Short Signatures from Regular Syndrome Decoding in the Head

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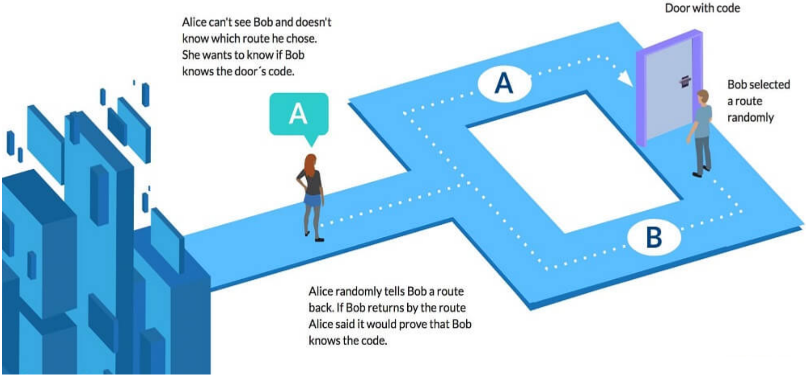
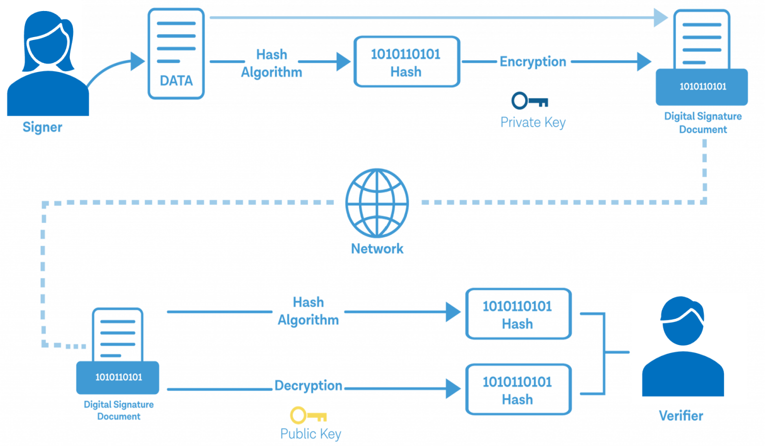
Zero Knowledge Proof of Knowledge

Bob wants to prove to Alice that he knows the door code without reveal it to her.



Signature scheme



Syndrome decoding problem

SD definition

The **Syndrome decoding problem** with parameters (K, k, w) is defined as follows:

- (Problem generation)
  Sample $H \leftarrow_r \{0,1\}^{k \times K}$ and $x \leftarrow_r \{x \in \{0,1\}^K : \mathsf{HW}(x) = w\}$.
  Set $y \leftarrow H \cdot x \bmod 2$. Output (H, y).
- (Goal) Given (H, y), find $x \in \{0,1\}^K$ such that
  - $H \cdot x = y \bmod 2$
  - $\mathsf{HW}(x) = w$ over $\mathbb{N}$.

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Our point of view

Let K, k, w be three integers, with $K > k > w$. Given $H \in \{0,1\}^{k \times K}$ and $y \in \{0,1\}^k$, find regular $x \in \{0,1\}^K$ s.t.:

- $H \cdot x = y$ over $\mathbb{F}_2$
- $\langle 1, x \rangle = w$ over $\mathbb{Z}_T$.

What is MPC?

Let $P_1, \cdots, P_n$ be n parties, each one with a private information $p_1, \cdots, p_n$. For a public function g, an n-party protocol allows them to compute

$$g(p_1, \cdots, p_n)$$

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The prover:

- Shares $[\![x]\!]_2 = (x_1, \cdots, x_n)$ s.t. $\sum_{i=1}^{n} x_i = x$ among n virtual parties,
- Computes $g(x_1, \cdots, x_n) = f\left(\sum_i x_i\right) = \left(\sum_i H \cdot x_i, \sum_i \langle 1, x_i \rangle\right)$ over appropriate ring.

ZKPoK from MPC

P

V

x_1
$y_1 = Hx_1 \mod 2$
$w_1 = \langle 1, x_1 \rangle \mod T$

P_1

x_n
$y_n = Hx_n \mod 2$
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P_n

P_2

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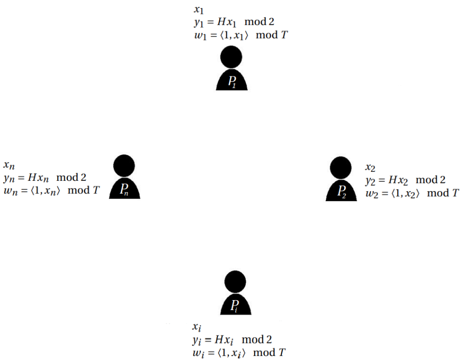

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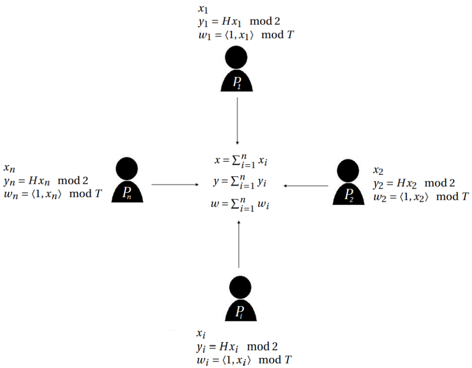

How to convert $[\![x]\!]_2$ into $[\![x]\!]_T$

$$[\![x]\!]_T = z \cdot [\![1-r]\!]_T + (1-z) \cdot [\![r]\!]_T$$

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Prepocessing material: $\mathbf{s} = [\![r]\!]_2, \mathbf{t} = [\![r]\!]_T$.

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⚠The higher is the weight, the lower is the cost!

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The prover computes the material himself in the preprocessing phase but he has to shuffle it using a uniformly random permutation chosen by the verifier before use it in the online phase of the MPC-in-the-head protocol.

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Definition

A real $\mathrm{p} \in (0,1)$ is a *combinatorial bound* if for every incorrect witness x, and every pair $(\mathbf{s}, \mathbf{t})$, the probability, over the random choice of π, that x satisfies:

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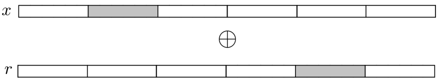

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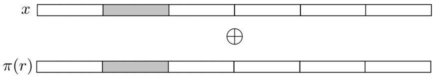

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⚠We chose parameters in an area s.t. the f-almost regular syndrome decoding is reduced to the standard regular syndrome decoding.

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Parameters (K, k, w, T) with $K > k > w$ and $T \leftarrow K/w$.
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- $([\![x]\!]_2, [\![\mathbf{r}]\!]_2, [\![\mathbf{r}]\!]_T) = ((\mathbf{x}_1, \cdots, \mathbf{x}_n), (\mathbf{s}_1, \cdots, \mathbf{s}_n), (\mathbf{t}_1, \cdots, \mathbf{t}_n))$.
- $c_i \leftarrow_r$ Commit$(\mathbf{x}_i, \mathbf{s}_i, \mathbf{t}_i)$ for $i = 1$ to n.

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- $[\![\mathbf{y}']\!]_2 = H \cdot [\![x]\!]_2$
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Round 4 (V) $d \in [n]$.
Round 5 (P) opens c_j for $j \neq d$.
Verification (V) checks

- all commitments were opened correctly;
- $\bigoplus_i y'_i = y$ and $\sum_i w'_i = w \mod T$;
- msg_j is consistent with $(\mathbf{x}_j,\mathbf{s}_j,\mathbf{t}_j)$.

THEOREM

Let

- Commit *be a non-interactive commitment scheme,*
- H *be collision-resistant hash function,*
- p *be the combinatorial bound previously discussed.*

Theorem

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Then our protocol is a gap honest-verifier zero-knowledge argument of knowledge for the relation $\mathscr{R}$ *such that*

$$((H,y),x) \in \mathscr{R} \text{ if } H \cdot x = y \bmod 2 \text{ and } x \text{ is a regular vector of weight } w$$

The gap relation $\mathscr{R}'$ *is such that*

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Soundness

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The soundness error of the proof is at most $\varepsilon = \mathsf{p} + 1/n - \mathsf{p}/n$.

Communication cost

Expected communication

$$4\lambda + \tau \cdot \left(\lambda(\log n + 1) + \left(\frac{2n-1}{n}\right) \frac{T-1}{T} (K-k) + \left(\frac{n-1}{n}\right) K \log_2 T/2 \right) \text{bits}$$

HOW DEFINE A SIGNATURE USING FIAT-SHAMIR TRANSORM?

The outputs of the first four round of our 5-round protocol are computed as follows:

- $h_1 = H(m, \mathsf{salt}, h)$,
- $\pi \leftarrow \mathsf{PRG}(h_1)$,
- $h_2 = H(m, \mathsf{salt}, h, h^{'})$,
- $d \leftarrow \mathsf{PRG}(h_2)$.

Final results

$f = 12, K = 1842,\ k = 1017, w = 307, \lambda = 128.$

Setting 1 – fast signature (RSD-F)

$\tau = 18,\ n = 193.$ Signature size $= 12.52$ KB. Runtime 2.7ms.

Setting 2 – medium signature 1 (RSD-M1)

$\tau = 13,\ n = 1723.$ Signature size $= 9.69$ KB. Runtime 17ms.

Setting 3 – medium signature 2 (RSD-M2)

$\tau = 12,\ n = 3391.$ Signature size $= 9.13$ KB. Runtime 31ms.

Setting 4 – short signature (RSD-S)

$\tau = 11,\ n = 7644.$ Signature size $= 8.55$ KB. Runtime 65ms.

Thank you for your attention!

Other developments in the paper:

- Combinatorial Analysis of the Construction
- Uniqueness Bound for Regular Syndrome Decoding
- Relation between SD, RSD and almost RSD
- Improvement of already known attacks against RSD
- Definition of a new attack based on an approximate birthday paradox

Comparison

Scheme	|sgn|	|pk|	t_{sgn}	Assumption
Wave	1.07 KB	3.2 MB	300	large-weight SD over $\mathbb{F}_3$, $(U, U+V)$-codes indist.
Durandal - I	3.97 KB	14.9 KB	4	Rank SD over $\mathbb{F}_{2^m}$
Durandal - II	4.90 KB	18.2 KB	5	Rank SD over $\mathbb{F}_{2^m}$
LESS-FM - I	9.77 KB	15.2 KB	-	Linear Code Equivalence
LESS-FM - II	206 KB	5.25 KB	-	Perm. Code Equivalence
LESS-FM - III	11.57 KB	10.39 KB	-	Perm. Code Equivalence
GPS - 256	24.0 KB	0.11 KB	-	SD over $\mathbb{F}_{256}$
GPS - 256	19.8 KB	0.12 KB	-	SD over $\mathbb{F}_{1024}$
FJR (fast)	22.6 KB	0.09 KB	13	SD over $\mathbb{F}_2$
FJR (short)	16.0 KB	0.09 KB	62	SD over $\mathbb{F}_2$
BGKM Sig1	23.7 KB	0.1 KB	-	SD over $\mathbb{F}_2$
BGKM Sig2	20.6 KB	0.2 KB	-	(QC)SD over $\mathbb{F}_2$
FJR - Var1f	15.6 KB	0.09 KB	-	SD over $\mathbb{F}_2$
FJR - Var1s	10.9 KB	0.09 KB	-	SD over $\mathbb{F}_2$
FJR - Var2f	17.0 KB	0.09 KB	13	SD over $\mathbb{F}_2$
FJR - Var2s	11.8 KB	0.09 KB	64	SD over $\mathbb{F}_2$
FJR - Var3f	11.5 KB	0.14 KB	6	SD over $\mathbb{F}_{256}$
FJR - Var3s	8.26 KB	0.14 KB	30	SD over $\mathbb{F}_{256}$
Our scheme - rsd-f	12.52 KB	0.09 KB	2.8*	RSD over $\mathbb{F}_2$
Our scheme - rsd-m1	9.69 KB	0.09 KB	17*	RSD over $\mathbb{F}_2$
Our scheme - rsd-m2	9.13 KB	0.09 KB	31*	RSD over $\mathbb{F}_2$
Our scheme - rsd-s	8.55 KB	0.09 KB	65*	RSD over $\mathbb{F}_2$
Our scheme - arsd-f	11.25 KB	0.09 KB	2.4*	f-almost-RSD over $\mathbb{F}_2$
Our scheme - arsd-m1	8.76 KB	0.09 KB	15*	f-almost-RSD over $\mathbb{F}_2$
Our scheme - arsd-m2	8.28 KB	0.09 KB	28*	f-almost-RSD over $\mathbb{F}_2$
Our scheme - arsd-s	7.77 KB	0.09 KB	57*	f-almost-RSD over $\mathbb{F}_2$