Exploiting Non-Full Key Additions: Full-Fledged Automatic Demirci-Selçuk Meet-in-the-Middle Cryptanalysis of SKINNY

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- Pull-fledged Framework with New Improvement Techniques
- 8 Results of SKINNY Block Cipher

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- Brief Description of Demirci-Selçuk Meet-in-the-Middle
- Previous Models Based on CP

2) Full-fledged Framework with New Improvement Techniques

3 Results of SKINNY Block Cipher

DS-MITM Attack

- Demirci-Selçuk MITM, FSE 2008 [DS08].
- Differential enumeration technique and key bridging technique, ASIACRYPT 2010 [DKS10].
- Improved differential enumeration technique, EUROCRYPT 2013 [DF13]
- Key dependent Sieve technique, FSE 2014 [LJW14]
- Tweak-difference cancellation technique, IET Inf. Secur 2019 [LJ19]
- Dedicated search algorithm, CRYPTO 2016 [DF16]
- Constraints programming based approach, Asiacrypt 2018 [SSD⁺18]

Introduction

DS-MITM Distinguisher

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DS-MITM Distinguisher

- $\mathcal{A} = [\mathbf{S}_0[j_0], \mathbf{S}_0[j_1], \dots, \mathbf{S}_0[j_s]]$ is a sequence of positions.
- δ(A): a set of messages that are all different in positions specified by A and all equal in other positions

 $\delta(\mathcal{A})$ -set: { $P^0, P^1, ..., P^{N-1}$ }



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- $\mathcal{B} = [\mathbf{S}_r[i_0], \dots, \mathbf{S}_r[i_t]]$ is a sequence of positions.

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- $\mathcal{B} = [\mathbf{S}_r[i_0], \dots, \mathbf{S}_r[i_t]]$ is a sequence of positions.
- ΔE_r(δ(A))[B]-sequence: an ordered sequence in positions specified by B of the associated δ(A)-set

 $\delta(\mathcal{A})$ -set: { P^0, P^1, \dots, P^{N-1} }



 $C^{0}[\mathcal{B}]\oplus C^{1}[\mathcal{B}]\parallel...\parallel C^{0}[\mathcal{B}]\oplus C^{N-1}[\mathcal{B}]$ $(\Delta \mathcal{E}_{r}(\delta(\mathcal{A}))[\mathcal{B}]\text{-sequence})$

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- Δ*E_{r₁}*(δ(*A*))[*B*] sequence is uniquely determined by several internal parameters.

 $\delta(\mathcal{A})$ -set: { P^0, P^1, \dots, P^{N-1} }



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 $C^{0}[\mathcal{B}] \oplus C^{1}[\mathcal{B}] \parallel \dots \parallel C^{0}[\mathcal{B}] \oplus C^{N-1}[\mathcal{B}]$ $(\Delta \mathcal{E}_{r}(\delta(\mathcal{A}))[\mathcal{B}]\text{-sequence})$

Attack Process

Precomputation phase.

• A lookup table is built to save all possible values of $\Delta \mathcal{E}_3(\delta(\mathcal{A}))[\mathcal{B}]$.

Online phase

- Guess round-keys involved in r_0 rounds to identify a $\delta(A)$ -set for the distinguisher.
- Guess round-keys involved in r_2 rounds to compute the output difference $\Delta \mathcal{E}_{r_1}(\delta(\mathcal{A}))[\mathcal{B}]$.
- Check whether the sequence in the lookup table, obtain the candidate of guessed round-keys involved in *r*₀, *r*₂ rounds that pass the test.



Basic Distinguisher on Toy Cipher

$$\bullet \ \mathbb{L} = [[0, 1, 1, 1], [1, 0, 1, 1], [1, 1, 0, 1]]$$

- $\mathcal{A} = [\mathbf{S}_0[3]], \mathcal{B} = [\mathbf{S}_3[1]]$
- $\Delta \mathcal{E}_3(\delta(\mathcal{A}))[\mathcal{B}] = P^0 \oplus P^1[\mathbf{S}_3[1]] \| \dots \| P^0 \oplus P^{255}[\mathbf{S}_3[1]]$ can be uniquely determined:

 $P^{0}[\mathbf{S}_{0}[3]], \{P^{0}[\mathbf{S}_{1}[j]] : j \in [0, 1, 2]\}, \{P^{0}[\mathbf{S}_{2}[j]] : j \in [0, 2, 3]\}.$

ΔE₃(δ(A))[B] can take at most 2^{7×8} possible values (2^{255×8} = 2²⁰⁴⁰ possibilities for a random 255-byte sequence)



Figure: A 3-round toy SPN block cipher

Improvement Techniques-Differential Enumeration Technique

Differential property of S-box

Given an input and output difference pair of $(\Delta_{in}, \Delta_{out})$ of an Sbox, the equation $\text{Sbox}(x) \oplus \text{Sbox}(x \oplus \Delta_{in}) = \Delta_{out}$ has one solution on average.

- Assume (P⁰, P') conforms to a truncated differential trail, P⁰ ∈ δ(A), many values of the internal parameters are not reached.
- $P^0[\mathbf{S}_0[3]], \{P^0[\mathbf{S}_1[j]] : j \in [0, 1, 2]\}, \{P^0[\mathbf{S}_2[j]] : j \in [0, 2, 3]\}$ can be determined by $P^0[\mathbf{S}_0[3]], \{P^0[\mathbf{S}_1[j]], j \in [0, 1, 2]\}, \{P^0 \oplus P'[\mathbf{S}_0^{\mathbb{SB}}[3]]\}, \{P^0 \oplus P'[\mathbf{S}_3[1]]\}.$



Figure: A truncated differential trail on toy cipher



Figure: A valid typeZ trail on toy cipher

Improvement Techniques

- Key-Dependent-Sieve Technique. Utilize the relations on round keys deduced from these internal parameters.
- **Tweak-Difference Cancellation Technique.** Utilize the tweak difference to cancel a difference in the state.
- **Key-Bridging Technique.** Utilize the dependent relations on keys involved in the key-recovery phase.

Modelling the Basic DS-MITM Distinguisher based on CP

Three types (typeX, typeY, typeZ) of 0-1 variables for each cell are introduced. $\mathbb{L} = [[0, 1, 1, 1], [1, 0, 1, 1], [1, 1, 0, 1]]$

- typeX variables (2) form a forward differential trail.
- Define $\overline{\mathcal{R}_i} = [\mathbf{S}_i[j] : \text{typeX-}\mathbf{S}_i[j] = 0]$. For each pair of (P, P') satisfying $P \oplus P'[j] = 0, \forall j \in \overline{\mathcal{R}_i}$, obtain $P \oplus P'[j] = 0, \forall j \in \overline{\mathcal{R}_{i+1}}$.
- Constraints over typeX variables follow the differential propagation rule with probability 1.



Figure: A valid forward differential trail on toy cipher

Modelling the Basic DS-MITM Distinguisher Based on CP

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- typeY variables (ID) form a backward determination trail.
- Define $\mathcal{B}_i = {\mathbf{S}_i[j] : \text{typeY-}\mathbf{S}_i[j] = 1}$. For any pair of (P, P'), each difference among ${P \oplus P'[j] : j \in \mathcal{B}_{i+1}}$ can be uniquely determined by ${P \oplus P'[j], P[j] : j \in \mathcal{B}_i}$.
- typeY-S_i[j] = 0 indicates that difference in each cell in B_{i+1} is independent of the knowledge of S_i[j].



Figure: A valid backward determination on toy cipher

Modelling the Basic DS-MITM Distinguisher based on CP

Three types (typeX, typeY, typeZ) of 0-1 variables for each cell are introduced. $\mathbb{L} = [[0, 1, 1, 1], [1, 0, 1, 1], [1, 1, 0, 1]]$

- typeZ-* (\boxtimes) equals 1 if and only if typeX-* = 1 (\square) and typeY-* = 1 (\square)
- $\Delta \mathcal{E}_{r_1}(\delta(\mathcal{A}))[\mathcal{B}]$ can be uniquely determined by the following internal parameters:

$$\{P^0[\mathbf{S}_i[j]] : type\mathbb{Z} \cdot \mathbf{S}_i[j] = 1, r_0 \le i \le r_0 + r_1 - 1, j \in [0, ..., n]\}.$$



Figure: A valid typeZ trail on toy cipher

Outline



Full-fledged Framework with New Improvement Techniques

- A High Level Overview
- Differential Enumeration Technique
- Key-Dependent-Sieve Technique
- Tweak-Difference Cancellation Technique



A High Level Overview

- Basic DS-MITM distinguisher. three types (typeX, typeY, typeZ) of 0-1 variables
- **Differential enumeration.** typeT variables describe the traditional truncated differential trail. typeGT variables describe the internal parameters whose values will be bounded by truncated differential trail. typeGZ variables describe the remaining internal parameters.
- **Key-dependent sieve.** typeGT and typeGZ variables are unified by typeV variables. typeK variables are introduced to describe the round-keys deduced from these internal parameters.
- **Tweak-difference cancellation.** typeX variables for each tweak cell and describe forward differential trail propagation for both tweak addition and tweak schedule.
- **Key-recovery phase** model the phase of obtaining a pair conforming to the truncated differential trail of the distinguisher. Impose variables to form *a backward differential trail* through the first *r*₀ rounds and *a forward differential trail* through the last *r*₂ rounds.

Three types (typeT (I), typeGT (I), typeGZ (I)) of 0-1 variables for each cell are introduced.

• typeT variables (I): truncated differential propagation

typeT-based backward determination trail

Define $\mathcal{G}_i = [\mathbf{S}_i[j] : \text{typeGT-}\mathbf{S}_i[j] = 1, j \in [0, ..., n-1]]$. (P, P') conforms to the truncated differential trail, obtain each difference in $\{P \oplus P'[j] : j \in \mathcal{G}_{i+1}\}$ can be uniquely determined by $\{P \oplus P'[j], P[j] : j \in \mathcal{G}_i\}$.



Figure: A valid backward determination on toy cipher



Figure: A typeT-based backward determination trail on toy cipher

Three types (typeT, typeGT, typeGZ) of 0-1 variables for each cell are introduced.

- typeT-based forward determination trail
- Each difference in $\{P \oplus P'[j] : j \in G_i\}$ can be uniquely determined by

 $\{\boldsymbol{P}\oplus\boldsymbol{P}'[j]:j\in\mathcal{G}_{i+1}\},\{\boldsymbol{P}[j]:j\in\mathcal{G}_i\}.$



Figure: A typeT-based forward determination trail on toy cipher

Three types (typeT, typeGT, typeGZ) of 0-1 variables for each cell are introduced.

Differential property of S-box

Given an input and output difference pair of $(\Delta_{in}, \Delta_{out})$ of an Sbox, the equation $\text{Sbox}(x) \oplus \text{Sbox}(x \oplus \Delta_{in}) = \Delta_{out}$ has one solution on average.

• Initialize typeGT variables in *R_M*-th round.

$$\mathsf{typeGT}\text{-}\mathbf{S}_{R_M}[j] = \begin{cases} 1, \text{ if } \mathsf{typeT}\text{-}\mathbf{S}_{R_M}[j] = 1 \text{ and } \mathsf{typeZ}\text{-}\mathbf{S}_{R_M}[j] = 1, \\ 0, \text{ otherwise.} \end{cases}$$



Three types (typeT, typeGT, typeGZ) of 0-1 variables for each cell are introduced.

 typeGZ variables are utilized to consider the remaining internal parameters except those covered by typeGT variables (typeGT-* = 1)

typeGZ-* =
$$\begin{cases} 1(\blacksquare) : \text{ if typeZ-*} = 1 \text{ and typeGT-*} = 0, \\ 0(\Box) : \text{ otherwise.} \end{cases}$$

Objective Function

 $\Delta \mathcal{E}_{r_1}(\delta(\mathcal{A}))[\mathcal{B}]$ -sequence can be uniquely determined by the following internal parameters:

$$\{P^{0} \oplus P'[\mathbf{S}_{r}[j]] : \text{typeGT-}\mathbf{S}_{r}[j] = 1, r \in \{r_{0}, r_{0} + r_{1}\}, j \in [0, ..., n - 1]\},\\ \{P^{0}[\mathbf{S}_{i}[j]] : \text{typeGT-}\mathbf{S}_{i}[j] = 1, r_{0} \le i \le r_{0} + r_{1} - 1, i \ne R_{M}, j \in [0, ..., n - 1]\},\\ \{P^{0}[\mathbf{S}_{i}[j]] : \text{typeGZ-}\mathbf{S}_{i}[j] = 1, r_{0} \le i \le r_{0} + r_{1} - 1, j \in [0, ..., n - 1]\}.$$

Modelling Key-Dependent-Sieve Technique

• Some round keys can be deduced from these internal parameters.

typeV-* =
$$\begin{cases} 1 : \text{if typeGZ-*} = 1 \text{ or typeGT-*} = 1, \\ 0 : \text{otherwise.} \end{cases}$$

• Assume
$$\mathbf{S}_{r+1} = \mathbb{L}(\mathbf{S}_r^{\mathbb{SB}}) \oplus RK_r$$
.

typeK-
$$RK_r[j] = \begin{cases} 1(\blacksquare) : \text{ if typeV-} \mathbf{S}_{r+1}[j] = 1, \text{typeV-} \mathbf{S}_r^{\mathbb{SB}}[j_i] = 1, \forall i, \\ 0(\Box) : \text{ otherwise.} \end{cases}$$

• The various relations for specified cipher can be included in the model dynamically.

Modelling the Tweak-Difference Cancellation Technique

- $\delta(\mathcal{A})$ -set: {(P^0, TW^0), ..., (P^N, TW^N)}.
- Tweak differences: cancel the state difference in one round.
- Constraints over typeX variables follows the forward differential propagation except for the round with tweak-difference cancellation.
- Assume tweak addition operation is expressed by $y = x \oplus rT$.

$$typeX-y = \begin{cases} 0 : typeX-x = typeX-rT = 0, \\ 1 : typeX-x \oplus typeX-rT = 1, \\ 0 \text{ or } 1 : typeX-x = typeX-rT = 1. \end{cases} \quad typeX-y = \begin{cases} 0 : typeX-x = typeX-rT = 0, \\ 1 : others. \end{cases}$$

Outline



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- 8 Results of SKINNY Block Cipher

Brief Description of SKINNY Block Cipher



- SubCells: Apply a non-linear substitution-box operation to each cell.
- AddConstants: update the state by XORing constants.
- AddRoundTweakey: update the state by XORing the first two rows with tweakey arrays.
- ShiftRows(SR): rotate *i*-th row to the right by *i* cells.
- MixColums: multiply each column by a binary matrix.

Non-full Key-addition Technique

- States between two consecutive rounds are not totally independent.
- Dependencies between the variables can be described by the rank of a matrix derived from the linear transformation.
- $(x_4, x_5, x_6, x_7) = \mathbb{L}(x_0 \oplus rk_0, x_1 \oplus rk_1, x_2, x_3).$
- For each subset $\{x_i : typeV-x_i = 1, i \in \{2, 3, ..., 7\}\}$, the degree of freedom β can be computed from the linear transformation.
- The reduced bytes are $\sum_{i=2}^{7}$ typeV- $x_i \beta$.

A Summary of the Results

Version	Approach	R_{attack}	Time	Data	Memory	CT	Ref.
SKINNY-128-128	ID	17	$2^{120.8}$	$2^{118.5}$	$2^{97.5}$		[23]
	ID	17	$2^{116.51}$	$2^{116.37}$	2^{80}	×	[15]
	DS-MITM	17	$2^{122.06}$	2^{96}	$2^{118.91}$		Sect. L, Fig. 35
SKINNY-128-256	ID	19	$2^{119.8}$	2^{62}	2^{110}		[23]
	ID	19	$2^{219.23}$	$2^{117.86}$	2^{208}		[15]
	DS-MITM	19	$2^{238.26}$	2^{96}	$2^{210.99}$	×	[14]
	DS-MITM	19	$2^{235.05}$	2^{96}	$2^{207.7}$		Sect. I, Fig. 29
	DS-MITM	20	$2^{254.28}$	2^{96}	$2^{250.99}$		Sect. H, Fig. 27
	DS-MITM	21	$2^{234.84}$	2^{96}	$2^{183.52}$		Sect. A. Fig. 13(8-bit tyeak)
	DS-MITM	21	$2^{234.99}$	2^{64}	$2^{231.86}$	1	Sect. C. Fig. 17(8-bit tweak)
	Int	22	2^{216}	$2^{113.58}$	2^{216}		[15]
SKINNY-128-384	TD	22	$2^{373.48}$	$2^{92.22}$	$2^{147.22}$		[22]
	ID	21	$2^{347.35}$	$2^{122.89}$	2^{336}		[15]
	MITM	23	2^{368}	2^{120}	2^{16}	×	[2]
	DS-MITM	22	$2^{366.28}$	2^{96}	$2^{370.99}$		[4]
	DS-MITM	23	2^{372}	2^{96}	$2^{352.46}$		Sect. G, Fig. 25
	DS-MITM	25	$2^{363.83}$	2^{96}	$2^{336.39}$	/	Sect. 5.2, Fig. 11(8-bit tweak)
	Int	26	2344	2^{121}	2^{340}	v	[15]
SKINNY-64-128	ID	18	2^{116}	2^{60}	2^{112}		[12]
	ID	19	$2^{119.8}$	2^{60}	2^{112}		[23]
	ID	19	$2^{110.34}$	$2^{60.86}$	2^{104}	×	[15]
	DS-MITM	18	$2^{126.32}$	2^{32}	$2^{61.91}$		[14]
	DS-MITM	19	$2^{123.43}$	2^{52}	$2^{126.95}$		Sect. N, Fig. 39
	DS-MITM	21	$2^{119.32}$	2^{60}	$2^{114.81}$		Sect. D, Fig. 19(8-bit tweak)
	ZC/Integral	20	$2^{97.5}$	$2^{68.4}$	2^{82}	~	[1]
	Int	22	2^{110}	$2^{57.58}$	2^{108}		[15]
SKINNY-64-192	ID	22	$2^{183.97}$	$2^{47.84}$	274.84		[22]
	ID	21	$2^{174.42}$	$2^{62.43}$	2^{168}		[15]
	MITM	23	2^{188}	2^{52}	2^{4}		[11]
	MITM	23	2^{188}	2^{28}	2^4	×	[2]
	MITM	23	2^{184}	2^{60}	2^{8}	^	[2]
	DS-MITM	21	$2^{186.63}$	2^{60}	$2^{133.99}$		[14]
	DS-MITM	21	$2^{180.01}$	2^{44}	$2^{191.55}$		Sect. K, Fig. 33
	DS-MITM	23	$2^{179.9}$	2^{32}	$2^{183.49}$		Sect. F, Fig. 23(8-bit tweak)
	DS-MITM	23	$2^{174.9}$	2^{56}	$2^{179.46}$	1	Sect. E, Fig. 21(16-bit tweak)
	ZC/Integral	23	2 ^{155.6}	2 ^{73.2}	2 ¹³⁸ 2 ¹⁷²		[1]
	int	26	2	Z.01	2		15

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Thanks for your attention.

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