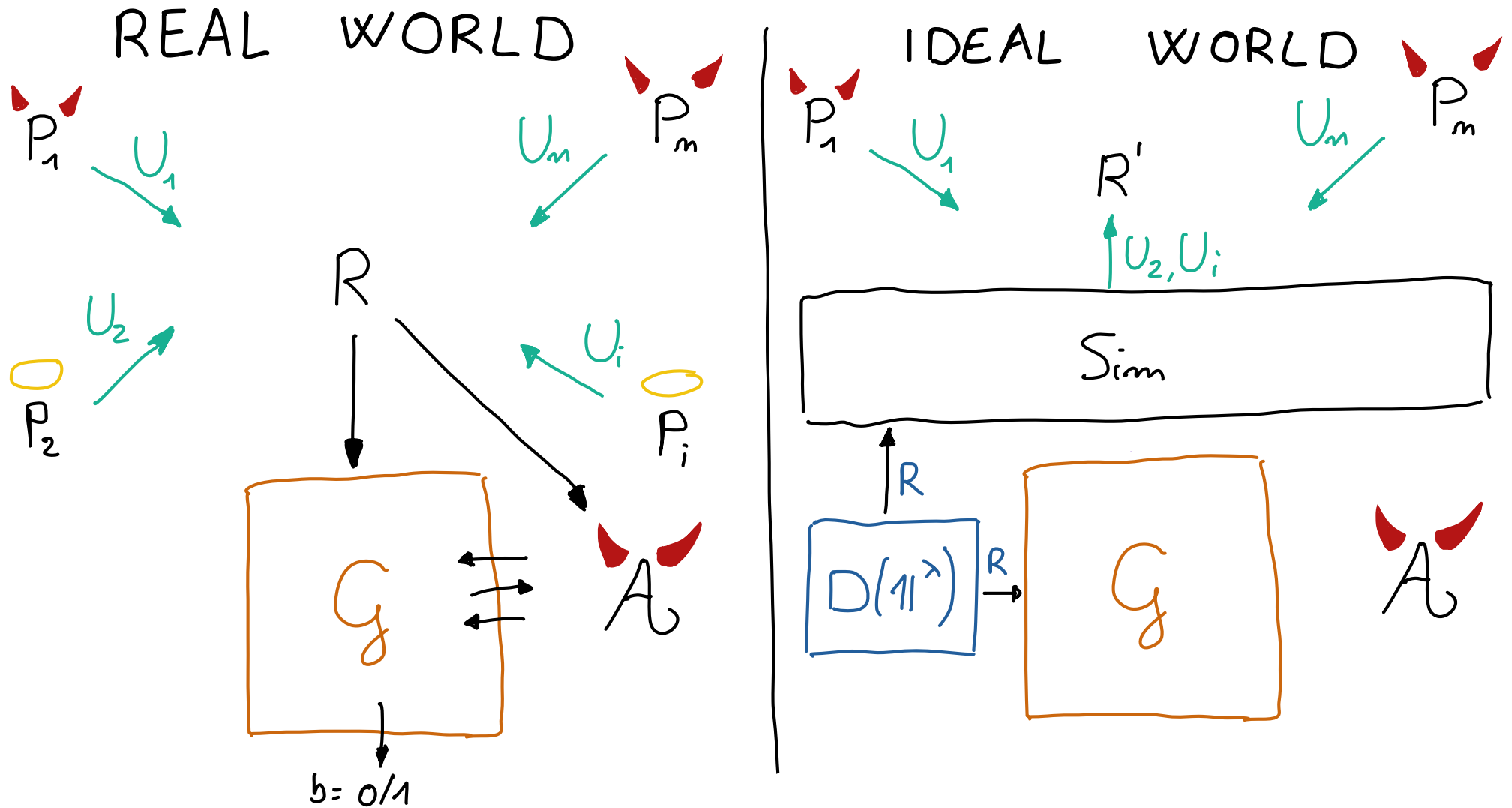
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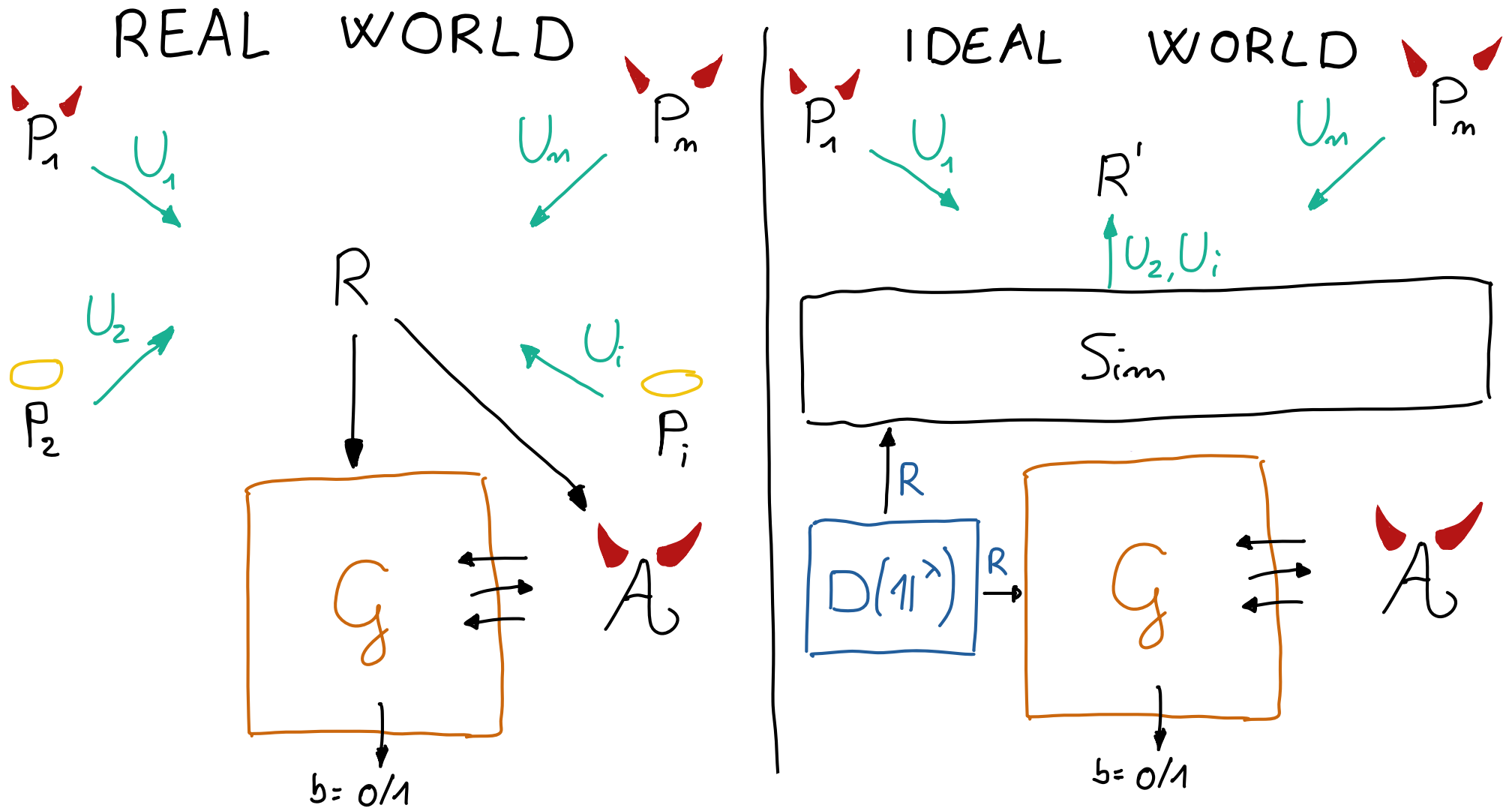
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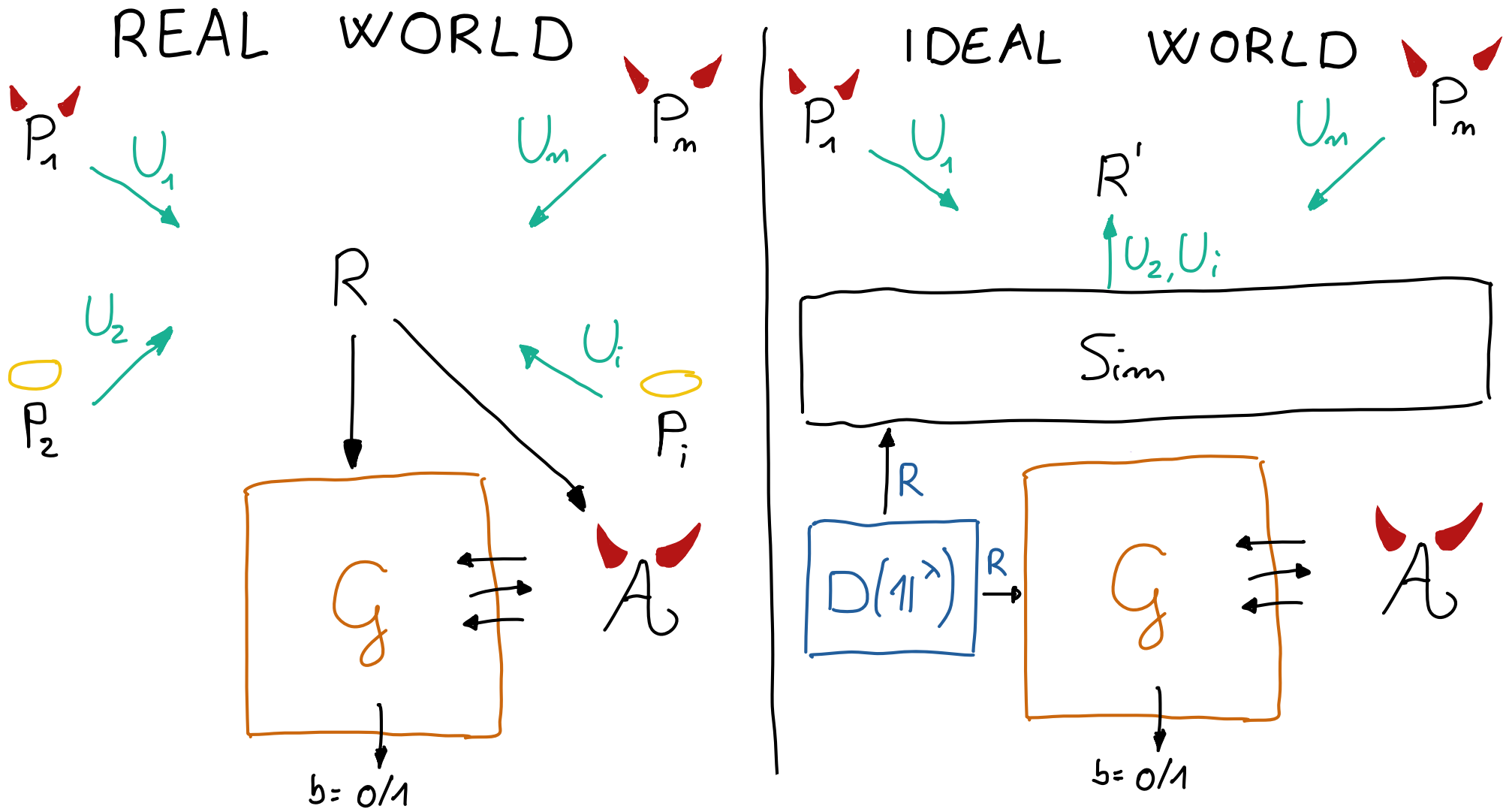


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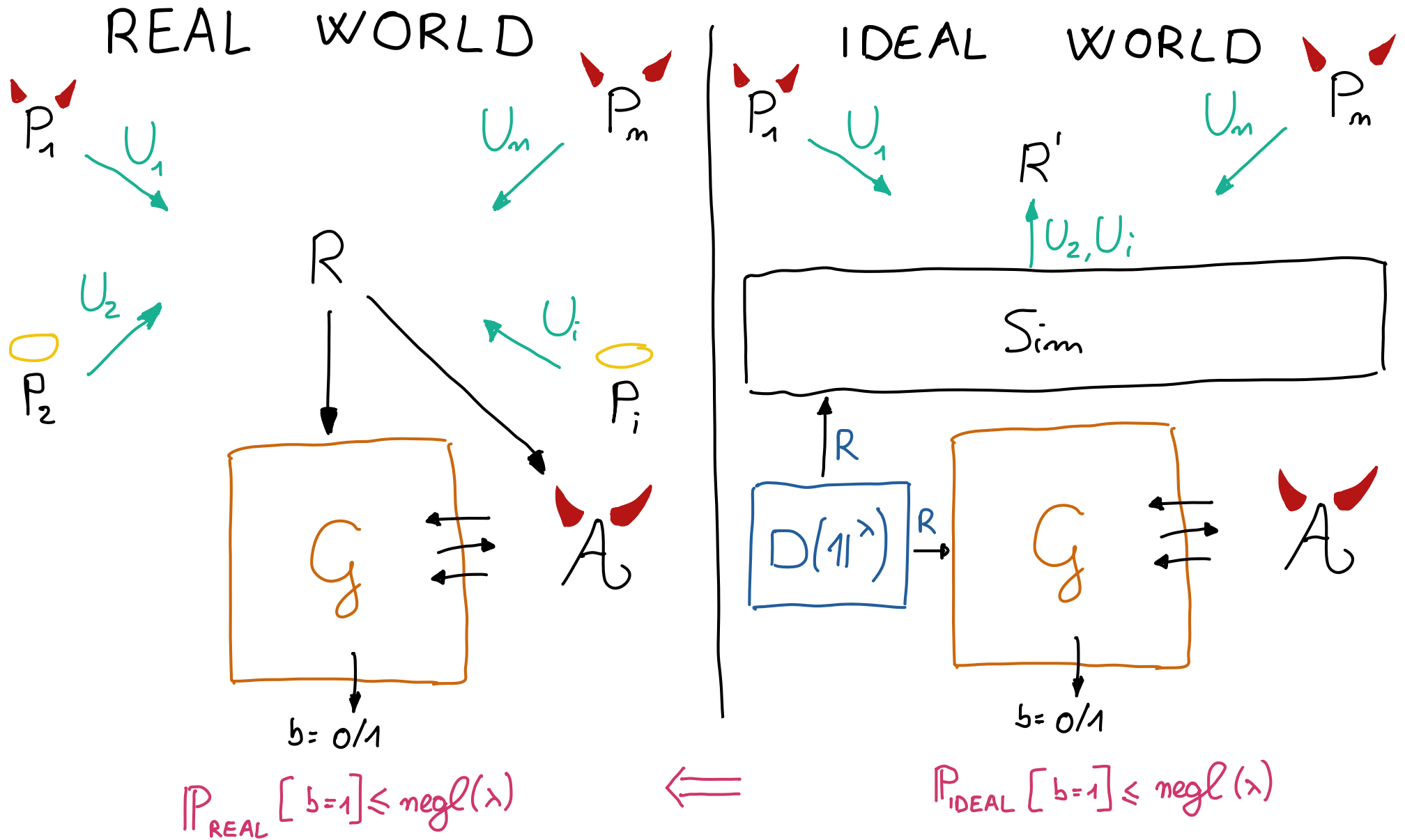


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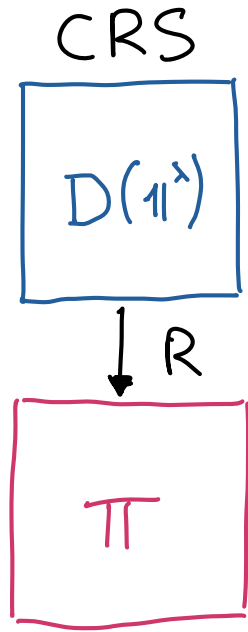


$$P_{IDEAL}[b=1] \leq \text{negl}(\lambda)$$

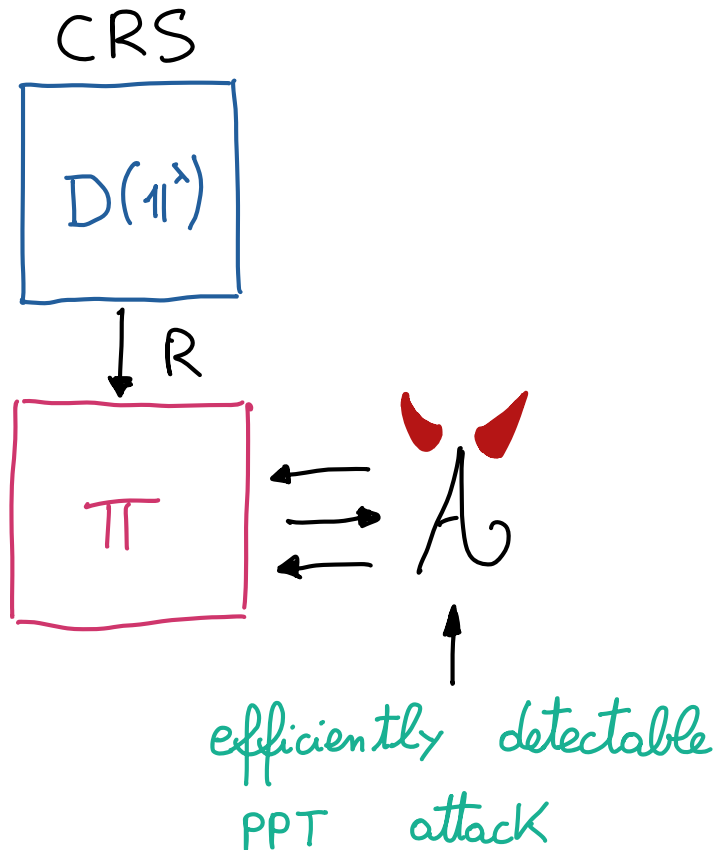
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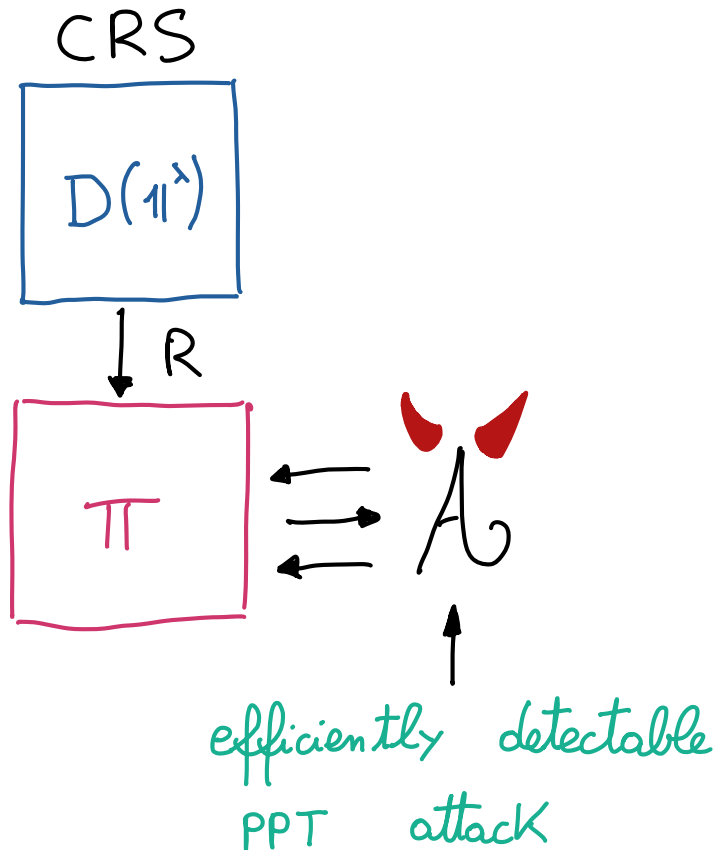
# SECURITY GUARANTEES OF HARDNESS-PRESERVING DISTRIBUTED SAMPLERS



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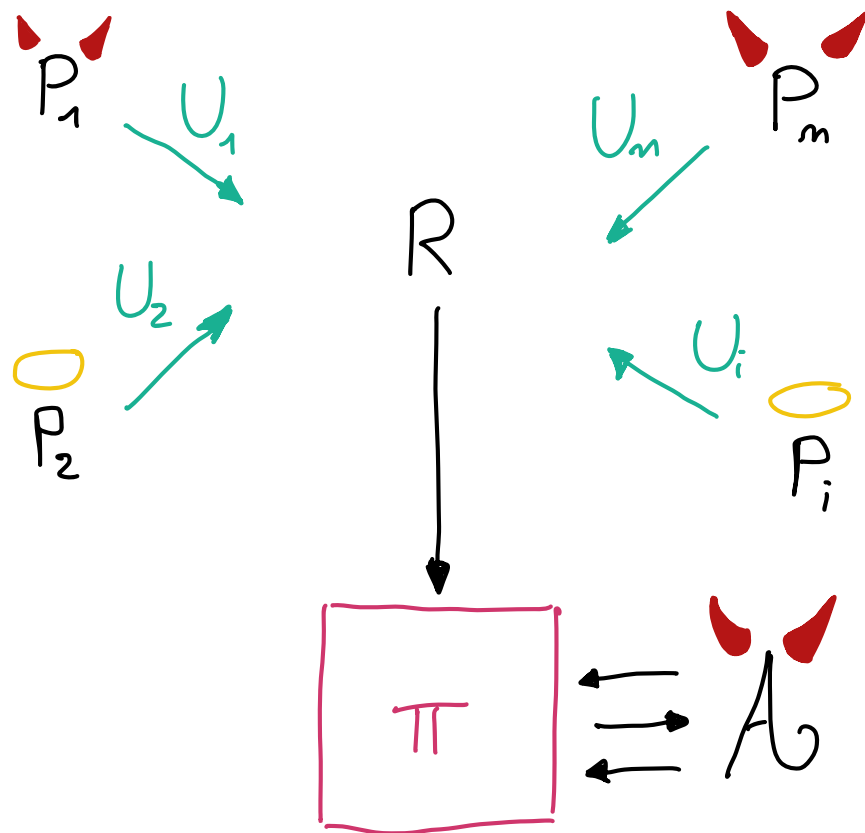
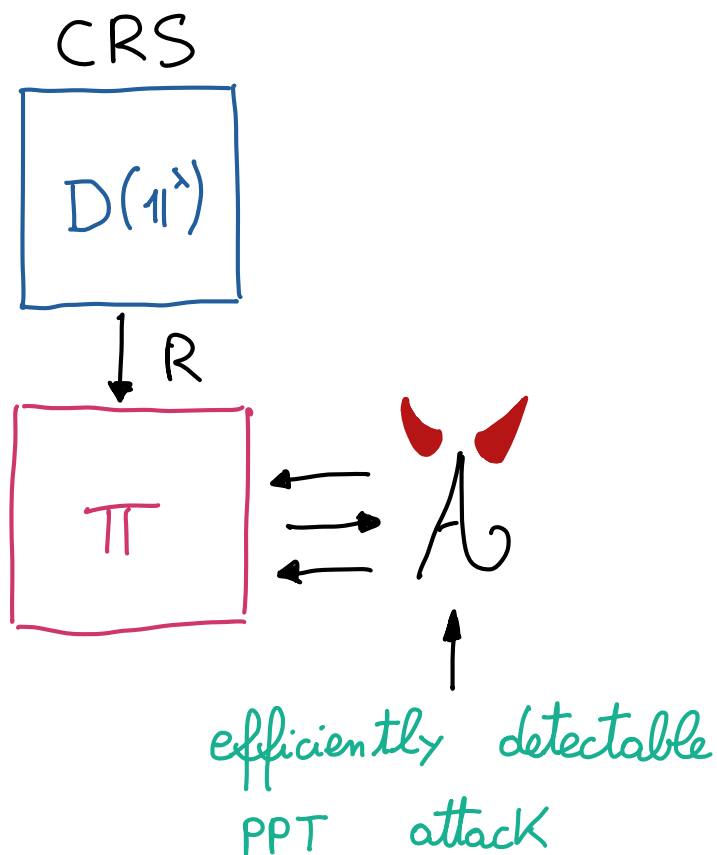


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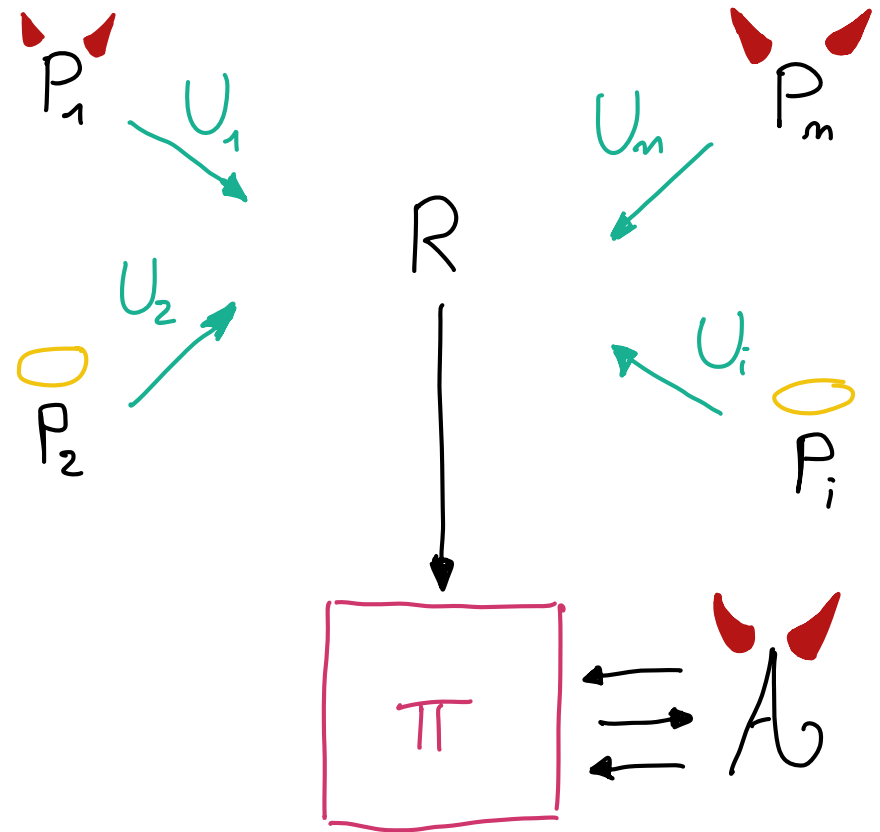
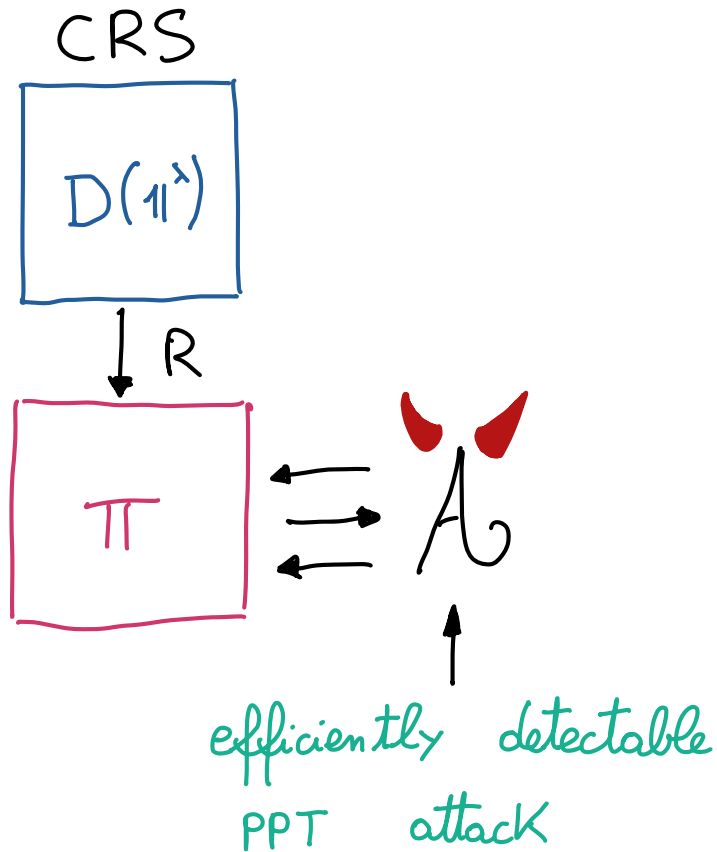
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$$\mathbb{P}_{\text{CRS}}[A \text{ succeeds}] < \text{negl}(\lambda) \Rightarrow \mathbb{P}_{\text{DS}}[A \text{ succeeds}] < \text{negl}(\lambda)$$

# LOSSY DISTRIBUTED SAMPLERS

distributed  
sampler message

$\forall U:$

$$\Omega_U := \{ \text{Sample}(U, U_2, \dots, U_{m-1}) \mid U_2, \dots, U_{m-1} \}$$



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STANDARD

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LOSSY

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polynomial



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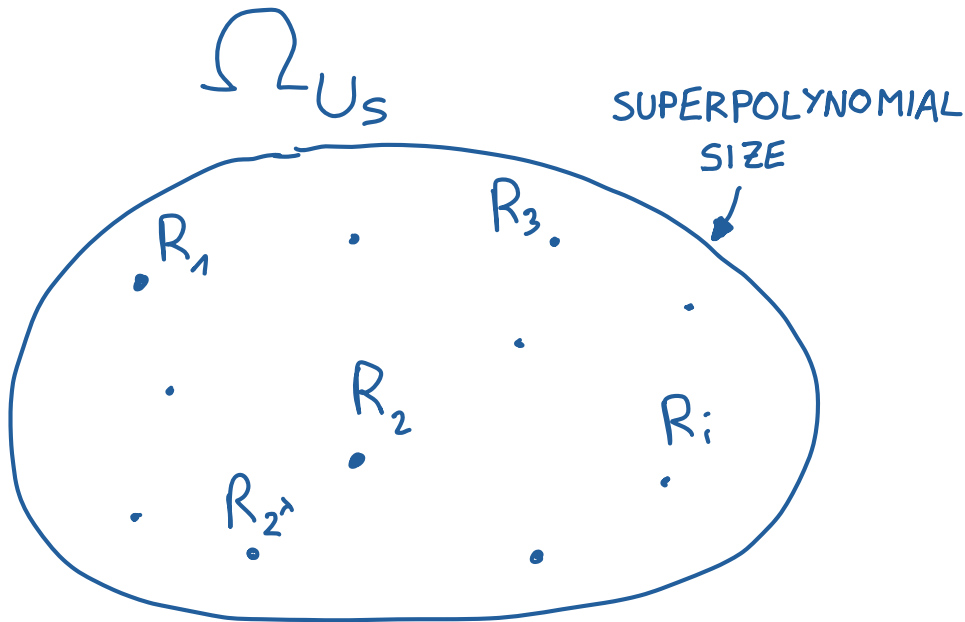


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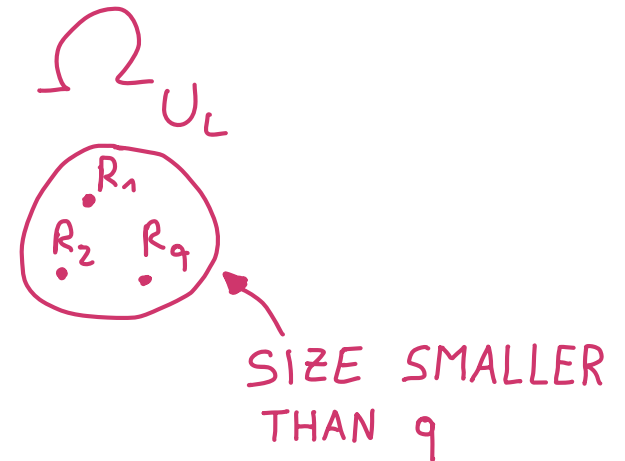
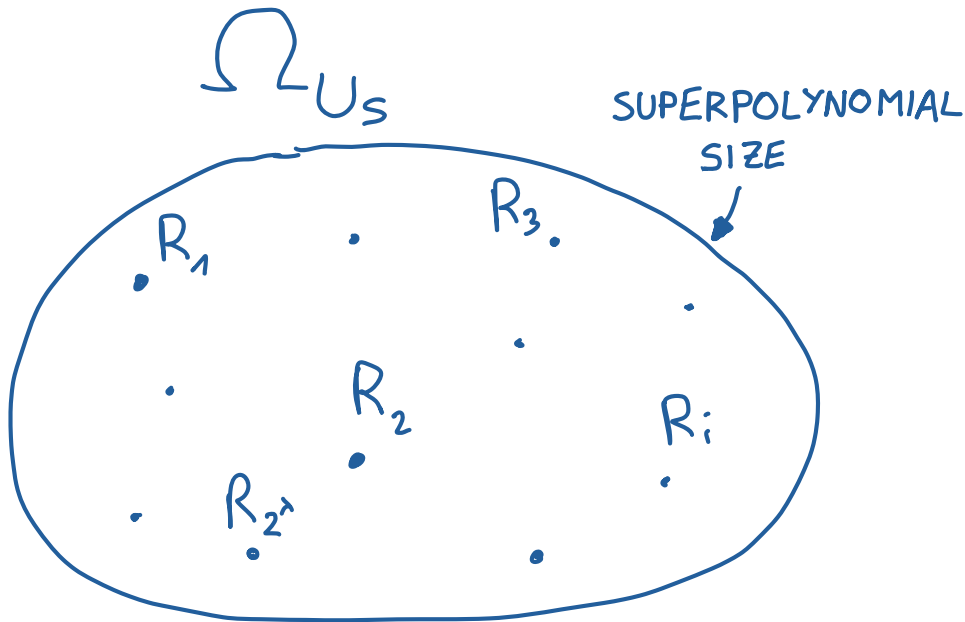
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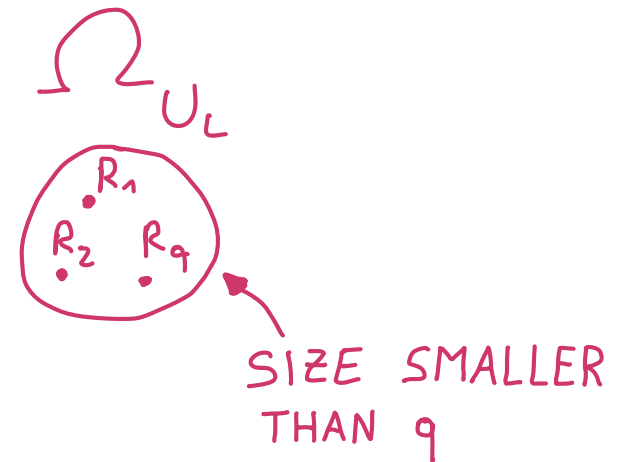
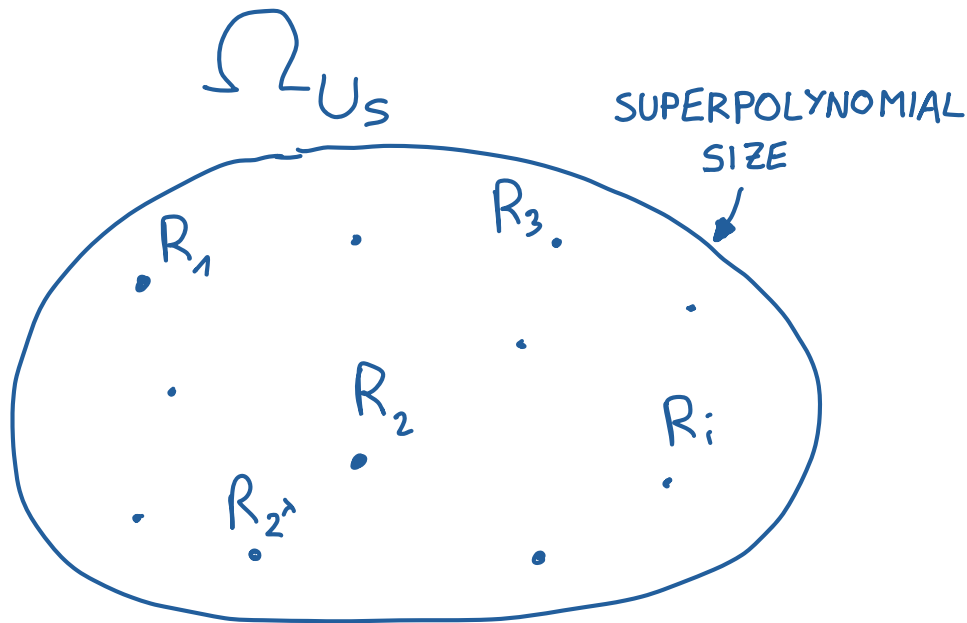
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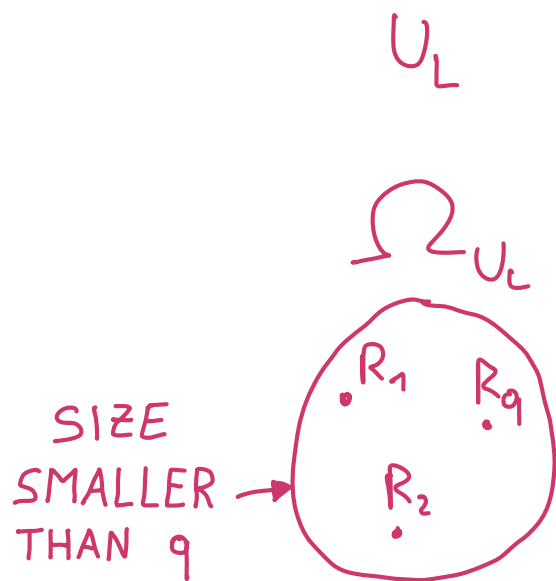


DISTINGUISHABLE WITH ARBITRARILY SMALL INVERSE-POLYNOMIAL ADVANTAGE!

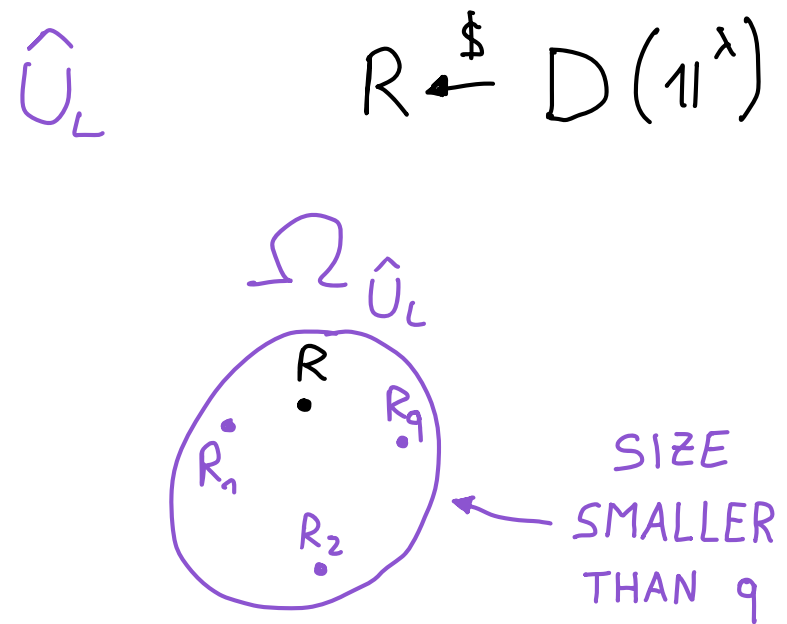
# PROGRAMMABILITY OF LOSSY DISTRIBUTED SAMPLERS

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LOSSY MODE ( $q$ )

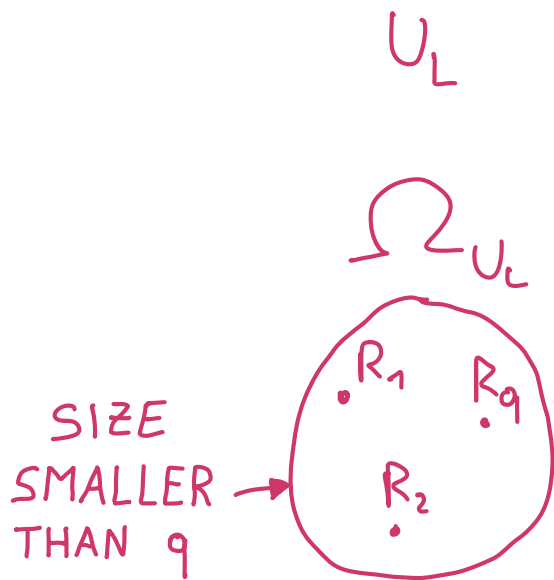


PROGRAMMED MODE ( $q$ )

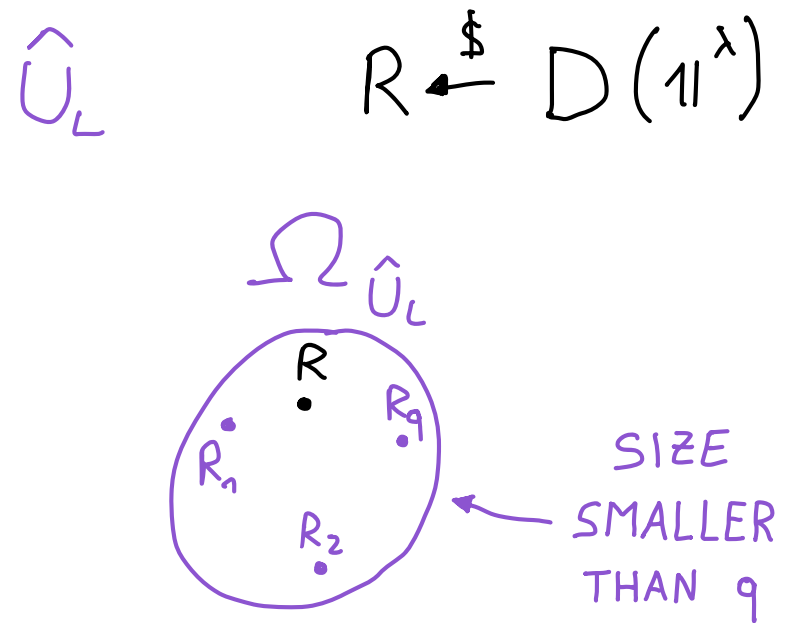


# PROGRAMMABILITY OF LOSSY DISTRIBUTED SAMPLERS

LOSSY MODE ( $q$ )



PROGRAMMED MODE ( $q$ )



INDISTINGUISHABLE



# BUILDING LOSSY DISTRIBUTED SAMPLERS

## THEOREM

Assume the existence of

- subexp iO
- subexp multi-Key FHE
- extremely lossy functions (ELFs)
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subexp injective OWFs  
perfectly correct IBE  
perfectly sound NIZK

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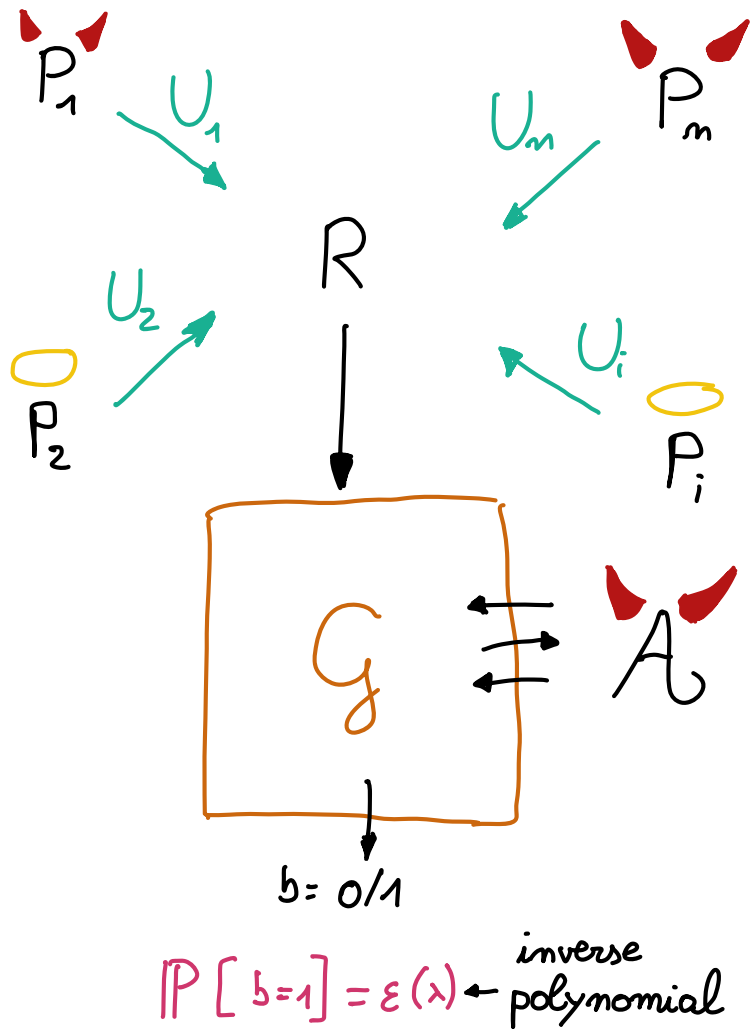
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perfectly correct IBE  
perfectly sound NIZK

Then, there exists a programmable lossy distributed sampler with a short ( $\text{poly } \lambda$ ), reusable CRS.

can be made unstructured

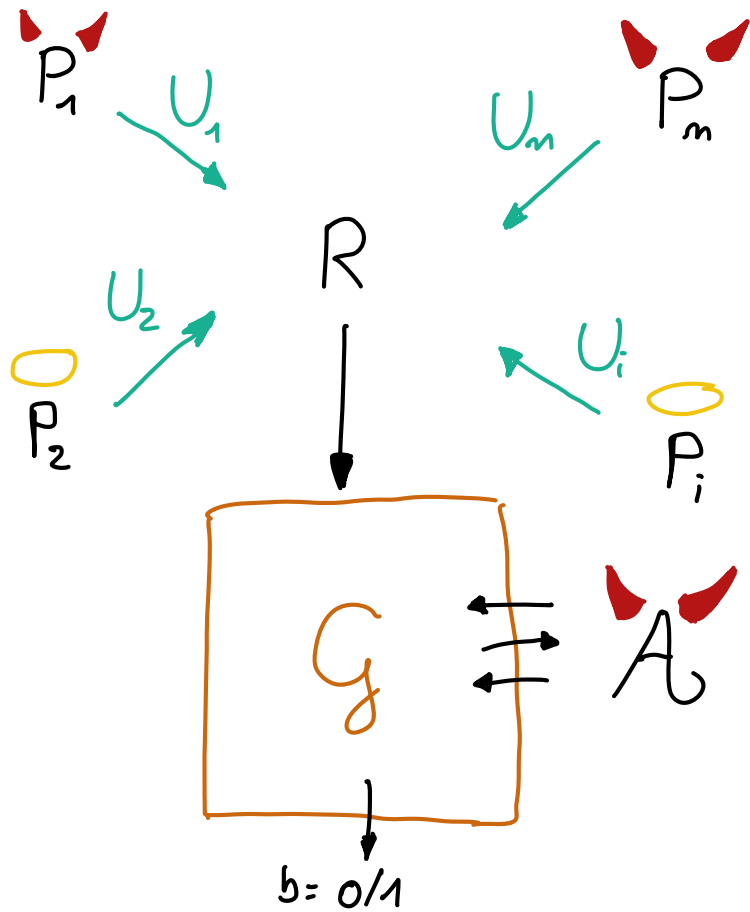
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

REAL WORLD

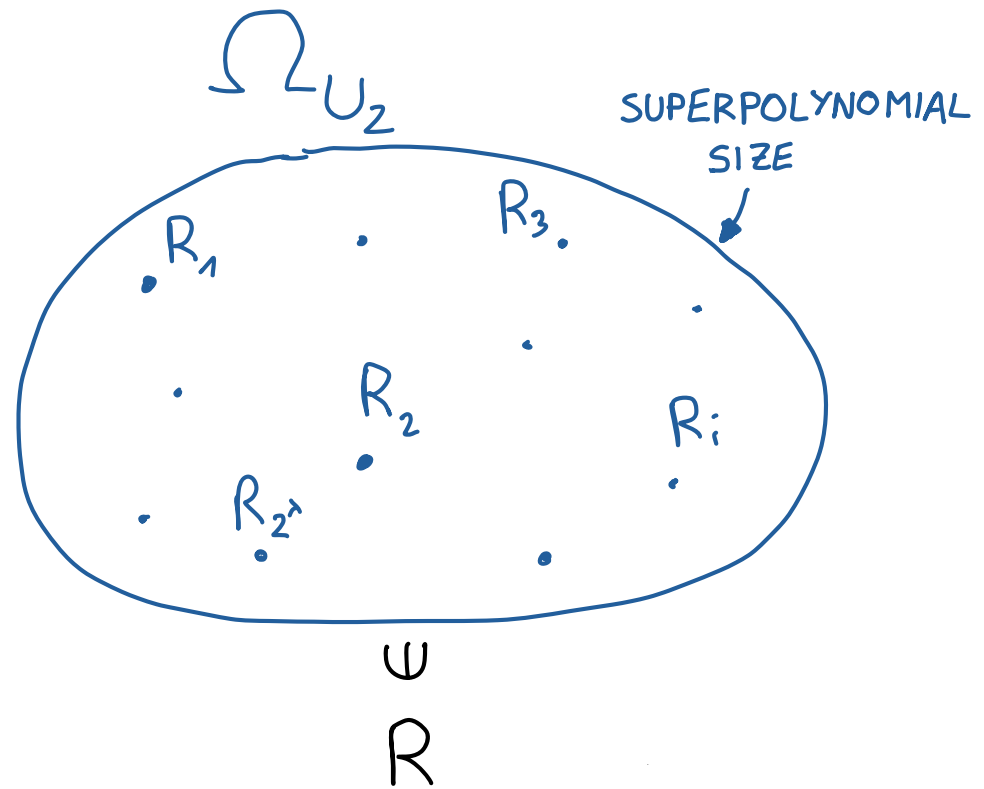


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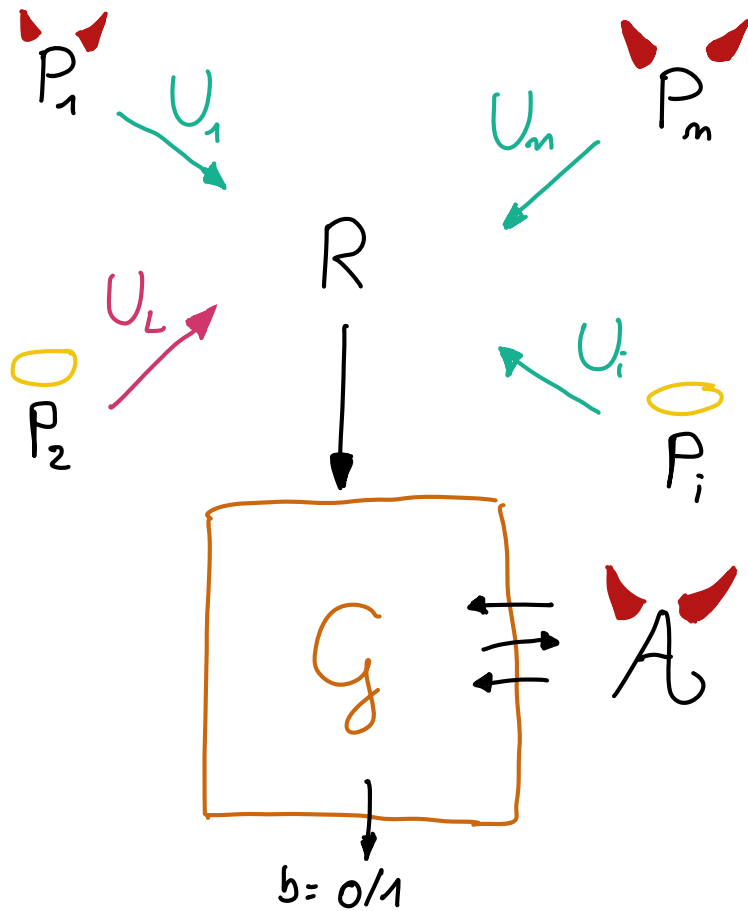


$P[b=1] = \epsilon(\lambda) \leftarrow \begin{matrix} \text{inverse} \\ \text{polynomial} \end{matrix}$



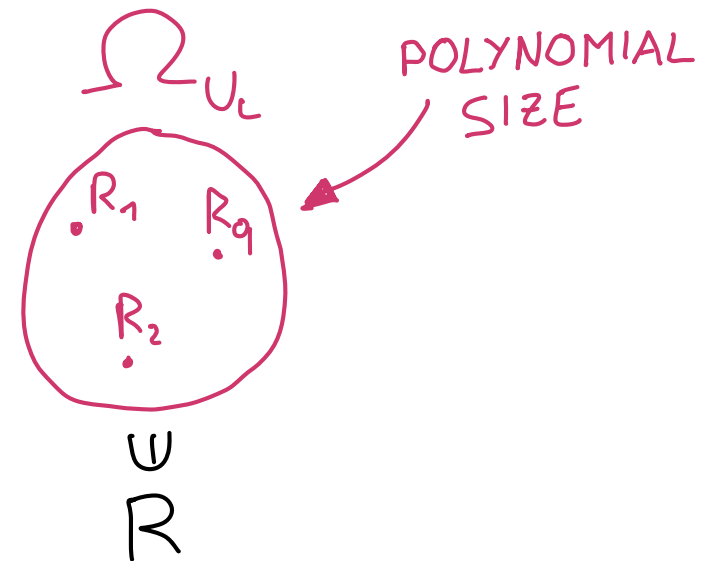
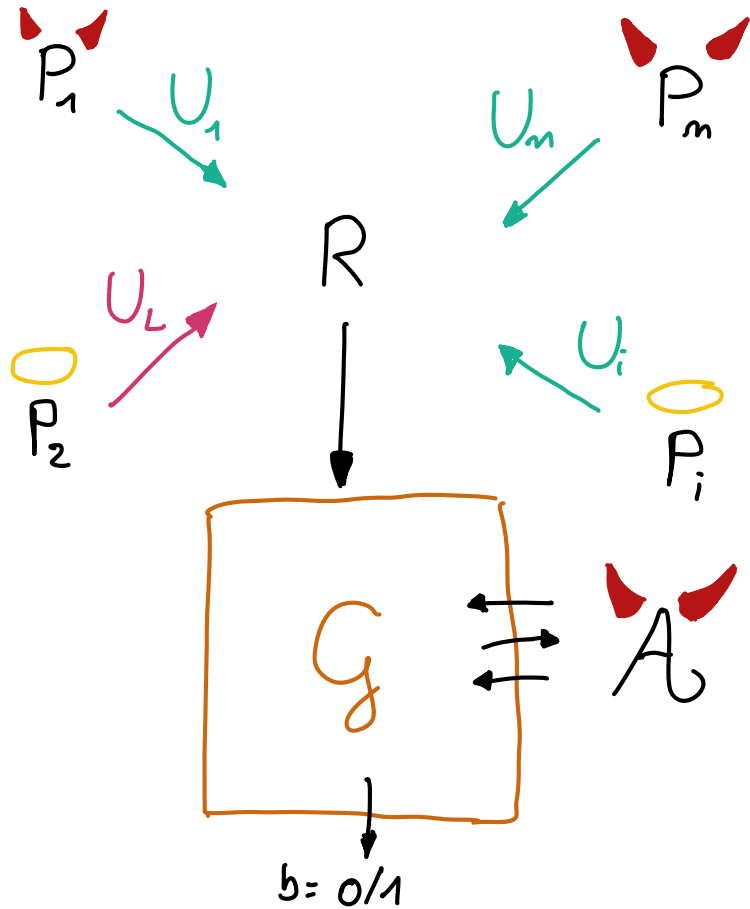
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

## HYBRID WORLD 1



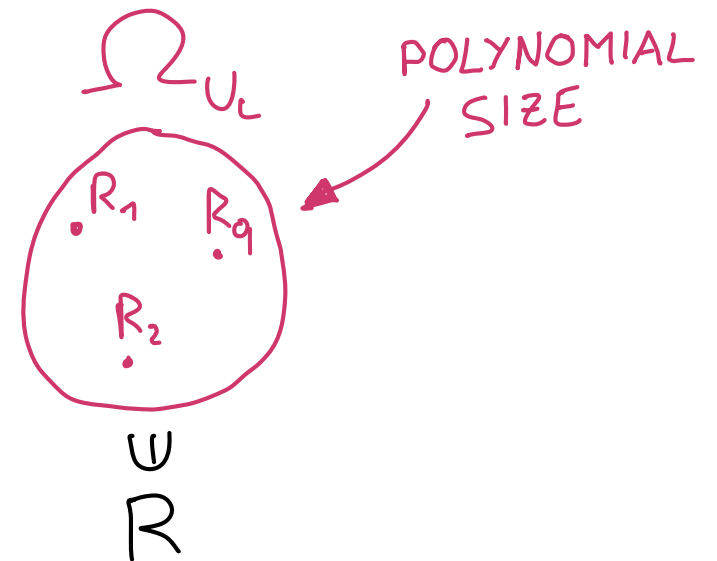
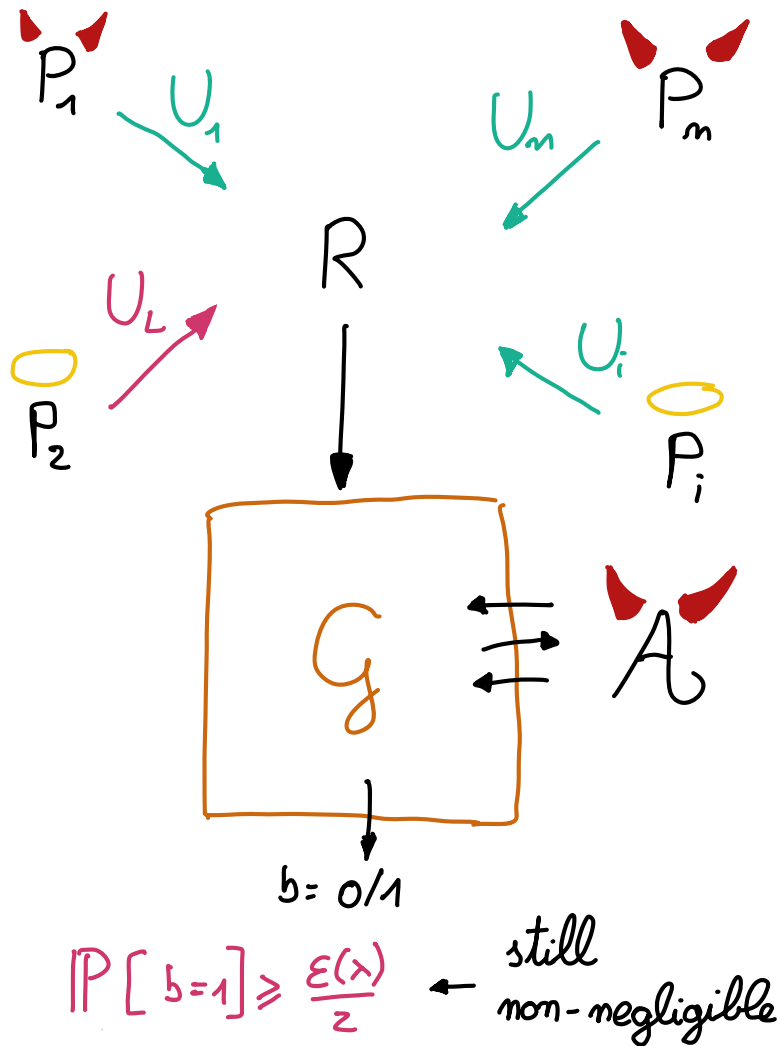
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

## HYBRID WORLD 1



# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

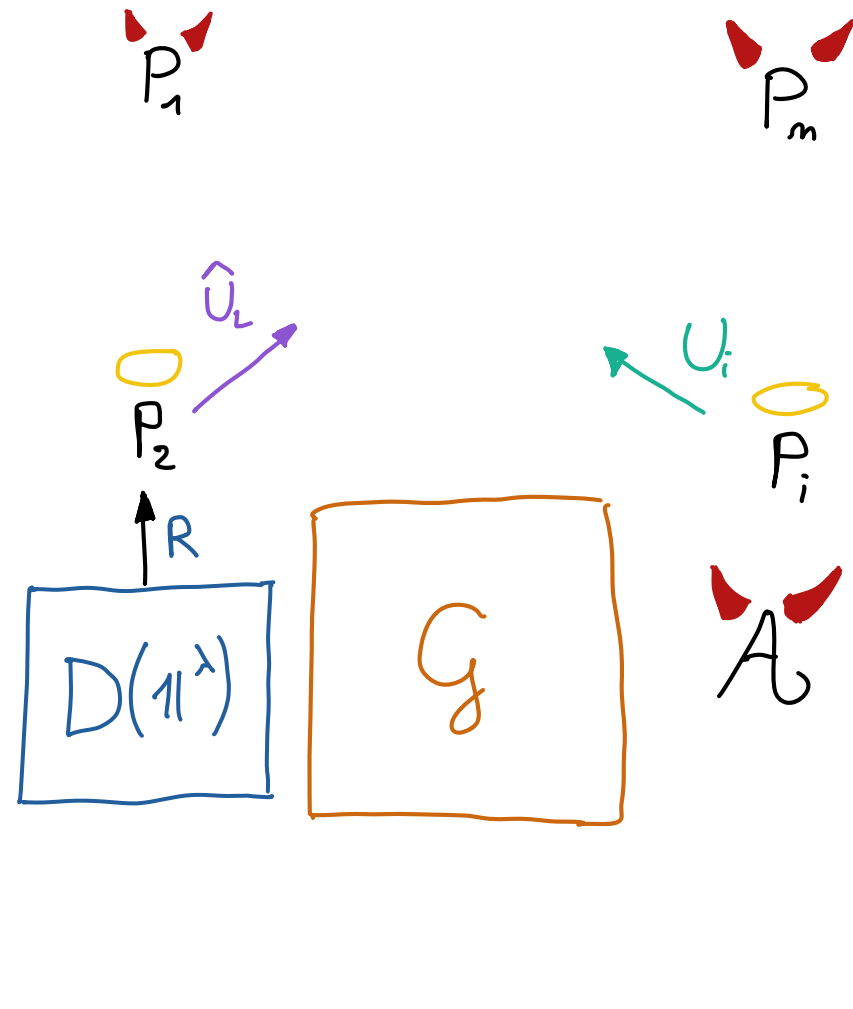
## HYBRID WORLD 1





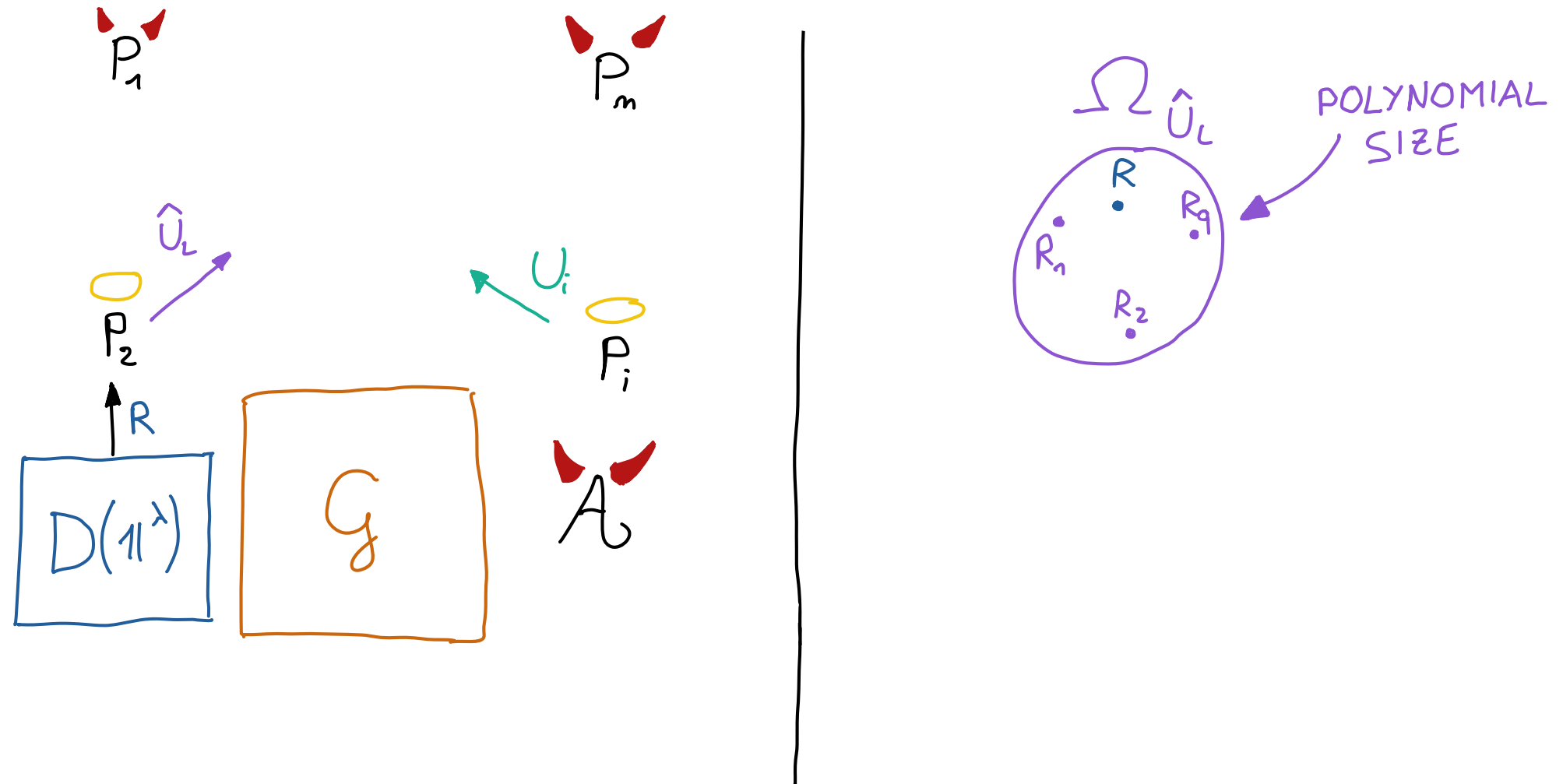
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

HYBRID WORLD 2



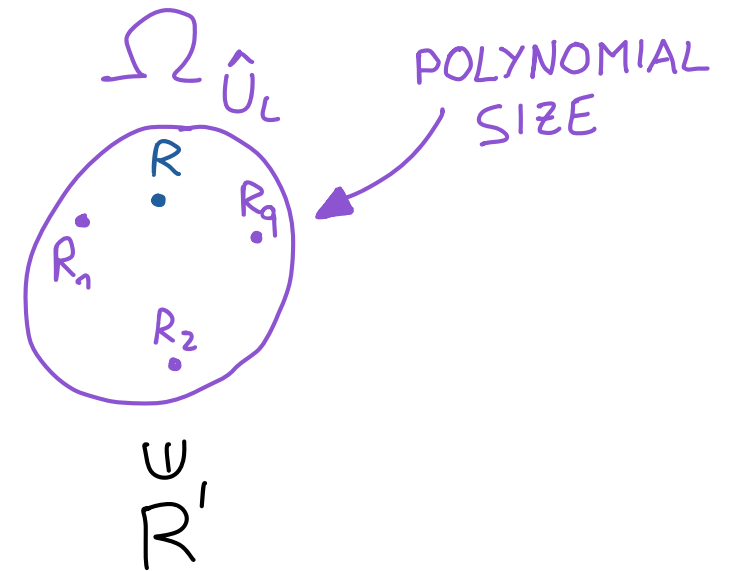
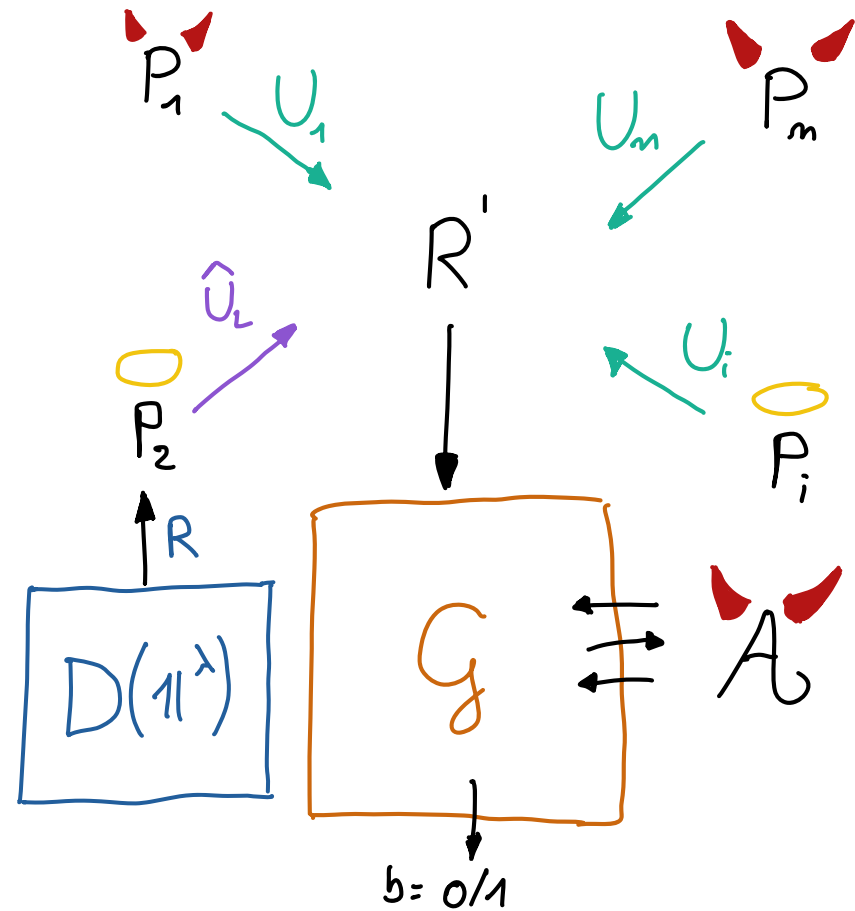
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

## HYBRID WORLD 2



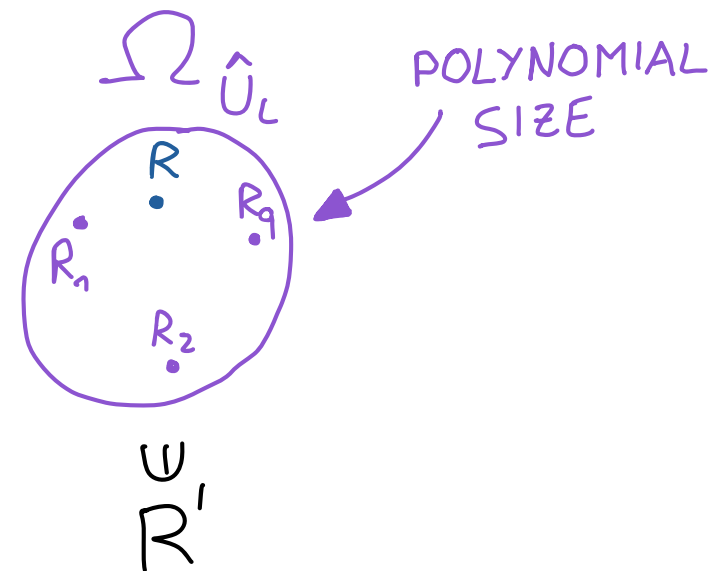
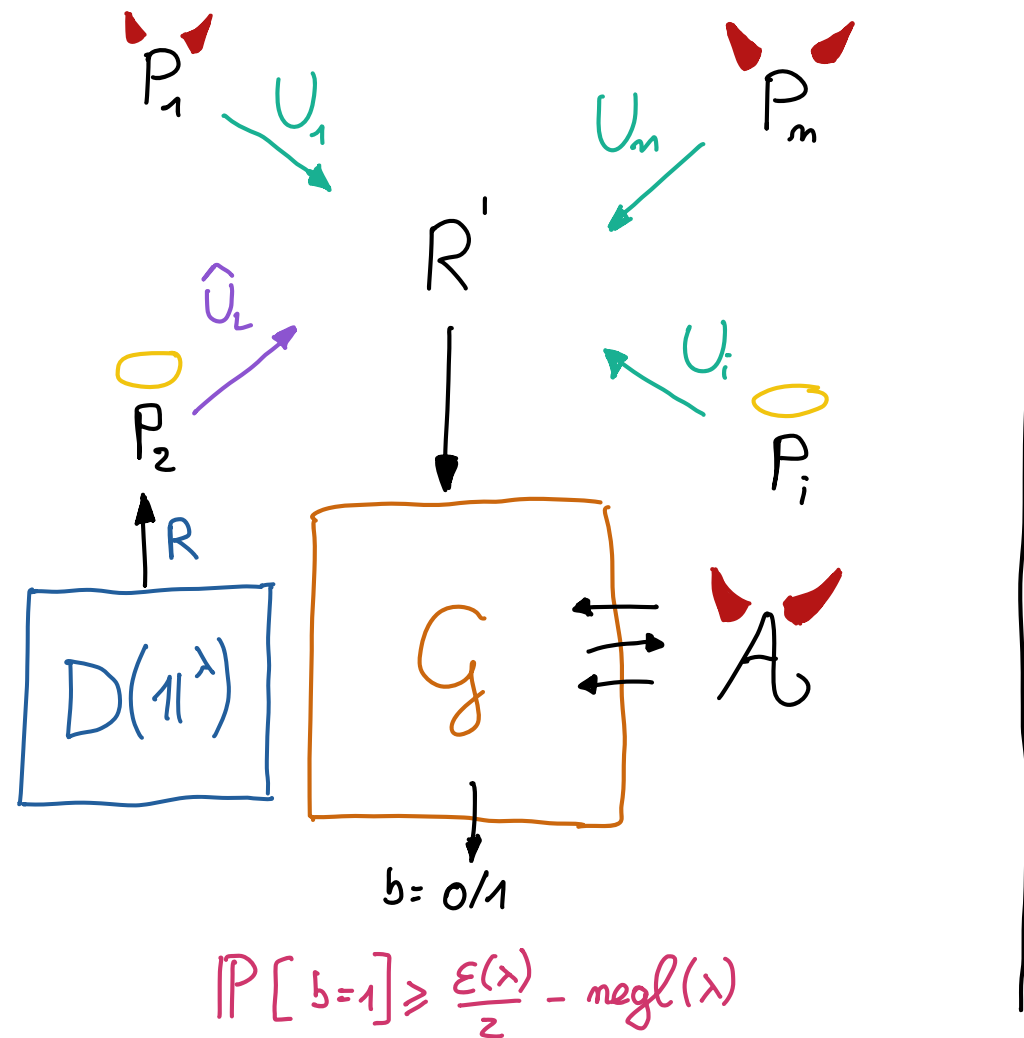
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

## HYBRID WORLD 2



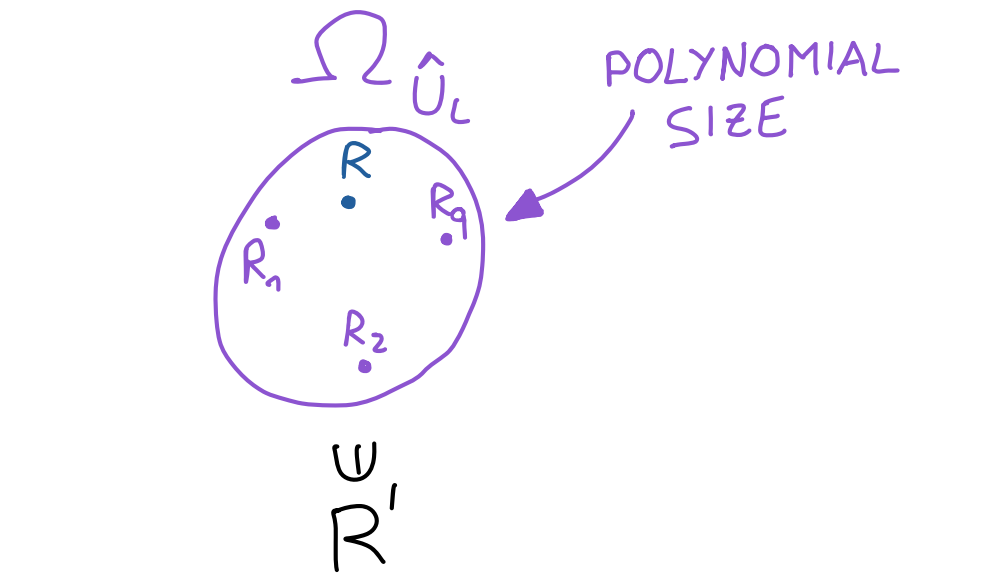
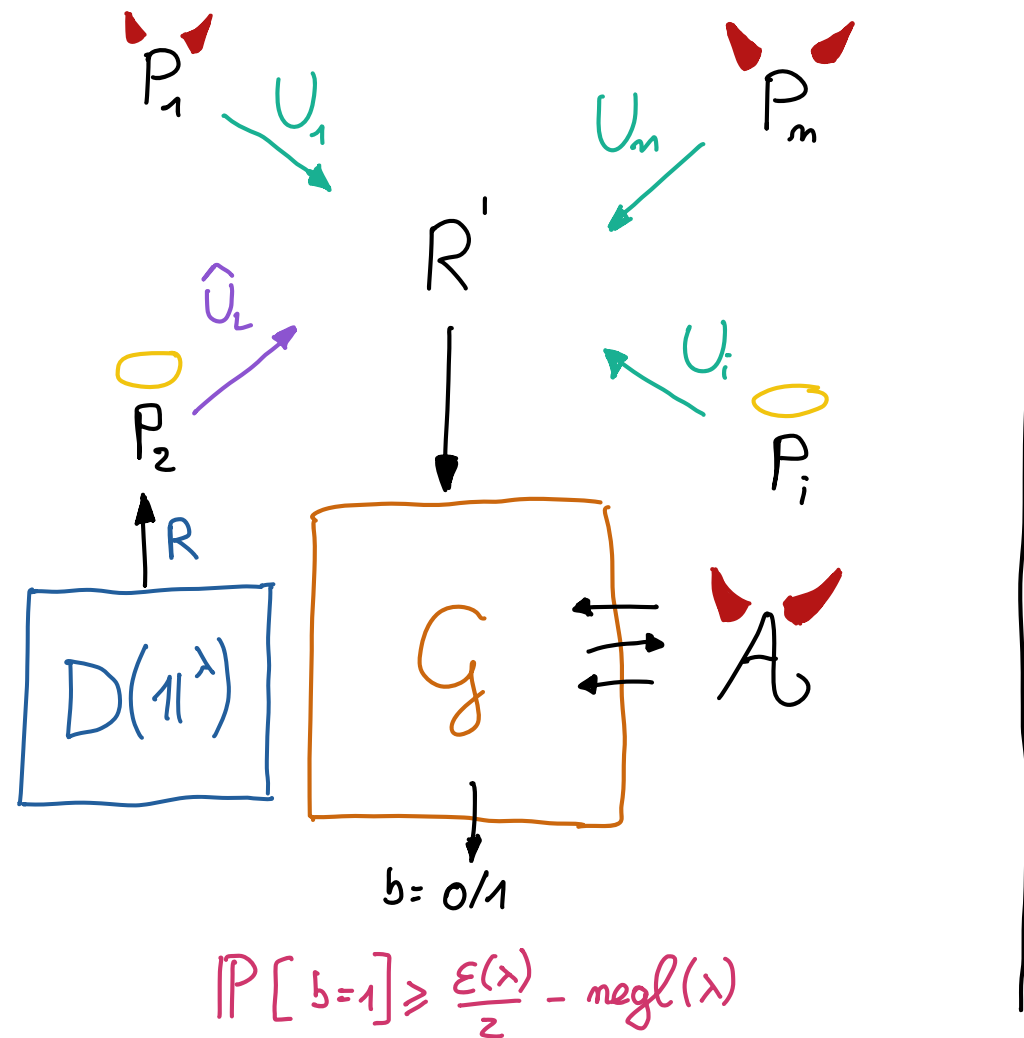
# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

## HYBRID WORLD 2



# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

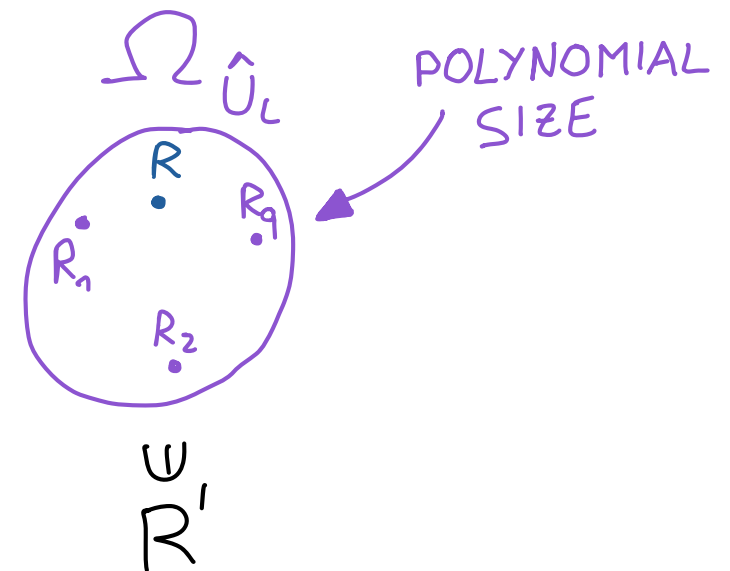
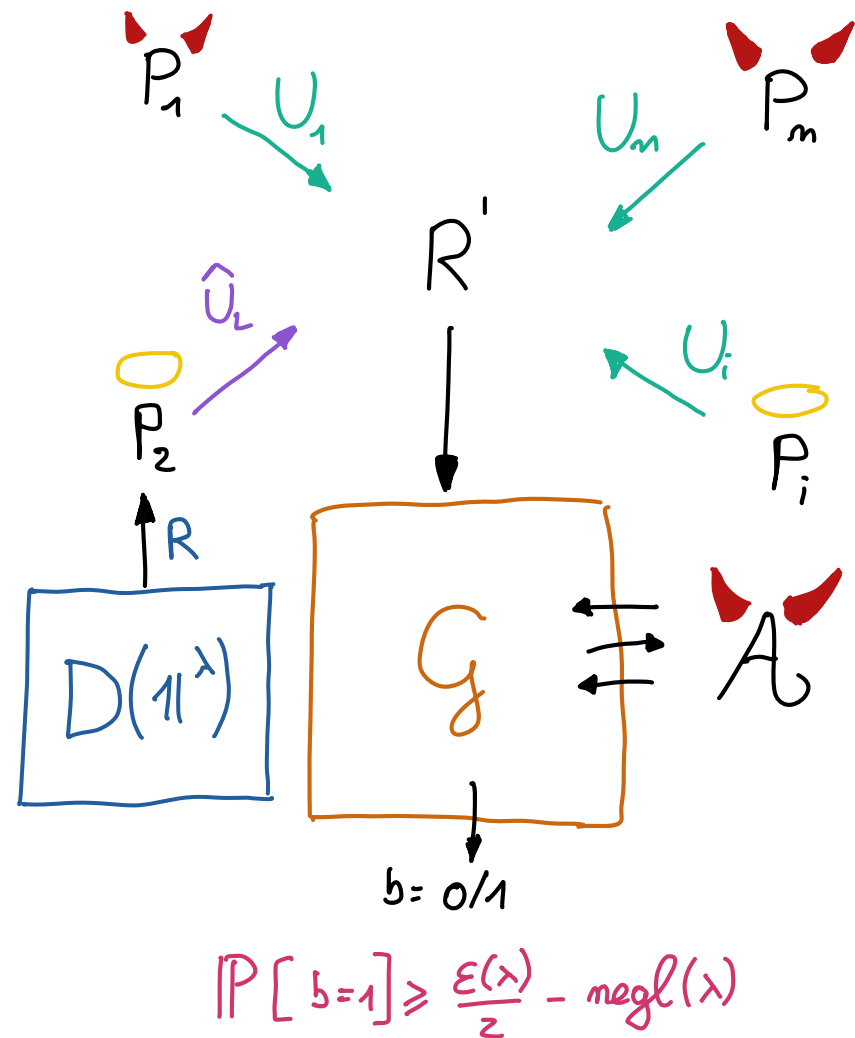
## HYBRID WORLD 2



$$P[b=1, R=R'] \geq \left(\frac{\epsilon(x)}{2} - \text{negl}(x)\right) \cdot \frac{1}{\text{poly}(x)}$$

# FROM LOSSY TO HARDNESS-PRESERVING DISTRIBUTED SAMPLERS

## HYBRID WORLD 2

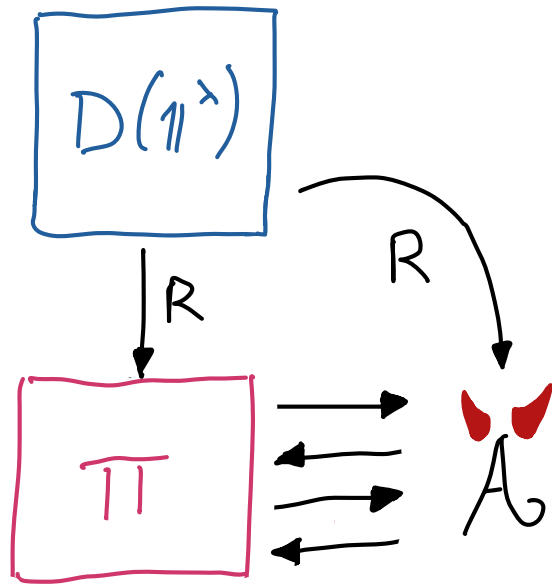


non-negligible

$$P[b=1, R=R'] \geq \left( \frac{\epsilon(x)}{2} - \text{negl}(x) \right) \cdot \frac{1}{\text{poly}(x)}$$

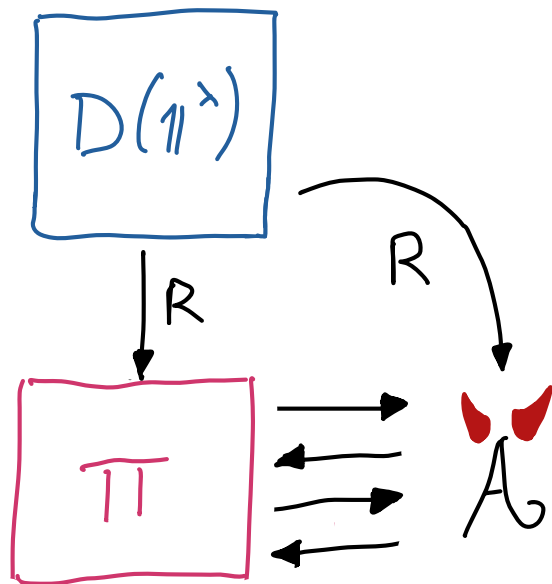
# INDISTINGUISHABILITY - PRESERVING DISTRIBUTED SAMPLERS

REAL WORLD

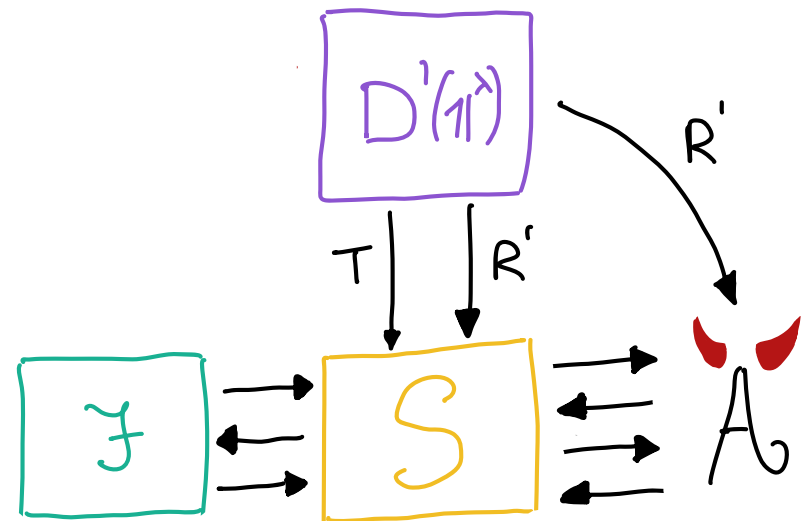


# INDISTINGUISHABILITY - PRESERVING DISTRIBUTED SAMPLERS

REAL WORLD



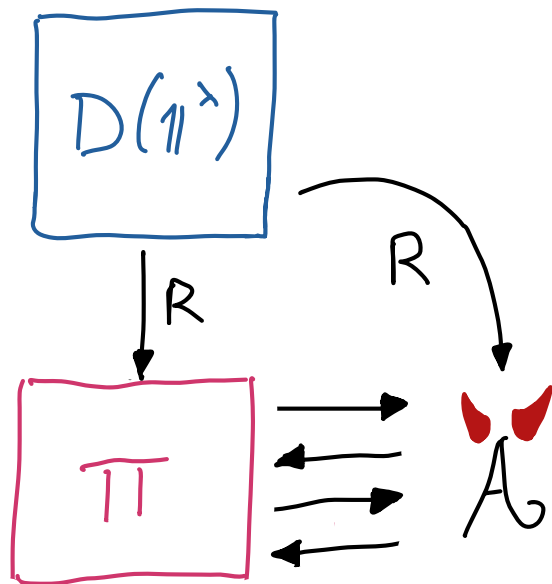
IDEAL WORLD



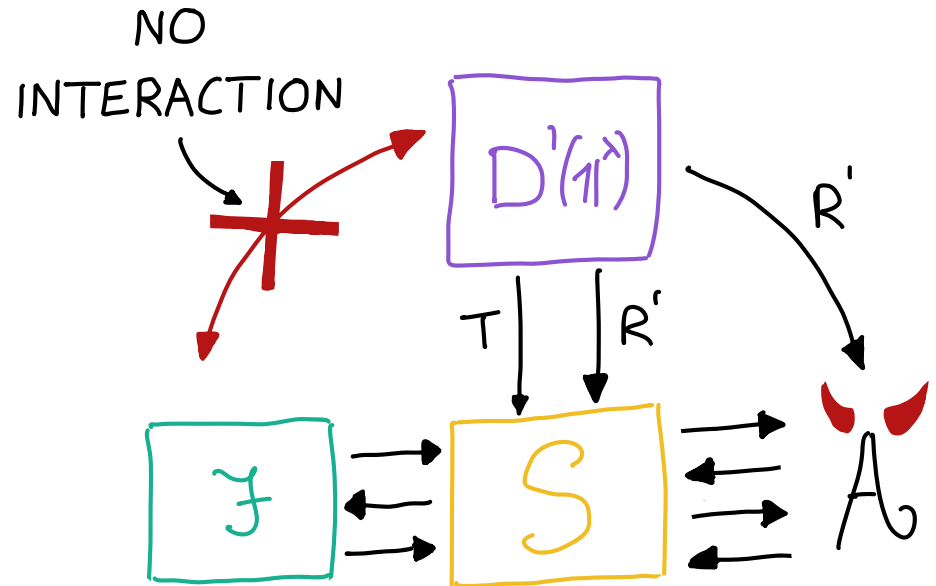


# INDISTINGUISHABILITY - PRESERVING DISTRIBUTED SAMPLERS

REAL WORLD



IDEAL WORLD



# BUILDING INDISTINGUISHABILITY- PRESERVING DISTRIBUTED SAMPLERS

## THEOREM

Our lossy distributed sampler is  
indistinguishability-preserving.

# ON THE NEED FOR CRS'S

Our distributed samplers have CRS's that are:

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- reusable

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- for indistinguishability - preserving distributed samplers **NO!**



# ON THE NEED FOR CRS'S

Our distributed samplers have CRS's that are:

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Can we get rid of CRSs?

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BY COMPILING [PVW08], WE WOULD GET  
3-ROUND ACTIVE OT IN THE PLAIN MODEL ⚡

# ON THE NEED FOR CRS'S

Our distributed samplers have CRS's that are:

- reusable
- short
- unstructured

Can we get rid of CRSs?

- for indistinguishability-preserving distributed samplers **NO!**

BY COMPILING [PVW08], WE WOULD GET  
3-ROUND ACTIVE OT IN THE PLAIN MODEL ⚡

- for hardness-preserving distributed samplers **OPEN!**

# ON THE NEED FOR CRS'S

We can build security-preserving distributed samplers without CRS if:

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- we allow non-uniform simulators

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We can build security-preserving distributed samplers without CRS if:

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We built CRS-less simulation-extractable NIZKs!

# SUMMARY

NEW DEFINITIONS OF ACTIVE DISTRIBUTED SAMPLERS THAT DON'T NEED RANDOM ORACLES:

HARDNESS-PRESERVING  
DISTRIBUTED SAMPLERS

↓  
preserving the hardness  
of search games

↓  
BUILT FROM SUBEXP IO, SUBEXP MK-FHE, ELF<sub>s</sub>, ...  
WITH REUSABLE, SHORT AND UNSTRUCTURED CRS.

INDISTINGUISHABILITY-PRESERVING  
DISTRIBUTED SAMPLERS

↓  
preserving the functionality  
for a large class of protocols.

NEW NIZK NOTIONS: • CRS-LESS NIZK<sub>s</sub>  
• ALMOST-EVERYWHERE-EXTRACTABILITY