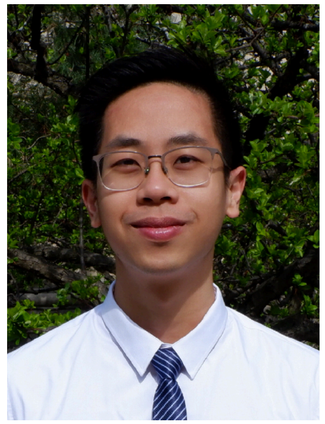
Spartan & Bulletproofs are Simulation-Extractable (for Free!)

Quang Dao
CMU



Paul Grubbs
Michigan

Eurocrypt 2023

zkSNARKs: Security & Use Cases

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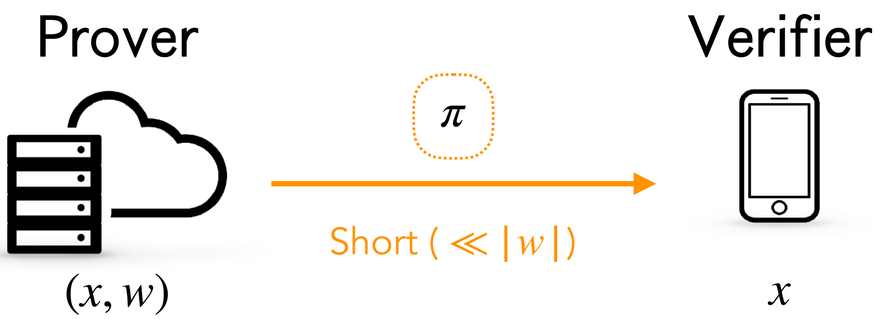
(Zero-knowledge Succinct Non-interactive ARguments of Knowledge)

short, non-interactive proofs



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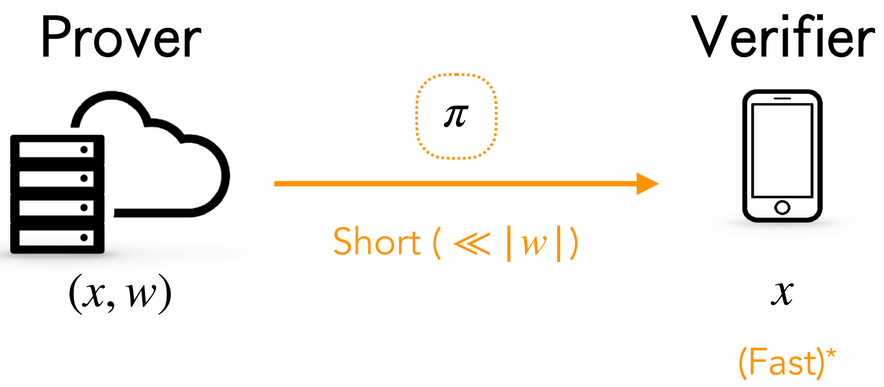

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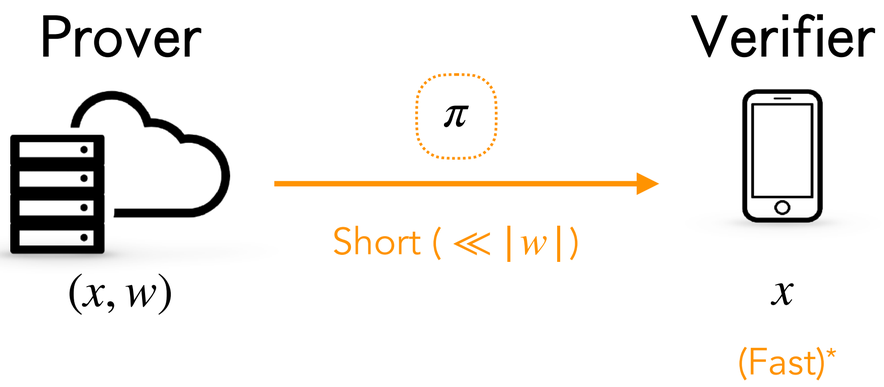

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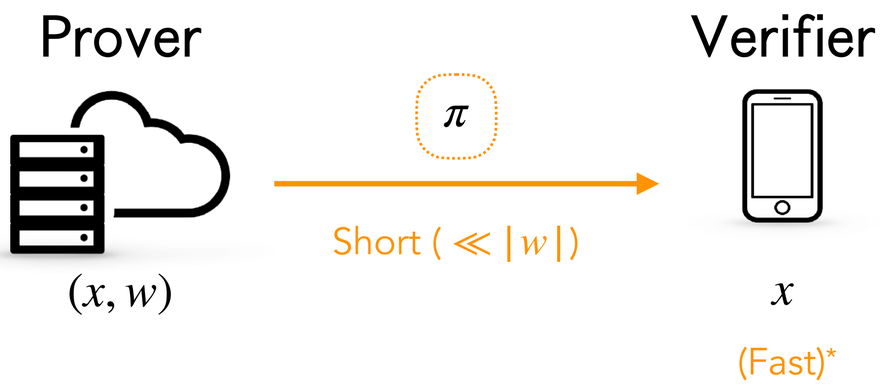

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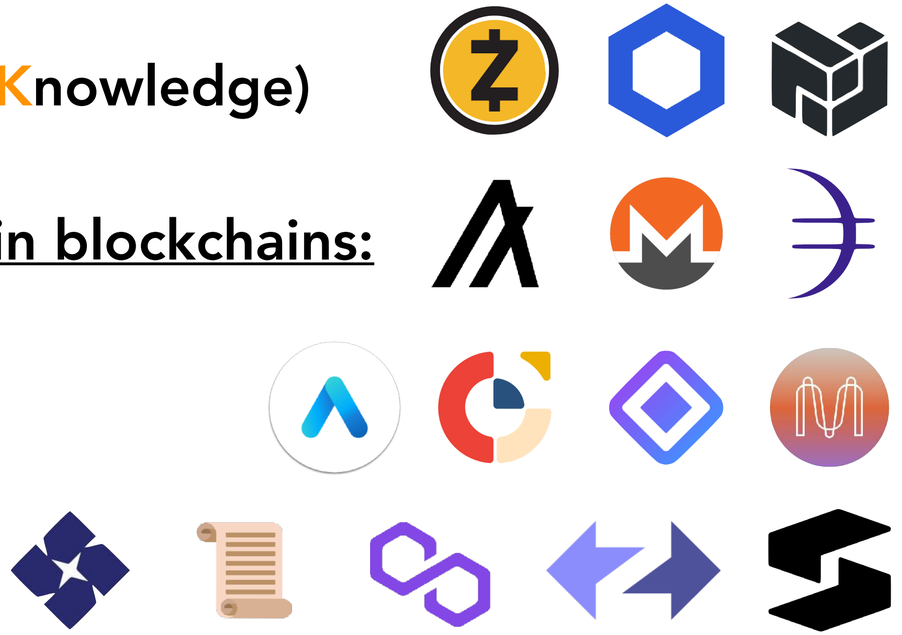

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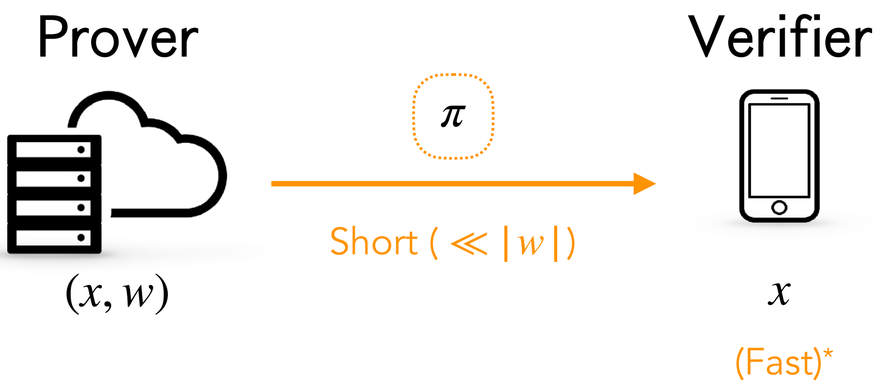

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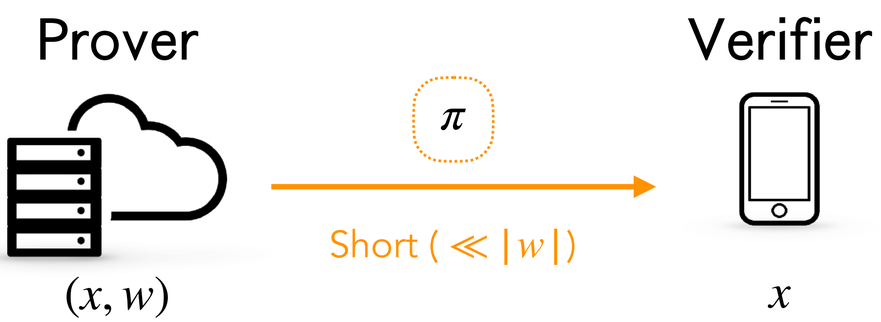

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Other applications:

- Proof of solvency [DBBCB15]
- Image provenance [NT16], [BD22], [KHSS22]
- Content moderation [RMM22], [GAZBW22]
- And many more!

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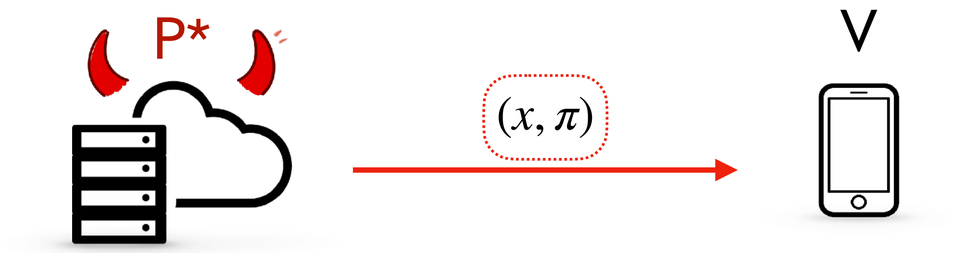
Adaptive attack: choose the statement adaptively based on the proof



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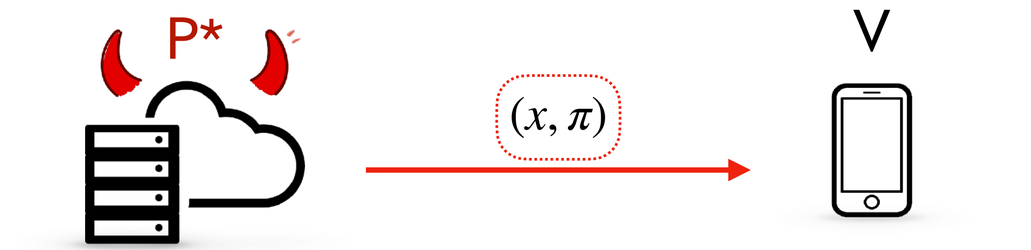

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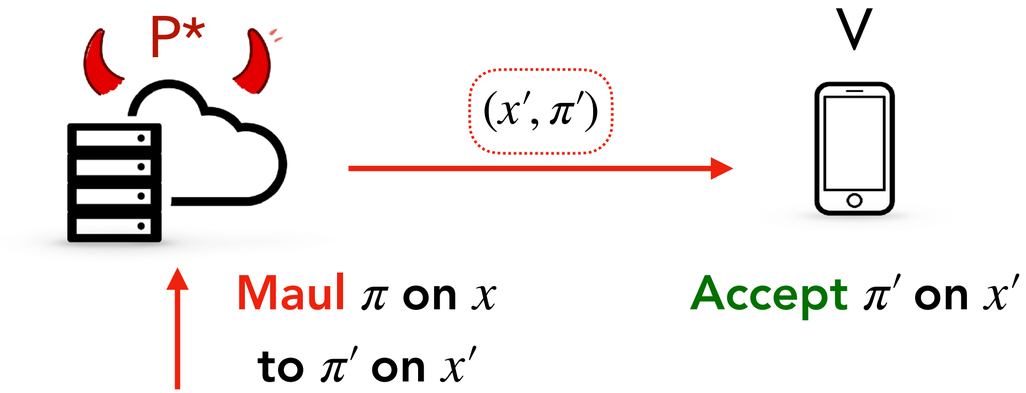

Maul π on x to π' on x'

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Valid (x, π)

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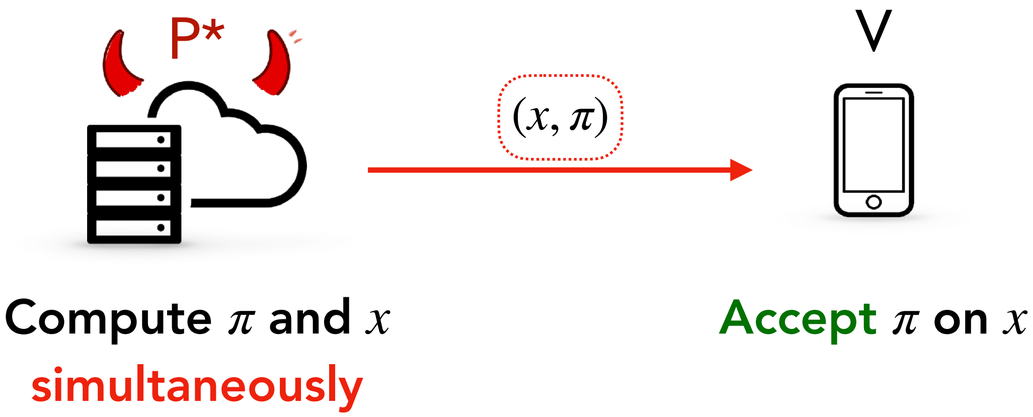

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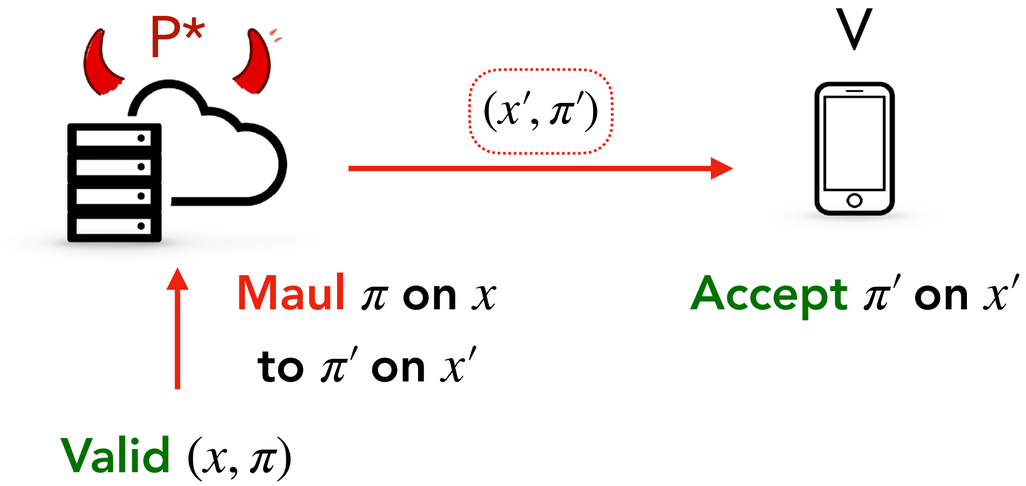

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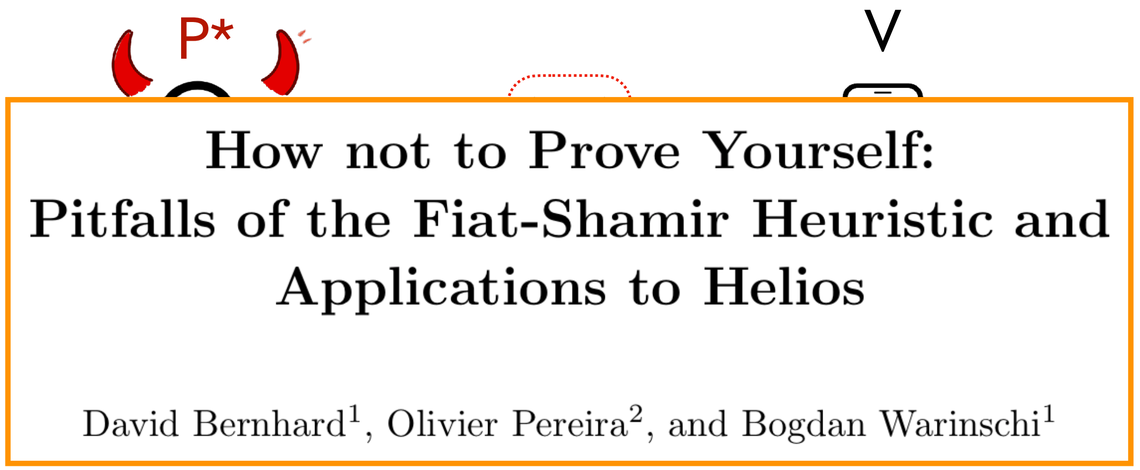

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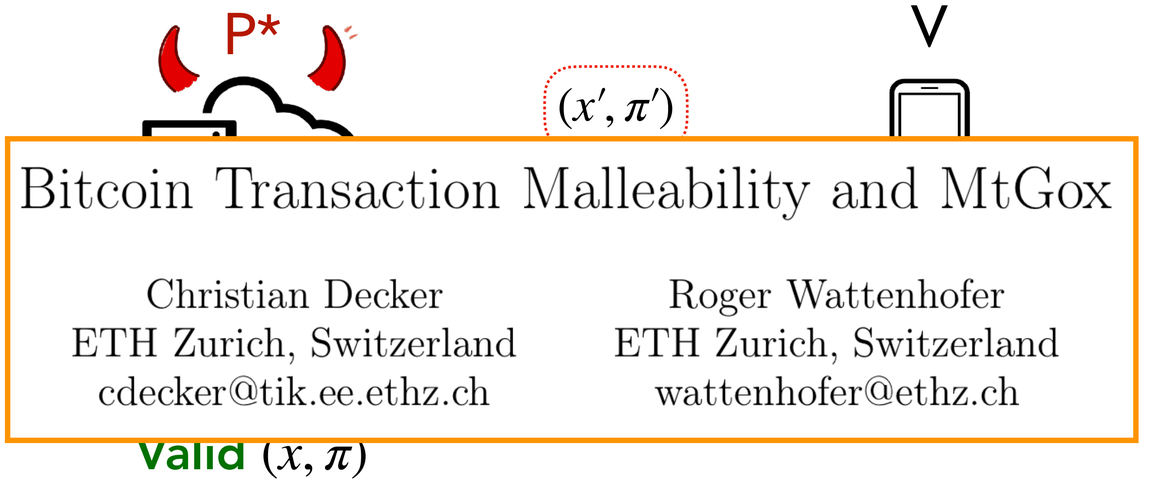

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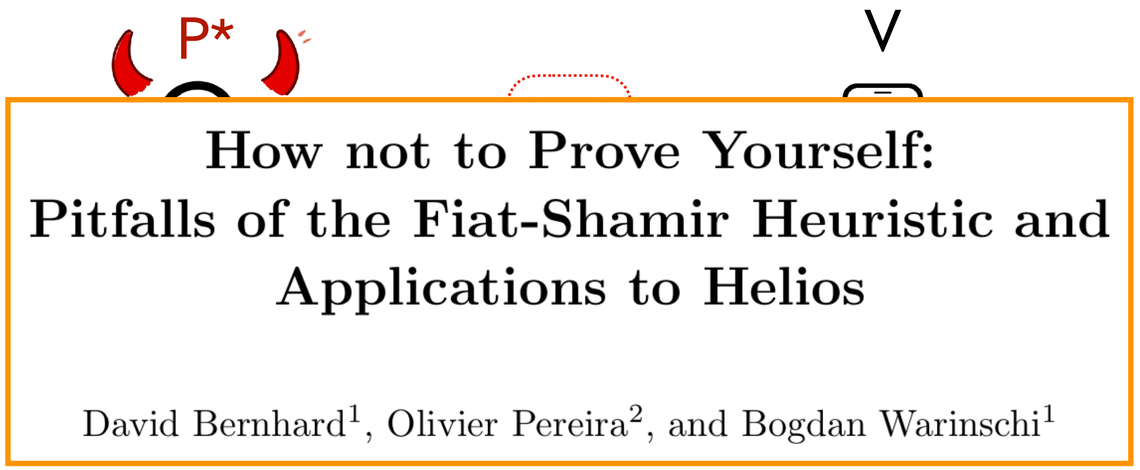

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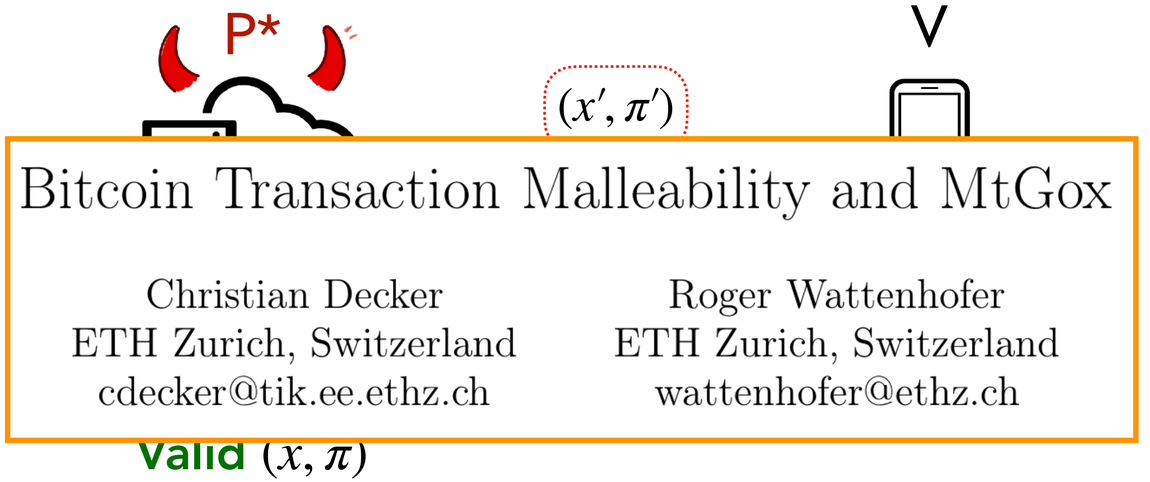

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$\Longrightarrow$ We need *stronger* security properties for deployment

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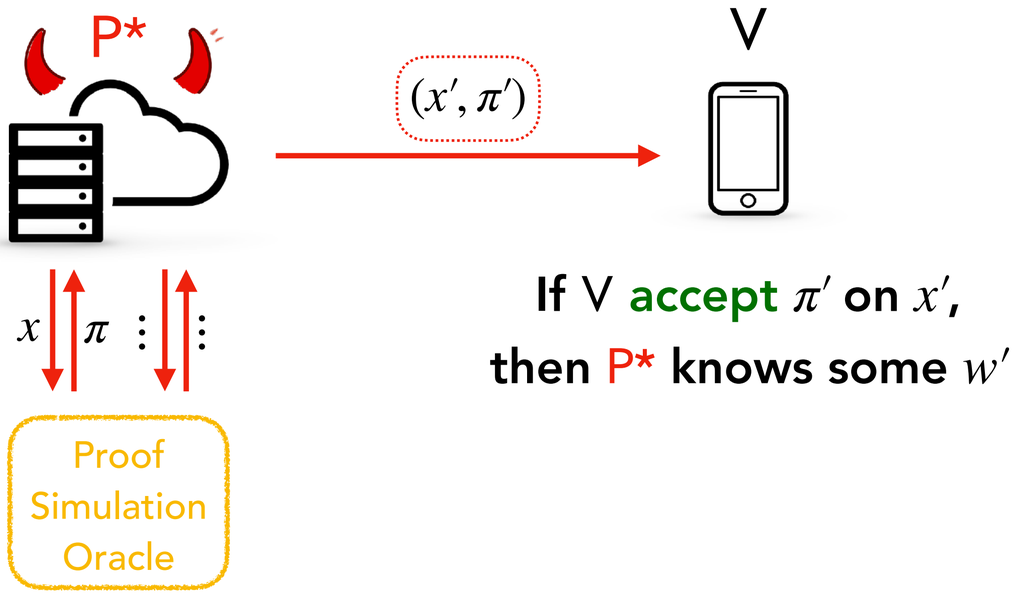
SIM-EXT (informal): [Sahai99], [DDOPS01]

Knowledge soundness holds *even when* P* gets *extra power*.



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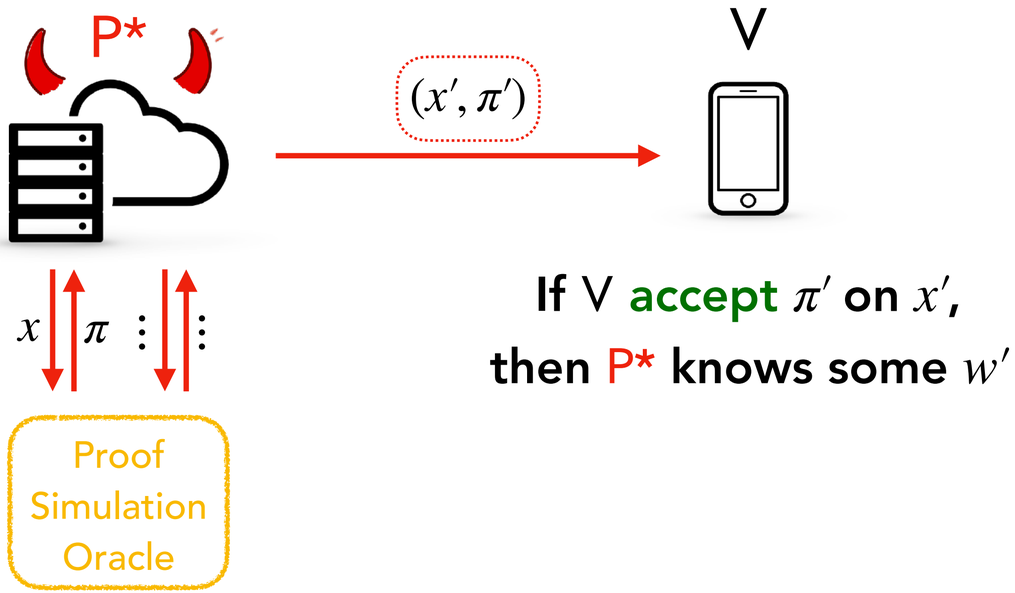

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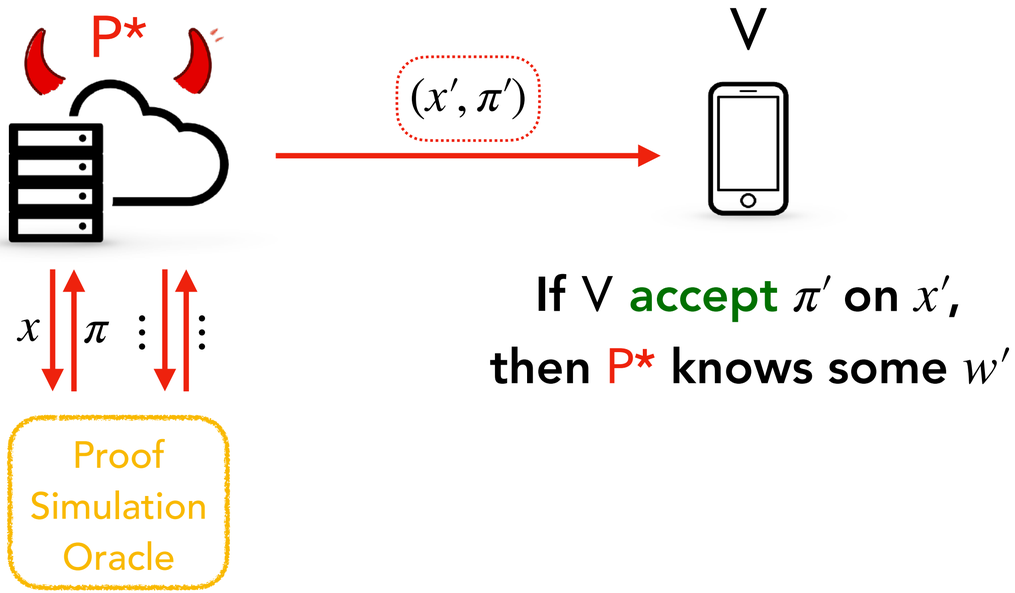

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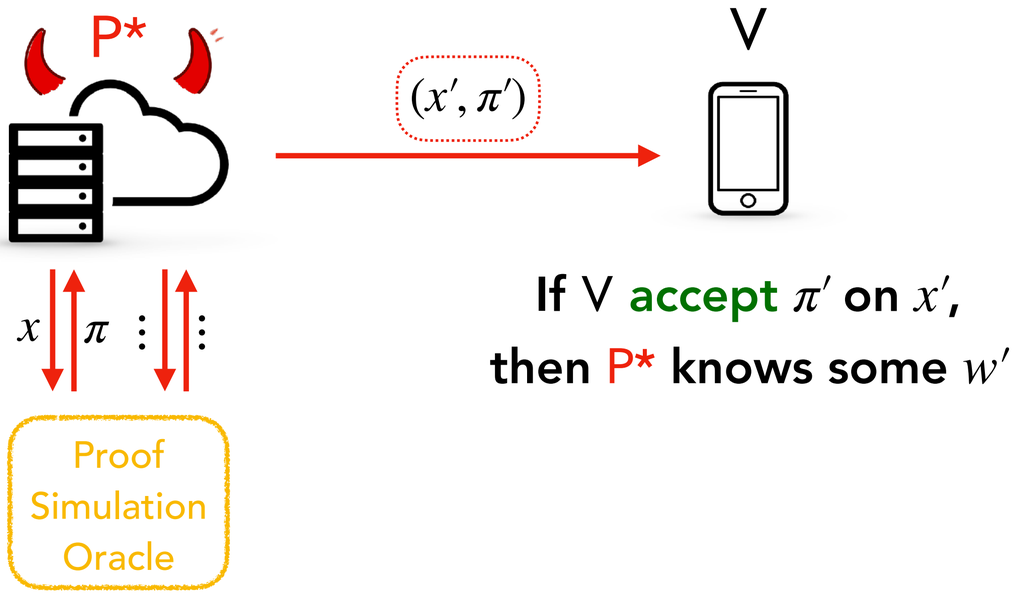

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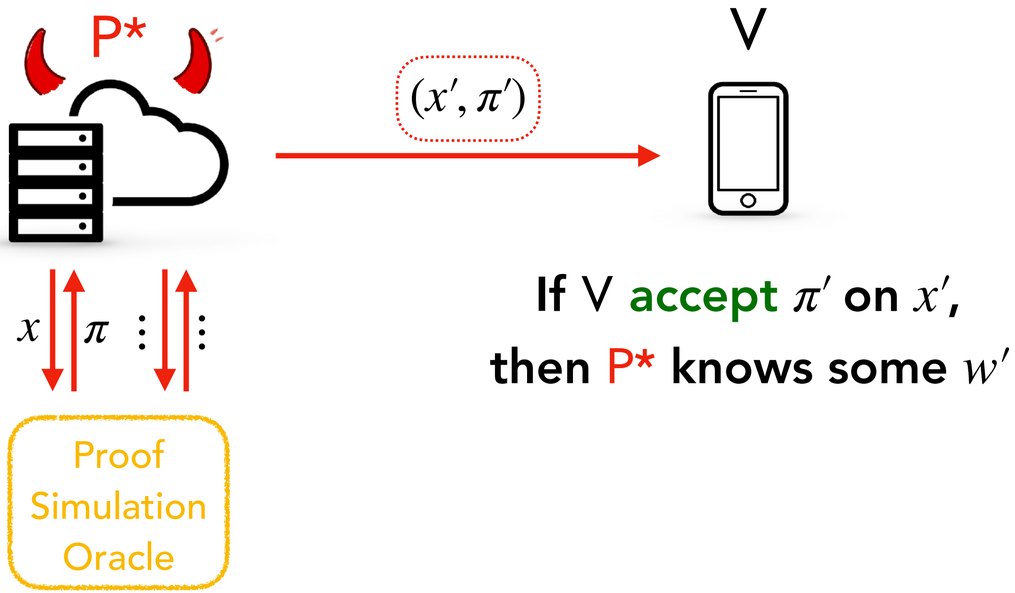

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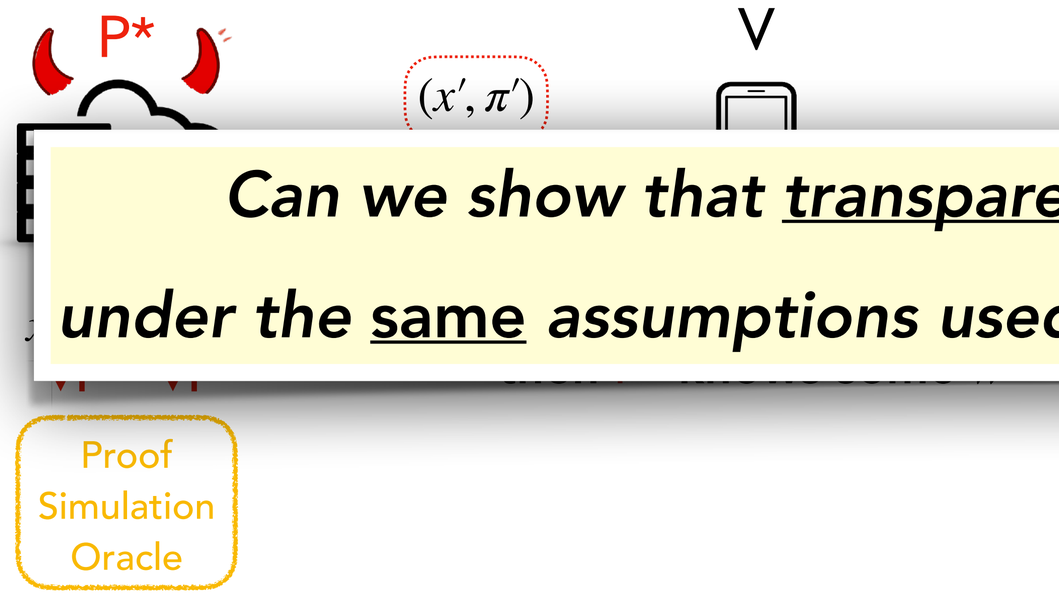

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Can we show that transparent zkSNARKs satisfy SIM-EXT under the same assumptions used to prove (knowledge) soundness?

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- A more general **tree extraction** lemma for proving knowledge soundness
  (building on the work of Attema, Fehr & Klooß [AFK22])

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1. Breaking SIM-EXT into smaller properties
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3. Knowledge Soundness via Generalized Tree Builder

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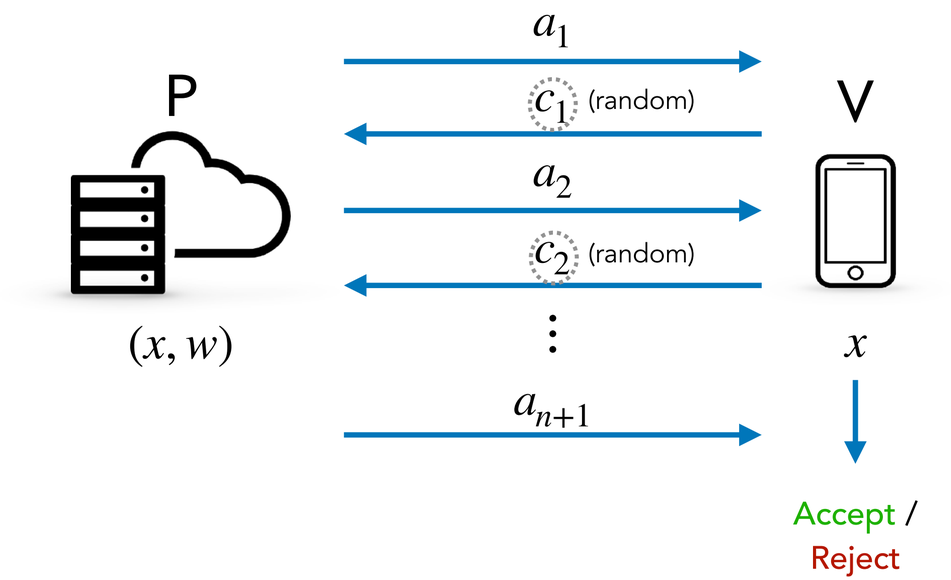

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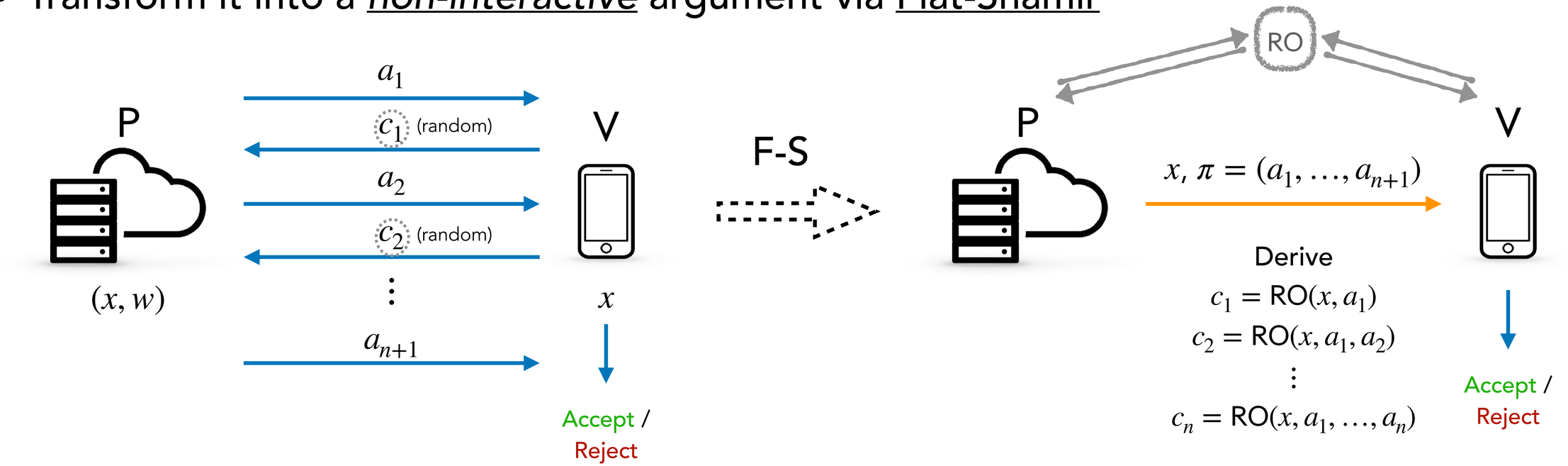

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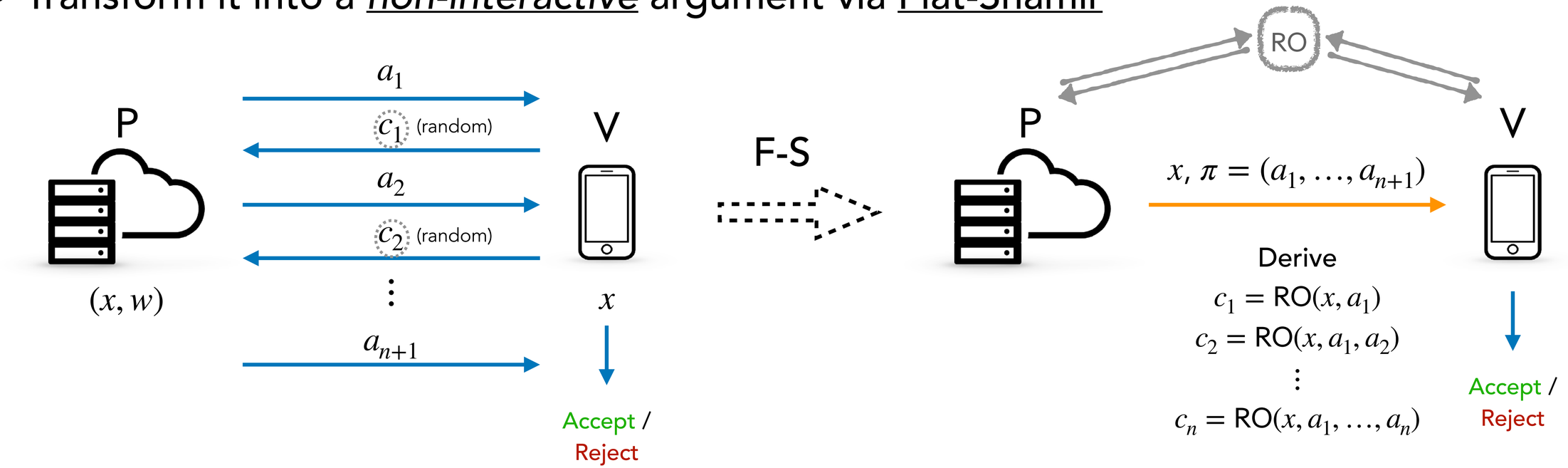

Insight: [GKKNZ22] ***Assuming 2 smaller properties, SIM-EXT of F-S argument may be reduced to its knowledge soundness (KS).***

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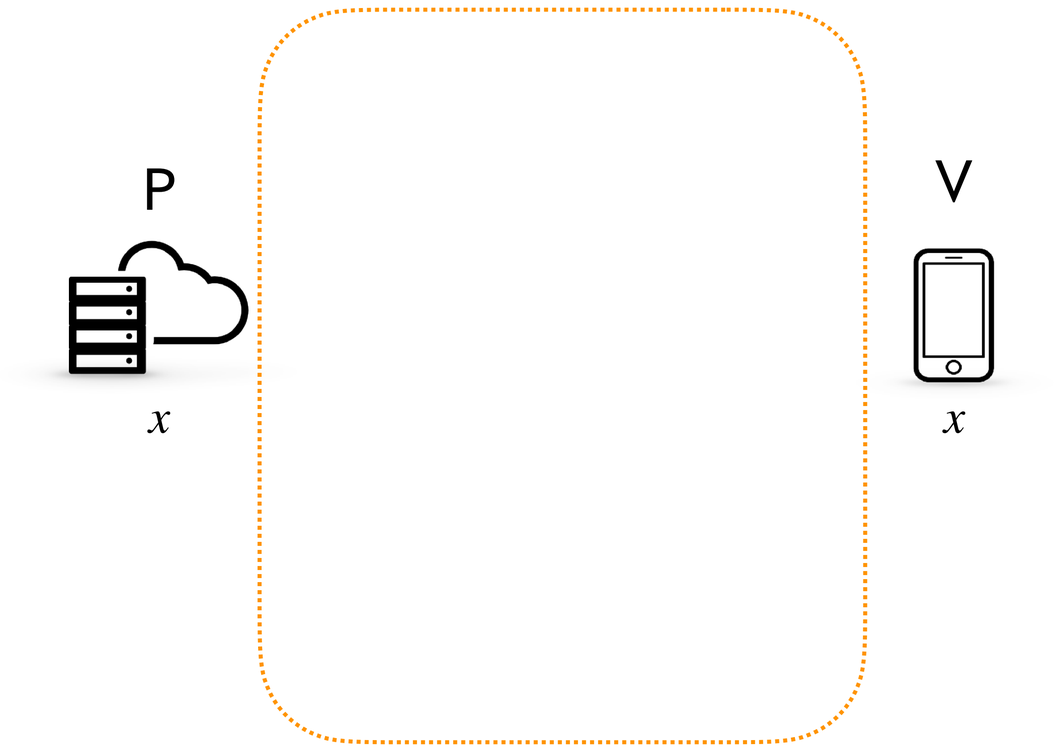

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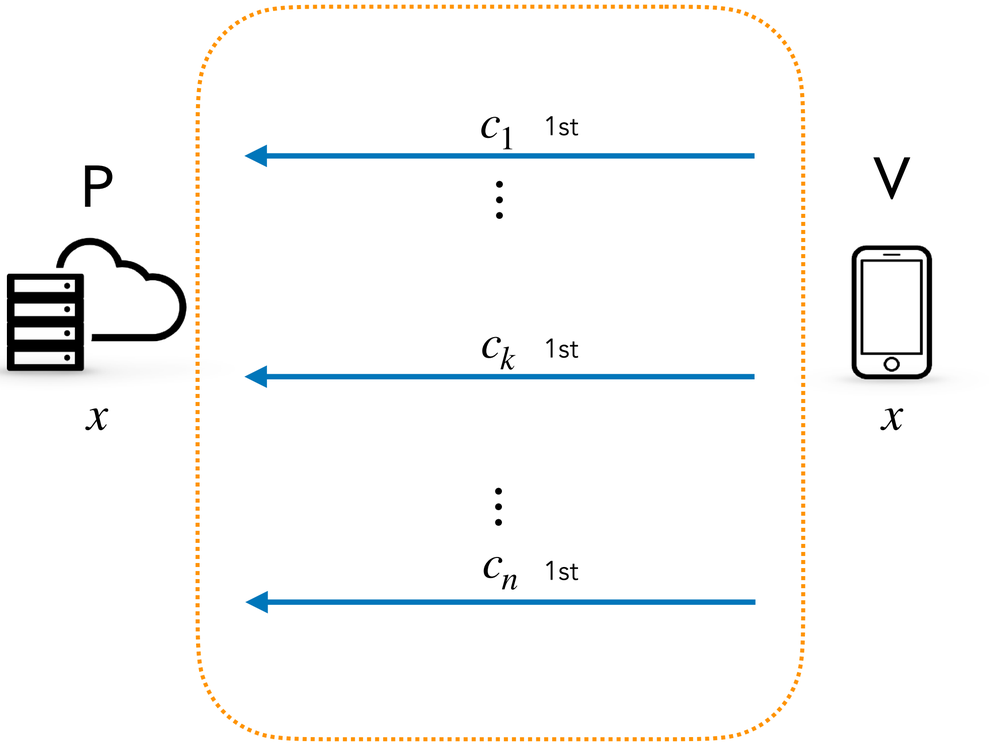

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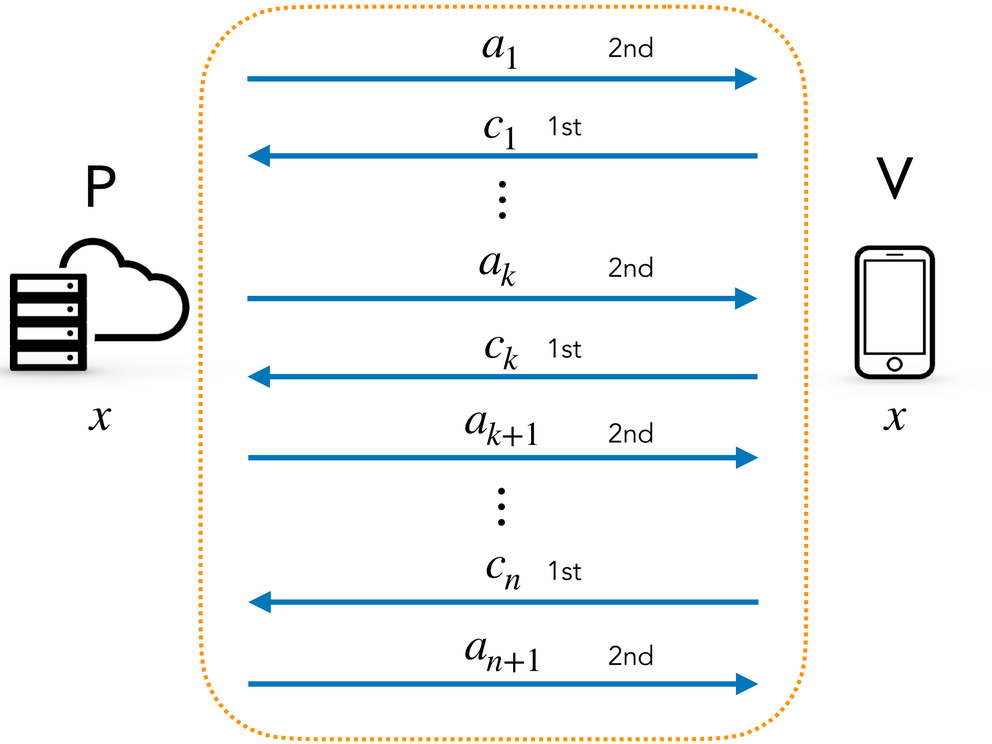

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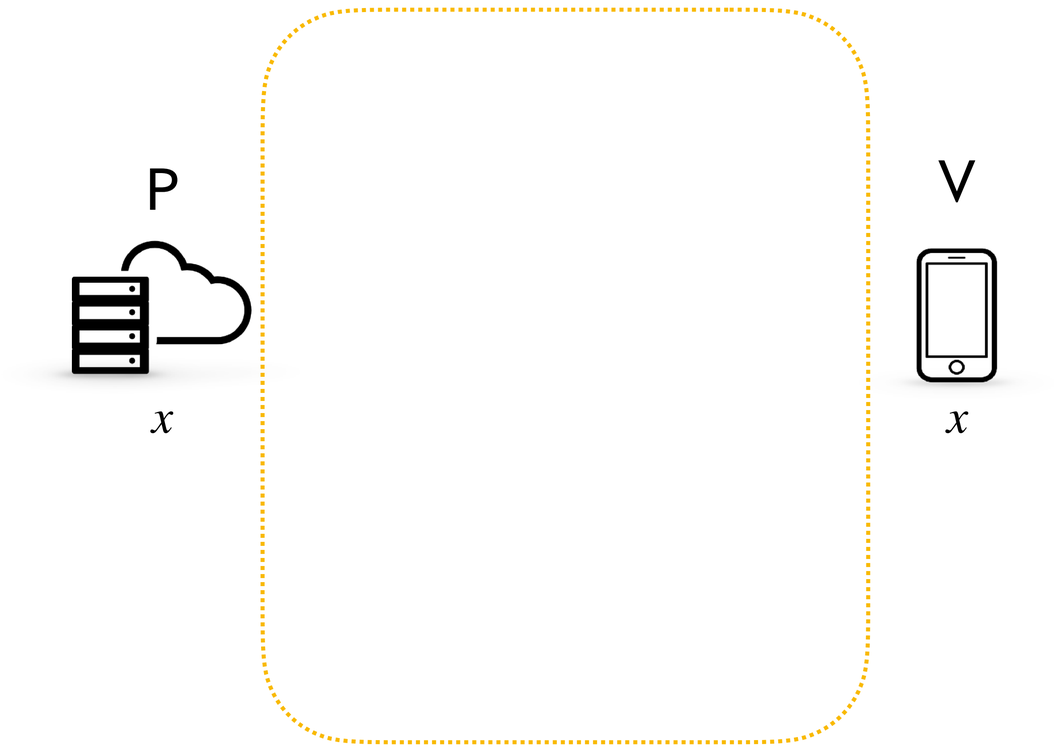

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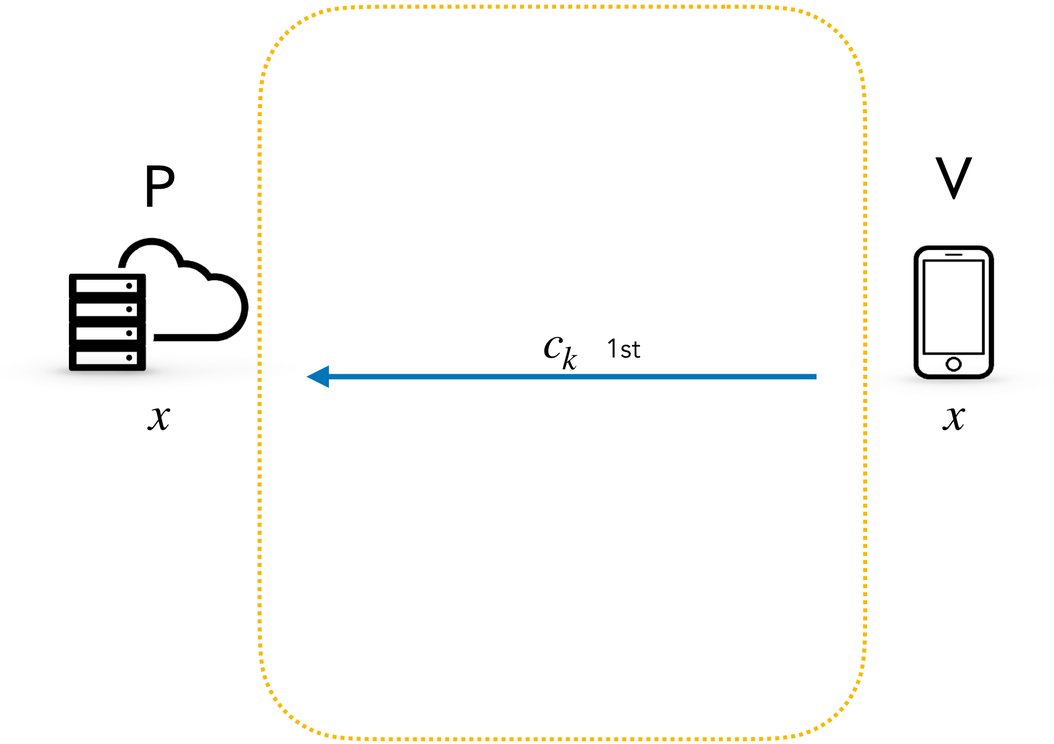

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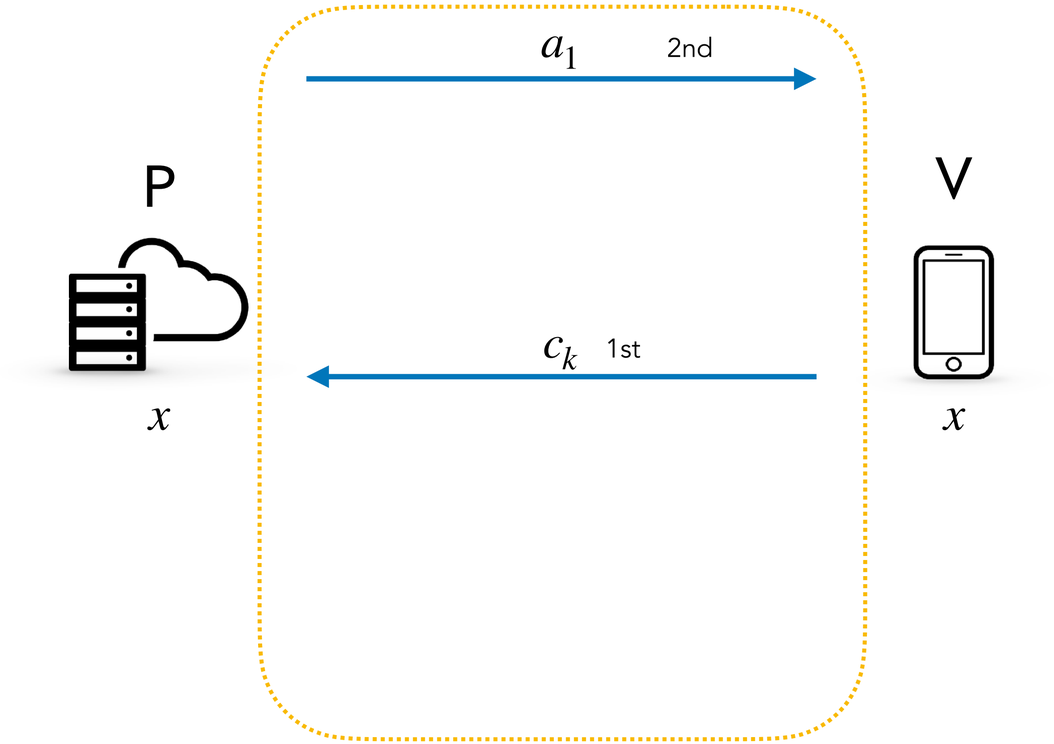

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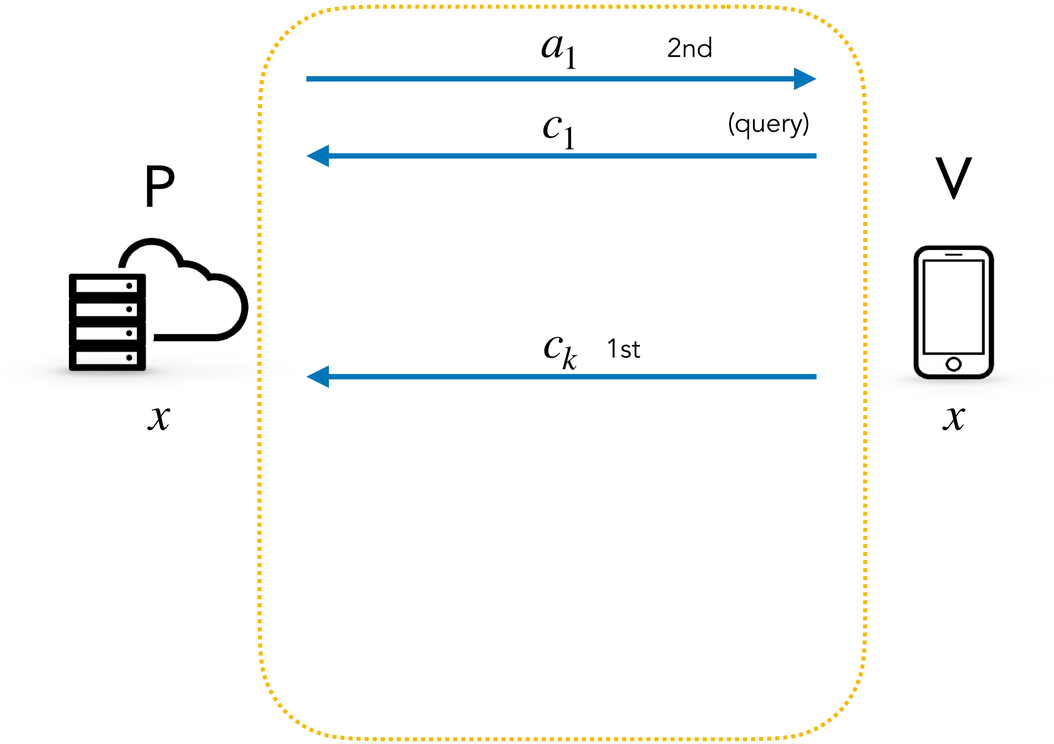

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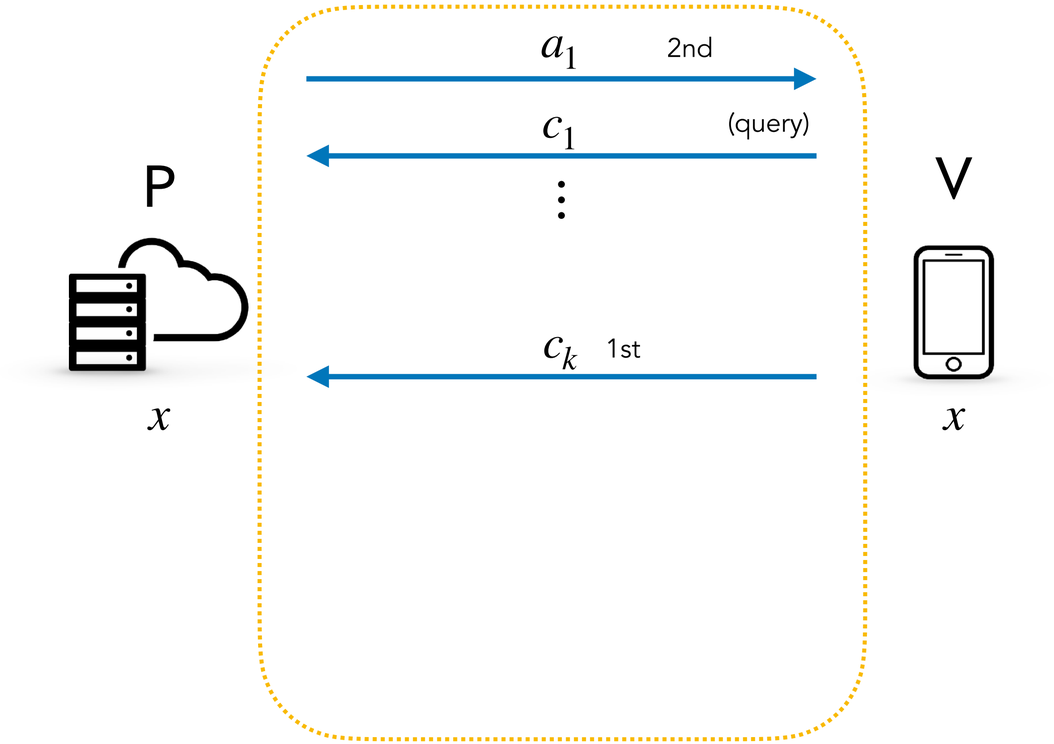

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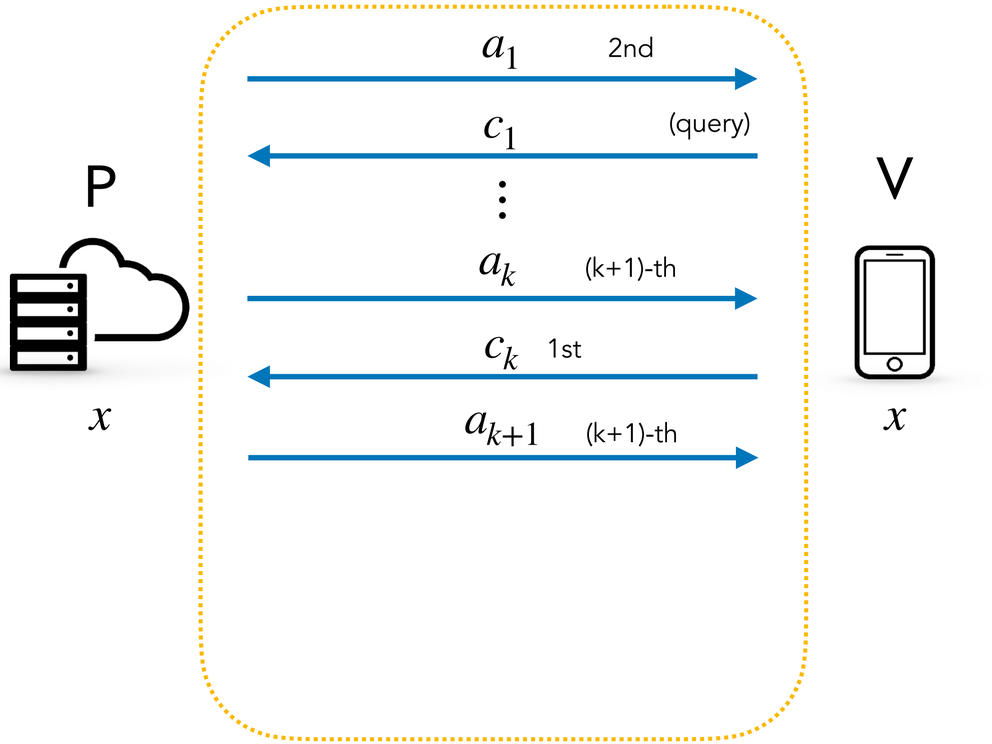

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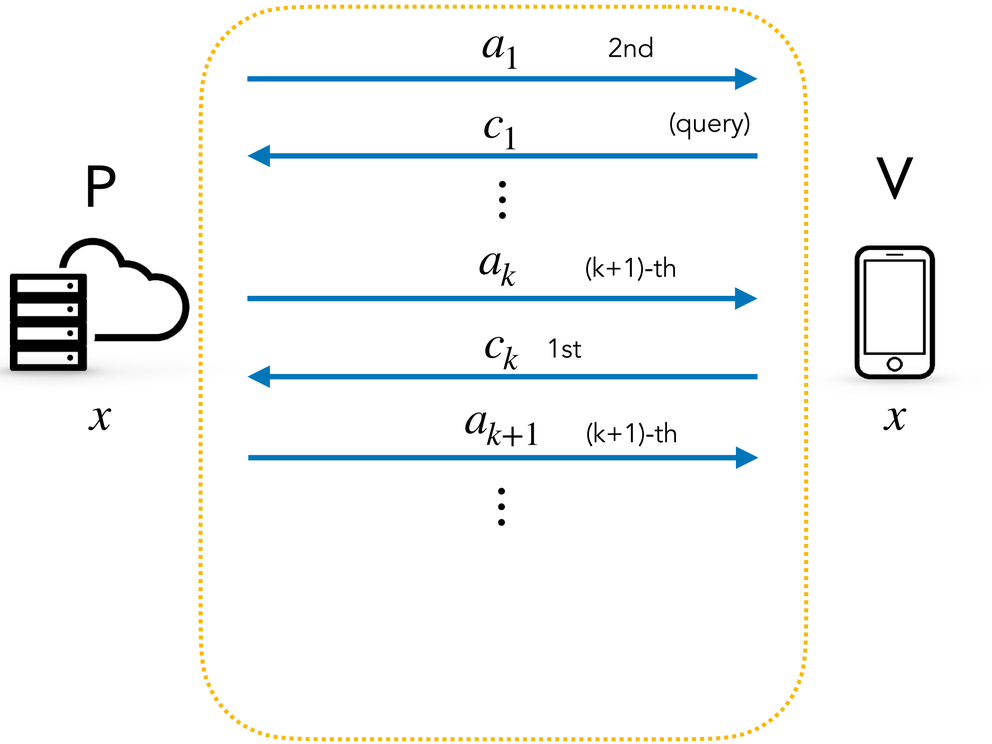

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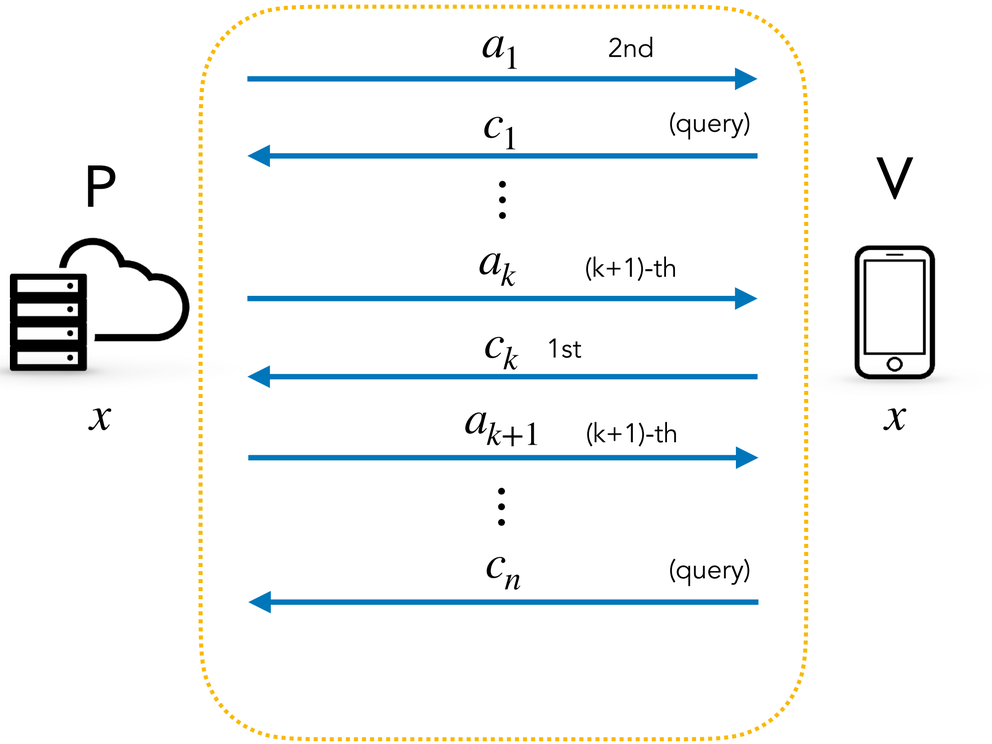

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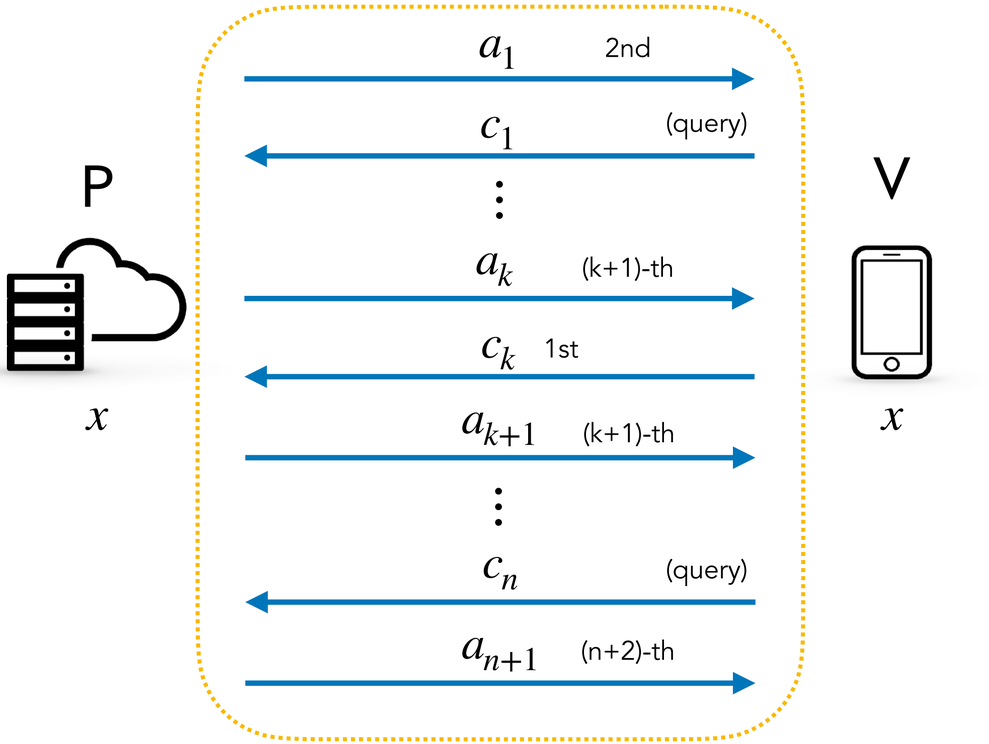

k-Zero-Knowledge and k-Unique Response

k-Zero-Knowledge (k-ZK): The simulator Sim_k may *only* choose k^{th} challenge, and compute other messages in order.

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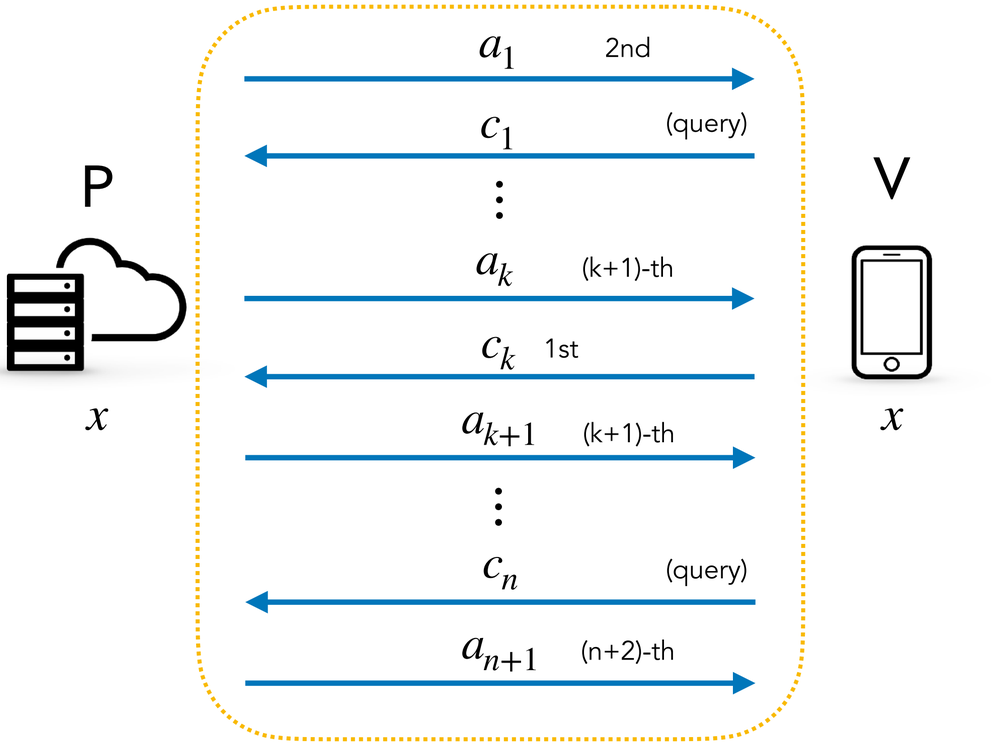

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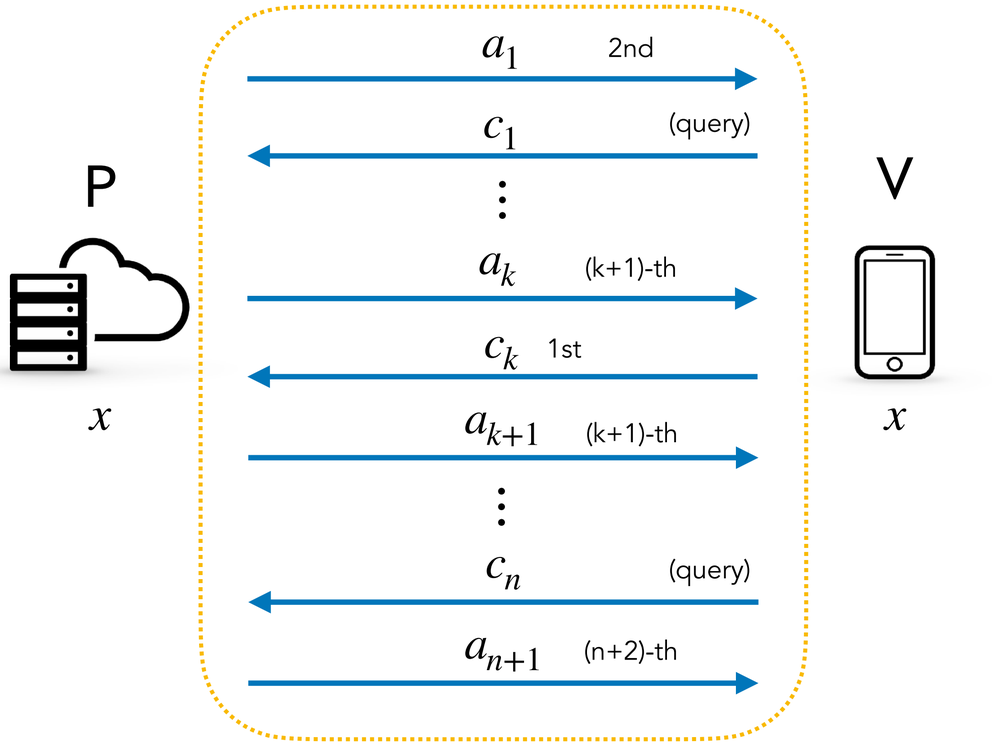

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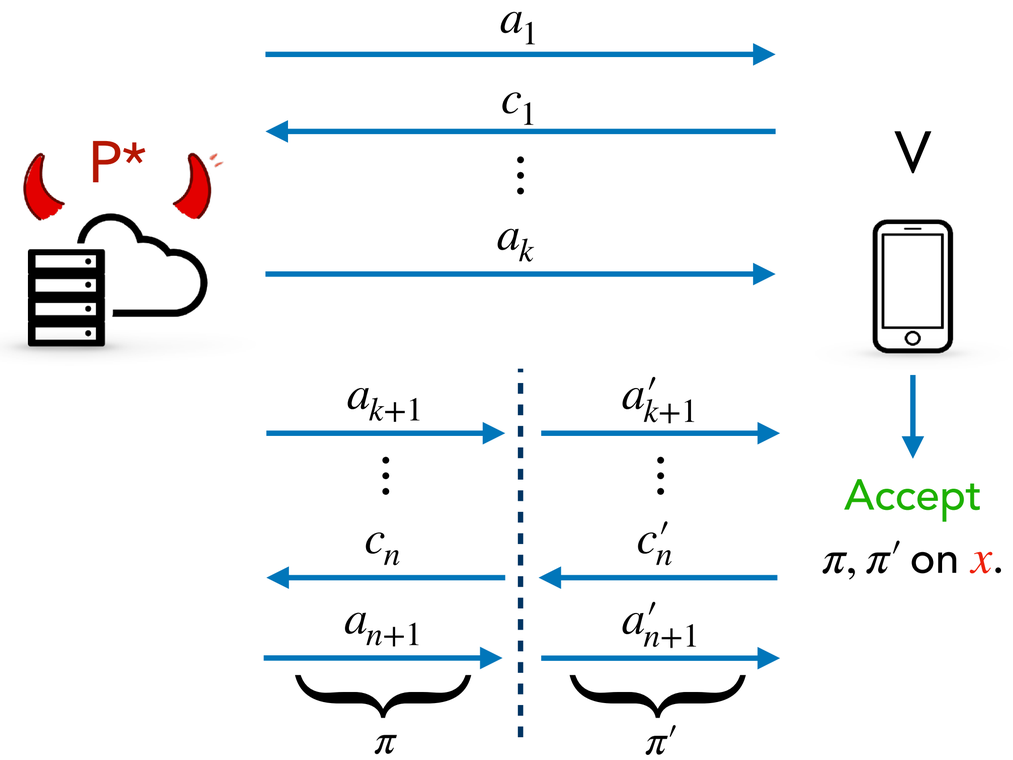

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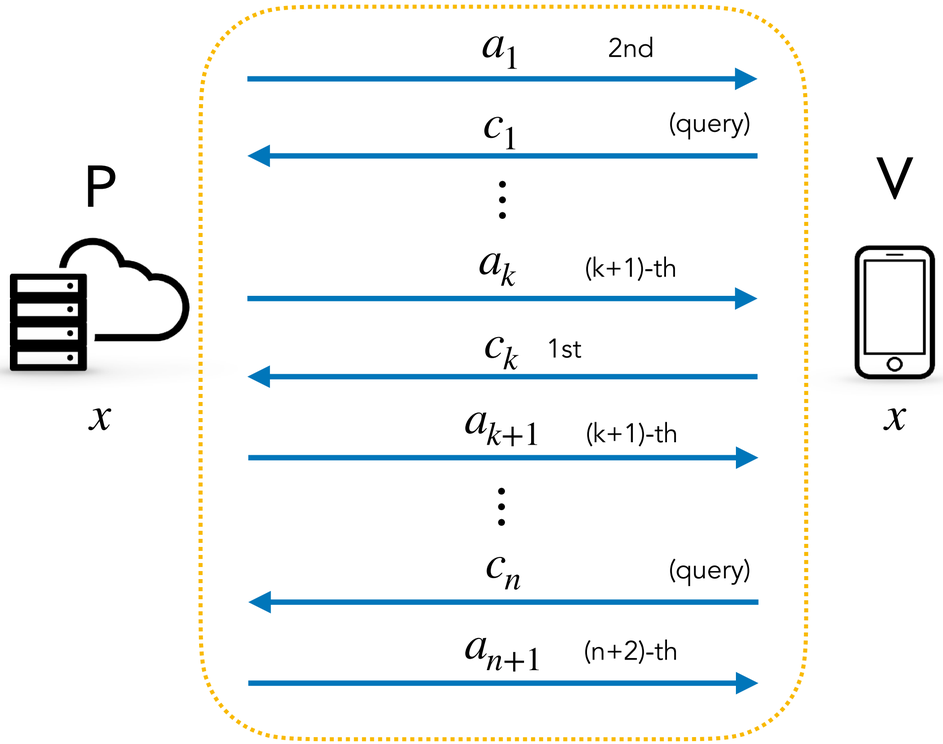

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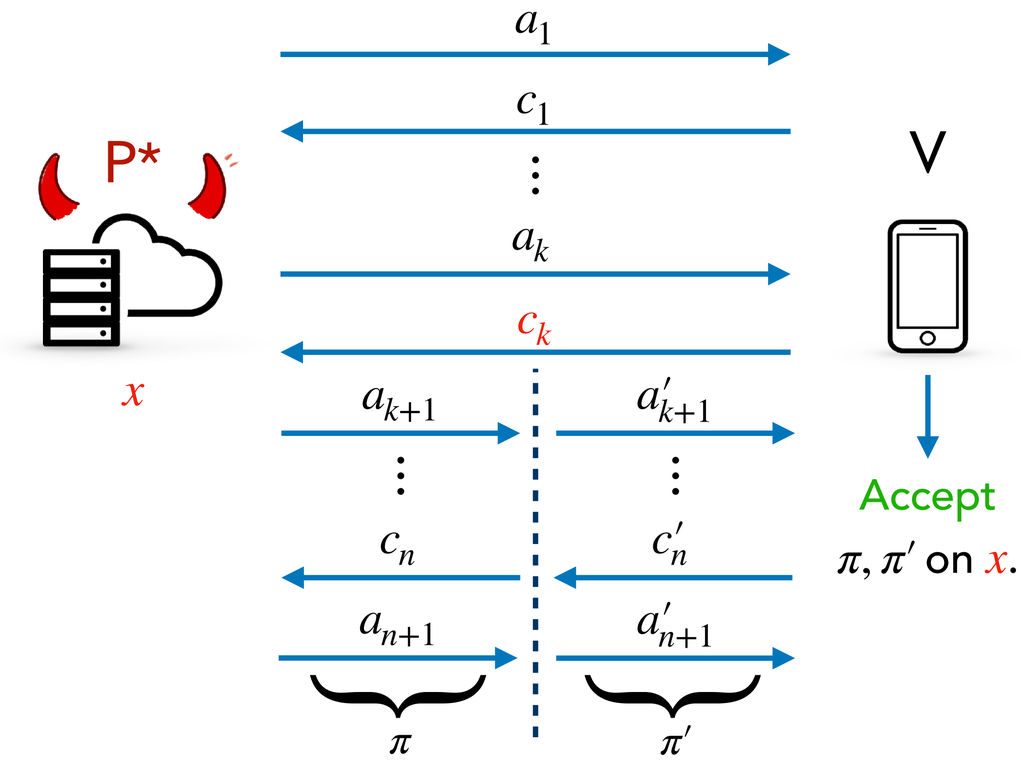

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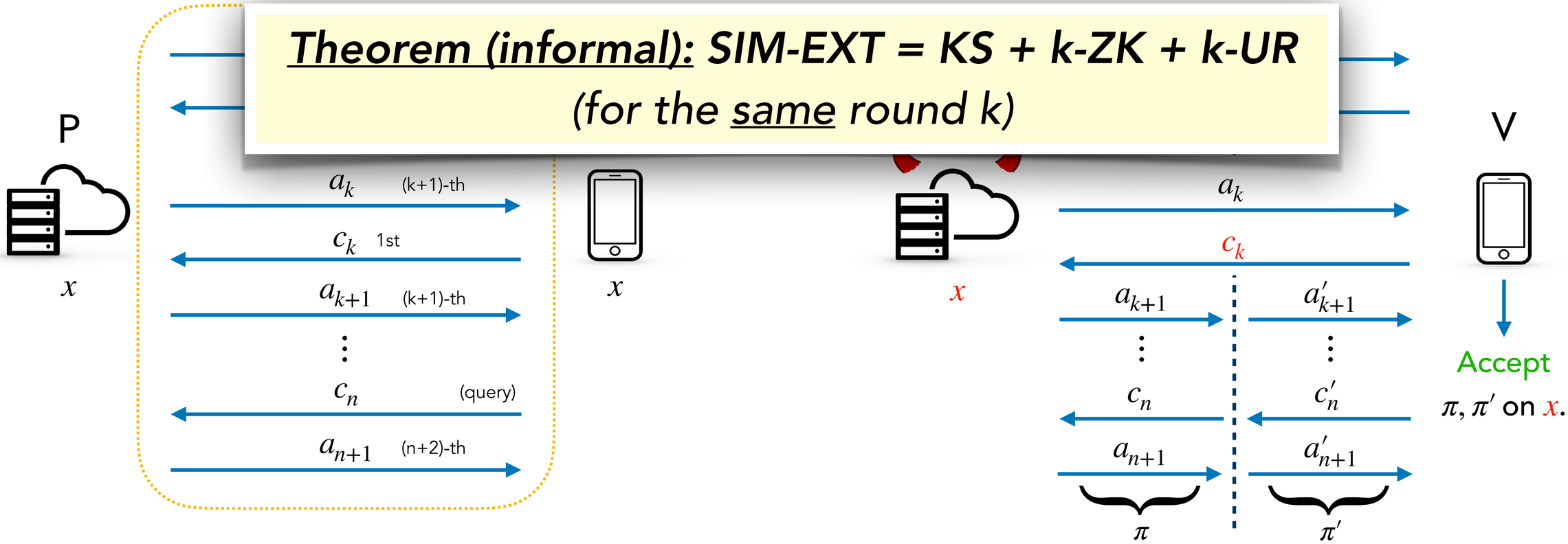
Theorem (informal): SIM-EXT = KS + k-ZK + k-UR
(for the same round k)



Agenda

1. SIM-EXT = KS + k-ZK + k-UR (for same k)
2. **Instantiating SIM-EXT template for Bulletproofs**
3. Knowledge Soundness via Generalized Tree Builder

Bulletproofs Range Proof

Bulletproofs Range Proof

Public Private

Relation: $V = g^v h^r$ and $0 \leq v \leq 2^n - 1$

Bulletproofs Range Proof

Let $\mathbf{a} = \text{BinDecomp}(v)$.

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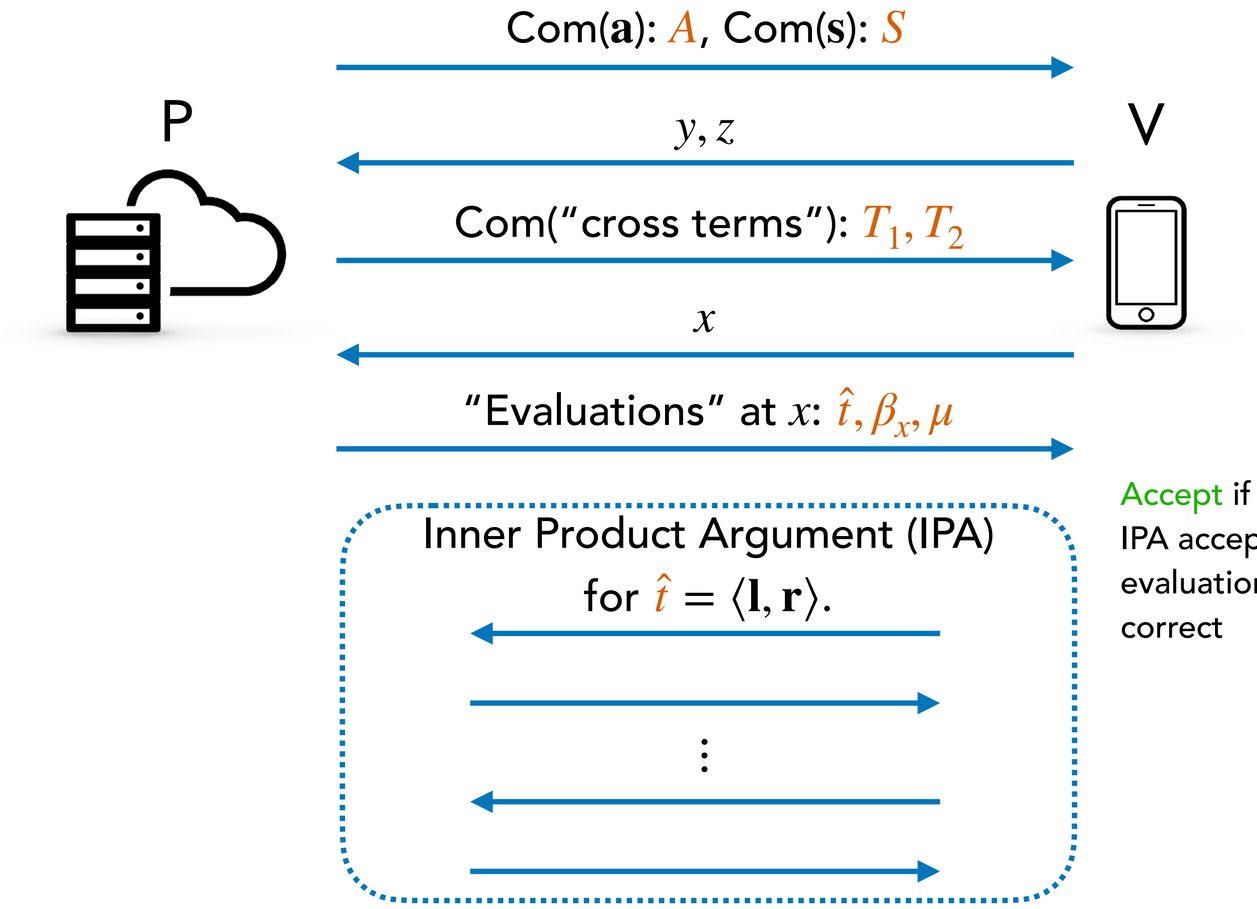

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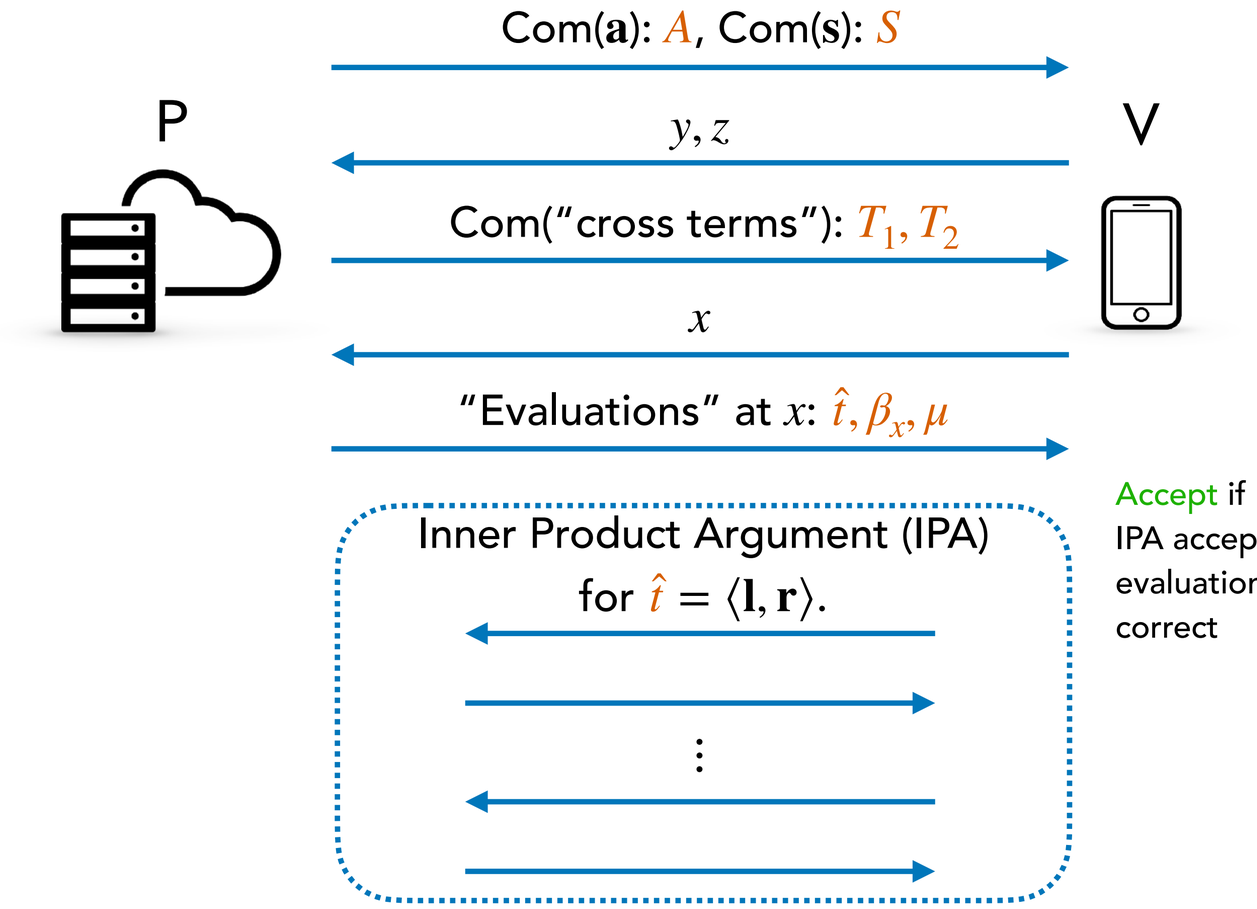
Recall: We need to show Bulletproofs satisfy KS, k-ZK, and k-UR for the same round k.



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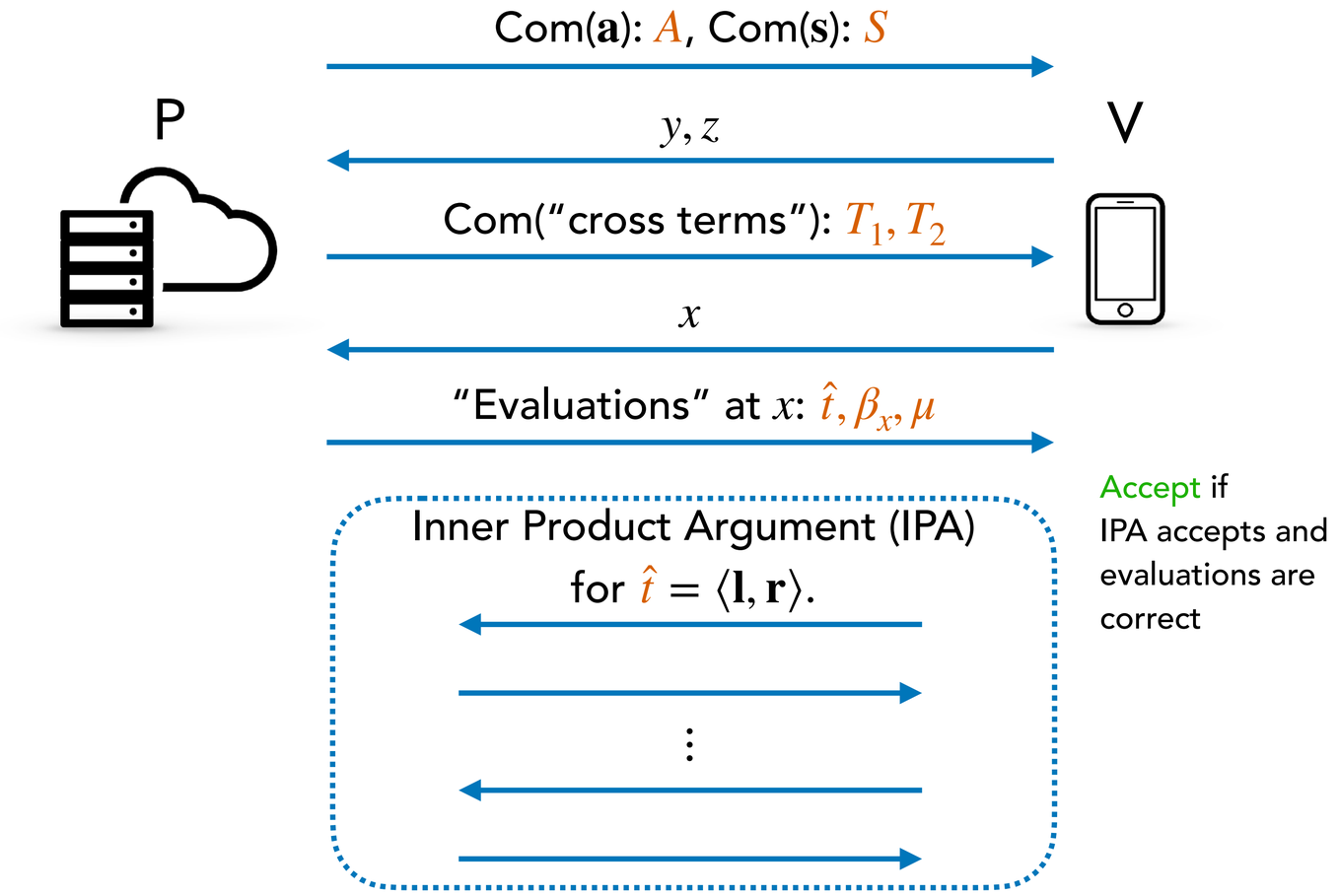

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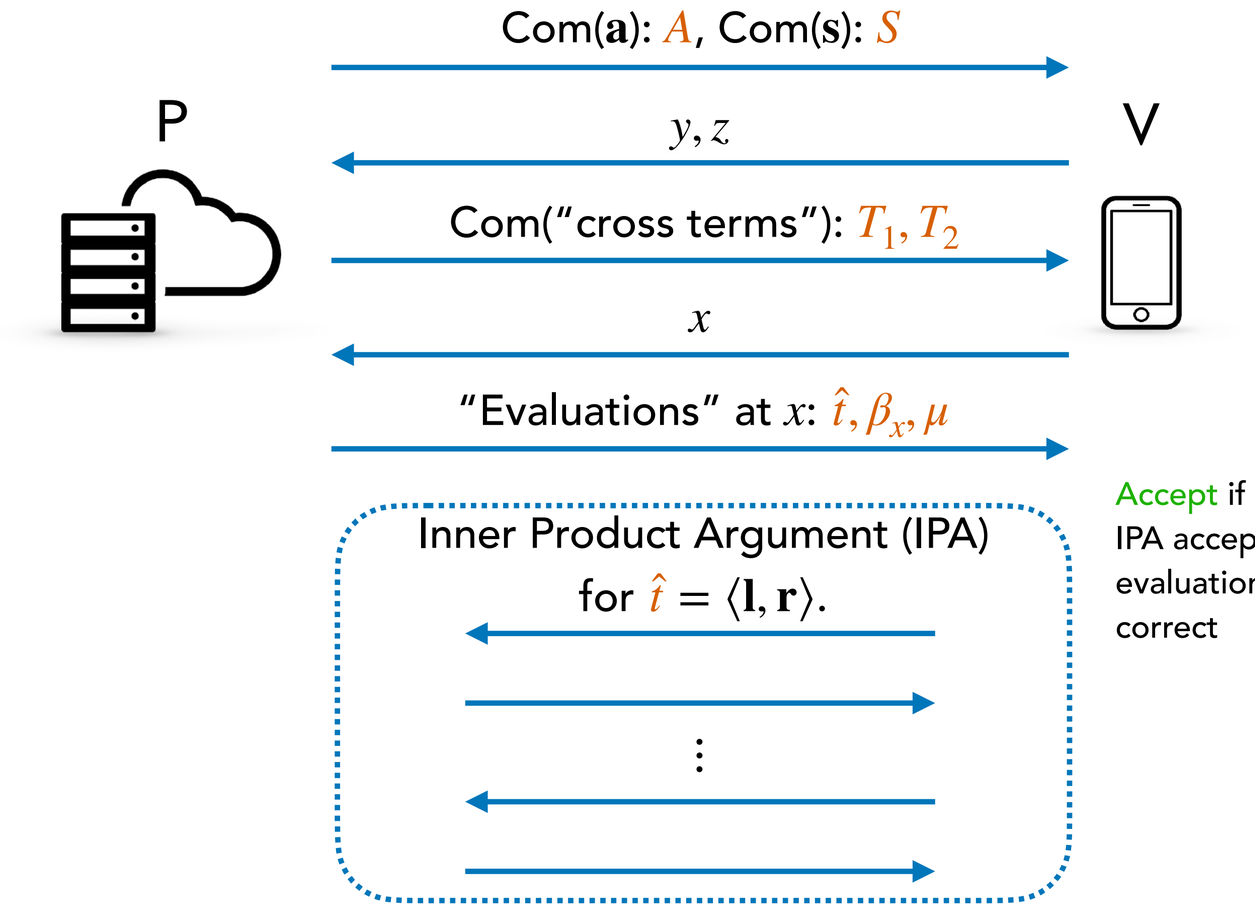
Q: Which round k to prove k-ZK and k-UR?



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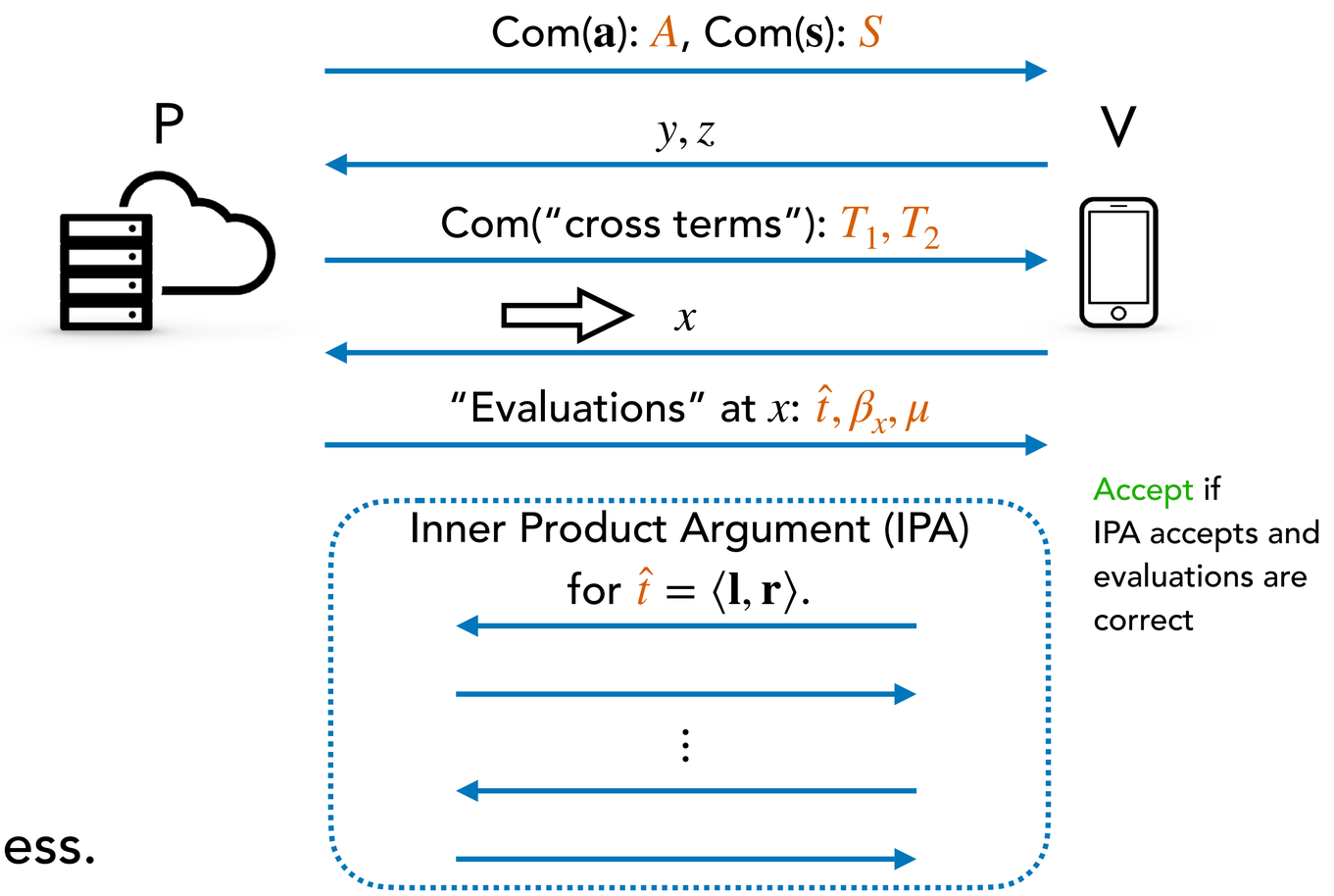
Q: Which round k to prove k-ZK and k-UR?

A: Choose the *last* round with P's randomness.
($k = 2$ in this case)



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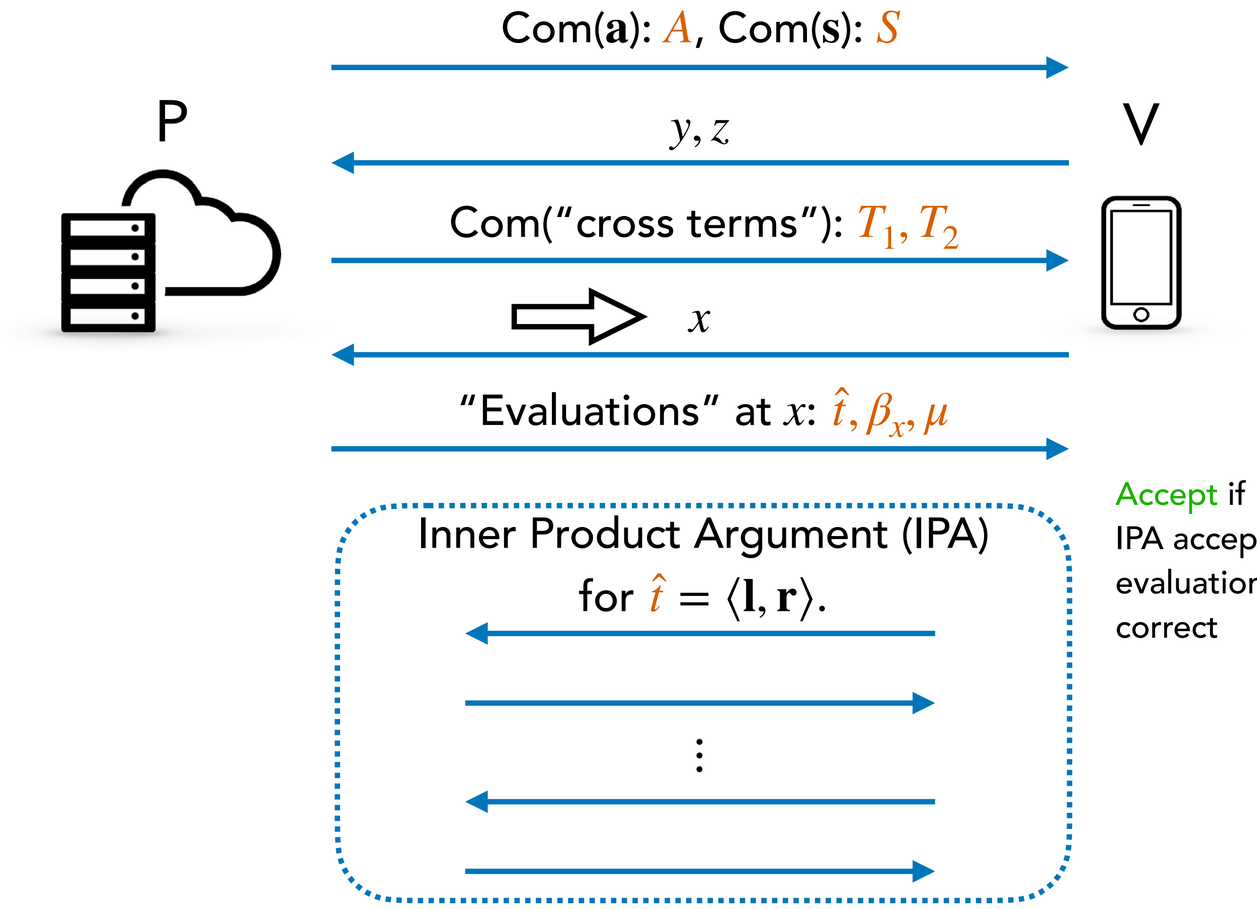
2-ZK: Simulator can only choose x first.



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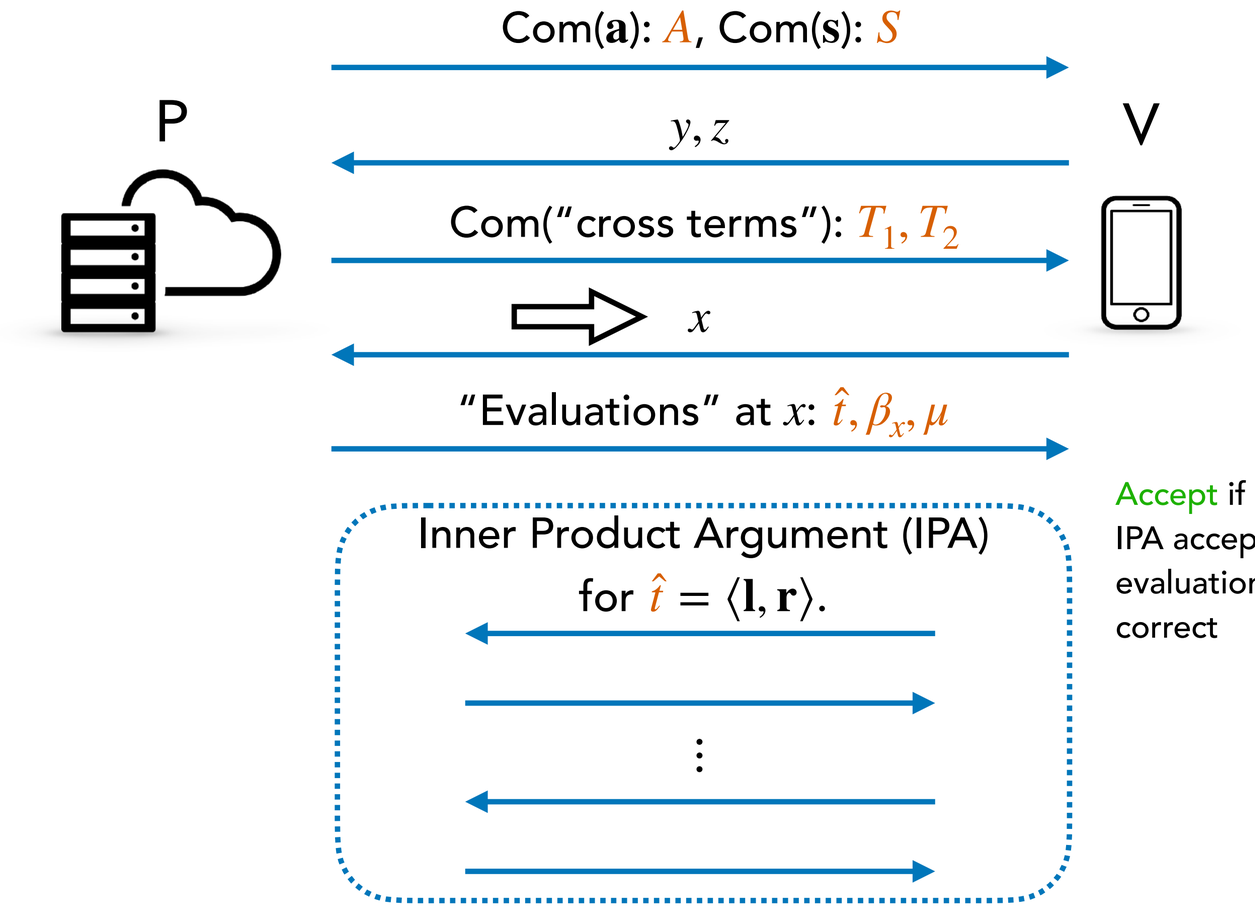

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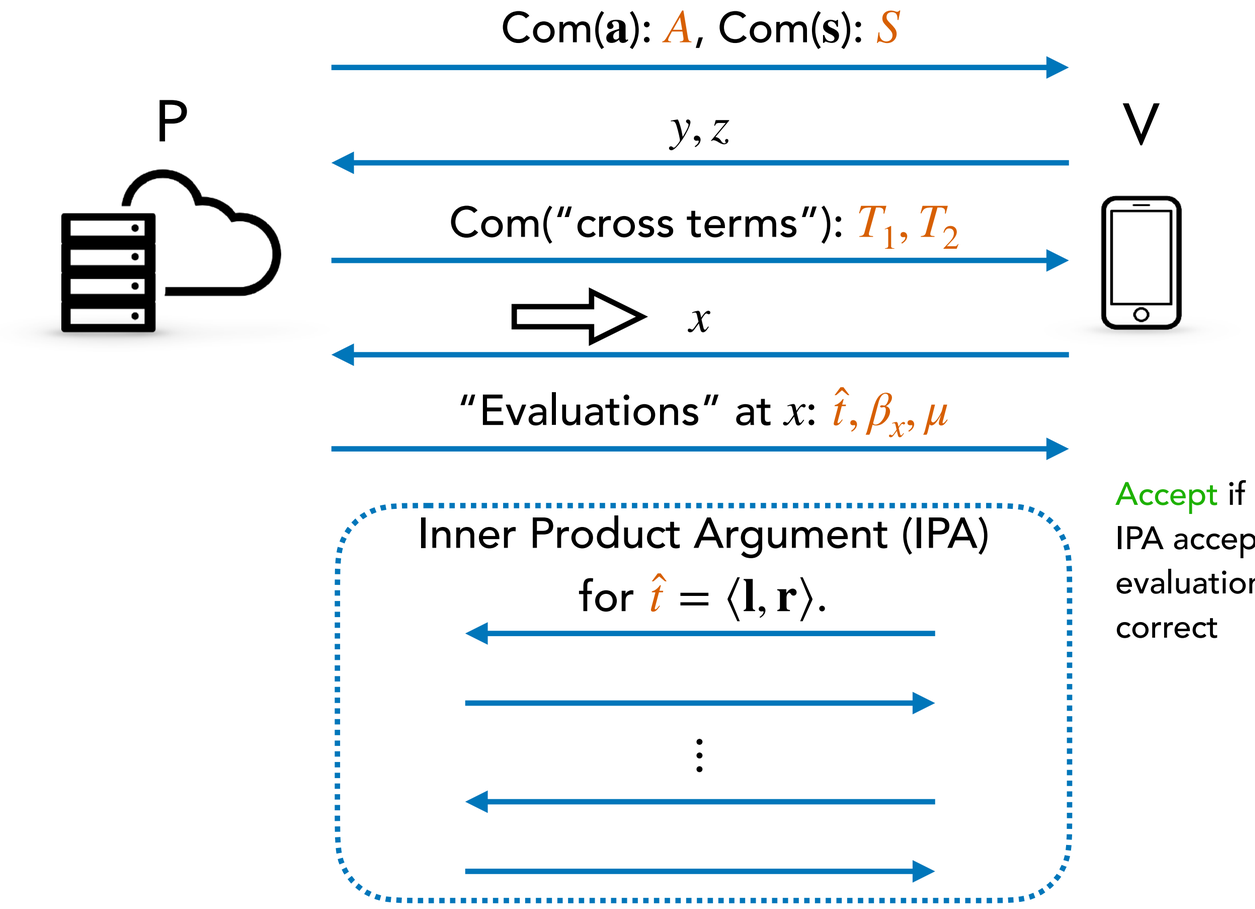

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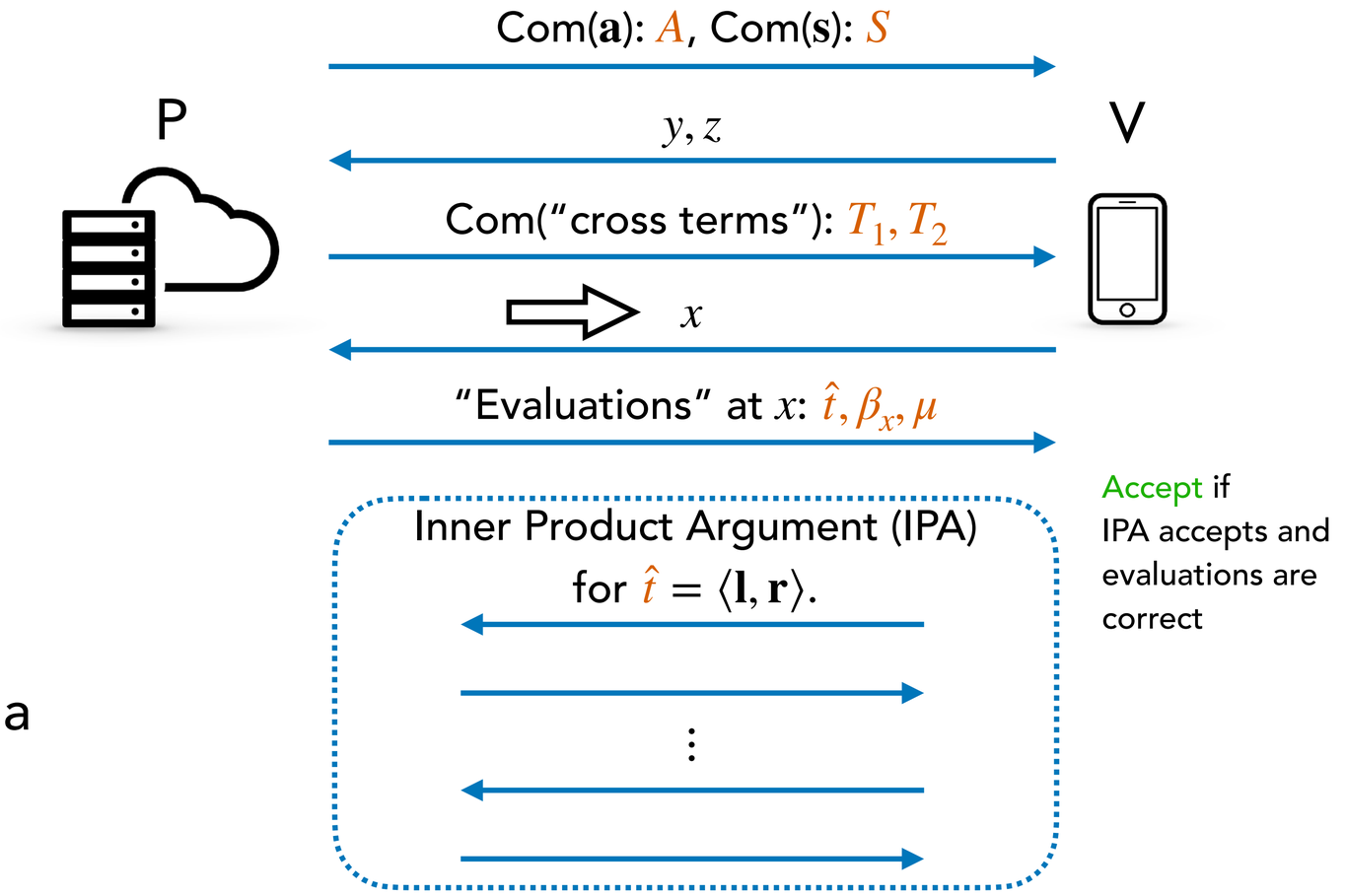
1. Run the honest prover's algorithm with a *"fake"* witness.



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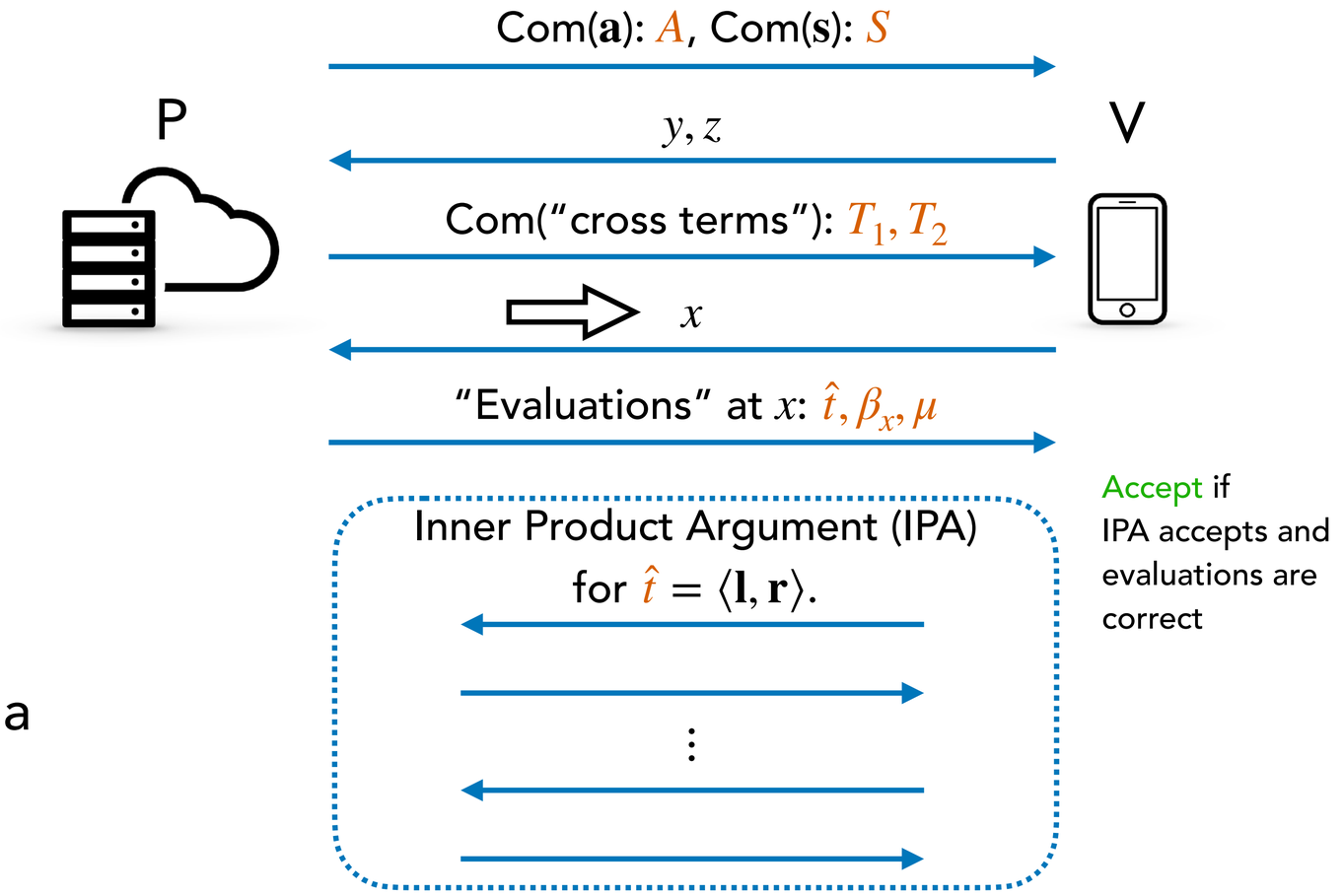
Idea:

1. Run the honest prover's algorithm with a *"fake"* witness.
2. Resolve contradiction via choosing k^{th} and $(k+1)^{th}$ message *at the same time*.



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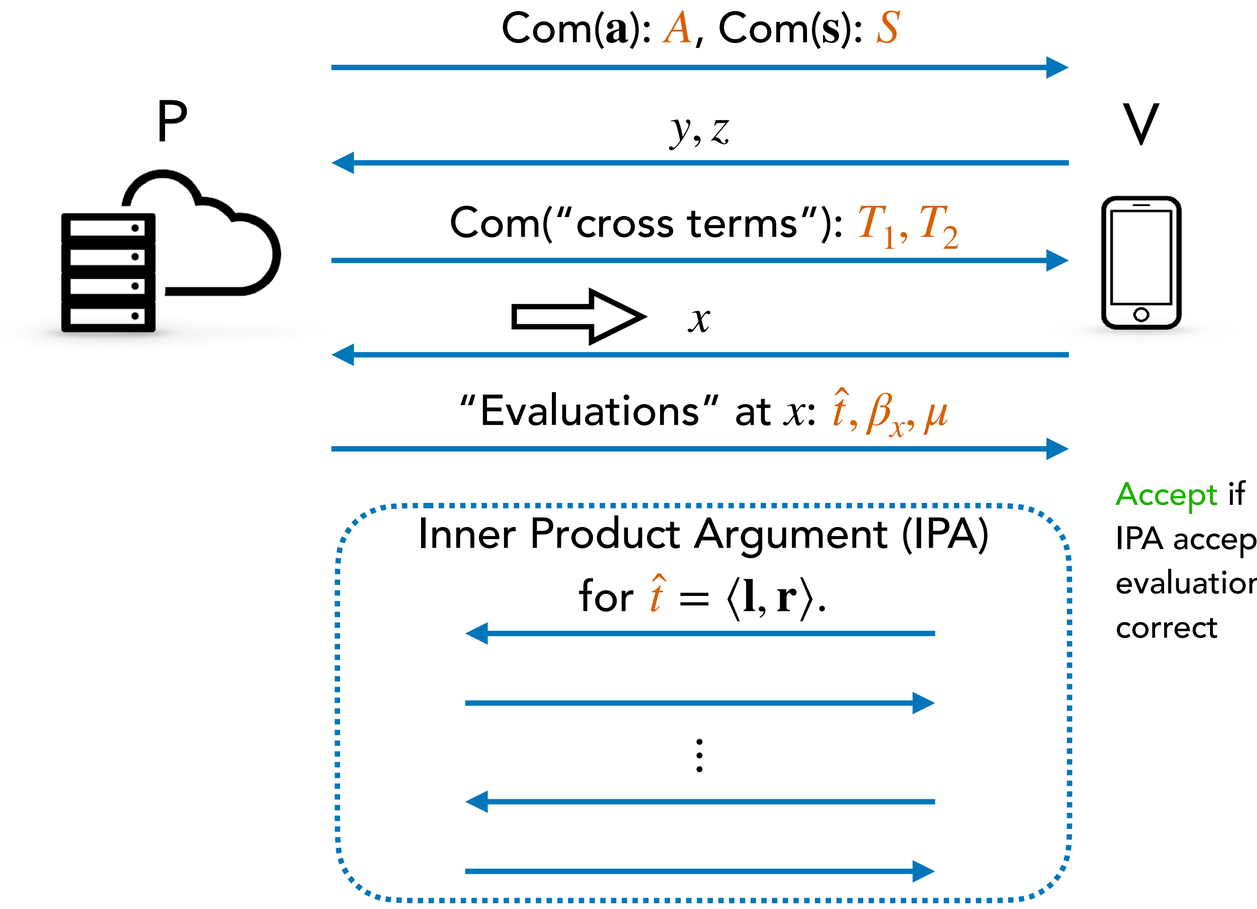

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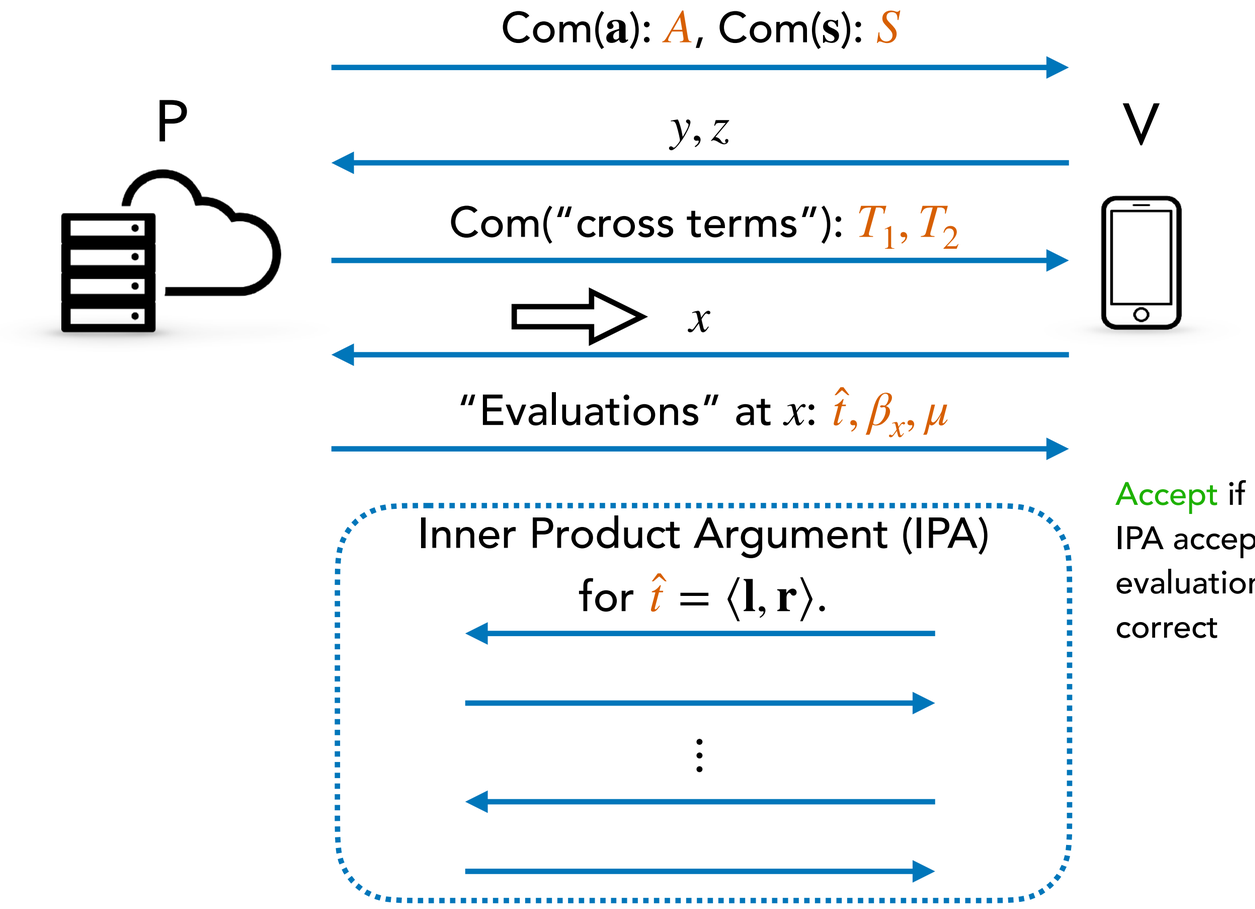

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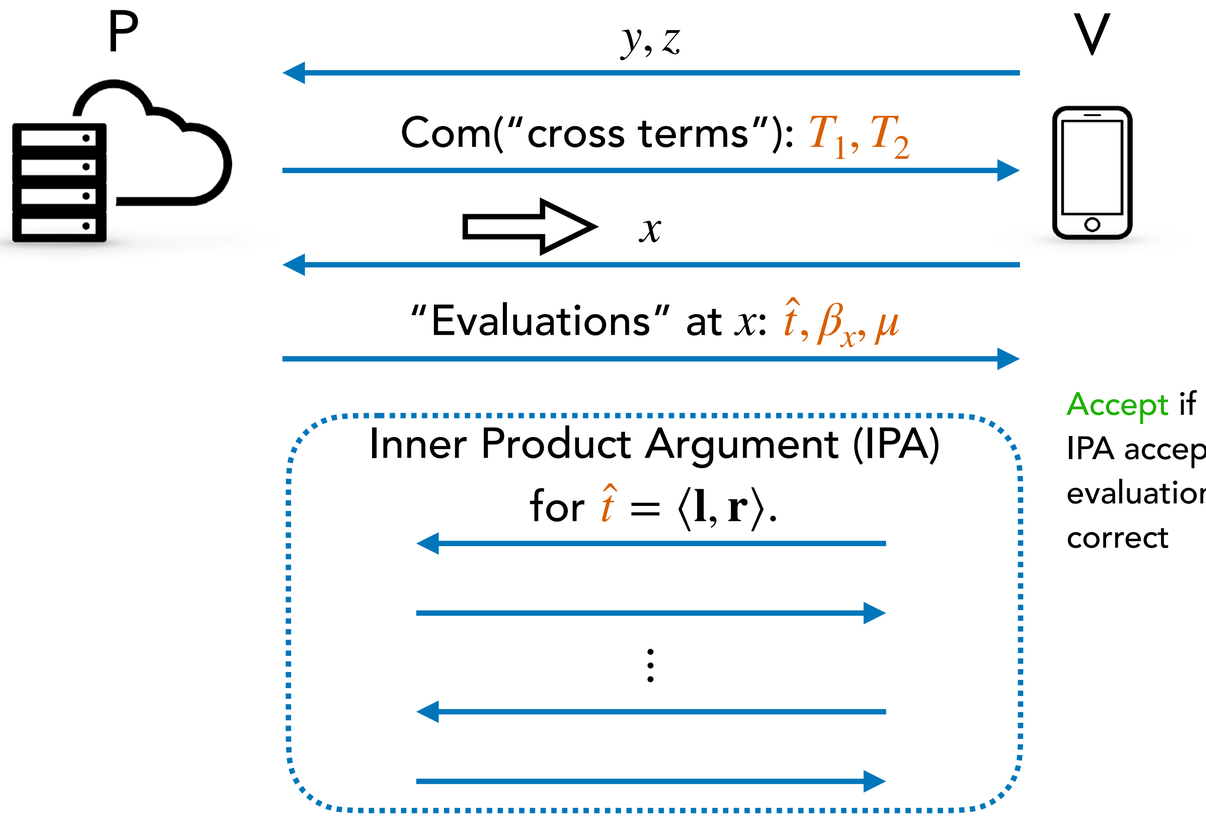

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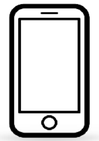
3. Pick *random* evaluations $\hat{t}, \beta_x, \mu$.
   Choose T_1, T_2 *consistent* with evaluations.

P

V



Com($\mathbf{a}$): A, Com($\mathbf{s}$): S

y, z

Com("cross terms"): T_1, T_2

x

"Evaluations" at x: $\hat{t}, \beta_x, \mu$

Inner Product Argument (IPA)
for $\hat{t} = \langle \mathbf{l}, \mathbf{r} \rangle$.

Accept if
IPA accepts and
evaluations are
correct

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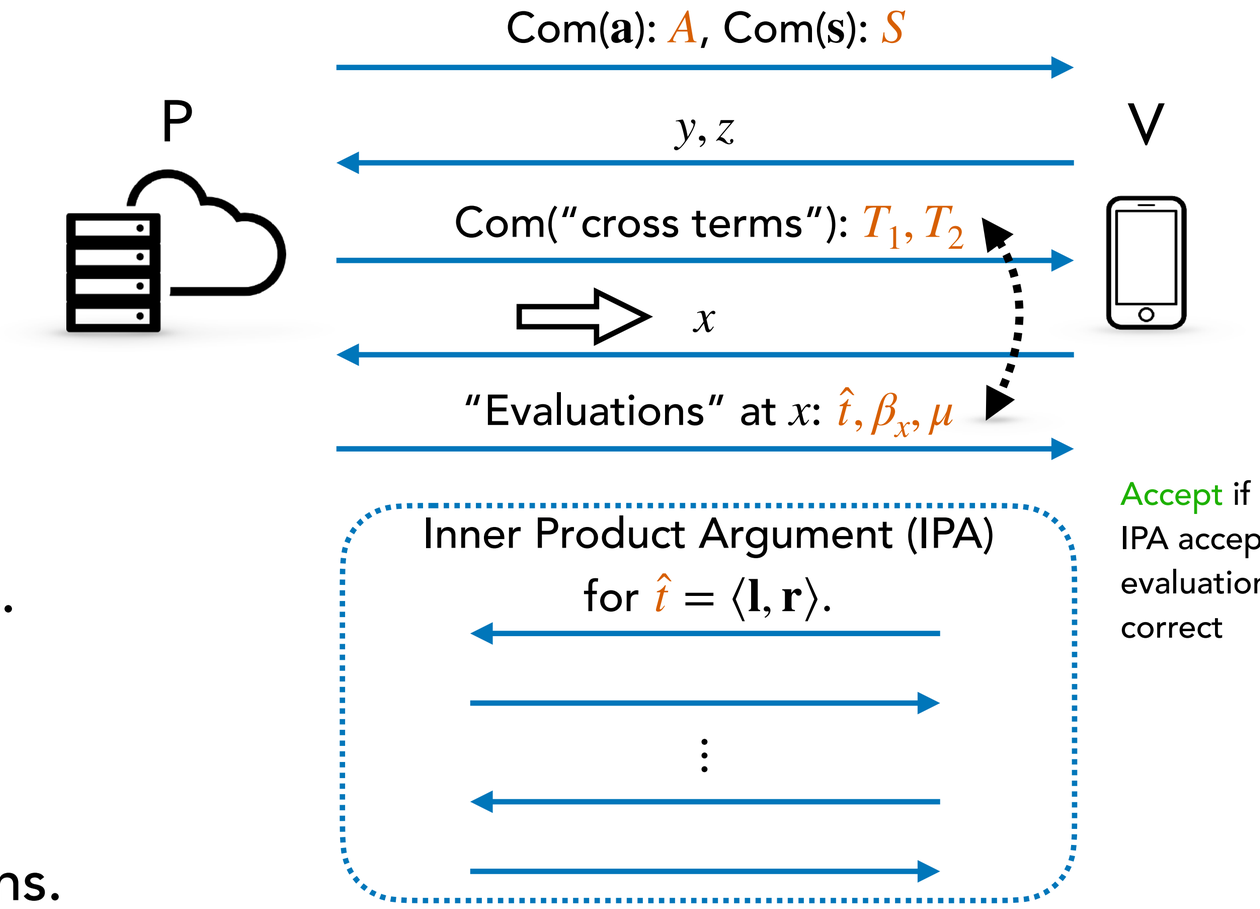

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$$g^{\hat{t}} \cdot h^{\beta_x} = V^{z^2} \cdot g^{\delta(y,z)} \cdot T_1^{\,x} \cdot T_2^{\,x^2}$$

(eval check)

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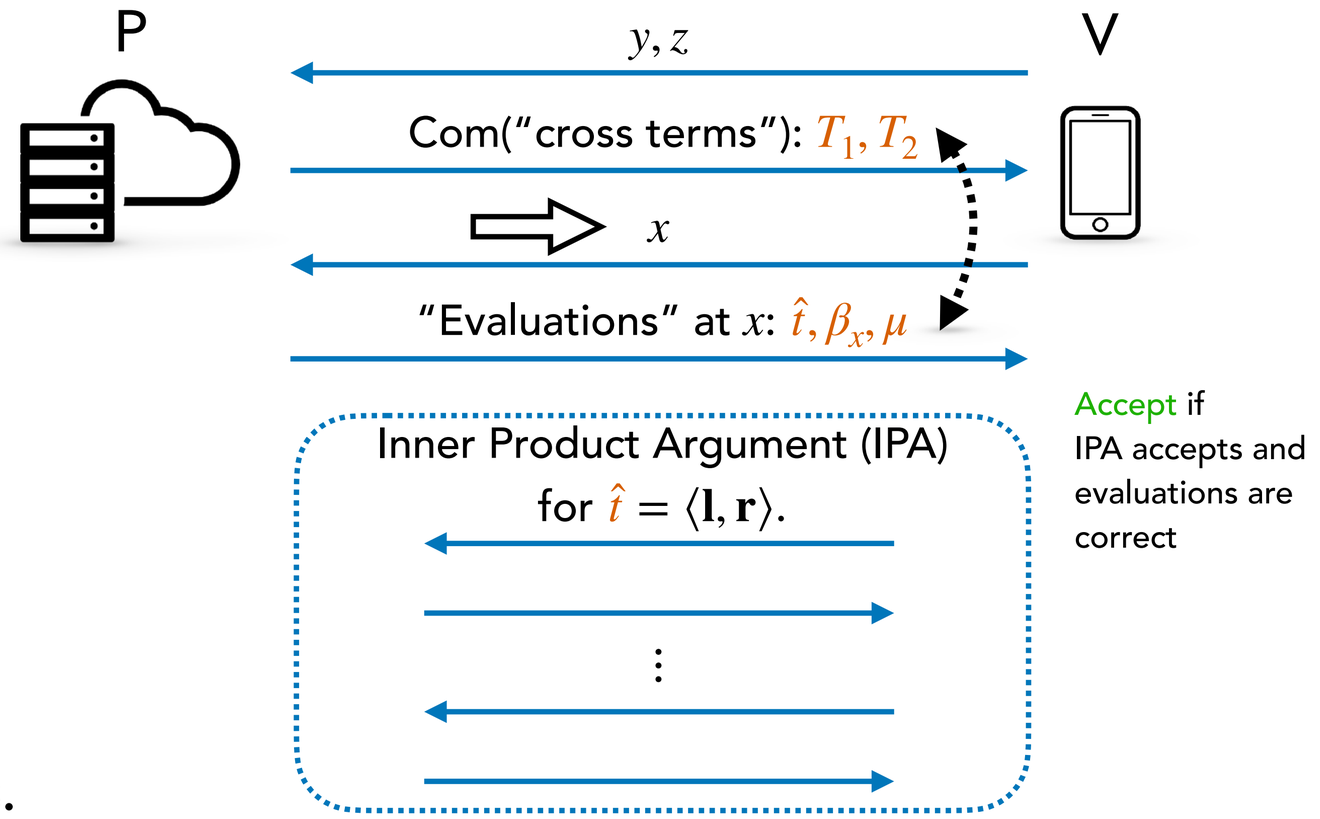
4. Execute IPA with satisfying witness $\mathbf{l}, \mathbf{r}$
   (derived from $\mathbf{a}, \mathbf{s}$).



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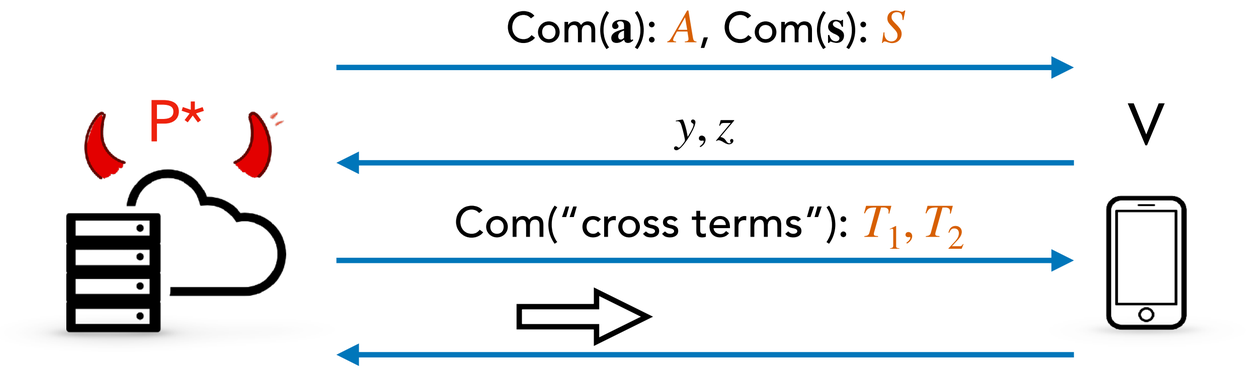

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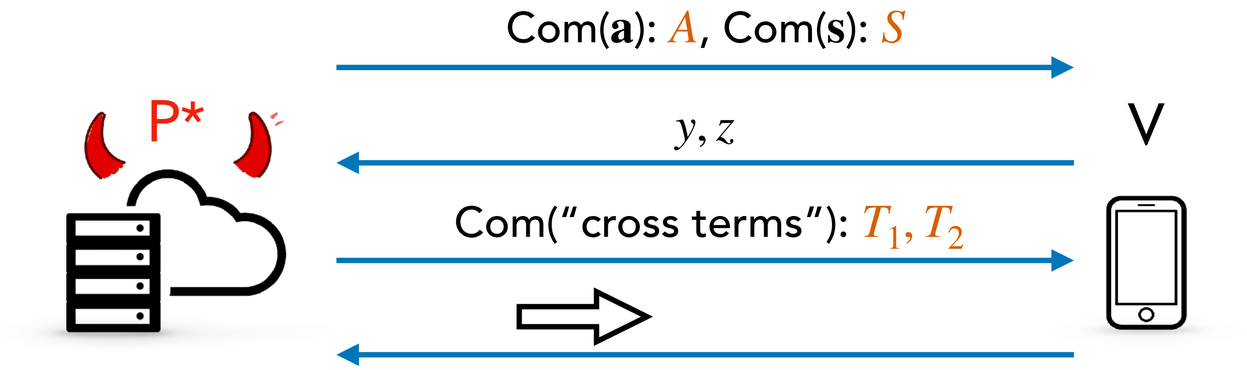

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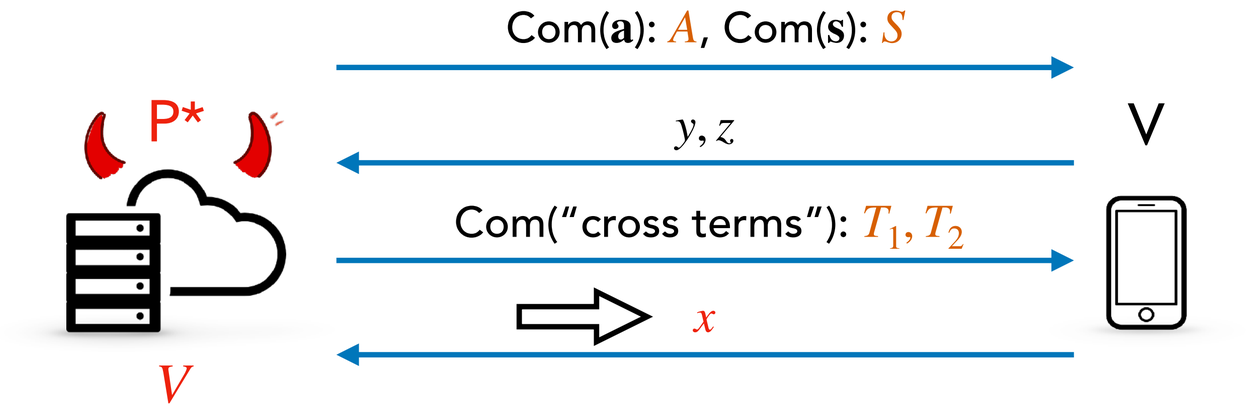

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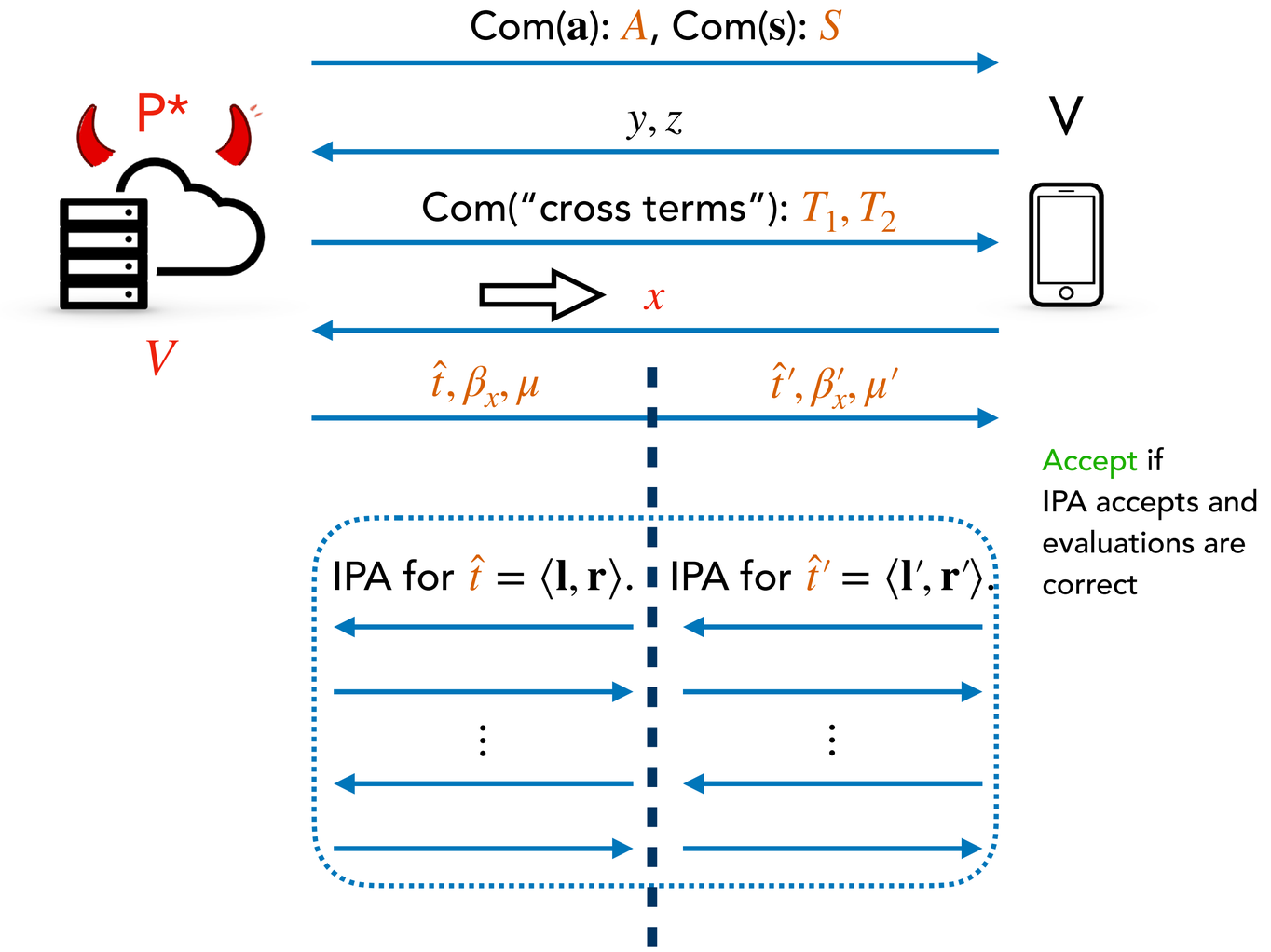

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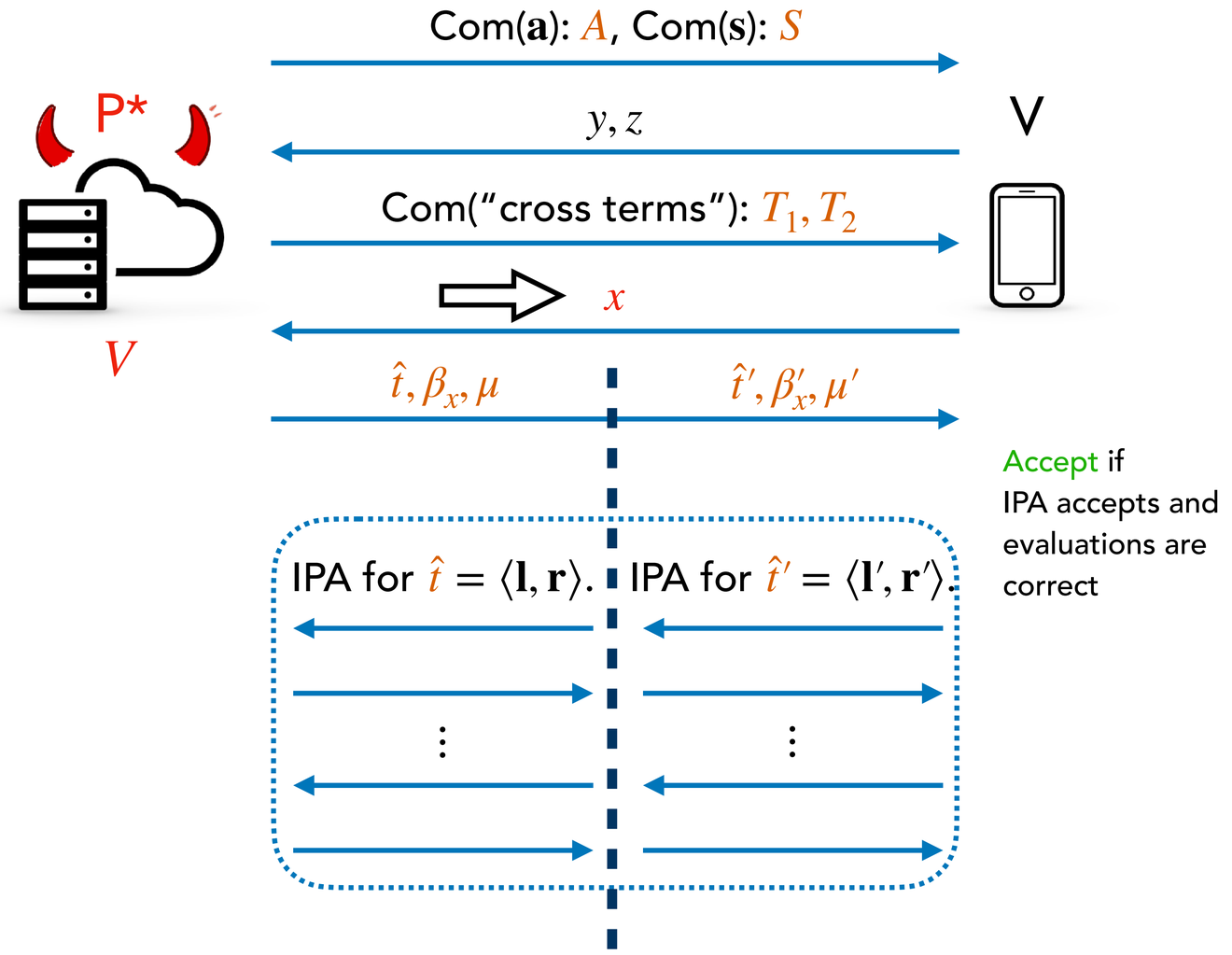
1. Use KS extractor for IPA to extract $(\mathbf{l}, \mathbf{r})$ from π_{IPA}, $(\mathbf{l}', \mathbf{r}')$ from π'_{IPA}.



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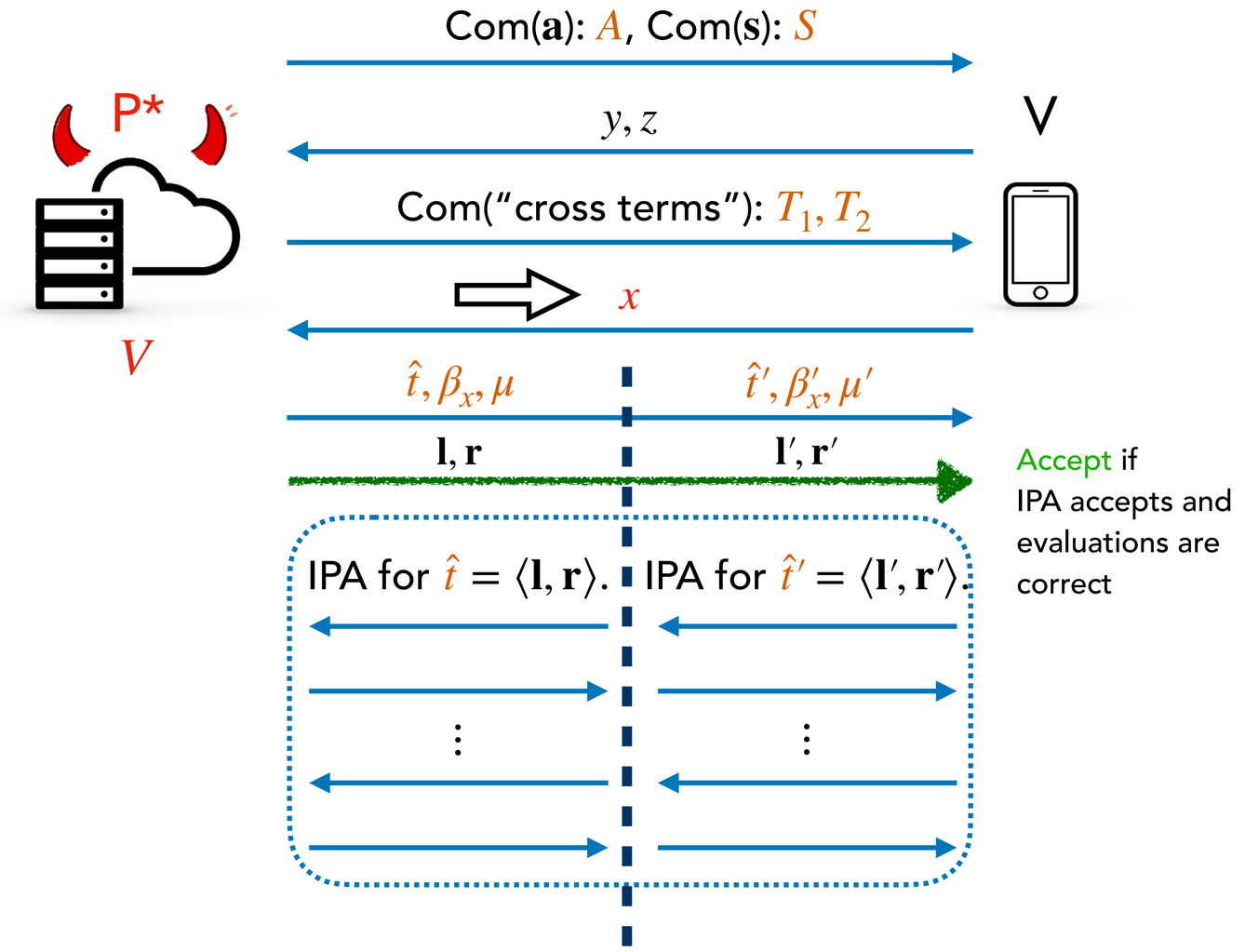

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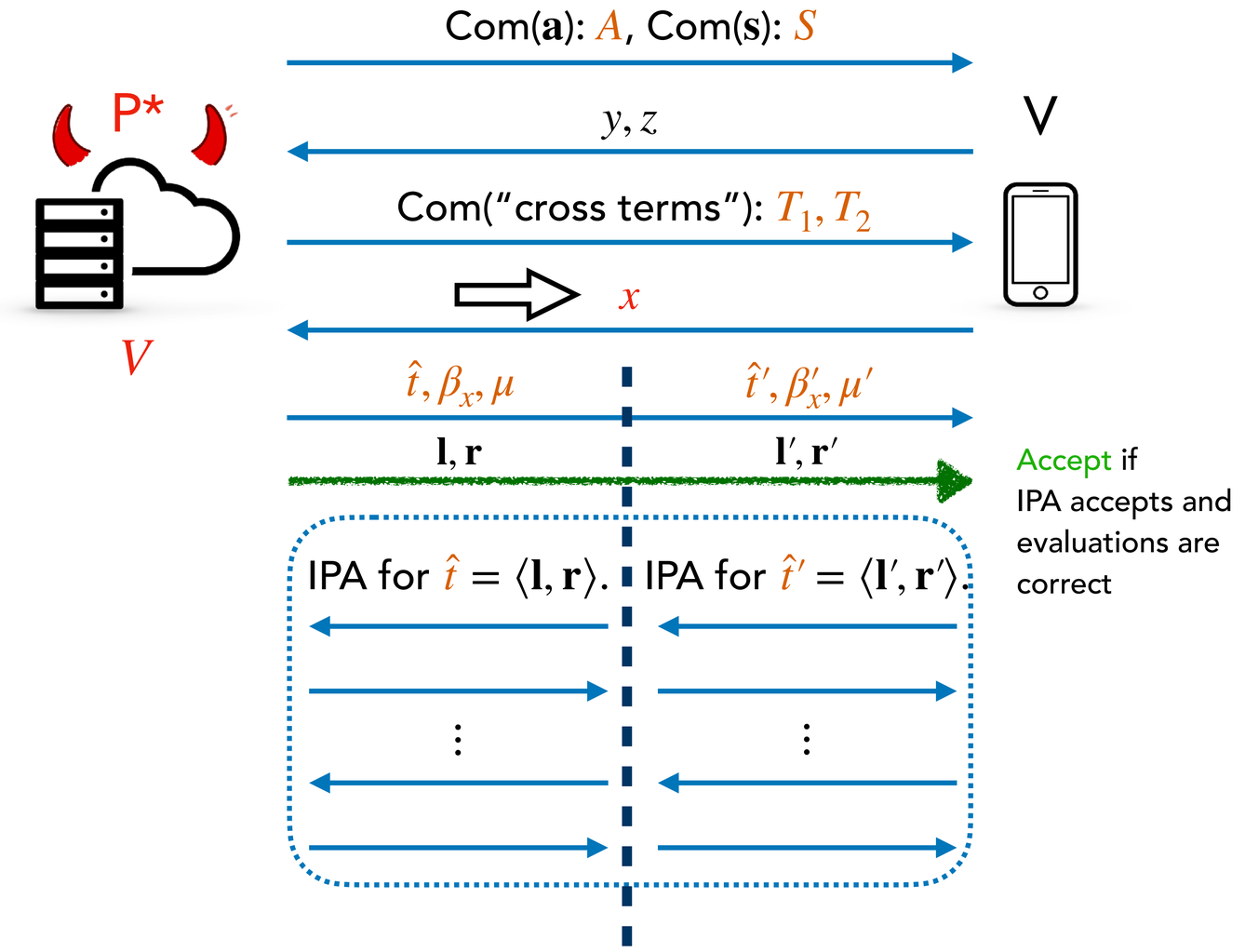
2. If $(\hat{t}, \beta_x) \neq (\hat{t}', \beta'_x)$, we have a non-trivial DLOG relation $\Longrightarrow$ P* breaks DLOG.



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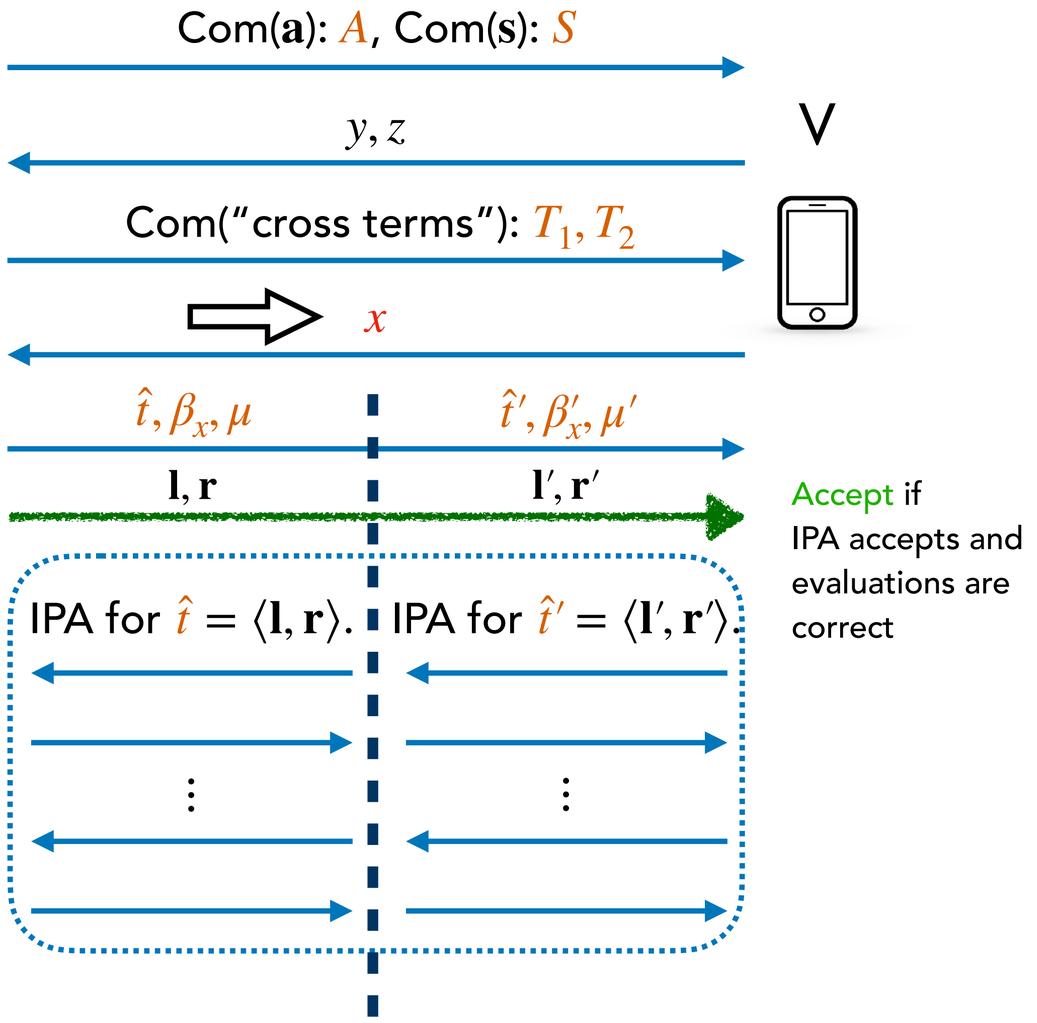
$$g^{\hat{t}} \cdot h^{\beta_x} = V^{z^2} \cdot g^{\delta(y,z)} \cdot T_1^{x} \cdot T_2^{x^2} = g^{\hat{t}'} \cdot h^{\beta'_x}$$

(eval check)



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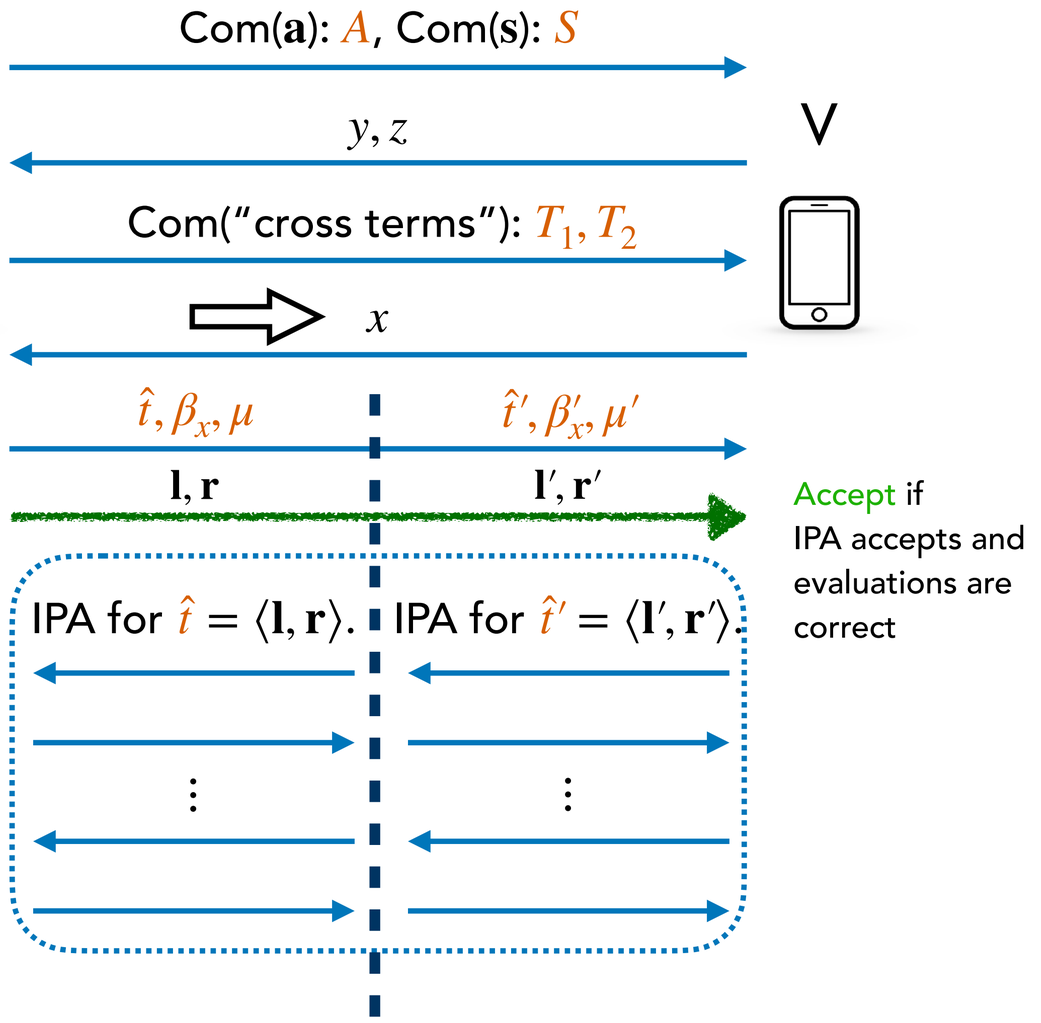
2. If $(\hat{t}, \beta_x) \neq (\hat{t}', \beta'_x)$, we have a non-trivial DLOG relation $\Longrightarrow$ P* breaks DLOG.
3. Else if $(\mathbf{l}, \mathbf{r}, \mu) \neq (\mathbf{l}', \mathbf{r}', \mu')$, we also get a non-trivial DLOG relation $\Longrightarrow$ P* breaks DLOG.

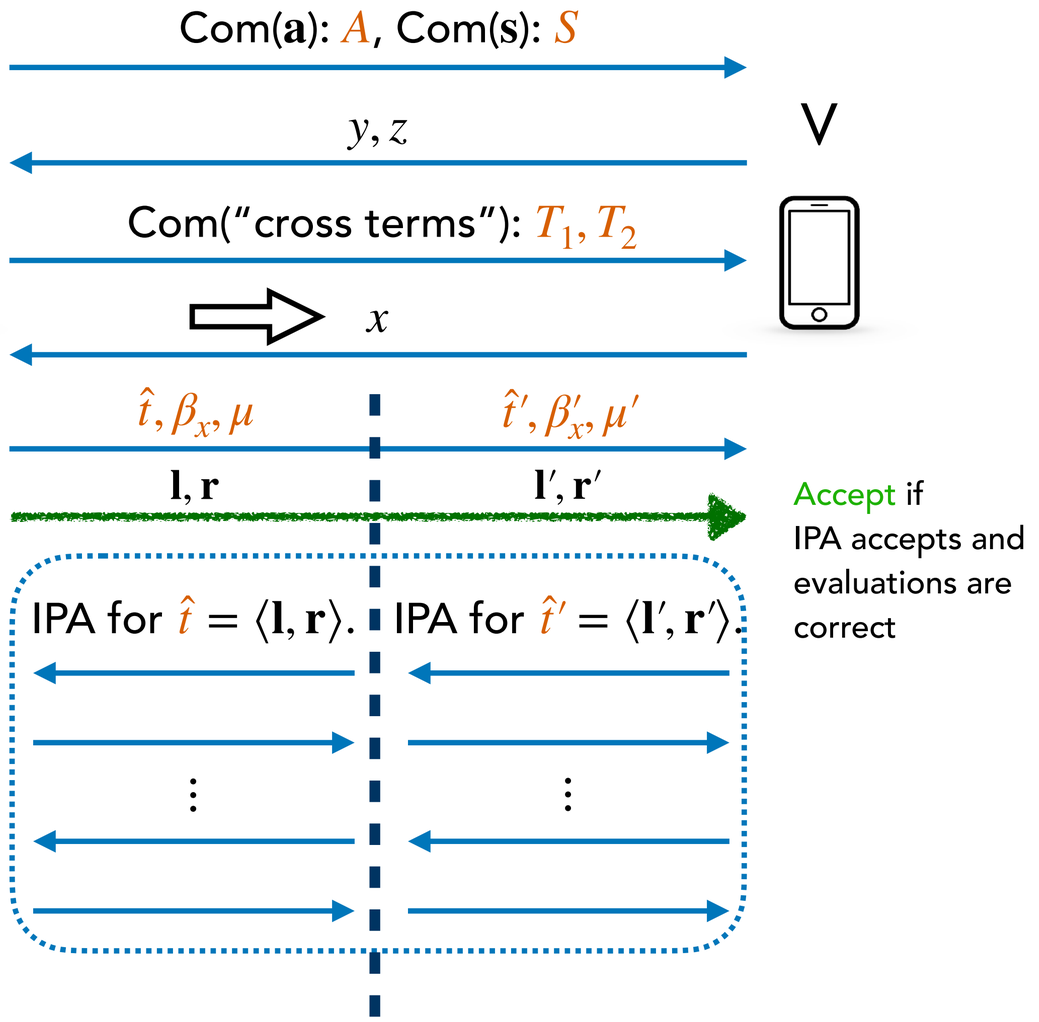


Bulletproofs Range Proof

Let $\mathbf{a} = \text{BinDecomp}(v)$.

2-UR: P* cannot produce two accepting proofs $\pi \neq \pi'$ that agree on A, S, T_1, T_2 (even if it can choose V and x).

1. Use KS extractor for IPA to extract $(\mathbf{l}, \mathbf{r})$ from π_{IPA}, $(\mathbf{l}', \mathbf{r}')$ from π'_{IPA}.
2. If $(\hat{t}, \beta_x) \neq (\hat{t}', \beta'_x)$, we have a non-trivial DLOG relation $\Longrightarrow$ P* breaks DLOG.
3. Else if $(\mathbf{l}, \mathbf{r}, \mu) \neq (\mathbf{l}', \mathbf{r}', \mu')$, we also get a non-trivial DLOG relation $\Longrightarrow$ P* breaks DLOG.
4. Else $(\mathbf{l}, \mathbf{r}) = (\mathbf{l}', \mathbf{r}')$ but $\pi_{\text{IPA}} \neq \pi'_{\text{IPA}} \Longrightarrow$ P* breaks DLOG.



Agenda

1. SIM-EXT = KS + k-ZK + k-UR (for same k)
2. k-ZK and k-UR for Bulletproofs
3. **Knowledge Soundness via Generalized Tree Builder**

Knowledge Soundness from Special Soundness

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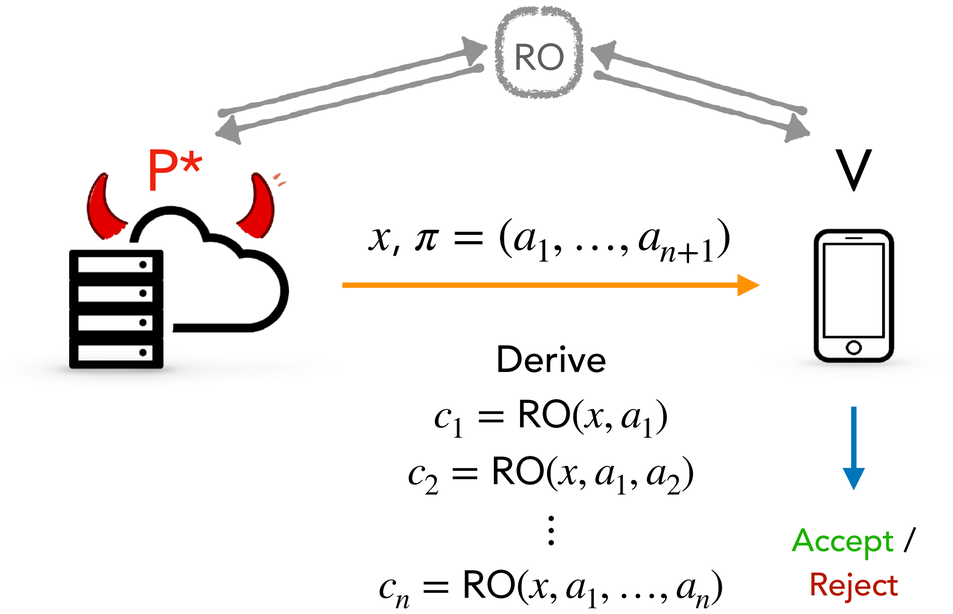
F-S Argument:



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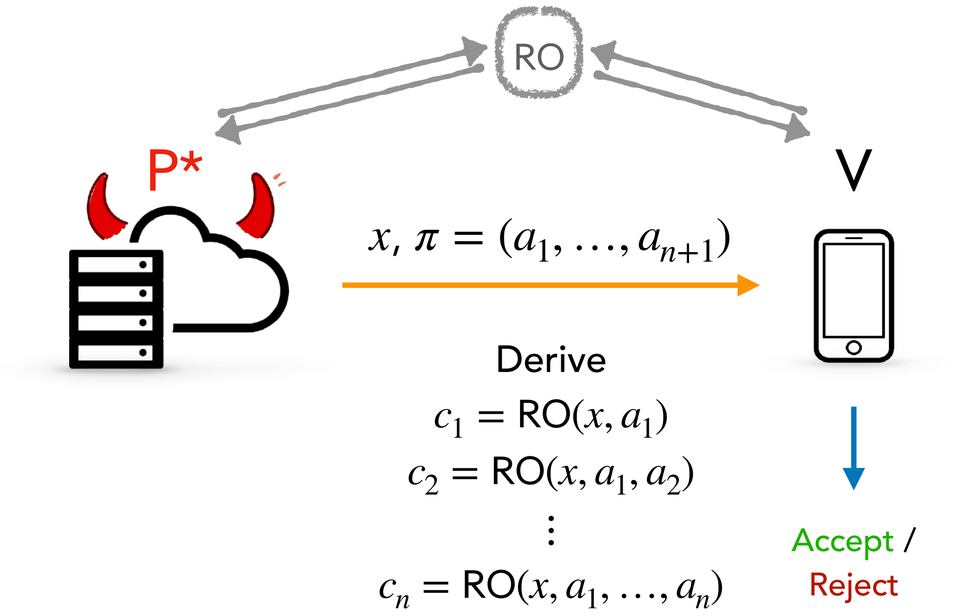

$(k_1, \ldots, k_n)$-Tree of Accepting Transcripts

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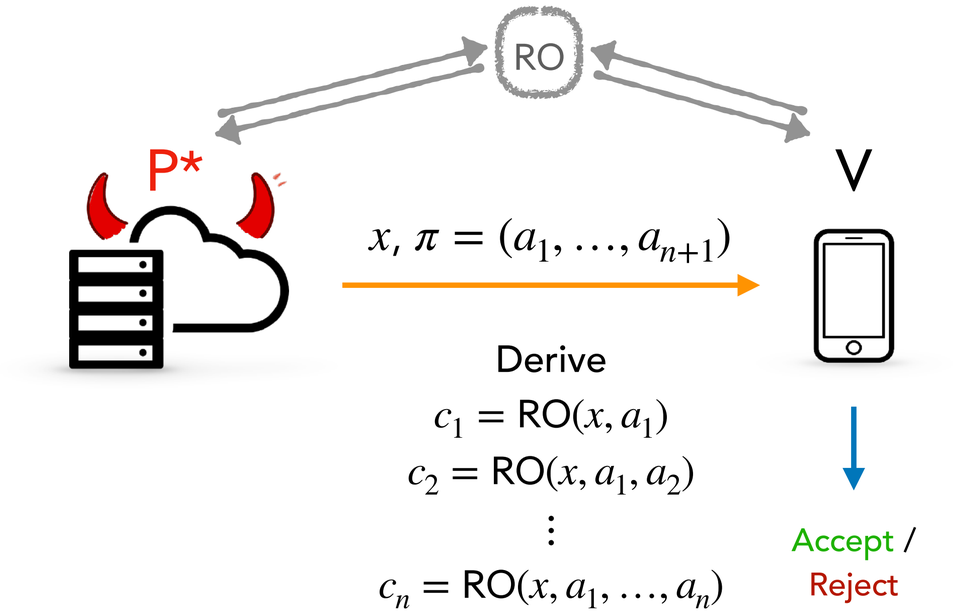

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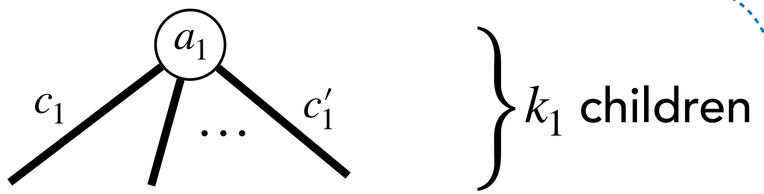

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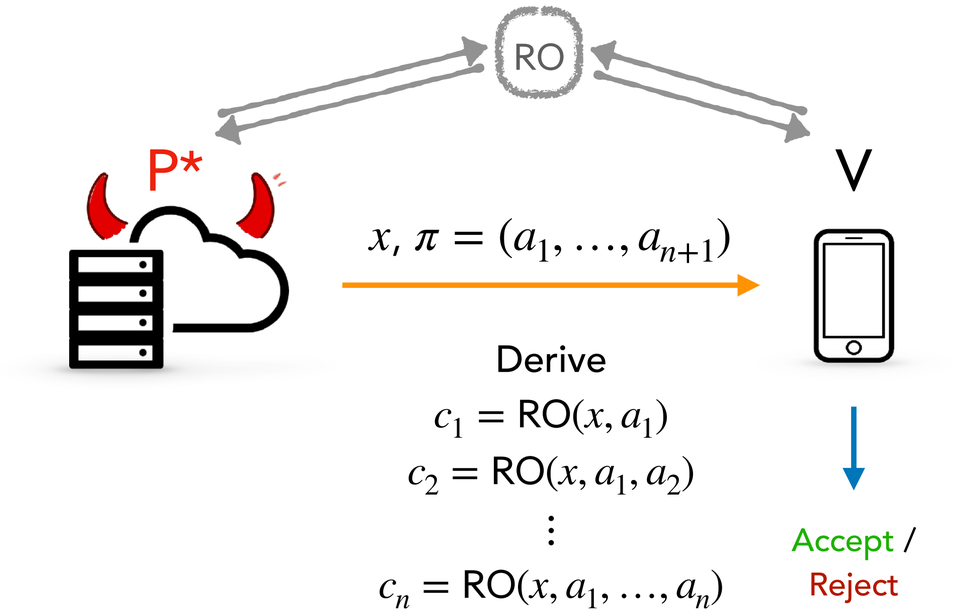

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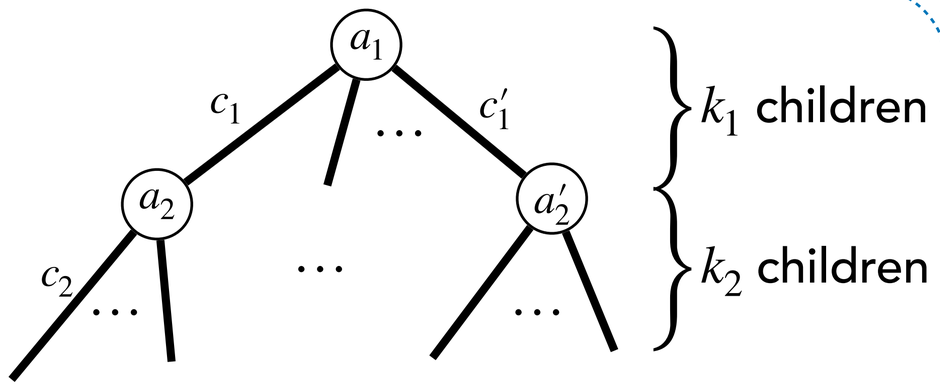

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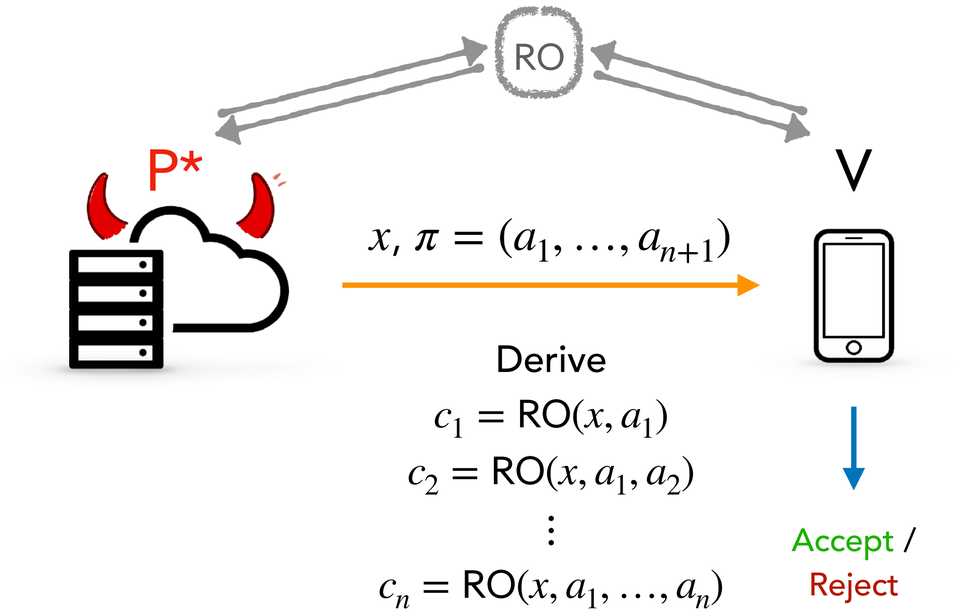

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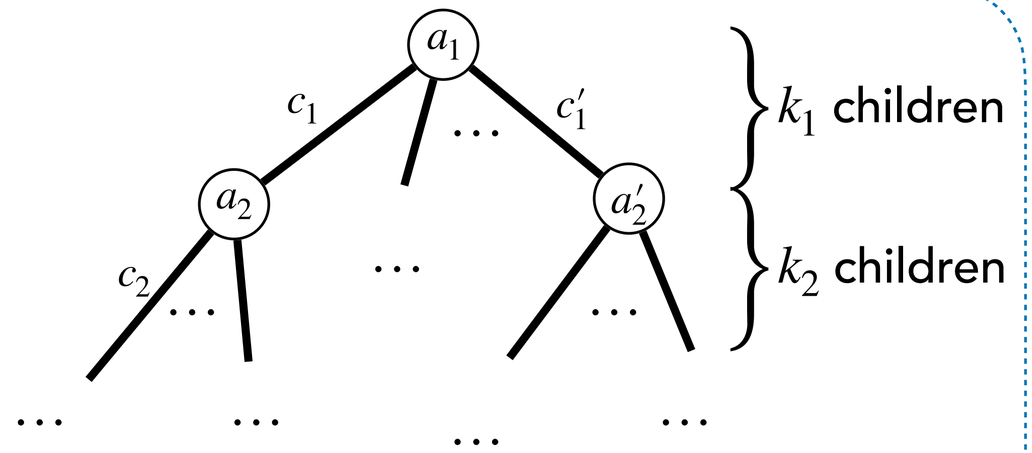

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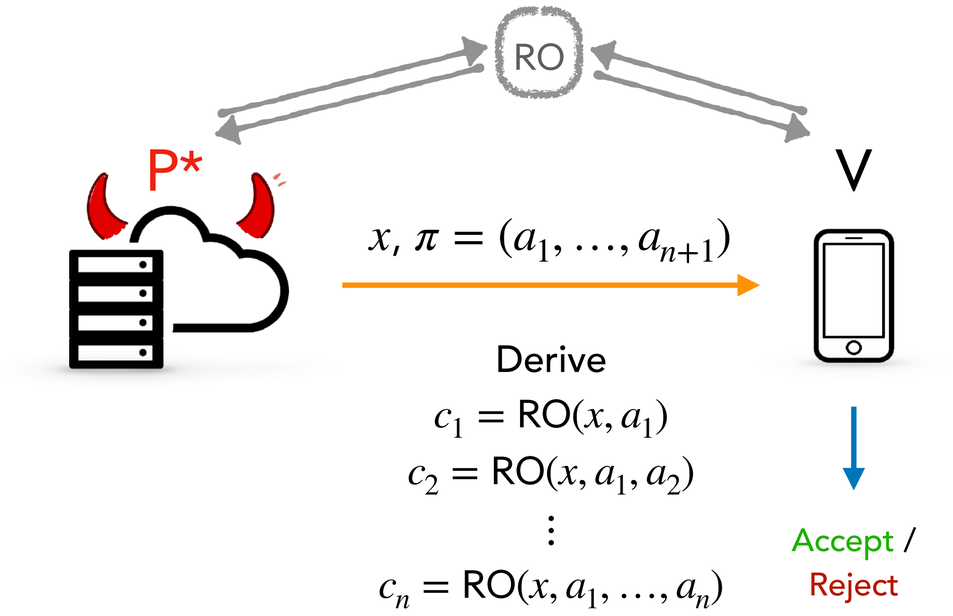

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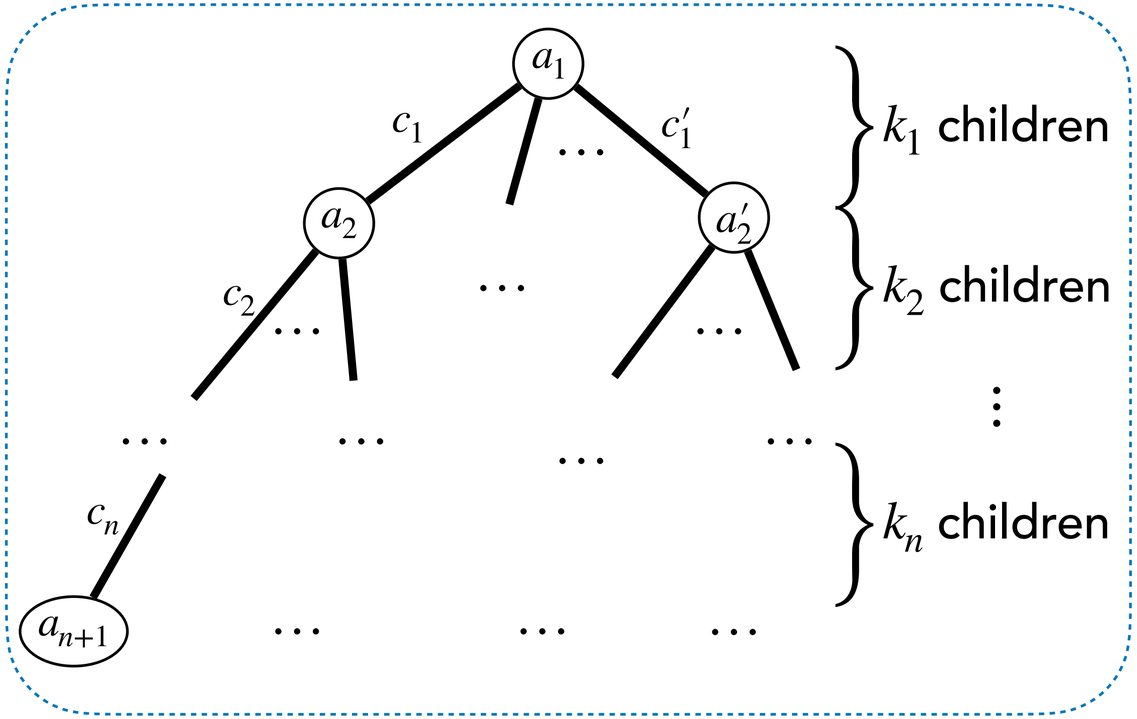

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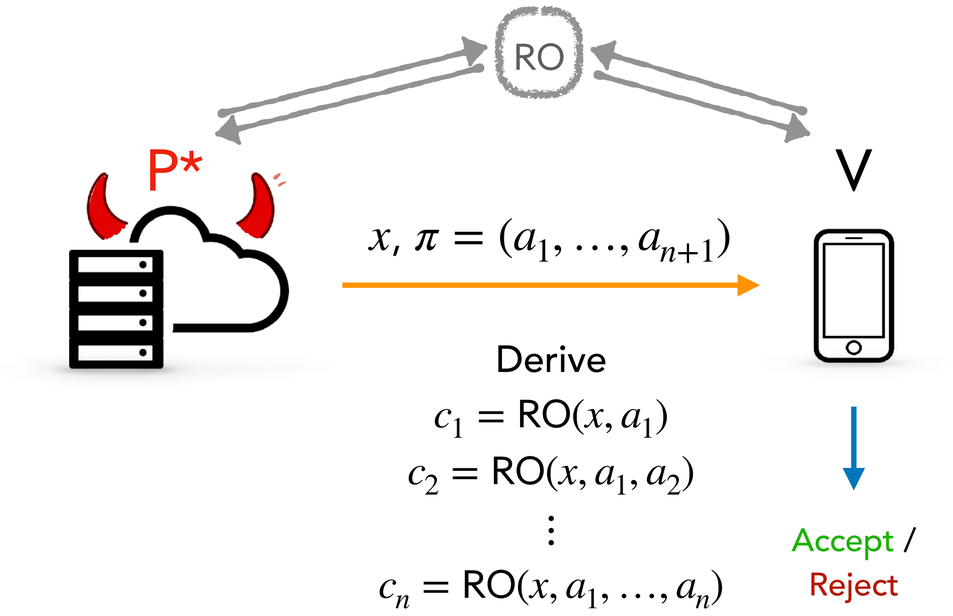

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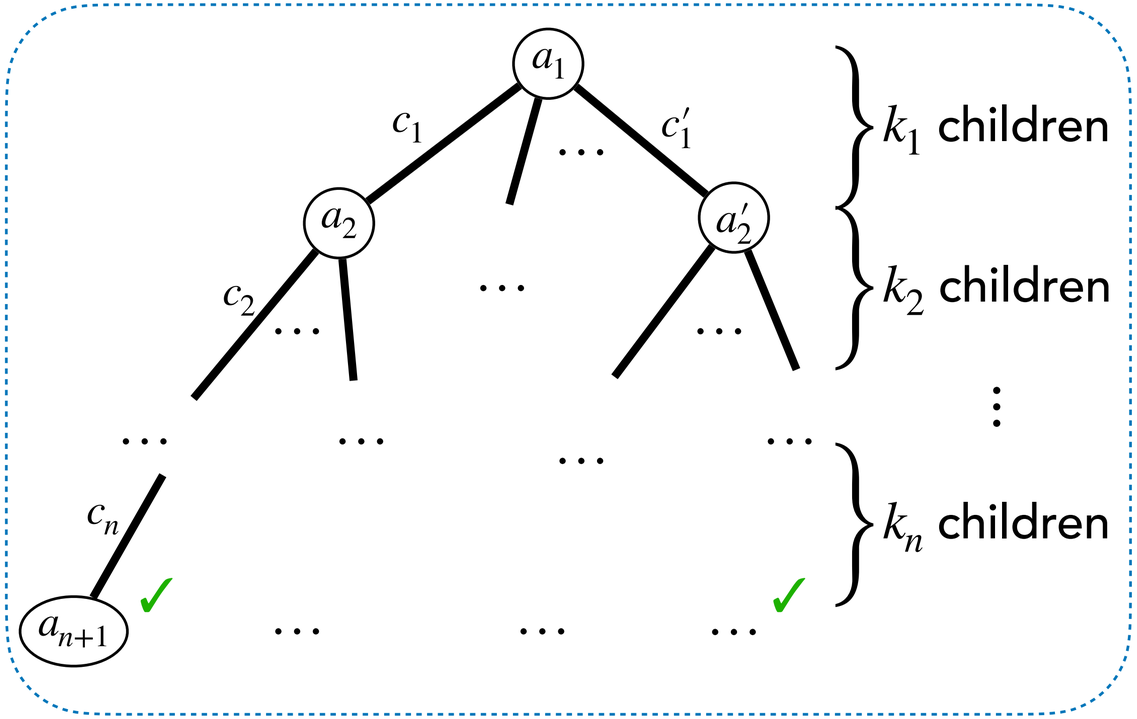

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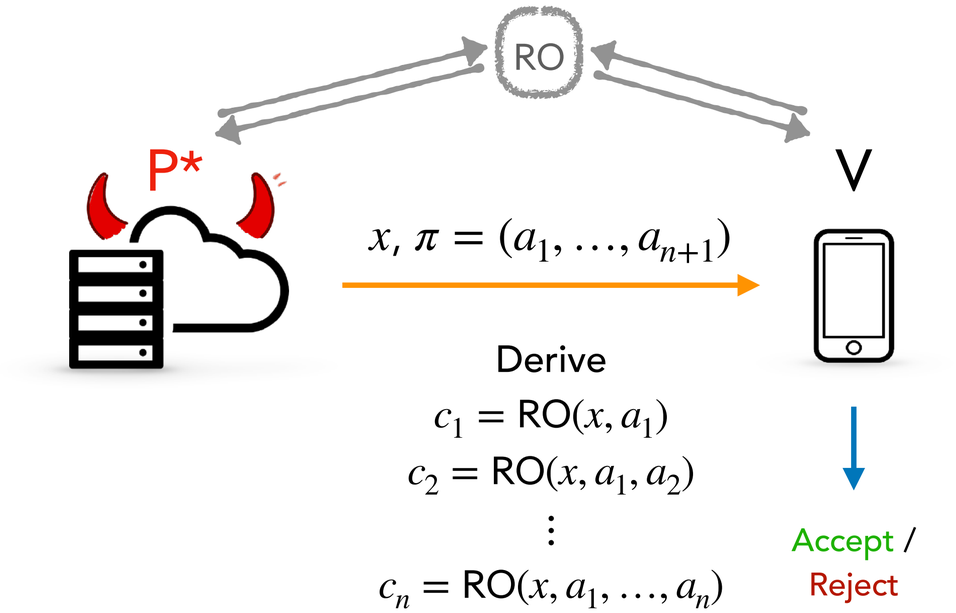

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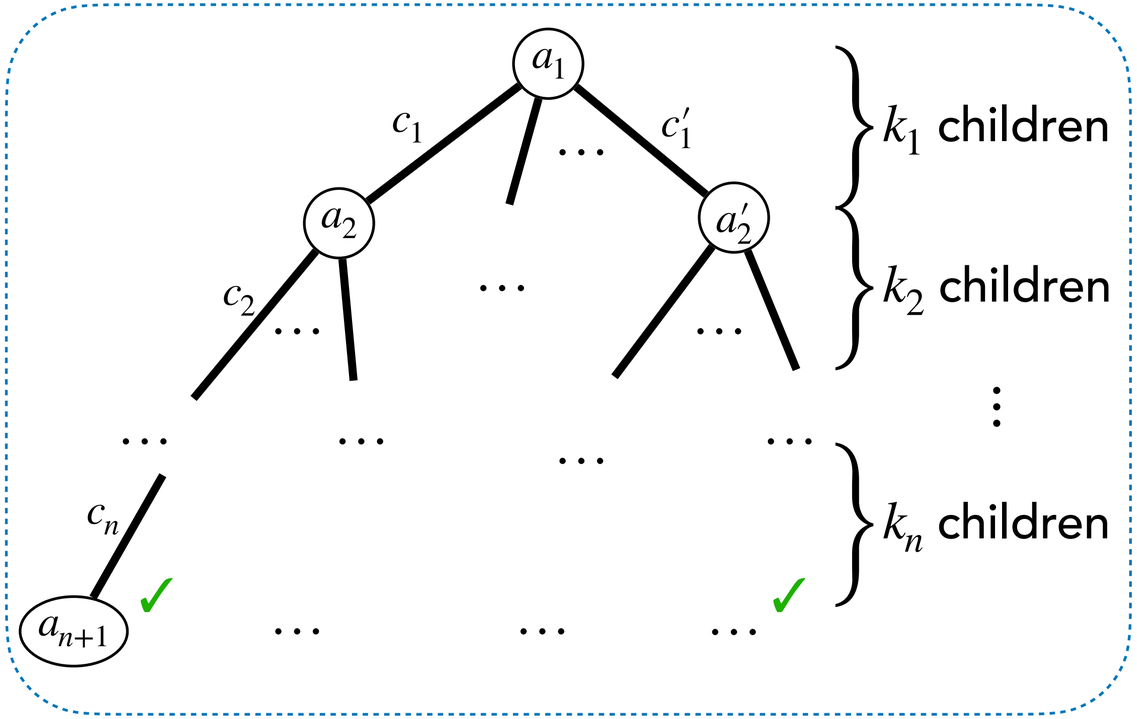

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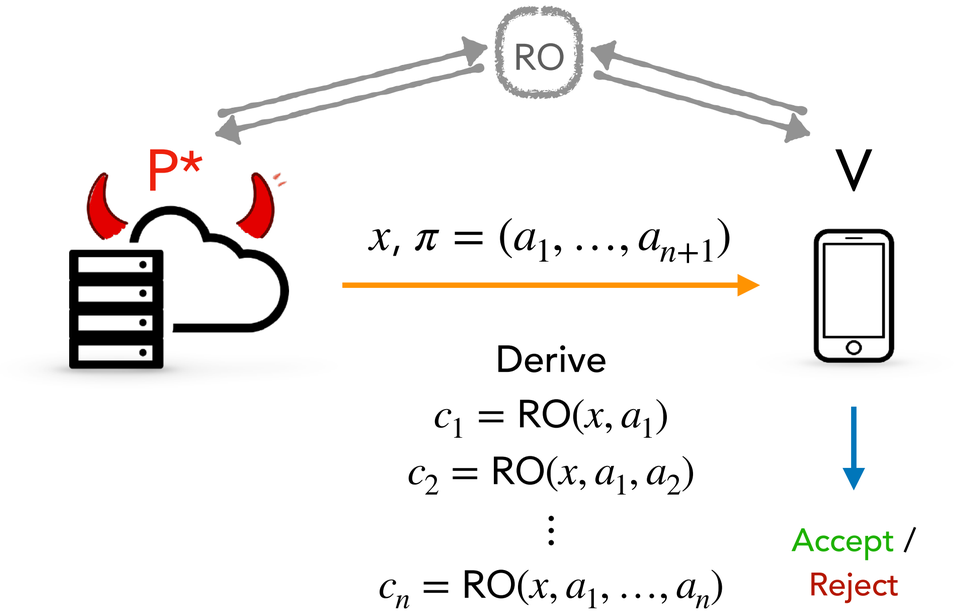

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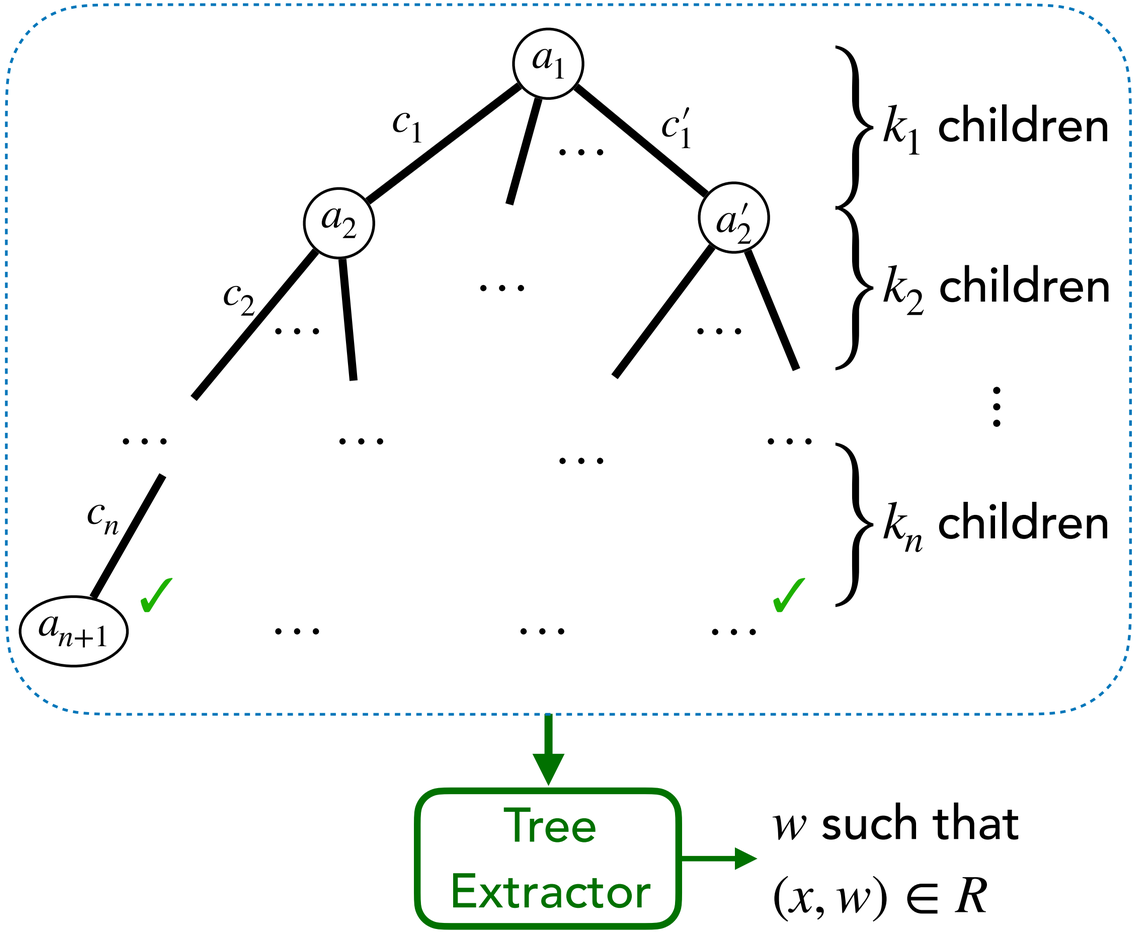

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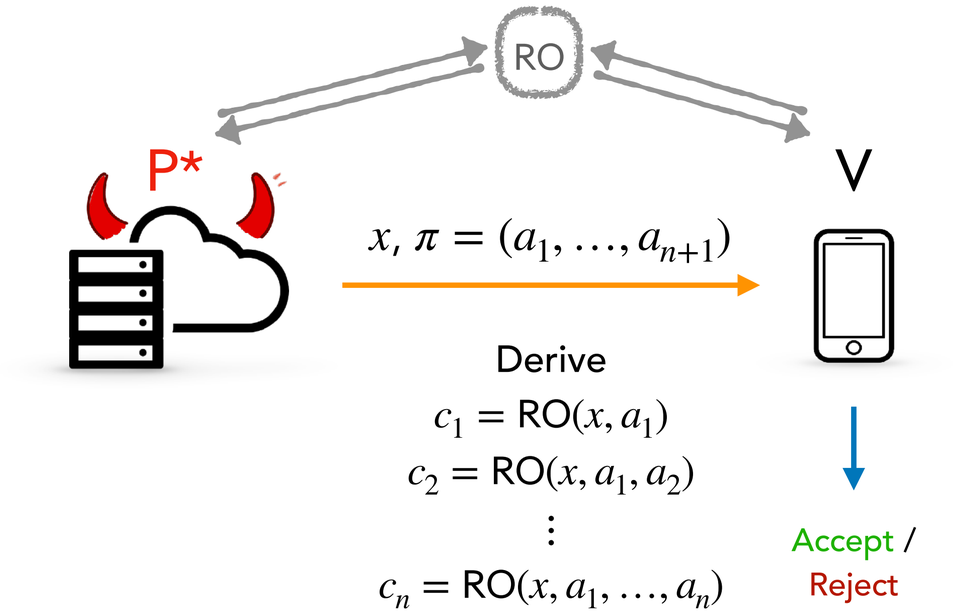

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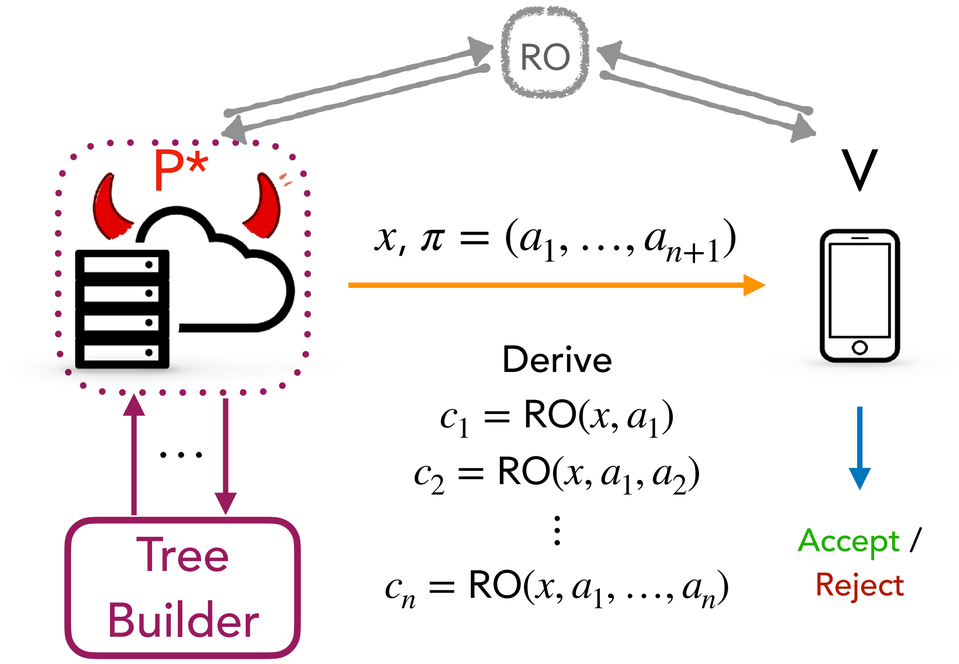

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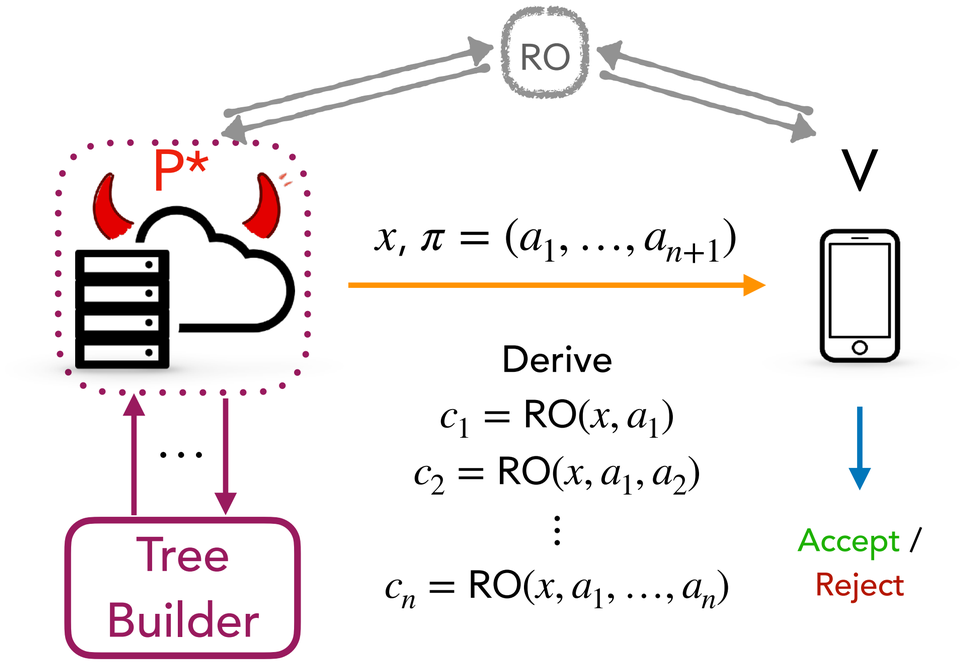

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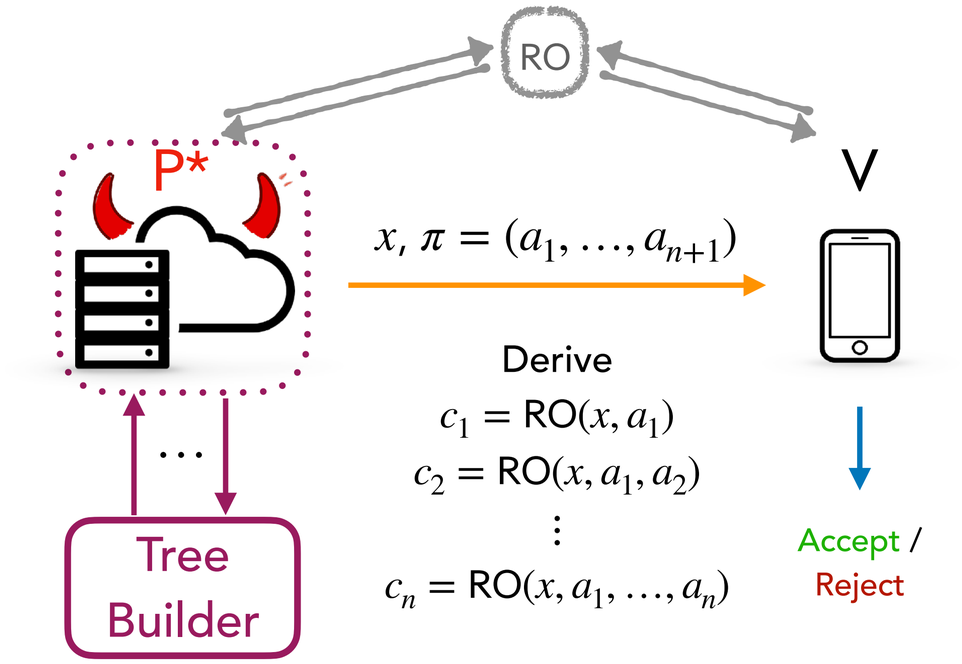

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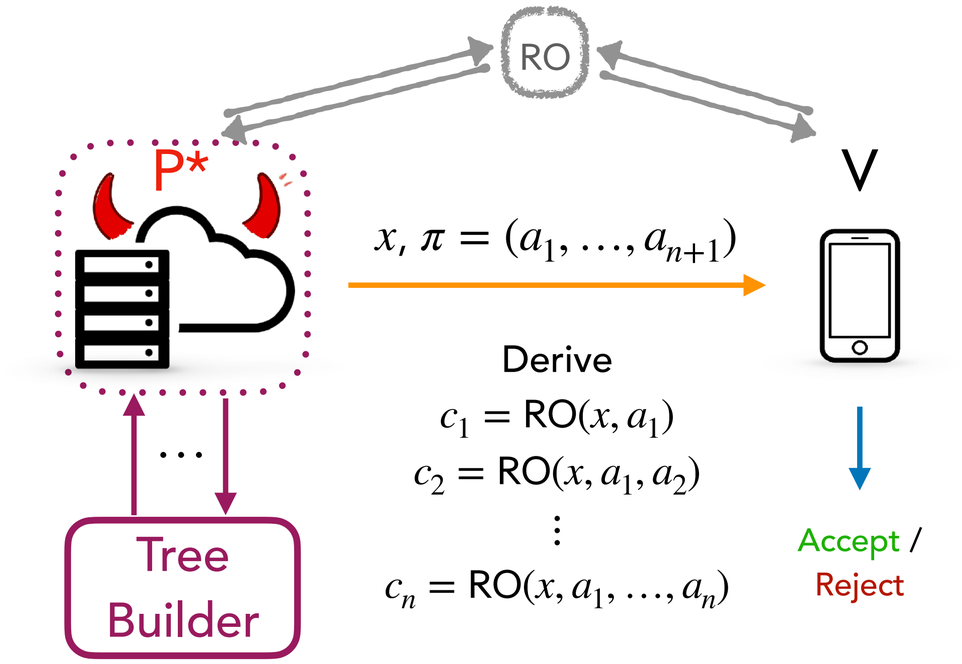

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Corollary: If a proof system satisfies special soundness, then it satisfies knowledge soundness.

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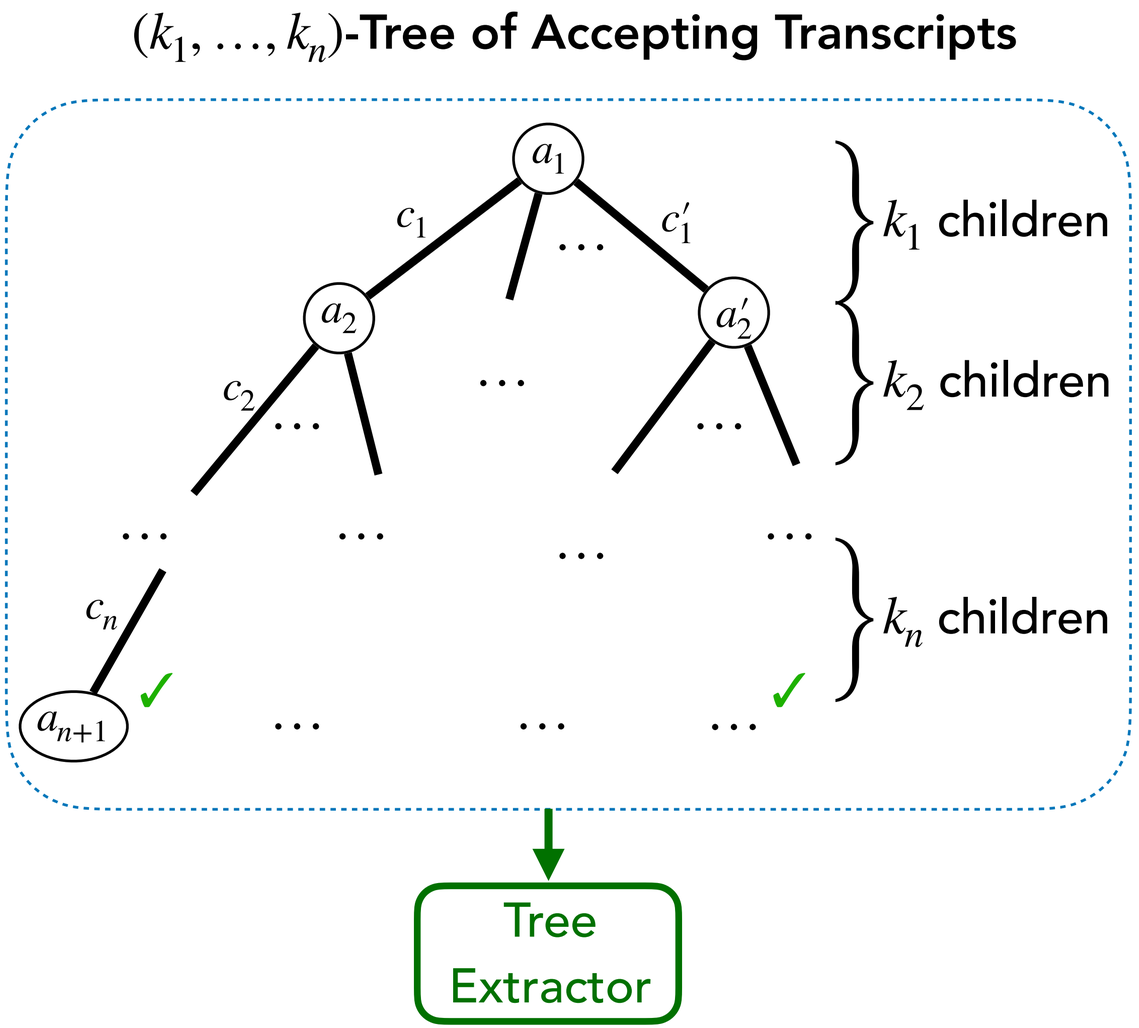

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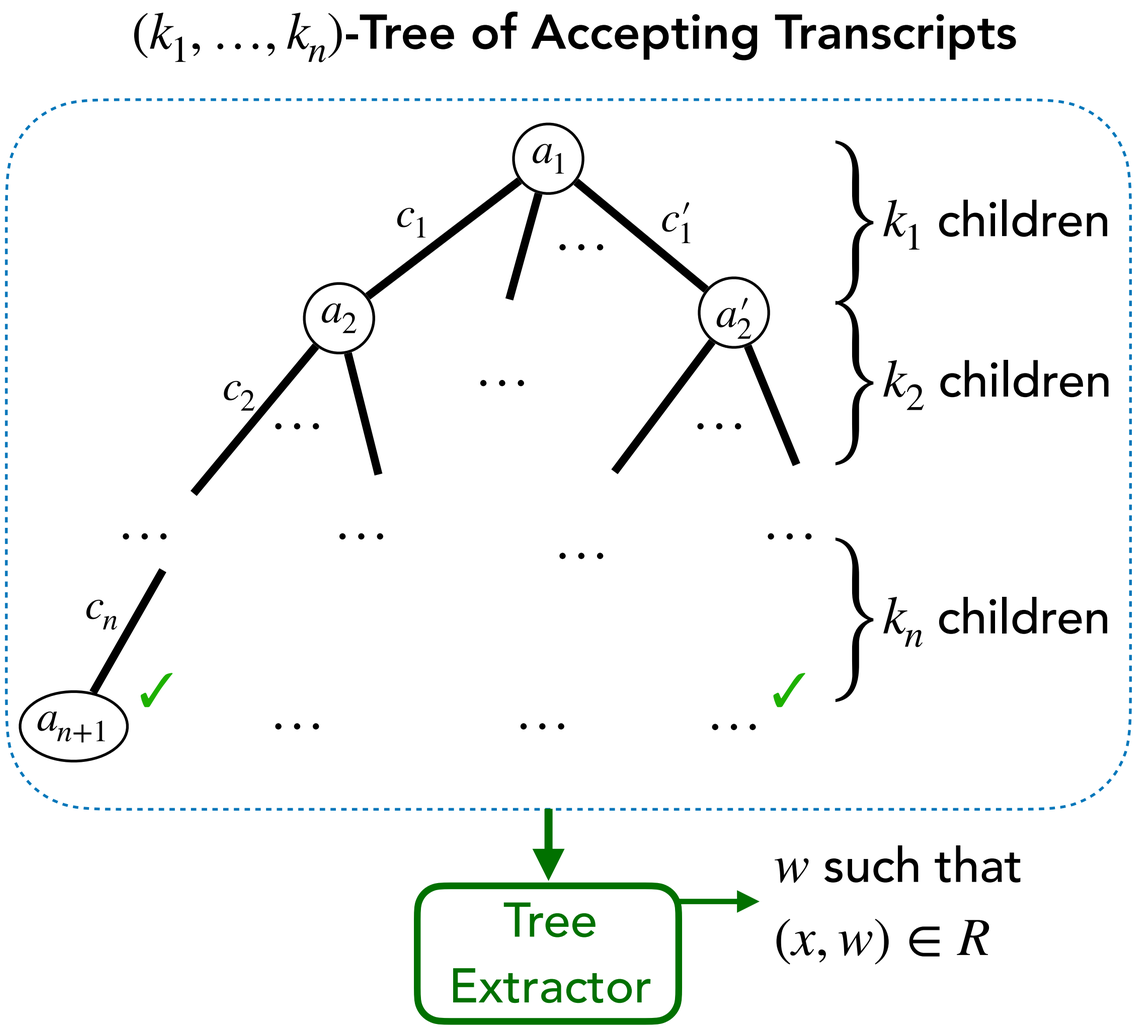

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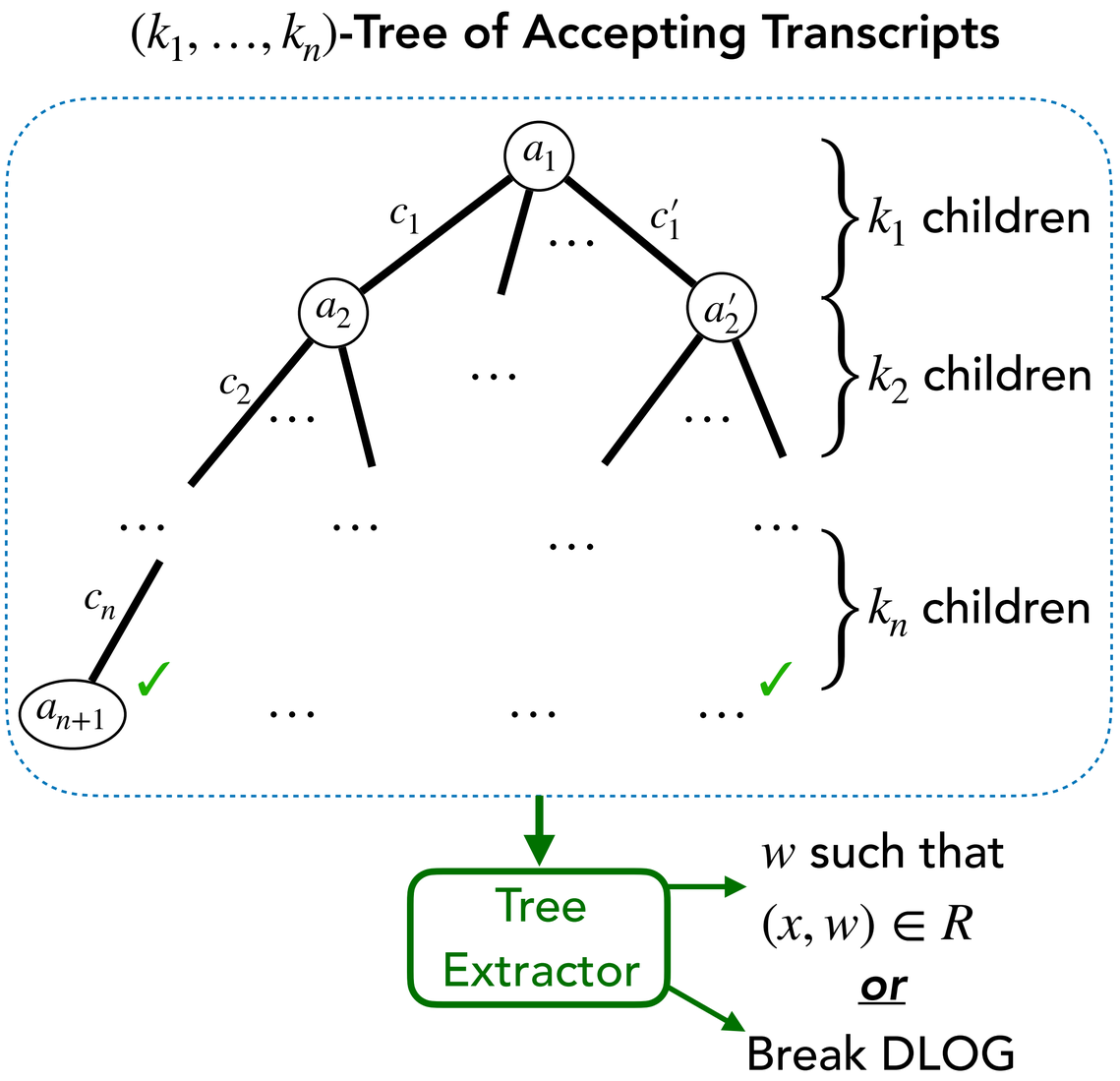

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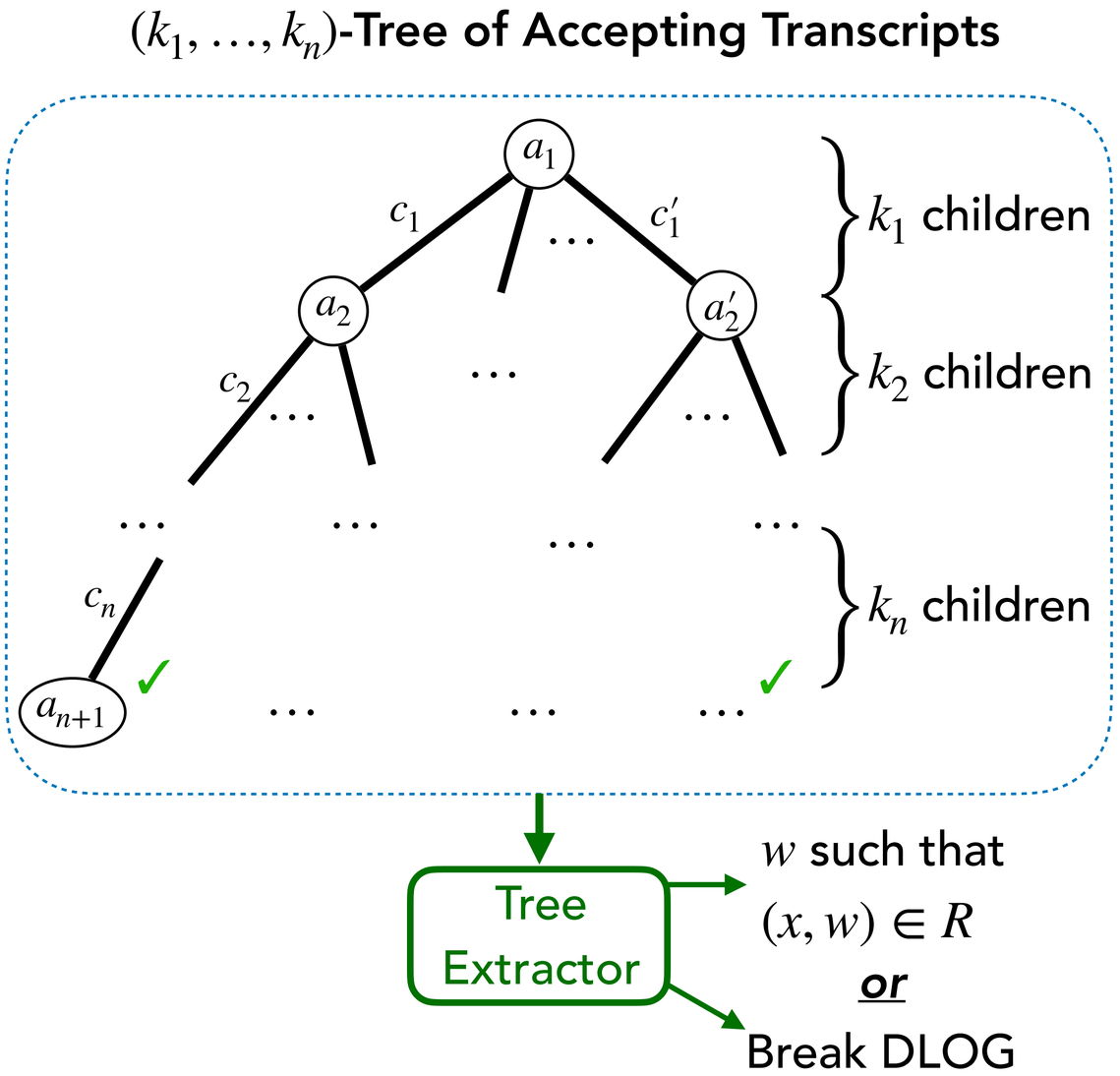

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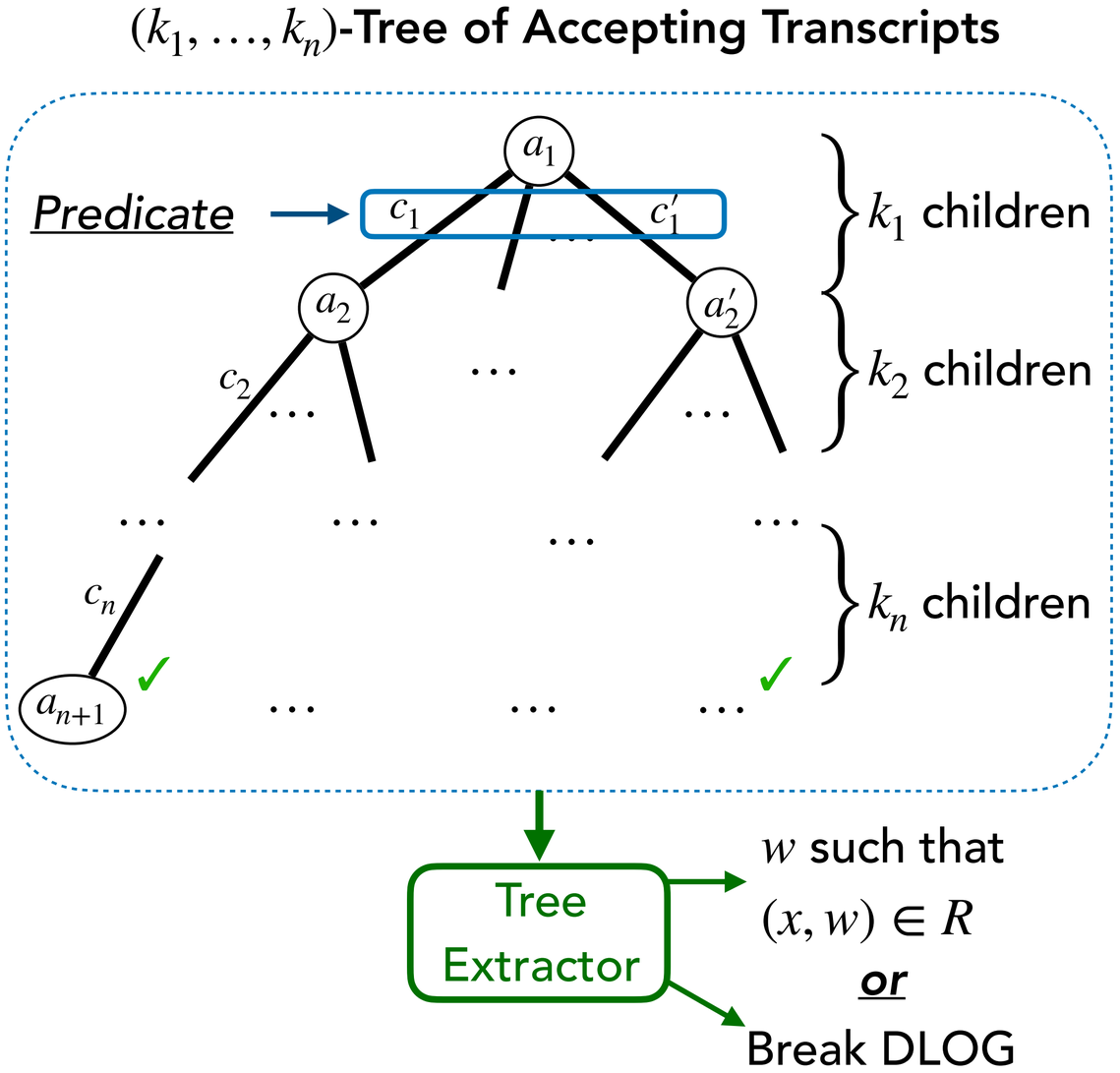

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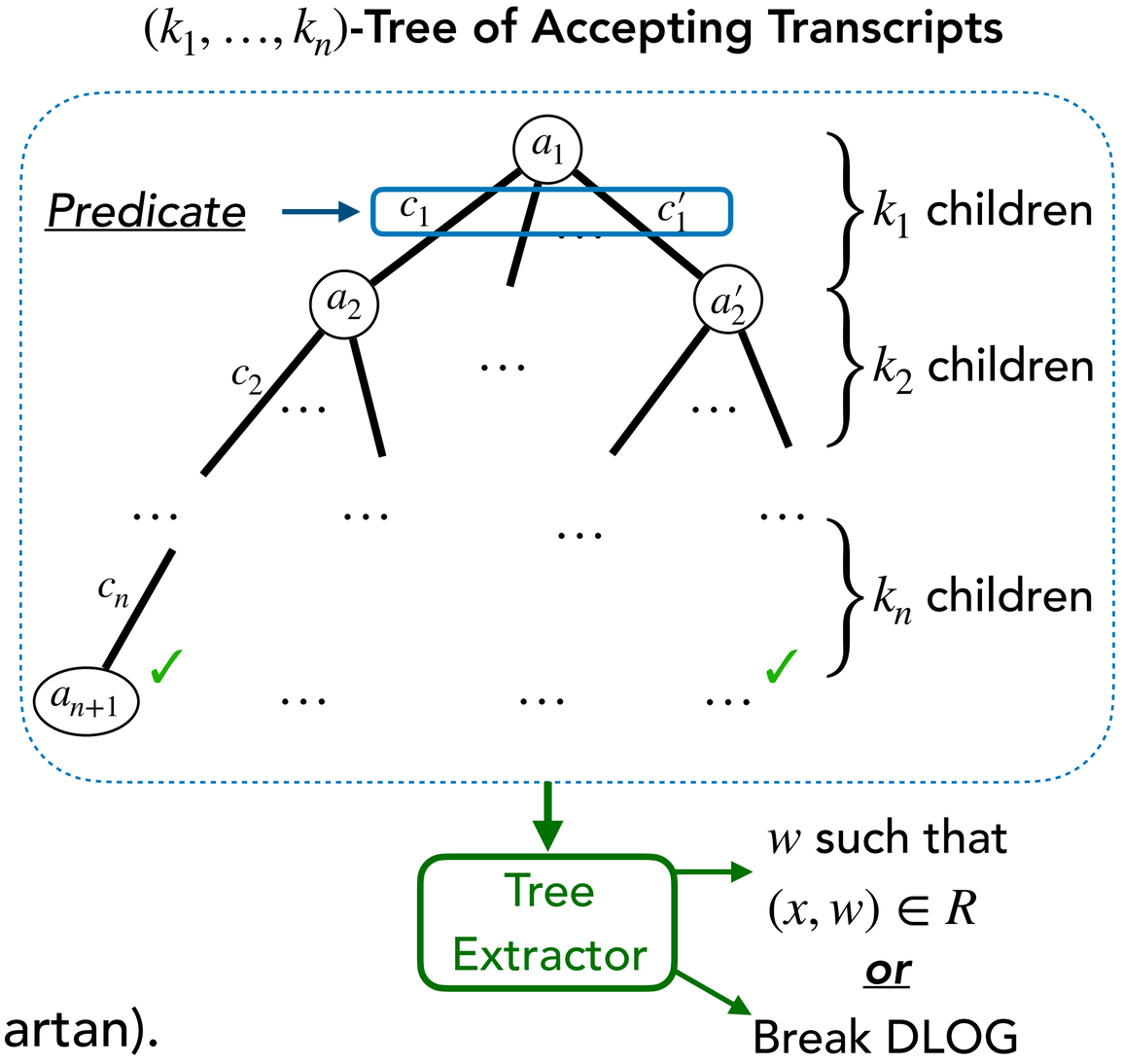

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Read our paper!
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Thank You!