Radbound University

Revisiting the Indifferentiability of the Sum of Permutations

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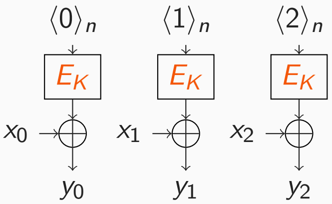
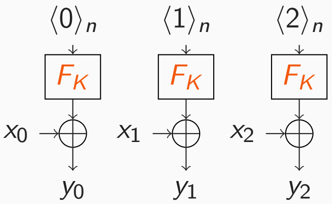
- Prominent example: CTR mode



$n/2$-bit security



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  - Conversions like summation achieve beyond birthday bound security

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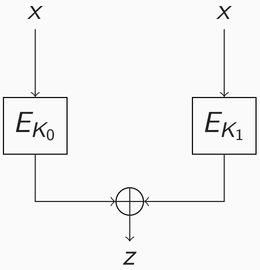

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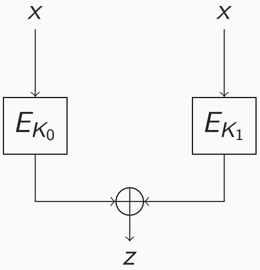

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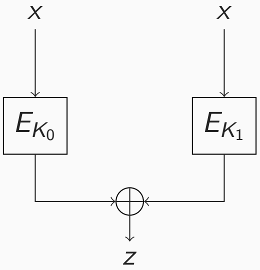

- Sums the output of two independent permutations
- Achieves n-bit security for private permutations

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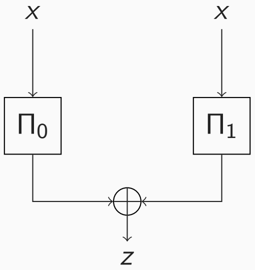

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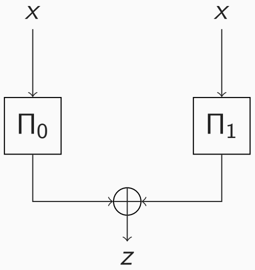

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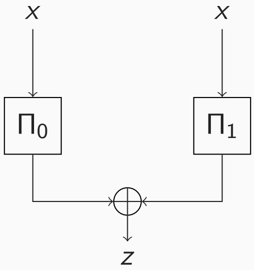

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- Moves to indifferentiability setting

Indifferentiability

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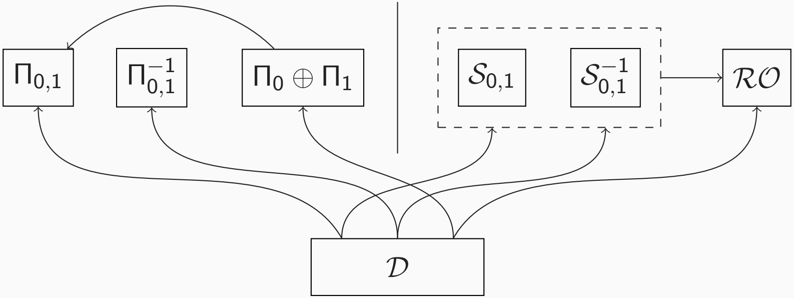

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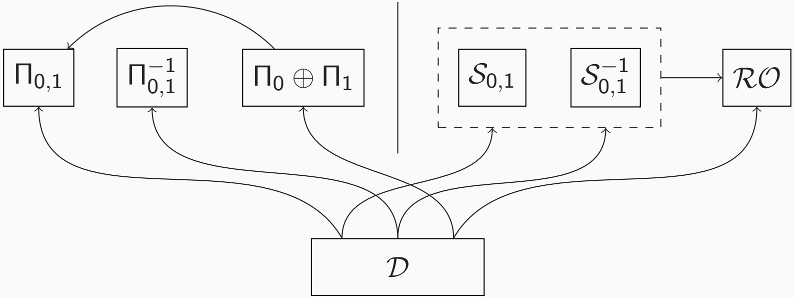

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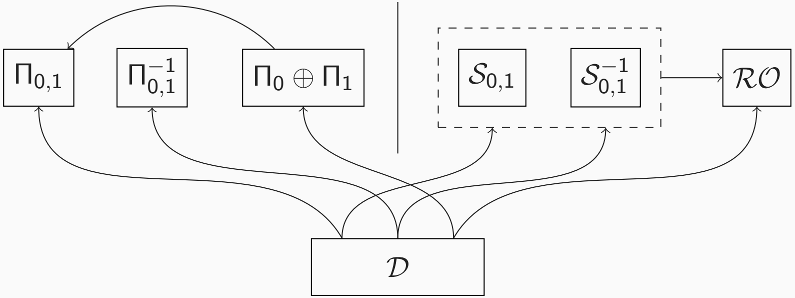

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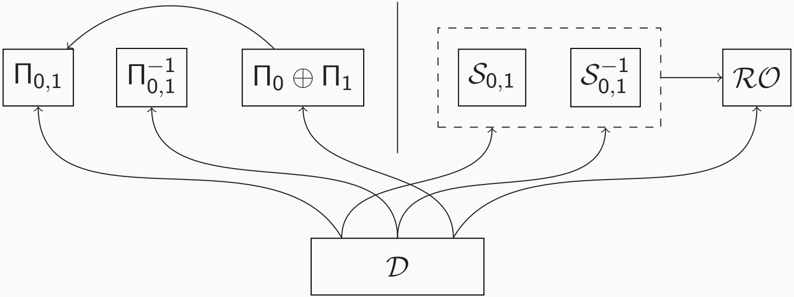

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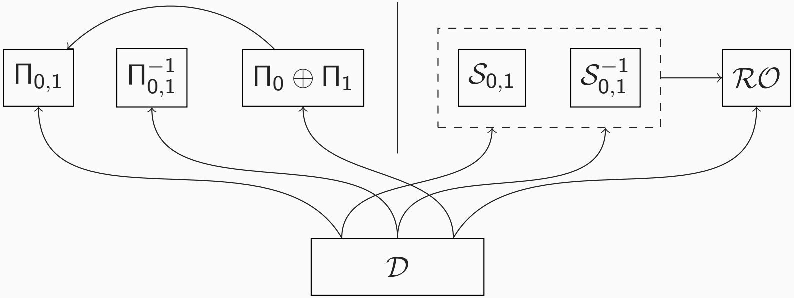
- Both forward and backward direction for the primitive queries



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  - Bhattacharya and Nandi [BN18] improved to n-bit security

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    - Otherwise, repeat the process up to ℓ times

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  - Attack on standard simulator using $\mathcal{O}(2^{5n/6})$ queries
  - Proof showing $(2n/3 - \log_2(n))$-bit security can be fixed using a new technique

Flaw 1: Random Range

- Let R_0 and R_1 be the ranges of the two primitives, i.e. in the real world we have

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- Fundamental problem, invalidating [MPN10, BN18]

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- Works in the weaker sequential indifferentiability setting, where all primitive queries have to be made before the construction queries

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- Simulator viewed as a stateless primitive
- A stateless primitive can be implemented by drawing all randomness at the start
- Most primitives are stateless: random permutations, random function, random oracle, etc.
- The simulator is stateful, making analysis more difficult

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- A stateless primitive allows queries to be made in any order: $P(x_1), P(x_2)$ has the same distribution as $P(x_2), P(x_1)$, simplifying analysis

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- The simulator is stateful and does not have this same behavior
- We show that the simulator partly has this property
- Queries can be reordered as necessary up to $2n/3$-bit security
- Re-establishes regular indifferentiability with $(2n/3 - \log_2(n))$-bit security using [MP15] for sequential indifferentiability

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- Comparison to illustrate the problem

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- A randomly drawn M&M has a probability of $1/5$ of being a specific color, even after other draws
- Suppose you do not like brown M&M's and do the following when grabbing one:

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- They are uniformly sampled from 5 colors: **red**, **brown**, **yellow**, **green** and **blue**
- A randomly drawn M&M has a probability of $1/5$ of being a specific color, even after other draws
- Suppose you do not like brown M&M's and do the following when grabbing one:
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- After this process, the probability that an M&M in the bag is brown becomes:

$$\frac{4}{5} \cdot \frac{1}{5} + \frac{1}{5} \cdot \left(\frac{8}{9} \cdot \frac{1}{5} + \frac{1}{9} \cdot 1 \right) = \frac{49}{225} > \frac{45}{225} = \frac{1}{5}$$

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- We give an attack that shows that this difference matters for more than $3n/4$-bit security

Attack: Standard Simulator Limited to $5n/6$-bit Security

- Recall that the forward simulator selects its output y_0 uniformly from all possibilities $Y = \{0,1\}^n \setminus (\mathrm{range}(\mathcal{S}_0) \cup (\mathrm{range}(\mathcal{S}_1) \oplus z))$

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- Surprisingly, in some cases the sampling in the real world does not behave uniformly
- Gives rise to an attack using $\mathcal{O}(2^{5n/6})$ queries
- Maybe possible to fix with a biased simulator, but gets very complicated

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Thank you for your attention!

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