# Exploiting Non-Full Key Additions: Full-Fledged Automatic Demirci-Selçuk Meet-in-the-Middle Cryptanalysis of SKINNY 

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## Outlines

(1) Introduction
(2) Full-fledged Framework with New Improvement Techniques
(3) Results of SKINNY Block Cipher

## Outline

(1) Introduction

- Brief Description of Demirci-Selçuk Meet-in-the-Middle
- Previous Models Based on CP
(2) Full-fledged Framework with New Improvement Techniques
(3) Results of SKINNY Block Cipher


## DS-MITM Attack

- Demirci-Selçuk MITM, FSE 2008 [DS08].
- Differential enumeration technique and key bridging technique, ASIACRYPT 2010 [DKS10].
- Improved differential enumeration technique, EUROCRYPT 2013 [DF13]
- Key dependent Sieve technique, FSE 2014 [LJW14]
- Tweak-difference cancellation technique, IET Inf. Secur 2019 [LJ19]
- Dedicated search algorithm, CRYPTO 2016 [DF16]
- Constraints programming based approach, Asiacrypt 2018 [SSD $\left.{ }^{+} 18\right]$


## DS-MITM Distinguisher

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- $\mathcal{A}=\left[\mathbf{S}_{0}\left[j_{0}\right], \mathbf{S}_{0}\left[j_{1}\right], \ldots, \mathbf{S}_{0}\left[j_{s}\right]\right]$ is a sequence of positions.



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\delta(\mathcal{A}) \text {-set: }\left\{P^{0}, P^{1}, \ldots, P^{N-1}\right\}
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- $\mathcal{B}=\left[\mathbf{S}_{r}\left[i_{0}\right], \ldots, \mathbf{S}_{r}\left[i_{t}\right]\right]$ is a sequence of positions.



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- $\Delta \mathcal{E}_{r}(\delta(\mathcal{A}))[\mathcal{B}]$-sequence: an ordered sequence in positions specified by $\mathcal{B}$ of the associated $\delta(\mathcal{A})$-set



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- $\Delta \mathcal{E}_{r_{1}}(\delta(\mathcal{A}))[\mathcal{B}]$ sequence is uniquely determined by several internal parameters.



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- $\Delta \mathcal{E}_{r_{1}}(\delta(\mathcal{A}))[\mathcal{B}]$ sequence is uniquely determined by several internal parameters.
- condition:



## Attack Process

- Precomputation phase.
- A lookup table is built to save all possible values of $\Delta \mathcal{E}_{3}(\delta(\mathcal{A}))[\mathcal{B}]$.
- Online phase
- Guess round-keys involved in $r_{0}$ rounds to identify a $\delta(A)$-set for the distinguisher.
- Guess round-keys involved in $r_{2}$ rounds to compute the output difference $\Delta \mathcal{E}_{r_{1}}(\delta(\mathcal{A}))[\mathcal{B}]$.
- Check whether the sequence in the lookup table, obtain the candidate of guessed round-keys involved in $r_{0}, r_{2}$ rounds that pass the test.



## Basic Distinguisher on Toy Cipher

- $\mathbb{L}=[[0,1,1,1],[1,0,1,1],[1,1,0,1]]$
- $\mathcal{A}=\left[\mathbf{S}_{0}[3]\right], \mathcal{B}=\left[\mathbf{S}_{3}[1]\right]$
- $\Delta \mathcal{E}_{3}(\delta(\mathcal{A}))[\mathcal{B}]=P^{0} \oplus P^{1}\left[\mathbf{S}_{3}[1]\right]\|\ldots\| P^{0} \oplus P^{255}\left[\mathbf{S}_{3}[1]\right]$ can be uniquely determined:

$$
P^{0}\left[\mathbf{S}_{0}[3]\right],\left\{P^{0}\left[\mathbf{S}_{1}[j]\right]: j \in[0,1,2]\right\},\left\{P^{0}\left[\mathbf{S}_{2}[j]\right]: j \in[0,2,3]\right\} .
$$

- $\Delta \mathcal{E}_{3}(\delta(\mathcal{A}))[\mathcal{B}]$ can take at most $2^{7 \times 8}$ possible values $\left(2^{255 \times 8}=2^{2040}\right.$ possibilities for a random 255-byte sequence)


Figure: A 3-round toy SPN block cipher

## Improvement Techniques-Differential Enumeration Technique

Differential property of S-box
Given an input and output difference pair of ( $\Delta_{\text {in }}, \Delta_{\text {out }}$ ) of an Sbox, the equation $\operatorname{Sbox}(x) \oplus \operatorname{Sbox}\left(x \oplus \Delta_{\text {in }}\right)=\Delta_{\text {out }}$ has one solution on average.

- Assume $\left(P^{0}, P^{\prime}\right)$ conforms to a truncated differential trail, $P^{0} \in \delta(\mathcal{A})$, many values of the internal parameters are not reached.
- $P^{0}\left[\mathbf{S}_{0}[3]\right],\left\{P^{0}\left[\mathbf{S}_{1}[j]\right]: j \in[0,1,2]\right\},\left\{P^{0}\left[\mathbf{S}_{2}[j]\right]: j \in[0,2,3]\right\}$ can be determined by $P^{0}\left[\mathbf{S}_{0}[3]\right],\left\{P^{0}\left[\mathbf{S}_{1}[j]\right], j \in[0,1,2]\right\},\left\{P^{0} \oplus P^{\prime}\left[\mathbf{S}_{0}^{\mathbb{S} \mathbb{B}}[3]\right]\right\},\left\{P^{0} \oplus P^{\prime}\left[\mathbf{S}_{3}[1]\right]\right\}$.


Figure: A truncated differential trail on toy cipher


Figure: A valid typeZ trail on toy cipher

## Improvement Techniques

- Key-Dependent-Sieve Technique. Utilize the relations on round keys deduced from these internal parameters.
- Tweak-Difference Cancellation Technique. Utilize the tweak difference to cancel a difference in the state.
- Key-Bridging Technique. Utilize the dependent relations on keys involved in the key-recovery phase.


## Modelling the Basic DS-MITM Distinguisher based on CP

Three types (typeX, typeY, typeZ) of $0-1$ variables for each cell are introduced. $\mathbb{L}=[[0,1,1,1],[1,0,1,1],[1,1,0,1]]$

- typeX variables ( $\boxtimes$ ) form a forward differential trail.
- Define $\overline{\mathcal{A}_{i}}=\left[\mathbf{S}_{i}[j]\right.$ : typeX- $\left.\mathbf{S}_{i}[j]=0\right]$. For each pair of $\left(P, P^{\prime}\right)$ satisfying $P \oplus P^{\prime}[j]=0, \forall j \in \overline{\mathcal{A}_{i}}$, obtain $P \oplus P^{\prime}[j]=0, \forall j \in \overline{\mathcal{F}_{i+1}}$.
- Constraints over typeX variables follow the differential propagation rule with probability 1.


Figure: A valid forward differential trail on toy cipher

## Modelling the Basic DS-MITM Distinguisher Based on CP

Three types (typeX, typeY, typeZ) of $0-1$ variables for each cell are introduced. $\mathbb{L}=[[0,1,1,1],[1,0,1,1],[1,1,0,1]]$

- type $Y$ variables $(\mathbb{\otimes})$ form a backward determination trail.
- Define $\mathcal{B}_{i}=\left\{\mathbf{S}_{i}[j]\right.$ : typeY- $\left.\mathbf{S}_{i}[j]=1\right\}$. For any pair of $\left(P, P^{\prime}\right)$, each difference among $\left\{P \oplus P^{\prime}[j]: j \in \mathcal{B}_{i+1}\right\}$ can be uniquely determined by $\left\{P \oplus P^{\prime}[j], P[j]: j \in \mathcal{B}_{i}\right\}$.
- typeY-S $\mathbf{S}_{i}[j]=0$ indicates that difference in each cell in $\mathcal{B}_{i+1}$ is independent of the knowledge of $\mathbf{S}_{i}[j]$.


Figure: A valid backward determination on toy cipher

## Modelling the Basic DS-MITM Distinguisher based on CP

Three types (typeX, typeY, typeZ) of $0-1$ variables for each cell are introduced. $\mathbb{L}=[[0,1,1,1],[1,0,1,1],[1,1,0,1]]$

- typeZ-* (\$) equals 1 if and only if typeX-* $=1$ (匹) and typeY-* $=1$ ( $\mathbb{\otimes}$ )
- $\Delta \mathcal{E}_{r_{1}}(\delta(\mathcal{A}))[\mathcal{B}]$ can be uniquely determined by the following internal parameters:

$$
\left\{P^{0}\left[\mathbf{S}_{i}[j]\right]: \text { typeZ-S } \mathbf{S}_{i}[j]=1, r_{0} \leq i \leq r_{0}+r_{1}-1, j \in[0, \ldots, n]\right\} .
$$



Figure: A valid typeZ trail on toy cipher

## Outline

(2) Full-fledged Framework with New Improvement Techniques

- A High Level Overview
- Differential Enumeration Technique
- Key-Dependent-Sieve Technique
- Tweak-Difference Cancellation Technique


## A High Level Overview

- Basic DS-MITM distinguisher. three types (typeX, typeY, typeZ) of 0-1 variables
- Differential enumeration. typeT variables describe the traditional truncated differential trail. typeGT variables describe the internal parameters whose values will be bounded by truncated differential trail. typeGZ variables describe the remaining internal parameters.
- Key-dependent sieve. typeGT and typeGZ variables are unified by typeV variables. typeK variables are introduced to describe the round-keys deduced from these internal parameters.
- Tweak-difference cancellation. typeX variables for each tweak cell and describe forward differential trail propagation for both tweak addition and tweak schedule.
- Key-recovery phase model the phase of obtaining a pair conforming to the truncated differential trail of the distinguisher. Impose variables to form a backward differential trail through the first $r_{0}$ rounds and a forward differential trail through the last $r_{2}$ rounds.


## Modelling the Differential Enumeration Technique

Three types (typeT ( $\square$ ), typeGT ( $\square$ ), typeGZ ( $\square$ )) of 0-1 variables for each cell are introduced.

- typeT variables (■): truncated differential propagation
typeT-based backward determination trail $\square$
Define $\mathcal{G}_{i}=\left[\mathbf{S}_{i}[j]\right.$ : typeGT- $\left.\mathbf{S}_{i}[j]=1, j \in[0, \ldots, n-1]\right]$. $\left(P, P^{\prime}\right)$ conforms to the truncated differential trail, obtain each difference in $\left\{P \oplus P^{\prime}[j]: j \in \mathcal{G}_{i+1}\right\}$ can be uniquely determined by $\left\{P \oplus P^{\prime}[j], P[j]: j \in \mathcal{G}_{i}\right\}$.


Figure: A valid backward determination on toy cipher


Figure: A typeT-based backward determination trail on toy cipher

## Modelling the Differential Enumeration Technique

Three types (typeT, typeGT, typeGZ) of 0-1 variables for each cell are introduced.

- typeT-based forward determination trail
- Each difference in $\left\{P \oplus P^{\prime}[j]: j \in \mathcal{G}_{i}\right\}$ can be uniquely determined by

$$
\left\{P \oplus P^{\prime}[j]: j \in \mathcal{G}_{i+1}\right\},\left\{P[j]: j \in \mathcal{G}_{i}\right\} .
$$



Figure: A typeT-based forward determination trail on toy cipher

## Modelling the Differential Enumeration Technique

Three types (typeT, typeGT, typeGZ) of 0-1 variables for each cell are introduced.
Differential property of S-box
Given an input and output difference pair of ( $\Delta_{\text {in }}, \Delta_{\text {out }}$ ) of an Sbox, the equation $\operatorname{Sbox}(x) \oplus \operatorname{Sbox}\left(x \oplus \Delta_{\text {in }}\right)=\Delta_{\text {out }}$ has one solution on average.

- Initialize typeGT variables in $R_{M}$-th round.

$$
\operatorname{typeGT}-\mathbf{S}_{R_{M}}[j]=\left\{\begin{array}{l}
1, \text { if typeT- } \mathbf{S}_{R_{M}}[j]=1 \text { and typeZ- } \mathbf{S}_{R_{M}}[j]=1, \\
0, \text { otherwise } .
\end{array}\right.
$$



## Modelling the Differential Enumeration Technique

Three types (typeT, typeGT, typeGZ) of 0-1 variables for each cell are introduced.

- typeGZ variables are utilized to consider the remaining internal parameters except those covered by typeGT variables (typeGT-* $=1$ )

$$
\text { typeGZ- } *=\left\{\begin{array}{l}
1(\square): \text { if typeZ- } *=1 \text { and typeGT- } *=0, \\
0(\square): \text { otherwise. }
\end{array}\right.
$$

Objective Function
$\Delta \mathcal{E}_{r_{1}}(\delta(\mathcal{A}))[\mathcal{B}]$-sequence can be uniquely determined by the following internal parameters:

$$
\begin{aligned}
& \left\{P^{0} \oplus P^{\prime}\left[\mathbf{S}_{r}[j]\right]: \text { typeGT- } \mathbf{S}_{r}[j]=1, r \in\left\{r_{0}, r_{0}+r_{1}\right\}, j \in[0, \ldots, n-1]\right\}, \\
& \left\{P^{0}\left[\mathbf{S}_{i}[j]\right]: \text { typeGT-S } \mathbf{S}_{i}[j]=1, r_{0} \leq i \leq r_{0}+r_{1}-1, i \neq R_{M}, j \in[0, \ldots, n-1]\right\}, \\
& \left\{P^{0}\left[\mathbf{S}_{i}[j]\right]: \text { typeGZ- } \mathbf{S}_{i}[j]=1, r_{0} \leq i \leq r_{0}+r_{1}-1, j \in[0, \ldots, n-1]\right\} .
\end{aligned}
$$

## Modelling Key-Dependent-Sieve Technique

- Some round keys can be deduced from these internal parameters.

$$
\text { typeV-* }=\left\{\begin{array}{l}
1: \text { if typeGZ- } *=1 \text { or typeGT }-*=1 \\
0: \text { otherwise } .
\end{array}\right.
$$

- Assume $\mathbf{S}_{r+1}=\mathbb{L}\left(\mathbf{S}_{r}^{\mathbb{S P}}\right) \oplus R K_{r}$.

$$
\text { typeK- } R K_{r}[j]=\left\{\begin{array}{l}
1(\square): \text { if typeV- } \mathbf{S}_{r+1}[j]=1, \text { typeV- } \mathbf{S}_{r}^{\mathbb{S B}}[j i]=1, \forall i, \\
0(\square): \text { otherwise. }
\end{array}\right.
$$

- The various relations for specified cipher can be included in the model dynamically.


## Modelling the Tweak-Difference Cancellation Technique

- $\delta(\mathcal{A})$-set: $\left\{\left(P^{0}, T W^{0}\right), \ldots,\left(P^{N}, T W^{N}\right)\right\}$.
- Tweak differences: cancel the state difference in one round.
- Constraints over typeX variables follows the forward differential propagation except for the round with tweak-difference cancellation.
- Assume tweak addition operation is expressed by $y=x \oplus r T$.
typeX- $y=\left\{\begin{array}{l}0: \text { typeX- } x=\text { typeX- } r T=0, \\ 1: \text { typeX- } x \oplus \text { typeX-rT }=1, \\ 0 \text { or } 1: \text { typeX- } x=\text { typeX-r } T=1 .\end{array}\right.$

$$
\text { typeX }-y=\left\{\begin{array}{l}
0: \text { typeX }-x=\text { typeX }-r T=0 \\
1: \text { others }
\end{array}\right.
$$

## Outline

(1) Introduction<br>2 Full-fledged Framework with New Improvement Techniques

(3) Results of SKINNY Block Cipher

## Brief Description of SKINNY Block Cipher



- SubCells: Apply a non-linear substitution-box operation to each cell.
- AddConstants: update the state by XORing constants.
- AddRoundTweakey: update the state by XORing the first two rows with tweakey arrays.
- ShiftRows(SR): rotate $i$-th row to the right by $i$ cells.
- MixColums: multiply each column by a binary matrix.


## Non-full Key-addition Technique

- States between two consecutive rounds are not totally independent.
- Dependencies between the variables can be described by the rank of a matrix derived from the linear transformation.
- $\left(x_{4}, x_{5}, x_{6}, x_{7}\right)=\mathbb{L}\left(x_{0} \oplus r k_{0}, x_{1} \oplus r k_{1}, x_{2}, x_{3}\right)$.
- For each subset $\left\{x_{i}: \operatorname{typeV}-x_{i}=1, i \in\{2,3, \ldots, 7\}\right\}$, the degree of freedom $\beta$ can be computed from the linear transformation.
- The reduced bytes are $\sum_{i=2}^{7}$ typeV- $x_{i}-\beta$.


## A Summary of the Results



## Thanks for your attention.

## Reference I

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