

# **Non-interactive Blind Signatures for Random Messages**

*Lucjan Hanzlik*

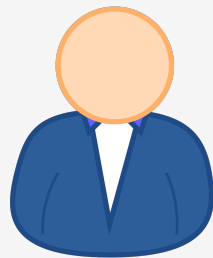
Eurocrypt 2023





# Two-move Blind Signatures

**User/Recipient**



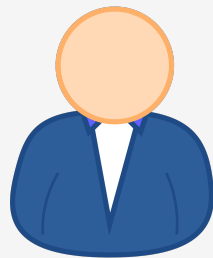
**Signer**





# Two-move Blind Signatures

**User/Recipient**



$(req, St) \leftarrow \text{Request}(m, pk)$

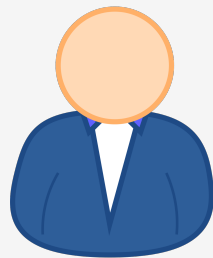
**Signer**





# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req



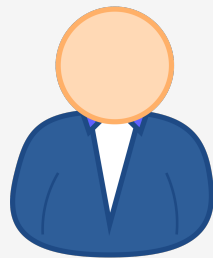
Signer





# Two-move Blind Signatures

**User/Recipient**



$(req, St) \leftarrow \text{Request}(m, pk)$

req



**Signer**

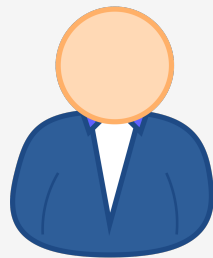


$pre \leftarrow \text{Issue}(req, sk)$



# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req

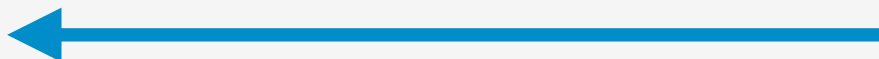


Signer



$pre \leftarrow \text{Issue}(req, sk)$

pre





# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req



Signer



$pre \leftarrow \text{Issue}(req, sk)$

pre

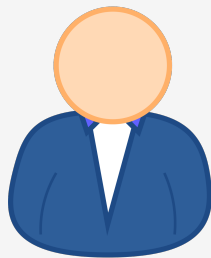


$sig \leftarrow \text{Obtain}(pre, St, pk)$



# Two-move Blind Signatures

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req

Signer

Unforgeability

$pre \leftarrow \text{Issue}(req, sk)$

pre

$sig \leftarrow \text{Obtain}(pre, St, pk)$





# Two-move Blind Signatures

User/Recipient



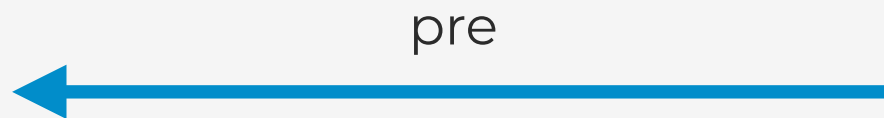
$(req, St) \leftarrow \text{Request}(m, pk)$



Signer



$pre \leftarrow \text{Issue}(req, sk)$



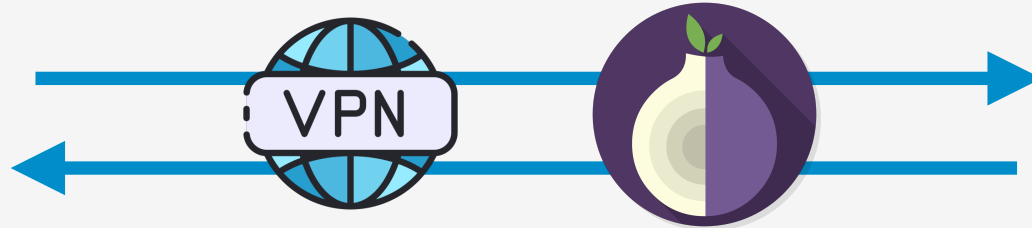
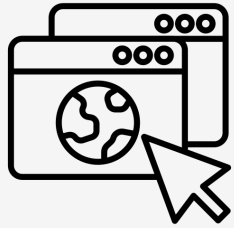
$sig \leftarrow \text{Obtain}(pre, St, pk)$



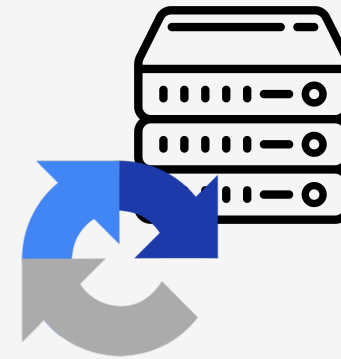
# Privacy Pass



Browser



WebServer



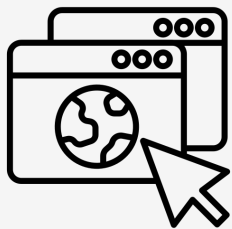
reCAPTCHA



# Privacy Pass



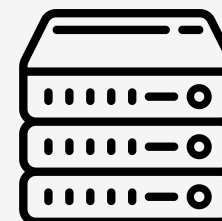
Browser



Issuer



WebServer

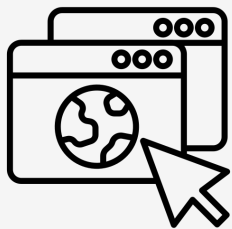




# Privacy Pass

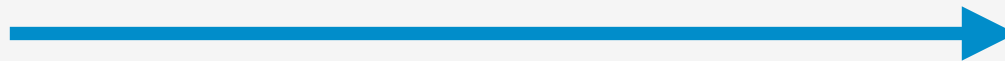


Browser

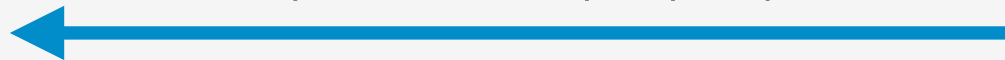


$\text{sig} \leftarrow \text{Obtain}(\text{pre}, \text{St}, \text{pk})$

$(\text{req}, \text{St}) \leftarrow \text{Request}(m, \text{pk})$



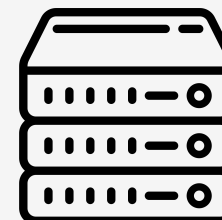
$\text{pre} \leftarrow \text{Issue}(\text{req}, \text{sk})$



Issuer



WebServer

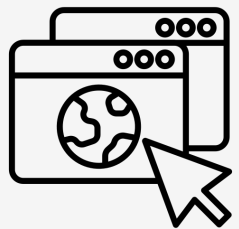




# Privacy Pass



Browser



$(req, St) \leftarrow \text{Request}(m, pk)$

Issuer



$pre \leftarrow \text{Issue}(req, sk)$

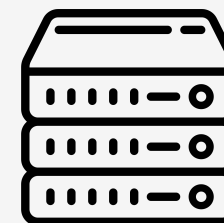
$sig \leftarrow \text{Obtain}(pre, St, pk)$

$(m, sig)$

WebServer

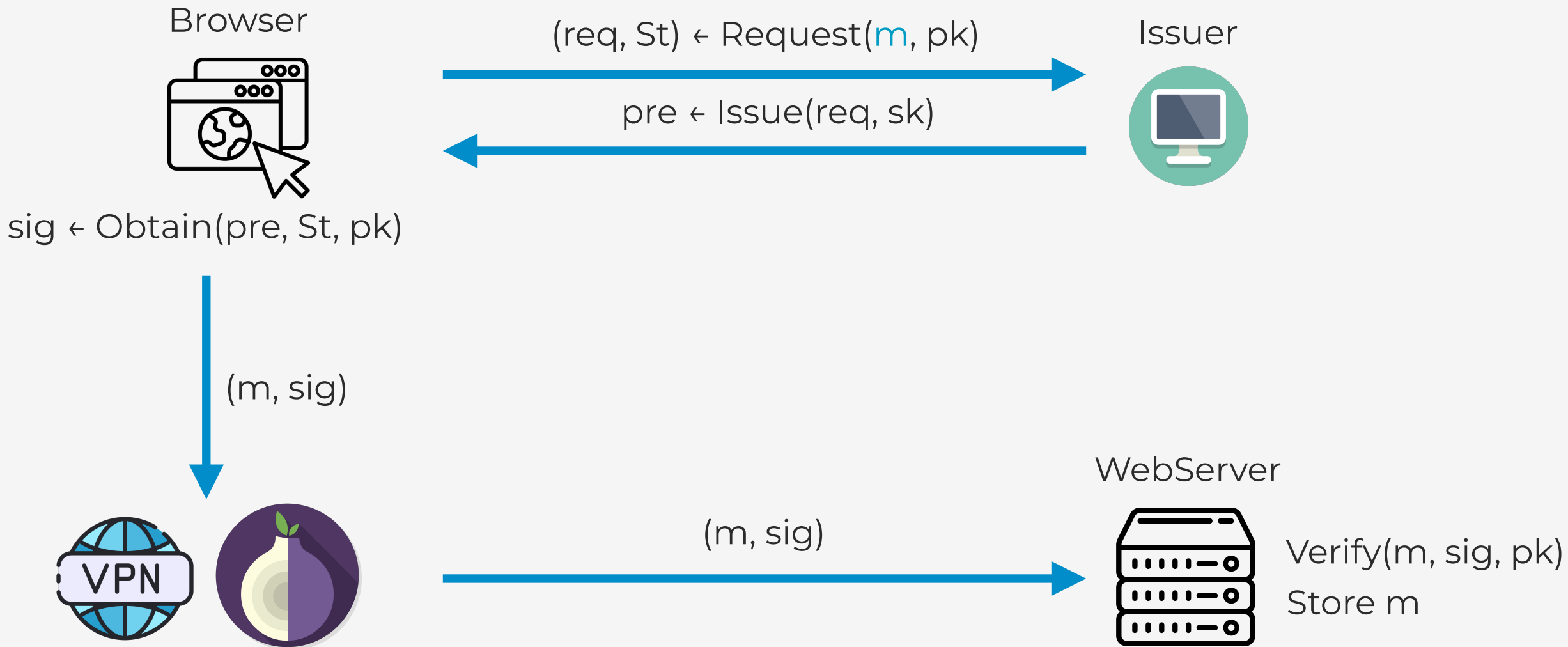


$(m, sig)$





# Privacy Pass

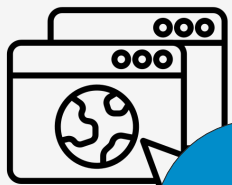




# Privacy Pass

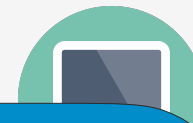


Browser



$(req, St) \leftarrow \text{Request}(m, pk)$

Issuer



$pre \leftarrow \text{Issue}(req, sk)$

$sig \leftarrow \text{Obtain}(pre)$

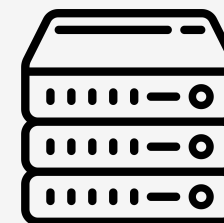
**Message is a random strings.**  
**Can we use this?**

$(m, sig)$

webServer



$(m, sig)$

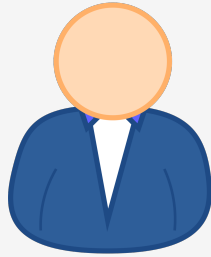


$\text{Verify}(m, sig, pk)$   
Store  $m$



# Blind Signatures for Random Messages

User/Recipient



$(req, St) \leftarrow \text{Request}(m, pk)$

req

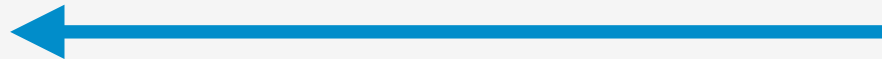


Signer



$pre \leftarrow \text{Issue}(req, sk)$

pre



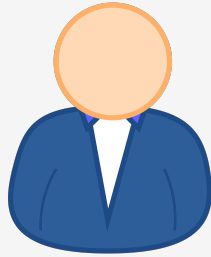
$(m, sig) \leftarrow \text{Obtain}(pre, St, pk)$





# Blind Signatures for Random Messages

User/Recipient



$(req, St) \leftarrow \text{Request}$

Signer



$e \leftarrow \text{Issue}(req, sk)$

**Not really interesting.**

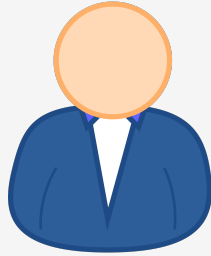
**Efficient two-move BS exist  
and provide more features.**

$(m, sig) \leftarrow \text{Obtain}(pre, St, pk)$



# Non-interactive Blind Signatures for Random Messages

User/Recipient ( $sk_r, pk_r$ )



Signer



$pre, nonce$

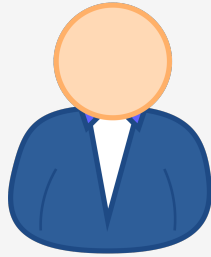
$pre \leftarrow \text{Issue}(sk, pk_r, nonce)$

$(m, sig) \leftarrow \text{Obtain}(sk_r, pk, pre)$



# Non-interactive Blind Signatures for Random Messages

User/Recipient ( $sk_r, pk_r$ )



Signer



$pre, nonce$

$pre \leftarrow \text{Issue}(sk, pk_r, nonce)$



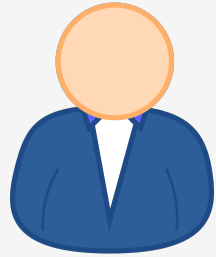
$(m, sig) \leftarrow \text{Obtain}(sk_r, pk, pre)$

**$m = f(sk_r, nonce)$**



# NIBS without PKI

**User/Recipient**



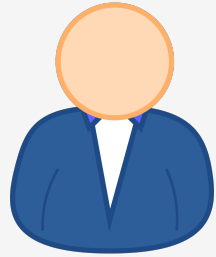
**Signer**





# NIBS without PKI

**User/Recipient**



$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

**Signer**





# NIBS without PKI

User/Recipient

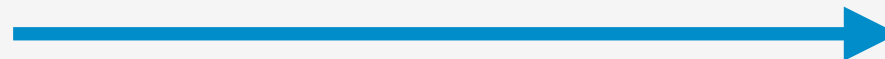


Signer



$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

pkr





# NIBS without PKI

User/Recipient



$(sk_r, pk_r) \leftarrow RKeyGen(secpar)$

Signer



$pk_r$

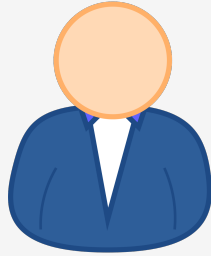
$pre, nonce$

$pre \leftarrow Issue(sk_s, pk_r, nonce)$



# NIBS without PKI

User/Recipient

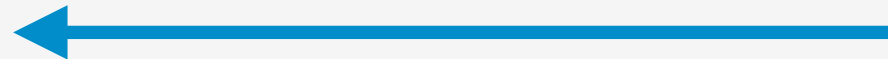


$(\text{skr}, \text{pkr}) \leftarrow \text{RKeyGen}(\text{secpar})$

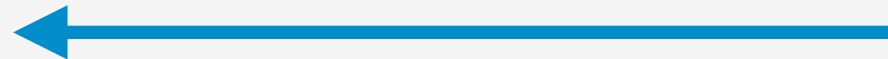
$\text{pkr}$



$\text{pre}, \text{nonce}$



$\text{pre}_2, \text{nonce}_2$



Signer



$\text{pre} \leftarrow \text{Issue}(\text{sk}, \text{pkr}, \text{nonce})$

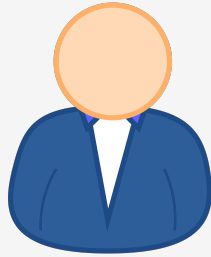
$\text{pre}_2 \leftarrow \text{Issue}(\text{sk}, \text{pkr}, \text{nonce}_2)$





# NIBS without PKI

User/Recipient



$(sk_r, pk_r) \leftarrow RKeyGen(s)$

Signer



**Can be used to batch and post issue tokens/e-cash.**

pre, nonce



$pre \leftarrow Issue(sk, pk_r, nonce)$

pre<sub>2</sub>, nonce<sub>2</sub>



$pre_2 \leftarrow Issue(sk, pk_r, nonce_2)$



# Unforgeability for NIBS

## Experiment

Adversary can make up to  $k$  queries to a presignature oracle.

Oracle outputs presignatures given nonce and recipients public key.

## Winning conditions

Adversary outputs at least  $k+1$  valid message-signature pairs.

Messages are distinct.



# Blindness for NIBS

## Recipient Blindness

Signatures obtained by different recipient are unlinkable.

Preserves the privacy across recipients.

Recipient Blindness

## Nonce Blindness

Signatures for the same recipient are unlinkable.

Allows to issue multiple presignatures without breaking blindness.

Nonce Blindness



# Building Blocks

## Signatures on Equivalence Classes



# Building Blocks

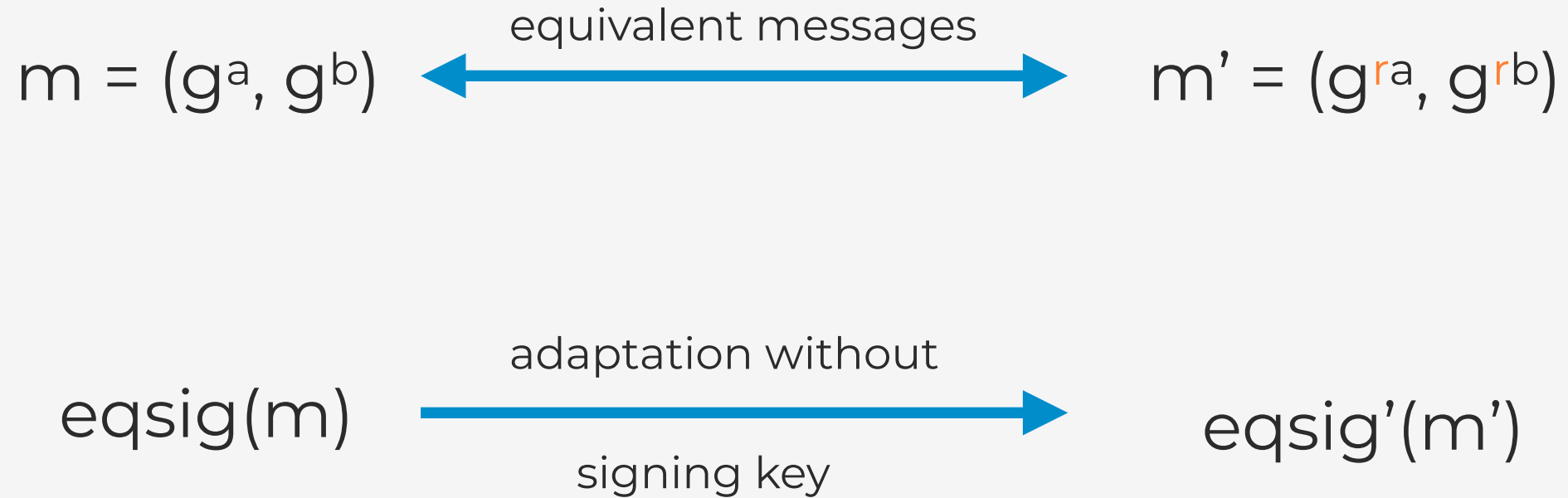
## Signatures on Equivalence Classes

$$m = (g^a, g^b) \quad \overset{\text{equivalent messages}}{\longleftrightarrow} \quad m' = (g^{ra}, g^{rb})$$



# Building Blocks

## Signatures on Equivalence Classes





# Building Blocks

## Signatures on Equivalence Classes

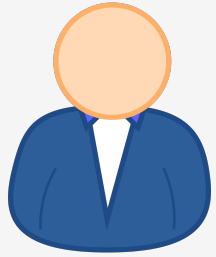
$$m = (g^a, g^b) \xleftrightarrow{\text{equivalent messages}} m' = (g^{ra}, g^{rb})$$

$$\text{eqsig}(m) \xrightarrow[\text{signing key}]{\text{adaptation without}} \text{eqsig}'(m')$$

**Random  
signature even if  
signer malicious**



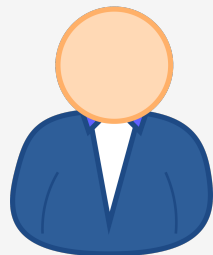
# How to efficiently construct NIBS?







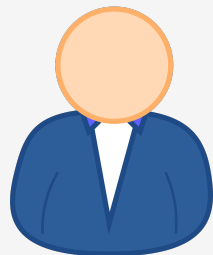
# How to efficiently construct NIBS?



$pre := eqsig( pkr, H(\text{nonce}) )$



# How to efficiently construct NIBS?



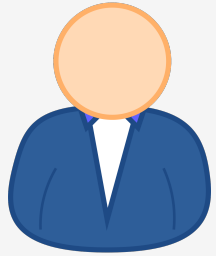
$$m := H(\text{nonce})^{skr^{-1}}$$



$$\text{pre} := \text{eqsig}(\text{pkr}, H(\text{nonce}))$$



# How to efficiently construct NIBS?



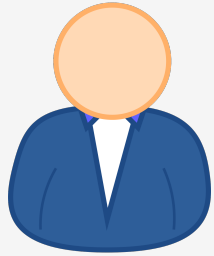
$$m := H(\text{nonce})^{s_{kr^{-1}}}$$
$$\text{sig} := \text{adapt}(\text{pre}, s_{kr^{-1}}) = \text{eqsig}(g, H(\text{nonce})^{s_{kr^{-1}}})$$



$$\text{pre} := \text{eqsig}(\text{pkr}, H(\text{nonce}))$$



# How to efficiently construct NIBS?



$$m := H(\text{nonce})^{\text{skr}^{-1}}$$

$$\text{sig} := \text{adapt}(\text{pre}, \text{skr}^{-1}) = \text{eqsig}(g, H(\text{nonce})^{\text{skr}^{-1}})$$



$$\text{pre} := \text{eqsig}(\text{pkr}, H(\text{nonce}))$$

**Recipients key is a  
standard DH key!**



# Security

- Unforgeability from signatures on equivalence signatures
- $H(\text{nonce})^{skr^{-1}}$  is a PRF for the recipient's key
- Blindness from inverse/strong DDH



# Summary and Open Problems

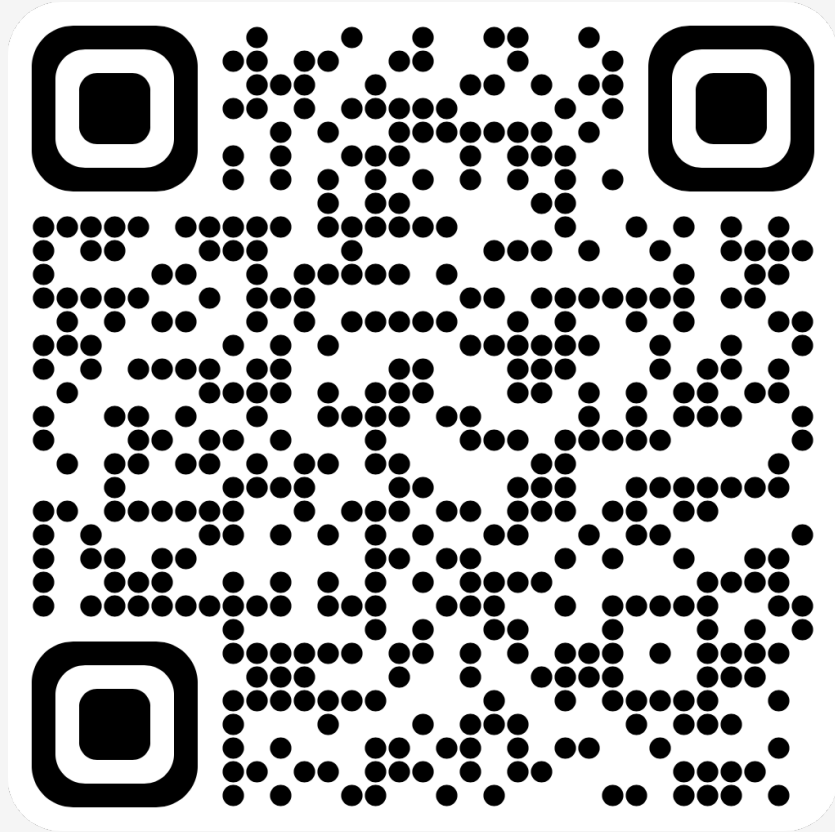
- NIBS and tag-based NIBS definitions
- Efficient constructions that works with standard PKI keys
- Generic construction from VRF and NIZK

## Can we construct:

- Post-quantum NIBS/TNIBS
- NIBS/TNIBS without ROM
- NIBS/TNIBS without pairings

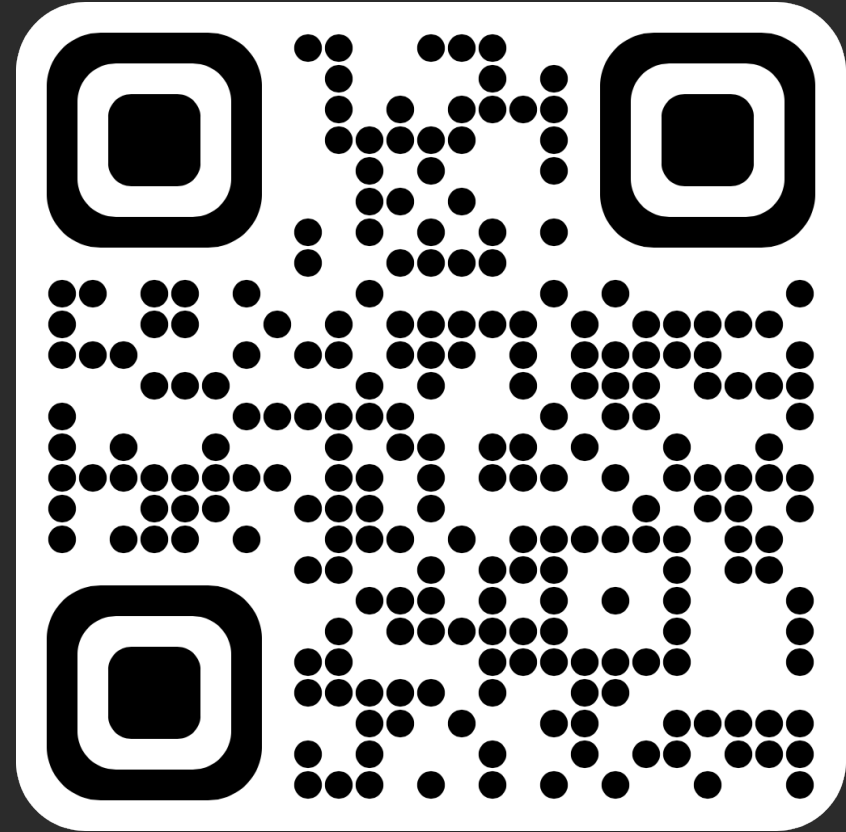


# Contact



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Eprint

[eprint.iacr.org/2023/388](https://eprint.iacr.org/2023/388)



# Unforgeability for NIBS

**Adversary**



**Challenger**





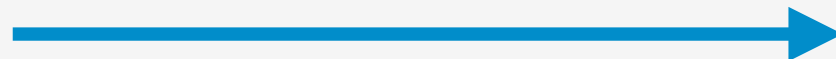


# Unforgeability for NIBS

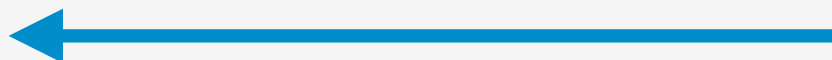
Adversary



$pk_{r_1}, nonce_1$



$pre_1$



Challenger



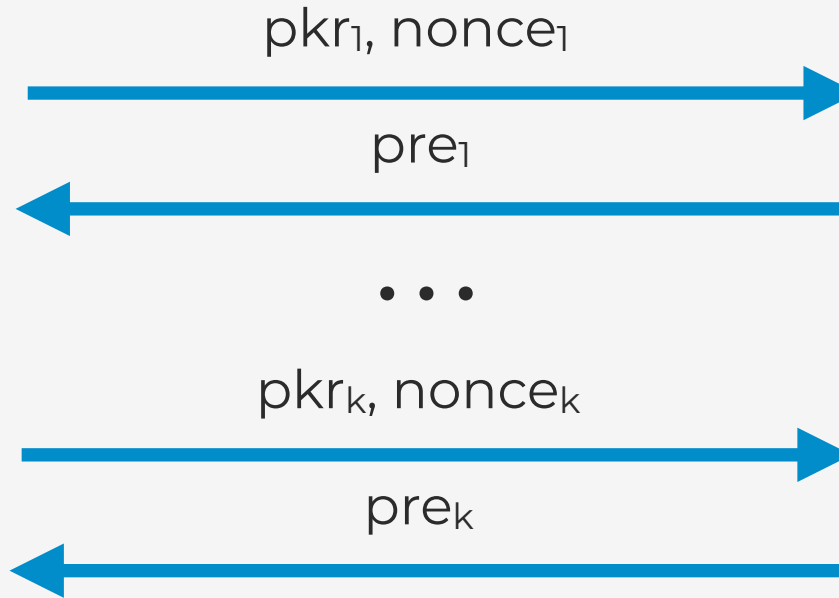


# Unforgeability for NIBS

Adversary



Challenger



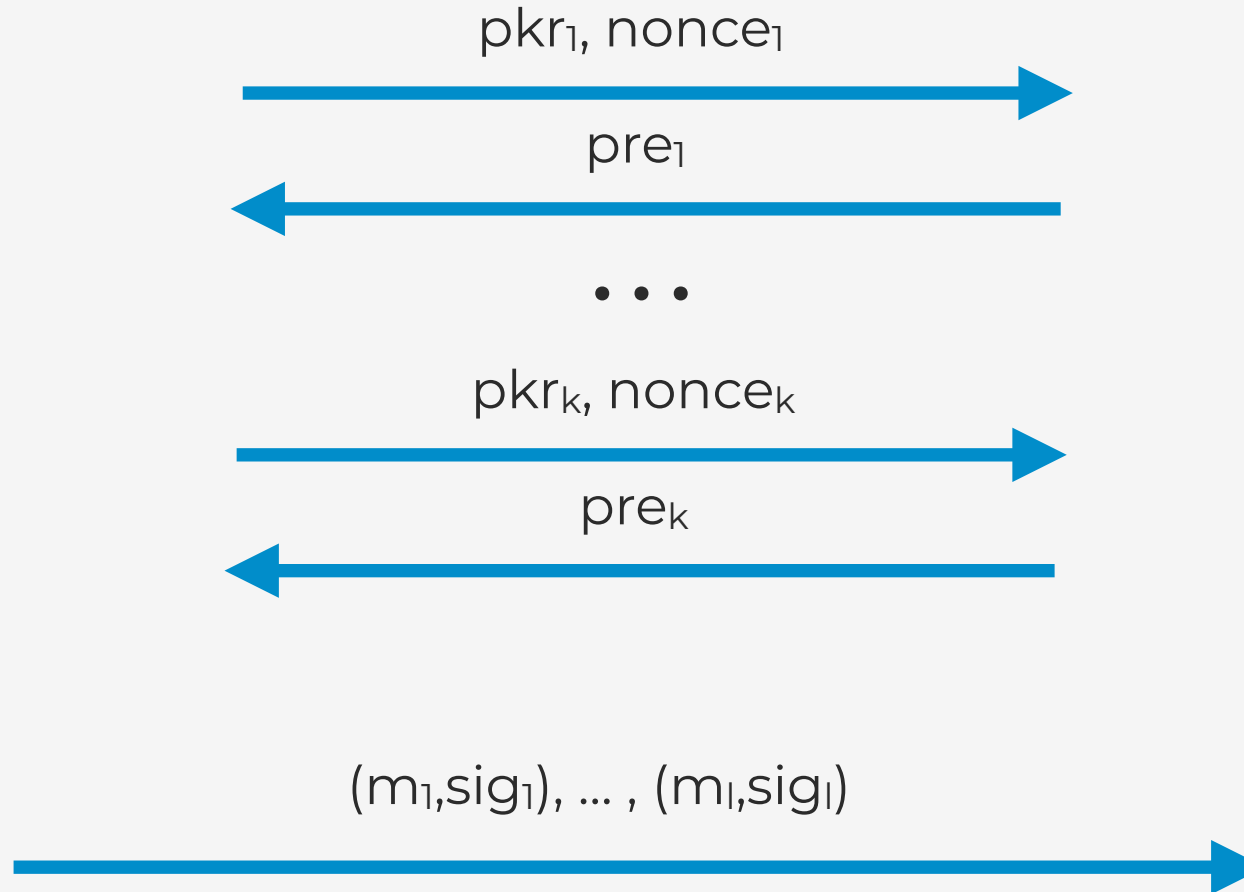


# Unforgeability for NIBS

Adversary



Challenger





# Unforgeability for NIBS

Adversary



Challenger



$pk_{r_1}, nonce_1$

$pre_1$

...

$pk_{r_k}, nonce_k$

$pre_k$

$(m_1, sig_1), \dots, (m_l, sig_l)$

- 1) valid signatures
- 2) distinct messages
- 3) queries  $k < l$



# Recipient Blindness

**Challenger**



**Adversary**





# Recipient Blindness

Challenger



$pk_{r_0}, pk_{r_1}$



Adversary





# Recipient Blindness

Challenger



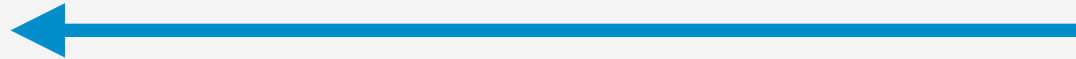
$pk_{r_0}, pk_{r_1}$



Adversary



$pre_0, nonce_0, pre_1, nonce_1, pk$





# Recipient Blindness

Challenger



$pk_{r_0}, pk_{r_1}$



Adversary



$pre_0, nonce_0, pre_1, nonce_1, pk$



$(m_0, sig_0) \leftarrow \text{Obtain}(skr_0, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr_1, pk, pre_1)$





# Recipient Blindness

Challenger



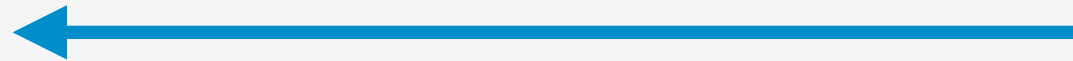
Adversary



$pk_{r_0}, pk_{r_1}$



$pre_0, nonce_0, pre_1, nonce_1, pk$



$(m_0, sig_0) \leftarrow \text{Obtain}(sk_{r_0}, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(sk_{r_1}, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$





# Recipient Blindness

Challenger



Adversary



$pk_{r_0}, pk_{r_1}$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(sk_{r_0}, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(sk_{r_1}, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$

$b'$



# Recipient Blindness

Challenger



Adversary



$pk_{r_0}, pk_{r_1}$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(sk_{r_0}, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(sk_{r_1}, pk, pre_1)$

**Adversary wins if  
 $b' = b$**



# Nonce Blindness

Challenger



Adversary





# Nonce Blindness

Challenger



pk<sub>r</sub>



Adversary





# Nonce Blindness

Challenger



Adversary



$pk_r$

$pre_0, nonce_0, pre_1, nonce_1, pk$



# Nonce Blindness

Challenger



Adversary



pk<sub>r</sub>

pre<sub>0</sub>, nonce<sub>0</sub>, pre<sub>1</sub>, nonce<sub>1</sub>, pk

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$



# Nonce Blindness

Challenger



Adversary



$pk_r$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$





# Nonce Blindness

Challenger



Adversary



$pk_r$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$

$(m_b, sig_b), (m_{1-b}, sig_{1-b})$

$b'$



# Nonce Blindness

Challenger



Adversary



$pk_r$

$pre_0, nonce_0, pre_1, nonce_1, pk$

$(m_0, sig_0) \leftarrow \text{Obtain}(skr, pk, pre_0)$

$(m_1, sig_1) \leftarrow \text{Obtain}(skr, pk, pre_1)$

Adversary wins if  
 $b' = b$