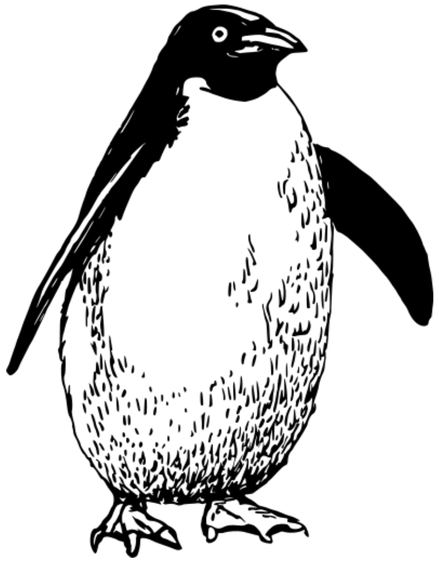
On Valiant's Conjecture

Eurocrypt 2023

Mathias Hall-Andersen, Jesper Buus Nielsen

Incrementally Verifiable Computation

Incrementally Verifiable Computation



Prover Penguin

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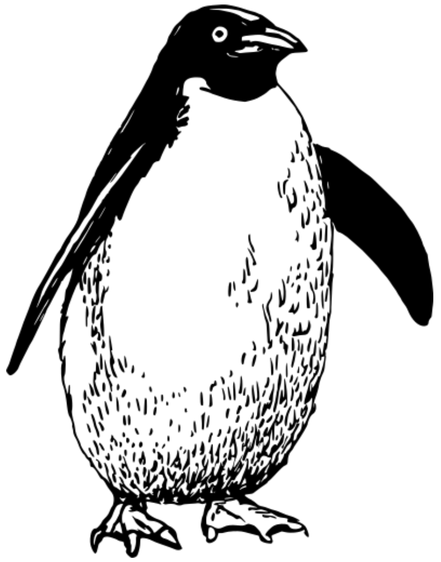

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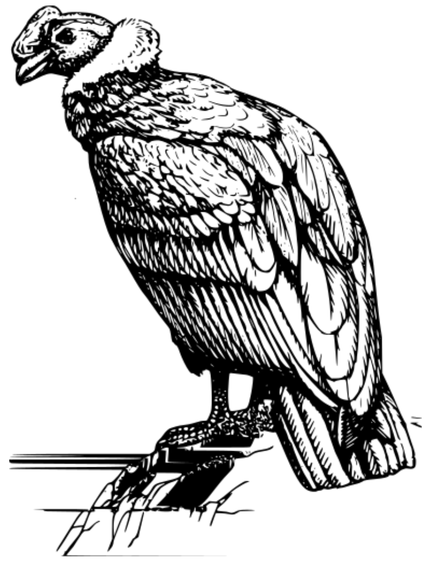
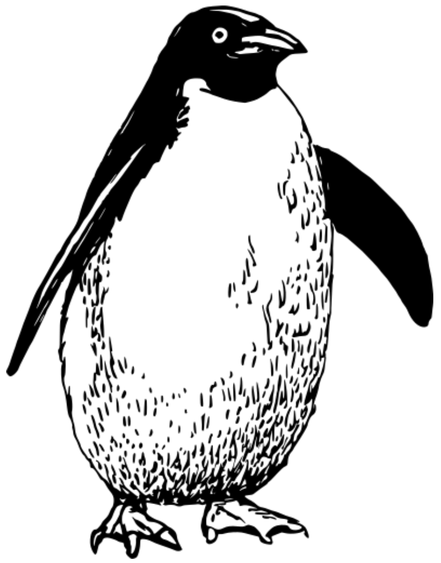


Verification Vulture

Incrementally Verifiable Computation

"I applied f to st_0 n times:"

$st_n = f^n(st_0)$



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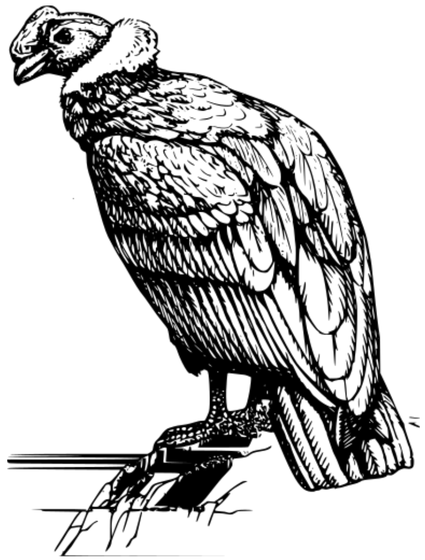

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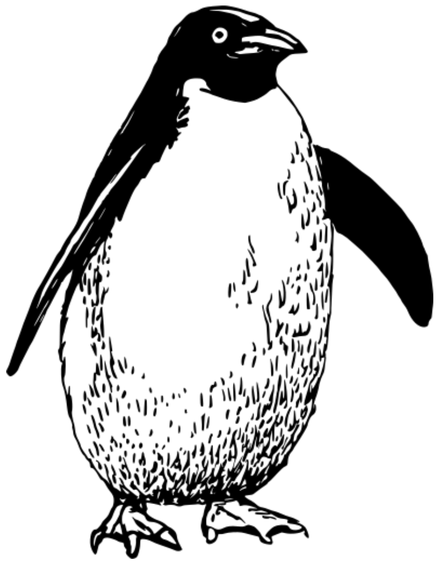
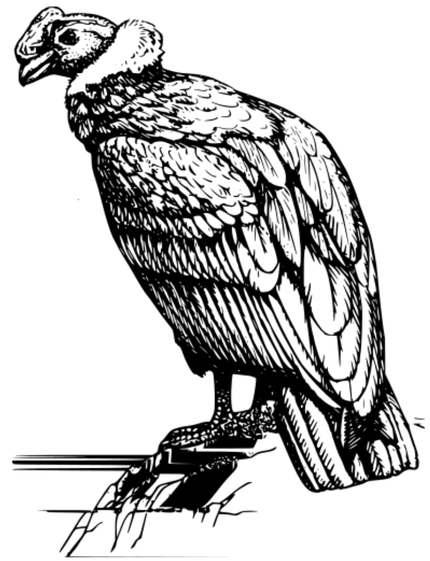


Prover Penguin

"Prove it!"



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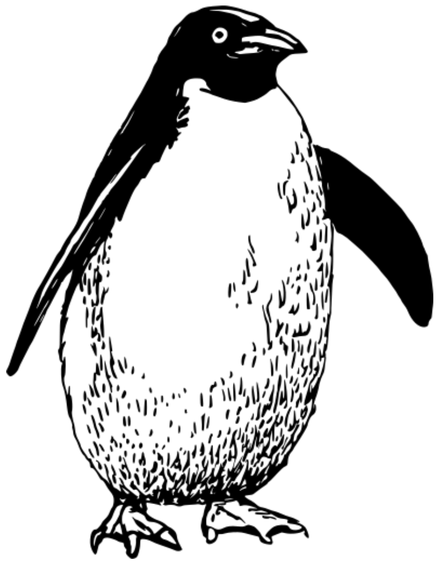

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$st_n, \tilde{\pi}_n$

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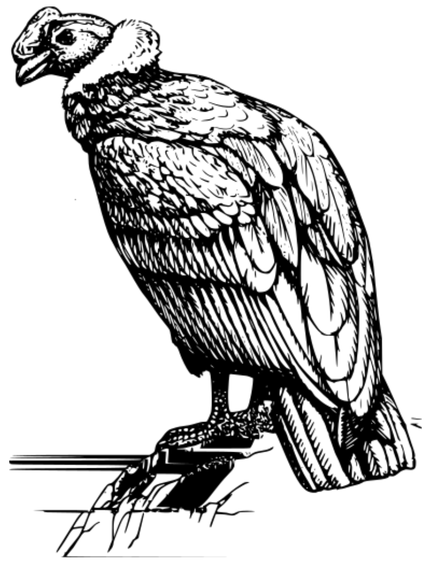

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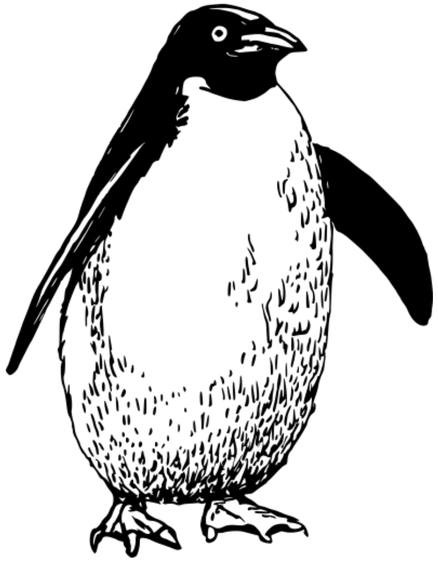

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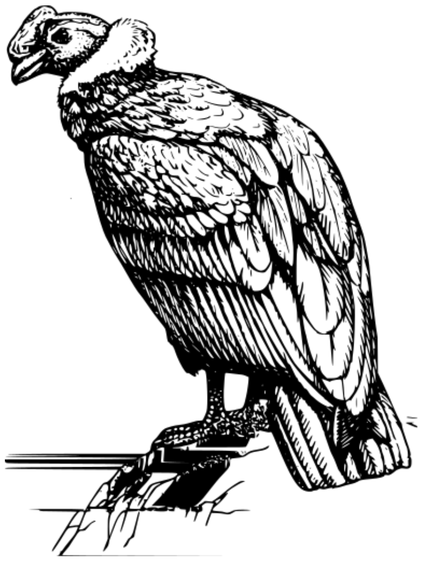

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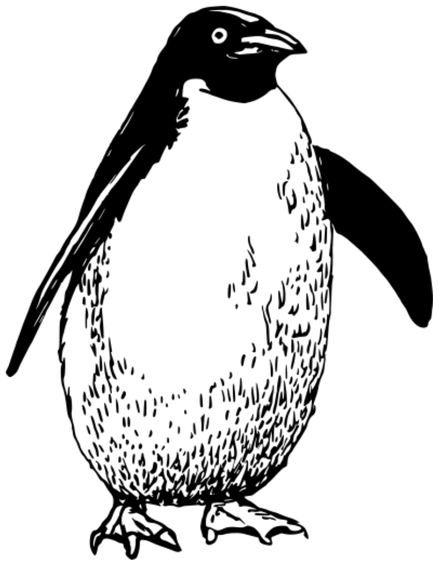

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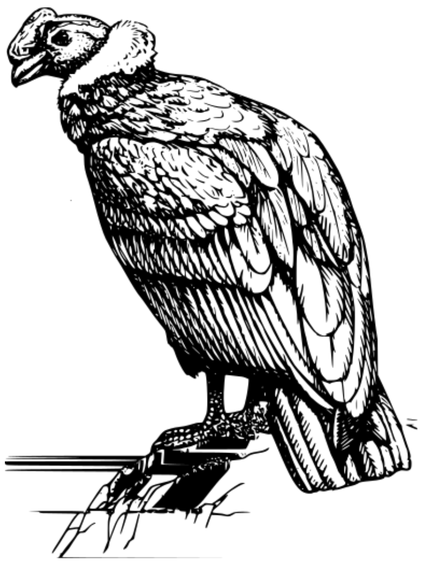

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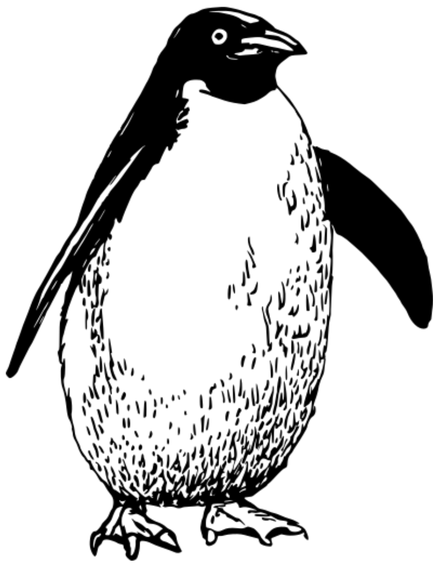

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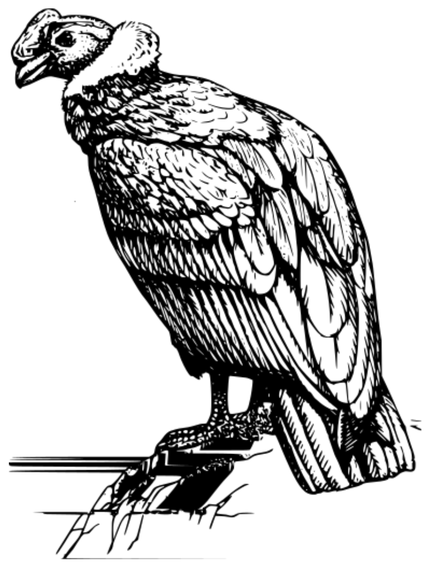

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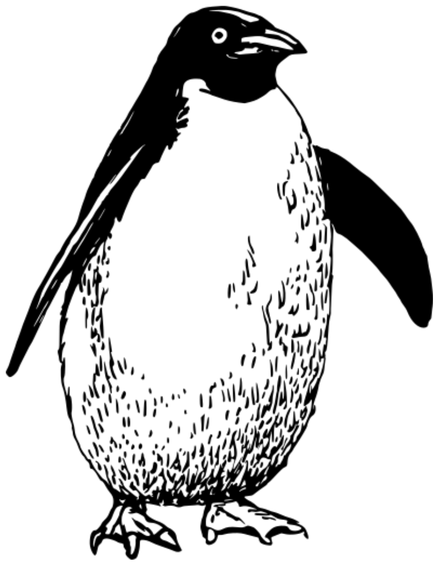
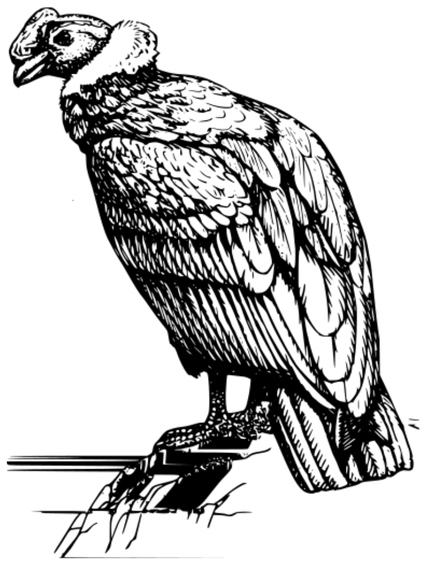


Prover Penguin

st_{n+1}, π_{n+1}



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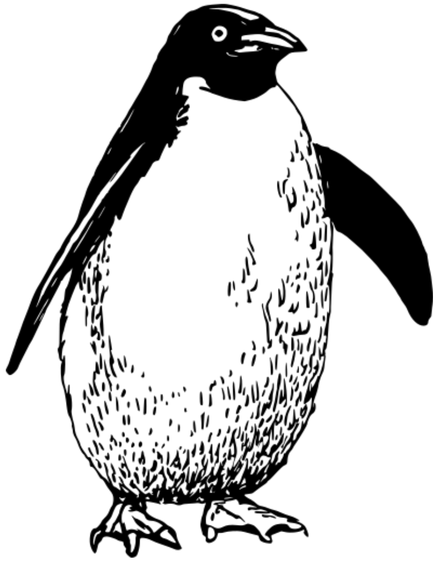
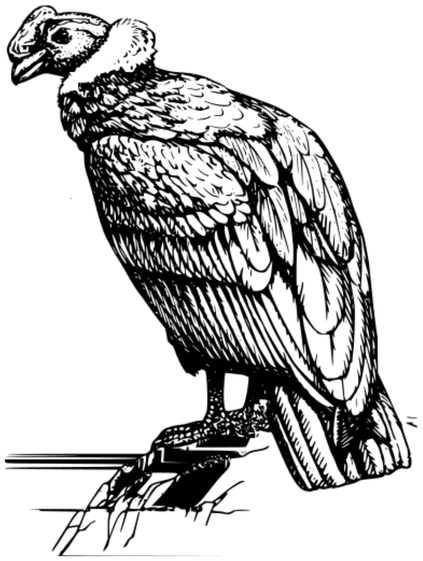


Prover Penguin

st_{n+1}, π_{n+1}

$st_{n+1} = f(st_n)$ computed from st_n.



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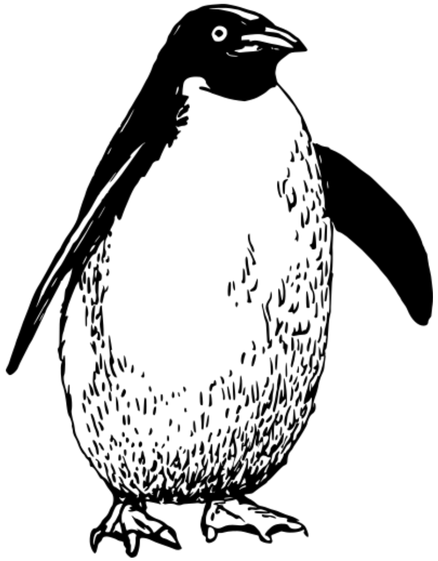


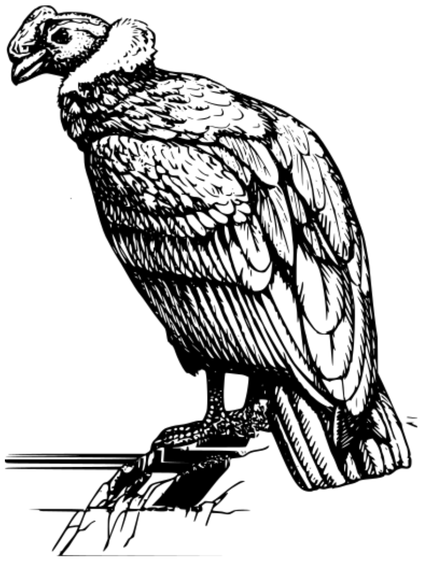
Prover Penguin

st_{n+1}, π_{n+1}

$st_{n+1} = f(st_n)$ computed from st_n.

What about computing π_{n+1} from π_n?

(with polylog(n) computation)



Verification Vulture

Incrementally Verifiable Computation

Incrementally Verifiable Computation

st_0

$\tilde{\pi}_0$

Incrementally Verifiable Computation

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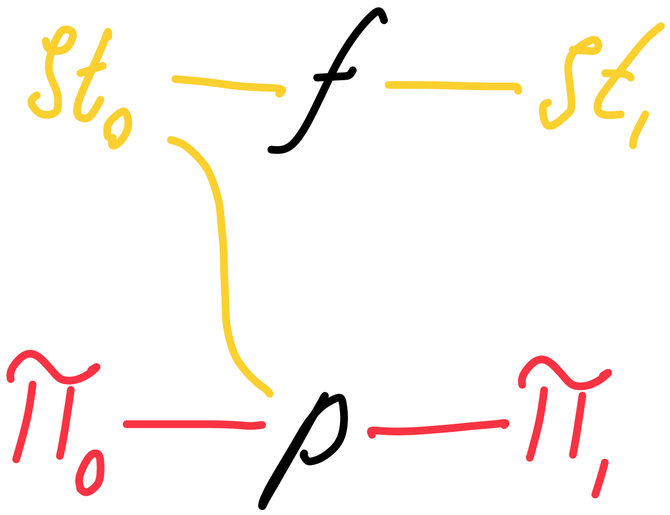

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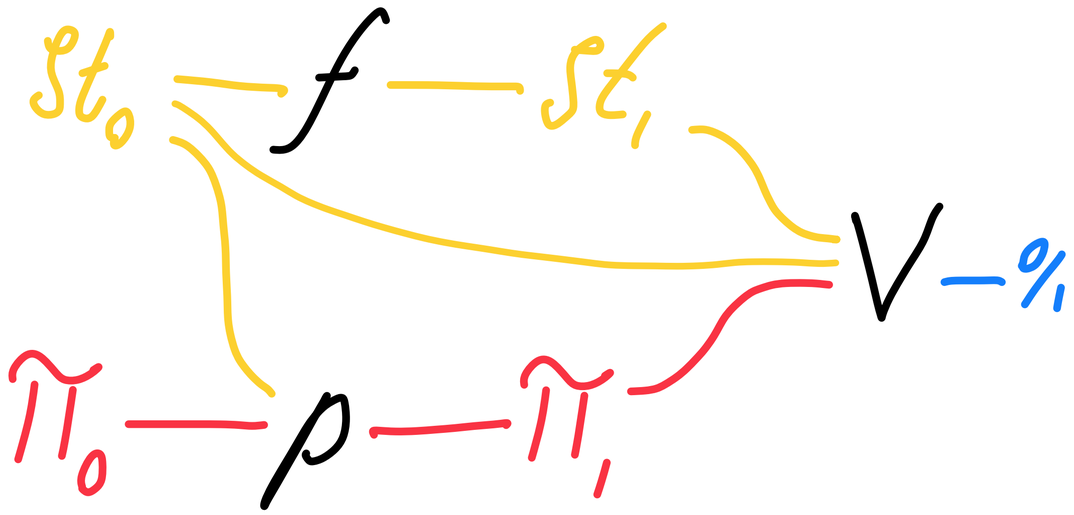

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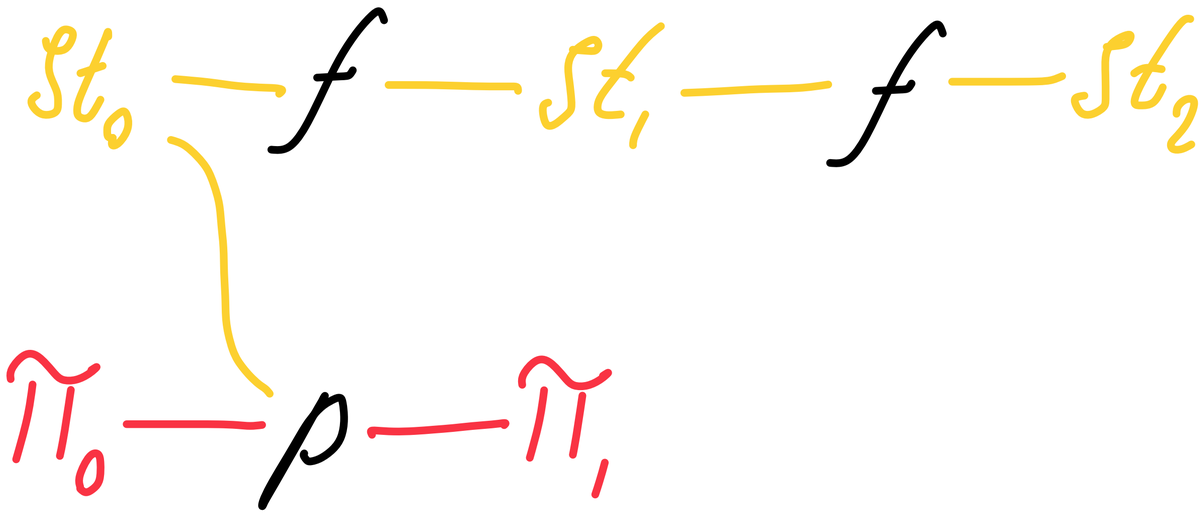

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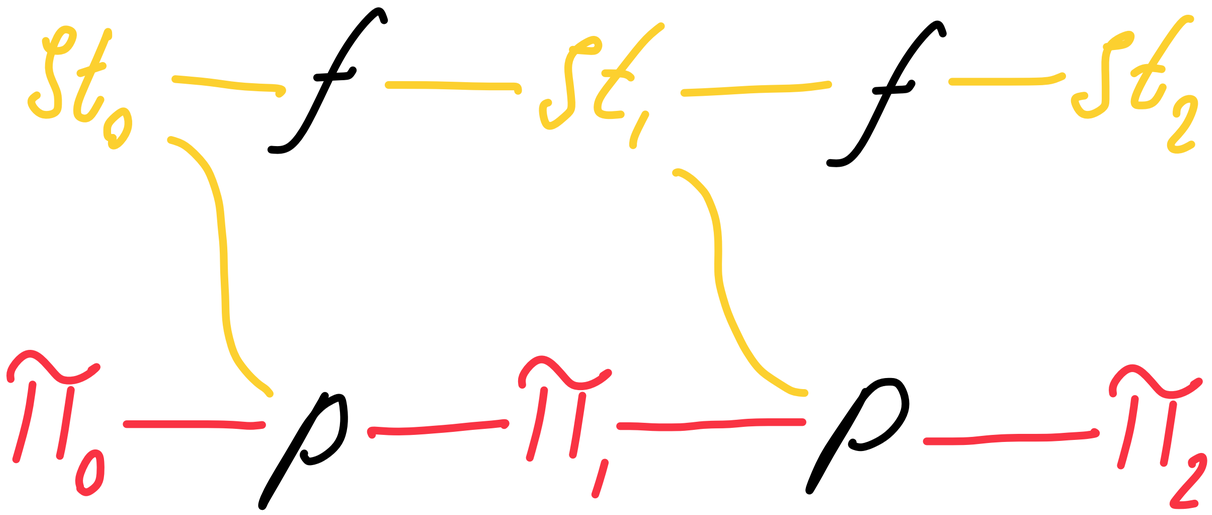

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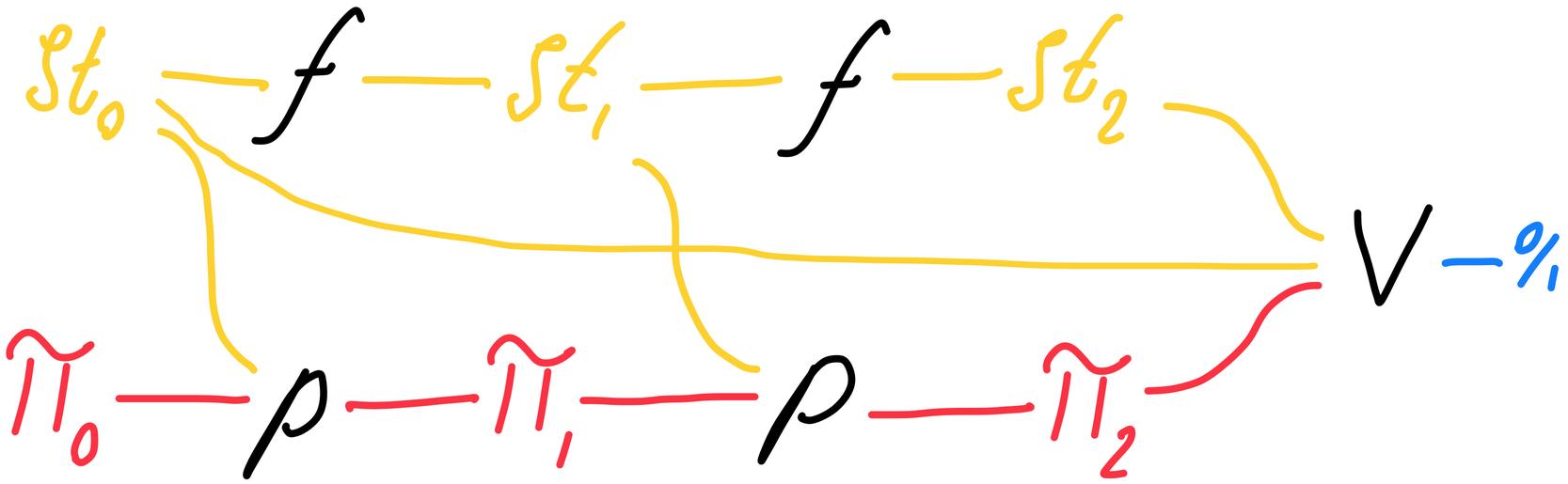

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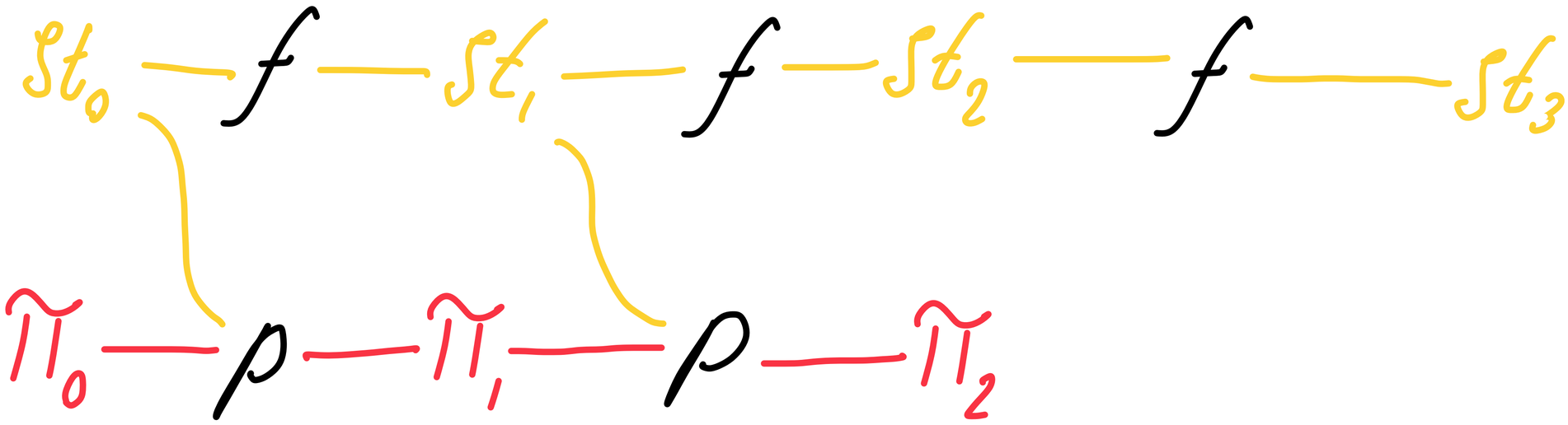

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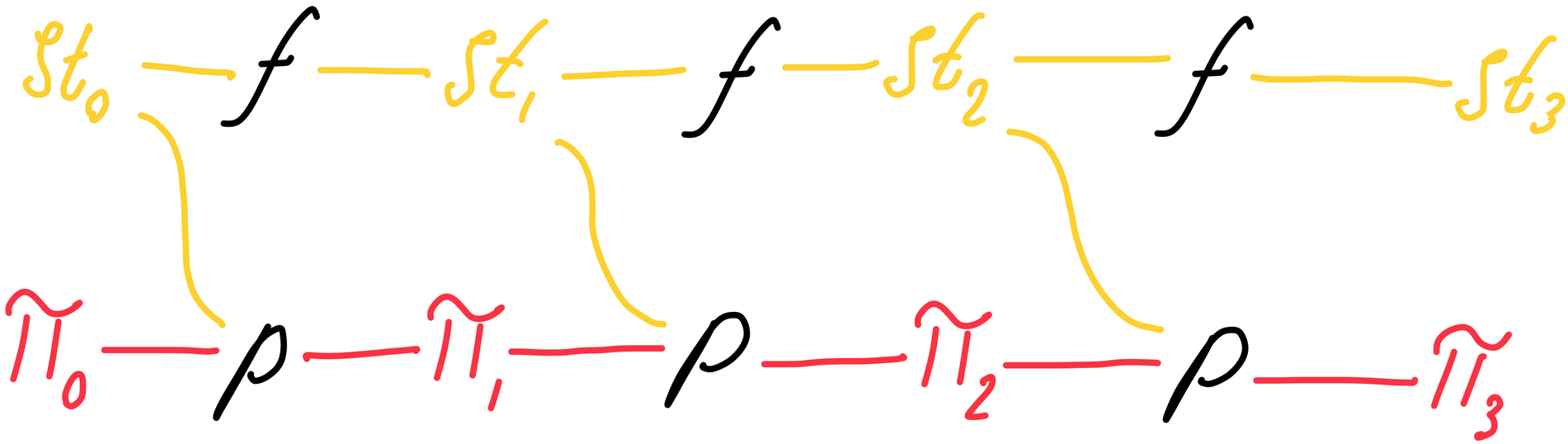

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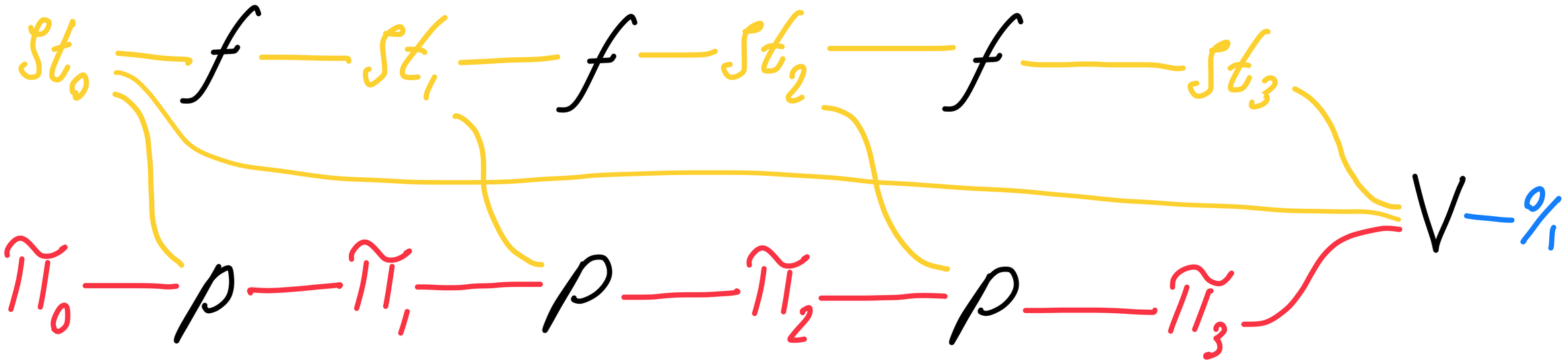

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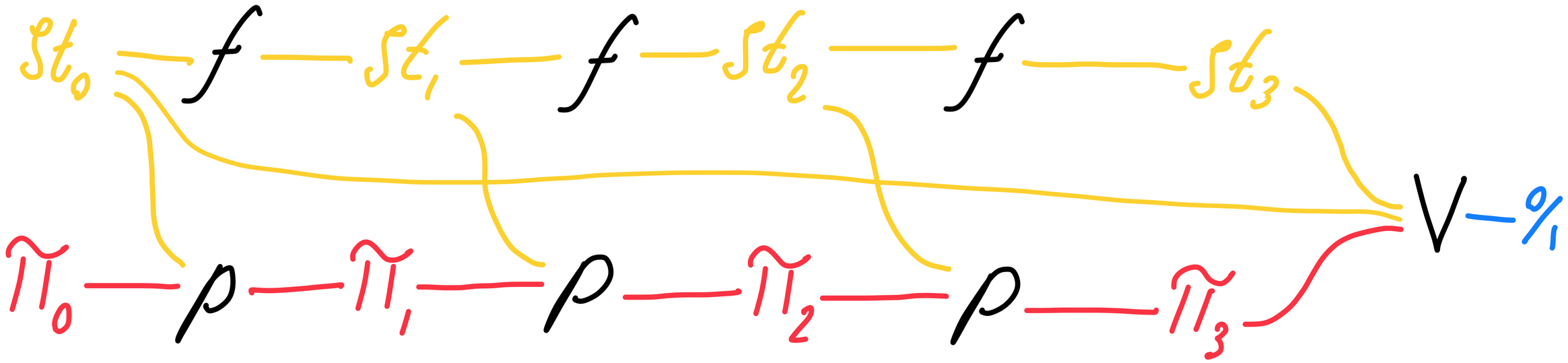
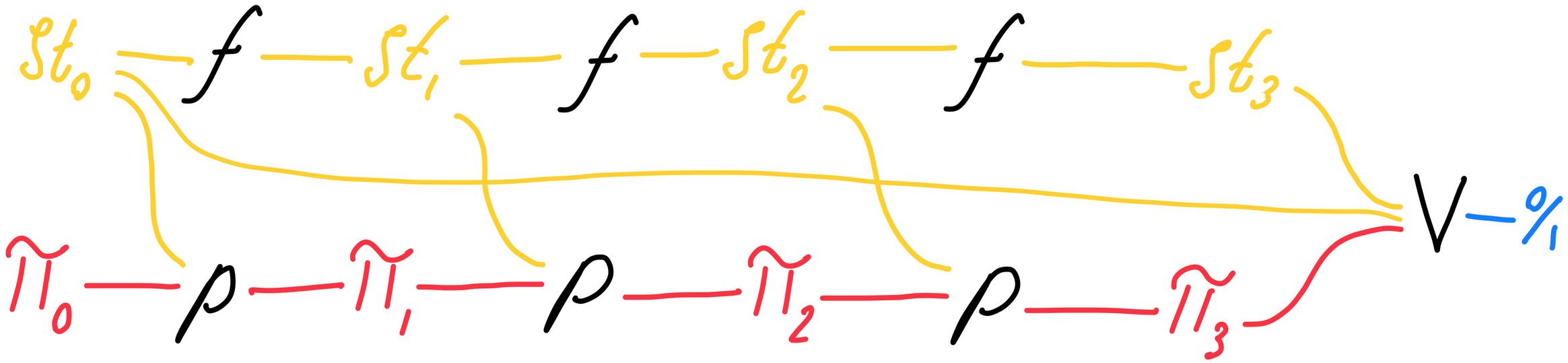


P Complexity: O(poly(|f|, log(n)))

Incrementally Verifiable Computation



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The Random Oracle Model

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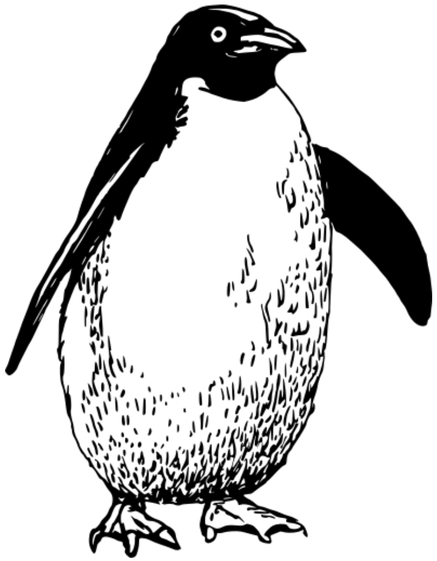

Prover Penguin

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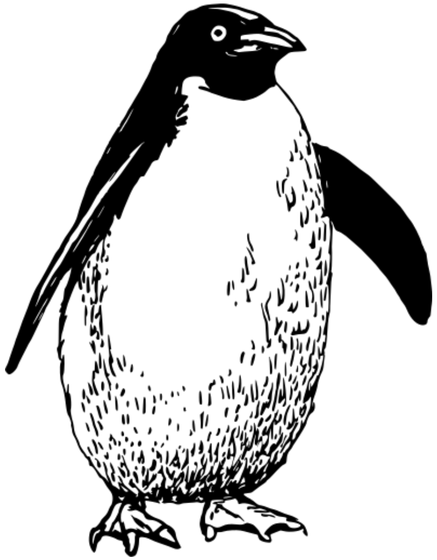

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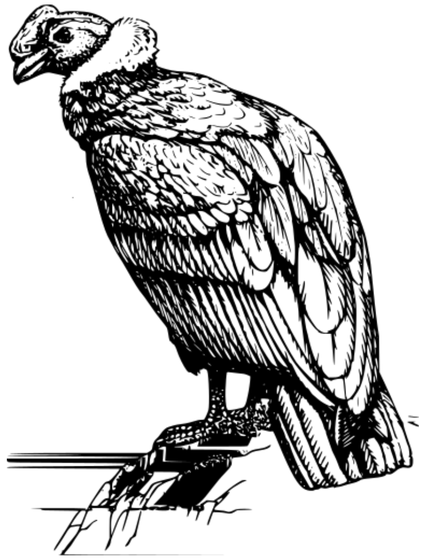

Verification Vulture

The Random Oracle Model

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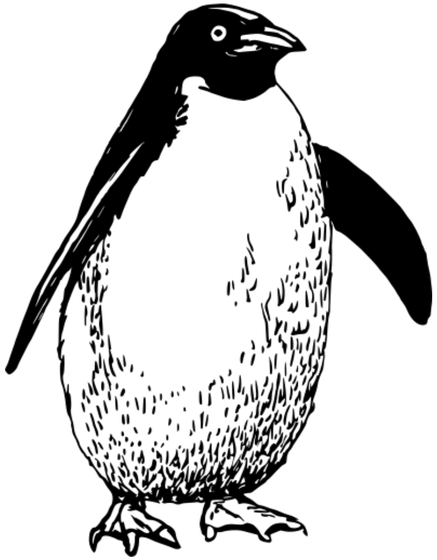

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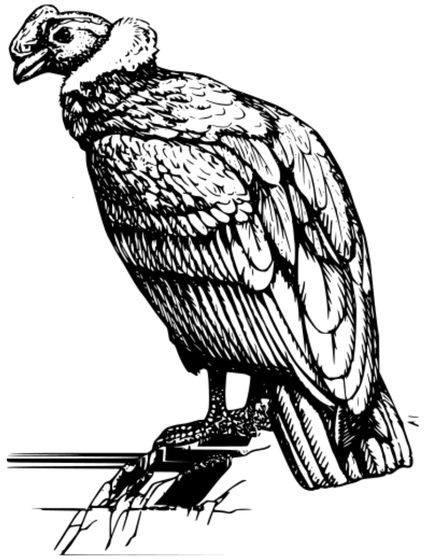

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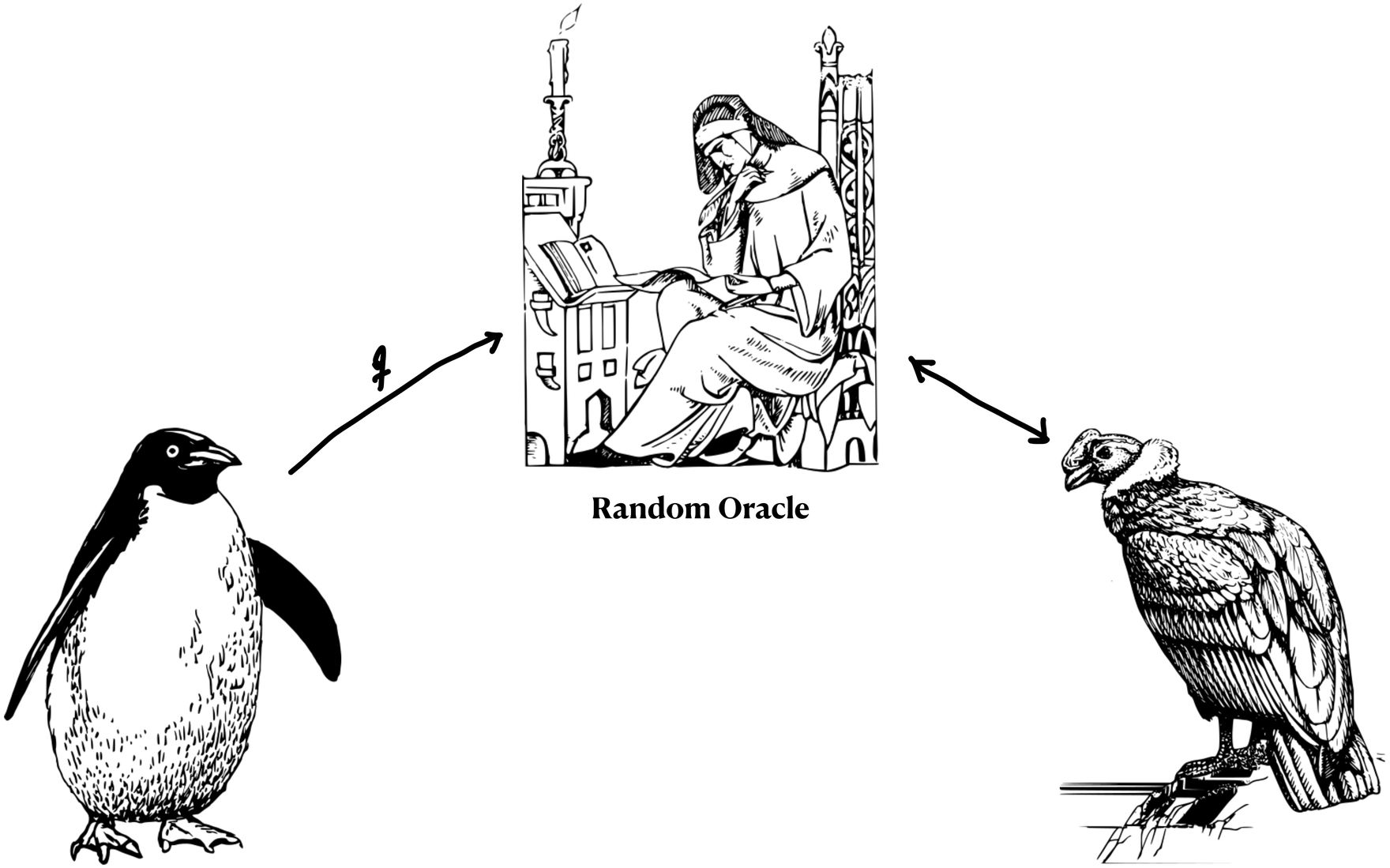

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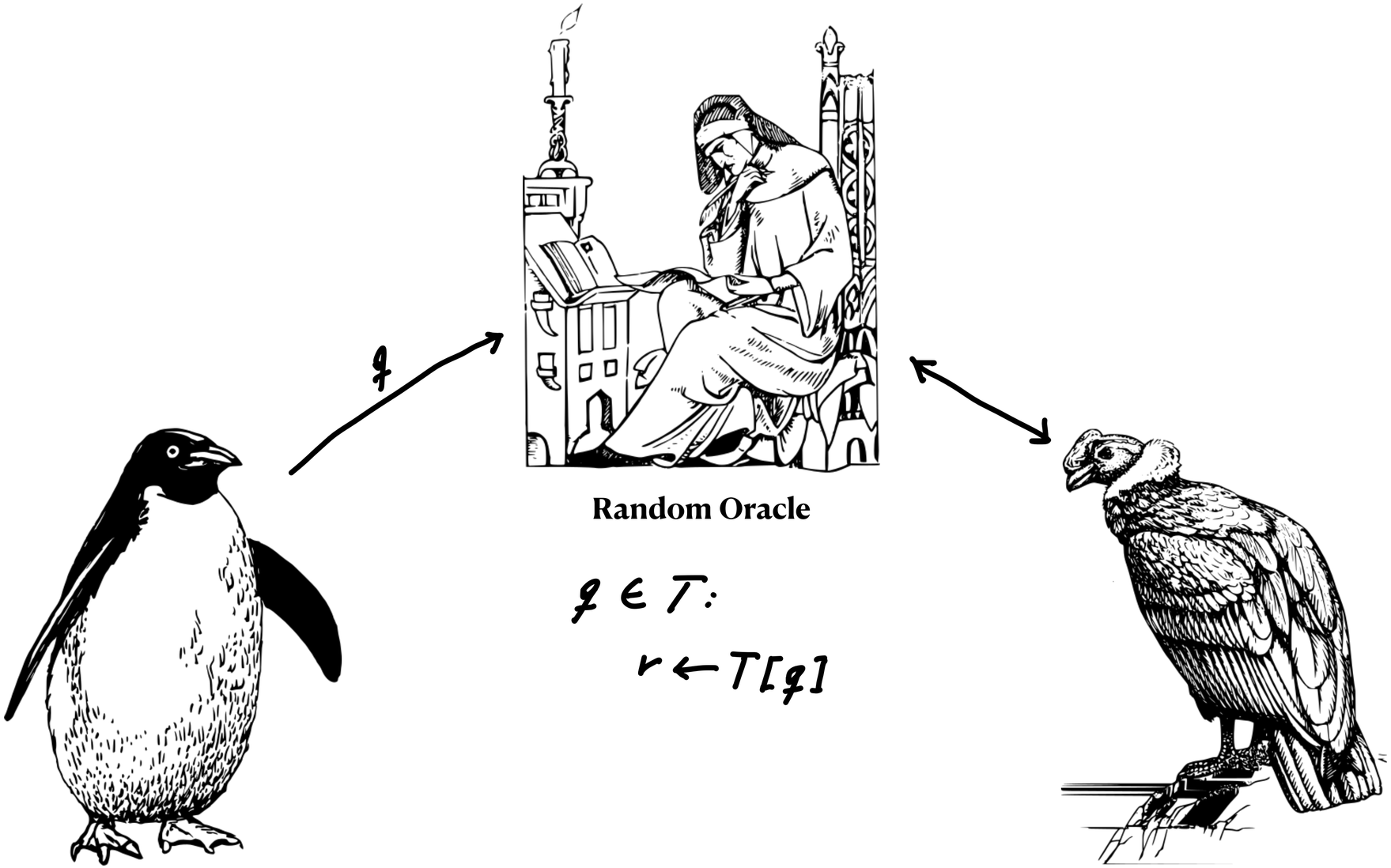

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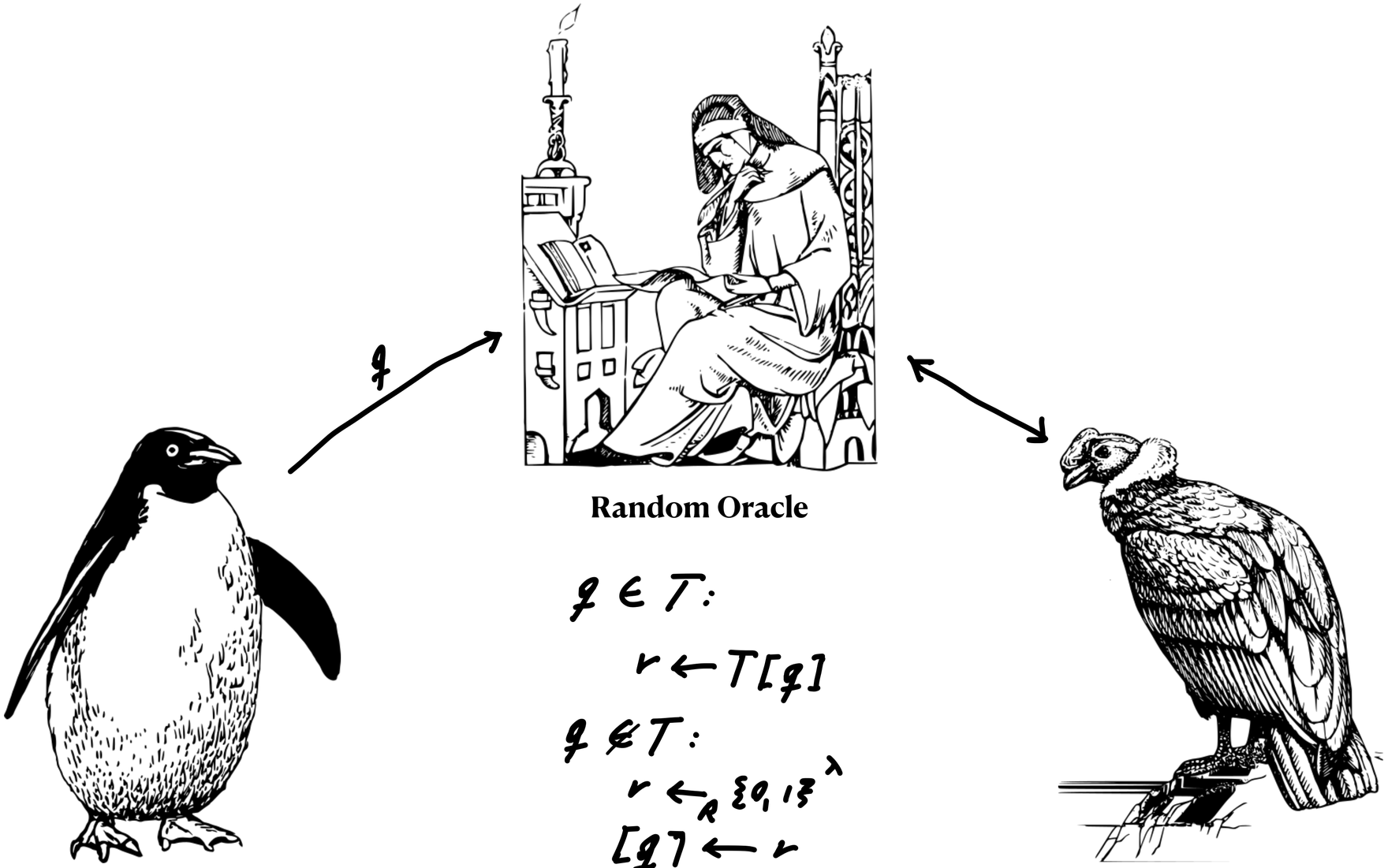

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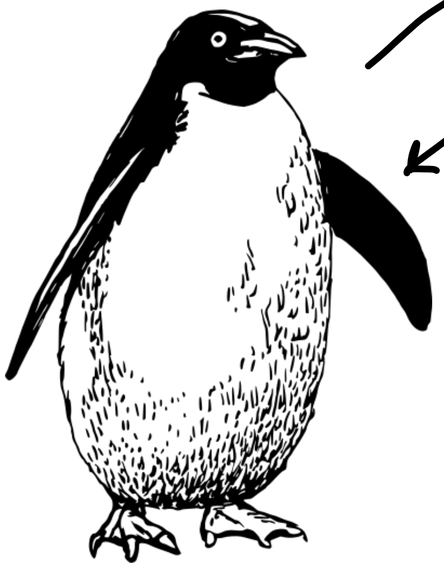

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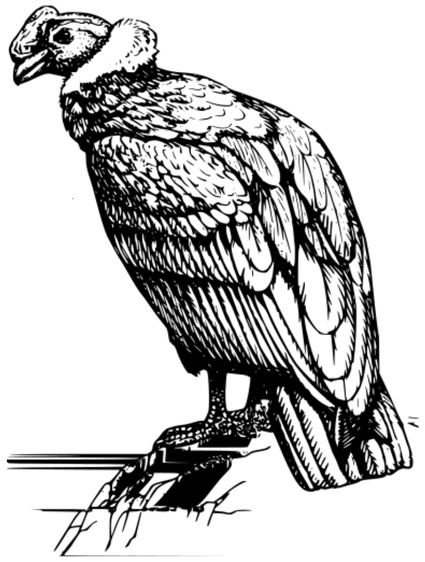

Verification Vulture

q

r

$q \in T$:

$r \leftarrow T[q]$

$q \notin T$:

$r \leftarrow_R \{0,1\}^\lambda$

$[q] \leftarrow r$

Valiant's Construction of IVC "in the ROM"

Valiant's Construction [2008] IVC From CS Proofs

Valiant's Construction of IVC "in the ROM"

Valiant's Construction [2008] IVC From CS Proofs

st_0

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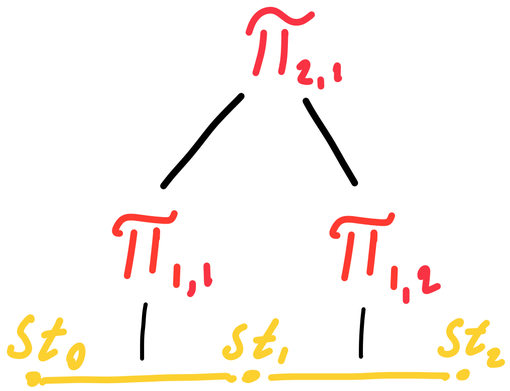

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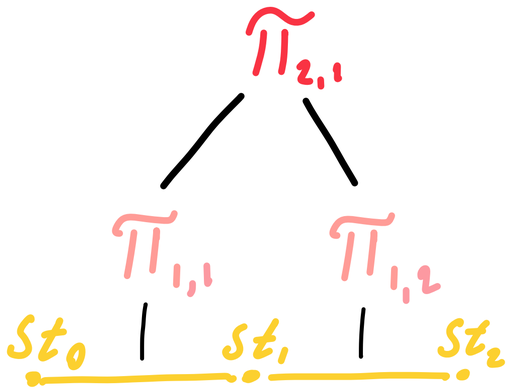

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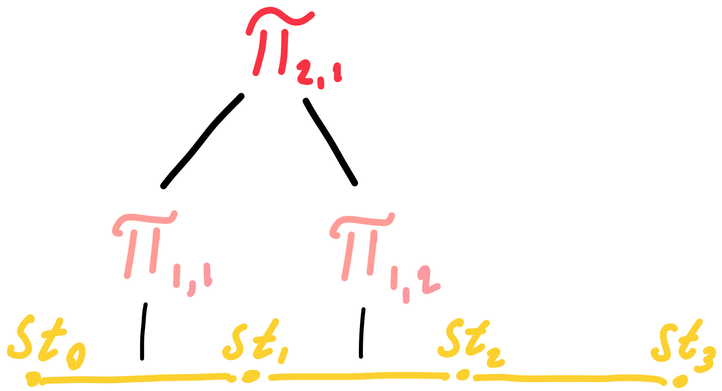

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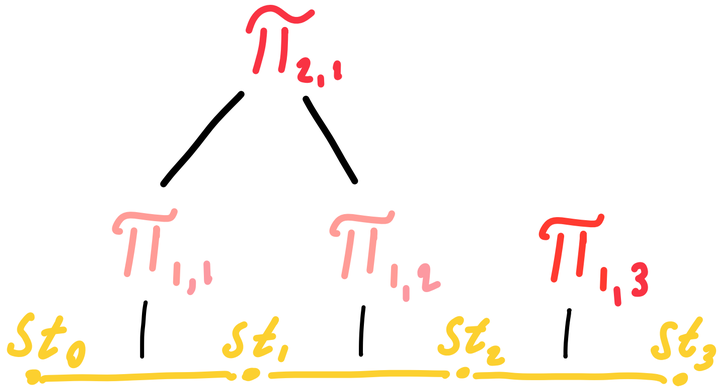

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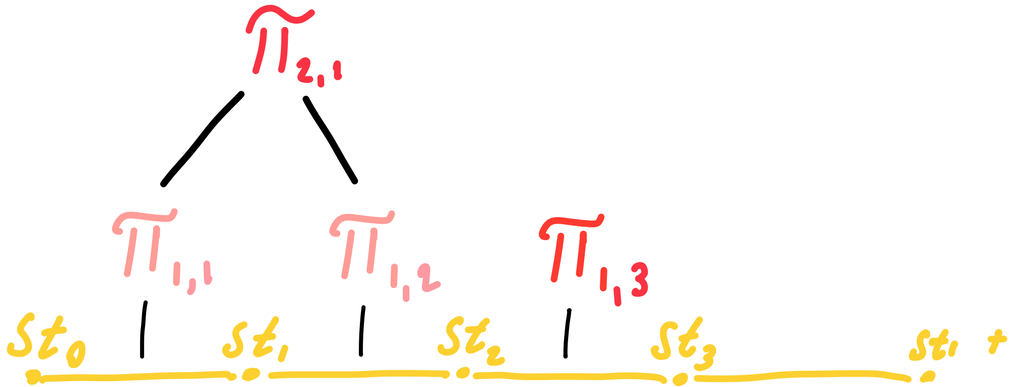

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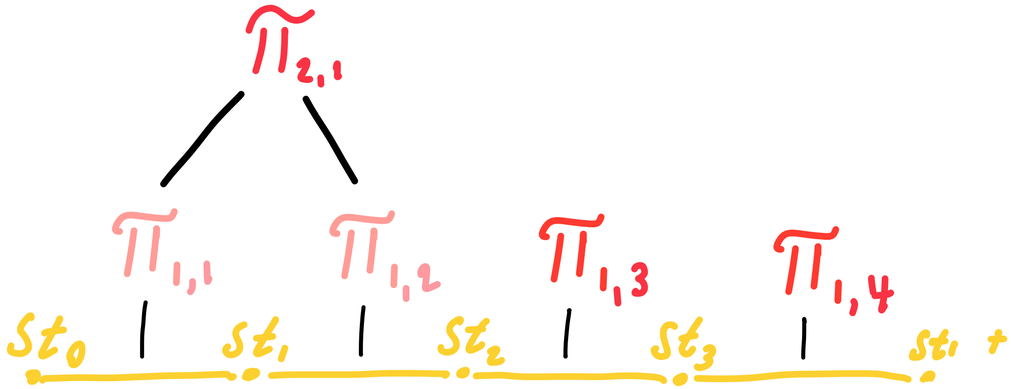

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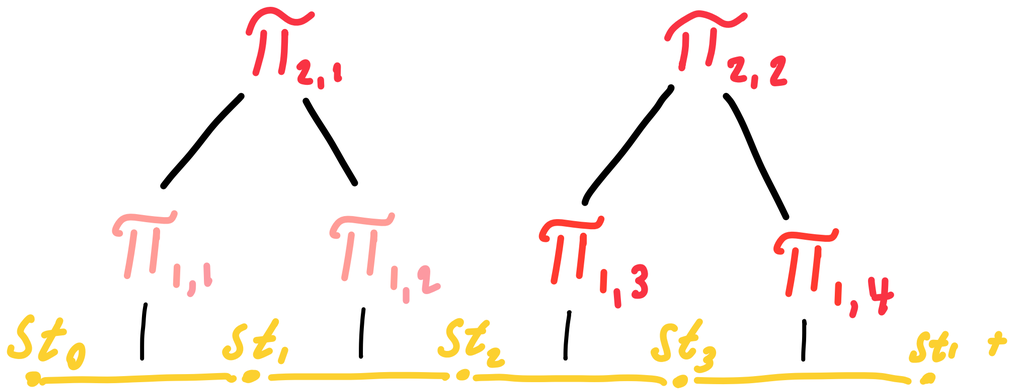

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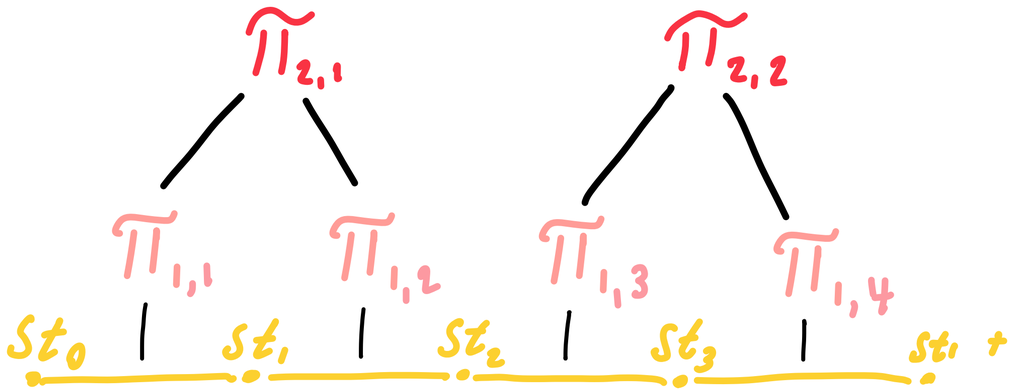

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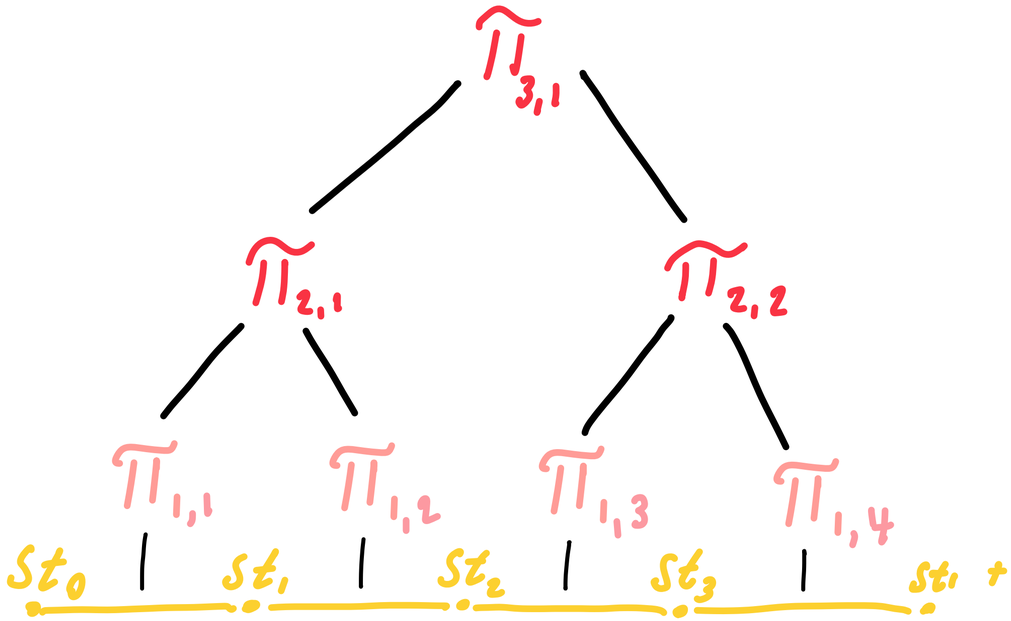

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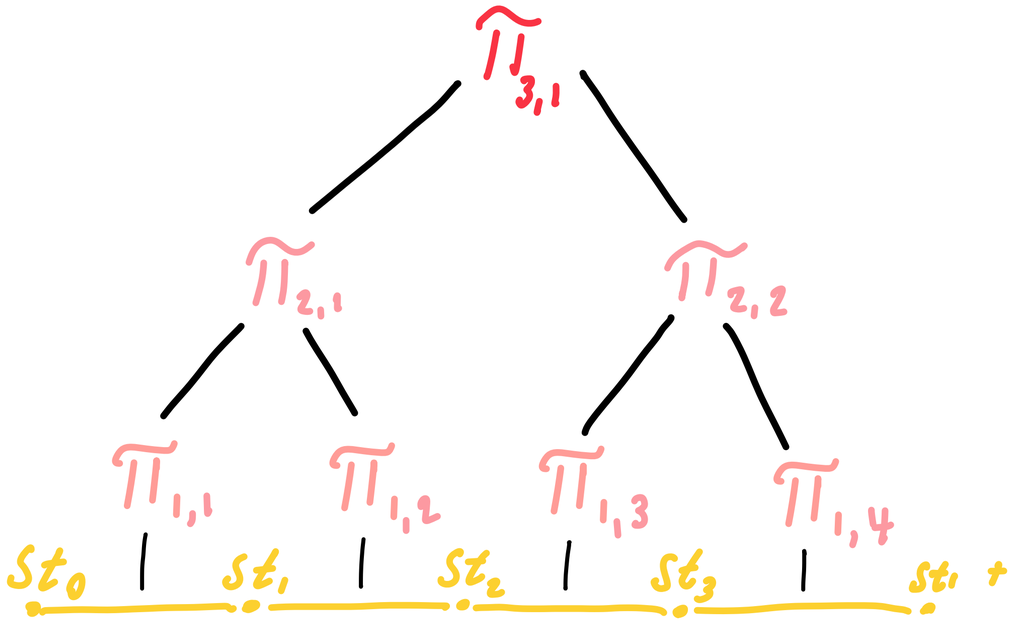

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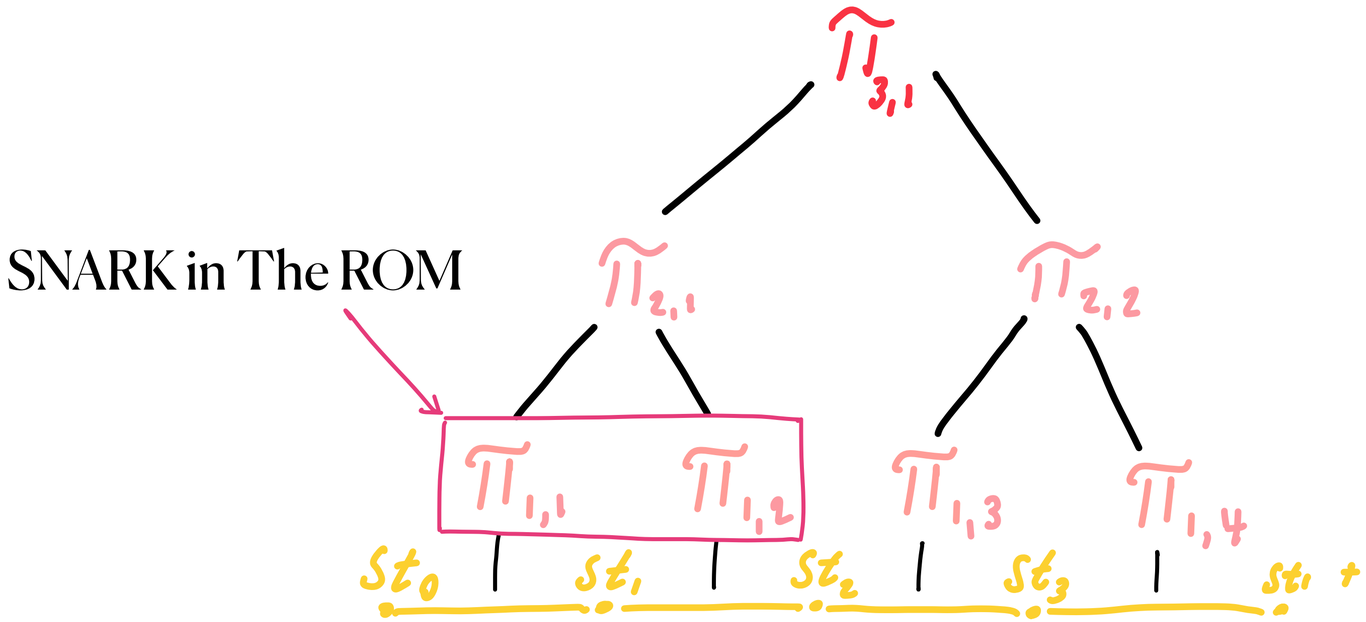

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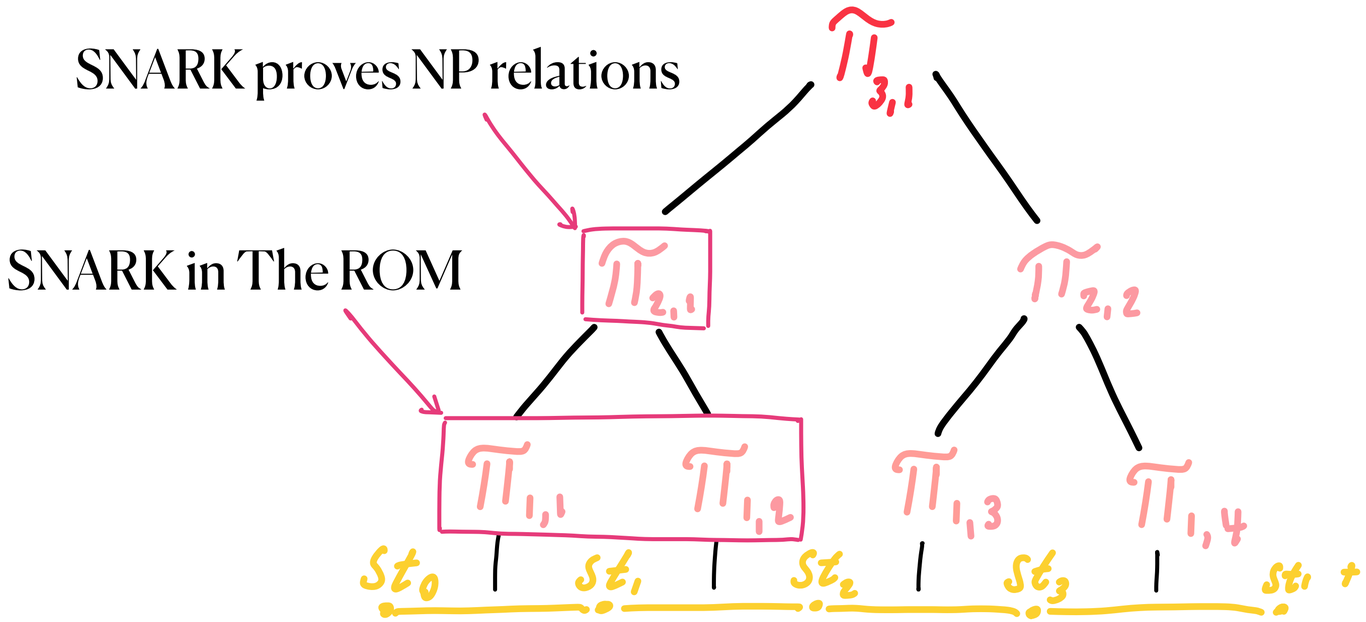

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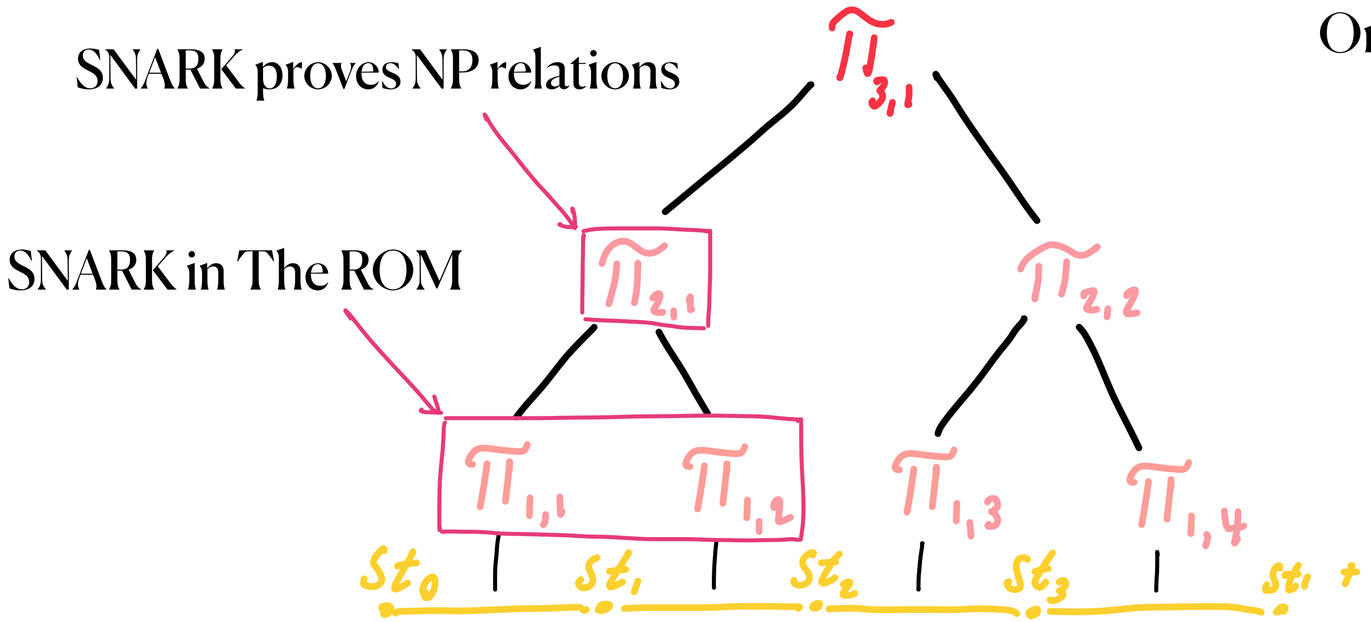

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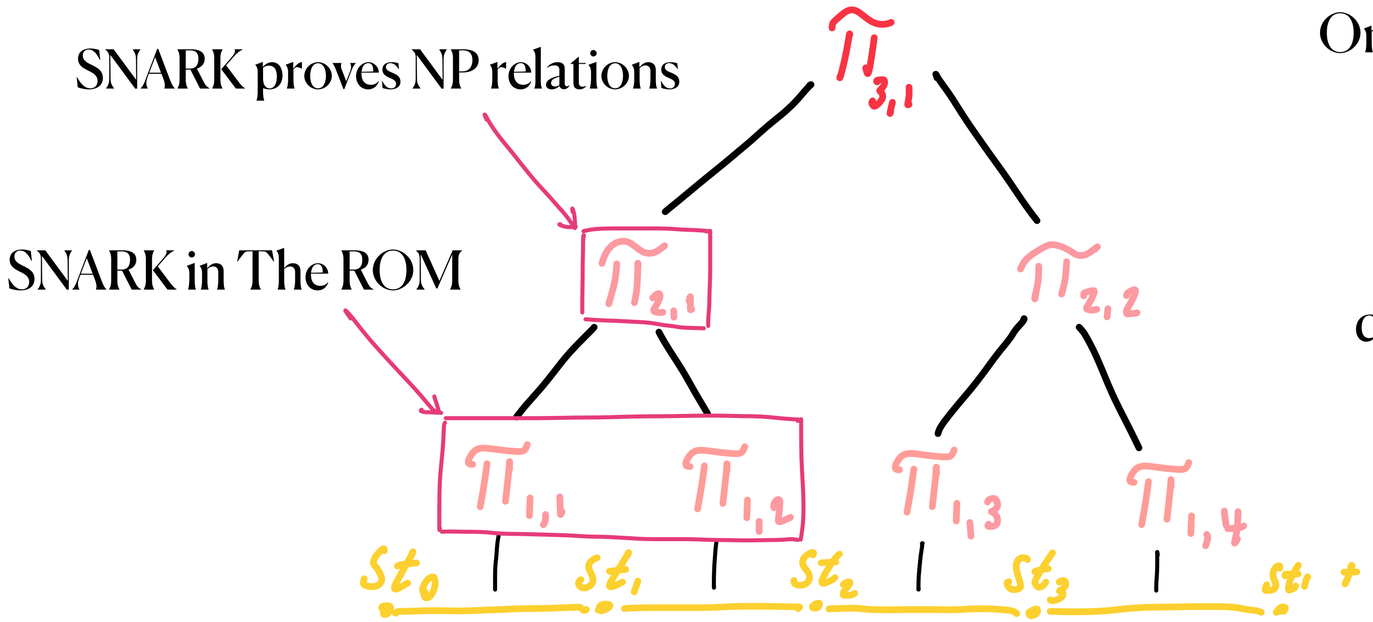

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The construction does not work in the ROM

Of Course Valiant Noted This.

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Can we prove this intuition?

Overview of Results

IVC to Non-Deterministic IVC

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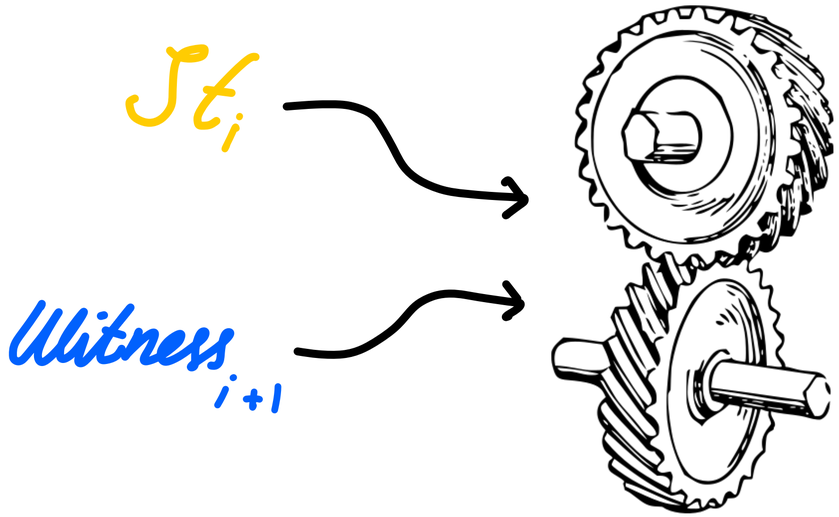

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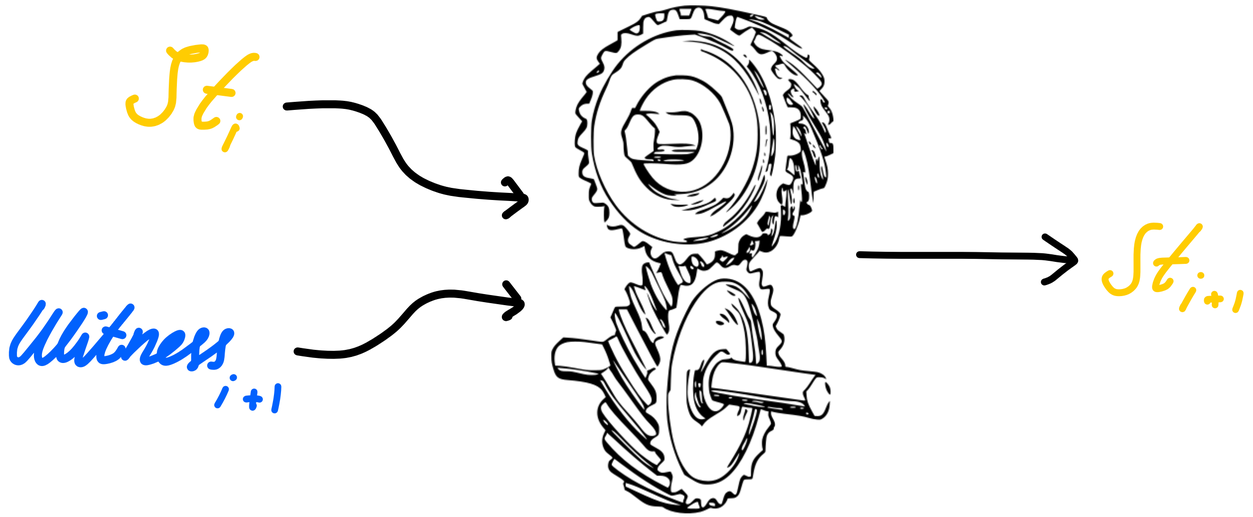

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- **Contrast with SNARKs in ROM:** Proving Incrementally is Harder.

Overview

Of All The Caveats

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can be Generated by Programming the RO

Intuition: Soundness Dependent on RO
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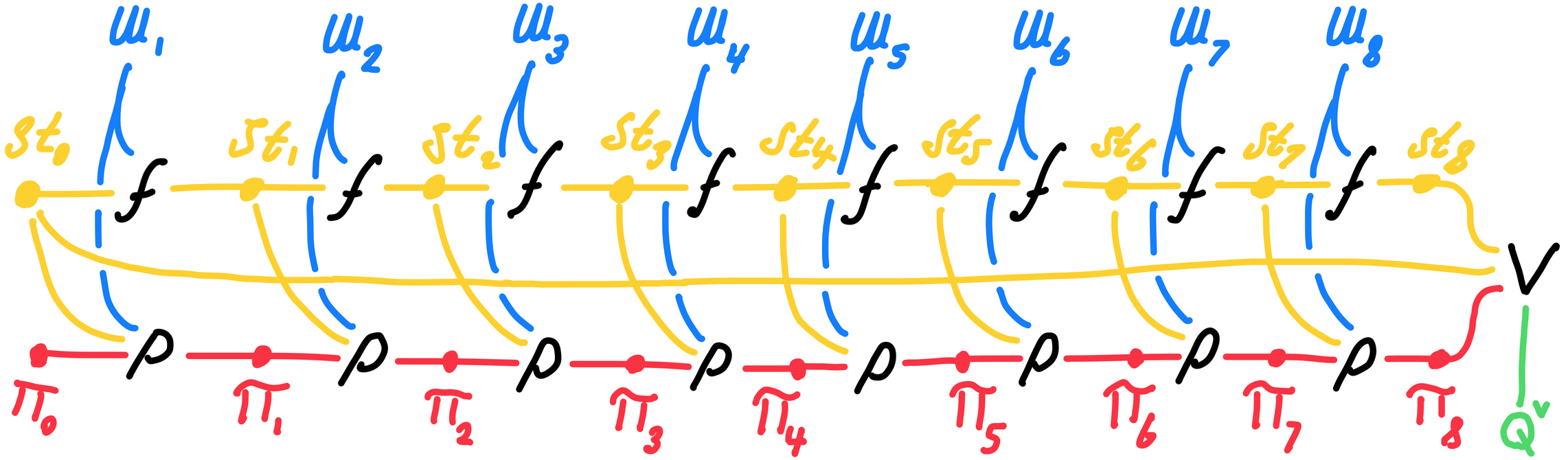
Note: Non-Deterministic IVC + CRH → SNARKs

Proof Sketch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w_2
w_3
w_4
w_5
w_6
w_7
w_8
st_0
st_1
st_2
st_3
st_4
st_5
st_6
st_7
st_8
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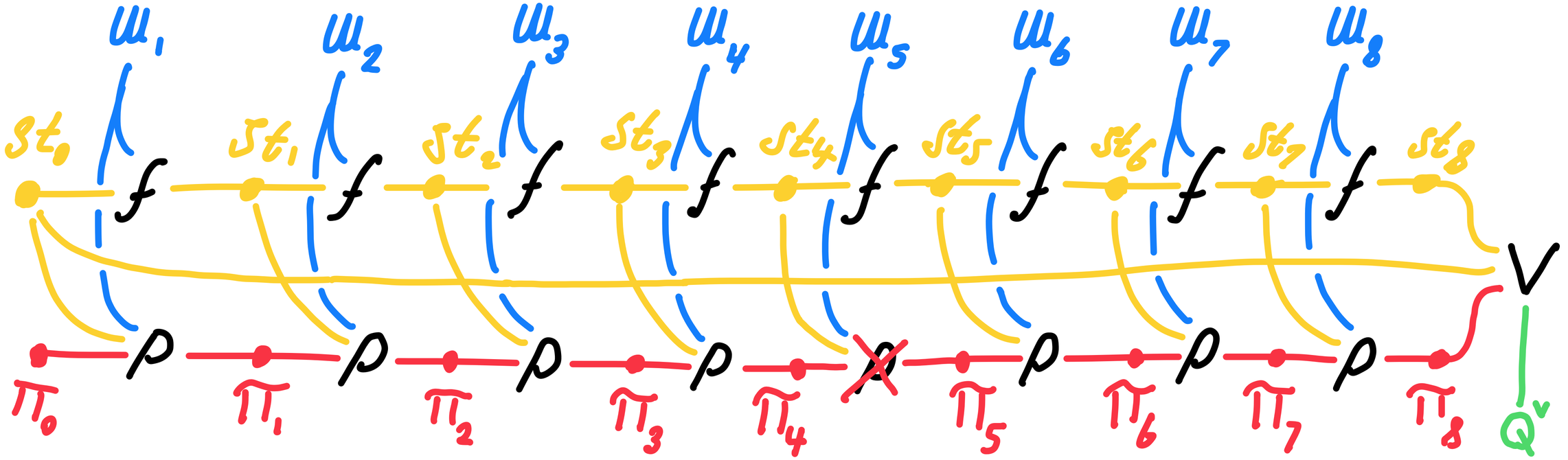
π_8
V
Q^V


w_1
w_2
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st_0
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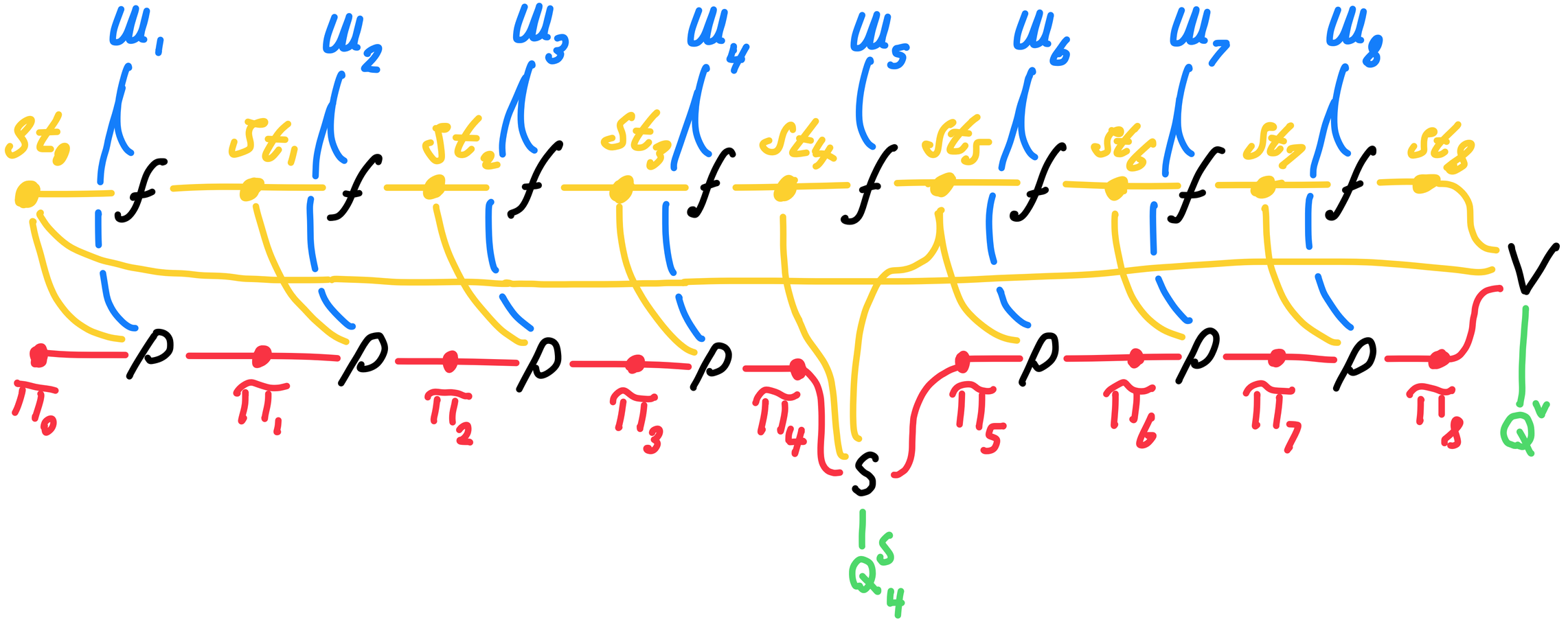
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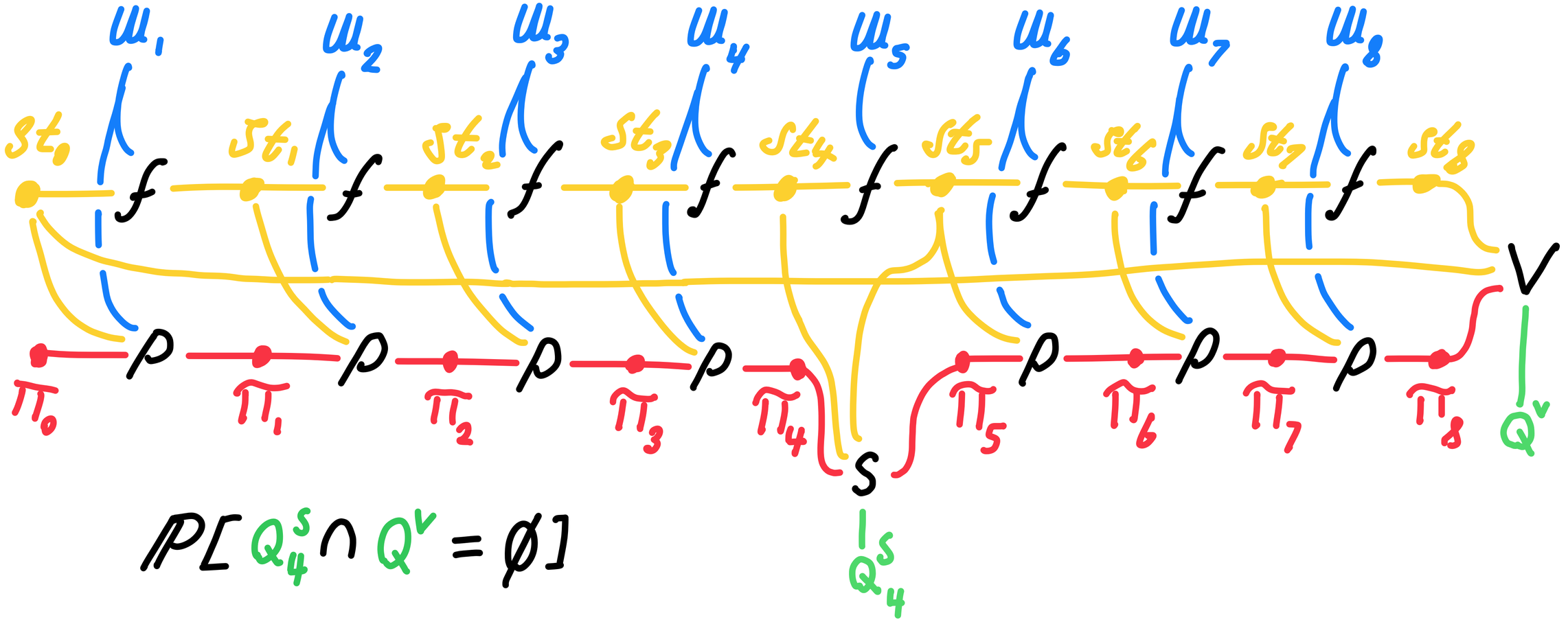
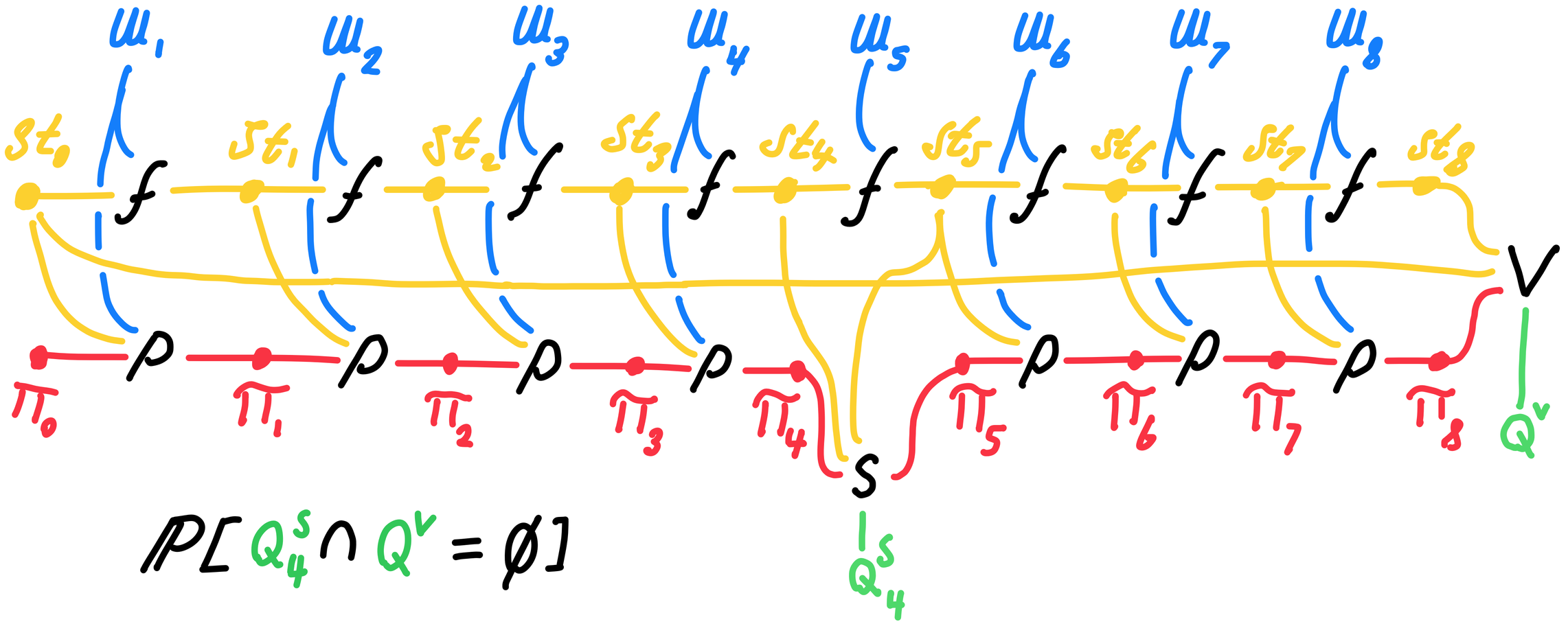


$$\mathbb{P}[\,Q_4^S \cap Q^V = \emptyset\,]$$



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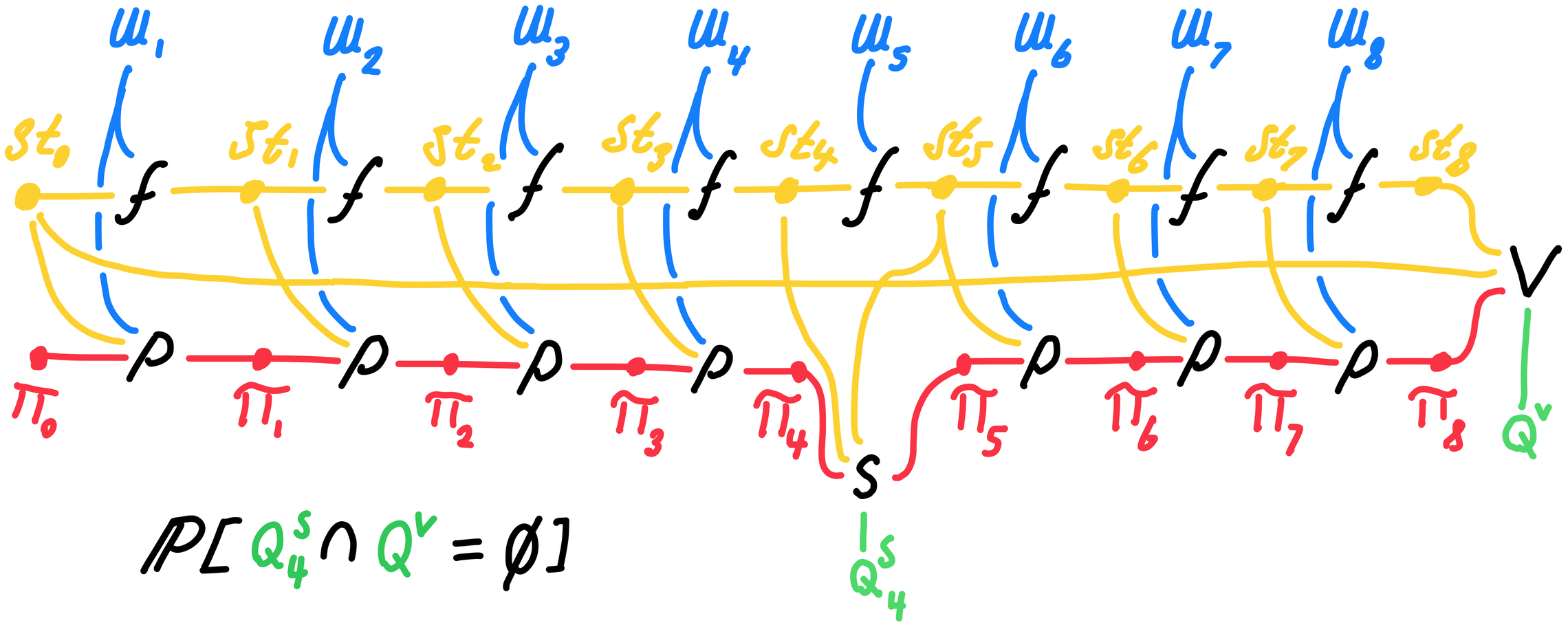

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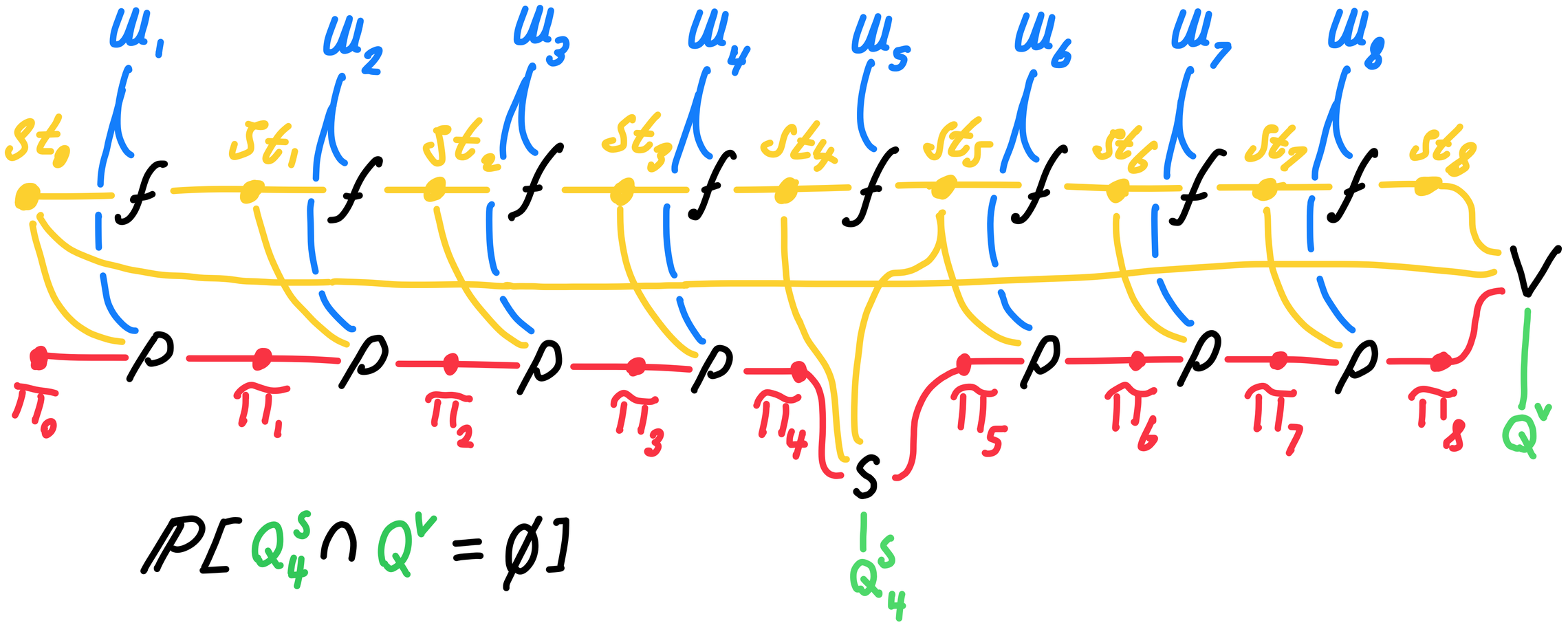

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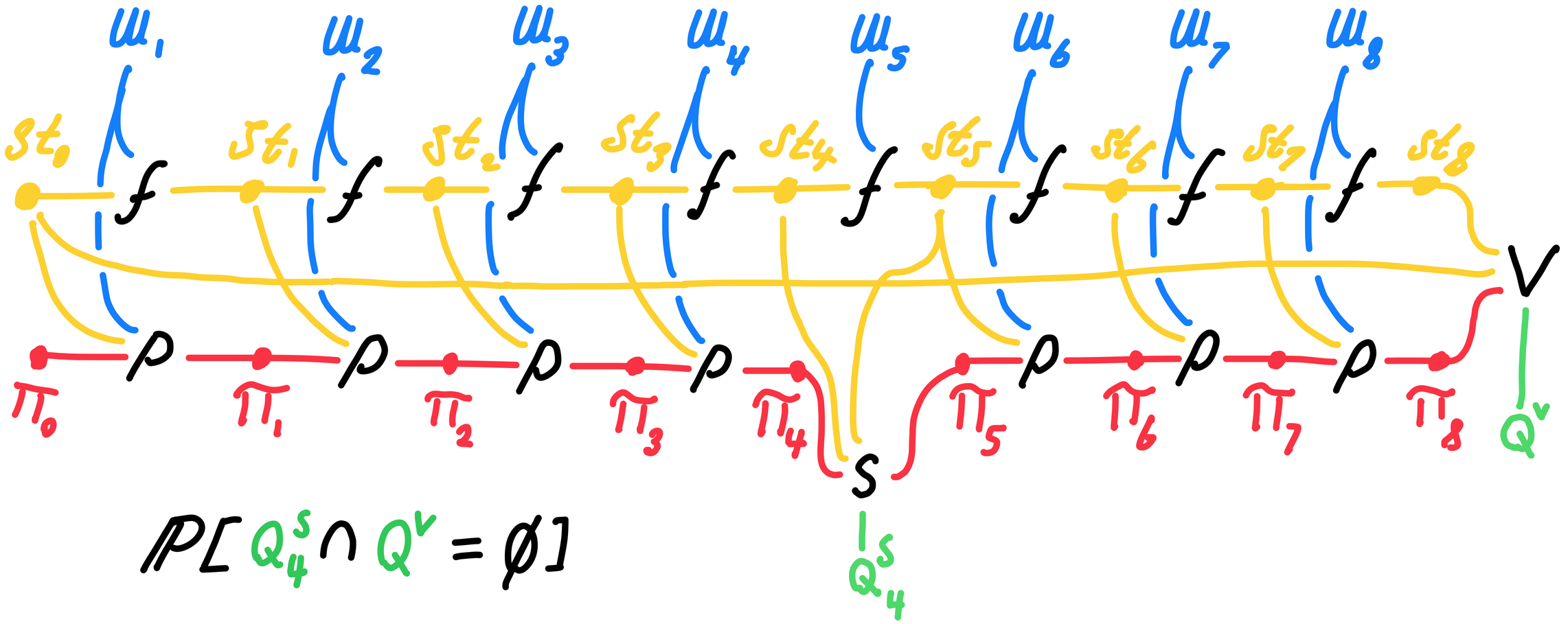

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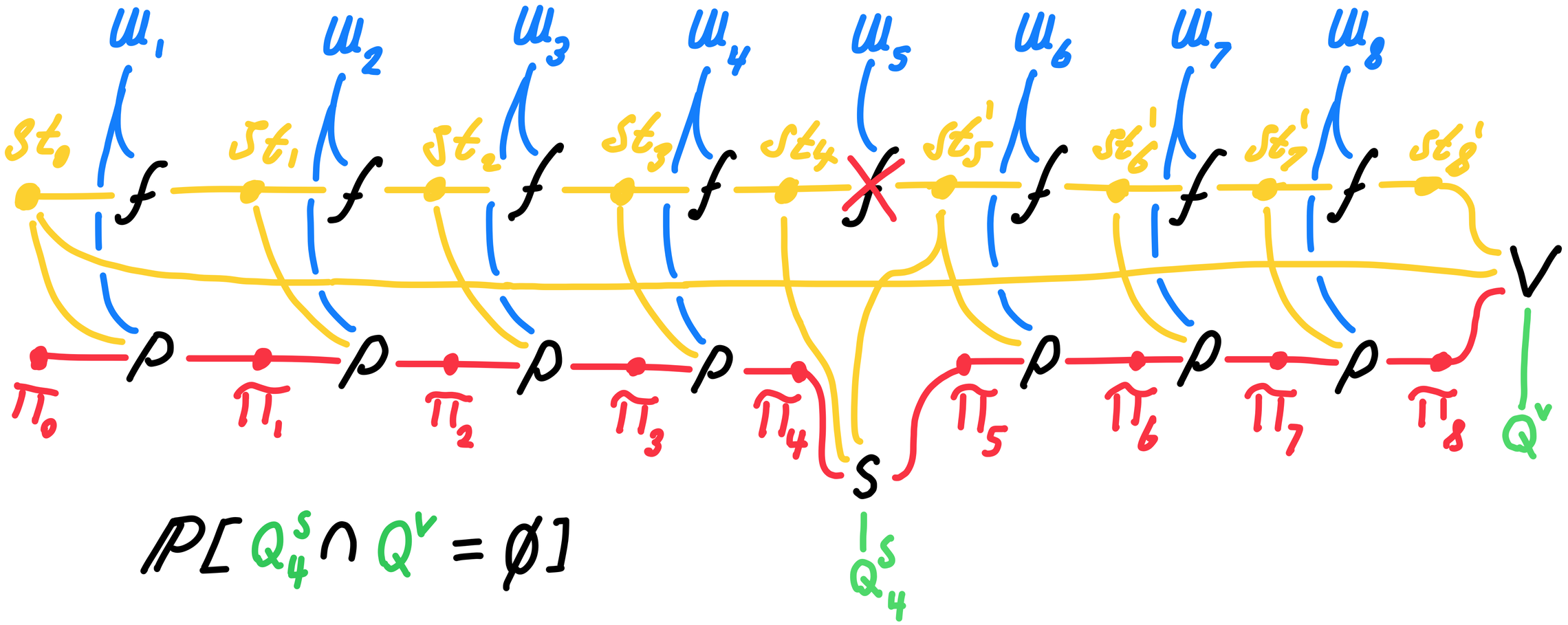

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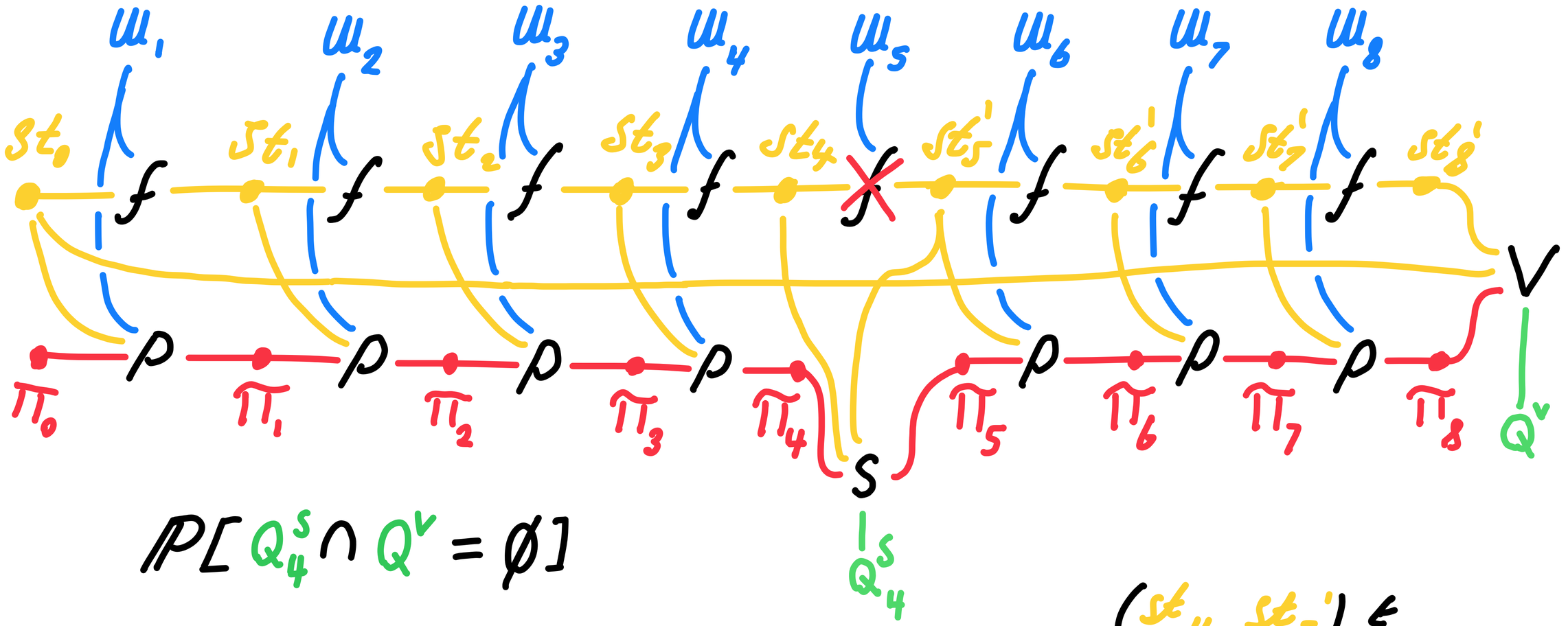

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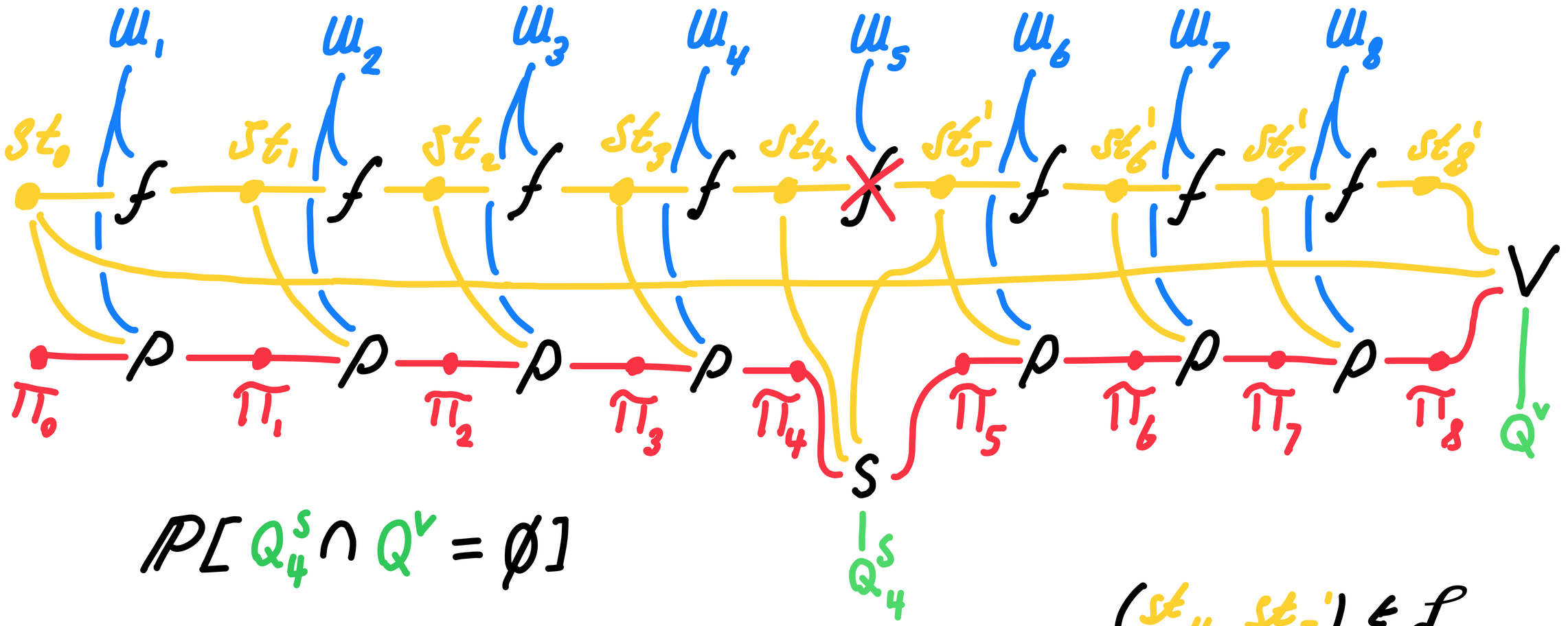

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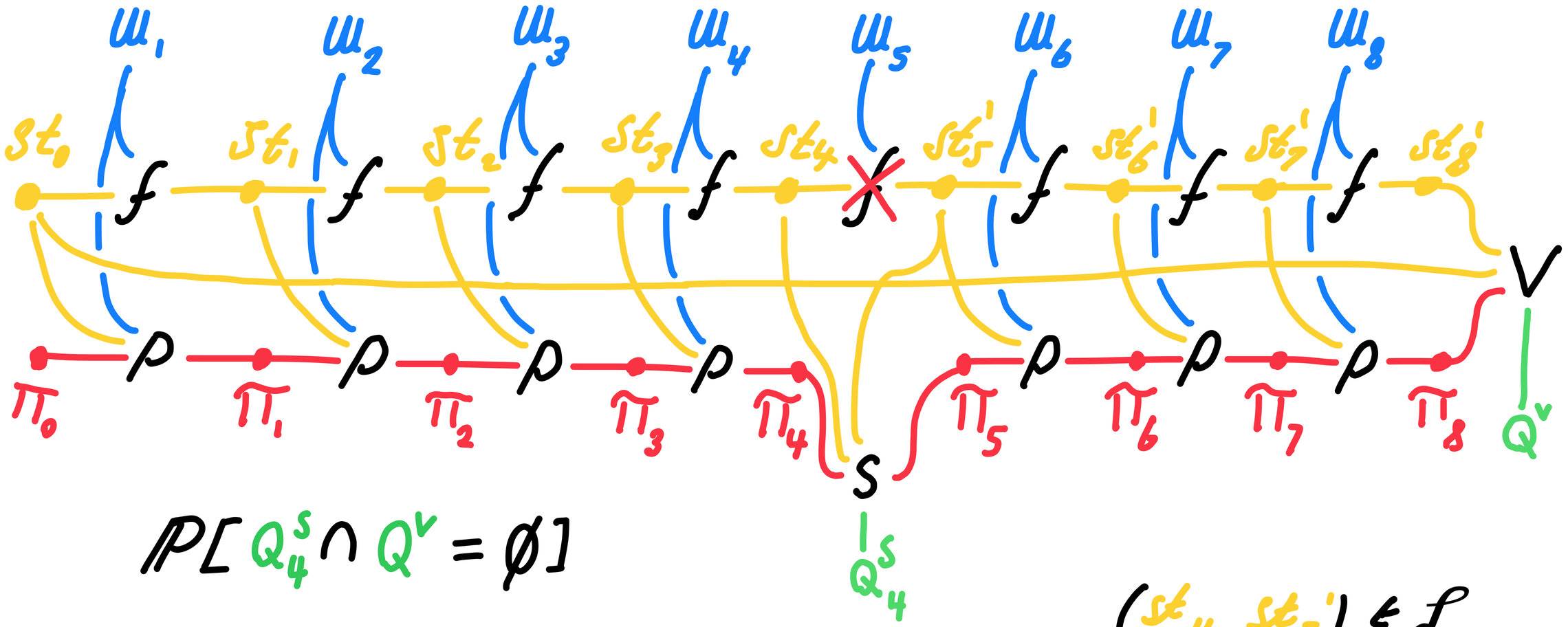

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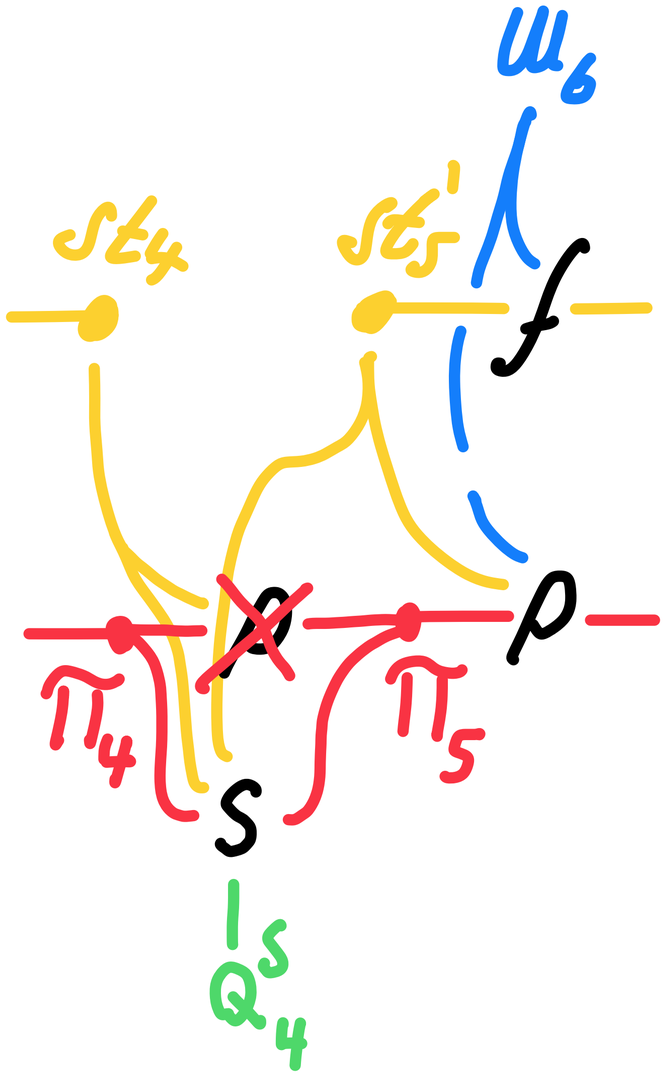

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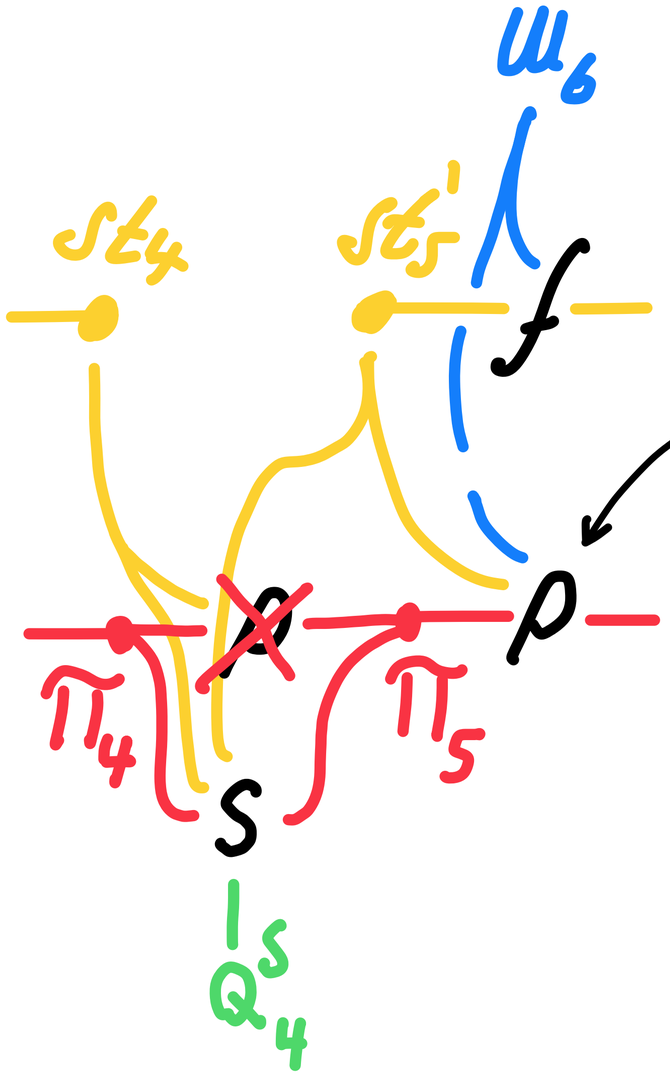

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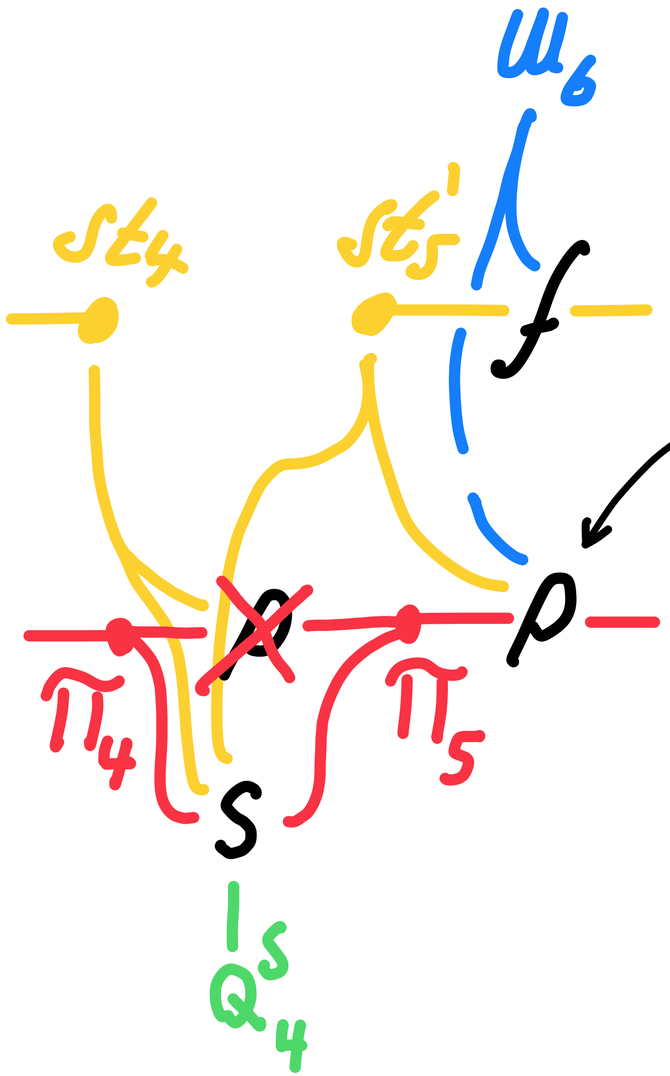

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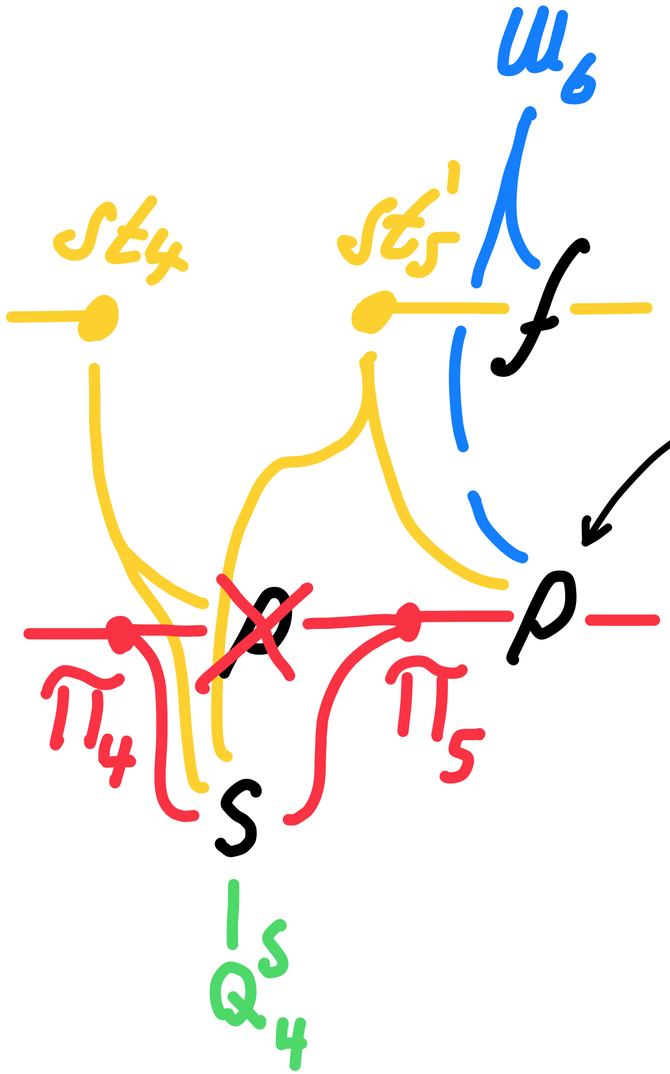

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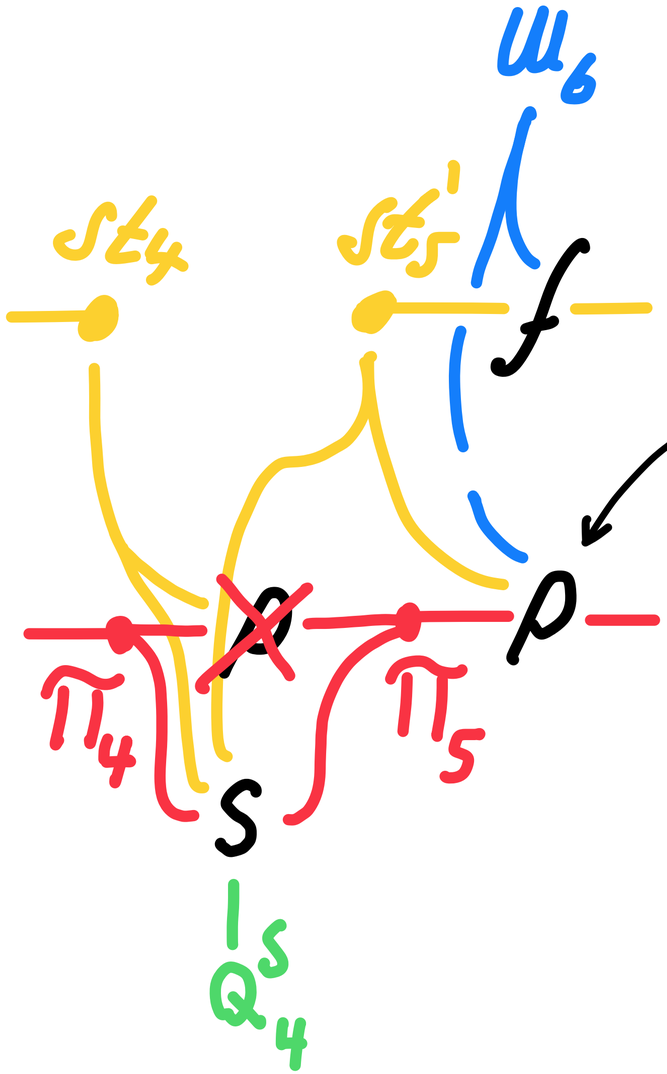

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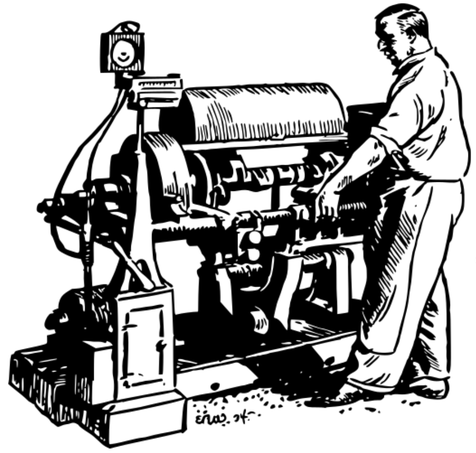

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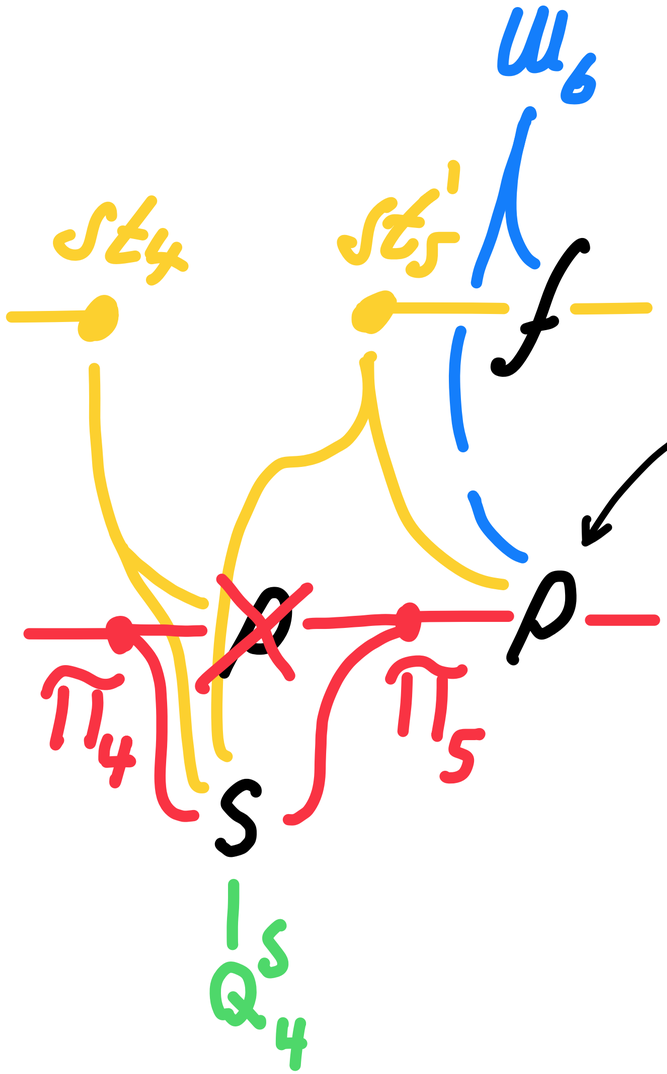

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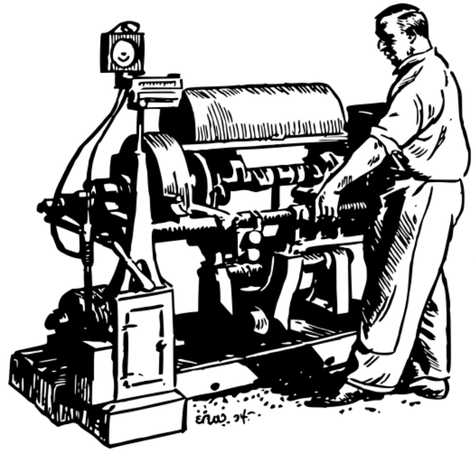

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Allowed to fail with arbitrary 1/p(n) Prob.

Questions?

"On Valiant's Conjecture"

Mathias Hall-Andersen
Jesper Buus Nielsen

Paintings By Bartholomeus Johannes van Hove