

# Improved Power Analysis Attacks on Falcon

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Shiduo Zhang, Xiuhan Lin, Yang Yu, Weijia Wang



# Overview

In this work, we develop several key recovery attacks exploiting **power leakage** on Falcon.

- a new effective key recovery using the half Gaussian leakage within the base sampler.
- the first side-channel analysis on Falcon taking the sign leakage into account.

# Outline

- ① Background
- ② The half Gaussian leakage and the sign leakage
- ③ Exploiting the half Gaussian leakage
- ④ Exploiting the sign leakage

# Background

# Falcon

Falcon is one of the three post quantum digital signatures to be standardized by NIST.

Falcon has a good performance especially it has the smallest bandwidth (public key size plus signature size) among the selected NIST signatures.

Falcon is a lattice-based hash-and-sign signature scheme.

# Hash-and-sign paradigm

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- signing: finding close vectors
- GGH, NTRUSign  $\rightarrow$  GPV  $\rightarrow$  Falcon

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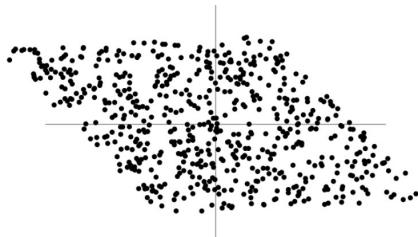
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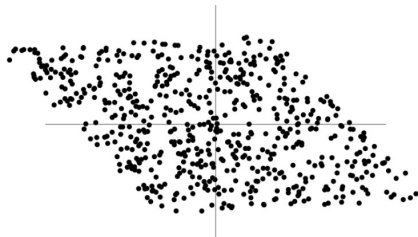


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- rounding based on random Gaussian sampling
- distribution of signatures is provably independent of the secret key

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Falcon is an efficient instantiation of the GPV framework by using optimal NTRU trapdoor.

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## Side-channel analysis

With PQC standardization and migration underway, security should be considered from both **algorithmic** and **implementation** aspects.

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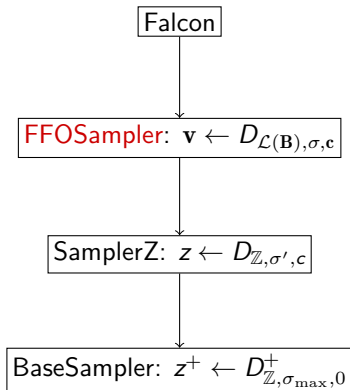
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We need to understand the connection between leakage and secret key itself.

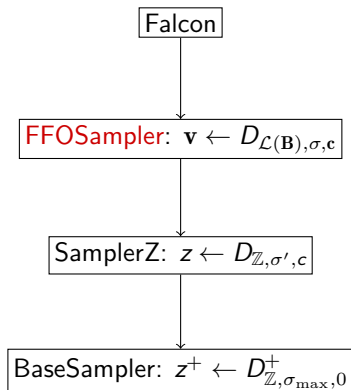
# Gaussian Samplers of Falcon

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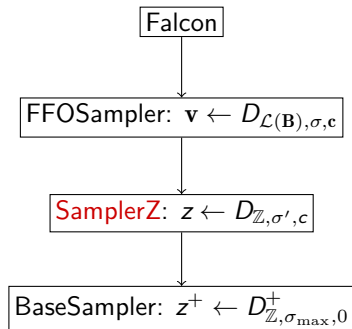
## The KGPV sampler

**Input:** a basis  $\mathbf{B} = (\mathbf{b}_0, \dots, \mathbf{b}_{n-1})$ , a center  $\mathbf{c}$  and  $\sigma \geq \|\mathbf{B}\|_{GS} \cdot \eta_\epsilon(\mathbb{Z})$

**Output:** a lattice point  $\mathbf{v}$  following a distribution close to  $D_{\mathcal{L}(\mathbf{B}),\sigma,\mathbf{c}}$ .

- 1:  $\mathbf{v} \leftarrow \mathbf{0}, \mathbf{c}' \leftarrow \mathbf{c}$
- 2: **for**  $i = n-1, \dots, 0$  **do**
- 3:  $\sigma_i = \sigma / \|\tilde{\mathbf{b}}_i\|$
- 4:  $c'_i = \langle \mathbf{c}', \tilde{\mathbf{b}}_i \rangle / \|\tilde{\mathbf{b}}_i\|^2$
- 5:  $z_i \leftarrow \text{SamplerZ}(\sigma_i, c'_i - \lfloor c'_i \rfloor) + \lfloor c'_i \rfloor$
- 6:  $\mathbf{c}' \leftarrow \mathbf{c}' - z_i \mathbf{b}_i, \mathbf{v} \leftarrow \mathbf{v} + z_i \mathbf{b}_i$
- 7: **end for**
- 8: **return**  $\mathbf{v}$

# Integer Gaussian sampler of Falcon



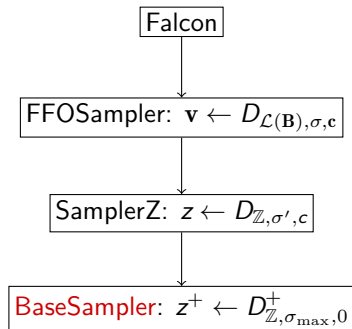
## SamplerZ( $\sigma, c$ )

**Input:**  $c \in [0, 1)$  and  $\sigma \in (\sigma_{\min}, \sigma_{\max})$ .

**Output:**  $z \in \mathbb{Z}$  following  $D_{\mathbb{Z},\sigma,c}$ .

- 1:  $z^+ \leftarrow \text{BaseSampler}()$
- 2:  $b \leftarrow U(\{0, 1\})$
- 3:  $z \leftarrow b + (2b - 1)z^+$
- 4:  $x \leftarrow -\frac{(z-c)^2}{2\sigma^2} + \frac{(z^+)^2}{2\sigma_{\max}^2}$
- 5: return  $z$  with probability  $\frac{\sigma_{\min}}{\sigma} \cdot \exp(x)$ , otherwise restart;

# Integer Gaussian sampler of Falcon



## BaseSampler()

**Output:**  $z^+ \sim D_{\mathbb{Z},\sigma_{\max},0}^+$ .

- 1:  $u \stackrel{\$}{\leftarrow} \{0, 1\}^{72}$
- 2:  $z^+ \leftarrow 0$
- 3: **for**  $i = 0 \dots 17$  **do**
- 4:      $z^+ \leftarrow z^+ + \llbracket u < RCDT[i] \rrbracket$
- 5: **end for**
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# The half Gaussian leakage and the sign leakage

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## Half Gaussian leakage

One can classify if  $z^+ = 0$  or not through the power consumption of the comparison  $\llbracket u < RCDT[i] \rrbracket$

# Sign leakages

## SamplerZ( $\sigma, c$ )

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## Sign leakage

One can determine  $b$  through the power of  $\llbracket z \leftarrow b + (2b - 1)z^+ \rrbracket$  and

$$\llbracket x \leftarrow -\frac{(z-c)^2}{2\sigma^2} + \frac{(z^+)^2}{2\sigma_{max}^2} \rrbracket$$



# Exploiting the half Gaussian leakage

## Parallelepiped-learning strikes again

In [GMRR22], Guerreau et al. proposed a key recovery attack exploiting the half Gaussian leakage.<sup>2</sup>

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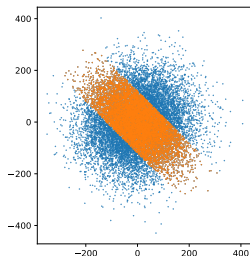
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## Fact

When  $z_0^+ = 0$ , the signature  $s = \sum_{i=0}^{2n-1} y_i \cdot \tilde{\mathbf{b}}_i$  with  $y_0 \in [-1, 1]$ .



## The attack of [GMRR22]

They reused the parallelepiped-learning technique to recover the key

The attack is **rather expensive**.

- $10^7$  traces for direct recovery
- $10^6$  traces and 1000h of computation

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# Our improved key recovery

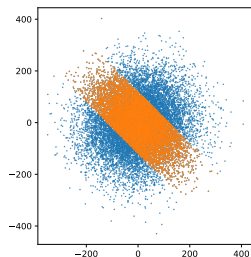
## Learning Slice Problem $LSP_{b,\sigma,N}$

Given  $\mathbf{b} \in \mathbb{R}^n$ , let  $\mathcal{S}_{\mathbf{b}}(b) = \{\mathbf{v} : |\langle \mathbf{v}, \mathbf{b} \rangle| \leq b\}$ . Let  $D_s$  be the conditional distribution of  $\mathbf{z} \sim (\mathcal{N}(0, \sigma^2))^n$  given  $\mathbf{z} \in \mathcal{S}_{\mathbf{b}}(b)$ . Given  $N$  independent samples drawn from  $D_s$ , find an approximation of  $\pm \mathbf{b}$ .

The geometric intuition: the projection of signatures in the slice on  $\mathbf{b}_0$  tends to be **unusually short**.

Our LSP algorithm

- 1 learning the direction of  $\mathbf{b}_0$
- 2 estimating  $\|\mathbf{b}_0\|$



## Step 1: Learning the slice direction

Let  $\mathbf{B} = (\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_{n-1})$  and  $\mathbf{D} = (\mathbf{d}_0, \dots, \mathbf{d}_{n-1})$  with  $\mathbf{d}_i = \tilde{\mathbf{b}}_i / \|\tilde{\mathbf{b}}_i\|$ .

For  $\mathbf{s} = \sum_i y_i \mathbf{d}_i \sim (\mathcal{N}(0, \sigma^2))^n$ ,  $y_i \sim \mathcal{N}(0, \sigma^2)$  and  $\mathbf{Cov}[\mathbf{s}] = \sigma^2 \mathbf{I}$ .

When  $\mathbf{s} \in \mathcal{S}_{\mathbf{b}_0}(b)$ , the variance of  $y_0$  is  $\sigma'^2 < \sigma^2$  and thus

$$\mathbf{Cov}[\mathbf{s} | \mathbf{s} \in \mathcal{S}_{\mathbf{b}_0}(b)] = \mathbf{D} \cdot \begin{pmatrix} \sigma'^2 & & \\ & \sigma^2 \mathbf{I} & \\ & & \end{pmatrix} \cdot \mathbf{D}^t.$$

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This analysis can be understood as principal component analysis rather than independent component analysis.



## Step 2: Learning the norm

The covariance  $\mathbf{Cov}[s | s \in \mathcal{S}_{\mathbf{b}_0}(b)]$  also leaks the information of  $\|\mathbf{b}_0\|$ :

$$\sigma'^2 = \frac{\int_{-b'}^{b'} x^2 \exp(-\frac{x^2}{2\sigma^2}) dx}{\int_{-b'}^{b'} \exp(-\frac{x^2}{2\sigma^2}) dx} \quad \text{where } b' = \frac{b}{\|\mathbf{b}_0\|}.$$

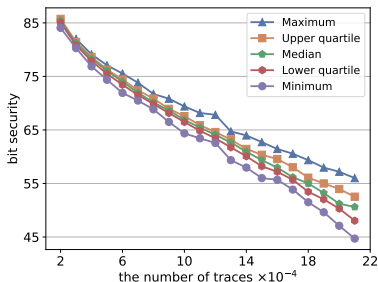
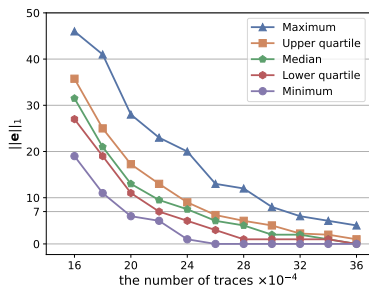
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This allows to numerically estimate  $\|\mathbf{b}_0\|$ !

# Experimental results



Our attack is much more efficient compared with [GMRR22]<sup>3</sup>!

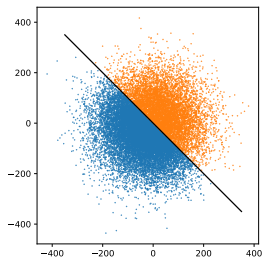
- direct recovery:  $10^7$  traces  $\rightarrow 3.6 \times 10^5$  traces
- $10^6$  traces + 1000h  $\rightarrow 2.2 \times 10^5$  traces + 0.5h of computation

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## Exploiting the sign leakage

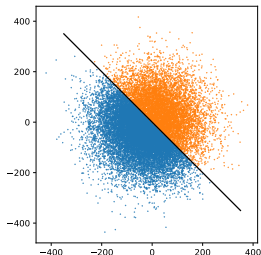
## Learning the halfspace

The sign leakage allows to determine whether a signature  $\mathbf{s}$  is in the halfspace  $\mathcal{H}^+ = \{\mathbf{v} : \langle \mathbf{v}, \mathbf{b}_0 \rangle \geq 0\}$  or  $\mathcal{H}^- = \{\mathbf{v} : \langle \mathbf{v}, \mathbf{b}_0 \rangle < 0\}$



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### Learning Halfspace Problem $\text{LHP}_{\sigma, N}$

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# Our LHP algorithm

At a high level, our algorithm can be seen as the reduction:

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Our LHP algorithm

- 1 learning a relatively rough direction  $\mathbf{v}$  of  $\mathbf{b}_0$  from samples in  $\mathcal{H}_{\mathbf{b}_0}^+$
- 2 filtering out those samples in  $\mathcal{S}_{\mathbf{v}}(b)$  using  $\mathbf{v}$
- 3 learning the direction of  $\mathbf{b}_0$  from the filtered samples in  $\mathcal{S}_{\mathbf{v}}(b)$



## Step 1: Learning a rough direction

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One can also learn the direction through the expectation of samples, but the expectation does not seem to improve the attack

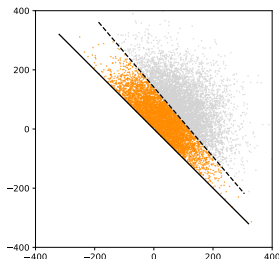
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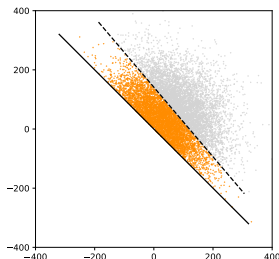
We propose to use the rough direction  $\mathbf{v}$  to classify all samples into two sets  $\mathcal{S} = \{\mathbf{s} \mid |\langle \mathbf{s}, \mathbf{v} \rangle| \leq b\}$  and  $\mathcal{C} = \{\mathbf{s} \mid |\langle \mathbf{s}, \mathbf{v} \rangle| > b\}$



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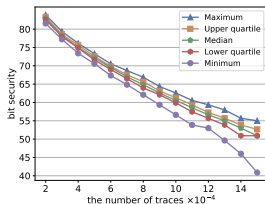
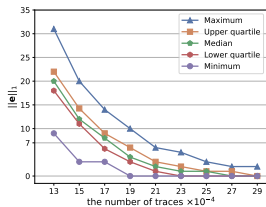
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Applying our LSP algorithm, we obtain a more accurate direction!

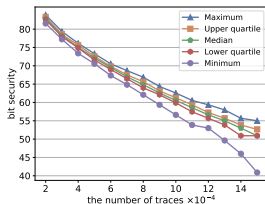
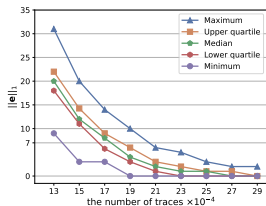
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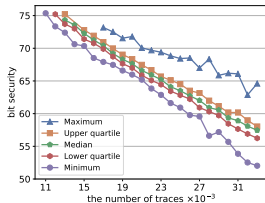
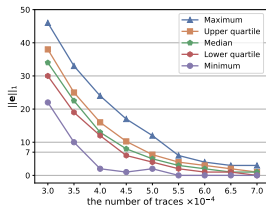


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The attack can be more efficient by using both two leakages!





# A practical countermeasure

## SamplerZ( $\sigma, c$ )

**Input:**  $c \in [0, 1)$  and  $\sigma \in (\sigma_{min}, \sigma_{max})$ .

**Output:**  $z \sim D_{\mathbb{Z}, \sigma, c}$ .

1:  $z^+ \leftarrow \text{BaseSampler}()$

2:  $b \leftarrow \mathbf{U}(\{0, 1\})$

3:  $z \leftarrow b + (2b - 1)z^+$

4:  $x \leftarrow -\frac{(z-c)^2}{2\sigma^2} + \frac{(z^+)^2}{2\sigma_{max}^2}$

5: return  $z$  with probability  $\frac{\sigma_{min}}{\sigma} \cdot \exp(x)$ , otherwise restart;

# A practical countermeasure

## SamplerZ( $\sigma, c$ )

**Input:**  $c \in [0, 1)$  and  $\sigma \in (\sigma_{min}, \sigma_{max})$ .

**Output:**  $z \sim D_{\mathbb{Z}, \sigma, c}$ .

1:  $z^+ \leftarrow \text{BaseSampler}()$

2:  $b \leftarrow U(\{0, 1\})$

3:  $z \leftarrow b + (2b - 1)z^+$

4:  $x \leftarrow -\frac{(z-c)^2}{2\sigma^2} + \frac{(z^+)^2}{2\sigma_{max}^2}$

5: return  $z$  with probability  $\frac{\sigma_{min}}{\sigma} \cdot \exp(x)$ , otherwise restart;

## Protected SamplerZ( $\sigma, c$ )

**Input:**  $c$  and  $\sigma \in (\sigma_{min}, \sigma_{max})$ .

**Output:**  $z \sim D_{\mathbb{Z}, \sigma, c}$ .

1:  $c' \leftarrow c - \lfloor c \rfloor$

2:  $z^+ \leftarrow \text{BaseSampler}()$

3:  $(\tilde{r}[0], \dots, \tilde{r}[15]) \leftarrow (2, 1, 1, 2, 2, 1, 1, 2, 2, 1, 2, 1, 1, 2, 1, 2)$

4:  $t \leftarrow U(\{0, \dots, 15\})$

5:  $b \leftarrow \tilde{r}[t]$

6:  $(\tilde{c}[0], \tilde{c}[1], \tilde{c}[2]) \leftarrow (0, c', 1 - c')$

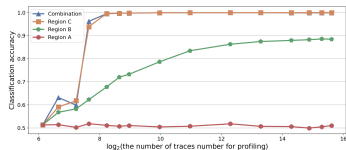
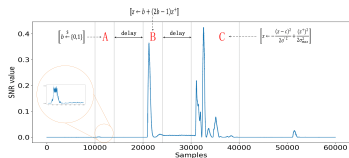
7:  $(\tilde{z}[0], \tilde{z}[1], \tilde{z}[2]) \leftarrow (0, \lfloor c \rfloor - z^+, \lfloor c \rfloor + 1 + z^+)$

8:  $x \leftarrow -\frac{(z^+ + \tilde{c}[b])^2}{2\sigma^2} + \frac{(z^+)^2}{2\sigma_{max}^2}$

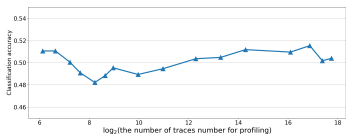
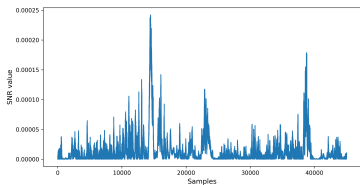
9: return  $\tilde{z}[b]$  with probability  $\frac{\sigma_{min}}{\sigma} \cdot \exp(x)$ , otherwise restart;

# Effectiveness

## Unprotected integer sampler



## Protected integer sampler



# Conclusion

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We provide an improved power analysis for Falcon.

- a new effective key recovery using the half Gaussian leakage within the base sampler
- the first side-channel analysis on Falcon taking the sign leakage into account.
- the above attacks also working with imperfect classification
- our attacks also working for the Mitaka signature scheme.

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We provide an improved power analysis for Falcon.

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- the above attacks also working with imperfect classification
- our attacks also working for the Mitaka signature scheme.

With the post-quantum standardization and migration underway, the side-channel security of post-quantum schemes needs more investigations.

# Thank you!

